Non-linear household earnings dynamics, self-insurance and welfare

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Motivation

- Nature of income inequality/risk: critical for many questions in economics. E.g.:
  - Consumption and wealth distribution
  - Ability to self-insure/welfare
  - Scope for social insurance and redistribution

- Better datasets and new methods are challenging long held views about labour income risk

- What are the implications for:
  - Consumption and wealth inequality
  - Self-insurance
  - Welfare
“Canonical” model of earnings dynamics

- Detrended labor earnings follow a (log-) linear process. E.g.

  \[
  \log Y_{it} = f(t) + \delta_i + \eta_{it} + \varepsilon_{it}
  \]

  \[
  \eta_{it} = \rho \eta_{i,t-1} + \nu_{it}
  \]

  with \( \delta_i, \eta_{i1}, \nu_{it}, \varepsilon_{it} \) normally distributed.

- Three main features:
  - Age-independence of conditional 2nd and higher moments
  - Normality: Shocks are symmetrically distributed + no fat tails
  - Linearity: conditional 2nd and higher moments independent of \( \eta_{i,t-1} \)
Individual, pre-tax earnings do not fit the canonical model

W2 Social Security Data (Guvenen et al. 2016):

Similarly in the PSID:
...nor do HH, disposable earnings

Rich features of individual, pre-tax earnings

are also present in disposable, HH earnings
This paper

- Estimate a flexible process à la Arellano, Blundell and Bonhomme (2017) for household post-tax labor earnings using the PSID.
- Use a structural life-cycle model to compare the implications of the flexible process against canonical permanent + transitory process.
Findings

- Allowing for a flexible earnings dynamics:
  - Significantly improves the fit of the growth of consumption dispersion over age
  - Implies a pass-through of “permanent” earnings shocks in line with the estimates in Blundell, Pistaferri and Preston (2008)
  - ... but does not improve the fit of the wealth distribution

- Lower welfare gains from removing earnings risk
Two strands of literature

- **Quantitative models of consumption and wealth inequality**
  (Huggett, 1996; De Nardi, 2004; Storesletten, Telmer and Yaron, 2006; ...)

- **Richer specifications of earnings dynamics**
  (Geweke and Keane, 2000; Meghir and Pistaferri, 2004; Browning, Ejrnaes and Alvarez, 2010; Altonji, Smith and Vidangos, 2013; Blundell, Graber and Mogstad, 2015; Arellano, Blundell and Bonhomme, 2017; Guvenen, Karahan, Ozkan and Song, 2016...)
Data

- PSID core sample, 1968-1992, joint earnings for all HH (25-60)
- Disposable earnings obtained by regression as in Guvenen and Smith (2014)
- Equivalization by regression on number of family members
- Residual disposable earnings net of time and age fixed-effects
A flexible but parsimonious model
Arellano, Blundell and Bonhomme (2017)

Let $Q_z(q|\cdot)$ denote the conditional quantile function for $z$

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad t = 1, \ldots, T$$
$$\eta_{it} = Q_\eta(v_{it}|\eta_{i,t-1}, t)$$
$$\varepsilon_{it} = Q_\varepsilon(u_{it}|t)$$
$$\eta_{i1} = Q_{\eta_1}(v_{i1})$$
$$u_{it}, v_{i1}, (v_{it}|\eta_{i,t-1}, \eta_{i,t-2}, \ldots) \sim U(0, 1)$$

Persistence

$$\rho_t(q, \eta_{i,t-1}) = \frac{\partial Q_\eta(q|\eta_{i,t-1}, t)}{\partial \eta_{i,t-1}}, \quad \rho_t(q) = \mathbb{E}_\eta \left[ \frac{\partial Q_\eta(q|\eta_{i,t-1}, t)}{\partial \eta_{i,t-1}} \right]$$
The flexible model: summary
Extra features

The flexible model considered allows for

- Age dependence of conditional 2nd and higher moments
- Non-normality of shocks
- Non-linearity in $\eta_{i,t-1}$ and its innovation
A flexible but parsimonious model

Parameterization

Let $\psi^k$, $k = 0, 1, \ldots$ denote a family of bivariate, polynomial fns.

$$Q_\eta(q \mid \eta_{i,t-1}, \text{age}_{it}) = \sum_{k=0}^{K} \alpha^\eta_k(q)\psi^k(\eta_{i,t-1}, \text{age}_{it})$$

$$Q_\varepsilon(q \mid \text{age}_{it}) = \sum_{k=0}^{K} \alpha^\varepsilon_k(q)\psi^k(\text{age}_{it})$$

$$Q_{\eta_1}(q \mid \text{age}_{i1}) = \sum_{k=0}^{K} \alpha^{\eta_1}_k(q)\psi^k(\text{age}_{i1})$$
Canonical benchmark

- Estimated by fitting variances and autocovariances of earnings over the life-cycle

\[ y_{it} = \eta_{it} + \varepsilon_{it} \]
\[ \eta_{it} = \rho \eta_{i,t-1} + \nu_{it} \]

<table>
<thead>
<tr>
<th>( \sigma^2_{\varepsilon} )</th>
<th>( \sigma^2_{\eta_1} )</th>
<th>( \sigma^2_{\nu} )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>0.0675</td>
<td>0.2363</td>
<td>0.0059</td>
</tr>
</tbody>
</table>
Features of NL vs canonical earnings processes

Second moments

**Persistent**

**Transitory**

<table>
<thead>
<tr>
<th>Age</th>
<th>Standard deviation NL process</th>
<th>Standard deviation Canonical</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>40</td>
<td>0.1</td>
<td>0.1</td>
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<tr>
<td>50</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.3</td>
<td>0.3</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Age</th>
<th>Persistence NL process</th>
<th>Persistence Canonical</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>40</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>50</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>60</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Features of NL vs canonical earnings processes

Skewness and Kurtosis

Skewness of transitory shock

Kurtosis of transitory shock

Skewness of persistent shock

Kurtosis of persistent shock

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**NL process** - - **Canonical**
Features of NL vs canonical earnings processes

Nonlinearity
So, these earnings dynamics are much richer. Does it matter for:

- Evolution of consumption inequality over the life cycle?
- Saving behavior and wealth inequality?
- Households ability to self-insure and welfare

Use these earnings process in a quantitative life-cycle model

Decompose the contribution of different features of the earnings process
Model implications
OLG model, key features

- Ex-ante identical agents. Idiosyncratic earnings shocks
- Working life age 25-60, then retirement until death
- Age-dependent probability of dying. Die for sure at age 86
- Infinitely-lived government, old age Social Security
- Single risk-free asset
Preferences and technology

- **Period utility**
  \[ u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}. \]

- **Discount factor** \( \beta \)

- **Agents supply labor inelastically**

- **Earnings follow, alternatively, the two empirical processes described**
Markets and government

- Incomplete assets markets: Agents can invest in a risk-free asset and can only borrow up to exogenous limit $a$.
- No annuity markets $\Rightarrow$ Flow of accidental bequests lost to the economy.
- Government
  - Provides old-age pensions
  - $r$ net of tax
  - Earnings process estimated on disposable income.
\[ V(t, z, \eta) = \max_{c, a'} \left\{ u(c) + \beta s_t E_t V(t + 1, z', \eta') \right\} \]

s.t. \( a' = z - c, \quad a' \geq a \)

\[ z = (1 + r)a + \eta + \epsilon \]
Retirees

\[ W(t, z, p) = \max_{c,a'} \left\{ u(c) + \beta s_t E_t W(t + 1, z', p) \right\} \]

s.t. \( a' = z - c, \quad a' \geq a \)

\[ z = (1 + r)a + p \]

- No utility from bequests \( W(T, z, p) = 0 \)
CRRA coefficient $\sigma = 2$

Risk-free rate $r = 0.04$

Survival probabilities are taken from Bell, Wade and Goss (1992)

$\beta$ calibrated to match $W/Y = 3.1$ in each of the economies
- Benchmark: $\beta = 0.957$
- NL process: $\beta = 0.939$

$a = 0.12$ (SCF average credit card limit)

Pension benefit: non-linear function of last period gross earnings (Kaplan and Violante 2010)

Discretization of NL earnings process by simulation
Discretization of NL earnings process

Grid points

![Graph showing the discretization of NL earnings process with grid points and percentile values.](image)
Discretization of NL earnings process

Grid points

![Graph showing the discretization of NL earnings process. The graph plots the residual log earnings against the percentile. The x-axis represents the percentile, ranging from 0 to 100, and the y-axis represents the residual log earnings, ranging from -1.5 to 1.5. The grid points are marked at intervals of 10.]
Discretization of NL earnings process

Grid points
Discretization of NL earnings process

Transition matrices

The elements $\pi^t_{mn}$ of the transition matrix $\Pi^t$ between age $t$ and $t+1$ are the proportion of individuals in bin $m$ at age $t$ that are in bin $n$ at age $t+1$.

\[
\begin{pmatrix}
\bar{Z}_t^1 \\ \bar{Z}_t^2 \\ \vdots \\ \bar{Z}_t^N \\
Z_t^1 \\
Z_t^2 \\
\vdots \\
Z_t^N
\end{pmatrix} \begin{pmatrix}
\pi^t_{11} & \pi^t_{12} & \cdots & \pi^t_{1N} \\
\pi^t_{21} & \pi^t_{22} & \cdots & \pi^t_{2N} \\
\vdots & \vdots & \cdots & \vdots \\
\pi^t_{N1} & \pi^t_{N2} & \cdots & \pi^t_{NN}
\end{pmatrix}
\]
Consumption implications
Benchmark generates too large increase by age

NL process generates substantially lower growth and captures (until age 47) non-monotonicity

Very hard to match without HIP (Guvenen 2007; Huggett, Ventura and Yaron 2011)
Opening the black box

Age-dependent second moments

[Graph showing age-dependent second moments with lines for CEX Data, Canonical, Normal, age-dependent, Non-normal, age-dependent, and NL process.]

[Graph showing standard deviation and rho values over age, with lines for Persistent shock, Transitory shock, and Persistence.]
Opening the black box
Age-dependent moments + non-normality

[Graphs showing age-dependent moments and non-normality for consumption and wealth variability.]
Opening the black box
Full NL

\[ \phi^x = 1 - \frac{\text{cov}(\Delta c_{it}, z_{it}^x)}{\text{var}(z_{it}^x)} \]

with \( z_{it}^x \) shock to \( x_{it} \).

Model true coefficients: earnings shocks are observed

\[ \phi^\eta = 1 - \frac{\text{cov}(\Delta c_{it}, y_{i,t+1} - y_{i,t-2})}{\text{cov}(\Delta y_{it}, y_{i,t+1} - y_{i,t-2})}, \quad \phi^\epsilon = 1 - \frac{\text{cov}(\Delta c_{it}, \Delta y_{i,t+1})}{\text{cov}(\Delta y_{i,t}, \Delta y_{i,t+1})} \]
BPP insurance coefficients

<table>
<thead>
<tr>
<th>Process/Coefficients</th>
<th>$\psi_{BPP}^p$</th>
<th>$\psi_{BPP}^{tr}$</th>
<th>$\psi^p$</th>
<th>$\psi^{tr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canonical (S.E. in parenthesis)</td>
<td>0.36</td>
<td>0.95</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Data: BPP (2008)</td>
<td>(0.09)</td>
<td>(0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canonical</td>
<td>0.13</td>
<td>0.89</td>
<td>0.31</td>
<td>0.92</td>
</tr>
<tr>
<td>Nonlinear process</td>
<td>0.43</td>
<td>0.82</td>
<td>0.46</td>
<td>0.91</td>
</tr>
<tr>
<td>Normal, age-dependent</td>
<td>0.42</td>
<td>0.83</td>
<td>0.46</td>
<td>0.88</td>
</tr>
<tr>
<td>Non-normal, age-dependent</td>
<td>0.42</td>
<td>0.83</td>
<td>0.46</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table: Insurance coefficients
$\phi^n$ by age

![Graph showing BPP coefficients by age]

- NL process
- Canonical
- Normal, age-dependent
## Wealth Inequality

<table>
<thead>
<tr>
<th>Wealth-income ratio</th>
<th>Wealth Gini</th>
<th>1%</th>
<th>5%</th>
<th>20%</th>
<th>40%</th>
<th>60%</th>
<th>80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data (SCF 1989)</td>
<td>3.1 .79</td>
<td>30</td>
<td>54</td>
<td>81</td>
<td>94</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>Benchmark</td>
<td>3.1 .64</td>
<td>9</td>
<td>29</td>
<td>65</td>
<td>88</td>
<td>97</td>
<td>100.1</td>
</tr>
<tr>
<td>NL Process</td>
<td>3.1 .61</td>
<td>7</td>
<td>25</td>
<td>61</td>
<td>85</td>
<td>96</td>
<td>99.9</td>
</tr>
</tbody>
</table>

- NL process does not help to improve the fit of the wealth distribution
- Not even with a process based on W2 administrative data (De Nardi, Fella, and Paz-Pardo, 2016)
Welfare costs of earnings risk

<table>
<thead>
<tr>
<th>Process</th>
<th>Welfare cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>28.3</td>
</tr>
<tr>
<td>NL process</td>
<td>26.2</td>
</tr>
</tbody>
</table>

- Lower persistence $\rightarrow$ lower relevance of initial realization
- With NL process, more equal distribution of lifetime income
- Easier to insure with precautionary savings
Conclusions

- Disposable, HH Earnings have much richer dynamics that traditionally assumed.
- In a life-cycle model these richer dynamics:
  - imply an age profile of consumption dispersion substantially closer to that in the data;
  - and a more realistic pass-through of persistent earnings shocks to consumption;
  - ... but do not improve the fit of the wealth distribution.
<table>
<thead>
<tr>
<th></th>
<th>$\sigma^2_{\epsilon}$</th>
<th>$\sigma^2_{\eta_1}$</th>
<th>$\sigma^2_{\nu}$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark (year effects)</td>
<td>0.0675</td>
<td>0.2363</td>
<td>0.0059</td>
<td>1</td>
</tr>
<tr>
<td>Cohort effects (Kaplan)</td>
<td>0.0655</td>
<td>0.2394</td>
<td>0.0057</td>
<td>1</td>
</tr>
<tr>
<td>STY04 (coh. effects)</td>
<td>0.063</td>
<td>0.2105</td>
<td>0.0166</td>
<td>0.9989</td>
</tr>
</tbody>
</table>