Investment Opportunities and the Sources of Lifetime Inequality*

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Abstract

The historically high college wage premium and the perennially high equity premium on stocks raise a natural question: Are college and the stock market (two high-yield, but risky, investment opportunities) currently acting to exacerbate initial disparities in household conditions, or are they allowing those initially disadvantaged to at least partially catch up? We approach this question with a rich life cycle model featuring empirically-plausible ex-ante heterogeneity in learning ability, initial human capital, and initial wealth and, importantly, detail on the features of college, the stock market, and the financing of investments in them. We show that the opportunity to invest in college, on net, increases inequality while, perhaps surprisingly, the stock market serves to lower it. Both college and the stock market reduce the contribution of initial conditions to lifetime inequality. College, however, raises the importance of learning ability relative to other initial conditions. Finally, we assess the importance of the magnitudes of college—and equity—premia for inequality and its sources. We find that the change in these premia relative to their levels in the 1970s alters inequality and the contributions of initial conditions, college, and stocks as follows [add results here].

JEL Codes: E21; G11; I24;

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*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

Are the stock market and higher education serving as vehicles to allow individuals to escape adverse initial conditions, or mainly *perpetuating* disparities in endowments? The goal of this paper is to provide a quantitatively compelling answer to this question.

We are motivated by the fact that, in recent decades, the wage premium for college-educated individuals relative to high-school completers (and those who have completed some college) has risen dramatically—increasing lifetime income by a factor of roughly two—and has stabilized at a historically high level (Goldin and Katz, 2007). This investment is thus lucrative for those who complete college and of a magnitude that may erase disparities in household conditions prevailing prior to enrollment. Moreover, a variety of policies (subsidized student loans and means-tested grants, most importantly) have been enacted precisely to mitigate the effect of initial conditions on access to college. As a result, one might suspect that college acts as an equalizer for household earnings by promoting mobility in income.

A second high-yield investment, especially in the long run, is stock-market equity. Famously of course, the return on this asset is so high that existing work has largely failed to account for it. While stock-market investment is less frequently emphasized than college investment as an agent of opportunity and upward mobility, it is not implausible that it is one. Household participation in the equity market has increased with the growth of defined contribution plans and, a priori, the returns to equity have been measured to be broadly similar to those on human capital (e.g., Cahuc et al., 2014, and references therein). As with college, these returns may well serve as a way to promote mobility and thereby shorten the shadow cast by initial conditions.

There are, however, several forces working against the ability of these two investments to limit overall inequality and the power of initial conditions for lifetime earnings, income, and wealth.[1] First, favorable initial conditions raise the expected returns to investment in both assets, while disadvantageous initial conditions lower them. In the case of college, initial conditions (e.g., in learning ability) are strongly correlated with the likelihood of completion. This force, all else equal, implies that the high return may accrue disproportionately to the already well-prepared and financially well-off. As a result, under current wage premia, college may be an engine of, rather than a brake on, growth in lifetime inequality.

In the case of stocks, those with higher initial wealth may again be disproportionately advantaged. While borrowing constraints for student loans may not be tight, given direct policy interventions aimed at broadening credit access for education, the same is not true for stocks. As a

[1] As discussed below, in our framework these initial conditions will take the form of initial human capital, financial wealth, and ability to learn.

result, those with low initial wealth will find it difficult to make leveraged investments in the stock market. Indeed, binding credit limits have been implicated in models aimed at understanding why younger households hold so few stocks: "junior can't borrow" (Constantinides et al., 2002). More recently, Davis et al. (2006) show that even if one allows borrowing to buy stocks, the applicable interest rate is very high, making the de facto return on stocks lowest for those with least initial wealth.

Second, investing in college and in the stock market means bearing risk. College, for its part, carries substantial risk associated with non-completion (Restuccia and Urrutia, 2004; Bound et al., 2010; Johnson, 2013), with partial completion offering relatively little reward. The risk of non-completion is, furthermore, negatively related to household wealth and measures of collegiate preparedness, which themselves are positively correlated. All else equal, this will disproportionately deter those with low wealth and preparedness and thereby increase inequality and the importance of initial conditions. That the stock market is risky needs less argument. The risk is in fact large, of an order of magnitude higher than that on treasury bills or bonds (Mehra and Prescott, 1985). The ability to bear this risk is likely not uniform in the population. Specifically, the wealth-rich may plausibly be more risk tolerant (in the sense of absolute risk aversion, certainly) than their less-wealthy counterparts, making them more willing to hold greater levels of risky equity. As a result, such agents may be able to remain in—and thereby reap the substantial long-run returns from—stocks. Their initially poorer counterparts will, for the same reasons, be unwilling to take on the risk needed to garner the high-yields, and so remain less wealthy.

Lastly, all individuals in the US appear to face significant uninsurable risk to earnings (Blundell et al., 2008). While this risk acts, mechanically, to limit the *proportion* of earnings, income, and wealth variation attributable to initial conditions, it will clearly cause the *level* of disparities to increase, all else equal. In particular, a large literature suggests that labor market risk looms largest for the least skilled (Blundell et al., 2008). Thus, if the least prepared are also the least able to acquire college and the labor market insurance it seems to offer, college will serve to intensify the effect of initial conditions. Labor market risk also compounds the risk of borrowing to invest in both college and the stock market, even if leveraged stock-purchases were feasible. As a result, labor market risk in adult life may once again imply that college and the stock market primarily allow the initially well-off to do even better as their poorer, less-prepared counterparts are forced to opt for safer, lower-return investments.

Our approach is conceptually straightforward, but to our knowledge, novel. We first construct a model economy in which individuals differ ex-ante in their initial human capital, financial wealth, and ability to learn. These agents have access to quantitatively disciplined representations of human capital and stock-market investment. In the case of human capital investment, our model

allows for college education, but where that investment is risky, and where the structure of financial aid captures existing need- and merit-based aid programs. Given initial heterogeneity in income and wealth, these features will affect the decisions of investors in college differently. With respect to life after college, our model allows for the accumulation of skill throughout life, something that reflects findings of Altonji et al. (2013), and many others, that earnings dynamics (wage growth) indeed reflect human capital accumulation. In the case of stock-market investment, our model allows again for risk, as well as for participation costs as measured in the literature. Due again to initial heterogeneity, even a constant cost of stock-market participation will matter differentially across agents. Heterogeneity in household incentives to make investments in either human capital or financial equity is thus at the heart of our approach. The initial heterogeneity in human capital, financial wealth, and learning ability are, in turn, quantitatively well-disciplined by the empirics of earnings over the life cycle. Moreover, our modeling of college includes variations in features such as college quality, costs, and majors. This ensures that we appropriately attribute observed differences in human capital to such factors and not just to initial endowments.

Once we have constructed the baseline model, we measure overall inequality, the contribution of initial conditions to inequality, and overall mobility in this environment. We then compare these measures to those obtaining in environments where one or more investment opportunity (college and stocks) is unavailable. We do so as follows: First, we directly compare the unconditional variance of earnings, income, and wealth across environments. Second, we gauge the extent to which initial conditions matter for inequality in each case, following the methodology of Storesletten et al. (2005). Third, we measure the role played by the dispersion of each initial condition individually: learning ability, initial human capital, and initial household wealth. We do this by measuring inequality in economies in which each of these is set in turn to its median value for all agents. Finally, to describe mobility, we construct transition probability matrices for each environment.

Our central findings are as follows. Investment in college, on net, increases inequality while, perhaps surprisingly, the stock market serves to lower it. Both college and the stock-market reduce the contribution of initial conditions to inequality. College, however, raises the importance of learning ability relative to other initial conditions, suggesting that the distribution of learning ability in the population is key to the role played by higher education in inequality. As for mobility [add discussion of results pertaining to mobility here]

Finally, to assess the importance of the magnitudes of college—and equity—premia for inequality and its decomposition, we use our model to answer two further questions. First, to what extent is the historically high college wage premium driving currently observed inequality? To answer this, we recalibrate our model to match economic conditions in the 1970s (including the equity premium prevailing at that time), a period in which the college premium was substantially

lower than it is at present. We then measure, just as we did with the baseline model, the level of inequality in the 1970s economy and the contribution of initial conditions, college, and stocks to measured inequality. Comparing the 1970s economy to the current baseline, we find that [add results here comparing the levels of inequality and the contribution of initial conditions, college, and stocks.]

Second, how would current inequality and its sources change if current premia were different? We answer this question by comparing scenarios in which agents in the baseline economy face lower or higher returns to college or stocks than they do in the baseline environment. Specifically, to measure the role of the equity premium, we recalculate our results under the assumption that the equity premium is one standard deviation lower (or higher) than it is in the baseline. As for the role of the college premium, we examine a fifty-percent increase, and then decrease, in the mean wage premium. As above, we change the premia that agents face, but in this case, unlike the previous one, we do not recalibrate the model. We find that [add results here].

1.1 Related work

The question of the determinants of inequality, economic mobility, and how uncertainty is resolved over the life-cycle has received enormous attention in the literature. We refer the reader to Heathcote et al. (2009) and the references therein for a comprehensive assessment of models that allow for heterogeneity and, especially, uninsurable risks. As for studies of the temporal resolution of inequality, important benchmarks include Keane and Wolpin (1997), who find that most inequality is determined very early in life, and Huggett et al. (2011), who find a much greater role for earnings shocks and, hence, a slower path to resolution of one's lifetime income. Our approach is strongly complementary to these two papers, especially the latter, in two ways. First, it can be seen as opening the "black box" of initial conditions. We do this by explicitly modeling the college investment decision and thereby substantially endogenizing what, in their work, is assigned to initial human capital. Nonetheless, an exogenous component remains, driven by exogenous distributions of learning ability and of pre-collegiate human capital and financial wealth. Second, our model follows Huggett et al. (2011) in allowing self-insurance via financial markets. While their work allows for trade in a low-return, risk-free bond, ours also allows for stock-market investment and the high mean returns (and risk) that this vehicle carries.

A second strand of work that we build on is aimed at understanding the role of human capital in inequality when the particulars of college education, in terms of its costs as a function of observable enrollee and household characteristics, are modeled explicitly. Important references in this literature include Arcidiacono (2005); Garriga and Keightley (2007); Chatterjee and Ionescu

(2012); Johnson (2013) and Altonji et al. (2015). Recent work of Abbott et al. (2013) is clearly relevant as well. They develop an extremely rich representation of higher education, allowing for a variety of salient features that have bearing on the measurement we are interested in: gender, labor supply during college, government grants and loans (including private loans), and heterogeneity in familial resources. An important distinction between our work and theirs is their primary focus on policy counterfactuals, which their detailed general equilibrium formulation permits.[2] Our work adopts a partial equilibrium perspective, but follows theirs in richly detailing the attributes of human capital investment through the collegiate education process (e.g., we allow for heterogeneity in college type, major type, and need- and merit-based aid). These features allow us to more precisely derive the (not-directly-observable) joint distribution of initial human capital and learning ability, which as argued above, plays a critical role in determining the net impact of these investment opportunities on inequality and mobility.

We are also informed by the work that emphasizes the bias imparted to measured returns to college by the possibility of noncompletion. Hendricks and Leukhina (2014) allow for selection effects and argue that two layers of selection are important: weakly-prepared students disproportionately fail to enroll in college, and those who enroll fail at high rates to complete.[3] Our model allows for both effects to operate, and thereby avoids overstating the payoff to college. With respect to failure risk, our work builds on earlier work of Restuccia and Urrutia (2004) and Akyol and Athreya (2005).[4] More recently, Athreya and Eberly (2013) demonstrate that college failure risk hinders low-wealth individuals, even relatively well-prepared ones, from enrolling in college.

We are also influenced by recent empirical insights of Hoxby and Turner (2015) who demonstrate the presence of an economically significant level of "undermatching" whereby good students from poorer households systematically attend (if at all) colleges that by objective measures are of lower quality and often high cost. This is important for the question of how college—as currently structured—contributes to observed inequality and its temporal resolution. Lastly, recent work of Stinebrickner and Stinebrickner (2013) argues that students may substantially overestimate their ability to complete a major in science. This work leads us to allow for major choice—which is clearly relevant for lifetime income—with attendant non-completion risk.[5]

With respect to stocks, our work follows the literature on portfolio choice in life-cycle models

[2]See also Epple et al. (2013) and Cestau et al. (2015) for analysis of higher education policies in the presence of substantial enrollee heterogeneity.

[3]See also Arcidiacono (2004).

[4]The possibility of college failure has also been evaluated in work of Stange (2012) and Ozdagli and Trachter (2011).

[5]On the issue of major choice and student information sets, see also recent work of Wiswall and Zafar (2015) that uses experimental information to measure the role of beliefs in major choice and argues that tastes play a significant role.

(see, for example, Cocco et al., 2005). In spirit, our work is also closely related to Kim et al. (2013), which also features both education and stock market investment. [6]

The remainder of the paper is organized as follows. Section 2 describes the model and Section 3 the data we use to calibrate it. Section 4 summarizes the calibration, with Appendix A providing details for the interested reader. The results are reported in Section 5 and Section 6 concludes.

2 Model

Our aim is to quantitatively assess the importance of two specific investments, college and stocks, for observed inequality. We are also interested in how the presence of these investment opportunities alters the relative contribution of initial endowments and shocks over the life cycle matter to observed inequality. We begin with a baseline model in which both investments are available. The model incorporates an array of salient features of both investments. The details are described below.

2.1 Environment

Time is discrete and indexed by $t = 1, ..., T$ where $t = 1$ represents the first year after high school graduation. We allow for three potential sources of heterogeneity across agents: their immutable learning ability, a, their initial stock of human capital, h^{HS}, and their initial assets, x_1. The these characteristics are drawn jointly according to a distribution $F(a, h, x)$ on $A \times H \times X$.

Each period, agents choose how much to consume and how to divide their time between learning and earning, as in Ben-Porath (1967). Agents also decide how much of their wealth to allocate to stocks, s, versus bonds, b. The latter may be used to either borrow or save. Debt is not defaultable and is subject to a borrowing limit, $-\underline{b}$, where $\underline{b} > 0$.

Agents work and accumulate human capital using the Ben-Porath technology until $t = J - 1$. Agents can also accumulate human capital by choosing to attend college in the first period. College can be financed using wealth, unsecured debt, b, and non-defaultable, unsecured student-loan debt, d. Agents retire in period $t = J$ after which they face a simple consumption savings problem.

To capture an important source of risk to human capital, we assume that agents may fail to complete college.[7] At the end of four years in college, the probability of completion—which

[6] Indeed, in Athreya et al. (2016), we incorporate the elements of Kim et al. (2013) in a model with human capital investment (though without 4-year college) and show that it can match important life-cycle observations on household stock market participation.

[7] For example, Bound et al. (2010) report, using NLS72 data, that only slightly over half of all college enrollees graduated within 8 years of enrollment.

depends on the agent's innate ability and human capital accumulation while in college—is realized. Those who complete college start their working life with human capital h^{CG} while those who fail to complete start life with human capital h^{SC}, where SC denotes "some college." Those who choose not to go to college start their working life with human capital h^{HS}.

[We will add college types, major choice, and differences in costs and completion probabilities across each. We will also add a stock-market participation cost.]

2.2 Preferences

Agents maximize the expected present value of utility over the life cycle:

$$\max E_0 \sum_{t=1}^{T} \beta^{t-1} u(c_t), \tag{1}$$

where $u(.)$ is strictly concave and increasing. Preferences are represented by a standard time-separable CRRA utility function over consumption. Agents do not value leisure.

2.3 Human Capital

Agents can invest in their human capital in two ways—by investing in a college education when young and by apportioning some of their available time to acquiring human capital using the Ben-Porath technology.

2.3.1 College Investment

Those who invest in college face the risk of noncompletion, which decreases with the level of human capital accumulated during college. Specifically, the probability of completion, $\pi(h_5(h_1, a, l^*_{1,\ldots,4}))$ is an increasing function of the amount of human capital accumulated after completing four years in college, h_5, which in turn increases with the initial human capital stock, h_1, the agent's learning ability, a, and the amount of time $l^*_{1,\ldots,4}$ that she chooses to allocate to human capital accumulation (versus working) while in college. [8]

Those who work in college earn a wage $w_{col}(a)$ per unit of time worked. We assume that the rate increases with ability in order to prevent low-ability students from enrolling in college only to enjoy earnings during college that are higher than what they would have earned had they not

[8] Modeling investment in four-year college and the risk of dropping out at the end of the fourth period in the model are justified by data: according to the BPS 1996/2001, 68.5% of students enroll in four-year colleges. Our findings also show that 89% of college dropouts are enrolled in college at least for three full years.

enrolled in college. We assume that the growth rate in earnings during college is 0. Working during college diverts time from human capital accumulation and therefore increases the probability of non-completion.

There are several possible sources of college financing: savings, x, borrowing, b, earnings from working while in college, and student loans. Agents are allowed to take out student loans up to $d(x) = \min[d_{max}, (\overline{d} - x)]$, which represents the full college cost, $\overline{d}$, minus any savings, x, up to a student loan limit d_{max}. They choose the loan amount, d, at the beginning of college and receive equal fractions of the loan each period in college. After college, they repay this loan in equal payments, p, which are determined by the loan size, $d(x)$, interest rate on student loans, R_g and the duration of the loan, P. Consistent with the data, the interest rate on student loans is $R_f < R_g < R_b$, where R_f is the risk-free savings rate and R_b the borrowing rate on unsecured debt.

[We will further enrich this environment to allow for at least two types of colleges and majors, with their attendant differences in cost. We will make completion conditional on these in addition to above-described parameters.]

2.3.2 Ben-Porath Human Capital Investment

During college and while working, agents accumulate human capital (as in the classic Ben-Porath, 1967, model):

$$h_{t+1} = h_t(1 - \delta_i) + a(h_t l_t)^\alpha \text{ with } \alpha \in (0, 1) \tag{2}$$

Human capital depreciates at a rate δ_i with $i \in \{cg, nc\}$ and $\delta_{cg} > \delta_{nc}$ where cg stands for individuals who have a college degree and nc stands for those without a college degree (high school graduates, college dropouts, and those enrolled in college).

The return to human capital is in the form of earnings during working life, which are subject to shocks as described below.

2.4 Labor Income

During an agent's working life, their earnings are given by:

$$y_t = w_t(1 - l_t)h_t z_{it}$$

where w_t is the rental rate of human capital, $(1 - l_t)$ is the time spent working and z_t is the stochastic component. The rental rate of human capital evolves over time according to

$$w_t = (1 + g_i)^{t-1}.$$

The growth rate for college graduates, g_{cg}, is greater than the growth rate for those with no college, g_{nc}, which includes college dropouts and high school graduates.[9]

The stochastic component, z_{it} also varies by education status ($i \in \{nc, cg\}$) and consists of a persistent shock $u_{it} = \rho u_{i,t-1} + \nu_{it}$, with $\nu_{it} \sim N(0, \sigma_\nu^2)$ innovation process and a transitory (iid) component $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$. The variables u_{it} and ϵ_{it} are realized at each period over the life cycle and are not correlated.

[We will modify this to allow earnings to depend on the major chosen in college]

2.5 Means-Tested Transfer and Retirement Income

Because these may have an impact on cross-sectional inequality, we allow agents to receive means-tested transfers, τ_t, which depend on age, income, and assets. Following Hubbard et al. (1994) we specify these transfers by

$$\tau_t(t, y_t, x_t) = \max\{0, \underline{\tau} - (\max(0, x_t) + y_t)\} \tag{3}$$

These transfers capture the net effect of the various U.S. social insurance programs that are aimed at providing a floor on consumption.

After period $t = J$, in which agents start retirement, they get a constant fraction of their income in the last period as working adults, $\phi^i(y_J)$, which they allocate between risky and risk-free investments. We allow the fraction for college graduates to differ from the fraction for all other agents.

2.6 Financial Markets

There are two financial assets in which the agent can invest, a risk-free asset, b_t, and a risky asset, s_t.

[9] The growth rates for wages are estimated from data. Evidence shows that wage growth rates for college dropouts and people with no college are similar and are lower than the growth rate for college graduates.

Risk-free assets

An agent can borrow or save using asset b_t. Savings will earn the risk-free interest rate, R_f. We assume that the borrowing rate, R_b, is higher than the savings rate: $R_b = R_f + \omega$. Debt is non-defaultable and comes with a borrowing limit $\underline{b} > 0$.

Risky assets

Risky assets, or stocks, earn stochastic return $R_{s,t+1}$ in period $t+1$, given by:

$$R_{s,t+1} - R_f = \mu + \eta_{t+1}, \tag{4}$$

where η_{t+1}, the period $t+1$ innovation to excess returns, is assumed to be independently and identically distributed (i.i.d.) over time and distributed as $N(0, \sigma_\eta^2)$. We assume that innovations to excess returns are uncorrelated with innovations to the aggregate component of permanent labor income.

Given asset investments at age t, b_{t+1} and s_{t+1}, financial wealth at age $t+1$ is given by

$$x_{t+1} = R_j b_{t+1} + R_{s,t+1} s_{t+1}$$

with $R_j = R_f$ if $b \geq 0$ and $R_j = R_b$ if $b < 0$.

2.7 Agent's Problem

The agent chooses how much to consume, how much time to allocate to learning, asset positions in stocks and bonds (or borrowing) and whether or not to invest in college to maximize lifetime utility.

We solve the problem backwards starting with the last period of life when agents consume their savings. The value function in the last period of life is set to $V_T^R(a, h, x) = u(x)$.

Retired agents do not accumulate human capital. They face a simple consumption-savings problem but may choose to invest in both risk-free and risky assets. The value function is given by

$$V^R(t, a, h, b, s) = \sup_{b', s'} \{ \frac{c^{1-\sigma}}{1-\sigma} + \beta V^R(t+1, a, h, b', s') \} \tag{5}$$

where

$$c + b' + s' \leq \phi^i(y_J) + R_j b + R_s s$$

In the above, $R_j = R_f$ if $b \geq 0$ and $R_j = R_b$ if $b < 0$.

2.7.1 Problem in Working Phase for those with No College

We use $V_J^R(t, a, h, b, s)$ from Equation 5 as a terminal node for the adult's problem on the no college path. We solve

$$V^{HS}(t, a, h, b, s, z) = \sup_{l, h', b', s'} \{\frac{c_t^{1-\sigma}}{1-\sigma} + \beta E V^{HS}(t+1, a, h^{'}, b^{'}, s^{'}, z^{'})\} \tag{6}$$

where

$$\begin{aligned} c + b^{'} + s^{'} &\leq w(1-l)hz + R_b b + R_s s + \tau(t, y, x) \text{ for } t = 1, .., J-1 \\ l &\in [0, 1] \\ h^{'} &= [h + a(hl)^{\alpha}] \end{aligned}$$

2.7.2 Problem in Working Phase for those who Attended College

As before, we use $V_J^R(t, a, h, b, s)$ from the retirement phase as a terminal node and solve for the set of choices in the working phase $j = 5, .., J-1$ of the life cycle. We further break down the working phase into a student loan post-repayment period and a repayment period. In the post-repayment period, $t = P+1, ..., J-1$, the problem is identical to the one for working adults on the no-college path.

During the repayment period , $t = 5, ..., P$, agents have to repay their student loans with a per-period payment

$$p = \frac{d(x)}{\sum_{t=1}^{P-5} R_g^t}.$$

The value function is given by

$$V^i(t, a, h, b, s, z) = \sup_{l, h', b', s'} \{\frac{c_t^{1-\sigma}}{1-\sigma} + \beta E V^i(t+1, a, h^{'}, b^{'}, s^{'}, z^{'})\}, i = CG, SC \tag{7}$$

where

$$\begin{aligned} c + b^{'} + s^{'} &\leq w(1-l)hz + R_j b + R_s s + \tau(t,y,x) \text{ for } t = P+1,..,J-1 \\ c + b^{'} + s^{'} &\leq w(1-l)hz + R_j b + R_s s + \tau(t,y,x) - p(x_1) \text{ for } t = 5,..,P \\ l &\in [0,1] \\ h^{'} &= [h + a(hl)^{\alpha}] \end{aligned}$$

$R_j = R_f$ if $b \geq 0$ and $R_j = R_b$ if $b < 0$.

2.7.3 Problem in College

For the college phase $t = 1,..,4$ of the life cycle we first take into account the risk of dropping out from college and use $V^C(5,a,h,b,s,z) = \pi(h_5)V^{CG}(5,a,h,b,s,z) + (1-\pi(h_5))V^{SC}(5,a,h,b,s,z)$ as the terminal node. The value function is given by

$$V^C(t,a,h,b,s,z) = \max_{l,h',b',d} \left[\frac{c^{1-\sigma}}{1-\sigma} + \beta V^C(t,a,h,b,s,z) \right] \tag{8}$$

where

$$\begin{aligned} c + b^{'} + s^{'} &= w_{col}(1-l) + R_b b + R_s s + d/4 - \hat{d} \\ l &\in [0,1] \\ h^{'} &= [h + a(hl)^{\alpha}] \\ d &= \min[d_{max}, (\overline{d} - x)]. \end{aligned}$$

Once the college and no-college paths are fully determined, agents then select between going to college or not by solving $\max[V^C(1,a,h,x), V^{HS}(1,a,h,x)]$.

[We will incorporate college choice into the set up here.]

3 Data

In order to map our model to data, we use data on annual earnings from the March Current Population Survey (CPS) and on financial assets from the Survey of Consumer Finances (SCF).

3.1 Life cycle earnings

As described in more detail in the next section, we calibrate our model to match the evolution of mean earnings, earnings dispersion, and earnings skewness over the lifecycle. To this end, we first estimate lifecycle profiles, for ages 23 to 60 (i.e. the "working life"), of mean earnings, the earnings Gini coefficient, and the mean/median earnings ratio using data from the March CPS, obtained through IPUMS at the University of Minnesota. We use data on annual wage and salary income for male heads of household with at least a high-school diploma (or equivalent) for calendar years 1963-2013 (corresponding to survey years 1964-2014). We restrict our sample to individuals who worked at least 12 weeks in the reference year and earned at least \$1,000 (in constant 2014 prices). We use the CPS weights to ensure that each year's sample is representative of the overall U.S. population; additionally, we renormalize the weights in each year in order to keep the population constant at its 2014 value; this way we abstract from issues related to population growth.

We use these data to construct lifecycle profiles for mean earnings, the earnings Gini coefficient, and the mean/median earnings ratio. Specifically, for each of these statistics, $s_{t,y}$, we compute $s_{t,y}$ in the data for each combination of age t and calendar year y, and regress $s_{t,y}$ against a full set of year and age indicators.[10] We then take the regression coefficients on the age indicators (we use calendar year 2013 as our base year), and normalize them so that at age 40 the coefficients profile goes through the unconditional average value of $s_{40,y}$ across all years y in our sample. The corresponding normalized age coefficients constitute the lifecycle profiles that we use in the calibration. Figure 1 shows the lifecycle profiles of mean earnings, the earnings Gini, and the mean/median earnings ratio obtained in this fashion.

3.2 Life cycle financial assets

We use data from the SCF to measure wealth and its composition. Our measure of wealth includes all financial assets. To be consistent with assumptions that we make later, we assume that wealth is comprised of one risky and one risk-free asset. Our measure of risky assets corresponds to a broad measure of households' equity holdings in the SCF, which includes directly held stocks as well as stocks held in mutual funds, IRAs/Keoghs, thrift-type retirement accounts, and other managed assets.

As in the case of earnings, we construct lifecycle profiles of asset holdings, controlling for time effects using 2013 as the base year. The results (in 2014 dollars) are reported in Figures 2-4.

[10] By using a full set of year indicators, this treatment controls for year effects in the construction of the age profiles. We have also computed age profiles controlling for cohort effects, rather than year effects. The behavior of the lifecycle profiles is qualitatively similar.

Figure 1: Life-cycle earnings statistics

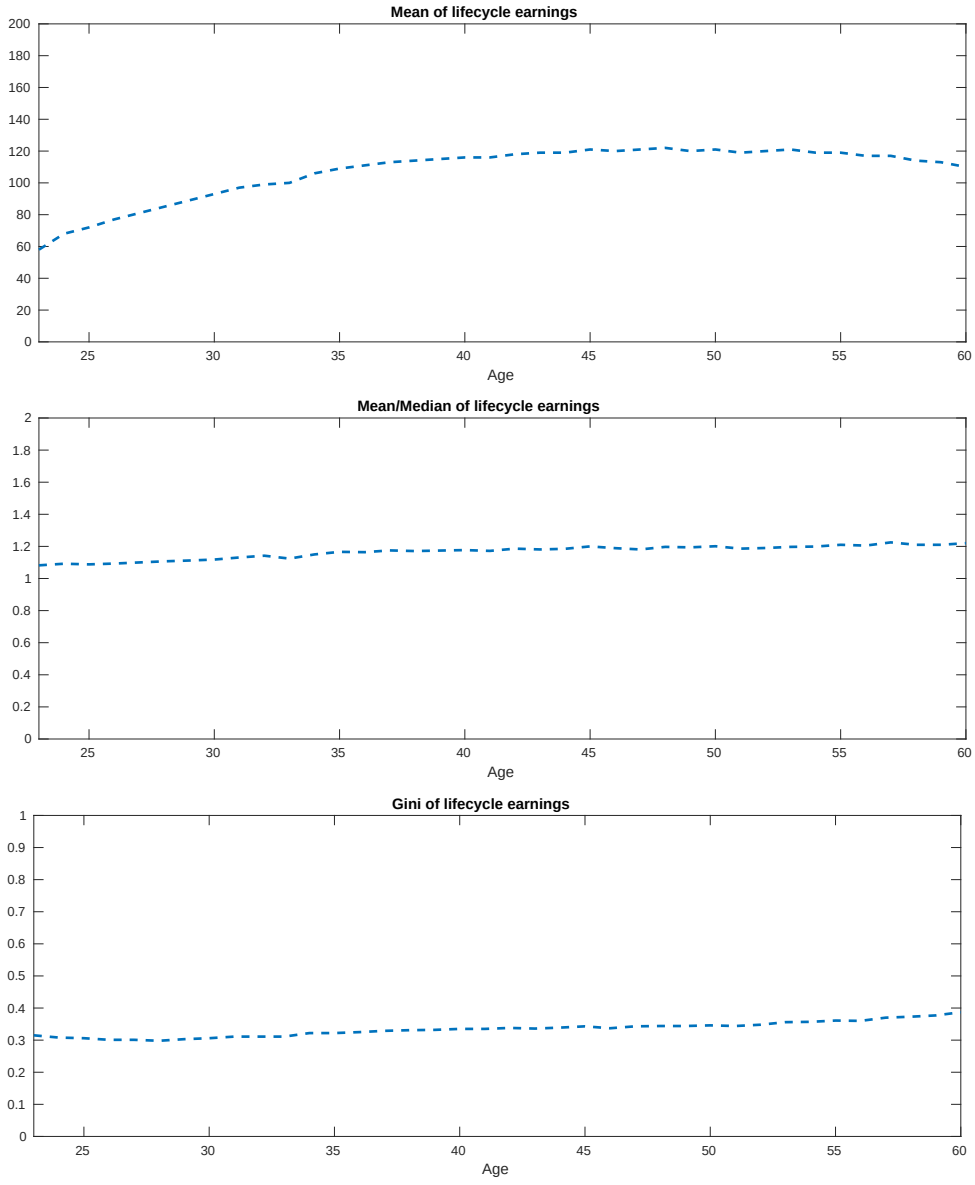


Figure 2: Average Life cycle Assets (SCF)

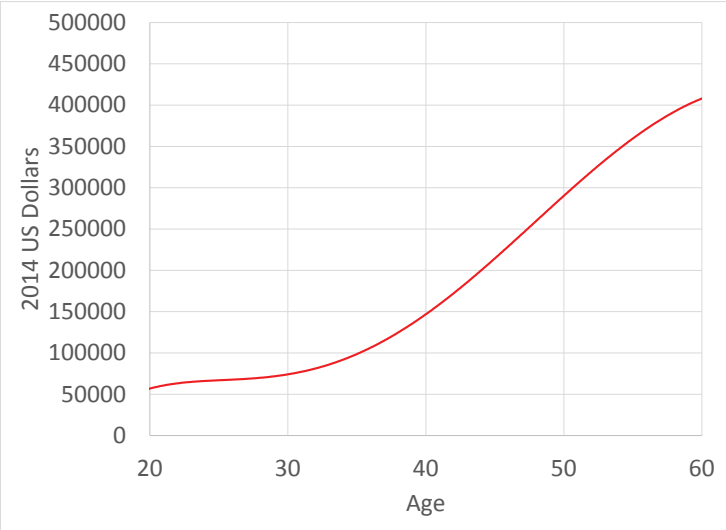


Figure 3: Average Risk-free Assets Conditional on Ownership (SCF)

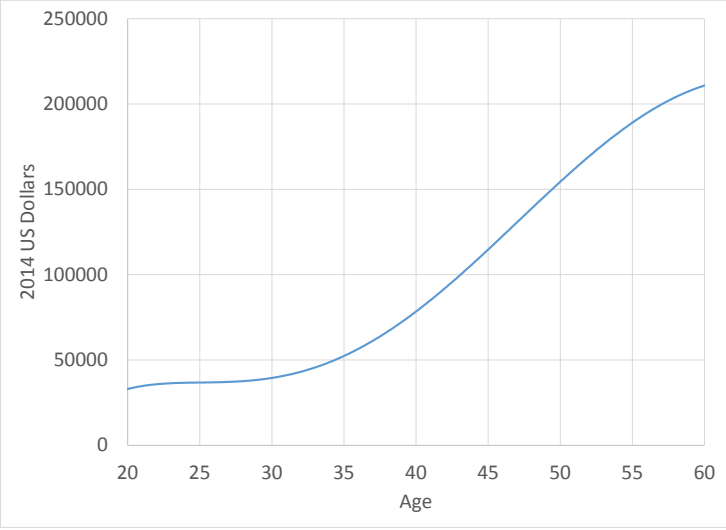
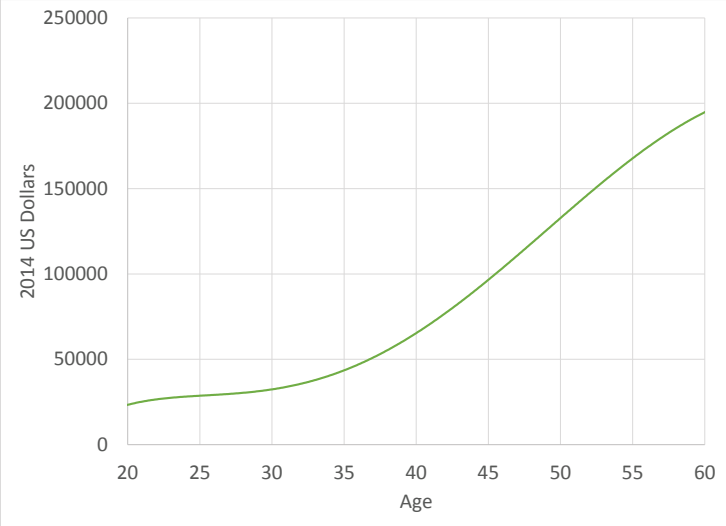


Figure 4: Average Risky Assets Conditional on Ownership (SCF)



4 Mapping the model to the data

The parameters in our model include: 1) standard parameters such as the discount factor and the coefficient of risk aversion; 2) parameters specific to human capital and to the earnings process; 3) parameters governing the distribution of initial characteristics; 4) parameters specific to college investment and financing; and 5) parameters specific to asset markets. Our approach involves a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments using several observable implications of the model. These are listed below in Table 1. The interested reader is directed to Appendix A for details on the calibration. model periods, which correspond to ages 18 to 80.

4.1 Model fit

We start by presenting measures of the goodness of fit for the baseline model. Figure 5 shows the earnings moments for a simulated sample of individuals in the model versus the CPS data.[11] As

[11] As a measure of goodness of fit, we use $\frac{1}{3J}\sum_{j=5}^{J}|log(m_j/m_j(\gamma))| + |log(d_j/d_j(\gamma))| + |log(s_j/s_j(\gamma))|$. This represents the average (percentage) deviation, in absolute terms, between the model-implied statistics and the data.We obtain a fit of 11% (where 0% represents a perfect fit).

Table 1: Parameter Values: Benchmark Model

Parameter	Name	Value
T	Model periods (years)	58
J	Working periods	34
β	Discount factor	0.96
σ	Coeff. of risk aversion	5
R_f	Risk-free rate	1.02
R_b	Borrowing rate	1.11
μ	Mean equity premium	0.06
σ_η	Stdev. of innovations to stock returns	0.157
α	Human capital production function elasticity	0.7
g_{nc}, g_{cg}	Growth rate of rental rate of human capital	$0.0013, 0.0065$
δ_{nc}, δ_{cg}	Human capital depreciation rate	$0.01, 0.0217$
ψ_{nc}, ψ_{cg}	Fraction of income in retirement	$0.682, 0.93$
$\underline{\tau}$	Minimal income level	$\$17,936$
$(\rho_{nc}, \sigma^2_{\nu nc}, \sigma^2_{\epsilon nc})$	Earnings shocks no college	$(0.951, 0.055, 0.017)$
$(\rho_{cg}, \sigma^2_{\nu cg}, \sigma^2_{\epsilon cg})$	Earnings shocks college	$(0.945, 0.052, 0.02)$
$(\mu_a, \sigma_a, \mu_h, \sigma_h, \varrho_{ah})$	Parameters for joint distribution of ability and initial human capital	$(0.44, 0.75, 77, 33, 0.71)$
$\bar{d}$	Total cost of college	\$53,454
d_{max}	Limit on student loans	\$23,000
w_{col}	Wage during college	\$17,700

the figure shows, the model does a reasonably good job of fitting the evolution of mean earnings over the lifecycle, though the model's profile is a bit less hump-shaped than in the data. The skewness of earnings is a touch lower in the model than in the data. And, for the Gini coefficient, the model matches the data quite well, except perhaps in the last few years of the lifecycle. Since we are interested in explaining the level of economic inequality over the lifecycle, this last statistic is particularly important. The bottom figure suggests that the model does a good job of matching the level of inequality.

We next test the model predictions for key non-targeted data moments. Figure 6 shows the mean wealth accumulation over the lifecycle for total assets over the lifecycle as well as for risky and risk-free assets, while Figure 7 shows participation in the stock market. Overall, the model is quite consistent with the observed financial investment behavior. In particular, despite not being targeted, our model produces empirically consistent estimates of lifecycle wealth and its allocation between risky and risk-free assets. In addition, our model's prediction for the stock-market participation rate is consistent with the data, over the *entire* lifecycle. This result is driven primarily by the presence of human capital investment, as shown by [add citation to AIN1].

Figure 5: Life-cycle earnings statistics

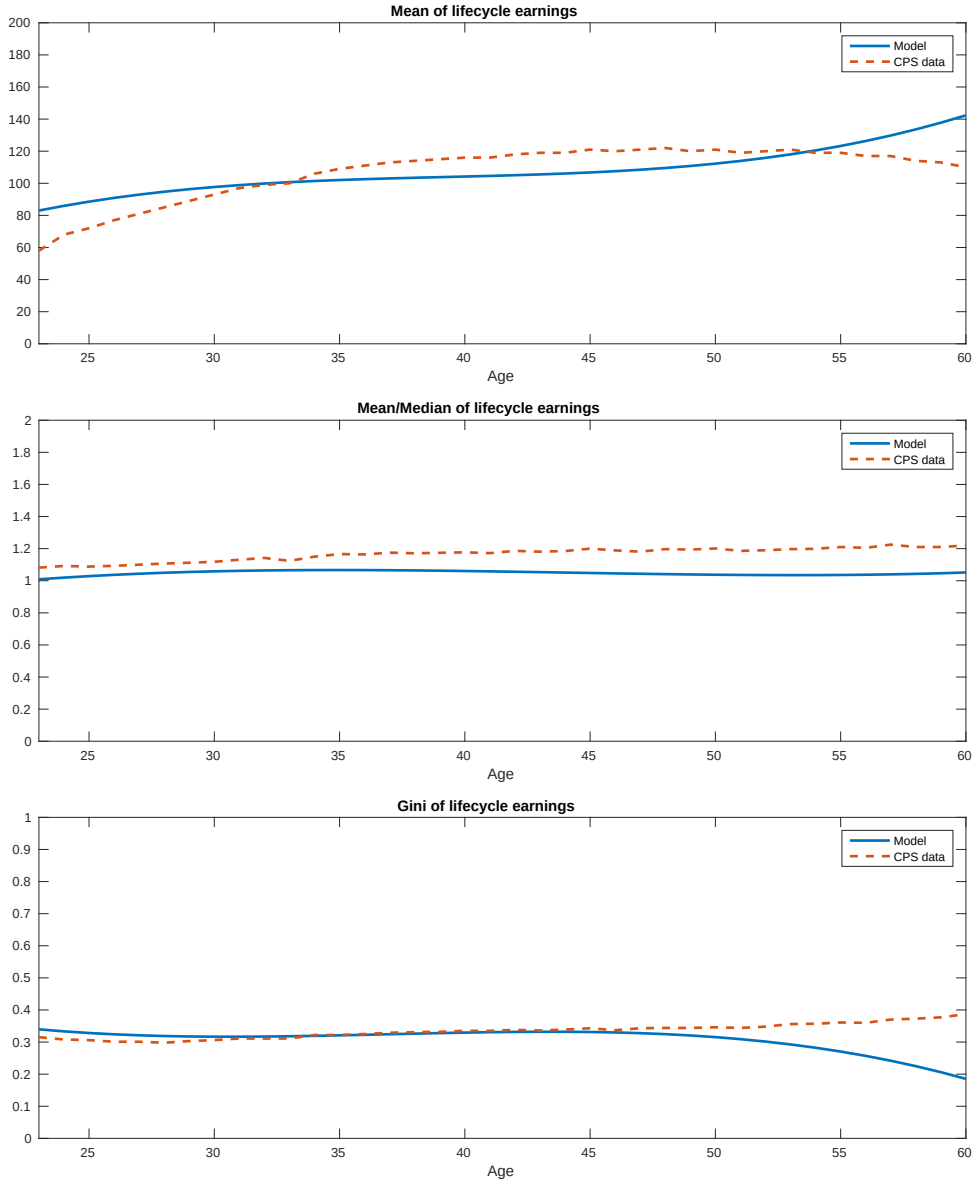


Figure 6: Life-Cycle Wealth Accumulation

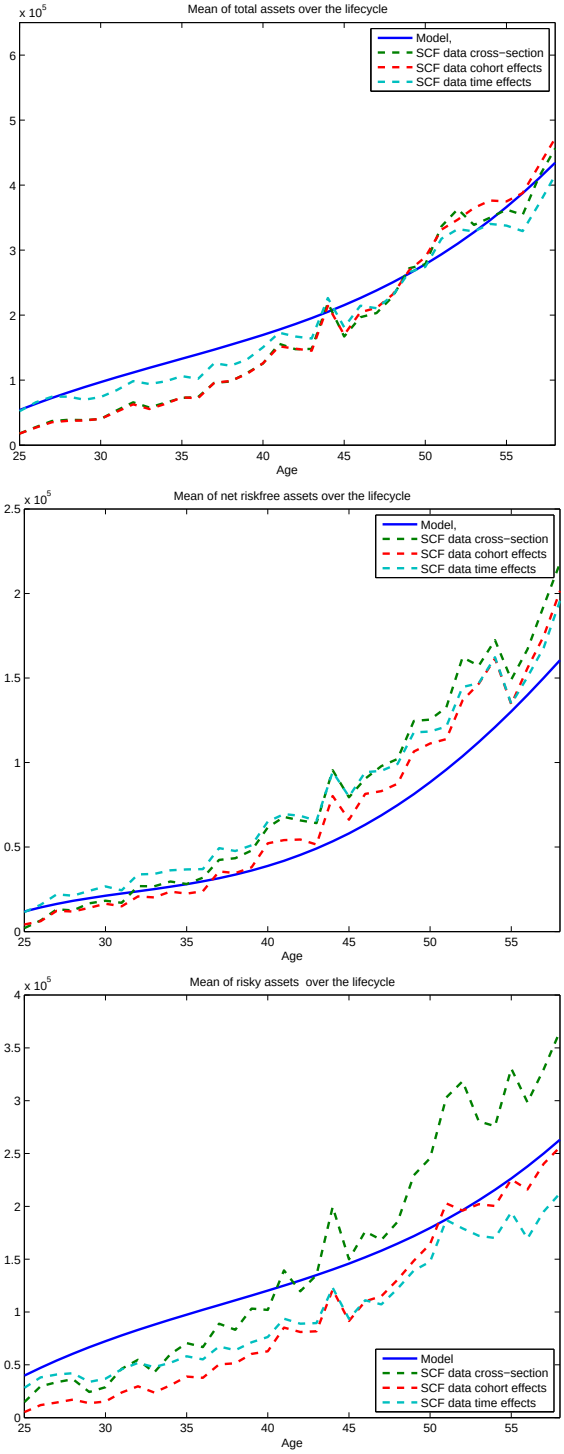
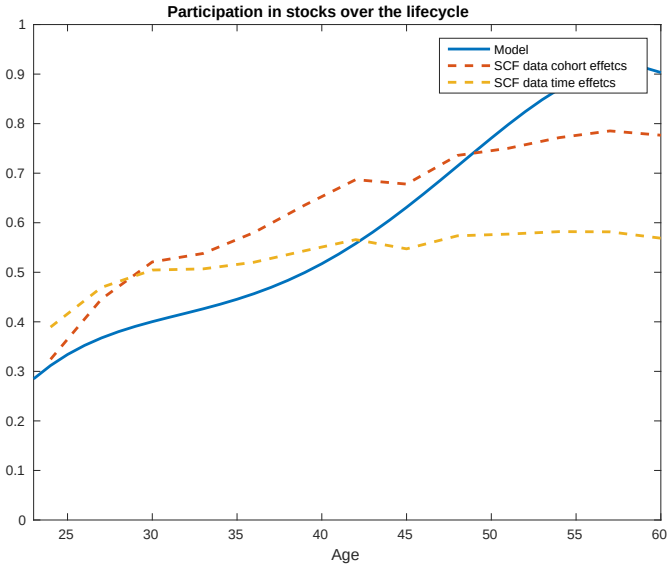


Figure 7: Investment in stocks over the life-cycle



5 Results

Our baseline model described above provides a rich characterization of the salient features of college education and stock market investment. We are interested in comparing outcomes from the baseline to outcomes in environments in which one or both of these investment opportunities are eliminated.

We first look at measures of inequality in each of these environments. Overall, we find that earnings inequality is 4% higher and wealth inequality is 3% higher in the baseline model (with both college and stocks) than in the model with only stocks but no college. Inequality is higher in the model with neither stocks nor college than in the model with stocks but no college: earnings inequality is 7% higher and wealth inequality is 6% higher. As Table 2 shows, the gap between top and median earners in the former environment than in the latter, whereas the gap between median and bottom earners is higher.

Table 2: Comparison of Earnings Inequality

Earnings differences	Stocks, No College	No College, No Stocks
Top to median earners	2.05	1.93
Median to bottom earners	2.55	2.84

[Add results for baseline model and for model with college but no stocks]

Table 3 shows that earnings in the top three quartiles of the population are substantially

higher in the model without stocks than in the model with stocks. However, earnings in the bottom quartile are lower.

Table 3: Comparison of Earnings Inequality Across Quartiles

Earnings quartiles	Earnings inequality: No Stocks, No College vs. Stocks, No College
Quartile 1	1.41
Quartile 2	1.16
Quartile 3	1.09
Quartile 4	0.97
All	1.07

[Add results for baseline model and for model with college but no stocks]

The presence of stocks results in two forces that work in opposite directions. On the one hand, those with higher earnings and wealth are more likely to participate in the stock market, making it possible for the rich to get richer. On the other hand, stocks offer an additional investment opportunity for those at the bottom of the income distribution for whom, by definition, the investment in the Ben-Porath human capital technology has relatively low returns. The presence of college, however, provides those with potentially high returns to human capital investment another avenue by which to further increase their human capital and therefore their lifetime earnings and wealth.

how much of observed inequality is determined by agents' endowments and the start of the model and how much is determined by events that occur over the course of the life cycle? To address questions such as this one, we now turn to a decomposition that follows Huggett and Kaplan (2011) and Storesletten et al. (2005). Specifically, we first simulate the model for a large number of individuals. For each simulated individual, we compute the present value of lifetime earnings and lifetime wealth (calculated as lifetime earnings plus the value of initial wealth), which we denote by $\Psi(a, h_0, x_0, z)$. We then compute the cross-sectional variance across the simulated individuals of lifetime earnings and lifetime wealth. As in Storesletten et al. (2005), we use the decomposition, $Var(\Psi) = Var(E[\Psi|a, h_0, x_0]) + E(Var[\Psi|a, h_0, x_0])$, where the contribution of the initial conditions is the ratio $Var(E[\Psi|a, h_0, x_0])/Var(\Psi)$.

The results of this calculation are reported in Table 4. In the baseline model, initial conditions account for about 70% of the variance in lifetime earnings and about 76% of the variance in lifetime wealth. When college is not available as an investment, initial conditions matter more, accounting for 76% of the variance in earnings and 84.6% of the variance in wealth. The numbers are even higher when neither investment is available (that is, when human capital is accumulated only via the Ben-Porath technology and savings can only be invested in the risky asset).

Our estimates fall in the middle of the range of results reported in the literature (a bit larger

than those found by Huggett et al. (2011) or Storesletten et al. (2005), and below those found by Keane and Wolpin (1997)). One important reason for the large role of initial conditions is that we obtain a very unequal initial distribution of both initial human capital and ability (e.g. the ability distribution we find is substantially more unequal than that found by Huggett et al. (2011)).[12]

[Add results for model with college but no stocks]

Table 4: Sources of lifetime inequality

Fraction due to initial conditions	Baseline	Stocks/No College	No Stocks/No College
of variance in lifetime earnings	70	76.0	78.2
of variance in lifetime wealth	76	84.6	86.5

Our exercise informs us about the role played by the particulars of college investment. There are two specific forces at play. On the one hand, college allows people to catch up by greatly enhancing their human capital in one step, so initial conditions may matter less. On the other hand, the risk of an investment in college may discourage some people from making this investment and this would increase the importance of initial conditions, particularly ability and wealth, which may interact strongly. For instance, wealth-poor households will have a relatively lower willingness to take the risk of college than their better-heeled counterparts, even if both have the same initial levels of ability and human capital. We find that the first effect dominates, and so the net effect is that the contribution of initial conditions decreases in the baseline economy.

We next decompose the contribution of initial conditions to lifetime inequality into the relative contributions of ability, initial human capital, and initial wealth. We first simulate our model economy without shocks and compute the cross-sectional variance of lifetime earnings and wealth. Next, we repeat the simulation, but each time setting one of ability, initial human capital, and initial wealth to their median value, and each time recomputing the cross-sectional variance of lifetime earnings and wealth. For each of the latter simulations, we report the variance of lifetime earnings and wealth as a fraction of the variance in the former simulation. The results are reported in Table 5.

[**In future drafts, we will focus on environments in which earnings shocks remain present and only the set of investments available to the household varies.**]

[12] Apart from the role played by formal education, which is embodied in initial conditions as of age 23, in particular in the form of the human capital stock at this age, we are exploring other mechanisms that contribute to the large role of initial conditions.

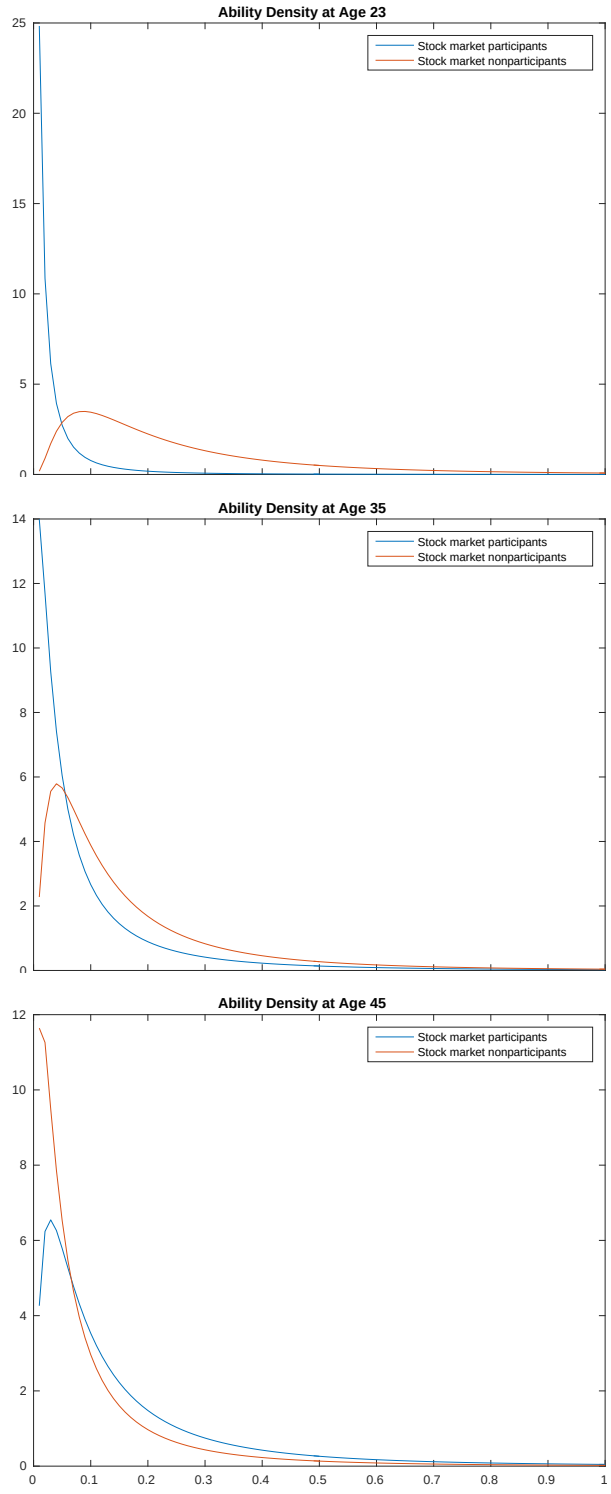
Table 5: Sources of lifetime inequality: initial conditions

Fraction due to h_0	Baseline	Stocks, No College	No College, No Stocks
of variance of lifetime earnings	0.37	0.55	0.47
of variance of lifetime wealth	0.24	0.34	0.29
Fraction due to a			
of variance of lifetime earnings	0.62	0.39	0.53
of variance of lifetime wealth	0.43	0.24	0.34
Fraction due to x_0			
of variance of lifetime earnings	0.01	0.07	0.00
of variance of lifetime wealth	0.34	0.42	0.37

First note that in all the environments we consider, both initial human capital and ability are quantitatively important for lifetime earnings inequality. This is because both characteristics shape the incentives to invest in human capital. At the same time, the contribution of initial wealth to lifetime earnings inequality is non substantial. This is because, unlike ability and initial human capital, initial wealth does not shape the incentives to invest in human capital and consequently does not affect the time allocated to work, which is what generates labor earnings. Initial wealth, on the other hand, does contribute quite a bit to lifetime wealth inequality since initial asset positions are relevant for wealth accumulation over life.

Comparing across environments, we find that the contribution of initial human capital to lifetime inequality is significantly lower in the baseline model (with college and stocks) than in the model with stocks but no college. The contribution of learning ability, however, is higher. This shows that the disadvantage of being endowed with low human capital early in life can be overcome—at least for those with high learning ability—by investment in college. In terms of the role played by stocks for the resolution of lifetime inequality, comparing the no-college models with and without stocks indicates that the relative importance of shocks over the life cycle (relative to initial conditions) increases somewhat in the economy where investment in stocks is allowed. As mentioned early, stocks offer an alternative investment opportunity to those agents whose initial endowments make investment in human capital a relatively unattractive option. Indeed, Figure 8 shows that in the model with stocks, individuals who are more likely to invest in human capital (and in particular early on in life) are less likely to participate in the stock market. As shown, when households are young, non-participants are those with substantially higher learning ability levels than participants. By middle age, as households have accumulated levels of human capital consistent with their ability, marginal returns to human capital are no longer substantially higher for them than the returns on stocks.

Figure 8: Ability Distribution of Participants and Non-Participants



5.1 The role of credit

When it comes to wealth and income (as opposed to earnings) inequality, a natural question is the extent to which borrowing constraints matter. Early in life, borrowing constraints may significantly affect human capital investment. Later in life, borrowing constraints may affect the ability of households to achieve desired financial investment positions. Moreover, because households who vary in their human capital attainment will also vary in their earnings, households' willingness to bear risk will differ in the population. Thus borrowing constraints may systematically change the distribution of risk-bearing capacity in the economy. Therefore, if access to credit is unequal, so too will be lifetime outcomes, not just for earnings but also for wealth.

We study the role of credit in a model where households may invest in both stocks and bonds. Specifically, we run a version of our where we shut down the possibility to borrow and a version of it where credit is cheap (the interest rate for credit is set to the riskfree interest rate). We find that access to credit lowers the contribution of (age-23) initial conditions. However, less expensive credit does not affect the role played by initial conditions for lifetime inequality.

We find that in the model people borrow primarily to smooth consumption while they invest in human capital rather than to invest in stocks. Therefore credit unavailability hinders households' efforts to smooth consumption intertemporally. This in turn diminishes the benefits to human capital accumulation as households' living standards can only rise once human capital payoffs are realized. This effect is particularly strong for households with low initial human capital levels who are the ones borrowing the most in the face of steep investment profiles in human capital, and therefore earnings. Taking away the option to borrow will therefore increase the importance of initial conditions, in particular that of initial human capital.

At the same time, cheap credit could induce borrowers to invest more in both human capital accumulation and stocks. We find that households only do the former: households who borrow allocate more time to human capital investment and use credit to smooth consumption in the face of lower current (and potentially higher future) earnings. This in turn, will lower the importance of initial conditions, but the quantitative effect is small given that households do not significantly change their stock market investment behavior despite having access to cheaper credit.

Our findings suggest that credit availability has important implications for the temporal resolution of lifetime inequality. Credit plays a significant role for the investment in human capital, in particular for those individuals for whom this investment gives the higher returns and/or supposes a relatively low opportunity cost. Consequently, an immediate question that arises is how does the role of credit change when we account for a richer, more realistic credit program to invest in human capital. In particular, we plan to study the implications of the current student loan program and

college-debt financing for lifetime inequality and for the role played by initial conditions. [Add results from college model here.]

5.2 The Role of Investment Returns

Once we have established a tight measure of the joint distribution of initial conditions, we will utilize this information to assess how the importance of initial conditions (in accounting for inequality and its life cycle evolution) depends on the size of payoffs associated with college and stock market. Specifically, we will examine the implications of one standard deviation changes in the wage premium for college, and in the returns to stock market investment for the questions at hand. [To be completed.]

6 Conclusion

Previous research has found that a significant fraction of inequality is determined early in life, with estimates ranging from half to as much as 90 percent. Our goal in this paper is to quantify the contribution of two prominent investments—human capital and stocks—to inequality and to the relative contribution of initial conditions and life cycle shocks. Consistent with previous work, we find that initial conditions contribute significantly to variations in lifetime earnings and wealth.

We find that inequality is highest in an environment where college is available as an investment option, though the role of initial conditions in lowest in this environment. Ability plays a larger role for inequality in a setting with college investment, while the role of initial human capital declines. When households cannot invest in stocks, inequality is slightly higher and the contribution of initial conditions to inequality is a bit larger overall relative to an economy where households can invest in stocks. Individual ability plays a larger role for inequality in a setting without stocks, whereas initial human capital plays a larger role when stock investment is available. Individuals with the highest opportunity cost of investing in human capital, namely those with relatively high levels of initial human capital, choose to invest in stocks instead starting early in life. Because they have high initial human capital, these individuals also have relatively high earnings and wealth. Stocks offer them an additional investment option by which to accumulate more wealth. When the option to invest in stocks is absent, this channel is shut down, which lowers the importance of initial human capital for lifetime inequality.

In all settings, we find that both initial human capital and ability are quantitatively important for lifetime inequality, whereas initial wealth matters for lifetime wealth inequality, but not earnings inequality. Finally, we assess the importance of the magnitudes of college—and equity—premia

for inequality and its sources. We find that the change in these premia relative to their levels in the 1970s alters inequality and the contributions of initial conditions, college, and stocks as follows [add results here].

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A Calibration

A.1 Preference Parameters

The per period utility function is CRRA as described in the model section. We set the coefficient of risk aversion, σ, to 5, which is consistent with values chosen in the financial literature. We conduct robustness checks on this parameter by looking at alternative values such the upper bound of $\sigma = 10$ considered reasonable by Mehra and Prescott (1985) as well as lower values such as $\sigma = 3$. The discount factor chosen ($\beta = 0.96$) is also standard in the literature. We set retirement income to be a constant fraction of labor income earned in the last year in the labor market. Following Cocco (2005) we set this fraction to 0.682 both for high school graduates and for those with some college education and to 0.93 for college graduates.

A.2 Human capital

We set the elasticity parameter in the human capital production function, α, to 0.7. Estimates of this parameter are surveyed by Browning et al. (1999) and range from 0.5 to 0.9. As previously noted, the rental rate of human capital in the model evolves according to $w_t = (1 + g_i)^{t-1}$. The growth rate g_i is calibrated to match the average growth rate in mean earnings observed in CPS data. Specifically, we obtain the values 0.0014 for the full sample, 0.0013 for individuals with no college degree, and 0.0065 for college graduates.

Given the growth rate in the rental rates, the depreciation rates are set so that the model produces the rate of decrease of average real earnings at the end of the working life cycle. The model implies that at the end of the life cycle negligible time is allocated to producing new human capital and, thus, the gross earnings growth rate approximately equals $(1 + g)(1 - \delta)$. We obtain $\delta = 0.0114$ for the full sample.

A.2.1 Earnings shocks

To parameterize the stochastic component of earnings, z_{it}, we follow Abbott et al. (2013) who use the National Longitudinal Survey of Youth (NLSY) data using CPS-type wage measures to estimate the autoregressive coefficients for the transitory and persistent shocks to wages. For the persistent shock, $u_{it} = \rho u_{i,t-1} + \nu_{it}$, with $\nu_{it} \sim N(0, \sigma_\nu^2)$ and for the idiosyncratic temporary shock, $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$, they report the following values for high school graduates: $\rho = 0.951$, $\sigma_\omega^2 = 0.055$, and $\sigma_\nu^2 = 0.017$ and college graduates: $\rho = 0.945$, $\sigma_\omega^2 = 0.052$, and $\sigma_\nu^2 = 0.02$. We use the first set of values for people with no college and some college education, and the second set of values for those who complete four years of college.

A.3 Distribution of Initial Characteristics: Assets, Ability and Human Capital

The distribution of initial characteristics (human capital, ability, and assets) is determined by seven parameters. These parameters are estimated to match the evolution of three moments of the earnings distribution over the life cycle: mean earnings, the Gini coefficient of earnings, and the ratio of mean to median earnings. The estimation proceeds as follows. First, for the distribution of assets, we use data from the Survey of Consumer Finances (SCF) to compute the mean and standard deviation of initial assets to be \$22,568 and \$24,256, respectively (in 2013 dollars). Second, we calibrate the joint distribution of ability and initial human capital to match the key properties of the earnings distribution over the lifecycle reported earlier using the March CPS.

The dynamics of the earnings distribution implied by the model are determined in several steps: i) we compute the optimal decision rules for human capital using the parameters described above for an initial grid of the state variable; ii) we simultaneously compute college and financial investment decisions and compute the lifecycle earnings for any initial pair of ability and human capital; and iii) we choose the joint initial distribution of ability and human capital to best replicate the properties of the CPS data.

We search over the vector of parameters that characterize the initial state distribution to minimize a distance criterion between the model and the data. We restrict the initial distribution to lie on a two-dimensional grid spelling out human capital and learning ability, and we assume that the underlying distribution is jointly log-normal. This class of distributions is characterized by five parameters.[13] We find the vector of parameters $\gamma = (\mu_a, \sigma_a, \mu_h, \sigma_h, \varrho_{ah})$ that characterizes

[13] In practice, the grid is defined by 20 points in human capital and in ability.

the initial distribution by solving the minimization problem

$$\min_{\gamma} \left(\sum_{j=5}^{J} |log(m_j/m_j(\gamma))|^2 + |log(d_j/d_j(\gamma))|^2 + |log(s_j/s_j(\gamma))|^2 \right)$$

where $m_j, d_j,$ and s_j are the mean, dispersion, and skewness statistics constructed from the CPS data on earnings, and $m_j(\gamma)$, $d_j(\gamma)$, and $s_j(\gamma)$ are the corresponding model statistics.[14]

A.4 College Parameters

We set the total college cost, $\overline{d} = \$53,454$. The limit and interest rate on student loans are $d_{max} = \$23,000$ and $R_g = 1.07$, respectively. We set the wage during college, $w_{col} = \$17,700$ (NCES data).

Lastly, the probability of college completion, $\pi(h_5)$ is based on mapping observed completion rates by cumulative GPA scores in the BPS 1996-2001 data set to h_5 in the model.[15] In the data, we observed what fraction of the student population obtained each of the sets of grades listed in the Table below. We divide the distribution of h_5 into group according to these percentages and assign each group the completion probability listed in the first column of the table. For example, an agent in the group with the highest levels of h_5 will face a 70% probability of completion.

Completion rate	Grades
0.07	grades C and D
0.30	mostly Cs
0.45	mostly Bs and Cs
0.56	mostly Bs
0.67	mostly Bs and As
0.70	mostly As

[Once we incorporate college choice, we will calibrate the difference in costs as well as completion probabilities for agents at all college types. This is not observed in the data, so we will rely on the empirical literature that estimates counterfactual completion probabilities if students had attended a different type of college from the one in which they ended up enrolling.]

[14]For details on the calibration algorithm see Huggett et al. (2006) and Ionescu (2009).

[15]We define the completion rate as the fraction of students who have earned a bachelor's degree by June 2001.

A.5 Financial Markets

We turn now to the parameters in the model related to financial markets. We fix the mean equity premium to $\mu = 0.06$, as is standard (e.g., Mehra and Prescott, 1985). The standard deviation of innovations to the risky asset is set to its historical value, $\sigma_\eta = 0.157$. The risk-free rate is set equal to $R_f = 1.02$, consistent with values in the literature (McGrattan and Prescott, 2000) while the wedge between the borrowing and

Borrowing limits in the model will be allowed to vary across households. We introduce heterogeneity in these limits as follows: We first group agents in the model by quartiles of initial human capital, then compute average earnings over the life cycle for each quartile. We then set the borrowing limit for all agents within a quartile to be a given percentage of the average life-cycle earnings for that quartile. We obtain the relevant percentages from the SCF by dividing the sample into income quartiles and calculating the average credit limit as a percentage of the average income within each quartile. The resulting borrowing limits as a percentage of average earnings by quartiles are: 55%, 48%, 35%, and 27%.[16] Lastly, in our baseline model, we assume that the returns to both risky assets (human capital and financial wealth) are uncorrelated. We assume for the time being that the returns to assets are uncorrelated. household portfolios held

[16]We extrapolate the first percentage from the other three rather than calculating it directly because of the large numbers of zeros in the earnings data for the lowest quartile.