Parallel Inverse Aggregate Demand Curves in Discrete Choice Models

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Abstract

This paper highlights a previously-unnoticed property of commonly-used discrete choice models, which is that they feature parallel demand curves. Specifically, we show that in random utility models, inverse aggregate demand curves shift in parallel with respect to variety if and only if the random utility shocks follow the Gumbel distribution. Using results from Extreme Value Theory, we provide conditions for other distributions to generate parallel demand asymptotically, as the number of varieties increase. We establish these results in the benchmark case of symmetric products, illustrate them using numerical simulations and show that they hold in extended versions of the model with correlated tastes and asymmetric products. Lastly, we provide a "proof of concept" of parallel demands as an economic tool by showing how to use parallel demands to identify the change in consumer surplus from an exogenous change in product variety.

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1).

Since the convergence $\Pi_{j=1}^{n} F(\alpha_n x + \beta_n - \delta_j) = F^n(\alpha_n x + \gamma_n) \to G(x)$ is uniform (see Resnick (1987) Chapter 0) and F^n is uniformly continuous, then for any $\epsilon > 0$ there exists η and $N(\eta, \epsilon)$ such that for all $x \in \mathbb{R}$ and all $J_0, J_1 > N(\eta, \epsilon)$ we have $\left|\frac{a_{J_1}}{a_{J_0}} - 1\right| \leq \eta$ and

$$
\begin{aligned}
\left|F^{J_0}(a_{J_0}x + \gamma_{J_0}) - F^{J_1}(a_{J_0}x + \gamma_{J_1})\right| &\leq \left|F^{J_0}(a_{J_0}x + \gamma_{J_0}) - F^{J_1}(a_{J_1}x + \gamma_{J_1})\right| \\
&\quad + \left|F^{J_1}(a_{J_1}x + \gamma_{J_1}) - F^{J_1}(a_{J_0}x + \gamma_{J_1})\right| \\
&< \epsilon
\end{aligned}
$$

there exists $d(J_0, J_1)$ such that for all $x \in \mathbb{R}$:

$$
\left|\mathbb{P}\left(WTP_i(J_0) \leq x\right) - \mathbb{P}\left(WTP_i(J_1) \leq x + d(J_0, J_1)\right)\right| < \epsilon.
$$

Therefore, for any $p \in \mathbb{R}$

$$
\begin{aligned}
&\left|\mathbb{P}\left(WTP_i(J_0) \leq x\right) - \mathbb{P}\left(WTP_i(J_1) \leq x + \frac{\gamma_{J_1} - \gamma_{J_0}}{\alpha}\right)\right| \\
&= \left|\mathbb{P}\left(\max_{j \in \{1,\ldots,J_0\}} \{\frac{\delta_j + \varepsilon_{ij} - \varepsilon_{i0}}{\alpha}\} \leq x\right) - \mathbb{P}\left(\max_{j \in \{1,\ldots,J_1\}} \{\frac{\delta_j + \varepsilon_{ij} - \varepsilon_{i0}}{\alpha}\} \leq x + \frac{\gamma_{J_1} - \gamma_{J_0}}{\alpha}\right)\right| \\
&= \left|\int_{\mathbb{R}} \Big(F^{J_1}\left(\alpha x + \varepsilon_{i0} - \delta_j + \gamma_{J_1} - \gamma_{J_0}\right) - F^{J_0}\left(\alpha x + \varepsilon_{i0} - \delta_j\right)\Big) f_0(\varepsilon_{i0}) d\varepsilon_{i0}\right| \\
&< \epsilon
\end{aligned}
$$

where f_0 is the probability density of ε_{i0}. We conclude that the willingness-to-pay densities are asymptotically parallel. □

Proof of Proposition 2

Proof. Assume parallel demands (Definition 1) and let $d(J_0, J_1)$ be such that $P(Q, J_0) + d(J_0, J_1) = P(Q, J_1)$. Then $\Lambda = \int_0^Q \Big(P(s, J_1) - P(s, J_0)\Big) ds = d(J_0, J_1) * Q$. □

Proof of Proposition 3

Proof. Observe:

$$\begin{aligned} d(J_0, J_1) &= p_1 - P(Q_1, J_0) \\ &= \left(\frac{p_1 - p_0}{Q_1 - Q_0} - \frac{P(Q_1, J_0) - p_0}{Q_1 - Q_0} \right) (Q_1 - Q_0) \end{aligned}$$

Now assume $(p(J), Q(J))_{J \in \mathbb{R}}$ is a continuously differentiable interpolation of $(p(J), Q(J))_{J \in \mathbb{N}}$ which exists by the Stone-Weierstrass theorem. Then by the Taylor approximation theorem:

$$\begin{aligned} d(J_0, J_1) &= \left(\frac{p_1 - p_0}{Q_1 - Q_0} - \frac{P(Q_1, J_0) - p_0}{Q_1 - Q_0} \right) (Q_1 - Q_0) \\ &= \left(\frac{\frac{dp}{dJ}}{\frac{dQ}{dJ}} - \frac{\partial P(Q, J)}{\partial Q} \right) \frac{dQ}{dJ} \triangle J + O((\triangle J)^2) \end{aligned}$$

□

Proof of Proposition 4

Proof. In the text. □

Proof of Proposition 5

Proof. Let $d = d(J_0, J_1)$. Observe by assumption $Q_{J_1}(\mathbf{p_{J_1}}) = Q_{J_0}(\mathbf{p_{J_0}} + (\rho - d)\mathbf{1_{J_0}})$, then the second part of the theorem follows directly from the first-order Taylor approximation:

$$Q_{J_1}(\mathbf{p_{J_1}}) = Q_{J_0}(\mathbf{p_{J_0}}) + (\rho - d) \frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt} + O\left((\rho - d)^2\right)$$

where $\frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt}$ is the directional derivative in the direction $\mathbf{1_{J_0}}$. And so

$$d = \left(\frac{\rho}{\Delta Q} - \left(\frac{dQ_{J_0}(\mathbf{p_{J_0}} + t\mathbf{1_{J_0}})}{dt} \right)^{-1} \right) \Delta Q + O\left((\rho - d)^2\right)$$

□