Consumption Inequality and Partial Insurance

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by Nicolò Russo
Question

*What is the link between income and consumption inequality?*

- The evolution of inequality can be explained by the degree of consumption insurance against income shocks
- Famous for consumption insurance, rather than inequality!
Mainstream approaches to consumption insurance

1. **Complete markets hypothesis**
   - Full insurance against idiosyncratic shocks
   - Rejected in the data $\rightarrow$ Attanasio and Davis (1996)

2. **Permanent income hypothesis**
   - Consumption reacts one-to-one to permanent shocks and is perfectly insured against transitory shocks
   - In the data:
     - Too little reaction to permanent shocks $\rightarrow$ Campbell and Deaton (1989)
     - Too much reaction against transitory shocks $\rightarrow$ Hall and Mishkin (1982)
This paper

• Studies **partial insurance** and estimates it

• Takes no a-priori stance on the insurance mechanisms

• Strategy:
  • Specifies income process
  • Uses covariance restrictions to identify insurance parameters

• Findings:
  1. Almost full insurance against transitory shocks
  2. Only partial insurance against permanent shocks
Insurance and inequality

• If there was **full insurance**:  
  • Consumption inequality would not react to income inequality

• If there was **no insurance**:  
  • Consumption inequality would perfectly track income inequality

• What happens in US data?
Income and consumption inequality

I. Characteristics of Consumption and Income Inequality

While there are large panel datasets that track the distribution of wages and incomes for households over time, the same is not true for broad measures of consumption. The PSID contains longitudinal income data, but the information on consumption is scanty (limited to food and a few more items). Indeed, one of the reasons why consumption inequality has not been studied as extensively as income and wage inequality is the nature of data availability. In this section we first document some basic features of the evolution of consumption and income inequality that motivate our study. Repeated cross-section data such as the CEX are not enough to uncover the degree of persistence in income shocks or to identify the partial insurance model. For that we need panel data, and in the second part of this section we describe our new panel data series.

A. The Evolution of Income and Consumption Inequality

There are two important features of the evolution of consumption and income inequality between the late 1970s and early 1990s which underpin our analysis. These are clearly evident from Figure 1, which uses PSID data on log income and CEX data on log consumption (see Section IB for details on sample selection and variable definitions). In this graph, we plot the actual estimates of the variances, as well as smoothing curves passing through the scatters (to ease legibility). In this figure the range of variation of the variance of PSID consumption is on the left-hand side; that of the variance of CEX consumption is on the right-hand side. The first distinct feature is that the slope of the income variance (the solid line) is greater than the slope of the consumption variance (the dashed line). The second feature of these inequality figures is that consumption inequality flattens out completely in the second part of the 1980s, whereas income inequality continues to rise, albeit at a much slower rate. Below we provide a framework for interpreting these changes. In particular, we show that the degree of detachment between consumption and income inequality depends on the persistence of income shocks and the availability of insurance to these shocks.

These overall patterns reflect what has also been found in previous analyses of inequality in income and consumption for this period, the most prominent study being that of Cutler and Katz (1992). See also the retrospective analysis in Johnson, Smeeding, and Boyle Torrey (2005).
Previous literature

- Blundell and Preston (1998)
  - Use growth in consumption inequality to identify growth in permanent income inequality
  - No panel data

- Krueger and Perri (2006)
  - Limited commitment explains the differences between consumption and income inequality
  - No distinction between permanent and transitory shocks

  - Study partial insurance in “simple” economies
Empirical observations

- Slope of income variance > Slope consumption variance
- Consumption inequality flattens out

- Consumption inequality should be monotonically increasing with age → Deaton and Paxson (1994)
- Broadly true in the sample
- Higher inequality for recent cohorts
What do these empirical observations tell us?

• Identified features of the evolution of inequality

• **But** how did these features come about?

• We do not know:
  1. Nature of changes in the income process
  2. Nature of insurance
A new panel dataset

- Need a panel with both income and consumption
- Not available for sample period!
- They combine PSID (panel) with CEX (cross-section)
- Sample selection:
  - Continuously married couples headed by a male age 30 to 65
  - No households with changes in head or spouse
Imputation procedure

• **Main idea:**
  • Use data from CEX to construct a measure of nondurable consumption for the PSID

• **Steps:**
  1. Start with food consumption → available in both datasets
  2. Estimate demand for food using CEX
  3. Invert demand to obtain nondurable consumption in the PSID
Imputation procedure

• Estimate demand for food in CEX:

\[ f_{i,t} = W'_{i,t} \mu + p_t \theta + \beta(D_{i,t})c_{i,t} + e_{i,t} \]  (1)

where:

• \( f := \) log of real food expenditure
• \( W := \) vector of demographic variables
• \( p := \) vector of relative prices
• \( c := \) log of nondurable expenditure
• \( e := \) unobserved heterogeneity and measurement error
• \( \beta(\cdot) := \) budget elasticity

• \( c \) is only in the CEX, all else is in both!

• Estimate and invert to get \( c \) in the PSID
Framework

• **Main object of interest:**
  - % response of consumption to a 1% change in income

• **Assumptions:**
  1. **Income**: net of taxes
  2. **Preferences**: separable between consumption and leisure
Income process

Real log-income:

\[ \log Y_{it} = Z_{it}' \varphi_t + P_{it} + \nu_{it} \]  

(2)

where:

- \( Z \) := observable known characteristics
- \( P \) := permanent component of income
- \( \nu \) := transitory component of income
Income components

- **Permanent component**: random walk

\[ P_{it} = P_{i,t-1} + \zeta_{it} \]  \hspace{1cm} (3)

where \( \zeta_{it} \) is serially uncorrelated

- **Transitory component**: MA(q)

\[ \nu_{it} = \sum_{j=0}^{q} \theta_j \epsilon_{i,t-j} \]  \hspace{1cm} (4)

where:
- \( \theta_0 = 1 \)
- \( q \) will be determined empirically

How flexible is this income process?
Unexplained income growth

- “Detrended” log-income:
  \[ y_{it} = \log Y_{it} - Z'_{it} \varphi_t \]

- Unexplained income growth:
  \[ \Delta y_{i,t} = \zeta_{i,t} + \Delta \nu_{i,t} \] (5)
Transmission of income shocks to consumption

- Unexplained change in log-consumption:

  \[ \Delta c_{it} = \phi_{it} \zeta_{it} + \psi_{it} \varepsilon_{it} + \xi_{it} \]  

- Partial insurance parameters:
  - \( \phi \): insurance against permanent shocks
  - \( \psi \): insurance against transitory shocks
Insurance benchmarks

- Full insurance
  \[ \phi_{it} = \psi_{it} = 0 \]

- No insurance
  \[ \phi_{it} = \psi_{it} = 1 \]

- Partial insurance
  \[ 0 < \phi_{it} < 1, \quad 0 < \psi_{it} < 1 \]
Models of partial insurance

1. PIH with self insurance through precautionary savings

2. Excess smoothness and “excess” insurance

3. Advance information
Identification of income process

- **WANT:** identification of $\phi$ and $\psi$

- Start from the income process

- **Assumptions**
  1. $\zeta$, $\nu$, $\varepsilon$ mutually uncorrelated
  2. $\nu$ is an MA(0) $\rightarrow \Delta y_{it} = \zeta_{it} + \Delta \varepsilon_{it}$

- Can show that:

  $\text{var}(\zeta_t) = \text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})$
  $\text{var}(\varepsilon_t) = -\text{cov}(\Delta y_t, \Delta y_{t+1})$
Identification of income process

- Can show that:

\[
\text{cov}(\Delta y_t, \Delta y_{t+s}) = \begin{cases} 
\text{var}(\zeta_t) + \text{var}(\Delta v_t) & \text{for } s = 0 \\
\text{cov}(\Delta v_t, \Delta v_{t+s}) & \text{for } s \neq 0 
\end{cases}
\]

- Use this to identify order of MA process for \( v \):
  - If \( v \) is an MA(\( q \)):
    \[
    \text{cov}(\Delta y_t, \Delta y_{t+s}) = 0 \quad \forall \quad |s| > q + 1
    \]
  - If \( v \) is serially uncorrelated (\( v_{it} = \varepsilon_{it} \)):
    \[
    \text{cov}(\Delta v_t, \Delta v_{t+s}) = -\sigma_\varepsilon^2, \quad \text{for } s = 1 \\
    \text{cov}(\Delta v_t, \Delta v_{t+s}) = 0, \quad \text{for } s \geq 2
    \]
Identification of insurance coefficients

• Can show that:

\[
\text{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} 
\phi_t \text{var}(\zeta_t) + \psi_t \text{var}(\varepsilon_t) & \text{for } s = 0 \\
\psi_t \text{cov}(\varepsilon_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 
\end{cases}
\]

• Can identify \(\phi\) and \(\psi\) with:

\[
\text{var}(\zeta_t) = \text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) \\
\text{var}(\varepsilon_t) = -\text{cov}(\Delta y_t, \Delta y_{t+1})
\]
Consumption growth inequality

• Recall that:

\[ \Delta c_{it} = \phi_{it} \zeta_{it} + \psi_{it} \varepsilon_{it} + \xi_{it} \]

• Can show that:

\[ \text{cov}(\Delta c_t, \Delta c_{t+s}) = \begin{cases} 
\phi_t^2 \text{var}(\zeta_t) + \psi_t^2 \text{var}(\varepsilon_t) + \text{var}(\xi_t) & \text{for } s = 0 \\
0 & \text{for } s \neq 0 
\end{cases} \]

• Consumption growth inequality \((s = 0)\) can increase because:

  1. Decline in insurance (increase in \(\phi\) and \(\psi\))
  2. Increase in the variance of income shocks
Autocovariances of income growth

Table 3—The Autocovariance Matrix of Income Growth

<table>
<thead>
<tr>
<th>Year</th>
<th>var(Δyt)</th>
<th>cov(Δyt+1, Δyt)</th>
<th>cov(Δyt+2, Δyt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.0832</td>
<td>−0.0196</td>
<td>−0.0018</td>
</tr>
<tr>
<td></td>
<td>(0.0089)</td>
<td>(0.0035)</td>
<td>(0.0032)</td>
</tr>
<tr>
<td>1981</td>
<td>0.0717</td>
<td>−0.0220</td>
<td>−0.0074</td>
</tr>
<tr>
<td></td>
<td>(0.0075)</td>
<td>(0.0034)</td>
<td>(0.0037)</td>
</tr>
<tr>
<td>1982</td>
<td>0.0718</td>
<td>−0.0226</td>
<td>−0.0081</td>
</tr>
<tr>
<td></td>
<td>(0.0051)</td>
<td>(0.0035)</td>
<td>(0.0026)</td>
</tr>
<tr>
<td>1983</td>
<td>0.0783</td>
<td>−0.0209</td>
<td>−0.0094</td>
</tr>
<tr>
<td></td>
<td>(0.0066)</td>
<td>(0.0034)</td>
<td>(0.0042)</td>
</tr>
<tr>
<td>1984</td>
<td>0.0805</td>
<td>−0.0288</td>
<td>−0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.0036)</td>
<td>(0.0032)</td>
</tr>
<tr>
<td>1985</td>
<td>0.1090</td>
<td>−0.0379</td>
<td>−0.0019</td>
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<tr>
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<td>(0.0180)</td>
<td>(0.0074)</td>
<td>(0.0038)</td>
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<td>0.1023</td>
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<td>−0.0115</td>
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<tr>
<td></td>
<td>(0.0077)</td>
<td>(0.0054)</td>
<td>(0.0038)</td>
</tr>
<tr>
<td>1987</td>
<td>0.1116</td>
<td>−0.0375</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.0097)</td>
<td>(0.0051)</td>
<td>(0.0046)</td>
</tr>
<tr>
<td>1988</td>
<td>0.0925</td>
<td>−0.0313</td>
<td>−0.0021</td>
</tr>
<tr>
<td></td>
<td>(0.0080)</td>
<td>(0.0042)</td>
<td>(0.0032)</td>
</tr>
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<td>1989</td>
<td>0.0883</td>
<td>−0.0280</td>
<td>−0.0035</td>
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<tr>
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<td>(0.0067)</td>
<td>(0.0059)</td>
<td>(0.0034)</td>
</tr>
<tr>
<td>1990</td>
<td>0.0924</td>
<td>−0.0296</td>
<td>−0.0067</td>
</tr>
<tr>
<td></td>
<td>(0.0095)</td>
<td>(0.0049)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>1991</td>
<td>0.0818</td>
<td>−0.0299</td>
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<tr>
<td></td>
<td>(0.0059)</td>
<td>(0.0040)</td>
<td>NA</td>
</tr>
<tr>
<td>1992</td>
<td>0.1177</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

- \( \text{var}(\Delta y_t) \uparrow \)
- \( \text{cov}(\Delta y_{t+1}, \Delta y_t) \uparrow \) until mid-80s
- \( \text{cov}(\Delta y_{t+2}, \Delta y_t) \) small, so MA(1)
Autocovariances of consumption growth

<table>
<thead>
<tr>
<th>Year</th>
<th>$\text{var}(\Delta c_t)$</th>
<th>$\text{cov}(\Delta c_{t+1}, \Delta c_t)$</th>
<th>$\text{cov}(\Delta c_{t+2}, \Delta c_t)$</th>
</tr>
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<tr>
<td>1980</td>
<td>0.1275</td>
<td>-0.0526</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>(0.0097)</td>
<td>(0.0076)</td>
<td>(0.0056)</td>
</tr>
<tr>
<td>1981</td>
<td>0.1197</td>
<td>-0.0573</td>
<td>0.0025</td>
</tr>
<tr>
<td></td>
<td>(0.0116)</td>
<td>(0.0084)</td>
<td>(0.0043)</td>
</tr>
<tr>
<td>1982</td>
<td>0.1322</td>
<td>-0.0641</td>
<td>0.0006</td>
</tr>
<tr>
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<td>(0.0110)</td>
<td>(0.0087)</td>
<td>(0.0060)</td>
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<tr>
<td>1983</td>
<td>0.1532</td>
<td>-0.0691</td>
<td>-0.0056</td>
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<tr>
<td></td>
<td>(0.0159)</td>
<td>(0.0100)</td>
<td>(0.0067)</td>
</tr>
<tr>
<td>1984</td>
<td>0.1869</td>
<td>-0.1003</td>
<td>-0.0131</td>
</tr>
<tr>
<td></td>
<td>(0.0173)</td>
<td>(0.0163)</td>
<td>(0.0089)</td>
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<td>1985</td>
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<td>-0.0872</td>
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</tr>
<tr>
<td></td>
<td>(0.0244)</td>
<td>(0.0194)</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>0.1628</td>
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<td>NA</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
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<td>NA</td>
<td>NA</td>
</tr>
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<td>1988</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>1989</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>1990</td>
<td>0.1751</td>
<td>-0.0602</td>
<td>-0.0057</td>
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<td>(0.0221)</td>
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<td>1991</td>
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<td>1992</td>
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<td>NA</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\text{var}(\Delta c_t)$ ↑ until 1985, then flattens
- $\text{var}(\Delta c_t)$ large
- $\text{cov}(\Delta c_{t+1}, \Delta c_t)$ large, so large imputation error
- $\text{cov}(\Delta c_{t+2}, \Delta c_t)$ very small
Income-Consumption growth covariance

Table 5—The Consumption-Income Growth Covariance Matrix

<table>
<thead>
<tr>
<th>Year</th>
<th>$\text{cov}(\Delta y_t, \Delta c_t)$</th>
<th>$\text{cov}(\Delta y_{t+1}, \Delta c_t)$</th>
<th>$\text{cov}(\Delta y_t, \Delta c_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.0040</td>
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<td>0.0053</td>
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<td>(0.0041)</td>
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<td>1981</td>
<td>0.0116</td>
<td>$-0.0056$</td>
<td>$-0.0043$</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.0032)</td>
<td>(0.0036)</td>
</tr>
<tr>
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<td>0.0165</td>
<td>$-0.0064$</td>
<td>$-0.0006$</td>
</tr>
<tr>
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<td>(0.0036)</td>
<td>(0.0031)</td>
<td>(0.0039)</td>
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<tr>
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<td>0.0215</td>
<td>$-0.0085$</td>
<td>$-0.0075$</td>
</tr>
<tr>
<td></td>
<td>(0.0045)</td>
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<td>(0.0043)</td>
</tr>
<tr>
<td>1984</td>
<td>0.0230</td>
<td>$-0.0030$</td>
<td>$-0.0119$</td>
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<tr>
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<td>(0.0043)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>1985</td>
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<td>$-0.0035$</td>
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<tr>
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<td>(0.0068)</td>
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<td>1986</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>1987</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>1988</td>
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<td>NA</td>
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<td>NA</td>
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<td>$-0.0016$</td>
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<td>1992</td>
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</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\text{cov}(\Delta y_t, \Delta c_t) \uparrow$ until 1985
- $\text{cov}(\Delta y_{t+1}, \Delta c_t)$ close to 0, so almost full insurance against transitory shocks
- Tests reject advance information
Estimation

• Objects of interest:
  • Variance of income shocks: $\sigma^2_\zeta, \sigma^2_\epsilon$
  • Insurance parameters: $\phi, \psi$

• Allow for:
  • Measurement error
  • Time varying variance in measurement error and shocks
  • MA(1) transitory component of income
  • Unobserved heterogeneity

• Three samples:
  1. Baseline
  2. Separated by education
  3. Separated by cohort

• Use diagonally weighted minimum distance (DWMD)
Insurance parameters

- MA parameter $\theta$ small
- Variance of unobserved heterogeneity small but significant
Insurance parameters

Table 6—Minimum-Distance Partial Insurance and Variance Estimates

<table>
<thead>
<tr>
<th></th>
<th>Whole sample</th>
<th>No college</th>
<th>College</th>
<th>Born 1940s</th>
<th>Born 1930s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.6423</td>
<td>0.9439</td>
<td>0.4194</td>
<td>0.7928</td>
<td>0.6889</td>
</tr>
<tr>
<td>(Partial insurance perm. shock)</td>
<td>(0.0945)</td>
<td>(0.1783)</td>
<td>(0.0924)</td>
<td>(0.1848)</td>
<td>(0.2393)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.0533</td>
<td>0.0768</td>
<td>0.0273</td>
<td>0.0675</td>
<td>-0.0381</td>
</tr>
<tr>
<td>(Partial insurance trans. shock)</td>
<td>(0.0435)</td>
<td>(0.0602)</td>
<td>(0.0550)</td>
<td>(0.0705)</td>
<td>(0.0737)</td>
</tr>
<tr>
<td>p-value test of equal ( \phi )</td>
<td>23%</td>
<td>99%</td>
<td>8%</td>
<td>81%</td>
<td>18%</td>
</tr>
<tr>
<td>p-value test of equal ( \psi )</td>
<td>75%</td>
<td>33%</td>
<td>29%</td>
<td>76%</td>
<td>4%</td>
</tr>
</tbody>
</table>

- \( \phi = 0.6423 \rightarrow \) partial insurance against permanent shocks
- \( \psi = 0.0533 \rightarrow \) almost full insurance against transitory shocks
- \( \phi \) changes by education
noting that from trough to peak the variance of the permanent shock more than doubles.

This evidence on permanent shocks is similar to that reported by Moffitt and Gottschalk (1995) using PSID data on male earnings. As we will document below, however, the precise evolution of inequality in transitory shocks depends on the source of income under study. Male labor earnings data will be shown to display a higher transitory variance in the earlier part of this time period.

Table 6 also reports the results of the model for two education groups (with and without college education), and for two representative birth cohorts (born in the 1940s and born in the 1930s).

The partial insurance parameter estimates point to interesting differences in insurance by type of household. In particular, there appears to be less insurance in response to permanent shocks among the group with no college education (indeed, we would not statistically reject the null hypothesis that there is no insurance in this group). In contrast, the evidence on college accords with a simple PIH model and we cannot reject the null that there is full smoothing with respect to transitory shocks ($c = 0$) for both education groups, though for the less well educated the point estimate is higher.

When the sample is stratified by year of birth, we find qualitatively similar results: there is evidence for full insurance with respect to transitory shocks and differences in the extent of insurance with respect to the permanent shocks.

It is worth considering whether the presence of precautionary asset accumulation is an explanation for the pattern of results. Recall that the insurance coefficients may reflect differences in $pi$, the share of future labor income in the present value of lifetime wealth, which in our framework reflects how close an individual is to retirement age. Thus, $pi$ is likely to be lower for older cohorts because they have both more accumulated financial wealth and lower prospective human capital wealth. Indeed, we find some evidence that...
It is interesting to note at this point the different pattern of transitory income inequality recovered from the baseline model versus the male earnings only specification. This is presented in Figure 6, which plots the path of the two variances over this period. Once total net income is considered, rather than male labor earnings alone, there is a much shallower rise in transitory income uncertainty. This reconciles the results with the results from the male earnings literature, in particular Moffitt and Gottschalk (1995) who, using male earnings in the PSID, document a much steeper rise in transitory inequality earlier in the 1980s. As noted above, their pattern of permanent inequality is closely in accord with Figure 4. The most interesting aspect of Figure 6 is that in the early 1980s there is little or no growth in the variance of the transitory shock to net income. Most of the growth occurs in the second half of the sample. This is in sharp contrast with the trends in the variance of the permanent shock to net income, which rises in the early 1980s and attenuates out afterward. Thus we may conclude that the increase in income inequality of the early 1980s is of a permanent nature, while the growth in the second half of the sample is more temporary.

D. A Variance of Consumption Growth Decomposition

At this point, we can go back to the decomposition of the variance of consumption growth proposed in Section IIC, and propose an explanation of our findings. We have argued that there is no evidence that insurance has changed over the sample period we examine. Thus $f_t^2 = c_t^2 = 0$. In the first half of our sample period there is a strong growth in the variance of permanent income shocks and little growth in the variance of transitory shocks, implying $\text{var}_1 \text{d}_c_t^2 \text{var}_1 z_t^2$. If there were no insurance with respect to permanent income shocks, $\text{var}_1 \text{d}_c_t^2 = \text{var}_1 z_t^2$, but in fact we find empirically that $f_t$, $1$, and so there is some attenuation, although as we saw earlier consumption inequality rises substantially. In the second half of the sample, the variance of

![Variance of Transitory Shocks](image-url)
Variance of consumption growth

\[ \Delta \text{var}(\Delta c_t) \approx \text{var}(\zeta_t) \Delta \phi_t^2 + \phi_{t-1}^2 \Delta \text{var}(\zeta_t) + \text{var}(\varepsilon_t) \Delta \psi_t^2 + \psi_{t-1}^2 \Delta \text{var}(\varepsilon_t) \]

- Evidence that \( \Delta \phi_t^2 = \Delta \psi_t^2 = 0 \), so:

\[ \Delta \text{var}(\Delta c_t) \approx \phi_{t-1}^2 \Delta \text{var}(\zeta_t) + \psi_{t-1}^2 \Delta \text{var}(\varepsilon_t) \]

- Early part of the sample:
  - Variance of permanent shock and of consumption ↑
  - But attenuation due to insurance

- Later part of the sample:
  - Variance of transitory shocks ↑
  - But \( \psi \) close to 0, so little effect on consumption inequality
I. Characteristics of Consumption and Income Inequality

While there are large panel datasets that track the distribution of wages and incomes for households over time, the same is not true for broad measures of consumption. The PSID contains longitudinal income data, but the information on consumption is scanty (limited to food and a few more items). Indeed, one of the reasons why consumption inequality has not been studied as extensively as income and wage inequality is the nature of data availability. In this section we first document some basic features of the evolution of consumption and income inequality that motivate our study. Repeated cross-section data such as the CEX are not enough to uncover the degree of persistence in income shocks or to identify the partial insurance model. For that we need panel data, and in the second part of this section we describe our new panel data series.

A. The Evolution of Income and Consumption Inequality

There are two important features of the evolution of consumption and income inequality between the late 1970s and early 1990s which underpin our analysis. These are clearly evident from Figure 1, which uses PSID data on log income and CEX data on log consumption (see Section IB for details on sample selection and variable definitions). In this graph, we plot the actual estimates of the variances, as well as smoothing curves passing through the scatters (to ease legibility). In this figure the range of variation of the variance of PSID consumption is on the left-hand side; that of the variance of CEX consumption is on the right-hand side. The first distinct feature is that the slope of the income variance (the solid line) is greater than the slope of the consumption variance (the dashed line). The second feature of these inequality figures is that consumption inequality flattens out completely in the second part of the 1980s, whereas income inequality continues to rise, albeit at a much slower rate. Below we provide a framework for interpreting these changes. In particular, we show that the degree of detachment between consumption and income inequality depends on the persistence of income shocks and the availability of insurance to these shocks.

These overall patterns reflect what has also been found in previous analyses of inequality in income and consumption for this period, the most prominent study being that of Cutler and Katz (1992). See also the retrospective analysis in Johnson, Smeeding, and Boyle Torrey (2005).
permanent shocks for the older cohort are smoothed to a greater extent than for younger cohorts, although these subgroup estimates are less precise. Whether this is due to the effect played by precautionary wealth accumulation remarked above or by greater availability of insurance (such as social security, disability insurance, or even insurance provided by adult children) in the group of persons born in the 1930s is something we cannot address in the absence of additional information, such as panel data on assets and age-specific estimates of human capital wealth. Later we provide some suggestive evidence that wealth accumulation is a potentially important explanation for the degree of insurance with respect to permanent income shocks.

How good is the fit of our model? In Figure 5 we plot the actual variance of income growth and its predicted value (the dashed line) from our baseline model. We repeat the exercise for the variance of consumption growth and the covariance between income and consumption growth. Our model appears to fit the model quite well in all three dimensions.

Before delving into more detail concerning the underlying mechanisms at work in our results, we ask the question: could these baseline results have been obtained using food data alone? With almost no exceptions, all the papers in the literature (including Hall and Mishkin 1982; Hayashi, Altonji, and Kotlikoff 1996) use the PSID data on food, so it is worth asking what is the value added of using our imputed measure of consumption. A possible argument in favor of this simpler approach is that food is a constant fraction of nondurable expenditure, so that the...
Taxes, transfers, and family labor supply

Table 7—Minimum-Distance Partial Insurance and Variance Estimates

<table>
<thead>
<tr>
<th>Consumption:</th>
<th>Nondurable net income baseline</th>
<th>Nondurable earnings only baseline</th>
<th>Nondurable male earnings baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.6423</td>
<td>0.3100</td>
<td>0.2245</td>
</tr>
<tr>
<td>(Partial insurance perm. shock)</td>
<td>(0.0945)</td>
<td>(0.0574)</td>
<td>(0.0493)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.0533</td>
<td>0.0633</td>
<td>0.0502</td>
</tr>
<tr>
<td>(Partial insurance trans. shock)</td>
<td>(0.0435)</td>
<td>(0.0309)</td>
<td>(0.0294)</td>
</tr>
</tbody>
</table>

- Replace net family income with family or male earnings
- \( \phi \downarrow \), so insurance \( \uparrow \)
- Important role for taxes, transfers, and family labor supply
Private transfers and low wealth

Table 8—Minimum-Distance Partial Insurance and Variance Estimates, Various Sensitivity Analyses

<table>
<thead>
<tr>
<th>Consumption: Income: Sample:</th>
<th>Nondurable net income baseline</th>
<th>Nondurable net income excluding help baseline</th>
<th>Nondurable net income low wealth</th>
<th>Nondurable net income high wealth</th>
<th>Total net income low wealth</th>
<th>Nondurable net income baseline+SEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi ) (Partial insurance perm. shock)</td>
<td>0.6423 (0.0945)</td>
<td>0.6215 (0.0895)</td>
<td>0.8489 (0.2848)</td>
<td>0.6248 (0.0999)</td>
<td>1.0342 (0.3517)</td>
<td>0.7652 (0.0354)</td>
</tr>
<tr>
<td>( \psi ) (Partial insurance trans. shock)</td>
<td>0.0533 (0.0435)</td>
<td>0.0500 (0.0434)</td>
<td>0.2877 (0.1143)</td>
<td>0.0106 (0.0414)</td>
<td>0.3683 (0.1465)</td>
<td>0.1211 (0.0354)</td>
</tr>
</tbody>
</table>

- Negligible impact of help from friends and relatives
- Low wealth individuals are less insured
- Durable purchases and timing of durable replacement might act as insurance for low wealth individuals
Conclusions

• The evolution of permanent and transitory income shocks can explain the disjuncture between income and consumption inequality

• Partial insurance against permanent shocks, almost full insurance against transitory shocks

• Less insurance of low-wealth, more insurance for more educated

• Tax and welfare system play important role for insurance
Comments

• Role of income process?
• Advance information and expectations?
• What are the insurance mechanisms?
• Role of borrowing constraints?
Extensions

- Kaplan and Violante (2010)
  - Advance information, borrowing constraints, performance of BPP estimator in incomplete markets model

  - Family labor supply and children

- Blundell, Borella, Commault, and De Nardi (2020) and Russo (2020)
  - Role of health
References I


References II


Food demand estimates

Table 2—The Demand for Food in the CEX

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Variable</th>
<th>Estimate</th>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln c</td>
<td>0.8503</td>
<td>ln c × 1992</td>
<td>0.0037</td>
<td>Family size</td>
<td>0.0272</td>
</tr>
<tr>
<td>(0.1511)</td>
<td>(0.0056)</td>
<td>(0.0812)</td>
<td>(0.0090)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × high school dropout</td>
<td>0.0730</td>
<td>ln c × one child</td>
<td>0.0202</td>
<td>In pfood</td>
<td>−0.9784</td>
</tr>
<tr>
<td>(0.0718)</td>
<td>(0.0336)</td>
<td>(0.0501)</td>
<td>(0.2160)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × high school graduate</td>
<td>0.0827</td>
<td>ln c × two children</td>
<td>−0.0250</td>
<td>In ptobacco</td>
<td>5.5376</td>
</tr>
<tr>
<td>(0.0890)</td>
<td>(0.0383)</td>
<td>(0.1501)</td>
<td>(0.0500)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1981</td>
<td>0.1151</td>
<td>ln c × three children+</td>
<td>0.0087</td>
<td>In pfuel+alcohol</td>
<td>−0.6670</td>
</tr>
<tr>
<td>(0.1123)</td>
<td>(0.0340)</td>
<td>(0.1200)</td>
<td>(4.7351)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1982</td>
<td>0.0533</td>
<td>One child</td>
<td>−0.1568</td>
<td>In pfuel+alcohol</td>
<td>−1.8684</td>
</tr>
<tr>
<td>(0.0837)</td>
<td>(0.3215)</td>
<td>(4.1425)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1983</td>
<td>0.0508</td>
<td>Two children</td>
<td>0.3214</td>
<td>Born 1955–59</td>
<td>−0.0385</td>
</tr>
<tr>
<td>(0.0704)</td>
<td>(0.3650)</td>
<td>(0.0554)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1984</td>
<td>0.0478</td>
<td>Three children+</td>
<td>0.0132</td>
<td>Born 1950–54</td>
<td>−0.0085</td>
</tr>
<tr>
<td>(0.0662)</td>
<td>(0.3259)</td>
<td>(0.0477)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1985</td>
<td>0.0304</td>
<td>High school dropout</td>
<td>−0.7030</td>
<td>Born 1945–49</td>
<td>−0.0060</td>
</tr>
<tr>
<td>(0.0638)</td>
<td>(0.6741)</td>
<td>(0.0406)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1986</td>
<td>0.0223</td>
<td>High school graduate</td>
<td>−0.8458</td>
<td>Born 1940–44</td>
<td>−0.0051</td>
</tr>
<tr>
<td>(0.0587)</td>
<td>(0.8298)</td>
<td>(0.0348)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1987</td>
<td>0.0524</td>
<td>Age</td>
<td>0.0122</td>
<td>Born 1935–39</td>
<td>−0.0044</td>
</tr>
<tr>
<td>(0.0599)</td>
<td>(0.0885)</td>
<td>(0.0273)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1988</td>
<td>0.0416</td>
<td>Age²</td>
<td>−0.0001</td>
<td>Born 1930–34</td>
<td>0.0032</td>
</tr>
<tr>
<td>(0.0458)</td>
<td>(0.0001)</td>
<td>(0.0193)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1989</td>
<td>0.0370</td>
<td>Northeast</td>
<td>0.0087</td>
<td>Born 1925–29</td>
<td>−0.0051</td>
</tr>
<tr>
<td>(0.0373)</td>
<td>(0.0065)</td>
<td>(0.0140)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1990</td>
<td>0.0187</td>
<td>Midwest</td>
<td>−0.0213</td>
<td>White</td>
<td>0.0769</td>
</tr>
<tr>
<td>(0.0295)</td>
<td>(0.0105)</td>
<td>(0.0129)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln c × 1991</td>
<td>−0.0094</td>
<td>South</td>
<td>−0.0269</td>
<td>Constant</td>
<td>−0.6404</td>
</tr>
<tr>
<td>(0.0318)</td>
<td>(0.0096)</td>
<td>(0.9266)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test of overidentifying restrictions: 20.92 (d.f. 18; χ² p-value 28%)
Test that income elasticity does not vary over time: 27.69 (d.f. 12; χ² p-value 0.6%)

Notes: This table reports IV estimates of the demand equation for (the logarithm of) food spending in the CEX. We instrument the log of total nondurable expenditure (and its interaction with time, education, and kids dummies) with the cohort-education-year specific average of the log of the husband’s hourly wage and the cohort-education-year specific average of the log of the wife’s hourly wage (and their interactions with time, education, and kids dummies). Standard errors are in parentheses, the Shea’s partial R² for the relevance of instruments in brackets. In all cases, the p-value of the F-test on the excluded instrument is < 0.01 percent.
How good is the imputation?

Figure 3. CEX and New PSID Compared

CEX and new PSID compared
How flexible is this income process?

- This is a **linear** income process
- Identification is relatively easy
- All shocks are associated to the same persistence
- Non-linear transmission of shocks is ruled out
PIH with self insurance

- $\pi_{it}$: share of future labor income in current human and financial wealth

- $\gamma_{tL}$: age-increasing annuitization factor

- One can show that:
  \[
  \phi_{it} \approx \pi_{it}, \quad \psi_{it} \approx \gamma_{tL} \pi_{it}
  \]

- Precautionary saving can only provide effective self-insurance if $\pi_{it}$ is small.
Excess smoothness

- Two alternative insurance configurations:
  1. Public information but limited enforcement of contracts
  2. Private information but full enforcement
- Self-insurance is Pareto inefficient
- More insurance than with a single noncontingent bond, but less than with complete markets.
- Relationship between income shocks and consumption depends on the degree of persistence of income shocks
- Another reason for partial insurance is moral hazard → Attanasio and Pavoni (2011) → when individuals have hidden access to a simple credit market, some partial insurance is possible.
Advance information

• If the agents knew in advance some parts of the shocks these would already be incorporated into current plans and would not directly affect consumption growth

• Estimated $\phi_{i,t}$ has to be interpreted as reflecting a combination of insurance and information.

• We would be overestimating insurance and thus underestimating parameters

• With no extra data, this combination cannot be untangled $\rightarrow$ BPP provide evidence that advance information is not a serious problem in their sample.
Identification of variances of shocks I

\[ \text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) = \]
\[ = \text{cov}(\zeta_t + \Delta \varepsilon_t, \zeta_{t-1} + \Delta \varepsilon_{t-1} + \zeta_t + \Delta \varepsilon_t + \zeta_{t+1} + \Delta \varepsilon_{t+1}) \]
\[ = \text{cov}(\zeta_t + \Delta \varepsilon_t, \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2} + \zeta_t + \varepsilon_t - \varepsilon_{t-1} + \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \]
\[ = \text{cov}(\zeta_t + \Delta \varepsilon_t, \zeta_{t-1} + \zeta_t + \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \]

Since \( \zeta_{i,t}, \nu_{i,t} \) and \( \varepsilon_{i,t} \) are assumed to be mutually uncorrelated and both \( \zeta \) and \( \varepsilon \) are serially uncorrelated, this yields:

\[ \text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) = \text{cov}(\zeta_t, \zeta_t) = \text{var}(\zeta_t) \]
Identification of variances of shocks II

Moreover, we have that:

$$-cov(\Delta y_t, \Delta y_{t+1}) = -cov(\zeta_t + \Delta \varepsilon_t, \zeta_{t+1} + \Delta \varepsilon_{t+1})$$

$$= -cov(\zeta_t + \varepsilon_t - \varepsilon_{t-1}, \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_t)$$

Since $\zeta_{i,t}$, $\nu_{i,t}$ and $\varepsilon_{i,t}$ are assumed to be mutually uncorrelated and both $\zeta$ and $\varepsilon$ are serially uncorrelated, this yields:

$$-cov(\Delta y_t, \Delta y_{t+1}) = -cov(\varepsilon_t, -\varepsilon_t)$$

$$= var(\varepsilon_t)$$
Identification of income process

Using $\Delta y_{it} = \zeta_{it} + \Delta \nu_{it}$:

$$cov(\Delta y_t, \Delta y_{t+s}) = cov(\zeta_t + \Delta \nu_t, \zeta_{t+s} + \Delta \nu_{t+s})$$

This implies:

$$cov(\Delta y_t, \Delta y_{t+s}) = cov[(\zeta_t + \Delta \nu_t)(\zeta_{t+s} + \Delta \nu_{t+s})]$$

$$= cov[\zeta_t, \zeta_{t+s}] + cov[\zeta_t, \Delta \nu_{t+s}] +$$

$$+ cov[\Delta \nu_t, \zeta_{t+s}] + cov[\Delta \nu_t, \Delta \nu_{t+s}]$$

Now, recall that $\zeta_{i,t}$, $\nu_{i,t}$ and $\varepsilon_{i,t}$ are mutually uncorrelated and that $\zeta_{i,t}$ is serially uncorrelated. Then, we have that

$$cov(\Delta y_t, \Delta y_{t+s}) = \begin{cases} 
var(\zeta_t) + var(\Delta \nu_t) & \text{for } s = 0 \\
\text{cov}(\Delta \nu_t, \Delta \nu_{t+s}) & \text{for } s \neq 0
\end{cases}$$ (7)
ACF for MA(1) process

\[ y = \varepsilon_t + 0.25\varepsilon_{t-1} \]
Identification of insurance coefficients

\[ \text{cov}(\Delta c_t, \Delta y_{t+s}) = \text{cov}
[(\phi_t \zeta_t + \psi_t \xi_t + \xi_t)(\zeta_{t+s} + \Delta \nu_{t+s})] \]
\[ = \phi_t \text{cov}[\zeta_t \zeta_{t+s}] + \phi_t \text{cov}[\zeta_t \Delta \nu_{t+s}] + \psi_t \text{cov}[\xi_t \zeta_{t+s}] + \psi_t \text{cov}[\xi_t \Delta \nu_{t+s}] + \text{cov}[\xi_t \zeta_{t+s}] + \text{cov}[\xi_t \Delta \nu_{t+s}] \]

which gives that:

\[ \text{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} 
\phi_t \text{var}(\zeta_t) + \psi_t \text{var}(\xi_t) & \text{for } s = 0 \\
\psi_t \text{cov}(\xi_t, \Delta \nu_{t+s}) & \text{for } s \neq 0
\end{cases} \]
Solution to identification problem

Start from:

\[
\text{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} 
\phi_t \text{var}(\zeta_t) + \psi_t \text{var}(\varepsilon_t) & \text{for } s = 0 \\
\psi_t \text{cov}(\varepsilon_t, \Delta \nu_{t+s}) & \text{for } s \neq 0
\end{cases}
\]

For \( s = 1 \) and using the fact that \( \nu \) is an MA(0):

\[
\text{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} 
\phi_t \text{var}(\zeta_t) + \psi_t \text{var}(\varepsilon_t) & \text{for } s = 0 \\
\psi_t \text{cov}(\varepsilon_t, \Delta \varepsilon_{t+1}) & \text{for } s = 1
\end{cases}
\]

which yields:

\[
\text{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} 
\phi_t \text{var}(\zeta_t) + \psi_t \text{var}(\varepsilon_t) & \text{for } s = 0 \\
-\psi_t \text{var}(\varepsilon_t) & \text{for } s = 1
\end{cases}
\]

Since you observe \( \text{cov}(\Delta c_t, \Delta y_{t+s}) \) from the data and you have identified the variances of the shocks before, this is a system of two equations in two unknowns, which you can solve to find \( \psi \) and \( \phi \).
Consumption growth inequality

\[
\text{cov}(\Delta c_t, \Delta c_{t+s}) = \text{cov}\left[\left(\phi_t \zeta_t + \psi_t \varepsilon_t + \xi_t\right)\left(\phi_{t+s} \zeta_{t+s} + \psi_{t+s} \varepsilon_{t+s} + \xi_{t+s}\right)\right] \\
= \phi_t \phi_{t+s} \text{cov}[\zeta_t \zeta_{t+s}] + \phi_t \psi_{t+s} \text{cov}[\zeta_t \varepsilon_{t+s}] + \\
+ \phi_t \text{cov}[\zeta_t \xi_{t+s}] + \psi_t \phi_{t+s} \text{cov}[\varepsilon_t \zeta_{t+s}] + \\
+ \psi_t \psi_{t+s} \text{cov}[\varepsilon_t \varepsilon_{t+s}] + \psi_t \text{cov}[\varepsilon_t \xi_{t+s}] + \\
+ \phi_{t+s} \text{cov}[\xi_t \zeta_{t+s}] + \psi_{t+s} \text{cov}[\xi_t \varepsilon_{t+s}] + \text{cov}[\xi_t \xi_{t+s}] \\
\]

This gives that:

\[
\text{cov}(\Delta c_t, \Delta c_{t+s}) = \begin{cases} 
\phi_t^2 \text{var}(\zeta_t) + \psi_t^2 \text{var}(\varepsilon_t) + \text{var}(\xi_t) & \text{for } s = 0 \\
0 & \text{for } s \neq 0
\end{cases}
\]
Imputation error

Suppose that consumption is measured with error. Then, we have:

$$c_{i,t}^* = c_{i,t} + u_{i,t}^c$$

where $c^*$ denotes measured consumption, $c$ is true consumption and $u^c$ is measurement error. Measurement error induces serial correlation in consumption growth. Now, suppose that $c_{i,t}$ is a random walk, that is:

$$c_{i,t} = c_{i,t-1} + \eta_{i,t}$$

where $\eta_{i,t}$ is i.i.d. Then $\Delta c_{i,t} = \eta_{i,t}$. Then, we have that $\Delta c_{i,t}^* = \Delta c_{i,t} + \Delta u_{i,t}^c = \eta_{i,t} + \Delta u_{i,t}^c$. This implies that:

$$E[\Delta c_{i,t}^* \Delta c_{i,t-1}^*] = E[(\eta_{i,t} + u_{i,t}^c - u_{i,t-1}^c)(\eta_{i,t-1} + u_{i,t-1}^c - u_{i,t-2}^c)]$$

$$= -E[u_{i,t-1}^c u_{i,t-1}^c]$$

$$= -\sigma_u^2$$

(Since $u \sim iid$)

Moreover we have that:

$$E[\Delta c_{i,t}^* \Delta c_{i,t+1}^*] = E[(\eta_{i,t} + u_{i,t}^c - u_{i,t-1}^c)(\eta_{i,t+1} + u_{i,t+1}^c - u_{i,t}^c)]$$

$$= -E[u_{i,t}^c u_{i,t}^c]$$

$$= -\sigma_u^2$$

(Since $u \sim iid$)
Insurance or pass-through coefficients?

Consider an idiosyncratic shock $x_{it}$. The pass-through coefficient measures the share of the variance of the shock that is passed to log-consumption (Kaplan and Violante (2010)):

$$
\phi^x = \frac{\text{cov}(\Delta c_{it}, x_{it})}{\text{var}(x_{it})}
$$

The insurance coefficient is the share of the variance of the shocks which is not passed to consumption, so $1 - \phi^x$.

BPP use the pass-through and call it insurance coefficient, be careful with reference values:

- Pass-through: 0 (full insurance), 1 (no insurance)
- Insurance: 1 (full insurance), 0 (no insurance)