The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents

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presented by
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Introduction
Research Question

Some Stylized Facts

- Income/Wealth distributions are skewed to the right.
  - Top 1% of the richest households in the US hold 33% of the wealth.
- Income/Wealth distributions have heavy upper tails.
  - Top wealth shares decline slowly.
  - Top end of wealth distribution follows a Pareto law.
- Models with uninsurable idiosyncratic labor income risk can generate some skewness, but not heavy tails.

Question

Which features of the wealth accumulation process explain these stylized facts, focusing on the heavy upper tail?
Pareto (power-law) distribution

\[ Pr(X > x) \sim kx^{-\alpha}, \quad \alpha > 1 \]
The Wealth of the Forbes 400
Klass, Biham, Levy, Malcai, and Solomon (2007)
This paper
OLG model

- “Standard” features:
  - Continuous time OLG
  - Finitely lived agents
  - “Joy of giving” bequest motive

- Non-standard features
  - Labor income has **uninsurable idiosyncratic component** and **trend-stationary component across generations**
  - Capital income is subject to **stationary idiosyncratic shocks**, possibly persistent across generations (in the data, due to housing and private business equity)

**Result**

Capital income risk, and not stochastic labor income, drives the properties of the right tail of the wealth distribution.
Model
Savings and Bequests

OLG structure

Generation $n$ of its dynasty faces rate of return $r_n$ and labor income $y_n$

$$(1 - b)w_{n-1}(T)$$
Savings and Bequests

Some notation

- Consumption and wealth of a household at $t$ depends on
  - the generation $n$ through $r_n$ and $y_n$
  - its age $\tau = t - s$
  - $r_n$ and $y_n$ are stochastic across generations and idiosyncratic across individuals
- Consumption for household of generation $n = \frac{s}{T}$ at time $t$: $c(s t) = c_n(t - s)$
- Wealth for household of generation $n = \frac{s}{T}$ at time $t$: $w(s t) = w_n(t - s)$
- Estate tax: $b < 1$
  - Household of generation $n$ inherits $w_n(0) = (1 - b)w_{n-1}(T)$. 
Savings and Bequests

Households

**Household Problem**

\[
\max_{c_n(\tau)} \int_{0}^{T} e^{-\rho \tau} u(c_n(\tau)) d\tau + e^{-\rho T} \phi(w_{n+1}(0))
\]

subject to

\[
\dot{w}_n(\tau) = r_n w_n(\tau) + y_n - c_n(\tau)
\]

\[
w_{n+1}(0) = (1 - b) w_n(T)
\]

Preferences satisfy

\[
u(c_n(\tau)) = \frac{c_n(\tau)^{1-\sigma}}{1 - \sigma}, \quad \phi(w_{n+1}(0)) = \chi \frac{w_{n+1}(0)^{1-\sigma}}{1 - \sigma}
\]
Savings and Bequests
Analytical Solution I

The Dynamics of Wealth Across Generations

Let \( w_n = w_n(0) \) denote the initial wealth of the \( n' \)th generation. Then,

\[
    w_{n+1} = \alpha_n w_n + \beta_n,
\]

where \((\alpha_n, \beta_n)_n = (\alpha(r_n), \beta(r_n, y_n)_n)\) is a stochastic process.

- \( \alpha_n \): lifetime rate of return on initial wealth from one generation to the next minus fraction of lifetime wealth consumed
- \( \beta_n \): lifetime labor income minus lifetime wealth consumed.
The Dynamics of Individual Wealth as a Function of Age

\[ w_n(\tau) = \sigma_w(r_n, \tau) w_n + \sigma_y(r_n, \tau) y_n \]

This is a deterministic map since \( r_n \) and \( y_n \) are fixed for any household.
Recall the dynamics of initial wealth:

\[ w_{n+1} = \alpha(r_n)w_n + \beta(r_n, y_n) \]

Suppose \( r_n \) and \( y_n \) (and therefore \( \alpha_n \) and \( \beta_n \)) are i.i.d. Then, wealth converges to stationary distribution with a Pareto law:

\[ Pr(w_n > w) \sim kw^{-\mu} \]

But the i.i.d assumption is very restrictive:

- **Autocorrelation in \( r_n \) and \( y_n \):** Captures variations in social mobility
- **Correlation between \( r_n \) and \( y_n \):** Higher labor income correlated with higher return on wealth in financial markets
The Stationary Distribution of Wealth

Initial Wealth II

Theorem 1

Consider,

\[ w_{n+1} = \alpha(r_n)w_n + \beta(r_n, y_n), \quad w_0 > 0. \]

Under certain assumptions on \((r_n, y_n)_n\) the tail of the stationary distribution of \(w_n\), 
\(Pr(w_n > w)\), is asymptotic to a Pareto law

\[ Pr(w_n > w) \sim kw^{-\mu} \]

where \(\mu > 1\) satisfies

\[ \lim_{N \to \infty} \left( \mathbb{E} \prod_{n=0}^{N-1}(\alpha_n)^\mu \right)^{1/N}. \]
We want to find distribution of wealth $w$ in the population.

- We need to aggregate over wealth of households of different ages, $\tau = 0, \ldots, T$.
- Recall the dynamics of wealth of generation $n$ at age $\tau$:
  \[ w_n(\tau) = \sigma_w(r_n, \tau)w_n + \sigma_y(r_n, \tau)y_n \]

- Define cdf of $w_n(\tau)$: $F(w; \tau) = 1 - Pr(w_n(\tau) > w)$
- Then, cdf of $w$ is $F(w) = \int_0^T F(w; \tau)\frac{1}{T}d\tau$
The Stationary Distribution of Wealth

Wealth in the Population II

**Theorem 2**

Suppose the tail of the stationary distribution of initial wealth $w_n = w_n(0)$ is asymptotic to a Pareto law, $Pr(w_n > w) \sim kw^{-\mu}$. Then the stationary distribution of wealth in the population has a power tail with the same exponent $\mu$. 
Wealth Inequality: Comparative Statics
Comparative Statics

The Tail Index

- Recall from Theorem 1 that initial wealth $w_n$ asymptotically follows a Pareto law:

$$Pr(w_n > w) \sim kw^{-\mu}$$

- The tail index $\mu$ is inversely related to wealth inequality
- Gini coefficient of the tail: $G = \frac{1}{2^{\mu-1}}$
- Four exercises: What is the relationship of $\mu$ with:
  - Capital and labor income risk
  - Preferences, particularly the bequest motive
  - Capital income and estate taxes
  - Social mobility
Comparative Statics
Capital and Labor Income Risk

Recall,

\[ w_{n+1} = \alpha_n w_n + \beta_n \]

Theorem 1 implies,

- \((\beta_n)_n\) has no effect on tail of stationary wealth distribution
- High capital income risk \(Pr(\alpha_n > 1) > 0\) is necessary for heavy tails in the distribution
  - If \(Pr(\alpha_n < 1) = 1\) the stationary wealth distribution is bounded at \(\frac{\beta}{1-\alpha}\).

**Proposition 1**

The tail index \(\mu\) decreases with the idiosyncratic risk on return on capital.
**Comparative Statics**

**Bequest Motive**

Recall,

\[ \phi(w_{n+1}(0)) = \chi \frac{w_{n+1}(0)^{1-\sigma}}{1-\sigma} \]

- If \( \chi \) is high, **households save more** and accumulate wealth faster
- Effective rate of return \( \alpha_n \) increases
- This increases wealth inequality

**Proposition 2**

The tail index \( \mu \) decreases with bequest motive \( \chi \).
Comparative Statics
Fiscal Policy

Let $\zeta$ be a tax on capital

- Post tax return on capital: $(1 - \zeta)r_n$

When $\zeta$ increases, capital income risk decreases

- By proposition 1, $\mu$ increases

- Bequests can partly offset this effect, but cannot change the direction of the response

Proposition 3

The tail index $\mu$ increases with the estate tax $b$ and capital income tax $\zeta$. 
Social mobility is higher, when \((r_n)_n\) and \((\beta_n)_n\) are less autocorrelated.

They consider the AR(1) and the MA(1) case:

\[
\log \alpha_n = \eta_n + \theta \eta_{n-1} \\
\log \alpha_n = \theta \log \alpha_{n-1} + \eta_n
\]

Proposition 4

The tail index \(\mu\) decreases with \(\theta\) in both, the AR(1) and the MA(1), cases.
Calibration
They calibrate these parameters following standard US data:

- $\sigma = 2$ ; $\rho = 0.04$ ; $\chi = 0.25$ ; $T = 45$
- $y_n$ has mean of 42000$ and standard deviation of 95000$. It grows at a rate $g$ of 1% per year.
- Data is from Diaz-Gimenez, Quadrini, Ríos-Rull, and Rodríguez (2002), who used the 1998 survey of consumer finances.

For the cross-sectional distribution of the rate of return on wealth - $r_n$, they:

- distinguish two components of $r_n$: a common economy-wide rate of return $r^E$ and an idiosyncratic component $r^I_n$
- According to the Survey of Consumer Finances: $r_n = \frac{r^E}{2} + \frac{r^I}{2}$
- They are set between 7% and 9%, and their processes follow from Angeletos (2007)
They model the variations in $r_n$ from generation to generation as a Markov Chain

- $r_n = (0.08, 0.12, 0.15, 0.32)$ and

$$Pr(r_{n+1} | r_n) = \begin{pmatrix}
0.8 + \epsilon_{low} & 0.12 - \frac{\epsilon_{low}}{3} & 0.07 - \frac{\epsilon_{low}}{3} & 0.01 - \frac{\epsilon_{low}}{3} \\
0.8 & 0.12 & 0.07 & 0.01 \\
0.8 & 0.12 & 0.07 & 0.01 \\
0.8 - \frac{\epsilon_{high}}{3} & 0.12 - \frac{\epsilon_{high}}{3} & 0.07 - \frac{\epsilon_{high}}{3} & 0.01 + \epsilon_{high}
\end{pmatrix}$$

- $\epsilon_{low}$ controls persistence of lowest rate of return
- $\epsilon_{high}$ controls persistence of highest rate of return
Calibration

Results

- Good match of top percentiles

<table>
<thead>
<tr>
<th>Economy</th>
<th>Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90th–95th</td>
</tr>
<tr>
<td>United States</td>
<td>.113</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = 0$</td>
<td>.118</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .01$</td>
<td>.116</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .02$</td>
<td>.105</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .05$</td>
<td>.087</td>
</tr>
</tbody>
</table>

TABLE II

PERCENTILES OF THE TOP TAIL; $\varepsilon_{\text{low}} = .01$
They claim that $\epsilon_{\text{high}} = 0.02$ has the best fit here, but it’s not that clear.

### TABLE III

_Tail Index, Gini, and Quintiles; $\epsilon_{\text{low}} = .01$_

<table>
<thead>
<tr>
<th>Economy</th>
<th>Tail Index $\mu$</th>
<th>Gini</th>
<th>Quintiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First</td>
</tr>
<tr>
<td>United States</td>
<td>1.49</td>
<td>.803</td>
<td>-.003</td>
</tr>
<tr>
<td>$\epsilon_{\text{high}} = 0$</td>
<td>1.796</td>
<td>.646</td>
<td>.033</td>
</tr>
<tr>
<td>$\epsilon_{\text{high}} = .01$</td>
<td>1.256</td>
<td>.655</td>
<td>.032</td>
</tr>
<tr>
<td>$\epsilon_{\text{high}} = .02$</td>
<td>1.038</td>
<td>.685</td>
<td>.029</td>
</tr>
<tr>
<td>$\epsilon_{\text{high}} = .05$</td>
<td>.716</td>
<td>.742</td>
<td>.024</td>
</tr>
</tbody>
</table>
Tax Experiments
Tax Experiments

Results

- Keeping $\epsilon_{high} = 0.02$ and $\epsilon_{low} = 0.01$, they run experiments with $b$ - estate tax and $\zeta$ - capital income tax.
- Taxes have a significant effect on the inequality of the wealth distribution as measured by the tail index. This is especially the case for the capital income tax.

<table>
<thead>
<tr>
<th>TABLE IX</th>
<th>TAX EXPERIMENTS—TAIL INDEX $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \backslash \zeta$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>.68</td>
</tr>
<tr>
<td>.1</td>
<td>.689</td>
</tr>
<tr>
<td>.2</td>
<td>.7</td>
</tr>
<tr>
<td>.25</td>
<td>.706</td>
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</table>

<table>
<thead>
<tr>
<th>TABLE X</th>
<th>TAX EXPERIMENTS—GINI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \backslash \zeta$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>.779</td>
</tr>
<tr>
<td>.1</td>
<td>.768</td>
</tr>
<tr>
<td>.2</td>
<td>.778</td>
</tr>
<tr>
<td>.3</td>
<td>.754</td>
</tr>
</tbody>
</table>
Castaneda, Diaz Gimenez, and Rios-Rull (2003) and Cagetti and De Nardi (2007) found very small (or even opposite) effects of eliminating bequest taxes in their calibrations in models with a skewed distribution of earnings but no capital income risk.

This paper has a different result.
Conclusion
Conclusion
Some comments

- The model results in a **good fit to the data** while still using **classical model structures**
- However:
  - The implication that **labour income** has little to no impact on wealth inequality at the tail is at odds with other modern papers, and seems unrealistic.
  - There is no role for **entrepreneurship** in this model, which has lately been shown to be a main factor in determining wealth distribution.
  - The study of **social mobility** is still limited.
  - **Age and dynasty size** are a determining factor, while data suggests that the super-rich are often self-made and even young.