1).

Since the convergence $\Pi_{j=1}^{n} F(\alpha_n x + \beta_n - \delta_j) = F^n(\alpha_n x + \gamma_n) \to G(x)$ is uniform (see Resnick (1987) Chapter 0) and F^n is uniformly continuous, then for any $\epsilon > 0$ there exists η and $N(\eta, \epsilon)$ such that for all $x \in \mathbb{R}$ and all $J_0, J_1 > N(\eta, \epsilon)$ we have $\left|\frac{a_{J_1}}{a_{J_0}} - 1\right| \le \eta$ and

$$\begin{aligned}\left|F^{J_0}(a_{J_0}x + \gamma_{J_0}) - F^{J_1}(a_{J_0}x + \gamma_{J_1})\right| &\le \left|F^{J_0}(a_{J_0}x + \gamma_{J_0}) - F^{J_1}(a_{J_1}x + \gamma_{J_1})\right| \\ &\quad + \left|F^{J_1}(a_{J_1}x + \gamma_{J_1}) - F^{J_1}(a_{J_0}x + \gamma_{J_1})\right| \\ &< \epsilon\end{aligned}$$

Therefore, for any $p \in \mathbb{R}$

$$\begin{aligned}&\left|\mathbb{P}\left(WTP_i(J_0) \le x\right) - \mathbb{P}\left(WTP_i(J_1) \le x + \frac{\gamma_{J_1} - \gamma_{J_0}}{\alpha}\right)\right| \\ &= \left|\mathbb{P}\left(\max_{j\in\{1,\ldots,J_0\}}\{\frac{\delta_j + \varepsilon_{ij} - \varepsilon_{i0}}{\alpha}\} \le x\right) - \mathbb{P}\left(\max_{j\in\{1,\ldots,J_1\}}\{\frac{\delta_j + \varepsilon_{ij} - \varepsilon_{i0}}{\alpha}\} \le x + \frac{\gamma_{J_1} - \gamma_{J_0}}{\alpha}\right)\right| \\ &= \left|\int_{\mathbb{R}} \left(F^{J_1}\left(\alpha x + \varepsilon_{i0} - \delta_j + \gamma_{J_1} - \gamma_{J_0}\right) - F^{J_0}\left(\alpha x + \varepsilon_{i0} - \delta_j\right)\right) f_0(\varepsilon_{i0}) d\varepsilon_{i0}\right| \\ &< \epsilon\end{aligned}$$

where f_0 is the probability density of ε_{i0}. We conclude that the willingness-to-pay densities are asymptotically parallel. □

Proof of Proposition 2

Proof. Assume parallel demands (Definition 1) and let $d(J_0, J_1)$ be such that $P(Q, J_0) + d(J_0, J_1) = P(Q, J_1)$. Then $\Lambda = \int_0^Q \Big(P(s, J_1) - P(s, J_0)\Big) ds = d(J_0, J_1) * Q$. □

Proof of Proposition 3

Proof. Observe:

$$\begin{aligned}d(J_0, J_1) &= p_1 - P(Q_1, J_0) \\ &= \left(\frac{p_1 - p_0}{Q_1 - Q_0} - \frac{P(Q_1, J_0) - p_0}{Q_1 - Q_0}\right)(Q_1 - Q_0)\end{aligned}$$

Now assume $(p(J), Q(J))_{J\in\mathbb{R}}$ is a continuously differentiable interpolation of $(p(J), Q(J))_{J\in\mathbb{N}}$

which exists by the Stone-Weierstrass theorem. Then by the Taylor approximation theorem:

$$\begin{aligned} d(J_0, J_1) &= \left(\frac{p_1 - p_0}{Q_1 - Q_0} - \frac{P(Q_1, J_0) - p_0}{Q_1 - Q_0} \right) (Q_1 - Q_0) \\ &= \left(\frac{\frac{dp}{dJ}}{\frac{dQ}{dJ}} - \frac{\partial P(Q, J)}{\partial Q} \right) \frac{dQ}{dJ} \triangle J + O((\triangle J)^2) \end{aligned}$$

□

Proof of Proposition 4

Proof. In the text. □

Proof of Proposition 5

Proof. Let $d = d(J_0, J_1)$. Observe by assumption $Q_{J_1}(\mathbf{p_{J_1}}) = Q_{J_0}(\mathbf{p_{J_0}} + (\rho - d)\mathbf{1_{J_0}})$, then the second part of the theorem follows directly from the first-order Taylor approximation:

$$Q_{J_1}(\mathbf{p_{J_1}}) = Q_{J_0}(\mathbf{p_{J_0}}) + (\rho - d)\frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt} + O\left((\rho - d)^2\right)$$

where $\frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt}$ is the directional derivative in the direction $\mathbf{1_{J_0}}$. And so

$$d = \left(\frac{\rho}{\Delta Q} - \left(\frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt} \right)^{-1} \right) \Delta Q + O\left((\rho - d)^2\right)$$

□