The Macroeconomic Consequences of Early Childhood Development Policies

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Early childhood investments increase education and income

- Effects can be large (e.g., Garcia, Heckman, Leaf, and Prados, 2020)
Motivation

**Early childhood investments increase education and income**

- Effects can be large (e.g., Garcia, Heckman, Leaf, and Prados, 2020)
- Based on small-scale and short-run programs
Motivation

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- Effects can be large (e.g., Garcia, Heckman, Leaf, and Prados, 2020)
- Based on small-scale and short-run programs

Consequences of large-scale and long-run policy depend on
- GE effects on capital and labor markets
- Deadweight loss of raising taxes
- Intergenerational dynamics
What is the impact of a permanent and universal early childhood government investment policy?

Particularly on: income, inequality, intergenerational mobility, and welfare

Use an overlapping generations (OLG) model

- with distortionary taxes
- in general equilibrium
What is the impact of a permanent and universal early childhood government investment policy?

Particularly on: income, inequality, intergenerational mobility, and welfare

Use an OLG model with distortionary taxes and in general equilibrium

GE Life-cycle Aiyagari + Endogenous Intergenerational Links

- Wage depends on skills
- Parental investments of time and money to build child’s skills

• Potential role for government investments because of:
  • Imperfect capital and insurance markets
  • Inability to write contracts with children
1. **Model**: GE Life-cycle Aiyagari + Endogenous Intergenerational Links
   - Wage depends on **skills**
   - Parental investments of time and money to build child's skills

2. **Estimation**:
   - Skill production function based on Cunha, Heckman, Schennach (2010)
   - Key moments on parental investments and transfers from PSID

3. **Validation**
   - Model replicates small-scale short-run RCT evidence
     (Garcia, Heckman, Leaf, and Prados, 2020)

4. **Policy**: large-scale government investments in early childhood
   - Long-run effects
   - Transition (with alternative ways to finance it)
   - Alternative policy in paper: parenting education
Preview of Results

Large long-run effects

- **Average income** grows by 7%
- **↓Inequality, ↑Int. mobility** \(\approx\) half of gap between US and Canada
- **Welfare** gains of 9%
  Welfare: Consumption equivalence for a newborn under veil of ignorance
Preview of Results

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**Short-run small-scale policy would underestimate gains by one-half**

- Large-scale tax increase reduces gains
- But long-run intergenerational dynamics more than compensate for the losses
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Investing in a child today will make him a better parent tomorrow

- **Transition**: Large increase in gains after first generation has its own children
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**Short-run small-scale policy would underestimate gains by one-half**
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**Investing in a child today will make him a better parent tomorrow**
- Transition: Large increase in gains after first generation has its own children

**Who does not benefit from the reform?**
- Older individuals at the time the policy is introduced
- But this depends on how the transition is financed
Related Literature

Inequality and social mobility

- **GE Quantitative Life-cycle Aiyagari**: De Nardi (2004); Conesa and Krueger (2006); Bakis, Kaymak, and Poschke (2015); Abbott, Gallipoli, Meghir, Violante (2019)...

- **Contribution**: Endogenous early childhood development

Early childhood development


- **Structural**: Cunha (2013); Del Boca, Flinn, and Wiswall (2014); Abbott (2016); Caucutt and Lochner (2017)...

- **Contribution**: Large-scale policy evaluation framework (labor and savings choices, general equilibrium, multiple generations)


- **Contribution**: alternative policies and transition (crucial to observe intergenerational dynamics)
Model

Estimation: USA 2000

Policy
Model: Timeline

- **Birth**
- **Independent**
- **Child born**
- **Transfer to child**
  - Child is independent
- **Retire**
- **Death**

- Live w/ parent
- Parent invests in skills
- Transfer at 16
- College or work
- College is costly but changes wage profile
- Work
  - Wage depends on: skill, education, age, and shock
  - Direct Investment on child: time and money
    - Multiple periods ⇒ Builds child’s skills
- Retirement
  - Retirement income:
    - Savings
    - Social Security
Model: Timeline

- **Birth**
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  - Direct investment on child: time and money in multiple periods ⇒ builds child’s skills
Model: Timeline

- **Birth**
  - Live w/ parent
  - Parent invests in skills
  - Transfer at 16

- **16**
  - College born
  - College is costly but changes wage profile

- **20**
  - Child born

- **28**
  - College or work

- **44**
  - Transfer to child
  - Child is independent

- **68**
  - Retire

- **80**
  - Death

- Retirement income:
  - Savings
  - Social Security
### Model: Timeline

- **Birth (0)**
  - Live w/ parent
  - Parent invests in **skills**
  - Transfer at 16

- **Child born (28)**
  - College or work
  - College is **costly** but changes wage profile

- **Child is independent (44)**
  - College
  - Work
  - Wage depends on: **skill**, education, age, and shock
  - **Direct Investment on child**: time and money
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- **Retire (68)**
  - Retirement income
    - Savings
    - Social Security

- **Death (80)**
Model: Timeline

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Live w/ parent
• Parent invests in skills
• Transfer at 16

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Retire
Retirement income:
• Savings
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Death

• Wage depends on: skill, education, age, and shock
• Direct Investment on child: time and money
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Stationary Equilibrium
\[ V_j(a, \theta, e, \eta, \theta_k) = \max_{c, a', h, m} u(c, h, a') + \beta \mathbb{E} \left[ V_{j+1}(a', \theta, e, \eta', \theta_k') \right] \]

\[ c + a' + m = y + a(1 + r) - T(y, a, c) \]

\[ y = w_e E_{e,j}(\theta, \eta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h + t \leq 1, \quad \eta' \sim \Gamma_{e,j}(\eta) \]

where

- \( a \): assets
- \( \theta \): agent’s skills
- \( t \): time with child
- \( e \): education
- \( \theta_k \): child’s skills
- \( m \): money towards child
- \( \eta \): wage shock
Early Childhood Investments

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### Early Childhood Investments

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In the paper: include child consumption \(c_k\) in utility, \(\delta u(c_k, 0)\)
## Early Childhood Investments

### Timeline

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### Equation

\[
V_j(a, \theta, e, \eta, \theta_k) = \max_{c, a', h, t, m} u(c, h, t) + \beta \mathbb{E} \left[ V_{j+1}(a', \theta, e, \eta', \theta'_k) \right]
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\[
c + a' + m = y + a(1 + r) - T(y, a, c)
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\[
y = w_e E_{e,j}(\theta, \eta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h + t \leq 1, \quad \eta' \sim \Gamma_{e,j}(\eta)
\]

### Parameters

\[
\theta'_k = \left[ \alpha_{1j} \theta^\rho_k + \alpha_{2j} \theta^\rho + \alpha_{3j} \frac{\rho_j}{\rho_j} \right]^{1/\rho_j} \exp(v), \quad v \sim N(0, \sigma_{j,v})
\]

\[
l = \tilde{A} \left[ \alpha_m (m + g)^\gamma + (1 - \alpha_m) t^\gamma \right]^{1/\gamma}
\]

Money \quad Time
Early Childhood Investments

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\[ \theta_k' = \left[ \alpha_{1j} \theta_{\rho j}^k + \alpha_{2j} \theta_{\rho j} + \alpha_{3j} l_{\rho j} \right]^{1/\rho_j} \exp(\nu), \nu \sim N(0, \sigma_{j,\nu}) \]

\[ l = \bar{A} \left[ \alpha_m (m + g)^{\gamma} + (1 - \alpha_m) t^{\gamma} \right]^{1/\gamma} \quad t, m \geq 0 \]
• Just before child becomes independent, choose transfer $\hat{a}$

$$V_{\text{Transfer}}(a, \theta, e, \eta, \theta_k) = \max_{\hat{a}} V_{44}(a - \hat{a}, \theta, e, \eta) + \delta E[V_{16}(\hat{a}, \theta_k, \phi_k)]$$

Parents’ Continuation

Child’s Utility

$\hat{a} \geq 0, \quad \varepsilon_k \sim N(\bar{\varepsilon}, \sigma_\varepsilon)$

Draw of school taste shock, depends on parent’s education
Role for Government Investments

Why may government investments increase welfare?
Welfare: Consumption equivalence for a newborn under veil of ignorance

1. **Parent can’t borrow against child’s income created by investing**
   - I. Lack of compensation mechanism
   - II. **Life-cycle borrowing constraints** $\Rightarrow$ Timing of compensation matters
Role for Government Investments

Why may government investments increase welfare?
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1. Parent can’t borrow against child’s income created by investing
   I. Lack of compensation mechanism
   II. Life-cycle borrowing constraints \( \Rightarrow \) Timing of compensation matters

2. Life-cycle borrowing constraints
   • Parent may not be able to use her own future income

3. Lack of insurance
   • Investing in child is risky, so more incentives to consume and invest in safe asset
Model: Timeline

Birth → Independent
  - Live w/ parents

Child → Transfer to child
  - College or work
  - Child is independent

Work
  - Earnings: wage life cycle by education + hours worked
  - Direct Investment on child: time and money
    Multiple periods ⇒ Builds child’s skills

Retirement → Death
  - Retirement income:
    - savings
    - social security

Parents invest in skill and transfer
Cobb-Douglas with constant returns to scale:

\[ Y = AK^\alpha H^{1-\alpha} \]

where \( H \) is the CES aggregator

\[ H = \left[ sH_0^\Omega + (1-s)H_1^\Omega \right]^{\frac{1}{\Omega}} \]
Outline

Model

Estimation: USA 2000

Policy
Child’s Skill Production Function

Based on Cunha, Heckman and Schennach (ECTA, 2010)

\[
\theta_k' = \left[ \alpha_1 j \theta_k + \alpha_2 j \theta_{\rho j}^p + \alpha_3 j I_{\rho j}^p \right]^{1/\rho_j} \exp(\nu), \quad \nu \sim N(0, \sigma_{j,\nu})
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- Investment’s productivity depends on child/parent’s skills
- Parameters can vary with child’s age
Child’s Skill Production Function

Based on Cunha, Heckman and Schennach (ECTA, 2010)

\[
\theta_k' = \left[ \alpha_{1j} \theta_k^{\rho_j} + \alpha_{2j} \theta^{\rho_j} + \alpha_{3j} \rho_j^{-1} \exp(\nu) \right]^{1/\rho_j} \\
\text{Next period} \quad \text{Current} \quad \text{Parent's} \quad \text{Parental} \quad \nu \sim N(0, \sigma_{\nu})
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Parameter values

- **Baseline estimation from CHS (2010)**
  - Estimated on a representative sample
  - Skills are more malleable when children are young
- **Estimation concerns** (e.g., Agostinelli and Wiswall, 2016)
  - Test robustness of results when we move away from CHS estimation

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Model requires specifying and estimating investment function \( I \)

\[ I = \bar{A} \left[ \alpha_m (m + g)^\gamma + (1 - \alpha_m) t^\gamma \right]^{1/\gamma} \]
Estimated to **match household level** data

**Important moments** for early childhood development

- **Parental investments**
  - **Hours:** Use *PSID Child Development Supplement* (CDS)
  - **Expenses:** CDS misses child care and school fees. Use CEX

- **Parental transfers**
  - Informative about altruism
  - Estimate from *PSID Rosters and Transfers Supplement*
### Estimation: Parameters

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<td>Parent-to-child transfer as share of avg. annual income</td>
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<td>32.4</td>
<td>(1.30)</td>
<td>Returns to investments</td>
<td>Average log(skill)</td>
<td>0.0</td>
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<td>$\alpha_m$</td>
<td>0.91</td>
<td>(0.02)</td>
<td>Money productivity</td>
<td>Ratio of money to hours</td>
<td>218</td>
<td>183</td>
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<td>$\gamma$</td>
<td>-0.20</td>
<td>(0.45)</td>
<td>Money-time substitutability</td>
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<td>$\iota$ ($\times 10^2$)</td>
<td>4.9</td>
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<td>Borrow-save wedge</td>
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<td>$\omega$ ($\times 10$)</td>
<td>2.05</td>
<td>(0.04)</td>
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<tr>
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<td>Money productivity</td>
<td>Ratio of money to hours</td>
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<td>183</td>
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<tr>
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<td>0.88</td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>( t \times 10^2 )</td>
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<td>(1.22)</td>
<td>Borrow-save wedge</td>
<td>Share of borrowers</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td><strong>Government</strong></td>
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<tr>
<td>( \omega \times 10 )</td>
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<td>(0.04)</td>
<td>Lump-sum transfer</td>
<td>Income variance ratio: Disposable to pre-gov</td>
<td>0.69</td>
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- **Moments’ Information**: [Link]
- **Non-targeted Moments**: [Link]
- **Back to Robustness**: [Link]
### Estimation: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std. Error</th>
<th>Description</th>
<th>Moment</th>
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</tbody>
</table>

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[Moments' Information] [Non-targeted Moments] [Back to Robustness] [Back to Robustness SR-PE]
Model

Estimation: USA 2000

Policy
Government investments in early childhood

- Government invests money $g$ directly:

\[
l = \bar{A} \left[ \alpha_m (m + g)^\gamma + (1 - \alpha_m) t^\gamma \right]^{1/\gamma}
\]
Validation: Experimental Evidence

Use **RCT to validate the estimated model**

- **Garcia, Heckman, Leaf, and Prados (2020):**
  - Two US early childhood programs (ABC, CARE) in 1970s
  - Cost ≈ $13.5k per year for 5 years, i.e., total $67.5k per child
  - Followed up into adulthood and observe education/income
Use **RCT to validate the estimated model**

- **Garcia, Heckman, Leaf, and Prados (2020):**
  - Two US early childhood programs (ABC, CARE) in 1970s
  - Cost $\approx$ $13.5k$ per year for 5 years, i.e., total $67.5k$ per child
  - Followed up into adulthood and observe education/income

- **Apply similar policy in model:**
  - **Small scale:** prices and taxes are not affected
  - **Target:** disadvantaged children of low-educated and low-income parents
  - **One-generation:** policy is not received by following generations
Validation: Experimental Evidence

Use **RCT to validate the estimated model**

- **Garcia, Heckman, Leaf, and Prados (2020):**
  - Two US early childhood programs (ABC, CARE) in 1970s
  - Cost $\approx 13.5k$ per year for 5 years, i.e., total $67.5k$ per child
  - Followed up into adulthood and observe education/income

- (a) College
- (b) Income (Age 30)
- (c) Return per Dollar
Large Scale and Permanent Policy

Evaluate universal version of policy
- **General Equilibrium:** Wages (and interest rate) adjust
- **Budget Balance:** Labor income tax adjusts

Outcomes of interest
- Average income, inequality, and intergenerational mobility
- Consumption equivalence under veil of ignorance
  
  *How much extra % consumption would an agent have to get in order to be indifferent between being born in initial SS and alternative?*

Outline

1. **Long-run effects**
   - (i) Alternative levels of $g$, (ii) Importance of long run, GE, budget-balance...
2. **Transition** (with alternative ways to finance it)
Intergenerational mobility: $\text{ChildRank}_i = \alpha + \beta \text{ParentRank}_i + \epsilon_i$
## Results Decomposition

<table>
<thead>
<tr>
<th>Alternative Exercises</th>
<th>Change from Baseline (%)</th>
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<tbody>
<tr>
<td></td>
<td>Consumption Equivalence</td>
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</tr>
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</table>

- Short-run small-scale policy would underestimate gains by one-half.
- Long-run intergenerational dynamics generate over 1/2 of welfare gains.
- Large-scale higher taxes reduce gains by 1/10th.
- Large-scale GE effects explain most of inequality reduction.
- Increase wage of HS-grads relative to college-grads.
  - Increase gains by 1/10th.
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<th>Mobility</th>
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<td>7.2</td>
<td>8.4</td>
<td>-7.9</td>
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**Short-run small-scale policy would underestimate gains by one-half**

- Long-run intergenerational dynamics generate over 1/2 of welfare gains
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- Large-scale higher taxes reduce gains by 1/10th

**Large-scale GE effects explain most of inequality reduction**

- Increase wage of HS-grads relative to college-grads
- Increase gains by 1/10th
Transition Dynamics

Many alternatives on how to transition to new steady state

First:

- Immediate introduction of investments $g$ and labor-income tax
- Balance budget every period using lump-sum tax
Intergenerational mobility: $\text{ChildRank}_i = \alpha + \beta \text{ParentRank}_i + \epsilon_i$
Intergenerational mobility: \( \text{ChildRank}_i = \alpha + \beta \text{ParentRank}_i + \epsilon_i \)
Transition Dynamics

Intergenerational mobility: $\text{ChildRank}_i = \alpha + \beta \text{ParentRank}_i + \epsilon_i$
Who Loses? Older Agents at Time of Introduction
Alternative Transitions

Two ways to reduce cost paid by older agents and earlier cohorts

- Government borrowing ⇒ Transfer costs to future cohorts
- Slow introduction of investments ⇒ Reduce earlier costs

Combination makes gains more homogenous across cohorts
Transition: Only Intervened Pay + Slow Intro
Move each parameter one std. dev. above and below

- Calculate steady-state and introduce same policy as before

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Cons. Equiv. Change from Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>Altruism</td>
<td>Down</td>
</tr>
<tr>
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<tr>
<td>( \alpha )</td>
<td>Avg. distaste for College</td>
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</tr>
<tr>
<td>( \alpha_{\theta_c} )</td>
<td>College taste-Cog Skills relation</td>
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<td>( \alpha_{\theta_{nc}} )</td>
<td>College taste-NonCog Skills relation</td>
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<tr>
<td>( \bar{\epsilon} )</td>
<td>Mean college taste shock</td>
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<tr>
<td>( \sigma_{\epsilon} )</td>
<td>SD of college taste shock</td>
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<tr>
<td>( \bar{A} )</td>
<td>Returns to investments</td>
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<tr>
<td>( \alpha_m )</td>
<td>Money productivity</td>
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<td>( \gamma )</td>
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<td>Parental time disutility</td>
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<td>( \iota )</td>
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Baseline 9.4
Move each parameter one std. dev. above and below

- Calculate steady-state and introduce same policy as before

<table>
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<th>Parameter</th>
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<td>$\bar{\varepsilon}$ Mean college taste shock</td>
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<td>$\sigma_{\varepsilon}$ SD of college taste shock</td>
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<td>$\bar{A}$ Returns to investments</td>
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<td>$\alpha_m$ Money productivity</td>
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<td>$\gamma$ Money-Time substitutability</td>
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Baseline: 9.4
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Baseline 9.4
## Results Robustness: Estimated Parameters Importance

Move each parameter one std. dev. above and below

- Calculate steady-state and introduce same policy as before

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<tbody>
<tr>
<td>$\delta$</td>
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<td>0.53</td>
<td>0.34 Down, -0.19 Up</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Labor Disutility</td>
<td>0.07</td>
<td>0.13 Down, -0.06 Up</td>
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<tr>
<td>$\alpha$</td>
<td>Avg. distaste for College</td>
<td>1.47</td>
<td>-0.66 Down, 0.81 Up</td>
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<tr>
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<td>College taste-Cog Skills relation</td>
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<tr>
<td>$\alpha_{\theta_{nc}}$</td>
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<td>0.01</td>
<td>-0.13 Down, -0.14 Up</td>
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<td>Mean college taste shock</td>
<td>0.02</td>
<td>-0.21 Down, -0.20 Up</td>
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<tr>
<td>$\sigma_{\varepsilon}$</td>
<td>SD of college taste shock</td>
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**Baseline**: 9.4
Move each parameter one std. dev. above and below

- Re-estimate, obtain steady-state, and introduce same policy as before

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<th>Cons. Equiv. Change from Baseline Long-Run GE</th>
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<tr>
<td>$\sigma_v$</td>
<td>Std. Dev. of Shock</td>
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</tr>
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<td>$\text{Var} (\theta_{k_0})$</td>
<td>Var of Initial Skills</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\theta, \theta_{k_0})$</td>
<td>IGE Corr of Initial Skills</td>
<td></td>
</tr>
</tbody>
</table>

Baseline: 9.4
Move each parameter one std. dev. above and below

- Re-estimate, obtain steady-state, and introduce same policy as before

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Cons. Equiv. Change from Baseline Long-Run GE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>Child’s Skills Importance</td>
<td>Down 1.64, Up -2.70, Total 4.34</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>Parents’ Skills Importance</td>
<td>Down 0.98, Up -1.48, Total 2.46</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>Investments Importance</td>
<td>Down 0.03, Up -0.89, Total 0.92</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Substitutability</td>
<td>Down -1.26, Up 0.96, Total 2.21</td>
</tr>
<tr>
<td>( \sigma_v )</td>
<td>Std. Dev. of Shock</td>
<td>Down 0.07, Up -0.66, Total 0.73</td>
</tr>
<tr>
<td>Var ( (\theta_{k0}) )</td>
<td>Var of Initial Skills</td>
<td>Down -0.66, Up -0.67, Total 0.01</td>
</tr>
<tr>
<td>Corr ( (\theta, \theta_{k0}) )</td>
<td>IGE Corr of Initial Skills</td>
<td>Down -0.69, Up -0.44, Total 0.25</td>
</tr>
</tbody>
</table>

Baseline 9.4
Results Robustness: CHS Parameters Importance

Move each parameter one std. dev. above and below

- Re-estimate, obtain steady-state, and introduce same policy as before

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<tr>
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Baseline: 9.4
Alternative Policy: Parenting Education Program

Parenting education program
- Extend model to allow parents to acquire minimum parenting skills
- Use experimental evidence to estimate costs and gains of programs

Two alternative implementations
1. Paid by Government
   - Welfare benefits of 8%
   - Reduces inequality by 5% and increases mobility by 15%

2. Paid by Households
   - Welfare benefits of 7%
   - Reduces inequality by 5% and increases mobility by 13%

As with ECD investments: long-run large-scale gains are larger than short-run small-scale ones
Conclusion

Consequences of large-scale early childhood policies depend on

- (i) GE effects; (ii) cost of raising taxes; (iii) intergenerational dynamics

Model

- Introduce endogenous parental investments into a GE OLG incomplete markets model with distortionary taxes

Government early childhood investments increase welfare by 9%

- Small-scale short-run programs underestimate gains
  - Large-scale higher taxes reduce gains by 1/10th
  - Large-scale GE reduces inequality and increases gains by 1/10th
  - Long-run intergenerational dynamics generate over 1/2 of welfare gains

- Effects on inequality and mobility
  - Large enough to close gap with Canada by 50%
Some suggestions

**Computation and data skills are very valuable**
- Software: your choice
- Guides: Judd’s or Miranda-Fackler’s books, Violante’s notes
- Practice is key so start early

**For heterogeneous-agents models**
- Endogeneous grid method–look at Pijoan-Mas notes
- Simulation using kronecker products
- But these methods evolve quickly...
  - Maybe approximation methods based on machine learning?

**Take advantage of HPC**
- Provides lots of computational power
- May need advisor/professor’s sponsorship
APPENDIX
Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
Early Childhood Development Programs around the world

Programs inspired by ABC/CARE around the world:

- Infant Health and Development Program (Spiker et al, 1997)
- John’s Hopkins Cerebral Palsy Study (Schneider and McDonald, 2007)
- Classroom Literacy Interventions and Outcomes (Sparling, 2010)
- Massachusetts Family Child Care Study (Collins, 2010)
- Many more in US, Manitoba, Australia (Garcia, Heckman, Leaf, and Prados, 2020)
Evidence on Early Childhood Programs

It is important to observe adult follow-ups (Garcia et al, 2020)

• Rather than using early measures to project adult outcomes

Most US evidence is from three programs:

• Large increases in education and income, and social gains

• Perry Preschool Program (ages 3–5)

• Carolina Abecedarian Project (ABC) and Carolina Approach to Responsive Education (CARE)

Head Start

• It is the largest program, between ages 4 (or 3) and 5

• Experimental evidence predicted smaller gains than non-experimental

• Larger gains if program substitution is accounted for (Kline and Walters, 2016)
Outline

Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
Preliminaries: Skills and Wages

**Labor income** of individual of age $j$, education $e$, and skills $\theta$ is product of:

1. Wage of your education group: $w_e$.
2. **Labor efficiency units**: $E_{i,e,j} = \epsilon_{e,j}\psi_{i,e,j}$.
3. Hours worked: $h$.

**Labor efficiency units** evolve stochastically as sum of three components:

$$\log(E_{i,e,j}) = \log(\epsilon_{e,j}) + \lambda_e \log(\theta_{ic}) + \eta_{i,e,j}$$

where

- $\lambda_e$ is education-specific return to skills.
- $\epsilon_{e,j}$ is education-specific age profile.
- $\psi_{i,e,j}$ is stochastic component with persistent cdf $\Gamma_{j,e}$. 
During working years

- Can borrow: limits by education group.
- Interest rate $r^b = r + \iota$ where $r$ is the returns to saving and $\iota$ is the wedge between borrowing and lending capital.
Preliminaries: Market Structure

During working years

- Can borrow: limits by education group.
- Interest rate $r^b = r + \iota$ where $r$ is the returns to saving and $\iota$ is the wedge between borrowing and lending capital.

College Loans

- Pay subsidized interest rate $r^c$: 
During working years

- Can borrow: limits by education group.
- Interest rate $r^b = r + \iota$ where $r$ is the returns to saving and $\iota$ is the wedge between borrowing and lending capital.

College Loans

- Pay subsidized interest rate $r^c$:

Today: Presentation of model abstracts from different interest rates.
## College Choice

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<th>20</th>
</tr>
</thead>
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<td></td>
</tr>
<tr>
<td></td>
<td><strong>College or work</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Work \( (e = 0) \)

\[
V_j^w (a, \theta, e, \eta) = \max_{c, a', h} u(c, h) + \beta \mathbb{E} \left[ V_{j+1}^w (a', \theta, e, \eta') \right],
\]

\[
c + a' = y + a (1 + r) - T(y, a, c),
\]

\[
y = w_e E_{e,j} (\theta, \eta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h \leq 1, \quad \eta' \sim \Gamma_{e,j}(\eta).
\]
College Choice

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\]

**College** \((e = 1)\)

\[
V_j^s (a, \theta, e) = \max_{c, a', h} u(c, h + \bar{h}) + \beta \mathbb{E}_{\eta|e} V_{j+1}^w (a', \theta, e, \eta)
\]

\[
c + a' + p^s = y + a (1 + r) - T(y, a, c)
\]

\[
y = w_0 E_{e,j}(\theta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h \leq 1 - \bar{h}
\]
**Work** ($e = 0$)

$$V_j^w (a, \theta, e, \eta) = \max_{c, a', h} u(c, h) + \beta \mathbb{E} \left[ V_{j+1}^w (a', \theta, e, \eta') \right],$$

$$c + a' = y + a(1 + r) - T(y, a, c),$$

$$y = w_e E_{e,j} (\theta, \eta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h \leq 1, \quad \eta' \sim \Gamma_{e,j}(\eta).$$

**College** ($e = 1$)

$$V_j^s (a, \theta, e) = \max_{c, a', h} u(c, h + \bar{h}) + \beta \mathbb{E}_{\eta|e} V_{j+1}^w (a', \theta, e, \eta)$$

$$c + a' + p^s = y + a(1 + r) - T(y, a, c)$$

$$y = w_0 E_{e,j} (\theta) h, \quad a' \geq a_{e,j}, \quad 0 \leq h \leq 1 - \bar{h}$$

**Work or college:**

$$V_j^{sw} (a, \theta, \phi) = \max \{ \mathbb{E}_{\eta|e=0} V_j^w (s, \theta, 0, \eta), V_j^s (s, \theta, 1, \varepsilon) - \kappa(\varepsilon, \theta) \}$$
Social Security: Received every period, relative to education $e$ and permanent skill $\theta$.

\[
V_j(a, \theta, e) = \max_{c, a'} u(c, 0) + \beta V_{j+1}^w(a', \theta, e),
\]

\[
c + a' = \pi(\theta, e) + a (1 + r) - T(0, a, c),
\]

\[
a' \geq 0
\]
Stationary Equilibrium

- **Distributions:**
  - Cross-sectional distribution of any cohort of age $j$ is invariant over time periods.
  - Distribution of initial states is determined by older generations.

- **Household optimize:** Household make choices of education, consumption, labor, parental time and expenditures, transfers such that maximize utility.

- **Firms maximize profits.**

- **Prices clear markets.**
Outline

Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
1. Standard parameters from literature.
   - e.g., discounting; intertemporal elasticity of substitution; Frisch elasticity...

2. Externally calibrated.
   - e.g., income process; borrowing limits...

3. Simulated Method of Moments.
   - Key moments to match novel elements of model (e.g., parental investments).
   - Estimated to match household level data.
Utility function is:

\[ u(c, h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} - \mu \frac{h^{1+\gamma_h}}{1+\gamma_h} \]
Utility function is:

\[ u(c, h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} - \mu \frac{h^{1+\gamma_h}}{1+\gamma_h} \]

Disutility of investing time \( t \) on children’s skills:

\[ v(t) = \xi t \]

- From literature: \( \gamma_c = 2, \gamma_h = 3 \).
- To estimate: \( \mu \) and \( \xi \).
### Parental investments

#### Sample Means

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Parents Together</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weekly Hours</strong></td>
<td>18.0</td>
<td>20.6</td>
</tr>
<tr>
<td>(0.3071)</td>
<td>(0.6721)</td>
<td></td>
</tr>
<tr>
<td><strong>Yearly Expenditures</strong></td>
<td>1,966</td>
<td>1,553</td>
</tr>
<tr>
<td>(35.53)</td>
<td>(57.31)</td>
<td></td>
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#### Regression Coefficients

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<td><strong>Hours on College</strong></td>
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<td><strong>Log(Hours) on Log(Income)</strong></td>
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<td>(0.0234)</td>
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<tr>
<td><strong>Expenditures on College</strong></td>
<td>732.4***</td>
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<td>(67.80)</td>
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<tr>
<td><strong>Log(Expenditures) on Log(Income)</strong></td>
<td>0.391***</td>
<td>0.634***</td>
</tr>
<tr>
<td>(0.0285)</td>
<td>(0.0624)</td>
<td></td>
</tr>
</tbody>
</table>

Expenditures: child-care expenditures in CEX.
Weekly Hours: based on time reading and playing in PSID-CDS.
Government Taxes

- **Tax function** has form: $T(y, a, c) = \tau_y y + \tau_k a r 1_{a \geq 0} + \tau_c c - \omega$.
- **Tax rates** from McDaniel (2014): $\tau_y = 0.22$, $\tau_c = 0.07$, and $\tau_k = 0.27$.
- Estimate lump-sum transfer $\omega$ such that ratio of the variances of disposable and pre-government log-income is 0.69 (PSID).
<table>
<thead>
<tr>
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<th>Non-Cognitive Skills</th>
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<tr>
<td></td>
<td>1st Stage</td>
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</tr>
<tr>
<td>Current Cognitive Skills</td>
<td>0.479</td>
<td>0.831</td>
</tr>
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Cunha, Heckman and Schennach (2010) — Only Cognitive

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|                                | Cognitive Skills | Non-Cognitive Skills |
|                                | 1st Stage        | 2nd Stage            | 1st Stage | 2nd Stage |
| Current Cognitive Skills       | 0.479            | 0.831                | 0.000     | 0.000     |
| Current Non-Cognitive Skills   | 0.070            | 0.001                | 0.585     | 0.816     |
| Investments                    | 0.161            | 0.044                | 0.065     | 0.051     |
| Parent’s Cognitive Skills      | 0.031            | 0.073                | 0.017     | 0.000     |
| Parent’s Non-Cognitive Skills  | 0.258            | 0.051                | 0.333     | 0.133     |
| Complementarity parameter      | 0.313            | -1.243               | -0.610    | -0.551    |
| Variance of Shocks             | 0.176            | 0.087                | 0.222     | 0.101     |
Outline

Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
Panel Study of Income Dynamics (PSID):
- Longitudinal household survey.
- Information on education, income, marriage, children, ... and expenditures on children: toys, vacations, school supplies, clothes, food and medical.
- Sampling: Core sample of approximately 5k families, in 1968. Over time it includes those born in these families.
Child Development Data: PSID + CDS

- **Panel Study of Income Dynamics (PSID):**
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  - Information on education, income, marriage, children,... and expenditures on children: toys, vacations, school supplies, clothes, food and medical.
  - Sampling: Core sample of approximately 5k families, in 1968. Over time it includes those born in these families.

- **Child Development Supplement (CDS):**
  - Multiple **Assessments of Child Skills:**
  - **Time Diary:** Detailed description of child’s activities (weekday and weekend). Information on active and passive participation of parents.
Active time with parents

- Using time diaries I calculate “active” time with parents.
- “Active:” parent is performing activity with kid.
  Assumption: If two parents are active, double the hours.
## Parental investments

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Expenditures: child-care expenditures in CEX.
Weekly Hours: based on time reading and playing in PSID-CDS.
Estimation: Labor income risk

**Labor income** of individual of age \( j \), education \( e \), and skills \( \theta \) is product of:

1. Wage of your education group: \( w_e \).
2. **Labor efficiency units**: \( E_{i,e,j} = \epsilon_{e,j} \psi_{i,e,j} \).
3. Hours worked: \( h \).

**Labor efficiency units** evolve stochastically as sum of three components:

\[
\log(E_{i,e,j}) = \log(\epsilon_{e,j}) + \lambda_e \log(\theta_{ic}) + \eta_{i,e,j}
\]

where

- \( \lambda_e \) is education-specific return to skills.
- \( \epsilon_{e,j} \) is education-specific age profile.
- \( \psi_{i,e,j} \) is stochastic component with persistent cdf \( \Gamma_{j,e} \).
## Estimation: Return to Skill

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<tr>
<th></th>
<th>(1) High School</th>
<th>(2) College</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>log(AFQT)</strong></td>
<td>0.471***</td>
<td>1.008***</td>
</tr>
<tr>
<td></td>
<td>(0.0335)</td>
<td>(0.0768)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>7,015</td>
<td>3,378</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.045</td>
<td>0.082</td>
</tr>
<tr>
<td><strong># of households</strong></td>
<td>988</td>
<td>487</td>
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</tbody>
</table>

*Source: NLSY. Robust standard errors in parentheses. *, **, *** denote statistical significance at the 10, 5, and 1 percent, respectively. log(AFQT) refers to the natural logarithm of the AFQT89 raw score. The regression includes year fixed effects. Methodology is explained in the main text.*

**Note:** The standard deviation of log-AFQT in the data is approximately 0.21.
## Age Profile

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) HS Grad</th>
<th>(2) College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.0312***</td>
<td>0.0557***</td>
</tr>
<tr>
<td></td>
<td>(0.00387)</td>
<td>(0.00577)</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>-0.000271***</td>
<td>-0.000530***</td>
</tr>
<tr>
<td></td>
<td>(4.65e-05)</td>
<td>(6.89e-05)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.084***</td>
<td>1.927***</td>
</tr>
<tr>
<td></td>
<td>(0.0779)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>Observations</td>
<td>9,130</td>
<td>6,015</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.051</td>
<td>0.093</td>
</tr>
<tr>
<td># of households</td>
<td>1357</td>
<td>864</td>
</tr>
</tbody>
</table>

Source: PSID.
\[ \eta_{i,e,j} = \rho_e \eta_{i,e,j-1} + Z_{i,e,j}, \quad Z_{i,e,j} \sim i.i.d. N(0, \sigma_{e,z}), \eta_0^e \sim N(0, \sigma_{\eta_0}^e) \]

<table>
<thead>
<tr>
<th></th>
<th>(1) High School</th>
<th>(2) College</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_e)</td>
<td>0.924</td>
<td>0.966</td>
</tr>
<tr>
<td>(\sigma_{e,z})</td>
<td>0.029</td>
<td>0.046</td>
</tr>
<tr>
<td>(\sigma_{e,\eta_0})</td>
<td>0.050</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Source: NLSY. A period is 4 years long. Methodology is explained in the main text.
Other elements of estimation

- Aggregate Production Function.
- Borrowing limits.
- Price of college.
- Retirement benefits.
Aggregate Production Function

- Cobb-Douglas Form with constant returns to scale:
  \[ Y = K^\alpha H^{1-\alpha} \]

  where \( H \) is the nested CES aggregator
  \[ H = \left[ sL_1^\Omega + (1 - s)L_2^\Omega \right]^{\frac{1}{\Omega}} \]

- Set \( \alpha = 1/3 \).
- Estimate using FOCs as in Katz and Murphy (1992) or Heckman et al (1998):
  - \( s = 0.53 \).
  - \( \frac{1}{1-\Omega} = 1.75 \).
Borrowing limits

Individuals can (unsecured) borrow **during working years**: 

- Interest rate $r^b = r + \iota$ where $r$ is the returns to saving and $\iota$ is the wedge between borrowing and lending capital.

- Borrowing limits estimated from self-reported limits by education in SCF: $20k$ and $34k$ for HS graduates and college graduates.
Borrowing limits

Individuals can (unsecured) borrow during working years:

- Interest rate $r^b = r + \iota$ where $r$ is the returns to saving and $\iota$ is the wedge between borrowing and lending capital.
- Borrowing limits estimated from self-reported limits by education in SCF: $20k$ and $34k$ for HS graduates and college graduates.

Borrowing is allowed for college at subsidized interest rate $r^c$:

- Pay interest rate $r^c = r + \iota^c$ where $\iota^c$ was estimated to be 1% annually in federal student loans (Mix of no interest rate loans and 2.6% loans). Note $\iota^c < \iota$.
- Borrowing limit estimated to be $23k$. 
College:

- Based on Delta Cost Project, yearly cost of college $\approx 6,588.
- This only considers tuition costs paid by individuals, i.e. it removes grants and scholarships.
• Replacement benefits are based on current US Social Security (OASDI).

• Use education and FE in model to estimate average lifetime income, on which the system is based.
Replacement rate

- $h$ is the last level of human capital before retirement. The average lifetime income is summarized by $\hat{y}(h,e)$.

- Progressive formula based on SSA

\[
\pi(h) = \begin{cases} 
0.9\hat{y}(h,e) & \text{if } \hat{y}(h,e) \leq 0.3\bar{y} \\
0.9(0.3\bar{y}) + 0.32(\hat{y}(h,e) - 0.3\bar{y}) & \text{if } 0.3\bar{y} \leq \hat{y}(h,e) \leq 2\bar{y} \\
0.9(0.3\bar{y}) + 0.32(2 - 0.3)\bar{y} + 0.15(\hat{y}(h,e) - 2\bar{y}) & \text{if } 2\bar{y} \leq \hat{y}(h,e) \leq 4.1\bar{y} \\
0.9(0.3\bar{y}) + 0.32(2 - 0.3)\bar{y} + 0.15(4.1 - 2)\bar{y} & \text{if } 4.1\bar{y} \leq \hat{y}(h,e)
\end{cases}
\]

where $\hat{y}(h,e) = [0.98\ 1.17\ 0.98] \times h$ and $\bar{y}$ is approximately $70,000$. 

Back to model
Back to calibration
Model Time Line
Model Inputs
## Estimation: Age

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jb</td>
<td>16</td>
<td>Independent - start with 12 years of education</td>
</tr>
<tr>
<td>Je</td>
<td>20</td>
<td>Max educ - average years of schooling 13.42</td>
</tr>
<tr>
<td>Jc</td>
<td>28</td>
<td>Fertility</td>
</tr>
<tr>
<td>Jk</td>
<td>36</td>
<td>Transfer to children</td>
</tr>
<tr>
<td>Jt</td>
<td>40</td>
<td>Transfers to parents</td>
</tr>
<tr>
<td>Jr</td>
<td>68</td>
<td>Retire</td>
</tr>
<tr>
<td>Jd</td>
<td>80</td>
<td>Death</td>
</tr>
</tbody>
</table>
Outline

Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
Estimation: 2-Steps Methodology

**Step 1: Target moments**
- Estimate target moments using whole sample
- Using bootstrap, obtain moments $M_n$ for $n = 1, \ldots, N$

**Step 2: Global estimation**
- Draw parameters from “large” uniform iid hypercube (sobol sequence)
- Trade-offs:
  - Obtain combination of parameters that best fits whole-sample moments
  - For moments $M_n$ ($n = 1, \ldots, N$), obtain an estimated parameters $P_n$
  - Calculate standard deviations or confidence intervals of $P_n$
  - But very costly to do if number of parameters is large
Preferences

Transfers to children

[Graph showing relationship between altruism (δ) and transfers to children with data points at δ = 0.46, 0.47, 0.48, 0.49, 0.68, 0.7, 0.72, 0.74, 0.76, 0.78, 0.8, 0.82.]

Altruism (δ)
Preferences

Transfers to children

Altruism ($\delta$)

0.46 0.47 0.48 0.49
0.68
0.7
0.72
0.74
0.76
0.78
0.8
0.82

Hours worked

Disutility of work ($\mu$)

0.58 0.6 0.62 0.64 0.66
63.5
64
64.5
65
65.5
66
66.5

Hours with child

Disutility of time w/child ($\xi$)

1 2 3
10
15
20
25
30

Back to Methodology  Back to Parameters
School Taste

Share of college grads (%)

Mean school taste ($\alpha$)

College: cog skills slope

School taste-cog skill relation ($\alpha_c$)

College: noncog skills slope

School taste-noncog skill relation ($\alpha_{nc}$)

College: residual variance

SD of taste shock ($\sigma_\phi$)
Skill Formation Productivity

High-Low skilled ratio

Ratio money-time

Prod. of Investments ($\bar{A}$)

Money multiplier ($\alpha_m$)

Money-time correlation

IGE persistence of education

Money-time substitutability ($\gamma$)

Mean school taste shock ($\bar{\varepsilon}$)
Tax Progressivity

Redistribution of income

Lump-sum transfer ($\omega$)
Financial Services

Share of borrowers

Borrowing-saving wedge ($i$)
Outline

Early Childhood Programs

Model: More Details

Estimation: More Details

Data

Moment’s Information

Additional Results
**Validation: NotTargeted Moments**

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regression of parental investments to parents’ characteristics</strong> (PSID-CDS and CEX)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Families</td>
<td>Homogeneous Families</td>
<td></td>
</tr>
<tr>
<td>Hours on college ed. parent</td>
<td>3.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Expenditures on college ed. parent</td>
<td>732</td>
<td>666</td>
</tr>
<tr>
<td>Log hours on log parent income</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>Log expenditures on log parent income</td>
<td>0.39</td>
<td>0.63</td>
</tr>
<tr>
<td><strong>Intergenerational Mobility</strong> (Chetty et al, 2016 and PSID-CDS)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rank-Rank coefficient</td>
<td>0.26–0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>Regression of college to log-parent income</td>
<td>0.24</td>
<td>0.18</td>
</tr>
<tr>
<td><strong>Inequality</strong> (PSID)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>Top-Bottom</td>
<td>3.7</td>
<td>3.1</td>
</tr>
<tr>
<td><strong>Savings</strong> (Inklaar and Timmer, 2013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital-Output Ratio (annualized)</td>
<td>≈ 3</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Return to College</strong> (PSID and Heckman et al, 2006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income Ratio: College – HS Graduate</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Yearly return</td>
<td>≈ 10%</td>
<td>12%</td>
</tr>
</tbody>
</table>
Consumption equivalence under veil of ignorance

Let utility under policy $P$ with extra % consumption $\lambda$ be:

$$
\tilde{V}_{ji}^P (a, \theta, \phi, \lambda) = \mathbb{E}^P \left\{ \sum_{j=J_i}^{j=J_d} \beta^{(j-J_i)} u(c_j^P (1 + \lambda), h_j^P) + \beta^{J_c} b \tilde{V}_{ji}^P (\varphi, \theta_k, \phi, \lambda) \right\}
$$

So average utility is:

$$
\bar{V}^P (\lambda) = \int_{a, \theta, \phi} \tilde{V}_{ji}^P (a, \theta, \phi, \lambda) \mu_P (a, \theta, \phi)
$$

Then, welfare gain from going from policy $P = 0$ to $P = p$ is given by $\lambda^p$ where:

$$
\bar{V}^0 (\lambda^p) = \bar{V}^p (0)
$$

By definition, welfare gains come from 2 sources

- **Changes in values** of becoming independent in each state, i.e., $\tilde{V}_{ji}^P (a, \theta, \phi, 0)$
- **Changes in probabilities** of each state, i.e., $\mu_P (a, \theta, \phi)$
By definition, welfare gains come from 2 sources

- Changes in values of becoming independent in each state, i.e., \( V(a, \theta, \varphi) \)
- Changes in probabilities of each state, i.e., \( \mu(a, \theta, \varphi) \)

Most welfare gains are driven by change in distribution \( \mu \)

- Fixing \( \mu \): Gains are 2.5%
- Fixing \( V \): Gains are 7.3%
Transition Dynamics

- Prices (%)
- Var Log-Lifetime-Earnings (%)
- Cons. Equiv. (%)
- Investment ($1,000)
- IGE Mobility (%)
- Share Change (%)
- Labor Tax (%)
- Lump-Sum Tax ($1,000)
- Gov. Deficit (%)
Early Childhood Investments

Graphs showing the relationship between budget per child ($1,000) and various metrics:
- Tax Rate (%)
- Consumption Equivalents (%)
- Variance of Log-Lifetime Earnings (%)
- IGE Mobility (%)
- Income: Mean (%)
- Labor Productivity (%)

Legends include:
- Ages 0-3: 100%
- Ages 0-3: 75%
- Ages 0-3: 50%
- Ages 0-3: 0%
Transition: Only Intervened Pay

Cohort

Cons. Equiv. (%)
Transition: Only Intervened Pay
Transition: Only Intervened Pay + Slow Intro

![Graph showing the changes in Cons. Equiv. (%) across Cohort from 0 to 40. The graph displays a steady increase in Cons. Equiv. (%) as Cohort increases.]
Transition: Only Intervened Pay + Slow Intro

Parent Types (Skills-Education)

Cons. Equiv. (%)
With Early Childhood Production Function
Assume early childhood good’s only input is college labor

- Price of early childhood is now wage of college graduate
Assume early childhood good’s only input is college labor

- Price of early childhood is now wage of college graduate

Short-run vs Long-run

1. **Short run**: scarcity of college graduates increases costs
2. **Long run**: increased supply of college reduces costs
With Early Childhood Production Function

![Graphs showing the impact of Early Childhood Development (ECD) on various economic metrics over different cohorts. The graphs illustrate changes in consumption equivalents, ECD/College attainment, wage gap, and variance in lifetime earnings when compared to baseline scenarios.](image-url)
Parenting Education
Endogenous parental investments allows for new policy:

- **Parenting Education**: teach techniques and games to solve discipline problems, foster confidence and capability,...

- **Estimated cost of program**: $11,400 per family
Recall production function is:

\[
\theta'_k = \left[ \alpha_{1j} \theta^p_{k} + \alpha_{2j} \theta^p_{j} + \alpha_{3j} I^p_{j} \right]^{1/\rho_j} \exp(\nu)
\]

Next period child's skills

\( \theta'_k \)

Current child's skills

\( \theta^p_{k} \)

Parent's skills

\( \theta^p_{j} \)

Parental investments

\( I^p_{j} \)
Recall production function is:

$$\theta_k' = \left[ \alpha_{1j} \theta_k^\rho_j + \alpha_{2j} \theta_j^\rho_j + \alpha_{3j} l_j^\rho_j \right]^{1/\rho_j} \exp(v)$$

Next period child’s skills

Current child’s skills

Parent’s skills

Parental investments

With parenting education:

$$\theta_k' = \left[ \alpha_{1j} \theta_k^\rho_j + \alpha_{2j} \max\{\theta, \theta_{PE}\}^\rho_j + \alpha_{3j} l_j^\rho_j \right]^{1/\rho_j} \exp(v)$$

Next period child’s skills

Current child’s skills

Program provides basic skills $\theta_{PE}$

Parental investments
Gertler et al (2013) study effect of parenting education in Jamaica

- **RCT** on growth-stunted and poor children, ages 0–2, in 1986
- Children around age 22 \(\Rightarrow\) income grew by 12% (at least)
Gertler et al (2013) study effect of parenting education in Jamaica

- **RCT** on growth-stunted and poor children, **ages 0–2**, in 1986
- Children **around age 22** ⇒ **income grew by 12%** (at least)

**Mimic RCT in model**

- **Small scale** and **one-time policy**
- Focus on children with low initial draws of skills
  And of low-income, low-skilled, low-educated parents

**Look for increase in productivity that increases income by 12%**
Benchmarking productivity of parenting education

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**Look for increase in productivity that increases income by 12%**

<table>
<thead>
<tr>
<th>$\theta_{PE}$</th>
<th>Change from Baseline (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev. of $\theta$</td>
<td>Income Bottom</td>
</tr>
<tr>
<td>-1.6 SD</td>
<td>0.00</td>
</tr>
<tr>
<td>-1.0 SD</td>
<td>2.13</td>
</tr>
<tr>
<td>-0.4 SD</td>
<td>5.22</td>
</tr>
<tr>
<td>0.0 SD</td>
<td>7.22</td>
</tr>
<tr>
<td>+0.4 SD</td>
<td>9.48</td>
</tr>
<tr>
<td>+0.8 SD</td>
<td>11.48</td>
</tr>
<tr>
<td>+1.0 SD</td>
<td>12.31</td>
</tr>
<tr>
<td>+1.2 SD</td>
<td>13.10</td>
</tr>
</tbody>
</table>
Benchmarking productivity of parenting education

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- **RCT** on growth-stunted and poor children, ages 0–2, in 1986
- Children **around age 22** ⇒ income grew by **12%** (at least)

Mimic RCT in model

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  And of low-income, low-skilled, low-educated parents

Look for increase in productivity that increases income by **12%**

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<thead>
<tr>
<th>$\theta_{PE}$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev. of $\theta$</td>
<td>Income Bottom</td>
</tr>
<tr>
<td>-2.6 SD</td>
<td>0.00</td>
</tr>
<tr>
<td>-2.0 SD</td>
<td>2.13</td>
</tr>
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<td>5.22</td>
</tr>
<tr>
<td>-1.0 SD</td>
<td>7.22</td>
</tr>
<tr>
<td>-0.6 SD</td>
<td>9.48</td>
</tr>
<tr>
<td>-0.2 SD</td>
<td>11.48</td>
</tr>
<tr>
<td>Benchmark = 0</td>
<td>12.31</td>
</tr>
<tr>
<td>+0.2 SD</td>
<td>13.10</td>
</tr>
<tr>
<td>$\theta_{PE}$ relative to benchmark</td>
<td>Cons. Equiv.</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>-1.4 SD</td>
<td>2.87</td>
</tr>
<tr>
<td>-1.0 SD</td>
<td>3.79</td>
</tr>
<tr>
<td>-0.6 SD</td>
<td>5.48</td>
</tr>
<tr>
<td>-0.2 SD</td>
<td>6.95</td>
</tr>
<tr>
<td>Benchmark</td>
<td>7.65</td>
</tr>
<tr>
<td>0.2 SD</td>
<td>8.19</td>
</tr>
</tbody>
</table>

Even if parenting education is 1.4 standard deviation less effective, it still has positive welfare effects in the long run. Large effects on intergenerational mobility and inequality.

Partial Equilibrium Back
### Parenting Education: Long Run, GE

<table>
<thead>
<tr>
<th>$\theta_{PE}$ relative to benchmark</th>
<th>Cons. Equiv.</th>
<th>Avg. Income</th>
<th>Inequality</th>
<th>Mobility</th>
<th>College</th>
<th>Tax Revenue</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.4 SD</td>
<td>2.87</td>
<td>2.29</td>
<td>-3.12</td>
<td>9.29</td>
<td>2.61</td>
<td>2.60</td>
<td>-0.28</td>
</tr>
<tr>
<td>-1.0 SD</td>
<td>3.79</td>
<td>2.85</td>
<td>-4.29</td>
<td>11.03</td>
<td>3.32</td>
<td>2.93</td>
<td>-0.44</td>
</tr>
<tr>
<td>-0.6 SD</td>
<td>5.48</td>
<td>4.36</td>
<td>-4.79</td>
<td>13.85</td>
<td>5.00</td>
<td>3.39</td>
<td>-0.76</td>
</tr>
<tr>
<td>-0.2 SD</td>
<td>6.95</td>
<td>5.39</td>
<td>-4.98</td>
<td>15.32</td>
<td>6.30</td>
<td>3.64</td>
<td>-1.05</td>
</tr>
<tr>
<td>Benchmark</td>
<td>7.65</td>
<td>5.68</td>
<td>-5.14</td>
<td>15.47</td>
<td>6.40</td>
<td>3.95</td>
<td>-1.16</td>
</tr>
<tr>
<td>0.2 SD</td>
<td>8.19</td>
<td>6.05</td>
<td>-5.35</td>
<td>16.70</td>
<td>6.87</td>
<td>4.06</td>
<td>-1.26</td>
</tr>
</tbody>
</table>

- Even if parenting education is **1.4 standard deviation less effective** it still has positive welfare effect in the long run.
### Parenting Education: Long Run, GE

<table>
<thead>
<tr>
<th>$\theta_{PE}$ relative to benchmark</th>
<th>Cons. Equiv.</th>
<th>Avg. Income</th>
<th>Change from Baseline (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inequality</td>
</tr>
<tr>
<td>-1.4 SD</td>
<td>2.87</td>
<td>2.29</td>
<td>-3.12</td>
</tr>
<tr>
<td>-1.0 SD</td>
<td>3.79</td>
<td>2.85</td>
<td>-4.29</td>
</tr>
<tr>
<td>-0.6 SD</td>
<td>5.48</td>
<td>4.36</td>
<td>-4.79</td>
</tr>
<tr>
<td>-0.2 SD</td>
<td>6.95</td>
<td>5.39</td>
<td>-4.98</td>
</tr>
<tr>
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<td>-5.14</td>
</tr>
<tr>
<td>0.2 SD</td>
<td>8.19</td>
<td>6.05</td>
<td>-5.35</td>
</tr>
</tbody>
</table>

- Even if parenting education is 1.4 standard deviation less effective, it still has positive welfare effect in the long run.
- Large effect on **Intergeneration mobility** and **inequality**
Now program can be purchased by families

<table>
<thead>
<tr>
<th>Change from Baseline (%)</th>
<th>Cons. Equiv.</th>
<th>Avg. Income</th>
<th>Inequality</th>
<th>Mobility</th>
<th>College</th>
<th>Tax Revenue</th>
<th>Tax Rate</th>
<th>Take-Up Low</th>
<th>Take-Up Medium</th>
<th>Take-Up High</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.4 SD</td>
<td>1.61</td>
<td>1.66</td>
<td>-2.08</td>
<td>5.63</td>
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<td>-5.29</td>
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- Market provided program provides slightly smaller gains.
Cost of parenting education program is hard to estimate
Cost of parenting education program is hard to estimate

- Estimate from Colombia (Attanasio et al, 2016) ⇒ US$450-750 per child.
- Program employed mostly women with high-school degree education. Assuming requires college graduate in US, would suggest costs per child of $3,400-5,700 in the US.
- **Choose upper bound**: $2 \times $5,700 per family (2 children).
## Parenting Education: Short Run, PE

<table>
<thead>
<tr>
<th>$\theta_{PE}$ relative to benchmark</th>
<th>Cons. Equiv.</th>
<th>Avg. Income</th>
<th>Inequality</th>
<th>Mobility</th>
<th>College</th>
<th>Tax Revenue</th>
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Robustness and Parameters Importance
**Results Robustness: Estimated Parameters Importance**

Move each parameter one std. dev. above and below

- Calculate steady-state and introduce same policy as before

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Cons. Equiv. Change from Baseline</th>
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<td>$\iota$</td>
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<tr>
<td>$\omega$</td>
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Baseline 3.9 9.4
Move each parameter one std. dev. above and below

- Re-estimate, obtain steady-state, and introduce same policy as before

<table>
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<th></th>
<th>Change from Baseline</th>
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<td>Cons. Equiv. LR-GE</td>
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<td>0.71</td>
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