Income Distribution Dynamics with Endogenous Fertility

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Introduction

This is joint work with Daniel Chen.

The most unequal developing countries, such as Brazil and South Africa, face an uphill battle in reducing inequality, since educated workers have birthrate lower than uneducated workers.

For example, in Brazil:

Table 1 -- Total Fertility Rate by Woman's Years of Education: Brazil, 1986.

Years of Education	None	1-3	4-6	7-9	10+
Total Fertility	6.7	5.2	3.4	2.8	2.2

Source: United Nations, 1995.

If children of educated workers more likely to become educated, demographic factors will increase proportion of unskilled workers from one generation to next.

Relative wage of unskilled workers will decline.

This reduces their opportunity cost of raising children.

Increase in fertility among unskilled, creates a vicious cycle.

This demographic force produces instability.

On other hand as proportion of unskilled increase, return to skills increases. This strengthens incentives for investment in education.

This force makes imbalances in supply of skilled workers self-correcting:

Dynamics depend on which of these forces dominates.

We combine these forces in model. Two-point distribution of cost of education. If initial proportion of skilled workers great enough, economy converges to steady state with low inequality.

If initial proportion of skilled is too low, inequality is self-reinforcing.

Economy converges to steady state with high inequality.

Expanding access to education expands basin of attraction of low-inequality steady state and may eliminate high-inequality steady state.

More generally, multiplier effects, even if steady state is unique.

Dahan and Tsiddon [1998] focus on transition dynamics and model demographic transition.

Kuznets curve.

In Dahan and Tsiddon, single steady state because all poor identical, whereas in this model, heterogeneous.

Creates possibility of multiple stable steady states.

Empirical evidence

Data is broadly consistent with model – fertility differentials between educated and uneducated women are greater in countries with more income inequality.

Not definitive proof of model

But cannot reject an implication of model

Organization of talk:

II. Theory

A. Model

Suppose

Wages will be

Utility is

$$V = \ln (n - \varepsilon) + X,$$

Raising each child requires time φ

Time endowment is 1

Budget constraint is $X = w (1 - n\varphi)$

Utility can be re-written as $V = \ln (n - \varepsilon) + w (1 - n\varphi)$

FOC for optimal fertility implies $n = \varepsilon + 1 / (w\varphi)$

Asymptotes to ε

If $w < 1 / (1 - \varepsilon\varphi)$, corner solution with no consumption

For now, consider case where A large enough that away from corner

and $\varepsilon = 0$ for tractability

Number of children of unskilled and skilled workers equals birthrate times population:

Under quasi-linear utility function, higher wages lead people to have fewer children.

Substitution effect > Income effect

Matches evidence: fertility declines with education across countries, within countries

Alternative explanation: quality-quantity tradeoff

Both may play role

Several papers give support for hypothesis that substitution effect important

Schultz (1981), Schultz and Razzaz (1998)

Female education reduces fertility while male education has no clear effect on fertility (United Nations, 1987 also has corroborating evidence).

Modeling choice of education:

Want model in which

(i) educational decisions are responsive to incentives provided by wage premia

(ii) children of unskilled parents face higher costs of education than children of skilled parents, due to either differences in home environments or capital market imperfections.

Group	Time to obtain skills
Children of skilled parents	units
Proportion θ of children of unskilled	units
Proportion $1 - \theta$ of children of unskilled	units

Obtain education if raises lifetime income;

for those with low cost, if $w_S (1 -) > w_U$.

Obtain education if L, H < D, where

$L \equiv 1 / (1 -)$ and $H \equiv 1 / (1 -)$

$D \equiv w_S / w_U$, anticipated wage differential when children become adults.

SKIP:

Note that both and are bounded by total time endowment, 1, and by 0, so L and H are unbounded above and bounded below by 1.

Note also that D is similarly unbounded above and bounded below by 1 (skilled workers should make more than unskilled workers).

B. Potential Steady States

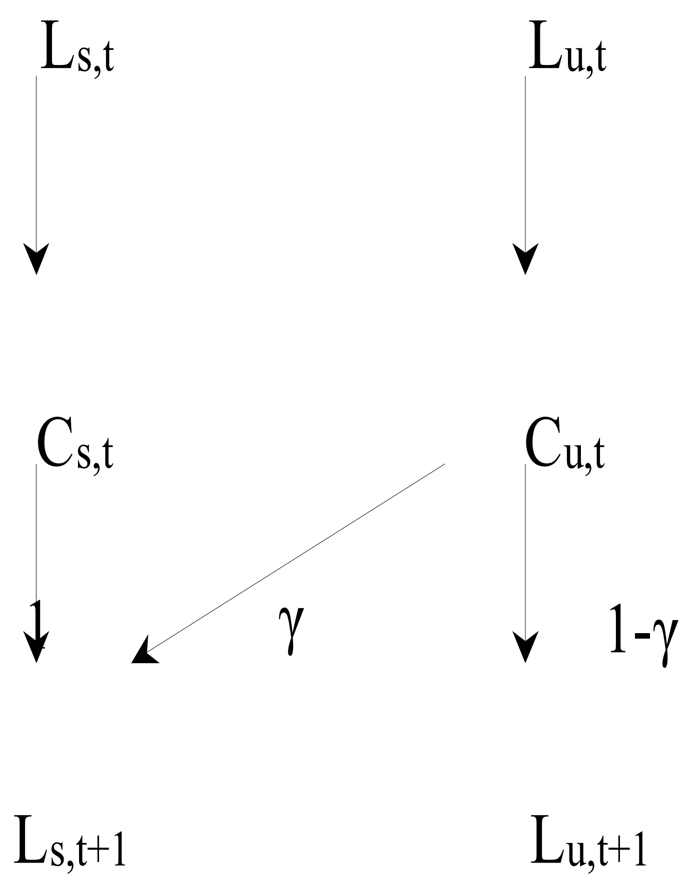
Define steady state as triplet (R^*, D^*, γ^*), such that if ratio of skilled to unskilled workers at time t is R^*, then

Wage differential = D^*

Proportion of children of unskilled who become skilled = γ^*

Ratio of skilled to unskilled in next generation = R^*.

To solve for steady states, find fixed points of $R_{t+1}(R_t)$.


Ls,t
Lu,t
Cs,t
Cu,t
1
γ
1-γ
Ls,t+1
Lu,t+1

If C_S and fraction γ of C_S become skilled, then:

$R_{t+1} = R_t$ implies steady state must satisfy:

Three possible types of solutions

(i) If R* induces D* = L, then $\gamma^* \leq \theta$

(ii) If R* induces D* = H, then $\gamma^* \geq \theta$

(iii) If R* induces L < D* < H, then $\gamma^* = \theta$.

No other solutions are possible since wage differential can never be expected to be above H or less than L. (Otherwise everyone or no one would become skilled, which would not be consistent with rational expectations.)

Note that quadratic equation in (5) yields more than two solutions because while there is a maximum of two solutions for a given γ^*, there may be more than one steady state γ^*.

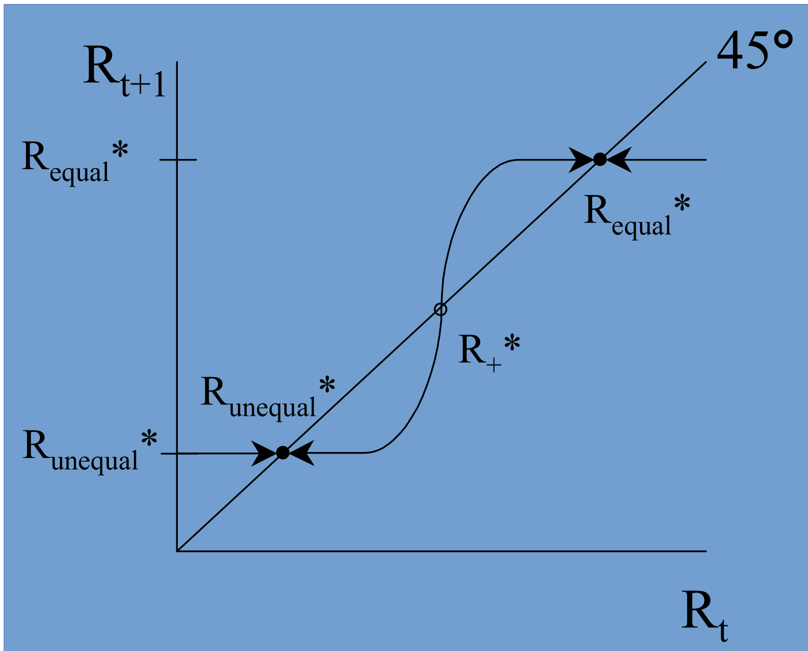


Figure 2: Multiple steady states

R_{equal}*, $R_{unequal}$*, and R_+* correspond to (i), (ii), and (iii) respectively.

PROPOSITION 1: *R_{equal}* is an admissible steady state where*

if and only if

PROOF:

$D_t = \alpha / [(1 - \alpha) R_t]$

$D^* = L \Leftrightarrow R^* = \alpha / [(1 - \alpha) L]$

To maintain R_{equal}*, γ must equal γ_{equal}* in (6).

Children with a low cost of education are indifferent to obtaining education, which gives inequality in (7).

■

For R_{equal}* to be admissible, θ cannot be too small relative to fertility differential between skilled and unskilled, or else too few C_U become skilled.

PROPOSITION 2: $R_{unequal}$* *is an admissible steady state where*

if and only if

PROOF: proof is similar to that of Proposition 1 except H replaces L.

$R_{unequal}$* is admissible if θ is low enough relative to fertility differential between skilled and unskilled.

In this case, even if all θ children become skilled, R* does not increase and D* does not fall below H.

COROLLARY 1: *The Gini coefficient for the equal steady state is less than the Gini coefficient for the unequal steady state.*

PROOF:

Only difference between $G_{unequal}$* and G_{equal}* is that L replaced by H.

Since derivative of G_{equal}* with respect to L is positive, $G_{unequal}^* > G_{equal}^*$.

■

Income distribution at unequal s.s. S.O.S.D. income distribution at equal ss.

Normalize to the same mean by fixing output and population.

Cobb-Douglas fixes income going to educated and uneducated.

At unequal ss, fewer educated so more income.

Must transfer

PROPOSITION 3: *R_+* is a steady state if and only if D_+* is between L and H and R_-* is a steady state if and only if D_-* is between L and H, where*

and the wage differential at R_- is at least as high as the wage differential at R_+*.*

PROOF: roots of quadratic equation.

C. Admissibility

LEMMA 1: *R_-* = R_+* if and only if $\theta = \theta_{critical}$, where*

If $\theta > \theta_{critical}$, R_+ and R_-* do not exist. If $\theta < \theta_{critical}$, $R_\pm$* exist and R_-* does not equal R_+*.*

LEMMA 2:

(i) θ_L and θ_H are between 0 and 1.

(ii) θ_L and θ_H attain their maximum, $\theta_{critical}$, when L and H = T, where $T = 1 + [1 / (1 - \alpha)]^{0.5}$. θ_L and θ_H have no other local maxima.

(iii) $\theta_L < \theta_H$ implies $L < T$. $\theta_H < \theta_L$ implies $H > T$.

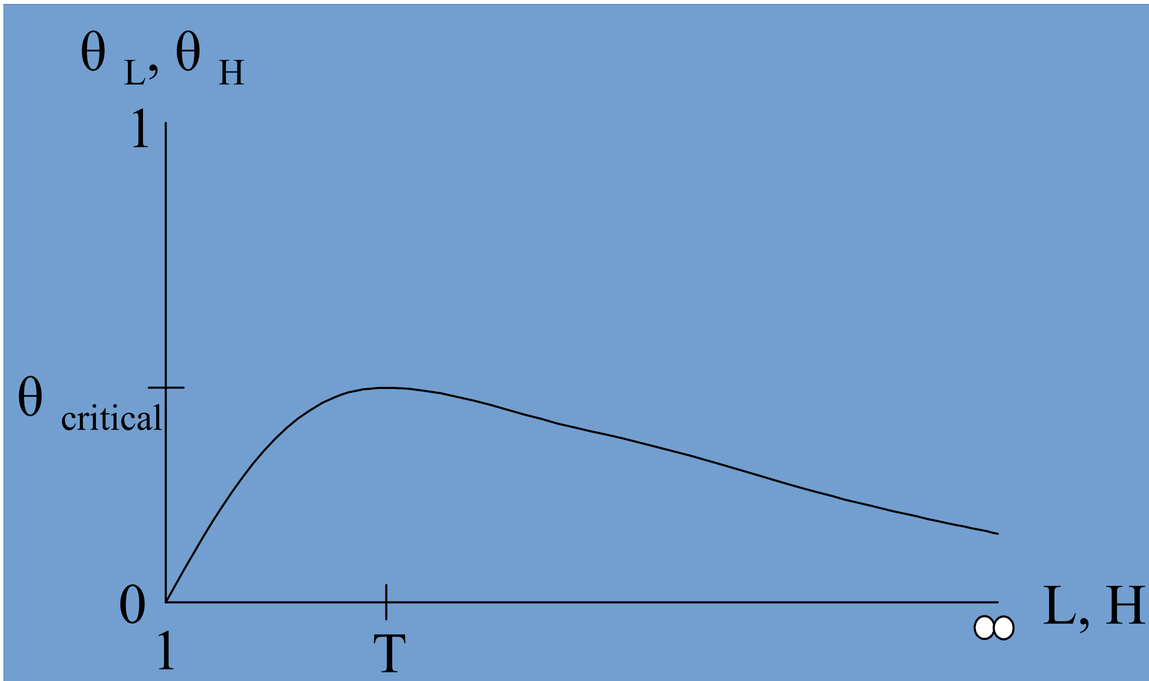


Figure 4: $\theta_L(L)$ and $\theta_H(H)$

Figure 4 shows $\theta_L(L)$ and is helpful to understand axes in Figure 3.

On two X-axes, θ varies from 0 to 1.

On Y-axis, L and H vary from 1 to infinity.

Intuition for Figure 3

Broadly confirms intuition that lower costs of education and higher proportion children of unskilled with low cost may reduce inequality.

Clearest for lowest and highest values of θ (Columns A and D).

If proportion θ of C_U with low cost is very low, economy converges to $R_{unequal}$* because R_{equal}* is inadmissible

For high θ, economy converges to R_{equal}*. $R_{unequal}$* is inadmissible

Multiple steady states for intermediate values of θ

Stability

When R_{equal}*, $R_{unequal}$*, R_-* are admissible, generically stable as well.

When R_+* is admissible, it is generically unstable.

E. Comparative Statics

When there are multiple steady states as in Figure 2, system converges to $R_{unequal}$* if $R_0 < R_+$*, and to R_{equal}* if $R_0 > R_+$*.

Increases in θ reduce R_+*, and expand basin of attraction of equal steady state. To see this, note that

which is negative since θ and α are less than 1.

Suggests that countries with R_0 just under R_+* may face a brief window of opportunity in which small and temporary increases in θ can move them into basin of attraction for R_{equal}*.

As time passes, and R falls, larger or longer-lasting increases in θ would be necessary to move to more equal steady state.

Temporary increase in opportunity –> permanently move country to greater equality.

III. Empirical Evidence

Some evidence that fertility differential between educated and uneducated women is greater in countries with more inequality.

However, since causality runs in both directions in feedback model, evidence is not conclusive

If only one direction of causality exists, still a positive association between differential fertility and inequality.

Cannot reject implication of model

remainder of section is organized as follows

III. Empirical Evidence

A. Data

B. Methodology

C. Results

A. Data

Table 1

Total fertility rates (TFR) by women's educational attainment	United Nations 1987, Jones 1982, United Nations 1995, and Mboup and Saha 1998

Interpret total fertility rate as expected number of children a woman would have if she lives until end of her reproductive years

Income inequality	Deininger and Squire (1996)
Mincer coefficients	Bils and Klenow (1998)
GDP data	Summers and Heston (1994)

1) 88 observations in a sample of 62 countries from 1974 to 1994 for which observations on total fertility rates (TFR) by women's educational attainment and Gini coefficients of inequality could be obtained

2) 30 observations of countries for which TFR by women's educational attainment and Mincer coefficients could be obtained.

Table 2

B. Methodology

Regress TFR on years of education.

(Use weights because sometimes very few women with extreme levels of educational attainment, which increases measurement noise of fertility at these levels

Empirical Model

Two econometric models

Error components model with country random effects.

Data comes from several surveys at three different points in time (1970s, 1980s, and 1990s), data set includes some countries up to three times

Since fertility differentials and income inequality are likely to be correlated for observations of same country due to unobservable characteristics of environment, we need to include country random effects.

Failing to account for these correlations leads to underestimates of standard errors of coefficients.

Model:

$F_{ct} = a + bG_{ct} + cX_{ct} + u_c + \varepsilon_{ct}$

where *c* indexes countries and *t* indexes time.

F_{ct} fertility differential

G_{ct} Gini coefficient

X_{ct} vector of country and time variables such as ln(GDP), a Latin America dummy, and year of survey

Second model takes into account fact that ε_{ct} is actually heteroskedastic.

Noise in approximation of F_{ct}, fertility differential, varies across observations.

Earlier regressions provide an estimate of their noise–the standard errors

Variance-weighted least squares

In effect, this regression puts more weight on observations with low variance.

Could not do both because of software limitations

C. Results

Table 3 -- Fertility Differential Regressed on Gini Coefficient, 1974-1995

Table 3 and Chart 1 refer to sample of countries for which observations on TFR by women's educational attainment and Gini coefficients of inequality could be obtained.

Positive and significant at 1% level for most specifications.

0.101*** indicates that going from a relatively equal country like Indonesia with a Gini coefficient of 0.320 in 1987 to a relatively unequal country like Brazil with a Gini coefficient of 0.545 in 1986 increases fertility differential by 0.023.

0.023 suggests that the ratio of the expected number of children between a woman with no schooling and a woman with 10 years of education is 1.26 children greater in a country like Brazil than it is in a country like Indonesia.

ln(GDP) and ln(GDP)^2 as additional covariates. No difference.

Latin America dummy

Decrease in magnitude and significance: some of increase in fertility differential could be due to other factors related to Latin America.

Sub-saharan africa dummy

asia dummy, ECE

support discussion in section about more general utility function

Table 4 -- Fertility Differential Regressed on Returns to Education

Table 4 refer to sample of countries for which TFR by women's educational attainment and Mincer coefficients of returns to education could be obtained.

Variance-weighted least squares: one Mincer coefficient per country 0.288 implies that increase in two standard deviations of returns to education –> increase fertility differential by 0.026.

Income effects: no difference

Mincer coefficient*Latin America dummy: suggests Latin American countries have large, significant negative relationship while non-Latin American countries have almost no relationship

Potentially problematic since wage differentials appear more directly related to differential fertility than income inequality, as measured by Gini coefficient, in model

But: very few observations, data quite mismatched by year for many countries, Mincer coefficients themselves have many problems.

(Ability differentials and mis-measurement of returns to education

because of varying quality of education across countries.)

(13 observations for the United States from 1925 to 1989 (See Table 5).

positive though somewhat weaker than the relationship in the other two

data sets.

small size of the US time-series sample.

hard to pick up in general for a high-income country

IV. A More General Utility Function

We use quasilinear utility function

Implies number of kids inversely proportional to wage

This approximation may be okay at some wage levels but not others

Malthus: if wages low enough, wage ↑ -> number of surviving children

↑

In Africa, at low education, n increases with education

Very high wages: wage ↑ –> reduce fertility only modestly.

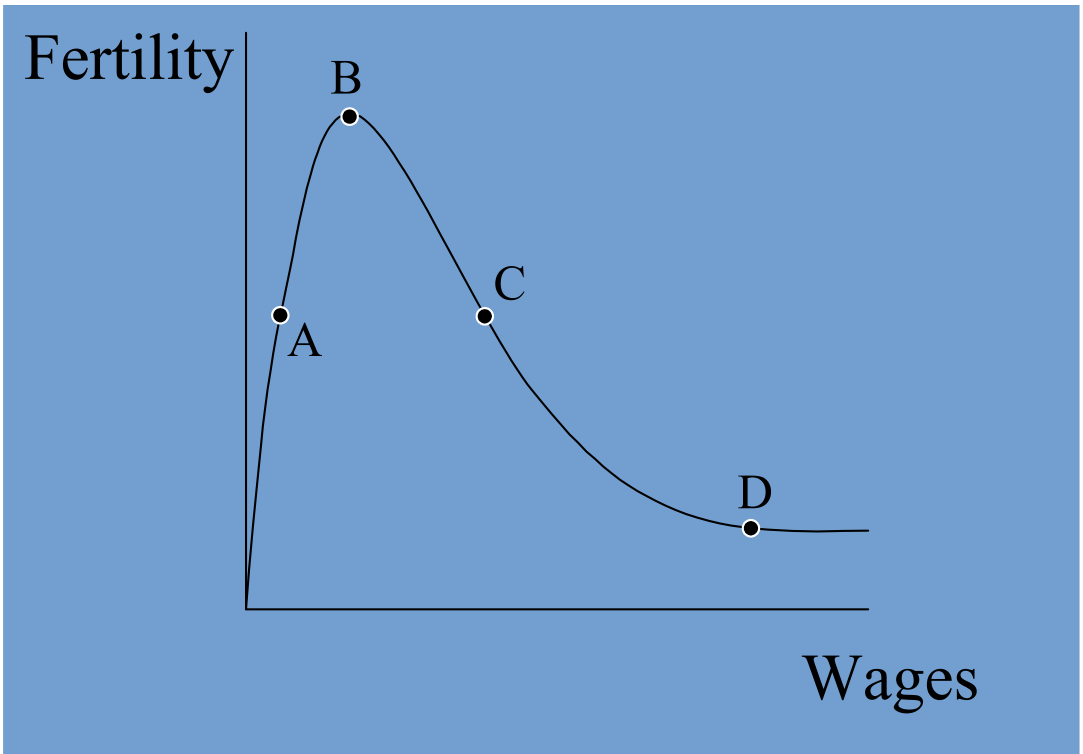


Figure 5: Fertility vs. Wages

Inverse relationship between fertility and wages: characteristic of middle-income countries: less strong for very low or very high income countries.

Can model positive asymptote with $\varepsilon > 0$.

Suppose actual reduced form like Figure 5

If wages are sufficiently low around A, then high wage have more kids than low wage, strengthens this

B - ambiguous

C - model applies

D - Wages are sufficiently high so that fertility of both skilled and unskilled labor is small –> differential fertility low –> Low variance –> Measurement noise.

Table 5 - relationship is in middle-Y countries

Cut-off between middle-income and low-income developing countries is

$950 GDP per capita while cut-off between middle-income and high-income countries is $9500 GDP per capita

Searched

V. Conclusion and Possible Extensions

We assume a two-point form for distribution of cost

Implies that supply of skilled labor is infinitely elastic when $D = L$ or H

–> stabilizing force dominates

completely inelastic for $L < D < H$. –> demographic force dominates

With a more general, continuous distribution of cost of education, either force could locally dominate in different areas.

R_{t+1} (R_t) curve could cross 45° line an arbitrary number of times, generating an arbitrary number of stable steady states.

Response of fertility differentials to wages should make R_{t+1} increase more steeply in R_t than if fertility were exogenous.

Suppose a program educates an additional 1,000,000 children –> unskilled wages will rise –> reducing fertility of unskilled –> further increase wages among unskilled. □ creating a multiplier effect on inequality.

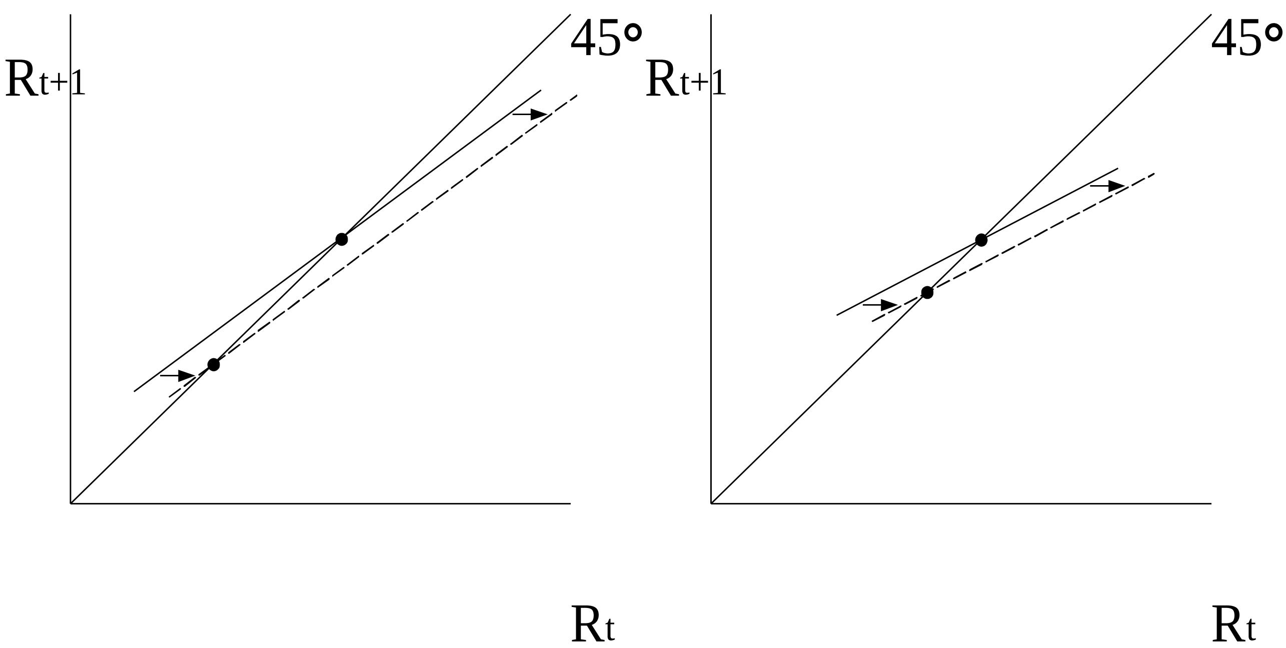


Figure 6: Multiplier effect in a local neighborhood of a stable steady state with and without endogenous fertility

Note that this does not apply in simple model with two-point distribution of cost

Limitation: relative wages depend on population ratio rather than market labor time ratio,

Market labor time depends on proportion of time each group spends on education and raising children.

Choice of whether to obtain education depends on people's expectations of fertility of skilled and unskilled when they become adults.

R_t would depend not only on R_{t-1} but also on R_{t+1}.

Conjecture: robust as long as

(i) skilled and unskilled workers are complements in production;

(ii) children of unskilled are more likely to be unskilled, and

(iii) higher wages reduce fertility because substitution effects outweigh income effects.