The Collective Model of the Household

Martín García Vázquez

University of Minnesota
Plan for today

1. Static models of the household
3. Some interesting applications of the collective model
5. Summary
Static Unitary model

\[
\max_{(X,x,l^1,l^2,d^1,D^1,d^2,D^2)} U^H(Q,q,l^1,l^2) \\
\text{s.t} \ (BC) \ p' \left( \sum_{k=1}^{K} X_k + \sum_{h=1}^{n} x_h \right) + \sum_{i=1}^{2} \omega^i \left( l^i + \sum_{k=1}^{N} D^i_k + \sum_{h=1}^{n} d^i_h \right) = Y \\
(HP) \ Q_k = F_k(X_k, D_k) \\
\text{and} \\
\sum_{i} q^i = f_h(x_h, d_h) \text{ for all } k \text{ and } h
\]

- \( q \ (Q) \): Quantity consumed of the private (public) good.
- \( l \): leisure.
- \( x \ (X) \): expenditures in private (public) good.
- \( d \ (D) \): time devoted to production of private (public) good.
Testable predictions

- Even in this very general form, this model generates some testable predictions.

- For instance, it predicts that only total household non-labor income, and not individual non-labor incomes, matters for allocations. This property is called **Income Pooling**.

- Standard consumer theory applies to this model. This implies that the Slutsky matrix generated by this model has to be symmetric and negative semi-definite.
  - Remember, the $i, j$ term of the Slutsky matrix is given by:

$$s_{ij} = \frac{\partial \xi_i}{\partial p_j} + \frac{\partial \xi_i}{\partial Y} \xi_i$$

  where $\xi$ is the Marshallian demand (solution to the Utility Maximization Problem)

- These testable restrictions (Income pooling and a symmetric negative semi-definite Slutsky matrix) are usually rejected in the literature.
Collective Model

- Acknowledges that a household is formed by many individuals.
- It does not assume a particular protocol for how decisions within the household are made.
- Instead only assumes that allocations are Pareto efficient.
- It allows each spouse’s position on the Pareto Frontier to depend on distribution factors.
- These are variables that do not affect preferences or the budget set, but rather ”bargaining power”.

Presented by: Martín García Vázquez

Dynamic models of the Household
Collective Model

\[
\max_{(X,x,l^1,l^2,d^1,d^2)} \mu_1(Z) U^1(Q, q^1, l^1) + \mu_2(Z) U^2(Q, q^2, l^2)
\]

s.t (BC) \[
p' \left( \sum_{k=1}^{K} X_k + \sum_{h=1}^{n} x_h \right) + \sum_{i=1}^{2} \omega^i \left( l^i + \sum_{k=1}^{N} D_k^i + \sum_{h=1}^{n} d^i_h \right) = Y
\]

(HP) \[
Q_k = F_k(X_k, D_k)
\]

and

\[
\sum_i q^i_h = f_h(x_h, d_h) \text{ for all } k \text{ and } h
\]
Income pooling and the properties of the Slutsky matrix predicted by the unitary model are rejected in a sample of married households.

The properties of the Slutsky matrix are neither rejected in a sample of single women, nor in a sample of single men.

For a collective household:

1. The Slutsky matrix is the sum of a symmetric negative semi-definite matrix $+$ a matrix of rank $\leq 1$

2. The Slutsky matrix is linear in the distribution factors

3. Let $\xi$ be the marshallian demand. The collective household model predicts:

$$\xi_{z_i} = \theta_i \xi_{z_1} \text{ for all } i \geq 2$$

The paper fails to reject all the restrictions implied by the collective model.
Why should we care?

- The collective model is more empirically supported than the unitary model.
- Why does this matter?
- Some policy recommendations are different for the unitary and the collective model.
- Example: Cash-transfer program to poor households, like Bolsa Familia in Brazil or Progresa in Mexico.
- Unitary model: It doesn’t matter whether the wife or the husband receives the transfer.
- Collective model: This may matter. Moreover, since the objective of this program is to improve outcomes of children, it is very likely that giving the money to mothers will be more effective.
So far, two different static models of the Household: unitary and collective.

Extending this to a dynamic setting may seem natural. However, for the collective model there are at least two ways of doing that:

1. Assume that upon household formation, individuals can commit to any budget-feasible contingent plan: Dynamic Collective Model with Full Commitment.

2. Alternatively, assume that individuals cannot commit to allocations that violate a Participation Constraint: Dynamic Collective Model with Limited Commitment.

Mazzocco (2007) shows how to empirically distinguish one model from the other using household-level consumption data (as opposed to individual level consumption-data).
Dynamic Unitary Model

\[
\max_{\{C_t, Q_t, s_t\}_{t \in T, \omega \in \Omega}} \mathbb{E}_0 \left[ \sum_{t=0}^{T} \beta^t U(C_t, Q_t) \right]
\]

s.t \quad C_t + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \quad \forall \ t, \omega

\quad s_T \geq 0, \forall \ \omega

where:

- \( C_t \) denotes total HH consumption of the private good.
- \( Q_t \) denotes total HH consumption of the public good.
- \( s_t \) denotes savings in riskless asset.
- \( R_t \) denotes the gross return of the riskless asset at \( t \).
- \( y_t^i \) denotes income of spouse \( i \) (exogenous labor supply).
Maximize \( \{c_1^t, c_2^t, Q_t, s_t\}_{t \in T, \omega \in \Omega} \)

\[
\max \mu_1(Z)E_0 \left[ \sum_{t=0}^{T} \beta_t u^1(c_1^t, Q_t) \right] + \mu_2(Z)E_0 \left[ \sum_{t=0}^{T} \beta_2^t u^2(c_2^t, Q_t) \right]
\]

Subject to

\[
s.t \quad \sum_{i=1}^{2} y_i^t + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_i^t + R_t s_{t-1}, \forall t, \omega \]

\[
s_T \geq 0, \forall \omega
\]

Where:

- \( \mu_i \) is the Pareto weight of individual \( i \).
- \( Z \) contains variables that affect the "bargaining power" of individuals within the HH, these are called *distribution factors*.
- Examples of distribution factors can be the relative income of each household member or the local sex ratio.
Full Commitment Collective Model

- For given values of $\mu_i$ we can construct a representative agent for the Household as follows:
  - First, for each time and state of nature you solve:
    \[ \hat{V}(C, Q, \mu(Z)) = \max_{c^1, c^2} \beta_1 \mu_1(Z) u^1(c^1, Q) + \beta_2 \mu_2(Z) u^2(c^2, Q) \]
    \[ \text{s.t} \quad \sum_{i=1}^{2} c^i = C \]
  - The representative agent solves:
    \[ \max_{\{C_t, Q_t, s_t\}_{t \in T, \omega \in \Omega}} \mathbb{E}_0 \left[ \sum_{t=0}^{T} \beta^t V(C_t, Q_t, \mu(Z)) \right] \]
    \[ \text{s.t} \quad C_t + P_t Q_t + s_t \leq Y_t + R_t s_{t-1}, \, \forall t, \omega \]
    \[ s_T \geq 0 \]

where $\hat{V}(C_t, Q_t, \mu(Z)) / \beta^t$
Limited Commitment Collective Model

$$\max_{\{c_t^1, c_t^2, Q_t, s_t\} t \in T, \omega \in \Omega} \mu_1(Z) E_0 \left[ \sum_{t=0}^{T} \beta_t^1 u^1(c_t^1, Q_t) \right] + \mu_2(Z) E_0 \left[ \sum_{t=0}^{T} \beta_t^2 u^2(c_t^2, Q_t) \right]$$

s.t. $E_T \left[ \sum_{t=0}^{T-\tau} \beta_t^i u^i(c_t^i, Q_t^i) \right] \geq u_{i,\tau}(Z), \forall \omega, \tau > 0, i = 1, 2$

$$\sum_{i=1}^{2} y_t^i + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \forall t, \omega$$

$s_T \geq 0, \forall \omega$

- The new constraint is the Participation Constraint.
- It captures that any of the two household members can decide to leave the Household at any point in time.
Limited Commitment Collective Model

- Following Marcet and Marimon (1992, 1988) we can re-write the LC model as:

\[
\max_{\{c_t^1, c_t^2, Q_t, s_t\}_{t \in T, \omega \in \Omega}} \sum_{t=0}^{T} \sum_{i=1}^{2} E_0 \left[ \beta_i^t M_i,t(Z) u_i^i(c_t^i, Q_t) - \lambda_i,t(Z) u_{i,t}(Z) \right] \\
\text{s.t.} \sum_{i=1}^{2} c_t^i + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \forall t, \omega \\
\text{where } M_{i,0} = \mu_i, M_{i,t,\omega} = M_{i,t-1,\omega} + \lambda_{i,t,\omega} \text{ and } \lambda_{i,t,\omega} \text{ is the Lagrangian multiplier corresponding to the participation constraint adjusted by the probability distribution of states and the discount factor.}
Limited Commitment Collective Model

- Again, we can find a Representative Agent for the LC Collective model:
  - For each date and state of nature you solve:

\[
\hat{V}(C, Q, M(Z)) = \max_{c^1, c^2} \beta_1 M_1(Z) u^1(c^1, Q) + \beta_2 M_2(Z) u^2(c^2, Q)
\]

\[
\text{s.t } \sum_{i=1}^{2} c^i = C
\]

- The RA solves:

\[
\max_{\{C_t, Q_t, s_t\} \in T, \omega \in \Omega} \left[ \sum_{t=0}^{T} \beta^t V(C_t, Q_t, M_t(Z)) - \sum_{i=1}^{2} \lambda_{i,t}(Z) u_{i,t}(Z) \right]
\]

\[
\text{s.t } C_t + P_t Q_t + s_t \leq Y_t + R_t s_{t-1}, \ \forall t, \omega
\]

\[
\text{s}_T \geq 0
\]

where \( \hat{V}(C_t, Q_t, M_t(Z))/\beta^t \)
Euler Equations

- The Euler Equation for the Unitary model is given by:

\[ U_C(C_t, Q_t) = \beta \mathbb{E} [U_C(C_{t+1}.Q_{t+1})R_{t+1}] \]

- We can exploit the existence of the Representative Agent for given distribution factors for the LC and the FC collective models to write the corresponding EE.

- The Euler Equation for the FC Collective model is given by:

\[ V_C(C_t, Q_t, \mu(Z)) = \beta \mathbb{E} [V_C(C_{t+1}.Q_{t+1}, \mu(Z))R_{t+1}] \]

- Assuming that savings are not in \( Z \), the Euler Equation for the LC Collective model is given by:

\[ V_C(C_t, Q_t, M_{t+1}(Z)) = \beta \mathbb{E} [V_C(C_{t+1}.Q_{t+1}, M_{t+1}(Z))R_{t+1}] \]
Euler Equations

- The paper will exploit differences in these EE to test one dynamic model of the household against the other.
- For example, if distribution factors do not show up in EE, then this supports the unitary model.
- Distinguishing the LC Collective model from the FC collective model is less straightforward conceptually, but still possible.
Rough intuition behind the tests

- EE differ for the Unitary, Collective with FC and Collective with LC models.
- These EE still differ when one takes a second order approximation.
- That is, the Unitary model EE is the FC Collective model EE when some of the coefficients are restricted to zero.
  - Test the Unitary model by estimating the FC Collective model and testing for parametric restrictions.
- The FC Collective Model EE is the L.C Collective model EE with some parametric restrictions
  - Test the FC Collective model by estimating the LC Collective model and testing for parametric restrictions.
Log-quadratic EE for $C$ (Expressions for $Q$ are similar)

- **FC Collective model:**

\[
\ln \frac{C_{t+1}}{C_t} = a_0 + a_1 \ln R_{t+1} + a_2 \ln \frac{Q_{t+1}}{Q_t} + \sum_{i=1}^{m} a_{i,3} \hat{z}_i \ln \frac{C_{t+1}}{C_t} + \sum_{i=1}^{m} a_{i,4} \hat{z}_i \ln \frac{Q_{t+1}}{Q_t} \\
+ a_5 \left[ \left( \ln \frac{C_{t+1}}{C_t} \right) - \left( \ln \frac{C_t}{C_t} \right) \right] + a_6 \left[ \left( \ln \frac{Q_{t+1}}{Q_t} \right) - \left( \ln \frac{Q_t}{Q_t} \right) \right] \\
+ a_7 \left[ \ln \frac{C_{t+1}}{C} \ln \frac{Q_{t+1}}{Q} - \ln \frac{C_t}{C} \ln \frac{Q_t}{Q} \right] + R(\hat{C}, \hat{Q}, \hat{Z}) + \ln (1 + e_{t+1,C})
\]

- **L.C Collective model:**

Same as EE for FC Collective Model + $\sum_{i=1}^{m} a_{8} \hat{z}_i + \sum_{i=1}^{m} a_{9} \hat{z}_i \ln \frac{C_{t+1}}{C_t}$

\[
+ \sum_{i=1}^{m} a_{i,10} \hat{z}_i \ln \frac{Q_{t+1}}{Q} + \sum_{i} \sum_{j} a_{i,j,11} \hat{z}_i \hat{z}_j + R_C(\hat{C}, \hat{Q}, \hat{Z}) + \ln (1 + e_{t+1,C})
\]
A bit about the Data

- **CEX**: Rotating panel with info for many categories of consumption.
- Short panel dimension.
- $C$: Sum of food at home, food away from home, tobacco, alcohol, public and private transportation, personal care, and clothing of wife and husband.
- $Q$: Sum of maintenance, heating fuel, utilities, housekeeping services, repairs, and children’s clothing.
- Both are deflated by an expenditure share-weighted average of individual price indexes.
- $y^i_t$: Sum of income categories that can be matched to each household individual in CEX.
- $R_t$: Quarterly average of the 20-year municipal bond deflated using the household-specific price.
Implementation of the Test for No Commitment.

1. Estimate No-Commitment Euler Equations using GMM and the efficient weighting matrix

2. Estimate the No-Commitment Euler Equations imposing the restrictions for full-commitment using GMM and the same matrix as before.

3. Calculate the distance $D$-statistic as follows:

$$ D_N = N(J_{LC} - J_{FC}) $$

where $J_{LC}$ and $J_{FC}$ are the sample GMM objectives under LC and FC respectively.

- Newey and West (1987) show that under the Null (in our case under FC) the statistic is asymptotically distributed $\chi^2$
- The test rejects Full Commitment for conventional significance levels.
- Using an analogous procedure the Unitary Model is rejected.
Comment on the test

- The test is done using consumption at the household-level and without tracking individuals over time.
- A test easier to interpret could be achieved if we could observe how individual shares of consumption respond to wage shocks.
- This kind of test requires household-level panel data with detailed information on individual-level consumption.
- This test is performed in Lise and Yamada (2018).
- They conclude that Pareto weights respond to large wage shocks but not to small wage shocks.
- This can be interpreted as a rejection of the Dynamic Collective Model with Full Commitment.
Lundberg, Startz and Stillman 2003: Marital Bargaining can rationalize the Consumption-Retirement puzzle if intra-household bargaining power depends on relative incomes and wives expect to live longer.

Lise and Seitz 2011: Adult-equivalent measures of consumption get the level and the trend of consumption inequality wrong.

1. The **level** of consumption inequality is higher than as measured by using equivalence scales because of intra-household inequality.
2. The **trend** is flatter because decreasing intra-household inequality offsets increasing within-household inequality.

Voena 2015: Examines how divorce laws affect intertemporal choices of couples and welfare.
Divorce laws are likely to affect the risk-sharing ability of households and bargaining power within the household. This matters for the level and the intra-household distribution of welfare.

1. Unilateral divorce introduces the possibility of walking away from marriage. This reduces the ability of couples to commit and therefore to share risks.

2. How property is divided upon divorce is likely to affect the bargaining power of household members. This effect on bargaining power is likely to depend on whether or not unilateral divorce is possible.

Given this, they also matter for household choices.
Institutional Framework

- **Divorce grounds:**
  1. Mutual consent: Divorce takes place if both partners agree to it.
  2. Unilateral Divorce: Any partner can end the marriage at any point in time.

- **Property Division:**
  1. Title based regimes: Assets are allocated after divorce according to ownership.
  2. Community property: Assets are divided equally upon divorce.
  3. Equitable distribution: Courts decide the division of assets upon divorce.
Husband and Wife live for $T$ periods and retire exogenously at age $R$. Each period, choose how much to save, how to allocate consumption and whether or not to stay together. Before retirement they also choose labor force participation for the wife.
Husband and wife derive utility from consumption $c^j_t$, participation $P^j_t$ and a subjective taste for marriage shock $\xi^j_t$

Period utilities:

$$u^j_{married} = u(c^j_t, P^j_t) + \xi^j_t$$  \hspace{1cm} $$u^j_{divorced} = u(c^j_t, P^j_t)$$

$\xi^j_t$ evolves according to:

$$\xi^j_t = \xi^j_{t-1} + \epsilon^j_t, \quad \xi^j_1 = \epsilon_1, \quad \epsilon^j_t \sim N(0, \sigma^2)$$

$$u(c, P) = \frac{c^{1-\gamma}}{1-\gamma} - \psi P$$, with $\gamma \geq 0$ $\psi \geq 0$
Model

Economies of scale and children

- Childbirth is exogenous and happens at predetermined ages.
- Let $x$ be the level of consumption expenditures. The inverse production function of consumption is given by:

$$x = F(c^H_t, c^W_t)e(k) = \left[(c^H_t)^\rho + (c^W_t)^\rho\right]^{\frac{1}{\rho}} e(k)$$

where $e(k)$ is an equivalence scale that depends on the number of children.
- This function tries to capture economies of scale in consumption.
- Key to include this if economies of scale are a big advantage of marriage.
Model

Income over the Life-Cycle

- Labor income of spouse $j$ ($y_{jt}^j$) depends on her human capital $h_{jt}^j$ and her permanent component of income ($z_{jt}^j$):

$$\ln y_{jt}^j = \ln h_{jt}^j + z_{jt}^j$$

where:

$$z_{jt}^j = z_{jt-1}^j + \zeta_{jt}^j$$

and $z_{j1}^j = \zeta_{j1}^j$, $\zeta_{jt}^j \sim N(0, \sigma_{\zeta j})$

with $\zeta_{jt}^j$ iid across time and correlated across spouses.

- Human capital evolves according to:

$$\ln h_{jt}^j = \ln h_{jt-1}^j - \delta(1 - P_{jt-1}^j) + (\lambda_0^j + \lambda_1^j t) P_{jt-1}^j$$
Model

Budget Constraints

- Married couple:

\[ A_{t+1} = (1 + r)A_t + x_t = y_t^H + (y_t^W - d_t^W)P_t^W \]

where \( A_t = A_t^H + A_t^W \)

- Keeping track of individual asset holdings matters only in title-based regimes.
- \( d_t^k \) captures child-care costs that the household has to incur if both parents work.

- Divorcee:

\[ A_{t+1}^j - (1 + r)A_t^j + c_t e(k_t) = \left( y_t^j - \frac{d_t^k}{2} \right) P_t^j \quad j = H, W \]

- Division of assets upon divorce:
  - In a title-based regime, each spouse keeps \( A_t^j \) after divorce.
  - In a community property regime, assets are divided equally.
  - In an equitable distribution regime, assets are divided randomly.
Model

Problem of the divorcee

- $\omega^H_t = \{A^j_t, y^j_t, \Omega_t\}$: State for the divorced husband.
  - $\Omega_t$: Divorce laws.
- $q^j_t = (c^j_t, A^j_{t+1})$ be the control.
- For the wife $h^W_t$ is added as a state and $P^W_t$ is added as a control.
- In each period, divorcees remarry with an exogenous probability $\pi^j_t$.
- Problem of the divorcee:
  \[
  V^{jD}_t(\omega_t) = \max_{q^j_t} u(c^{jD}_t, P^{jD}_t) + \beta \left\{ \pi^{jD}_{t+1} E \left[ V^{jR}_{t+1}(\omega_{t+1}|\omega_t) \right] \right. \\
  \left. \quad + (1 - \pi^{j\Omega}_{t+1}) E \left[ V^{jD}_{t+1}(\omega_{t+1}|\omega_t) \right] \right\}
  \]
  subject to Divorsee B.C for $j = H, W$

- Let $V^{jDR} = \pi^{j\Omega}_t V^{jR}_t(\omega) + (1 - \pi^{j\Omega}_t) V^{jD}_t(\omega)$ be the expected value of entering $t$ as a divorcee.
Model
Problem of the married couple under Mutual Consent

- State: $\omega_t = (A^H_t, A^W_t, y^H_t, y^W_t, \xi^H_t, \xi^W_t, h^W_t, \Omega_t)$
- Choice vector: $q_t = \{c^H_t, c^W_t, P^W_t, A^H_{t+1}, A^W_{t+1}, D_t\}$
- Value function:

$$V_t(\omega_t) = \max_{q_t} (1 - D_t) \left\{ \theta u(c^H_t, P^H_t; \xi^H_t) + (1 - \theta) u(c^W_t, P^W_t; \xi^W_t) \\
+ \beta E[V_{t+1}(\omega_{t+1}|\omega_t)] \right\}$$

$$+ D_t \left\{ \theta \left[ u(c^H_t, P^H_t) + \beta E[V_{t+1}^{HDR}(\omega_{T+1}|\omega_T)] \right] \\
+ (1 - \theta) \left[ u(c^W_t, P^W_t) + \beta E[V_{t+1}^{WR}(\omega_{t+1}|\omega_t)] \right] \right\}$$

s.t B.C in marriage holds if $D_t = 0$

s.t B.C in divorce holds if $D_t = 1$

$$u(c^j_t, P^j_t) + \beta E[V_{t+1}^{jDR}(\omega_{T+1}|\omega_T)] > V_t^{jM} \text{ for } j = H, W \text{ if } D_t = 1$$
Model
Problem of the married couple under Unilateral Divorce

- $\omega_t$ now includes the within-period Pareto weights, $\tilde{\theta}^j_t$, which change over time to ensure that the participation constraint is satisfied.
- These evolve according to:

$$\tilde{\theta}^j_{t+1} = \tilde{\theta}^j_t + \mu^j_t \text{ for } j = H, W$$

- Other difference with respect to the Mutual Consent Regime is that now the Participation Constraints say that the value of marriage now has to be higher than the value of divorce for both partners.
- Upon divorce, assets are divided according to the division rule in place.
Divorce Laws and Household Outcomes

- Mutual Consent:
  \[
  \frac{u_c(c_t^H, P_t^H)}{u_c(c_t^W, P_t^W)} = \frac{1 - \theta}{\theta}
  \]

- Unilateral Divorce:
  \[
  \frac{u_c(c_t^H, P_t^H)}{u_c(c_t^W, P_t^W)} = \frac{\tilde{\theta}_t^W + \mu_t^W}{\tilde{\theta}_t^H + \mu_t^H}
  \]

- Mutual consent: allocations in marriage are not affected by the division rule upon divorce.
- Unilateral divorce: This is no longer the case. Allocations in marriage depend on division rule through the Lagrange multipliers.
- Finally, note that this is partial equilibrium (\(\theta\) is fixed across institutional arrangements).
Data and Estimation

- **Data:**
  1. Panel Study of Income Dynamics (PSID)

- **Three groups of model parameters:**
  1. Some parameters are taken from external sources.
  2. Earnings processes: Estimated without solving the model. These are estimated using Non-linear least squares and using a correction procedure to account for female selection into the labor force.
  3. \((\theta, \psi, \sigma)\) estimated via indirect inference.
# Preset parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
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<tbody>
<tr>
<td>Initial age</td>
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<td>McClements scale</td>
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<td>Years in each period</td>
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<td>Age at death</td>
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<td>Retirement age</td>
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<td>McClements scale</td>
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<td>Economies of scale for children ($e(k)$)</td>
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<td>McClements scale</td>
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<td>RRA ($\gamma$)</td>
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<td>Market returns on assets ($r$)</td>
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<td>Discount factor ($\beta$)</td>
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<td>Retirement income</td>
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<td>Attanasio et al. (2008)</td>
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<td>Child care costs ($g^k$)</td>
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<td>Remarriage probabilities $\pi^i_j$</td>
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<td>Cost of divorce ($CD$)</td>
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Parameters of Income Processes

Table 4—Parameters of the Income Process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$’s returns to experience (constant)</td>
<td>$\lambda_0^W$</td>
<td>0.083</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$W$’s returns to experience (age)</td>
<td>$\lambda_1^W$</td>
<td>-0.004</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$W$’s human capital depreciation</td>
<td>$\delta$</td>
<td>0.080</td>
<td>(0.043)</td>
</tr>
<tr>
<td>$H$’s returns to experience (constant)</td>
<td>$\lambda_0^H$</td>
<td>0.055</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>$H$’s returns to experience (age)</td>
<td>$\lambda_1^H$</td>
<td>-0.0066</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Offer wage gender gap</td>
<td>$\frac{y_1^W}{y_1^H}$</td>
<td>0.59</td>
<td>(0.0365)</td>
</tr>
<tr>
<td>Variance of $W$’s income shock</td>
<td>$\sigma_{\zeta^W}^2$</td>
<td>0.074</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Variance of $H$’s income shock</td>
<td>$\sigma_{\zeta^H}^2$</td>
<td>0.042</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Covariance of $H$’s and $W$’s income shocks</td>
<td>$\sigma_{\zeta^H\zeta^W}$</td>
<td>0.007</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Notes: Income process parameters estimated by nonlinear least squares using PSID data of couples married before divorce law reforms and of divorcees. Standard errors in parentheses computed by bootstrap to account for first-stage estimation errors.
Parameters estimated by indirect inference

1. \( \sigma \): The standard deviation of the marriage quality shock.
2. \( \psi \): The utility cost of participation.
3. \( \theta \): The Pareto weight of the husband.

Auxiliary models (estimated on the sub-sample of couples living in community property states):

\[
\begin{align*}
\text{assets}_{i,s,t} &= \beta \text{Unilateral}_{s,t} + \gamma' Z_{i,t} + \delta_t + f_i + v_{1,i,s,t} \\
\phi_1 &= \frac{\beta}{\text{average assets}} \\
\end{align*}
\]

\[
\begin{align*}
\text{employment}_{i,s,t} &= \phi_2 \text{Unilateral}_{s,t} + \gamma' Z_{i,t} + \delta_t + f_i + v_{2,i,s,t} \\
\text{employment}_{i,s,t} &= \phi_3 + v_{3,i,s,t} \\
\text{ever divorced} &= \phi_4 + v_{4,i,s} \\
\end{align*}
\]
Identification

- \( \phi_2 \) is informative of \( \theta \).
  - When \( \theta \) is high, wife is more likely to be better off when divorced. When unilateral divorce is introduced, her Pareto weight shifts. Since she values leisure, her labor supply goes down.
  - When \( \theta \) is low, wife is better off in marriage. When unilateral divorce is introduced, divorce and the associated drop in consumption are more likely. This increases wife’s labor supply due to a consumption-smoothing motive.

- \( \phi_3 \) is informative of \( \psi \) and \( \theta \).
  - \( \psi \) The higher the disutility of working, the lower the participation rate.
  - \( \theta \) The higher the Pareto weight of the wife, the larger her consumption of leisure and the lower her participation rate.

- \( \phi_4 \), (share of women ever divorced) is informative of \( \sigma \). The higher \( \sigma \), the larger the probability of a very negative marriage quality shock, and the more likely divorce is.
IdentIFICATION

Panel A. $\phi_2$ and $\theta$

Panel B. $\phi_3$ and $\theta$

Panel C. $\phi_3$ and $\psi$

Panel D. $\phi_4$ and $\sigma$

Figure 2. Identification of the Parameters

Notes: Relationship between a parameter of the structural model and a parameter of the auxiliary model obtained by simulation. The solid line is computed using the other estimated parameters. The dotted lines are computed using random values of the other structural parameters.
## Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of preference shocks</td>
<td>$\sigma$</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>Disutility from labor market participation</td>
<td>$\psi$</td>
<td>0.0107</td>
<td>0.0025</td>
</tr>
<tr>
<td>Husbands’ Pareto weight</td>
<td>$\theta$</td>
<td>0.7</td>
<td>0.0155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Auxiliary model parameter</th>
<th>Symbol</th>
<th>Target</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of uni. divorce on savings in CP</td>
<td>$\phi_1$</td>
<td>13.54 percent</td>
<td>13.43 percent</td>
</tr>
<tr>
<td>Effect of uni. divorce on participation in CP</td>
<td>$\phi_2$</td>
<td>$-6.93$ pcpt</td>
<td>$-6.86$ pcpt</td>
</tr>
<tr>
<td>Baseline participation rate in CP</td>
<td>$\phi_3$</td>
<td>55.97 percent</td>
<td>56.03 percent</td>
</tr>
<tr>
<td>Baseline divorce probability in CP</td>
<td>$\phi_4$</td>
<td>19.44 percent</td>
<td>19.44 percent</td>
</tr>
</tbody>
</table>

*Notes: Parameters of the dynamic model \{\sigma, \psi, \theta\} estimated by indirect inference. The parameters of the auxiliary model are \{\phi_1, \phi_2, \phi_3, \phi_4\}.*
Use model to evaluate how introduction of unilateral divorce affected distribution of resources within marriage and upon divorce.

At the estimated value of $\theta = 0.7$, the average share of resources that goes to the wife increases from 39% to 41%.

This is driven by the 19% of couples for whose Pareto weights are modified after the introduction of unilateral divorce.

The distribution of assets upon divorce also changes. In a title-based regime, the average share of assets upon divorce is 40%, while this share is 42% under equitable distribution and 50% under community property by construction.
Divorce laws and consumption insurance

- From the risk-sharing with limited commitment literature we know that the possibility of walking out of the marriage reduces the ability to share risk.
- In this models the introduction of unilateral divorce worsens the commitment technology.
- If we had individual consumption data we could just run BPP regressions on individual consumption under mutual consent and unilateral divorce and see how those coefficients change.
- We do not have data on individual consumption, but we can use simulations from the model.
- The paper also examines how individual consumption responds to divorce in both regimes.

\[ \Delta \log(c_{jt}^j) = \kappa^j + \mu^j \Delta \log(y_{it}^j) + \nu^j' X_{it}^j + \epsilon_{it}^j \]

\[ \log(c_{it}^j) = \chi^j + \eta^j \text{Divorced}_{it} + \psi^j' X_{it}^j + \rho^j_i + \nu_{it}^j \]
As expected, consumption is more responsive to the husband’s income under unilateral divorce for all property division regimes.

This is not true for the wife’s income, because under unilateral divorce wives work less.

Moreover, divorce is associated with a drop in consumption in all institutional arrangements for both wife and husband.

Husbands suffer the least from divorce under unilateral divorce and a title-based regime.

Wives suffer the least under mutual consent and a community property regime.
Summary

- Introduced the static unitary and collective models of the household, and revised some literature that suggests that the collective model has more empirical support.

- Use an influential paper to see how these static models can be extended to a dynamic context.

- When dynamics are introduced, commitment becomes an issue. Revised how to empirically distinguish a model with commitment from a model without commitment.

- Revised an interesting application of the dynamic collective model.

- From this application, we learned that divorce laws affect intertemporal choices of married couples and their welfare by changing their ability to commit and the distribution of resources upon divorce.