Entrepreneurship, Frictions and Wealth

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Abstract

Entrepreneurship is a very important determinant of the distribution of wealth. In the data entrepreneurs are a small fraction of the population, but have a high saving rate and hold a large share of total wealth. Conversely, the distribution of wealth has a significant effect on entrepreneurship: initial assets have great influence on entrepreneurial decisions. We construct a model that generates these two facts from two features: an overlapping-generations structure and the limited ability to enforce repayment of the loans to entrepreneurs. These factors are sufficient to match wealth inequality very well, both for entrepreneurs and non-entrepreneurs. We find that less restrictive borrowing constraints generate less wealth concentration and more entrepreneurs. We also find that there would be fewer large firms and the distribution of wealth would be less concentrated if all of the bequests that we do observe were accidental, rather than a mix of accidental and voluntary bequests.

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1 Introduction

Many microeconomic studies show that entrepreneurs face borrowing constraints and the decision to become entrepreneurs depends on own assets, availability of collateral, and receipt of bequests. For example, Evans and Jovanovic [11] estimate a structural model of entrepreneurial choice and find evidence of liquidity constraints. Gentry and Hubbard [14] show that external financing to start or expand a business is very costly, and initial wealth plays a role in the choice of becoming an entrepreneur. Part of these funds may be generated by own savings: the possibility of becoming an entrepreneur may induce people to save more to build up the required funds. Part of it may also come from intergenerational transfers, such as bequests. Holtz-Eakin et al. [15], for instance, show that the decision to become entrepreneurs is affected by the receipt and the size of an inheritance.

In presence of borrowing constraints, the decision to invest, the fraction of entrepreneurs and the size distribution of firms depend on the distribution of assets in the economy. The data show that wealth holdings are very concentrated, even more so than labor earnings and income, with a small fraction of people owning huge fortunes (see Díaz-Giménez, Quadrini and Ríos–Rull [10]). While entrepreneurs are a small fraction of the population, they have high saving rates and hold a large share of total wealth. For instance, in the 1989 Survey of Consumer Finances entrepreneurs are 8.7% of the sample, but hold 39% of total net worth. Both Gentry and Hubbard [14] and Quadrini [24] document that the large wealth holdings of entrepreneurs are due not only to the fact that entrepreneurs earn more income, but that they also save a larger fraction of their income than non entrepreneurs. Because of the interaction between borrowing constraints and asset holdings, it is key to study entrepreneurial choice in a model that matches well the distribution of wealth.

We construct and solve numerically a quantitative life-cycle model with entrepreneurial choice in an environment in which debt repayment cannot be perfectly enforced. The amount that entrepreneurs can borrow depends on their observable characteristics and the entrepreneurs' assets act as collateral for their debts. Since the implicit rate of return for entrepreneurs is higher than for workers, they have a higher saving rate. To quantify the importance of inheriting the family's wealth and entrepreneurial activity, we also study the effect of intergenerational altruism and voluntary bequests. We calibrate the key parameters of the model, such as entrepreneurial ability and degree of enforcement, to match some key moments of the data, and discuss the implications of the model and its components for entrepreneurial choice and wealth inequality. We show that our model with entrepreneurial choice matches very well the the observed distribution of wealth, both for entrepreneurs and non-entrepreneurs. We find that less restrictive borrowing constraints generate less inequality in wealth holdings and increase the number of people engaging in entrepreneurial activities. Our results also indicate that entrepreneurial wealth and firm size would be smaller, and wealth would be less concentrated, when the degree of intergenerational altruism is zero and all bequests are accidental.

This paper is related to various works that have studied wealth accumulation, entrepreneurial choice, and imperfectly enforceable contracts.

Most models of wealth accumulation have difficulties in explaining the extreme concentration observed in the upper tail of the wealth distribution and the large saving rates of the richest households[2]. Among the exceptions, Castañeda, Díaz-Giménez and Ríos–Rull [6] adopt a dynastic model with idiosyncratic shocks and perfect altruism and reconstruct an income process that matches earnings and wealth dispersion. De Nardi [9] evaluates the importance of bequest motives in explaining wealth dispersion in a life cycle optimization model, and shows that while these motives are quantitatively important to explain the distribution of wealth, they are not sufficient to explain the wealth holdings of the very richest. Neither of these papers models the entrepreneurial decisions, and thus cannot study the differences in behavior between workers and entrepreneurs and the interdependence between wealth inequality and entrepreneurship. Quadrini [25] was the first to show that modelling entrepreneurship is important in explaining the distribution of wealth. He did so by assuming an exogenous learning process that enables the entrepreneurs to run larger and larger ventures and, given exogenous borrowing constraints, determines the firms size distribution in the aggregate economy. We model explicitly a market friction that generates endogenously the firm size distribution as well as the wedge between the market interest rate and the implicit rate of return on the entrepreneur's own assets. Accounting endogenously for these key factors in the distribution of wealth is likely to play a major role in any policy experiment. We also explicitly model intergenerational transmission of wealth and entrepreneurial ability to study the importance of intergenerational transfers.

[2] See Quadrini and Ríos–Rull [26] for a discussion of the shortcomings of most computable models of wealth dispersion.

Albuquerque and Hopenhayn [1] characterize optimal contracts and their qualitative implications for firm growth and survival in an environment in which firms face limited liability and repayment of debt cannot be perfectly enforced. They consider both long- and short-term contracts. Compared to their work, we assume short-term contracts in a model with capital and investigate its quantitative implications for entrepreneurial choice, wealth inequality and firm size distribution. Monge [22] analyzes a model in which income and wealth distributions coincide, and financial market institutions are crucial in determining it. In contrast to our paper, Monge is not interested in focusing on wealth inequality and the quantitative implications of the model. The role of limited contract enforceability is also studied in Cooley et al. [7], who focus on the role of these constraints in retarding the diffusion of new technologies.

2 Empirical evidence on entrepreneurship, borrowing constraints and wealth

This section discusses the evidence indicating that entrepreneurs are liquidity constrained and have a higher saving rate. It also shows the key role of entrepreneurship in generating a skewed wealth distribution. The data are from the 1989 Survey of Consumer Finances (SCF).[3] Unlike other datasets (such as the Panel Study of Income Dynamics and the Health and Retirement Survey), the SCF oversamples rich households and thus provides important advantages. First, it gives a better picture of the concentration of wealth and of the asset holdings of richer households, that include a large share of entrepreneurs. Second, as shown by Curtin et al. [8] and Juster et al. [18], the total wealth implied by the SCF is very close to the total wealth implied by aggregate data (such as the Federal Reserve Board flow of funds accounts); the SCF can thus be used to calibrate aggregates (for instance, the share of entrepreneurial wealth and the percentage of entrepreneurs) in a general equilibrium model such as the one developed in this paper.

We can use different criteria to classify a household as an entrepreneur, based on business asset ownership or on self-declared employment status. In our model, an entrepreneur must invest his own wealth in the entrepreneurial activity, and his income is primarily the return from this activity. Therefore,

[3]The data for the 1992 and 1995 waves are similar. The results are available from the authors upon request.

we consider as entrepreneurs the households that declare owning a business (or a share of one), and having an active management role in it.[4] In this, we follow Gentry and Hubbard[5] [14], who use the same SCF question, and who further restrict the definition of entrepreneurship only to households who own at least $ 5,000 in actively managed businesses, in order to isolate people who have made a significant up-front investment in their business.[6] Even with these restrictions, we are most likely overestimating the number of entrepreneurs, since some of these households have another, non-entrepreneurial main occupation, or are either only temporarily self-employed.

2.1 Entrepreneurship and borrowing constraints

A crucial characteristic of the entrepreneurs' portfolio is that business wealth is a large share of their total wealth, and own assets are often used as a collateral. The median ratio of business wealth to net worth (for the entrepreneurs who have more than $5000 in business assets in 1989) is 48%, the third quartile is 77% and the top decile is 96%. Therefore, most of the people classified as entrepreneurs have a significant share of their own wealth invested in their business, and are poorly diversified. As shown in table 1, the share is high for all quantiles of the wealth distribution. Approximately half of the net worth is constituted by business wealth both for entrepreneurs in the top and in the bottom of the distribution. The last two rows of table 1 show that the percentage of entrepreneurs who have more than 75% and 90% of net worth invested in own assets decreases slightly as we move to higher percentiles. This suggests that some entrepreneurs in higher quantiles may be able to avoid very large ratios and at least partially diversify. However, as pointed out, the share of business assets tends to be large. Moskowitz and Vissing-Jørgensen [23] also document the poor diversification of the entrepreneurs' portfolio.

Unfortunately, from the SCF it is difficult to isolate exactly business debts, and the characteristics of these debts (conditions, interest charged, whether

[4] The exact question is: "Do you (or anyone in your family living here) have an active management role in any of these businesses?"

[5] Some of the results reported in this section had already been computed by Gentry and Hubbard [14]. Quadrini [24] also reports similar statistics, but computed using the Panel Study of Income Dynamics.

[6] In the tables entrepreneurs refers to those who answered yes to the question mentioned above, and Gentry and Hubbard refers to the more restricted definition, which give very similar results.

	1	5	10	20
$> 50\%$	0.53	0.56	0.52	0.51
$> 75\%$	0.19	0.27	0.27	0.28
$> 90\%$	0.04	0.08	0.10	0.12

Table 1: Each row is the fraction of entrepreneurs (GH) with a business wealth to total net worth ratio higher than the given percentage, among those that are in the top percent (column) of the wealth distribution.

the amount was limited, and so on). However, the survey asks explicitly about whether some of the debts are explicitly collateralized with own private assets. 33% of entrepreneurs declare that they currently use own assets as collateral. Within this group, the median amount of collateral is $36,000, the top decile is $300,000 and the top 5% is $570,000. The median ratio of collateral to business value is 21%, the top decile is 77% and the top 5% is 100% (these fractions do not change significantly across quantiles of the wealth distribution). This suggests that several business need to put up collateral in order to borrow. As just mentioned, these numbers are just an indication, because they only include the use of personal assets (other than the business itself), and do not indicate the relation between the amount borrowed and the size of the business, nor the amount of borrowing desired by the entrepreneur.

Several papers have documented the importance of collateral, the correlation between own assets and external financing, and the relation between wealth and entrepreneurial entry. For instance, Gentry and Hubbard [14] analyze SCF data and argue that costly external financing (coupled with potentially high returns on those investments) has important implications for the saving, investment, and entry decisions of continuing and potential entrepreneurs. Using the National Longitudinal Survey of Young Men, Evans and Jovanovic [11] estimate a structural model of entrepreneurial choice and find evidence of liquidity constraints. Their results indicate that entrepreneurs can borrow only up to 50% of their own current assets. Evans and Leighton [12], in the same dataset, find that the probability of switching into self-employment increases with assets.[7]

Using tax returns, Holz-Eakin et al. [15] (and Blanchflower [5] for the

[7] More recently, however, Hurst and Lusardi [17] argue that this correlation is much lower than what shown in those previous papers.

UK) show that the receipt of a bequest (and thus an increase in own wealth) increase the probability of starting a business.

This evidence therefore suggests that entrepreneurs face borrowing constraints and that the possibility of becoming entrepreneurs and level of possible borrowing is related to the level of own wealth. The need to accumulate assets in the presence of such constraints may also generate high savings rate among entrepreneurs (or household planning to become entrepreneurs). Using the 1983-1989 panel of the SCF, Gentry and Hubbard [14] find higher saving rates for entrepreneurs than for the rest of the population. A similar conclusion arises also from Quadrini's [24] analysis of the Panel Study of Income Dynamics.

In our paper, we generate both of these mechanisms by modelling an endogenously determined borrowing constraint, and we show that its consequences on the distribution of wealth match those presented in the following subsection.

2.2 Entrepreneurship and the wealth distribution

Even though entrepreneurs are only a small fraction of the population (8.7%), they hold 39% of total net worth. Table 2 displays wealth quantile cutoffs for the whole population (first column) and for various subpopulations (other columns) and shows that entrepreneurs are much richer than non entrepreneurs: the median household in the population has a net worth of around $50,000, the median entrepreneur three to four times as much, and the difference is similar for other quantiles.

Table 3 shows what fraction of wealth is held by the top quantiles of the distribution of wealth, and the composition (entrepreneurs or non entrepreneurs) of these groups. The households in the top 1% of the wealth distribution hold around 30% of total net worth, and those in the top 5% hold more than half of the total. Entrepreneurs represent a large share of the households in these quantiles. Around two thirds of those in the top 1%, and one half of those in the top 5% are entrepreneurs, and they hold respectively 69% and 60% of the wealth held by household in those quantiles. As table 4 shows, the corresponding statistics for the self employed are close to the ones for entrepreneurs.

As an additional measure of inequality, we computed the Gini indices of the wealth distribution various subgroups. The Gini index of the wealth distribution for the whole population is 0.78, which is much higher than that for

Quantile	Population	Entr.	GH	S.E.
99	2,321	7,783	8,751	8,202
95	691	2,391	3,059	2,391
90	368	1,386	1,661	1,388
75	147	599	734	557
50	47.3	200	308	169
25	5.5	78.7	125	37.7
10	0	22.4	61.7	7.0

Table 2: Cutoffs for the quantiles of the wealth distribution in the SCF ($ ’000).

Top %	1	5	10	20
share wealth (population)	29.8	54.1	66.9	80.5
percentage GH	65.1	51.9	42.0	30.0
share wealth GH	69.3	59.9	54.8	48.7

Table 3: Percentage of total net worth held by top % of the wealth distribution (first line), percentage of entrepreneurs among the household in the top % of the wealth distribution (second line), and share of entrepreneurial wealth in the total wealth held by households in those quantiles (third line) .

Top %	1	5	10	20
share wealth (population)	29.8	54.1	66.9	80.5
percentage self-employed	62.1	46.6	38.2	26.4
share wealth self-employed	69.1	57.3	52.4	46.1

Table 4: Percentage of total net worth held by top % of the wealth distribution (first line), percentage of self-employed among the household in the top % of the wealth distribution (second line), and share of self-employed’s wealth in the total wealth held by households in those quantiles (third line) .

	Percentage in population	Share of total wealth
Entrepreneurs	11.5	41.6
Gentry and Hubbard	8.7	38.8
Have business assets but no management role	1.7	7.2
Self employed	11.1	39.0

Table 5: Percentage of entrepreneurs in the population and corresponding share of the total wealth held.

Self employed who are entrepreneurs	67.5
Self employed who are GH entrepreneurs	52.7

Table 6: Fraction of self-employed in the SCF.

earnings (0.42). The index of wealth dispersion among entrepreneurs is 0.69 and among non entrepreneurs is 0.73. Therefore, the distribution is very concentrated also within each group. Even without assuming that entrepreneurs differ in their ability level, our model matches very well also the within group inequality. This is due to the interaction between financial constraints and wealth holdings.

2.3 More about entrepreneurs

To conclude this section, it is interesting to note some more facts about entrepreneurs. The left panel of table 5 uses the SCF to compares various definitions of entrepreneurship. The percentage of households whose head declares himself self-employed is around 10%, similar to the percentage of entrepreneurs. However, only around two thirds of the self employed have business assets, and only slightly more than a half of them have more than $5,000 invested in a business (table 6). There is thus a difference between being self employed and owning business assets. Some self-employed households do not invest any of their (non-human) wealth in their activity, or invest only a very small amount. The difference between those two groups is highest in the lower

	Percentage	Share of total wealth
partnership	25.3	16.0
sole proprietorship	49.0	32.8
corporation	25.7	51.2

Table 7: Percentage of businesses by type (second column) and fraction of entrepreneurial wealth held by each type (third column).

quantiles of the wealth distribution, where the self-employed tend to be poorer than the entrepreneurs, and many of them have no business assets. For the higher quantiles, however, the two groups are almost the same. For instance, most (from 85% to 90%, depending on the year) of the self-employed who are in the top 5% of the overall wealth distribution are also entrepreneurs according to our definition. Therefore, if one is interested only in the total wealth held by those groups, or in the right tail of the wealth distribution, there is little difference in using either definition. Since, as mentioned in the previous section, a key aspect of entrepreneurship is ownership of and investment in business assets, we use the first definition.

The group of households that we classify as entrepreneurs is of course very heterogeneous. Table 7 reports the distribution by type of business. Roughly a half of businesses are sole proprietorships, a quarter partnership, and a quarter are incorporated. The larger the business, the more likely it is to be incorporated. For instance, 65% of the entrepreneurs in the top 1% and 56% in the top 5% of the wealth distribution are part of a corporation, while only 26% and 32%, respectively, have sole proprietorship. As a result, the 25% of business that are incorporated comprise more than half of the total entrepreneurial wealth. While a few of those that have a management role in a corporation should be classified as managers rather than entrepreneurs who invest their own wealth, the previous section shows that most of the entrepreneurs have a significant share of own wealth invested in their business. In fact, the ratio of business assets to total net worth, and the fraction of entrepreneurs who have collateralized loans is very similar across these different groups, as well as across different types of business activities.

There is significant heterogeneity in the type of activity of the business, as reported in table 8. A quarter are farms or other agricultural firms, 22% are restaurant or stores, 25% are in manufacturing or contracting, and 27% in

	Percentage	Share of total wealth
agriculture	25.8	16.8
retail	22.0	23.2
manufacturing and contracting	25.1	23.6
services	27.1	36.4

Table 8: Percentage of businesses by activity (second column) and fraction of entrepreneurial wealth held by each type (third column) .

services. Farmers tend to be in the lower quantiles of the distribution, while entrepreneurs with a service firm are over-represented in the higher quantiles. For instance, among the top 1% of the wealth distribution, 10% are in agriculture, and 36% in services, and as a consequence, only 16% of total entrepreneurial wealth is in the hand of the first group, while 36% belongs to the second. Since we do not focus on specific differences within the group, all of these households are qualified as entrepreneurs.

3 The model

3.1 Demographics

The households go through two stages of life: young and old age. To reduce the computational burden while having short time periods we assume that people age stochastically: a young person faces a constant probability of aging every period $(1 - \pi_y)$ and an old person faces a constant probability of dying every period $(1 - \pi_o)$.[8]

The government is infinitely lived, taxes labor income and pays social security benefits to the retirees. Social security benefits are a fixed fraction of the

[8] While we replicate the average length of the young and old age, the stochastic aging structure generates a very small number of agents who live for a large number of years. For the parameter values used in our calibration, .6% of the population are young people who have been in the model for 50 years, and .2% who have been in the model 100 years. However, these fractions are extremely small, and adding age as a state variable would make the problem extremely difficult to solve numerically. Note that, because of the relatively high probability of death, the number of old people who remain old for more than 50 years is basically zero.

average worker's income. The government balances its budget at every period.

3.2 Preferences

Households have standard CRRA preferences on consumption, the instantaneous utility function is thus $\frac{c^{1-\sigma}}{1-\sigma}$. They discount the future at rate β and, in addition, they discount the utility of their offspring at rate η. To study the role of bequests, our model nests life-cycle and fully altruistic households as two extreme cases. In the purely life-cycle version of the model individuals put no weight on the utility of their descendants ($\eta = 0$). In the perfectly altruistic version individuals care about their descendants as much as themselves ($\eta = 1$). We assume an exogenous labor supply.

3.3 Technology

Each person possesses two different types of ability, which we take to be exogenous, positively correlated over time and, as a starting point, uncorrelated with each other.

Entrepreneurial ability (θ) is the capacity to invest capital more or less productively. Entrepreneurs can borrow and invest capital in a technology whose return depends on their own entrepreneurial ability: those with higher ability levels have higher average and marginal returns from capital. When the entrepreneur invests some working capital k, production net of depreciation is $(1-\delta)k + \theta k^{\nu}$. Entrepreneurs face decreasing returns from investing in capital ($0 < \nu < 1$) as their managerial skills become gradually stretched over larger and larger projects. This implies that while the level of entrepreneurial ability is exogenously given, the entrepreneurial rate of return from investing in capital is endogenous and is a function of the size of the project that the entrepreneur implements. In this model economy, therefore, there is a endogenous distribution of returns to entrepreneurial activity which depends on the project size distribution.

Note that there is no uncertainty regarding the returns of the entrepreneurial project. θ is observable and known by all at the beginning of the period. We ignore therefore problems arising both from partial observability and costly state verification, and from diversification of entrepreneurial risk. The simplification is adopted in order to focus only on the effect of the borrowing constraint.

Working ability (y) pertains to the capacity to produce income out of labor. Workers earn y and can save (but not borrow) at a riskless, constant, rate of return.

Most large firms are not controlled by a single entrepreneur and are likely not to face the same financing restrictions that we stress in our model. Therefore, as in Quadrini [25], we model two sectors of production: one populated by the entrepreneurs, and a second sector ("non-entrepreneurial"), populated by large firms. We assume that the the non-entrepreneurial sector can be described by a Cobb-Douglas technology:

$$F(K_c, L_c) = AK_c^{\alpha} L_c^{1-\alpha} \tag{1}$$

where K_c and L_c are the total capital and labor inputs in the non-entrepreneurial sector and A is a constant.

We assume that the entrepreneurs work on their own project without hiring labor and that all of the workers are hired by the non-entrepreneurial sector. In equilibrium the prices are given by the marginal products of each factor of production and the rate of return from investing in capital in the non-entrepreneurial sector must equate the risk free rate that equates savings and investment.

3.4 Credit market constraints

We model contracts as imperfectly enforceable: the entrepreneurs who borrow can either invest the money in the entrepreneurial activity as promised and repay their debt with interest at the end of the period, or run away without investing it and become workers. In the latter case, they retain a fraction f of their working capital k (which includes own assets) and their creditors seize the rest.

These assumptions lead to an endogenous borrowing constraint in the spirit of Kehoe and Levine [19] and Banerjee and Newman [4]: since no creditor lends to a person who has the incentive to escape with the borrowed money, a person can borrow only up to an amount (possibly zero) such that her utility of investing and repaying is at least as large as the utility from running away with the money. The incentive to default is increasing in the amount borrowed because the dishonest borrower gets to keep a fixed fraction of the amount borrowed. In the absence of market imperfections, the optimal level of capital is $k^* = \left(\frac{\bar{r}+\delta}{\nu\theta}\right)^{\frac{1}{\nu-1}}$, and does not depend on initial assets. However, in our setup, the higher the amount of own wealth invested in the business, the larger the amount that the creditor is able to recover, and therefore the larger the amount that the entrepreneur is able to borrow. Own wealth, therefore, acts as collateral, whose amount we endogenize as explained above.

This modelling aspect also implies that households with high entrepreneurial ability but low wealth are not able to start a business. By becoming an entrepreneur, a household is foregoing the potential earnings as a worker, and thus has the incentive to keep the money borrowed, and earn his labor wage. To enforce repayment, the lender must be able to seize at least part of the entrepreneur's wealth, and thus the entrepreneur must invest a suitably large amount of wealth in the business.[9]

3.5 Households

At the beginning of each period, before taking any economic decisions, the current ability levels are known with certainty, while next period's ones are uncertain.

Each young individual starts the period with assets a, entrepreneurial ability θ, and worker ability y, and chooses whether to be an entrepreneur or a worker during the current period.

An old entrepreneur can decide to keep the activity going or retire, while a retiree cannot start a new entrepreneurial activity. We allow entrepreneurs to remain active when old to capture the fact that, while most workers retire before age 65, entrepreneurs often continue their activity until much later.

3.5.1 The young's problem

The young's state variables are his current assets a, earnings ability y, and entrepreneurial ability θ. His value function is:

$$V(a, y, \theta) = \max\{V_e(a, y, \theta), V_w(a, y, \theta)\}, \tag{2}$$

$V_e(a, y, \theta)$ is the value function of a young individual that manages an entrepreneurial activity during the current period. In order to invest k the

[9] Note that we do not impose exogenous minimum firm size or investment level, nor startup costs. We experimented adding a fixed startup cost and a minimum firm size (both of the order of $5,000-20,000), but doing so did had no significant impact on our numerical results.

young entrepreneur borrows $(k-a)$ from a financial intermediary at the interest rate $\bar{r}$. $\bar{r}$ is the risk-free interest rate at which people can borrow and lend in this economy. Consumption c is enjoyed at the end of the period.

$$V_e(a,y,\theta) = \max_{c,k,a'}\{u(c) + \beta\pi_y EV(a',y',\theta') + \beta(1-\pi_y)EW(a',\theta')\} \tag{3}$$

$$a' = (1-\delta)k + \theta k^{\nu} - (1+\bar{r})(k-a) - c \tag{4}$$

$$V_e(a,y,\theta) \geq V_w(f \cdot k, y, \theta) \tag{5}$$

$$a \geq 0 \tag{6}$$

$$k \geq 0 \tag{7}$$

The expected value of the value function is taken with respect to (y',θ'), conditional on (y,θ). $F(y',\theta'|y,\theta)$ is a first order Markov process. $W(a',\theta')$ is the value function of the old entrepreneur at the beginning of the period, before deciding whether he wants to stay in business or retire.

$V_w(a,y,\theta)$ is the value function if he chooses to be a worker during the current period. We have:

$$V_w(a,y,\theta) = \max_{c,a'}\{u(c) + \beta\pi_y EV(a',y',\theta') + \beta(1-\pi_y)W_r(a')\} \tag{8}$$

subject to eq. (6) and

$$a' = (1+\bar{r})a + (1-\tau)\,w\,y - c \tag{9}$$

Where w is the wage and τ is the tax rate on labor. When the worker becomes old, he is retired, and $W_r(a')$ is the corresponding value function.

3.5.2 The old's problem

The old entrepreneur can choose to continue the entrepreneurial activity or retire. The old's person state variables are therefore his current assets a, entrepreneurial ability θ, and whether he was a retiree or an entrepreneur during the previous period.

The value function of an old entrepreneur is:

$$W(a,\theta) = \max\{W_e(a,\theta), W_r(a)\} \tag{10}$$

$W_e(a,\theta)$ is the value function for the old entrepreneur that stays in business. $W_r(a)$ is the value function of the old, retired person. η is the weight on the utility of the descendants: if $\eta = 0$, the household behaves as pure life-cycle, if $\eta = 1$ the household behaves as a dynasty.

$$W_e(a,\theta) = \max_{c,k,a'}\{u(c) + \beta\pi_o EW(a',\theta') + \eta\beta(1-\pi_o)EV(a',y',\theta')\} \tag{11}$$

subject to eq. (4), eq. (7) and

$$W_e(a,\theta) \geq W_r(f \cdot k) \tag{12}$$

The child of an entrepreneur is born with ability level (θ', y'). The expected value of the child's value function with respect to y' is computed using the invariant distribution of y, while the one with respect to θ' is conditional on the parent's θ and evolves according to the same Markov process that each person faces for θ while alive. This is justified by the assumption that the child of an entrepreneur inherits the parent's firm.[10]

A retired person (who is not an entrepreneur) receives pensions and social security payments (p) and consumes his assets. His value function is:

$$W_r(a) = \max_{c,a'}\{u(c) + \beta\pi_o EW_r(a') + \eta\beta(1-\pi_o)EV(a',y',\theta')\} \tag{13}$$

subject to eq. (6) and

$$a' = (1+\bar{r})a + p - c \tag{14}$$

3.6 Equilibrium

Let $x = (a, y, \theta, s)$ be the state vector for our economy, where s distinguishes young workers, young entrepreneurs, old entrepreneurs, and old retired. From the decision rules that solve the maximization problem and the exogenous Markov process for income and entrepreneurial ability, we can derive a transition function $M(x, \cdot)$, which provides the probability distribution of x' (the state next period) conditional on x.

A stationary equilibrium is given by

$$\begin{cases} \text{a risk free interest rate } \bar{r} \text{ and wage rate } \bar{w} \\ \text{taxes and social security payments } \tau, p \\ \text{allocations } c(x), a(x), \text{ and } k(x) \\ \text{and a constant distribution of people over the state variables } x\text{: } m^*(x) \end{cases}$$

such that, given $\bar{r}$, $\bar{w}$, τ and p:

[10] We intend to experiment with lower degrees of intergenerational persistence, although this would add an additional free parameter.

Parameter	Value	Source(s)
σ	1.5	Attanasio et al. [2]
δ	.06	see text
k_s	60%	Quadrini [25]
π_y	.98	see text
π_o	.91	see text
P_y	see text	Huggett [16], Lillard et al. [21]
p	40% average yearly income	Kotlikoff et al. [20]
η	1.0	Perfect Altruism

Table 9: Fixed parameters and their sources.

Calibrated Parameter	Value
β	.87
θ	[0, 0.5]
P_θ	see text
ν	.88
f	75%

Table 10: Calibrated parameters.

- the functions c, a and k solve the maximization problem described above.

- the capital and labor markets clear. The total labor supplied by the workers equal the total labor employed in the non-entrepreneurial sector. The total savings in the economy equal the sum of the total capital employed in the non-entrepreneurial and in the entrepreneurial sectors.

- the government budget constraint balances (the payroll taxes collected on labor income equate social social security payments to the retirees).

- m^* is the invariant distribution for the economy.

3.7 Calibration

Tables 9 and 10 list the parameters of the model. Table 9 shows the set of parameters that we take from other studies and do not use to match moments of the data.

We take the coefficient of relative risk aversion to be 1.5, a value close to those estimated, among others, by Attanasio et al. [2]. As standard in the business cycle literature, we choose a depreciation rate δ of 6%. The share of total capital invested in the non-entrepreneurial sector is fixed at 60%, as computed by Quadrini [25]. The probability of aging and of death are such that the average length of the working life is 45 years, and the average length of the retirement period is 11 years. The logarithm of the income y process for working people is assumed to follow an AR(1). We take its persistence to be .95, as estimated, for instance, by Storesletten et al. [28]. The variance is chosen to match the Gini coefficient for earnings of .38, the average found in the PSID. We assume that the income process and the entrepreneurial ability processes evolve independently; the exact values for the income and ability processes are described in appendix A. The social security replacement rate is 40% of average income, net of taxes (see Kotlikoff et al. [20]). Finally, we set $\eta = 1$, perfect altruism, in the baseline case, and then experiment with different degrees of altruism.

Table 10 lists the remaining parameters of the model: β, θ, P_θ, ν, f and their corresponding values in the baseline calibration. We consider only two values of entrepreneurial ability: zero (no entrepreneurial ability) and a positive number. This implies that P_θ is a two by two matrix. Since its rows have to sum to one, this gives us two parameters to calibrate. We also have to choose values for ν, the degree of decreasing returns to scale to entrepreneurial ability, and f, the fraction of working capital the entrepreneur can keep in case he defaults. This gives us a total of six parameters to calibrate to the data.

We use these six parameters to pin down the following moments generated by the model: the capital to GDP ratio, the fraction of total capital invested in the non-entrepreneurial and entrepreneurial sector, total assets bequeathed every period as a fraction of GDP, the fraction of entrepreneurs becoming workers during each period, the share of income that goes to capital in the non-entrepreneurial sector, and the number of entrepreneurs as a fraction of the population.

Given the features matched in the calibration, we analyze how well the model matches the overall distribution of wealth and the distributions of wealth for entrepreneurs and workers. We then study the role of borrowing constraints and voluntary bequests.

3.8 Results

The first row in table 1 displays various statistics on wealth and wealth distribution in the U.S economy. In the other rows of the table we report the corresponding statistics generated by the simulations of our model economy.

Let us discuss the statistics on the U.S. data first. The notion of capital that we use includes residential structures, plant, equipment, land and consumer durables, and implies a capital output ratio of about 3 for the period 1959-1992 (Auerbach and Kotlikoff [3]). The bequests to GDP ratio is computed by Gale and Scholz [13], who use direct measures of intergenerational flows from the Survey of Consumer Finances. The data on the wealth distribution are from the 1989 SCF. As we discussed in section 2, the various definitions of entrepreneurship we adopted still tend to overestimate the number of those that are entrepreneurs as we model them. In fact, many people owning business assets are not self-employed and have another main occupation. The fraction of households who own more than 5,000 dollars in business assets is 8.7%; the fraction of self-employed in the population is around 11%, but only two thirds of them own business assets, and slightly more than one half own more than 5,000 dollars in business assets. Therefore we calibrate the benchmark simulation to have about 7% of entrepreneurs.

Capital output ratio	Bequest GDP ratio	Wealth Gini	Perc. entr.	Percentage wealth in the top 1%	5%	20%	40%	80%
U.S. data								
3.0	1.5%-1.7%	.78	7.0%	30	54	81	94	100
Baseline without entrepreneurs								
3.0	1.0%	.58	0.0%	5	20	57	84	100
Baseline with entrepreneurs								
3.0	1.6%	.77	6.9%	28	55	79	92	100

Table 11: U.S. calibration.

3.8.1 The model without entrepreneurs

The second row of table 1 shows the same statistics for the model economy without entrepreneurs. In this run, we assign zero entrepreneurial ability to everyone, set the interest to the one generated by the benchmark economy (with entrepreneurs) and change the household's discount factor to match the same capital to GDP ratio. All other parameters are the same as in the benchmark economy. These results thus refer to a model economy with labor earnings risk and a simplified life-cycle structure. As we can see from the table, this model economy produces a distribution of wealth that is much less concentrated than in the data, and in particular, does not explain the emergence of the large estates that characterize the upper tail of the distribution of wealth. Figure 1 compares the data on the distribution of wealth (SCF, 1989 in thousands of dollars) with the one implied by the model without entrepreneurial choice. While the data on wealth display a fat tail, in the model without entrepreneurial choice all households hold less than \$600,000.

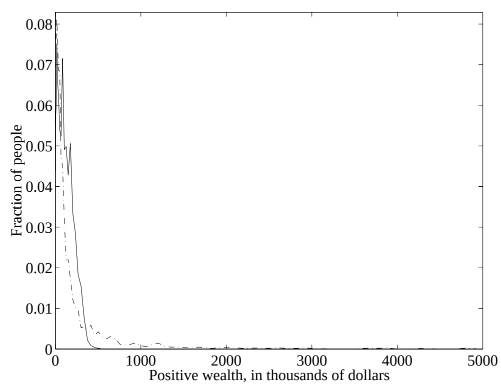

Figure 1: Wealth, dash-dot line: data. Solid: model without entrepreneurs

3.8.2 The model with entrepreneurs

The third row of table 1 refers to the benchmark economy with entrepreneurs. In our baseline simulation $\bar{r}$ is 6.8%, and the share of income that goes to capital in the non-entrepreneurial sector is 33% The fraction of entrepreneurs that during each period switch to being workers is 21%, as in the data. Total bequests as a fraction of GDP are 1.6%.

This parameterization matches the distribution of wealth very well (see figure 2). Figure 3 compares the wealth distributions generated by the model

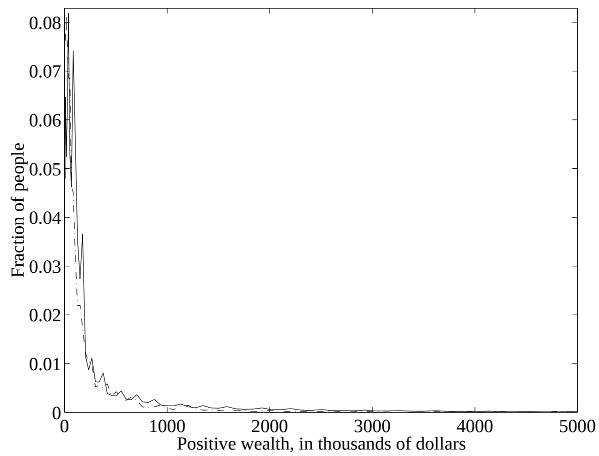

Figure 2: Wealth, dash-dot line: data. Solid: baseline model

for entrepreneurs and workers. Figure 4 shows the wealth distribution for the subpopulation of entrepreneurs, for the model and the data. These pictures reveal two important features of the baseline model. First, consistent with the data, the distribution of wealth for the population of entrepreneurs displays a much fatter tail than the one for workers. Second, contrary to the model without entrepreneurial choice, the baseline model generates distributions of wealth for both entrepreneurs and non entrepreneurs with a significant mass of people that own more than \$600,000. In the model, the non-entrepreneurs in the right tail of the wealth distribution are former entrepreneurs, or descendant of entrepreneurs who have not continued the business of the parents.

In order to understand entrepreneurial behavior, figure 5 displays the saving rate[11] for people that have the highest ability level as workers during the current period. The solid line refers to the people that get the high entrepreneurial ability level during the current period, while the dash-dot line refers to those that get the low entrepreneurial ability draw. Given the same asset level (and potential earnings as workers), the people with high entrepreneurial ability have a much higher saving rate.

[11]The savings rate in the graph is defined as assets in a given period minus assets in the previous period, divided by total income during the period.

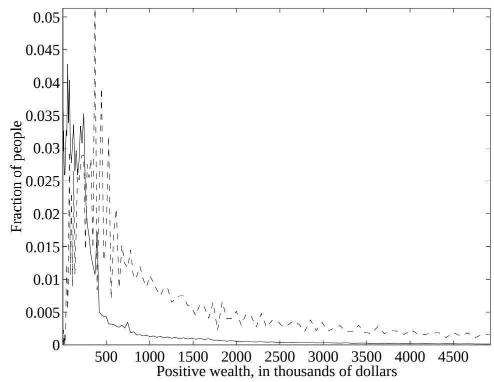

Figure 3: Distribution of wealth, baseline. Solid: workers, dash-dot entrepreneurs

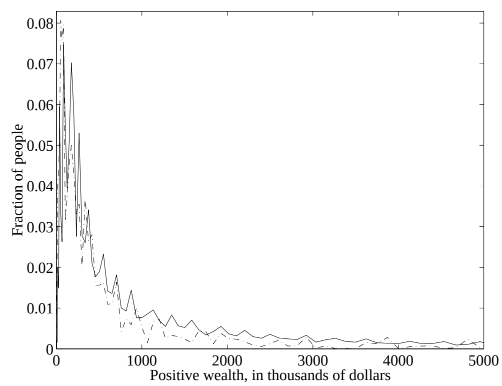

Figure 4: Entrepreneur's wealth, dash-dot line: data. Solid: baseline model

Those with low entrepreneurial ability (and are thus workers) exhibit buffer-stock saving behavior: if their assets are low they save because they are experiencing a high ability level as workers and want to build up their buffer-stock. If their assets are high enough, they dissave and the rate of dissaving is larger, the richer they are. In this simulation, the asset level at which the saving rate goes from positive to negative is below one million dollars.

The people with high entrepreneurial ability, as explained in section 3.4, become entrepreneurs only if their wealth is above a certain level, denoted in the graph by a vertical line. The saving rate of those with high entrepreneurial ability that do not own enough assets to become entrepreneurs is higher than

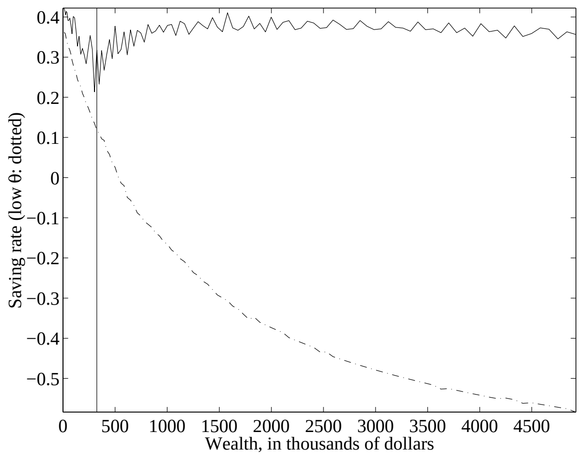

Figure 5: Saving rate, highest ability y

the one for the workers because ability is persistent, and the workers with high entrepreneurial ability save to have a chance to start a business in the future. In this region, the distance between the solid line and the dash-dot line is solely due to the higher implicit rate of return from saving that one could obtain becoming an entrepreneur in the future: all households become workers in this range and earn the same income, but the desire to become entrepreneurs generates higher savings rate for those who have such ability.

The saving rate of those with high entrepreneurial ability and enough assets to become entrepreneurs is positive and considerably higher than that for workers. The return on the entrepreneurial activity is high, and the entrepreneur would like to increase the size of the firm by borrowing capital. However, the borrowing constraint limits the size of the firm. In order to expand the business, the entrepreneur must in part self finance the increase in capital. The combination of higher returns from the business together with the budget constraint thus generates a very high saving rate for entrepreneurs. As the firm expands, the returns decrease. Therefore, also the savings rate will eventually decrease.

In our calibration, with only one positive value for the entrepreneurial ability, there is a unique optimal firm size in the absence of frictions, as explained in section 3.4. However, in the presence of the borrowing constraint, the distribution of firm sizes is non-degenerate, as shown in figure 6. The distribution exhibits high dispersion and a fat tail; the tail is generated by the entrepreneurs

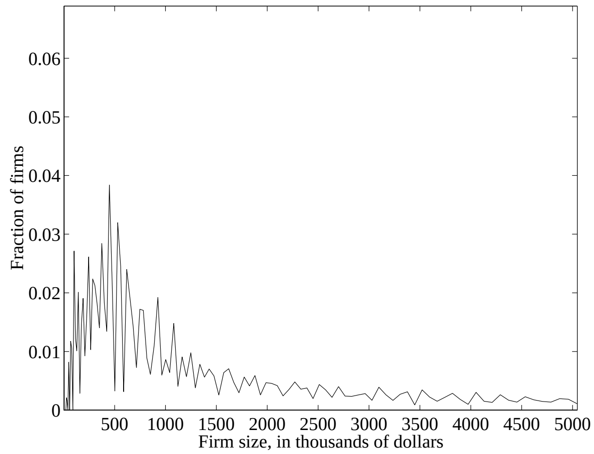

Figure 6: Firm size distribution, baseline simulation

who have remained in business for a long period (and have possibly inherited the firm from the parents) and have had thus time to save and increase the firm size.

3.8.3 The borrowing constraints

In this section, we examine the effect of relaxing the borrowing constraint. To relax the borrowing constraint, we decrease f, the fraction of working capital that the entrepreneur can run away with if he decides to default, from .75 to .70. We consider two experiments. In the first one, we only change f, keeping the other parameters fixed. In the second, we recalibrate the discount factor so that the capital to GDP ratio is 3, as in the baseline case. The latter case thus compares two economies with the same aggregate capital, but with different credit market constraints.

Figure 7 shows the maximum amount of investment (including own assets and borrowed funds) for an entrepreneur that has the highest ability level as a worker, as a function of his own assets. The solid line refers to the baseline model, while the dash-dot line refers to the model with less restrictive borrowing constraints (and recalibrated β). In both economies the entrepreneurs with little assets cannot borrow. The amount of collateral necessary to borrow a positive amount in the two economies coincides at low levels of assets and borrowing. The entrepreneur with lowest ability level as a worker must own at

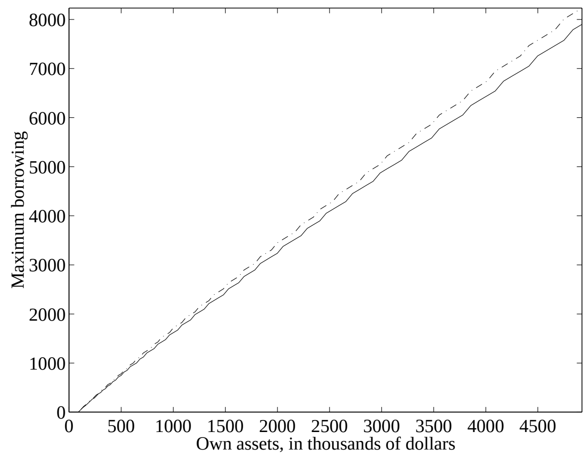

Figure 7: Maximum borrowing, solid=baseline, dash-dot=less restrictive b.c.

least \$10,000 in order to borrow; this amount increases to \$87,000 for the entrepreneur with highest ability level as a worker. This happens because a more able worker is better off in case of default therefore he has to provide more collateral. The key difference in the two economies is that richer entrepreneurs can borrow and invest more in the economy with less restrictive borrowing constraints. For this reason they need less initial assets to implement a project of a given size. If the entrepreneur is rich enough, he is unconstrained.

The third line of table 12 reports the effect of less restrictive borrowing constraints on the aggregates. The capital to GDP ratio increases to 3.4 because the average firm size increases. The distribution of wealth becomes less concentrated, with a lower Gini coefficient, and a smaller percentage of wealth held by the richest (for instance, the richest 1% now owns 23%, compared with 28%). The decrease in wealth concentration is due to the fact that initial assets become less important and poorer entrepreneurs can build larger firms. The percentage of entrepreneurs increases because some of the poor individuals with high entrepreneurial ability can now borrow and start a firm. Similar results remain when we recalibrate the discount factor β to obtain a capital to GDP ratio of 3, as shown in the fourth line.

Capital output ratio	Bequest GDP ratio	Wealth Gini	Perc. entr.	Percentage wealth in the top 1%	5%	20%	40%	80%
U.S. data								
3.0	1.5%-1.7%	.78	7.0%	30	54	81	94	100
Baseline with entrepreneurs								
3.0	1.60%	.77	6.9%	28	55	79	92	100
Less stringent borrowing constraints: $f = 0.7$								
3.4	1.6%	.75	8.1%	23	50	77	91	100
$f = 0.7$, recalibrated β								
3.0	1.6%	.73	7.5%	22	48	75	98	100
No altruism: $\eta = 0$, only involuntary bequests								
2.4	1.1%	.74	5.7%	23	46	74	92	100
$\eta = 0$, recalibrated β								
3.0	1.2%	.75	7.1%	23	49	75	92	100

Table 12: Borrowing constraints and bequests.

3.8.4 Bequests

In the baseline economy households are altruistic towards their children, therefore the total amount of bequests includes both voluntary and accidental bequests due to life-span risk. We use our model to study what happens to entrepreneurial choice and to wealth inequality when households do not care about their descendants and all bequests are accidental.

The fifth line of table 12 displays how the aggregates change when we set to zero the degree of intergenerational altruism. The total capital of the economy decreases considerably (the capital to GDP ratio is now 2.4), and the aggregate flow of bequests as a fraction of GDP drops from 1.6% to 1.1%. The absence of the bequest motive reduces the incentives to accumulate capital. Younger people are bequeathed less wealth, and in the presence of borrowing constraints, this means that young potential entrepreneurs have less resources to start and increase their businesses (the fraction of entrepreneur in fact decreases to 5.7%). Both effects reduce capital accumulation in this economy.

The concentration of wealth decreases: the Gini coefficient of inequality decreases from .77 to .74 and the fraction of wealth held by the richest is reduced, for example from 27% to 23% for the richest 1%. As shown also in other papers, such as De Nardi [9] and Castañeda et al. [6], voluntary bequests are fundamental to explain the concentration of wealth.

To match the initial capital to GDP ratio of 3, we increase the discount factor β from .86 to .88 (last line of the table). The flow of bequests is still much lower than in the case with altruism (the ratio of bequests to GDP is 1.2%). This reduces the formation of large estates and larger businesses, which implies lower wealth concentration. The fraction of entrepreneurs increases slightly compared to the baseline model, from 6.9% to 7.1%. This effect is due to the increase in patience. In the recalibrated model, households have no bequest motive, but are more patient. The effect of patience is more relevant for younger households, who will accumulate more wealth than in the baseline model; however the old will decumulate faster, and keep less wealth, because of the lack of altruism. More people of working age will thus be able to become entrepreneurs. However, the old have less incentives to continue and expand the entrepreneurial activity, and will pass to their offspring less wealth, and smaller firms. This reduces the number and the size of large firms.

4 Conclusions

We developed and solved numerically a model of wealth accumulation and bequests in which entrepreneurs face an endogenous borrowing constraint that limits the amount that they can borrow. The entrepreneur's wealth acts as collateral, so that the richer the entrepreneur, the higher the amount that he can borrow. We show that this setup can generate a wealth distribution that matches the one observed in the data, with a small number of very rich households, many of whom are entrepreneurs. Because of the relation between wealth and borrowing limits, entrepreneurs, although richer, have higher saving rate than workers. We also show that the tightness of borrowing constraints and voluntary bequests are key forces in determining the number of entrepreneurs and the size of their firms, as well as the overall wealth concentration in the population.

These results have implications for policy analysis, such as subsidized loans to entrepreneurs and estate taxes. Subsidized loans would make it cheaper for the entrepreneurs to borrow, but also change their incentives to default, making the effects of this policy a priori ambiguous. Taxing bequests may decrease inequality, while at the same time reduce the amount of entrepreneurial wealth that could be used as a collateral, and thus reduce both the number of entrepreneurs, and the total capital of the economy. We leave this issues for future research.

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A Income and entrepreneurial ability processes

As explained in section 3.7, we assume that the income process is AR(1), and approximate it with a five point discrete Markov chain, using the method described in Tauchen and Hussey [29]. We use an autocorrelation coefficient of .95 (in line with the high persistence found in many microeconomic estimates, such as Storesletten et al. [27]), and choose the variance to match the Gini coefficient of earnings of .38. The resulting gridpoints y for the income process (normalized to 1) are:

$$\begin{bmatrix} 0.2468 & 0.4473 & 0.7654 & 1.3097 & 2.3742 \end{bmatrix}$$

and the transition matrix P_y is:

$$\begin{bmatrix} 0.7376 & 0.2473 & 0.0150 & 0.0002 & 0.0000 \\ 0.1947 & 0.5555 & 0.2328 & 0.0169 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0169 & 0.2328 & 0.5555 & 0.1947 \\ 0.0000 & 0.0002 & 0.0150 & 0.2473 & 0.7376 \end{bmatrix}$$

We assume that the entrepreneurial ability process is uncorrelated with the income process. The two values for ability θ are 0 (meaning no entrepreneurial ability) and a positive value (.5), and the transition matrix P_θ is

$$\begin{bmatrix} .97 & .03 \\ .2 & .8 \end{bmatrix}$$

θ and P_θ are calibrated as explained in section 3.7.

B The algorithm

The algorithm proceeds as follows.

- Construct a grid for the state variables. The maximum asset level is chosen so that it is not binding for the household's saving decisions.
- Fix an interest rate $\bar{r}$ and wage rate w. Taking $\bar{r}$ and w as given, solve for the value functions using value function iteration.
- Construct the transition matrix M. Compute the associated invariant distribution of wealth, starting from a guess π and iterating on $\pi' = M\pi'$ until $(\pi' - \pi)$ is smaller than a given convergence criterion.
- Compute total savings and total capital invested in the two sectors implied by the invariant distribution.
- Iterate on $\bar{r}$ and w until total savings equal total capital, and the ratio of capital invested in the two sectors is a given quantity (see calibration section).

The computation of the value functions is non standard because of the presence of the endogenous borrowing constraints. For each state x, the endogenous borrowing constraint specifies a maximum amount $\hat{k}(x)$ that an entrepreneur can borrow. The specific function $\hat{k}$ depends however on the value functions themselves. In the algorithm we exploit the fact that, for a given set of state variables, if an entrepreneur runs away with a given level of capital $\tilde{k}$, he would also run away with any $\tilde{k} + \epsilon$, where $\epsilon \geq 0$. We adopt the following algorithm: initialize $\hat{k}(x) = k_{max}$, the maximum investment level in the economy. We solve the value functions, iterating until convergence, conditional on this borrowing constraint. For each value of x, we compare the value function associated with remaining an entrepreneur and repaying the debt with the value function associated with default; we find the maximum level of investment (and borrowing) for which the entrepreneur would not default and set the new $\hat{k}(x)$ to this new value, and compute again the value functions conditional on this updated constraint. This procedure is iterated until $\hat{k}$ does not change across iterations.

As we do not constrain the $\hat{k}(x)$ functions to be decreasing when we iterate on them, we are not imposing convergence. Together with the initialization of these functions at the maximum possible level of borrowing, this implies that if the model has more than one solution, and if the algorithm converges monotonically, then we converge to the "best" solution, i.e. the one that allows for the borrowing in the economy. In all of our simulations the algorithm did converge monotonically.