Testing Axiomatizations of Ambiguity Aversion

Daniel Chen & Martin Schonger

ETH Zurich - Center for Law and Economics
www.nber.org/~dlchen
www.n.ethz.ch/~scmartin

Economic Science Association, Fri 27 June 2014

Introduction

- Many theories to explain Ellsburg paradox
- Ambiguity Aversion applied to finance, health, law
- Machina's thought experiment challenges axiomatizations of ambiguity aversion (2014)

What is your Certainty Equivalent ?

What is your c?

$\$c \sim (\frac{1}{2}, \$0; \frac{1}{2}, \$100)$

Machina thought experiment

$$\overbrace{\begin{matrix}\text{black} & \text{white}\end{matrix}}^{\text{2 balls}} \quad \overbrace{\text{red}}^{\text{1 ball}} \qquad\qquad \overbrace{\text{red}}^{\text{1 ball}} \quad \overbrace{\begin{matrix}\text{black} & \text{white}\end{matrix}}^{\text{2 balls}}$$

$$\begin{matrix}\$0 & \$c & \$100\end{matrix} \qquad \text{vs.} \qquad \begin{matrix}\$0 & \$c & \$100\end{matrix}$$

What might people prefer?

Act 1		
2 balls		1 ball
black	white	red
\$0	\$$c$	\$100

vs.

Act 2		
1 ball	2 balls	
red	black	white
\$0	\$$c$	\$100

Machina (2014): "If ambiguity aversion somehow involves “pessimism,” mightn’t an ambiguity averter have a strict preference for [Act] II over [Act] I, just as a risk averter might prefer bearing risk about higher rather than lower outcome levels?"

Machina's thought experiment

- Classic Ellsberg urns have 2, 3 or 4 states, but only **2 outcomes**.
- Machina (2014) proposes acts with **3 outcomes**.
- Proceeds to show that 4 major theories of ambiguity aversion predict indifference between the 2 acts:
  - Multiple Priors (Gilboa/Schmeidler 1989)
  - Rank-Dependent/Choquet model (Schmeidler 1989)
  - Smooth Ambiguity Preferences Model (Klibanoff, Marinacci, Mukerjii 2005)
  - Variational Preferences Model (Maccheroni, Marinacci and Rustichini 2006)
- for any prior: SEU $\Rightarrow Act I \sim Act II$
- Recursive Non-EU (Disappointment Aversion) (Dillenberger/Segal 2014): $Act II \succ Act I$ (example)

Machina's thought experiment - our observations

Machina's thought experiment: 2 observations

- Machina: major theories of ambiguity aversion predict indifference => test of these major theories of ambiguity aversion
- We observe:
  1. Probabilistically sophisticated non-EU DM can fail to be indifferent
  2. Any non-probabilistically sophisticated EU DM is indifferent
- Machina thought experiment at least as much a test of independence as of ambiguity aversion
- One reason we we propose a richer experiment

Obs.1: Probabilistically sophisticated non-EU DM can fail to be indifferent

Goal: Example of probabilistically sophisticated DM with $ActI \nsim ActII$
Assumptions:

1. Probabilistic sophistication: $p_B = \frac{2}{3}$,$p_W = 0$
2. Non-EU Gul's (1991) disappointment aversion ($\beta > 0$)

- For any lottery with 2 outcomes $\underline{x} < \overline{x}$ Gul's functional is simply: $v(lottery) = \frac{(1+\beta)p(\underline{x})u(\underline{x})+p(\overline{x})u(\overline{x})}{1+\beta p(\underline{x})}$
- Normalize $u(0) = 0$, $u(100) = 100$
  $\Rightarrow u(c) = v(\$0;\frac{1}{2},\$100;\frac{1}{2}) = \frac{\frac{1}{2}100}{1+\frac{1}{2}\beta} = \frac{100}{2+\beta}$
- $v(I) = \frac{(1+\beta)\frac{2}{3}u(0)+\frac{1}{3}u(100)}{1+\beta\frac{2}{3}} = \frac{100}{3+2\beta}$
- $v(II) = \frac{(1+\beta)\frac{1}{3}u(0)+\frac{2}{3}u(c)}{1+\beta\frac{1}{3}} = \frac{2u(c)}{3+\beta} = \frac{200}{(2+\beta)(3+\beta)}$

$\Rightarrow v(II) < v(I) \Rightarrow ActI \succ ActII$

Obs. 2: Non-probabilistically sophisticated EU DM indifferent

Act 1		
2 balls		1 ball
black	white	red
$\$0$	$\$c$	$\$100$

vs.

Act 2		
1 ball	2 balls	
red	black	white
$\$0$	$\$c$	$\$100$

Replace \$c with the lottery it is induced by.

Obs. 2: Replace c by underlying lottery

	Act 1		
	$\overbrace{\text{black} \quad \text{white}}^{\text{2 balls}}$		$\overbrace{\text{red}}^{\text{1 ball}}$
	black	white	red
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

vs.

	Act 2		
	$\overbrace{\text{red}}^{\text{1 ball}}$	$\overbrace{\text{black} \quad \text{white}}^{\text{2 balls}}$	
	red	black	white
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

Obs. 2: Replace c by underlying lottery

	Act 1		
	2 balls		1 ball
	black	white	red
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

vs.

	Act 2		
	1 ball	2 balls	
	red	black	white
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

Obs. 2: Replace c by underlying lottery

	Act 1		
	2 balls		1 ball
	black	white	red
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

vs.

	Act 2		
	1 ball	2 balls	
	red	black	white
$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100

Experimental challenges and design

3 balls vs. 60 balls

- Machina parsimoniously fills his opaque urn with 1 known and 2 unknown balls
- Experience shows that then some subjects assume some symmetric objective probability distribution is implied and mechanically start calculating the resulting distribution of this compound lottery
- We avoid this by having 20 known and 40 unknown balls:
  - makes mechanic thoughtless calculation harder
  - makes examples better "for example 7 black and 33 white balls"
  - Ellsberg also proposed a large number of balls

Certainty equivalent c

- Machina experiment requires knowledge of an individual's certainty equivalent c for the lottery $(\frac{1}{2},\$0;\frac{1}{2},\$100)$
- Existence & uniqueness is guaranteed under mild assumptions (continuous preference relation and monotonicity in money)

How to elicit CE

Becker De Groot Marschak
2 problems:
1.) Becker De Groot Marschak truth-telling is optimal only if Independence holds (Karni/Safra 1987)
2.) Even if independence holds, if we later use the result from BDM against the DM, truth telling is not optimal
What do we do?

Solution: PRINCE & not use it against subjects

- PRINCE = PRior INCEntive system (Johnson/Baillon/Bleichrodt/Li/van Dolder/Wakker)
  - like the list method formally about equivalent to BDM
  - advantage over list method: allows any answer, not just answer on list
  - envelope already there and framing as give us instructions might lessen concerns of seeing this as a big lottery when eliciting CE
  - we use PRINCE for FOSD-task (to familiarize subjects) and for CE
- For Machina experiment we use combination of PRINCE and list method
  - Why? It is a priori not clear that people have a unique switching point nor direction

List method: X instead of unknown c

Act 1		
2 balls		1 ball
black	white	red
$\$0$	$\$x$	$\$100$

vs.

Act 2		
1 ball	2 balls	
red	black	white
$\$0$	$\$x$	$\$100$

Different ways to switch

- Goal: Simple example to show how people can switch
- Assumptions: Probabilistic sophistication, EU
- $EU(I) = \frac{1}{3}100 + p_W u(x)$
- $EU(II) = p_W 100 + p_B u(x)$
- Example 1 $p_W = 0$:
  $EU(I) = \frac{1}{3}100$ and $EU(II) = \frac{2}{3}u(x)$, thus Act I $\succsim$Act II iff $x < c$
- Example 2 $p_B = 0$:
  $EU(I) = \frac{1}{3}100 + \frac{2}{3}u(x)$ and $EU(II) = \frac{2}{3}100$, thus Act II $\succsim$Act I iff $x < c$
- Example 3 $p_W = p_B = \frac{1}{3}$:
  $EU(I) = \frac{1}{3}100 + \frac{1}{3}u(x)$ and $EU(II) = \frac{1}{3}100 + \frac{1}{3}u(x)$, thus Act I $\sim$Act II for all x

3 tasks in experiment

1. FOSD taks (PRINCE)
2. CE elicitation (PRINCE)
3. Machina experiment (PRINCE and list method)

FOSD task (experiment 1 of 3)

Envelope content:

Option A	
Balls in drum	Money you get when a ball of this color is drawn
4 red balls	CHF X
3 white balls	CHF 9
3 black balls	CHF 7

Option B	
Balls in drum	Money you get when a ball of this color is drawn
2 red balls	CHF X
3 white balls	CHF 9
5 black balls	CHF 7

Answer sheet design:

□ If X is below or equal to CHF _ _, _ _ then I want **Option A,** else I will receive **Option B.**

□ If X is below or equal to CHF _ _, _ _ then I want **Option B,** else I will receive **Option A.**

FOSD => lower row, CHF 7,00

Certainty equivalent (experiment 2 of 3)

Envelope content:

> The drum is filled with 20 balls, 10 of which are white and 10 of which are black. The experimenter will draw one ball from the drum at random.
>
> **Option A**
> If the ball is black you get CHF 20. If the ball is white you get nothing.
>
> **Option B**
> Regardless of what color is drawn, you get **CHF X**.

Answer sheet design:

IF X is below or equal to CHF [__ __,__ __] I want Option A, otherwise I will receive Option B.

Risk- aversion: lower than CHF 10,00

Machina (experiment 3 of 3)

Envelope content:

The ball will be drawn from a drum with 20 red balls and 40 balls which may be any combination of white and black balls. You do not know exactly how many white/black balls are in the drum. There are 60 balls in total in the drum. You can choose one of two options. The option payoff is dependent on the color of the ball drawn from the drum:

	20 balls	40 balls	
	Red ball drawn	Black ball drawn	White ball drawn
Option A	CHF 0	CHF **X**	CHF 20
Option B	CHF 20	CHF 0	CHF **X**

Answer sheet design:

If X is....	...I want Option A	...I want Option B	I am indifferent
CHF 0			
CHF 1			
CHF 2			
CHF 3			
CHF 4			
CHF 5			
CHF 6			
CHF 7			
CHF 8			
CHF 9			
CHF 10			
CHF 11			
CHF 12			
CHF 13			
CHF 14			
CHF 15			
CHF 16			
CHF 17			
CHF 18			
CHF 19			
CHF 20			

SEU: unique switch at CE

Pilot Results

Urn A

	2/3		1/3
	Black	White	Red
	\$c	\$100	\$0

Urn B

	2/3		1/3
	Black	White	Red
	\$0	\$c	\$100

- We replaced \$c with the lottery it is induced by
- Among 432 subjects across 4 MTurk experiments, 64% preferred Urn B (Ambiguity at Low)
  - Republicans 22% points more likely to prefer Urn B
  - Americans 48% points more likely to prefer Urn A
  - Asians 27% points more likely to prefer Urn A
    - ⋆ We randomized options but gave no option to report indifference

Experimental results

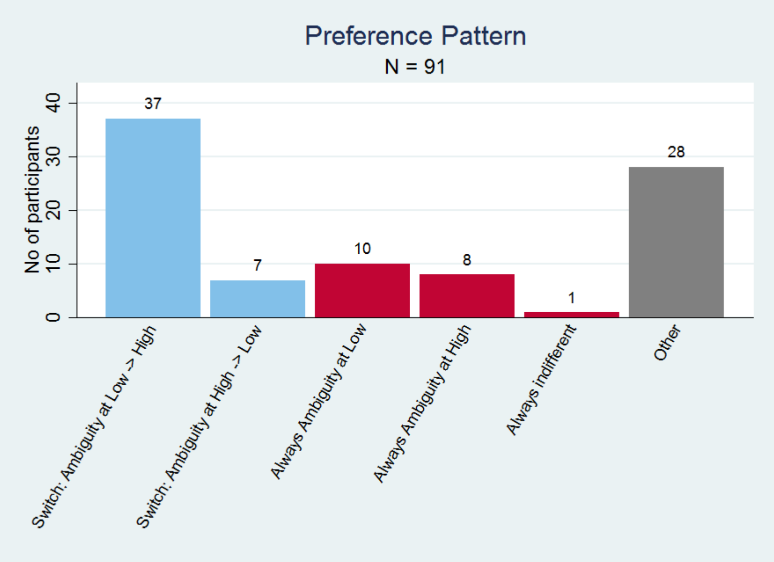
Overview

- 6 sessions with up to 20 participants per session
- ETH Zurich laboratory (subject pool shared with U Zurich)
- additional data collection possible

Overview

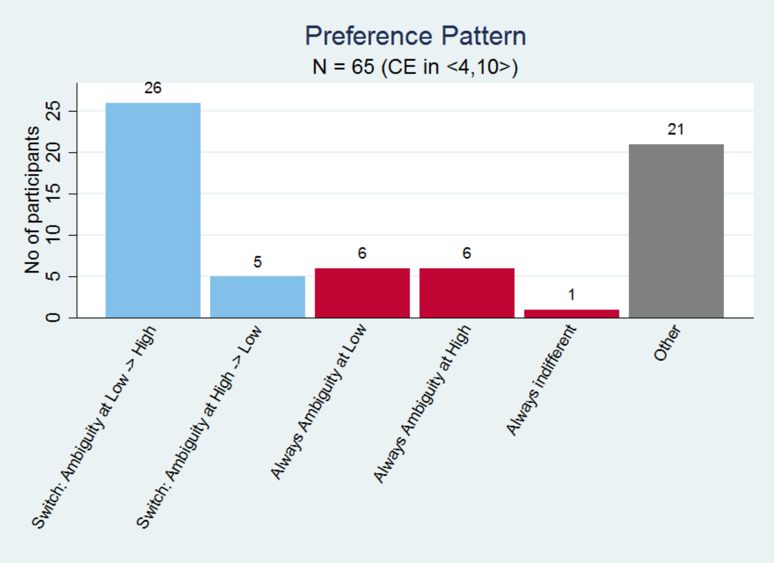
- People should be indifferent when X=their CE.
  - prefer one option below/above their CE and switch to the other one when X crosses CE
  - or simply report indifference
- Do people switch? (if not, they are not indifferent)
- When they switch, does the switch occur at their CE?
- What direction do they switch?

Preference Pattern: All participants

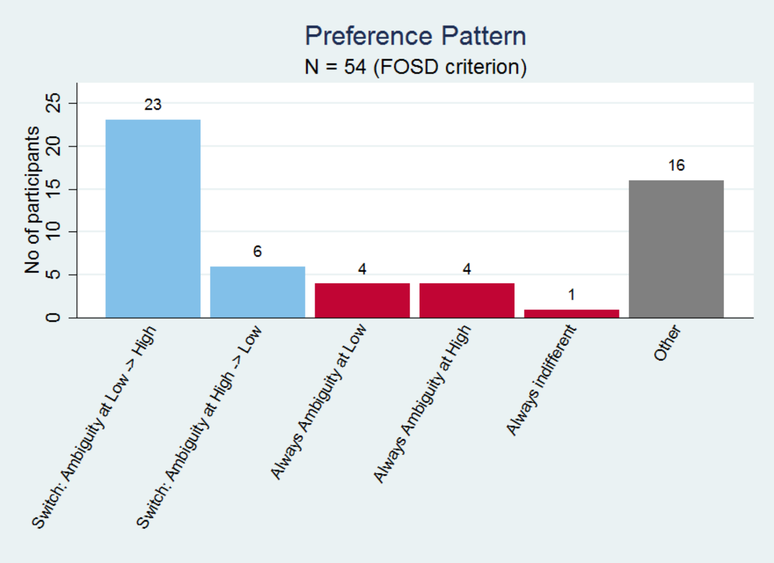


- Many people don't switch, slight preference for Ambiguity at Low
- Switchers switch from Ambiguity at Low to Ambiguity at High

Preference Pattern: Reasonable CE



Preference Pattern: FOSD



Calculating Indifference at X = CE

- Indifferent := report indifference at $CE \pm 1$ or if $CE \in \{S - 1.96 \cdot SD([CE - S]); S + 1.96 \cdot SD([CE - S])\}$
  - null hypothesis: $CE = S$
  - any difference is measurement error
  - $\exists$ people for whom CE strongly differs from S
  - Overestimate indifference
- S := average value between the last A/B and first B/A for single-switchers

Are People Indifferent at X = CE? No

Whole Sample

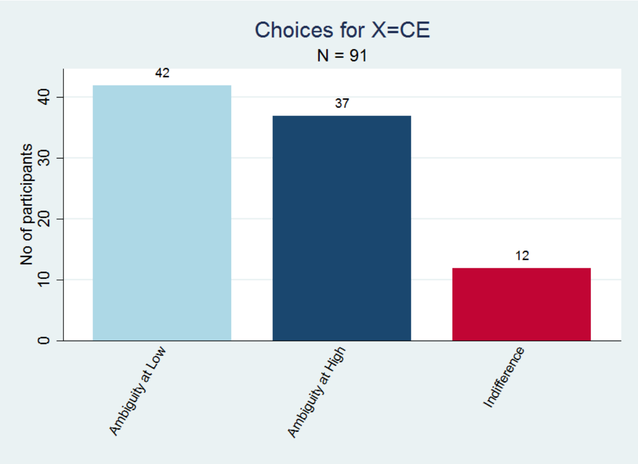
	Count	Share in %
Indifferent	49	53.8
Non-indifferent	42	46.2
Total	91	100

Keep only people with $4 \leq CE \leq 10$ and satisfy FOSD

	Count	Share in %
Indifferent	21	53.8
Non-indifferent	18	46.2
Total	39	100

Choices at X = CE?

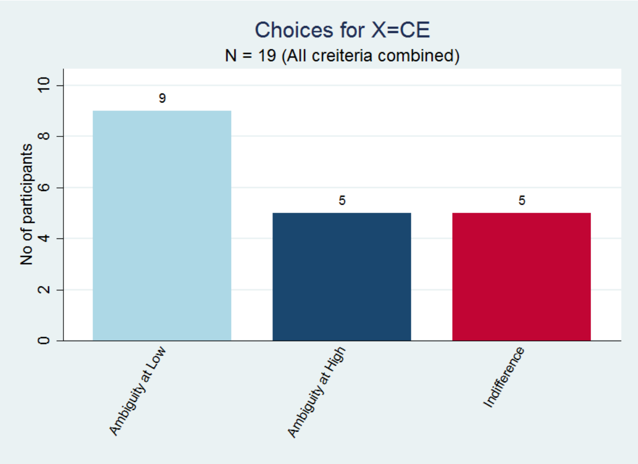
Full Sample



- Ambiguity at Low is Preferred

Choices at X = CE?

Single-Switchers with $4 \leq CE \leq 10$ and satisfying FOSD



- Ambiguity at Low is Preferred

CE vs. Switching point

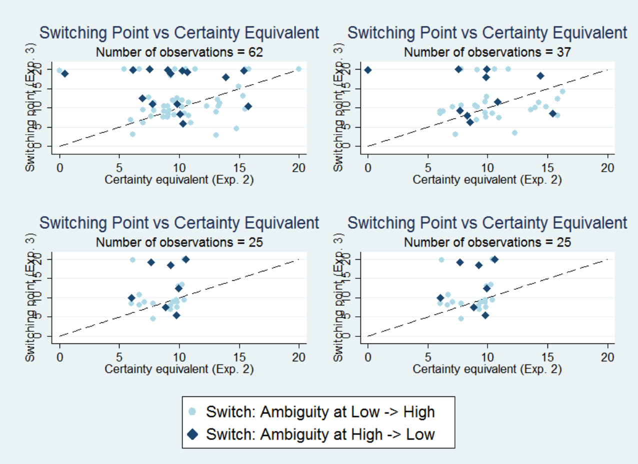


Figure: Raw data with $CE = S$ line. Sample includes people who always prefer A or B (their switching point represented as 20). Clockwise: Full, $CE \in [4, 10]$, FOSD, both.

CE vs. Switching point

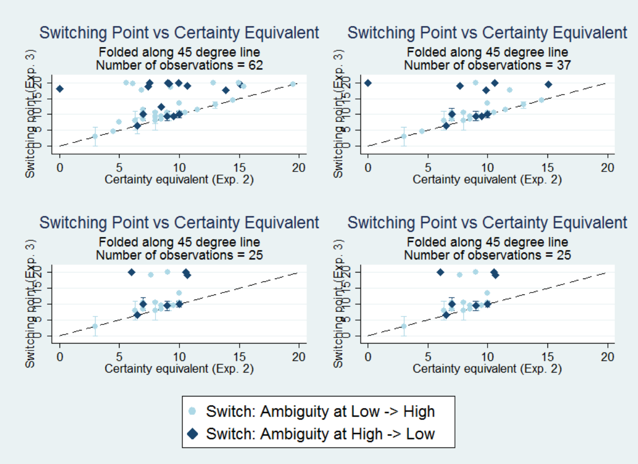
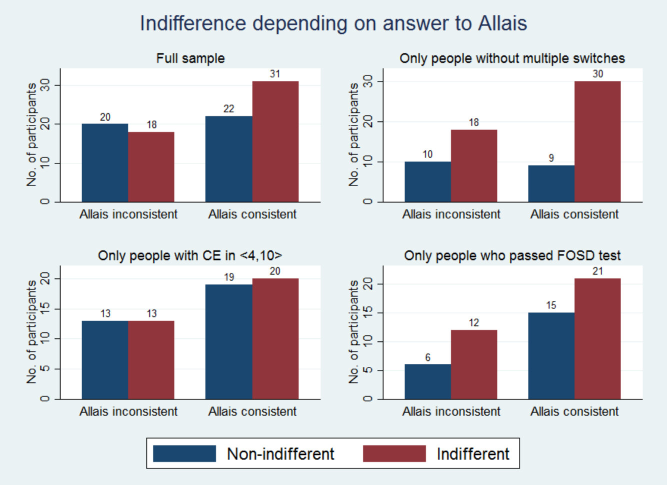


Figure: The data is folded over 45 degree line. Sample includes people who always prefer A or B (their switching point represented as 20). Clockwise: Full, $CE \in [4, 10]$, FOSD, both.

Allais and Machina paradoxes



- As people become more mathematical, they become more indifferent

Does order matter for switch direction?

- Yes, but people still Switch from Ambiguity at Low to Ambiguity at High.
- Fraction of switches from Ambiguity at High Outcome to Ambiguity at Low Outcome depending on the order of options on the answer sheet (normal order lists Ambiguity at Low Outcome first):

Group	Obs	Mean	Std Dev
Normal Order	27	.07	.27
Reversed order	17	.29	.47

H0: means are equal; p-value for two-sided test: 0.054

Predictions about Direction of Switch

$$W_{URNI} = q_{BB} \cdot V_{URNI}(BB) + q_{BW} V_{URNI}(BW) + q_{WW} V_{URNI}(WW)$$

$$W_{URNII} = q_{BB} \cdot V_{URNII}(BB) + q_{BW} V_{URNII}(BW) + q_{WW} V_{URNII}(WW)$$

Gul's disappointment aversion model with β as disappointment aversion parameter:

$$V_{URNI}(BB) = \frac{\frac{2}{3}(1+\beta)\cdot 0 + \frac{1}{3}\cdot 100}{1+\frac{2}{3}\beta} = \frac{100}{3+2\beta}$$

$$V_{URNI}(WW) = \frac{\frac{2}{3}(1+\beta)\cdot X + \frac{1}{3}\cdot 100}{1+\frac{2}{3}\beta} = \frac{100+2(1+\beta)X}{3+2\beta}$$

$$V_{URNII}(BB) = \frac{\frac{1}{3}(1+\beta)\cdot 0 + \frac{2}{3}\cdot X}{1+\frac{1}{3}\beta} = \frac{2X}{3+\beta}$$

$$V_{URNII}(BB) = \frac{\frac{1}{3}(1+\beta)\cdot 0 + \frac{2}{3}\cdot 100}{1+\frac{1}{3}\beta} = \frac{200}{3+\beta}$$

If $\beta > 0$, then if $X < 50$, URNI is preferred over URNII. Therefore as X increases we should observe switch from URNI to URNII, which we find.

Conclusion

- Many theories give a sharp point prediction
- Is the point prediction of indifference about right?
  - No
- Find strong pattern of which way people shift
- This shift could be used to test theories

Thank you

Thank you!

We appreciate comments now or by email.

Daniel Chen: chendan@ethz.ch
Martin Schonger: mschonger@ethz.ch

ETH Zurich Center for Law and Economics