Exchange Rate Disconnect in General Equilibrium*

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UNDER CONSTRUCTION

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Abstract

We propose a dynamic general equilibrium model of exchange rate disconnect, which simultaneously accounts for all major puzzles associated with the nominal and real exchange rates. This includes the Meese-Rogoff puzzle, the PPP puzzle, the terms-of-trade puzzle, the Backus-Smith puzzle, and the UIP puzzle. The model has two main building blocks — the driving force (or the exogenous shock process) and the transmission mechanism — both crucial for the quantitative success of the model. The transmission mechanism — which relies on strategic complementarities in price setting, low degree of substitutability between domestic and foreign goods, and home bias in consumption — is tightly disciplined by the micro-level empirical estimates in the recent international macroeconomics literature. The driving force is an exogenous small but persistent shock to international asset demand, which result in a *wedge* in the Euler equation for international bonds. We prove formally that this is the only shock that can generate exchange rate disconnect in the autarky limit. We then show that a model with this financial shock alone is quantitatively consistent with the moments describing the dynamic comovement between exchange rates and the macro variables. The addition of heterogeneous firms and sticky wages and prices improves the quantitative performance of the model, but does not affect its qualitative properties. Importantly, sticky prices are not necessary to generate exchange rate disconnect, as the driving force does not rely on the monetary shocks.

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1 Introduction

Exchange rate disconnect is among the most challenging international macro puzzle (Obstfeld and Rogoff 2001). The term disconnect narrowly refers to the lack of the correlation between exchange rate and other macro variables, but the broader puzzle is more pervasive and nests a number of additional empirical patterns, which stand at odds with the conventional international macro models. We define the broader exchange rate disconnect to include:

1. Meese and Rogoff (1983) puzzle: *nominal exchange rate* follows a random walk and is not robustly correlated, even contemporaneously, with the fundamentals (see also Engel and West 2005).
2. PPP puzzle (Rogoff 1996): *real exchange rate* tracks very closely the nominal exchange rate at most frequencies (in particular, exhibiting similarly large persistence and volatility) and, if it mean reverts, its half-life is as long as 3–5 years, much in excess of the typical duration of sticky prices (see also Chari, Kehoe, and McGrattan 2002; henceforth CKM).
3. *Terms of trade* are positively correlated with the real exchange rate, yet exhibit a markedly lower volatility, in contrast with the predictions of the standard models, suggesting a particular pattern of *law of one price* violations and *pricing to market* (Atkeson and Burstein 2008). In addition, the real exchange rate dynamics at most horizons is almost fully accounted for by the law of one price violations for tradable goods, and the relative non-tradable prices explain almost no variation in the real exchange rate (Engel 1999).
4. Backus and Smith (1993) international risk-sharing condition that relative consumption of the country should be positively and strongly correlated with the real exchange rates (i.e., high consumption when low prices) is sharply violated in the data, with a mildly negative correlation and markedly lower volatility of relative consumption (see also Kollmann 1995, Benigno and Thoenissen 2008 and CKM).
5. Forward premium puzzle, or the violations of the *uncovered interest rate parity* (UIP) condition, that the interest rate differentials should predict the nominal exchange rate devaluation is violated in the data with an opposite sign, but a nearly zero R^2 (see Fama 1984, Engel 1996).

In Section 2 we summarize and review the empirical patterns in greater detail, and use them as the quantitative targets for our model in later sections.

The absolute majority of the modern general equilibrium international macro models feature these puzzles, or attempt to address one puzzle at a time, resulting in a lack of a unified framework that exhibits satisfactory exchange rate properties. This is a major challenge for the academic and policy discussion, since exchange rates are the core prices in any international macro model, and failing to match their basic properties jeopardizes the conclusions one can draw from a model, in particular when the focus is on the international transmission of shocks.

A successful model of *disconnect* must satisfy two properties. First, it should specify the *driving force* (shock process) of nominal and real exchange rates, which cannot simultaneously have a sharp effect on the contemporaneous consumption, output, prices and interest rates. Second, it should specify the *transmission mechanism*, which mutes the effect of volatile exchange rate fluctuations on local prices

and quantities. Violation of either of the properties would break the disconnect. While the literature has provided a lot of empirical evidence on the transmission mechanism, taking the exchange rate movements as exogenous, we currently lack direct empirical information on the details of the shock process, which accounts for the bulk of the exchange rate fluctuations.

Given this state of the literature, we adopt the following strategy. On one hand, we tightly discipline the transmission mechanism with the empirical estimates from the recent literature, which leaves us with little uncertainty, or degrees of freedom. In contrast, we initially impose no restrictions on the nature of the shock process, saturating the model with all possible shocks, or wedges. We then show that there exists a single type of shocks, which can satisfy the exchange rate disconnect properties.

In particular, we impose the requirement that the shock should produce a volatile exchange rate with a vanishing effect on the economy's quantities, prices and interest rates, as the economy becomes closed to trade. Indeed, in the limit of the closed economy, any exchange rate volatility (real or nominal) should be completely inconsequential for the allocations. Not surprisingly, productivity and monetary shocks, as well as the majority of other shocks, violate this intuitive requirement. The one shock that satisfies this requirement is the shock to the international asset demand. We later show that this shock can have a variety of microfoundations in the financial market, including noise trading with limits to arbitrage, heterogeneous beliefs, and financial frictions.

We further show that the model with a single international asset demand shock is consistent both qualitatively and quantitatively with the exchange rate disconnect. In particular, small persistent shocks to international asset demand result in a volatile random-walk-like behavior of the nominal exchange rate, needed to equilibrate the international asset market, simultaneously being consistent with the intertemporal budget constraints of the countries. As the economy becomes more closed to international trade, this shock still generates volatile nominal exchange rate fluctuations, which however have a vanishingly small effect on the rest of the economy.

Furthermore, the transmission mechanism in the model ensures that the nominal exchange rate exhibits the empirically relevant comovement properties with the other variables, even when the economy is open to trade. The transmission mechanism relies on three main features:

1. pricing to market and law of one price violations due to strategic complementarities in price setting (using estimates from Amiti, Itskhoki, and Konings 2016), which limit the response of prices (terms of trade) to the exchange rate movements;
2. low elasticity of substitution between home and foreign goods (using estimates from Feenstra, Luck, Obstfeld, and Russ 2014), which limit the extent of expenditure switching conditional on the terms of trade movements;
3. significant home bias in consumption (consistent with the trade share in GDP for the US, Japan, and the European Union as a whole), which limits the effect of expenditure switching on aggregate consumption, employment and output.

This provides the quantitative discipline for the transmission mechanism in our model. In particular, it allows the model to reproduce the real exchange rate behavior that tracks closely that of the nominal exchange rate, including the volatility and persistence, which are in line with the PPP puzzle literature.

Importantly, this does not rely on price or wage stickiness, which we assume away in the baseline model. Indeed, the nominal stickiness is not needed, as the driving process does not rely on the monetary shocks. The model, instead, relies on strategic complementarities in price setting, which generate the empirical patterns of deviations from the law of one price. The firms in the model adjust markups in response to the exchange rate movements, absorbing part of the shock in their profit margins. In an extension, we show that sticky wages further improve the quantitative performance of the model, but do not change any of our qualitative conclusions.

The mechanism in the model is as follows. A small persistent increase in demand for foreign bonds (e.g., due to its liquidity or safety properties, which we leave outside the model) results in a sharp depreciation of the home currency and a slow but persistent appreciation thereafter, as the shock gradually dies out. Such behavior of the exchange rate ensures that both the asset demand and the intertemporal budget constraint of the country are satisfied. The more persistent is the shock, the closer is the behavior of the nominal exchange rate to a random walk, with the unexpected exchange rate innovation accounting for all of the exchange rate movements in the limit.[1]

In response to the exchange rate devaluation, there is domestic price inflation and a reduction in the home real wage, and simultaneously an expenditure switching towards the goods produced at home. To sustain a simultaneous drop in wages and an increase in labor demand, home consumption falls, supported by the increase in the home interest rate. Since the exchange rate is devalued, but expected to appreciate, the model reproduces both low consumption in periods of low prices (i.e., the deviation from the Backus-Smith condition) and expected depreciation in periods of high interest rates (i.e., the UIP puzzle). The transmission mechanism—with substantial home bias and low pass-through into prices and quantities—ensures that the movements in consumption and interest rates are very mild, much smaller than those in the nominal exchange rate, consistent with the empirical patterns. In particular, the model reproduces the close to zero R^2 in the Fama regression. In an extension, we show that adding small productivity shocks largely leaves unchanged the quantitative properties of the model, and additionally allows it to match the close to zero correlations between the exchange rate and the other macro variables.

Literature review [TO BE COMPLETED]

- Kouri (1976, 1983), Masson (1981), Henderson and Rogoff (1982), Branson and Henderson (1985), Blanchard, Giavazzi, and Sa (2005), Gourinchas (2008): balance-of-payment view of the exchange rate, international demand shocks for assets and goods
- Jeanne and Rose (2002), Gourinchas and Tornell (2004), Bacchetta and van Wincoop (2006), Alvarez, Atkeson, and Kehoe (2009), Brunnermeier, Nagel, and Pedersen (2009), Farhi and Gabaix (2016), Gabaix and Maggiori (2015), Valchev (2015), Lustig and Verdelhan (2016), Lustig, Stathopoulos, and Verdelhan (2016), Engel (2016): financial models of exchange rate

[1] In the limit of the closed economy, the exchange rate devaluation simply ensures the drop in the country's wealth relative to the foreign, as there cannot be actual trade in assets (non-zero capital account) in the absence of the offset trade in the goods market (zero current account). The physical allocations, however, remain unchanged despite the reduction in the nominal wealth of the country relative to the rest of the world.

- Jeanne and Rose (2002), Kollmann (2005), Monacelli (2004) on *Mussa (1986, 1990) puzzle*
- Kollmann (2005), Devereux and Engel (2002), Farhi and Werning (2012): risk-premia (Euler equation) shocks
- Benigno and Thoenissen (2008), Corsetti, Dedola, and Leduc (2008): Backus-Smith under incomplete markets
- Chari, Kehoe, and McGrattan (2007), Eaton, Kortum, and Neiman (2015): wedge accounting
- DSGE literature: ...

The rest of the paper is organized as follows. In Section 2 we review the stylized facts and puzzles related to the exchange rates, and re-calculated the main empirical moments in the recent samples to use as the targets for the model to match in later section. In Section 3 we describe the baseline model and prove that only the international asset demand shocks can be consistent with the exchange rate disconnect. In Section 4 then explores the qualitative and quantitative properties of the model with the international asset demand shocks alone. In Section 5.1 we allow for multiple shocks and estimate the model using the moments documented in Section 2; using the estimated model, we provide a variance decomposition of the exchange rate process into the contribution of various shocks. Section 5 describes the extensions, including a sticky wage model, a model with heterogeneous firms, and a multi-country model. Section 6 concludes, and the appendix provides detailed derivations and proofs.

2 Stylized Facts and Puzzles

[TO BE COMPLETED]

3 Modeling Framework and Shocks

We start with a flexible modeling framework that can nest most standard international macro models, which allows us in what follows to consider various special cases and extensions. There are two countries, home (Europe) and foreign (US, denoted with a $*$). Each country has a nominal unit of account, with the wage rates given by W_t euros at home and W_t^* dollars in foreign, and $\mathcal{E}_t$ denotes the nominal exchange rate, i.e. the price of one dollar in euros. Therefore, an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the home currency (the euro). We allow for a variety of shocks hitting the economy, with some of the shocks taking the form of wedges (following Chari, Kehoe, and McGrattan 2007), proxying in some cases for unmodelled market imperfections. We then explore which of these shocks and wedges can and which cannot account for the exchange rate disconnect, as we formally define it below in Section 3.2.

3.1 Model setup

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{1}$$

where (χ_t, κ_t) are the utility shocks, σ is the relative risk aversion parameter and ν is the Frisch elasticity of labor supply.[2] Table 1 below summarizes all shocks/wedges and model parameters. The flow budget constraint is given by:

$$P_t C_t + \frac{B_{t+1}}{R_t} + \frac{B^*_{t+1}\mathcal{E}_t}{e^{\psi_t} R^*_t} \leq B_t + B^*_t \mathcal{E}_t + W_t L_t + \Pi_t + T_t, \tag{2}$$

where P_t is the consumer price index, (B_t, B^*_t) are the quantities of the home and foreign bonds paying out next period one unit of the currency of the issuing country, and (R_t, R^*_t) are their discounts (i.e., $1/R_t$ and $1/R^*_t$ are their prices); Π_t are the dividends and T_t are lump-sum transfers. Lastly, ψ_t is the (inverse) cost shock to the international bond trading ability of the domestic households, e.g. a capital controls tax/wedge, as we further discuss below.[3]

The households are active in three markets. First, they supply labor according to the standard static optimality condition:

$$e^{\kappa_t} C_t^{\sigma} L_t^{1/\nu} = \frac{W_t}{P_t}, \tag{3}$$

where the preference shock κ_t can be alternatively interpreted as the *labor wedge*, playing an important role in the closed-economy business cycle literature and capturing the departures from the neoclassical labor market dynamics due to search frictions or sticky wages (see e.g. Shimer 2009). In addition, we denote $W_t \equiv e^{w_t}$ and interpret w_t as the shock to the nominal value of the unit of account, which captures monetary shocks in our framework.[4]

Second, the households choose their bond positions according to the dynamic optimality conditions:

$$1 = R_t \, \mathbb{E}_t \Theta_{t+1} \qquad \text{and} \qquad 1 = e^{\psi_t} R^*_t \, \mathbb{E}_t \left\{ \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}, \tag{4}$$

$$\text{where} \qquad \Theta_{t+1} \equiv \beta e^{\Delta \chi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

is the stochastic discount factor. The change in the time preference shock, $\Delta\chi_{t+1}$, affects the consumption-savings decision and acts as the intertemporal demand shifter for both types of bonds. The ψ_t shock instead acts as the demand shock for the foreign bond. We discuss in Section 4.1 the various models of

[2] Our results are robust to alternative utility specifications, such as the GHH utility without income effects on labor supply.

[3] Incompleteness of international asset markets is important for some results, as we explain in Section 4. However, the assumption that a risk-free bond is the only internationally-traded asset is not essential, and we adopt it only because we rely on a log-linearization for the analytical solution of the model. Some of the results, in particular Proposition 1, do not rely at all on the assumptions about the (in)completeness of the international asset market.

[4] In Section 5.1 we provide an explicit model with nominal wage and local-currency price stickiness and conventional Taylor rules, which offers an example of one typical source of $(w_t, \kappa_t, \mu_t, \eta_t)$ shocks (with μ_t and η_t defined below).

Table 1: Model parameters and shocks/wedges

Shocks/wedges		Parameters†	
w_t	nominal wage rate (shock to the value of the unit of account)	$\beta = 0.99$	discount factor
a_t	productivity shock	$\sigma = 2$	relative risk aversion (inverse of IES)
g_t	government spending shock	$\nu = 1$	Frisch elasticity of labor supply
χ_t	intertemporal preference shock	$\gamma = 0.07$	foreign share (home bias) parameter
κ_t	labor wedge (sticky wages)	$\theta = 1.5$	elasticity of substitution
μ_t	markup shock (sticky prices)	$\alpha = 0.4$	strategic complementarity elasticity
η_t	law-of-one-price shock (local currency pricing)	$\phi = 0.5$	intermediate share
ξ_t	international good demand shock	$\rho = 0.97$	persistence of the shock
ψ_t	international asset/currency demand shock		

† We report the baseline parameter values used in Sections 4–5.

asset demand which result in a similar reduced form as in (4), and in what follows we refer to ψ_t as the *international asset/currency demand shock*.[5]

Lastly, the households allocate their within-period expenditure between home and foreign goods:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft},$$

and we assume the good demand is homothetic and symmetric, and given by:

$$C_{Ht} = (1-\gamma) e^{-\gamma \xi_t} h\left(\frac{P_{Ht}}{P_t}\right) C_t \qquad \text{and} \qquad C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t}\right) C_t, \tag{5}$$

where ξ_t is the relative *demand shock for the foreign good* and γ is the *home bias* parameter, with the demand for the foreign good collapsing to zero as $\gamma \to 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, satisfies $h'(\cdot) < 0$ and $h(1) = 1$, and we denote its point elasticity by $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1}$. The demand formulation in (5) emerges from a homothetic and separable Kimball (1995) demand aggregator, as we show in Appendix A.1, where we also derive an explicit expression for the price index P_t. In our analysis, we focus on the behavior of the economy around a symmetric steady state with $\xi = 0$ and $P = P_H = P_F$, and make use of the following three properties of this demand system (see Appendix A.1):

Lemma 1 (Properties of Demand) *In a symmetric steady state with $\xi = 0$ and $P = P_H = P_F$:*

(i) The expenditure share on foreign goods (the foreign share *for brief), defined as $\frac{P_{Ft} C_{Ft}}{P_t C_t}$, equals the home bias parameter γ.*

[5] One simple story, following Dekle, Jeong, and Kiyotaki (2014), directly places foreign bonds into the utility function of home households (in parallel with the money-in-the-utility models). In particular, if we add $e^{\psi_t} B_t^*$ to the flow utility in (1) and drop ψ_t term from the budget constraint (2), we obtain an identical log-linearized reduced form of the model. More spelled-out micro theories of asset demand model explicitly (a) the demand for liquidity or safety, (b) limits to arbitrage, heterogeneous beliefs or financial frictions in the asset markets, or (c) risk premia due to disasters or long-run risk (reflected in the second moments of the stochastic discount factor). We view the ψ_t shock/wedge as a stand-in for all such mechanisms.

(ii) The log-linear approximation to demand (5) around the steady state is given by:

$$c_{Ht} = -\gamma\xi_t - \theta(p_{Ht} - p_t) + c_t \qquad \textit{and} \qquad c_{Ft} = (1-\gamma)\xi_t - \theta(p_{Ft} - p_t) + c_t. \tag{6}$$

Therefore, $c_{Ft} - c_{Ht} = \xi_t - \theta(p_{Ft} - p_{Ht})$, and the elasticity of substitution between home and foreign goods, defined as $\frac{\partial \log(C_{Ft}/C_{Ht})}{\partial \log(P_{Ht}/P_{Ft})}$, equals the point elasticity of the demand schedule θ.

(iii) The log-linear approximation of the consumer price index around the steady state is given by:

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}, \tag{7}$$

where small letter $p_t \equiv \log P_t - \log \bar{P}$ denotes the log deviation of the price index from its steady state value P, and similarly for the other variables.

We show below that the values of γ (trade openness) and θ (elasticity of substitution between home and foreign goods) play the central role in the quantitative properties of the transmission mechanism.

Production and prices Output is produced by a given pool of identical firms according to a Cobb-Douglas technology in labor L_t and intermediate inputs X_t:

$$Y_t = e^{a_t} L_t^{1-\phi} X_t^{\phi}, \tag{8}$$

where a_t is the *productivity shock* and ϕ is the elasticity of output with respect to intermediates, which determines the equilibrium expenditure share on intermediate goods. Intermediates are the same bundle of home and foreign varieties as in the final consumption, and hence their price index is also given by P_t. Therefore, the marginal cost of production is:

$$MC_t = e^{-a_t} \left(\frac{W_t}{1-\phi}\right)^{1-\phi} \left(\frac{P_t}{\phi}\right)^{\phi}, \tag{9}$$

and the firms optimally allocate expenditure between labor and intermediates according to the following input demand conditions:

$$W_t L_t = (1-\phi) MC_t\, Y_t \qquad \text{and} \qquad P_t X_t = \phi MC_t\, Y_t. \tag{10}$$

The expenditure on intermediates X_t is further split between the domestic and foreign varieties, X_{Ht} and X_{Ft}, in parallel with the consumption expenditure by the households in (5). The profits of the domestic firms (distributed to the domestic households) are given by:

$$\Pi_t = (P_{Ht} - MC_t)\, Y_{Ht} + (P^*_{Ht}\mathcal{E}_t - MC_t)\, Y^*_{Ht}, \tag{11}$$

where total production is split into output delivered to the home and foreign markets, $Y_t = Y_{Ht} + Y^*_{Ht}$.[6]

[6]For analytical tractability, we focus on a constant-return-to-scale production without capital, until Section 5 where we show the robustness of our results to an extension with capital and adjustment costs. We further assume no entry or exit

We postulate the following price setting:

$$P_{Ht} = e^{\mu_t} MC_t^{1-\alpha} P_t^{\alpha}, \tag{12}$$

$$P_{Ht}^{*} = e^{\mu_t+\eta_t} \left(MC_t/\mathcal{E}_t\right)^{1-\alpha} P_t^{*\alpha}, \tag{13}$$

where $\alpha \in [0,1)$ is the *strategic complementarity elasticity*, μ_t is the *markup shock*, and η_t is the *law of one price (LOP) shock*. Given these prices, the firms satisfy the resulting demand in both markets. Equations (12)–(13) are *ad hoc* yet general pricing equations, as the markup terms (together with a flexible choice of α) allow them to be consistent with a broad range of price setting models, including both monopolistic and oligopolistic competition models under both CES and non-CES demand. Furthermore, if the time path of (μ_t, η_t) is not restricted, these equations are also consistent with dynamic price setting models, and in particular the sticky price models (with either producer, local or dollar currency pricing).[7]

Strategic complementarities in price setting ($\alpha > 0$) reflect the tendency of the firms to set prices closer to their local competitors, a pattern which is both pronounced in the data and emerges in a variety of models (see Amiti, Itskhoki, and Konings 2016), and we emphasize in our analysis below its role for the international transmission of shocks. Appendix A.1 discusses a model of Kimball (1995) demand that is simultaneously consistent with our choices of elasticity of substitution θ and strategic complementarity elasticity α. Lastly, we note that the violations of the law of one price:

$$\mathcal{Q}_{Ht} \equiv \frac{P_{Ht}^{*}\mathcal{E}_t}{P_{Ht}} = e^{\eta_t} Q_t^{\alpha}, \qquad \text{where} \qquad \mathcal{Q}_t \equiv \frac{P_t^{*}\mathcal{E}_t}{P_t}, \tag{14}$$

arise either due to the LOP shock η_t (capturing, for example, local currency pricing) or due to $\alpha > 0$ (capturing pricing-to-market), where $\mathcal{Q}_t$ is the *real exchange rate*, which reflects the differences in the price levels across the two markets.

Government and transfers The government uses lump-sum taxes to finance an exogenous stochastic path of government expenditure $P_t G_t$, where $g_t \equiv \log G_t$ is the *government spending shock*. For simplicity, we assume that government expenditure is allocated between the home and foreign goods in the same way as the final consumption in (5). The government runs a balanced budget, which in view of Ricardian equivalence is without loss of generality. As a result, the overall transfers to the households are given by:

$$T_t = \left(e^{-\psi_t} - 1\right) \frac{B_{t+1}^{*}\mathcal{E}_t}{R_t^{*}} - P_t e^{g_t}. \tag{15}$$

of firms, as our model is a medium-run one (for the horizons of up to 5 years), where empirically extensive margins play negligible roles (see e.g. Bernard, Jensen, Redding, and Schott 2009). Instead of introducing variable trade costs and non-tradables, we simply adjust the exogenous home bias parameter γ to match the trade shares in the data. The baseline model features symmetric representative firms, and we discuss the extension with heterogeneous firms in Section 5.

[7] Note that η_t can also stand in for the trade cost shocks in the goods market, playing an important role in the recent quantitative analyses of Eaton, Kortum, and Neiman (2015) and Reyes-Heroles (2016). A combination of η_t and ξ_t shocks can stand in for the world commodity price shocks, playing an important role as terms of trade shocks for the commodity-exporting countries such as Canada, Australia, South America, Brazil and Chile (see e.g. Chen and Rogoff 2003).

The first terms reflects the proceeds from the capital controls tax, but is also consistent with the alternative interpretations of the ψ_t wedge.[8]

Foreign The foreign households are symmetric, except that the home (euro) bonds are not available to them, and their budget constraint is given by:

$$P_t^* C_t^* + \frac{B_{t+1}^{*F}}{R_t^*} \leq B_t^{*F} + W_t^* L_t^* + \Pi_t^* + T_t^*,$$

where B_t^{*F} are the holdings of the foreign (dollar) bond by foreign households.[9] The foreign households supply labor and demand home and foreign goods according to the optimality condition parallel to (3) and (5) respectively. In particular, the goods demand by the foreign households is given by:

$$C_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h \left(\frac{P_{Ht}^*}{P_t^*}\right) C_t^* \qquad \text{and} \qquad C_{Ft}^* = (1-\gamma) e^{-\gamma \xi_t^*} h \left(\frac{P_{Ft}^*}{P_t^*}\right) C_t^*, \tag{16}$$

where ξ_t^* is the foreign demand shock for home goods. The bond demand of foreign households is given by:

$$1 = R_t^* \, \mathbb{E}_t \Theta_{t+1}^*, \qquad \text{where} \qquad \Theta_{t+1}^* \equiv \beta e^{\Delta \chi_{t+1}^*} \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}. \tag{17}$$

Lastly, the foreign firms are also symmetric, demand foreign labor and home and foreign intermediates, and charge prices according to the counterparts of (12)–(13) with their own markup and LOP shocks μ_t^* and η_t^* (see Appendix A.2).

Equilibrium conditions ensure equilibrium in the asset, product and labor markets, as well as the intertemporal budget constraints of the countries. The labor market clears when L_t is consistent simultaneosly with labor supply in (3) and labor demand in (10), and symmetrically for L_t^* in foreign. The goods market clearing requires $Y_t = Y_{Ht} + Y_{Ht}^*$, where

$$Y_{Ht} = C_{Ht} + X_{Ht} + G_{Ht} = (1-\gamma) e^{-\gamma \xi_t} h \left(\frac{P_{Ft}}{P_t}\right) \left[C_t + X_t + G_t\right], \tag{18}$$

$$Y_{Ht}^* = C_{Ht}^* + X_{Ht}^* + G_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h \left(\frac{P_{Ht}^*}{P_t^*}\right) \left[C_t^* + X_t^* + G_t^*\right], \tag{19}$$

and symmetric conditions hold for $Y_{Ft} + Y_{Ft}^* = Y_t^*$. The bonds market clearing requires $B_t = 0$ for the home-currency bond, as it is in zero net supply and not traded internationally, and $B_t^* + B_t^{*F} = 0$ for the foreign-currency bond, which is in zero net supply internationally.

Lastly, we combine the household budget constraint (2) with profits (11) and taxes (15) to obtain

[8] For example, if ψ_t is the cost of the financial intermediation, this term reflects the profit transfer from intermediaries to the households. The model with bonds in the utility function is also exactly isomorphic to this formulation. Alternatively, if the international cost ψ_t is purely waisted, it needs to be additionally subtracted from the resource constraint, but it changes neither qualitative, nor quantitative results.

[9] We consider this asymmetric formulation between home and foreign for simplicity, but provide a symmetric version of the model with a financial sector in Section 5.

the country budget constraint:

$$\frac{B^*_{t+1}\mathcal{E}_t}{R^*_t} - B^*_t\mathcal{E}_t = NX_t, \qquad \text{where} \qquad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft} \tag{20}$$

is the net exports of the home country (in the home currency). Note that the relative prices at which the home country exchanges its exports for imports is the *terms of trade*:

$$\mathcal{S}_t \equiv \frac{P_{Ft}}{P^*_{Ht}\mathcal{E}_t}. \tag{21}$$

This completes the description of the model environment and the equilibrium system (further summarized in Appendix A.2), and we now turn to the selection of shocks.

3.2 Shock selection: disconnect in the limit

This section uses the general modeling framework to prove two theoretical results, which narrow down the set of the shocks that can be consistent with the empirical exchange rate disconnect properties. In particular, we study the behavior of the equilibrium system around the autarky limit, which we use as our diagnostic device. The autarky limit (as foreign share $\gamma \to 0$) is interesting for two reasons:[10]

1. Autarky ($\gamma = 0$) offers a model of *complete exchange rate disconnect*. When countries are in autarky (in both good and asset markets), the nominal exchange rate is of no consequence, and can take any values as an outcome of arbitrary sunspot equilibria. Therefore, it can be arbitrary volatile, yet have no relationship with any macro variables in the two economies (the Meese-Rogoff puzzle). Since price levels do not respond to this volatility, the real exchange rate comoves perfectly with the nominal exchange rate, and in particular can have arbitrary persistence (the PPP puzzle). This is possible because in autarky the real exchange rate does not affect allocations.
2. As the economies become open and move further away from autarky (i.e., γ increases), the responses of macro variables to exchange rate increase together with openness γ, and the macro variables become more volatile and less disconnected. Therefore, if the economy does not exhibit exchange rate disconnect properties near autarky (for $\gamma \approx 0$), it cannot exhibit such properties away from autarky (for $\gamma \gg 0$).

We now extend the autarky logic to study circumstances under which a near-closed economy features a *near-complete exchange rate disconnect*. While such continuity requirement may appear natural as the equilibrium dynamics is continuous in γ, it nonetheless offers a sharp selection criterion for exogenous shocks. This is because a limiting economy with $\gamma > 0$ acts as a refinement on equilibria when $\gamma = 0$, as it rules out the sunspot equilibria with volatile exchange rate dynamics.[11]

We start by formalizing the notion of the exchange rate disconnect in the autarky limit:

[10] Note that $\gamma = 0$ implies autarky in both goods and assets, while financial autarky alone is insufficient for our results.

[11] Indeed, for $\gamma > 0$, the equilibrium in the model of Section 3.1 is unique and excludes the possibility of sunspot dynamics in the nominal and real exchange rates.

Definition 1 (Exchange rate disconnect in the limit) *Denote with* $\mathbf{Z}_t \equiv (W_t, P_t, C_t, L_t, Y_t, R_t)$ *a vector of all domestic macro variables (wage rate, price level, consumption, employment, output, interest rate) and with*

$$\varepsilon_t \equiv \mathbf{V}'\Omega_t, \qquad \Omega_t = \{w_t, w_t^*, \chi_t, \chi_t^*, \kappa_t, \kappa_t^*, a_t, a_t^*, g_t, g_t^*, \mu_t, \mu_t^*, \eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t\}$$

an arbitrary combination of shocks hitting the economy, where $\mathbf{V}$ *is a vector of loadings. We say that an open economy (with* $\gamma > 0$*) exhibits exchange rate disconnect in the autarky limit (as* $\gamma \to 0$*), if the impulse responses have the following properties:*

$$\lim_{\gamma \to 0} \frac{\mathrm{d}\mathbf{Z}_{t+j}}{\mathrm{d}\varepsilon_t} = 0 \quad \textit{for all } j \geq 0 \qquad \textit{and} \qquad \lim_{\gamma \to 0} \frac{\mathrm{d}\mathcal{E}_t}{\mathrm{d}\varepsilon_t} \neq 0. \tag{22}$$

A corollary of condition (22) is that $\lim_{\gamma \to 0} \frac{\mathrm{d}\log \mathcal{E}_{t+j} - \mathrm{d}\log \mathcal{Q}_{t+j}}{\mathrm{d}\varepsilon_t} = 0$ *for all* $j \geq 0$.

A model, defined by its structure and the set of shocks, exhibits exchange rate disconnect in the autarky limit if the shocks have a vanishingly small effect on the macro variables, yet result in a volatile equilibrium exchange rate, as captured by the two conditions in (22).[12] This property captures the exchange rate disconnect in its narrow Meese-Rogoff sense. However, as the corollary points out, this property also implies the PPP-puzzle behavior for the real exchange rate, which in the limit comoves perfectly with the nominal exchange rate.

Definition 1 immediately allows us to exclude a large number of candidate shocks:

Proposition 1 *The model of Section 3.1 cannot exhibit exchange rate disconnect in the autarky limit (22), if the combined shock* $\varepsilon_t = \mathbf{V}'\Omega_t$ *has a weight of zero on the subset of shocks* $\{\eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t\}$.

In other words, this proposition states that the following shocks alone:

$$\Omega_t^{\varnothing} \equiv \{w_t, w_t^*, \chi_t, \chi_t^*, \kappa_t, \kappa_t^*, a_t, a_t^*, g_t, g_t^*, \mu_t, \mu_t^*\},$$

in any combination and with arbitrary correlations, cannot reproduce an exchange rate disconnect property even as the economy approaches autarky. We provide a formal proof in Appendix A.3, yet the intuition behind this result is straightforward: Any of the shocks in $\Omega_t^{\varnothing}$ will have a direct effect on real allocations, prices, and/or interest rates, and thus cannot result in a volatile exchange rate without having a direct effect on the macro variables of the same order of magnitude.[13] Therefore, as an economy subject to these shocks approaches autarky, the disconnect property (22) is necessarily violated. The proof of this proposition does not rely on the international risk sharing condition, and therefore this result is robust to the (in)completeness of the international asset markets.

[12] A weaker, yet still suitable, condition is $\lim_{\gamma \to 0} \frac{\mathrm{d}\mathbf{Z}_{t+j}/\mathrm{d}\varepsilon_t}{\mathrm{d}\mathcal{E}_t/\mathrm{d}\varepsilon_t} = 0$ for all $j \geq 0$.

[13] Intuitively, the unit of account w_t shocks results in wage inflation, the markup μ_t shocks result in price inflation, the labor wedge κ_t shocks result in changes in either employment or consumption, the productivity a_t shocks result in changes in either employment or output, the government spending g_t shocks result in changes in either consumption or output, and the intertemporal preference shocks χ_t result in changes in the interest rate. We illustrate this logic in Figure **??**. The formal proof in Appendix A.3 establishes further that no combination of these shocks can be consistent with the disconnect property (22).

Proposition 1 can be viewed as pessimistic news for both the International RBC and the New Open Economy Macro (NOEM) models of the exchange rate. However, it does *not* imply that, for example, productivity cannot be an important source of exogenous shocks. Instead, it suggests that productivity shocks a_t are unlikely to be the dominant drivers of exchange rate movements if the model is to exhibit exchange rate disconnect.[14] The same applies to monetary shocks in a model with nominal stickiness, which we study in detail in Section 5.1.

In other words, we view Proposition 1 as a diagnostic tool suggesting that the shocks in $\Omega_t^{\varnothing}$ are unlikely to be successful at reproducing the empirical exchange rate behavior even away from the autarky limit. Therefore, we should first explore the other three types of shocks — namely, the LOP deviation (or trade cost) shocks η_t, the international good demand shocks ξ_t, and/or the international asset demand shocks ψ_t — as the likely key drivers of the exchange rate dynamics. The distinctive feature of these shocks is that they affect the equilibrium system exclusively through the *international equilibrium conditions*: ψ_t affects international risk sharing (condition (**??**) below), while η_t and ξ_t affect the country budget constraint (20) through their impact on export prices (13) and export demand (19) respectively.[15] The impact of shocks to these equilibrium conditions on the macro variables is vanishingly small as the economy becomes closed to international trade in goods and assets, yet such shocks can have substantial effect on the equilibrium exchange rates and terms of trade even when γ is close to zero.

Proposition 1 does not allow us to discriminate between the remaining three types of shocks, as they all satisfy the autarky-limit disconnect condition (22). Yet, these shocks differ in the implied comovement between exchange rates and macro variables, which we now use as a further selection criterion. In particular, we explore the comovement between the exchange rates and respectively terms of trade, relative consumption, and the interest rate differential, near the autarky limit (as $\gamma \to 0$). Since these shocks are already consistent with the Meese-Rogoff and the PPP puzzles by virtue of Proposition 1, the additional moments correspond to the three remaining exchange rate puzzles. We prove the following result (see Appendix A.3):

Proposition 2 *Near the autarky limit (for $\gamma \to 0$), the international asset demand shock ψ_t is the only shock among $(\eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t)$ that simultaneously and robustly produces:*

(i) *a positive correlation between the terms of trade and the real exchange rate;*

(ii) *a negative correlation between relative consumption growth and real exchange rate depreciation;*

(iii) *deviations from the UIP and a negative Fama coefficient.*

The main conclusion is that both the LOP deviation (trade cost) shocks η_t and the international good demand shocks ξ_t produce the conventional *counterfactual* comovement between the exchange

[14] Productivity shocks can have two additional indirect effects, either acting as *news shocks* about future productivity or by affecting the *risk premium* (e.g., rare-disaster or long-run-risk shocks). As the direct effect of the productivity shock becomes vanishingly small relative to indirect effects, this shock becomes indistinguishable from an Euler-equation shock (as it does not affect the static equilibrium conditions), and we view it as a special case of the ψ_t shock. Proposition 1 nonetheless applies as long as the direct effect of the productivity shock is non-trivial. See Section 4.1 and Appendix A.6.1 for further discussion.

[15] The ξ_t and η_t shocks are additionally featured in the goods market clearing (18)–(19) and in the price level (7), but in both cases their effect on these conditions is proportional to trade openness γ, and thus vanishes in the autarky limit.

rates and respectively relative consumption (the Backus-Smith puzzle) and interest rate differential (the Forward Premium puzzle). In contrast, the international asset demand shock ψ_t is instead consistent with both of these empirical patterns, as we explain in detail in Section 4.

To summarize, Propositions 1 and 2 explain why almost any shock has hard times at reproducing the empirical exchange rate properties, and hence why these properties are labeled as *puzzles* in the literature. These propositions favor the international asset demand shock ψ_t as the only likely shock to generate exchange rate disconnect in an equilibrium model. While these propositions are concerned with the autarky limit, the continuity of the model in trade openness γ suggests that the near-disconnect properties should hold for $\gamma > 0$ but small. In what follows, we explore both qualitative and quantitative properties of the model away from the autarky limit.

4 Baseline Model of Exchange Rate Disconnect

Our baseline model features a single shock — the financial shock ψ_t — and the transmission mechanism, which emphasizes home bias in expenditure (low γ), strategic complementarities in price setting ($\alpha > 0$), and low elasticity of substitution between home and foreign goods ($\theta > 1$, but small). The other parameters, including the intertemporal elasticity of substitution (IES) and the elasticity of labor supply, prove to be less consequential for the results, as we discuss below. Even more surprisingly, nominal rigidities turn out to be of little importance for generating the quantitative exchange rate disconnect properties (in response to a ψ_t shock). Therefore, our baseline model does not feature any nominal rigidities, and for simplicity we assume that the monetary authorities adopt policy rules which fully stabilize respective wage inflations at zero (specifically, we have $W_t \equiv 1$ and $W_t^* \equiv 1$). We show the robustness of our results to nominal frictions and conventional Taylor rules in Section 5.1.

The model of this section is analytically tractable (upon log-linearization), and all our results can be easily obtained with pen and paper, which allows us to fully explore the intuitions behind various mechanisms. At the same time, we emphasize the quantitative objective of this section. That is, our goal is to establish whether a simple one-shock model can be *quantitatively* consistent with a rich set of moments describing the comovement between exchange rates and macro variables. In doing so, we tie our hands from the start, and calibrate the parameters of the model on which we have direct and reliable empirical evidence. In particular, we set $\gamma = 0.07$ to be consistent with the 0.28 trade (imports plus exports) to GDP ratio of the United States, provided the intermediate input share $\phi = 0.5$.[16] We further use the estimate of Amiti, Itskhoki, and Konings (2016) of the elasticity of strategic complementarities $\alpha = 0.4$, which is also in line with much of the markup and pass-through literature and corresponds to the own cost shock pass-through elasticity of $1 - \alpha = 0.6$ (see survey in Gopinath and Itskhoki

[16] This value of the trade-to-GDP ratio is also characteristic of the other large developed economies (Japan and the Euro Zone). Appendix A.2.1 derives the relationship between the value of the trade-to-GDP ratio and the value of γ (steady state imports-to-expenditure ratio), which is typically four times smaller. Intuitively, imports in a symmetric steady state are half of total trade (imports plus exports), and GDP (final consumption) is about one half of the total expenditure with the other half allocated to intermediate inputs ($\phi = 0.5$). This value of ϕ is consistent with both aggregate input-output matrices and firm level data on intermediate expenditure share in total sales. The decomposition of gross exports for the U.S., the E.U. and Japan in Koopman, Wang, and Wei (2014) suggests this proportion holds for trade flows as well.

2011). In contrast, the value of the elasticity of substitution between home and foreign goods θ is more contested. We follow here the recent estimates of Feenstra, Luck, Obstfeld, and Russ (2014) and set $\theta = 1.5$, which is also the number used in the original calibrations of Backus, Kehoe, and Kydland (1994) and Chari, Kehoe, and McGrattan (2002).[17]

For the remaining parameters, we set the relative risk aversion $\sigma = 2$, the Frish elasticity of labor supply $\nu = 1$ and the (quarterly) discount factor $\beta = 0.99$, as we summarize in Table 1.[18] We show robustness to alternative parameter values in Appendix A.4.2, while our theoretical results require only weak parameter restrictions, easily met in conventional calibrations, as we discuss in Appendix A.4.1.

4.1 Models of financial shock ψ_t

Before turning to our results, we offer a brief discussion of the financial shock ψ_t, and the various theories microfounding it. In view of the parity condition between home and foreign bonds (arising from (4) upon log-linearization), the ψ_t is commonly referred to as the *UIP shock*:

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \psi_t, \tag{23}$$

where $i_t - i_t^* = \log R_t - \log R_t^*$ and $e_t = \log \mathcal{E}_t$. It follows that the (uncovered) interest rate parity $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$ deviates from zero by the magnitude of the financial shock ψ_t, which may have a number of origins explored in the macro-finance literature (see also Cochrane 2016):

1. Exogenous preference for international assets, where ψ_t is an *ad hoc* shock to the utility from holding the foreign bond, as a result of which domestic households are willing to hold the foreign asset with a negative excess return (UIP deviation), as in Dekle, Jeong, and Kiyotaki (2014). This approach additionally requires incompleteness of international asset markets, and is closely related to the non-optimizing *portfolio balance models* of the 1970-80s (e.g. Kouri 1976, 1983, Henderson and Rogoff 1982, Branson and Henderson 1985), revived recently by Blanchard, Giavazzi, and Sa (2005) and Gourinchas (2008).
2. Noise traders and limits to arbitrage in currency markets, as in Jeanne and Rose (2002), where all international trade in assets needs to go through risk-averse intermediaries, who are willing to be exposed to the currency risk only if they are offered a sufficient compensation in the form of a positive expected return. A noise trader shock ψ_t, thus, needs to be accommodated by a UIP deviation. The two useful features of this model is that it generates endogenous volatility of UIP deviations, and hence can speak to the Mussa puzzle (see Section 5.1.1), and can also be easily nested in a general equilibrium environment (see Section 5.2).
3. A related class of models relies on financial frictions to generate upward sloping supply in the currency market, where ψ_t represents shocks to the risk-bearing capacity of the financial sector,

[17] The *macro* elasticity of substitution between the aggregates of home and foreign goods is indeed the relevant elasticity for our analysis, while the estimates of the *micro* elasticity at more disaggregated levels are typically larger (around 4). The quantitative performance of our model does not deteriorate significantly for elasticities of substitution as high as 3.

[18] We normalize and fix the shocks/wedges of Section 3.1, other than ψ_t, as follows: $w_t = a_t = \chi_t = \kappa_t = \eta_t = \xi_t \equiv 0$, $g_t \equiv g$ and $\mu_t \equiv \mu$, and symmetrically for foreign. Our results, however, are robust to any common shock processes in the two countries ($a_t = a_t^*$, etc.). In our baseline, we set $g = \mu = 0$, and show robustness to $g = \log(0.3 \cdot GDP)$ and $\mu = 0.2$.

for example a shock to the net worth of the financial intermediaries limiting the size of the positions that they can absorb (see e.g. Hau and Rey 2006, Brunnermeier, Nagel, and Pedersen 2009, Gabaix and Maggiori 2015, Adrian, Etula, and Shin 2015).

4. Incomplete information, heterogeneous beliefs and expectational errors in the currency market, as in Evans and Lyons (2002), Gourinchas and Tornell (2004) and Bacchetta and van Wincoop (2006), where ψ_t represents the deviations from the full-information rational expectations. The advantage of heterogeneous beliefs models is that they can additionally explain the large volumes of daily currency trade, much in excess of fundamental trade flows. These models, however, often come at a substantial cost in terms of tractability, especially in general equilibrium.

5. Time-varying risk premia models, where ψ_t corresponds to shocks to the *second moments* of the stochastic discount factor, such as rare disasters (Farhi and Gabaix 2016), long-run risk (Colacito and Croce 2013), or habits (Verdelhan 2010). For convenience, these models typically assume complete markets, while an alternative approach relies on modeling segmented markets, where the SDF shocks emerge from the participation margin (see e.g. Alvarez, Atkeson, and Kehoe 2009).

Given our focus on the set of moments characterizing the empirical exchange rate disconnect, all the approaches to modeling the financial shock ψ_t listed above are isomorphic, as they all result in a version of equation (23), and hence we cannot distinguish between them. The specific models of ψ_t can be discriminated based on a richer set of asset market moments, e.g. a term structure of carry trade returns, or a comovement of exchange rate with returns across various asset classes, which we leave outside the scope of this paper (see e.g. Engel 2016, Lustig and Verdelhan 2016, Du, Tepper, and Verdelhan 2016).

For concreteness and tractability, we assume the financial shock ψ_t follows an exogenous AR(1) process with persistence $\rho \in [0, 1]$:

$$\psi_t = \rho\psi_{t-1} + \varepsilon_t, \tag{24}$$

and the variance of innovations given by σ_ε^2. For our quantitative analysis, we assume that ψ_t shocks are small, but persistent, with ρ close to but smaller than 1 and $\sigma_\psi^2 = \sigma_\varepsilon^2/(1-\rho^2)$ close to 0.[19] In particular, we set $\rho = 0.97$ (quarterly) as our benchmark, consistent with the persistence of the interests rate differential in the data, as we explain in Section 4.6.

4.2 Equilibrium exchange rate dynamics

The model admits a convenient recursive structure, which allows us to solve for the relationship between exchange rate and macro variables (prices, quantities and interest rates) in partial equilibrium, as we do in turn in Sections 4.3–4.6 and Appendix A.4. We start, however, with the characterization of the general equilibrium exchange rate dynamics.

The dynamics of the exchange rate is determined by the combination of the country budget constraint (20) and the international risk sharing condition, captured by the UIP condition (23). Using the static equilibrium conditions, we solve for the equilibrium relationships between net exports, interest

[19] The model admits a solution as long as $\beta\rho < 1$, and the $\beta\rho \to 1$ limit offers a useful limiting case, as will become clear.

rates and exchange rate, which allows us to rewrite (23) and the log-linearized version of (20) as:[20]

$$\mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1+\gamma\lambda_1}\psi_t, \tag{25}$$

$$\beta b^*_{t+1} - b^*_t = nx_t = \gamma\lambda_2 e_t. \tag{26}$$

The coefficients λ_1 and λ_2 depend on the structural model parameters other than β and ρ, and are positive under mild parameter restrictions which we assume are satisfied (e.g., a variant of the Marshall-Lerner condition; see Appendix A.4.1). Equation (26) simply reflects that trade surplus (and hence net foreign asset accumulation) increases with an exchange rate depreciation, and more so the more open is the country to international trade. Equation (25), in turn, suggests that a financial shock $\psi_t > 0$ — that is, an increase in foreign asset demand — requires an expected appreciation of the home currency to balance the depressed relative demand for the domestic bond.[21]

We can now solve the dynamic system (25) and (26), together with the shock process (24), for the equilibrium exchange rate process:

Proposition 3 (Equilibrium exchange rate process) *In the baseline model with the financial shock $\psi_t \sim$ AR(1) with persistence ρ and innovation ε_t, the equilibrium nominal exchange rate e_t follows an ARIMA(1,1,1), or equivalently $\Delta e_t \sim$ ARMA(1,1), with an AR root ρ and a non-invertible MA root $1/\beta$:*

$$\Delta e_t = \rho \Delta e_{t-1} + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\left(\varepsilon_t - \frac{1}{\beta}\varepsilon_{t-1}\right). \tag{27}$$

This describes the unique equilibrium exchange rate path, and non-fundamental solutions do not exist.

An analytical proof of this proposition in Appendix A.4 relies on the Blanchard and Kahn (1980) solution method, while here we offer a more intuitive explanation. Asset market equilibrium condition (25) determines the expected path of the future exchange rate changes, as emphasize by Engel and West (2005), and can be iterated forward to obtain:

$$e_t = \lim_{T\to\infty}\left\{\mathbb{E}_t e_{t+T} + \frac{1}{1+\gamma\lambda_1}\sum\nolimits_{j=0}^{T}\mathbb{E}\psi_{t+j}\right\} = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\lambda_1}\frac{\psi_t}{1-\rho}. \tag{28}$$

In contrast with Engel and West (2005), however, the general version of the risk sharing (or UIP) condition features no discounting, and therefore admits a multiplicity of no-bubble solutions parametrized by the long-run expectation $\mathbb{E}_t e_\infty$. Nonetheless, the equilibrium path of the exchange rate is uniquely pinned down by the intertemporal budget constraint, which obtains from (26) by iterating it forward and imposing the No Ponzi Game Condition on net foreign assets $\lim_{T\to\infty}\beta^T b^*_{t+T+1} = 0$. This pins

[20] In Appendix A.2 we log-linearize the equilibrium system around a symmetric steady steady, and by default use the small letters to denote the log deviations of the respective variables from their steady state values (e.g., $x_t \equiv \log X_t - \log \bar{X}$ for any variable X). The NFA position B^*_t and the net exports NX_t are zero in a symmetric steady state, and we adopt an alternative definition for their deviations: $b^*_t \equiv \frac{\bar{\mathcal{E}}}{P_H Y} B^*_t$ and $nx_t \equiv \frac{1}{P_H Y} NX_t$.

[21] Observe from (23) that $\psi_t > 0$ must be accommodated either by $i_t - i^*_t > 0$, or by $\mathbb{E}_t \Delta e_{t+1} < 0$ (expected appreciation), both of which moderate the increased demand for the foreign bond. We show in Section 4.6 that both of this occurs in equilibrium, with the relative importance of the interest rate adjustment decreasing as the countries become more closed (see (43) below). In the autarky limit ($\gamma \to 0$), the interest rates do not move, and we have $\mathbb{E}_t \Delta e_{t+1} = -\psi_t$, consistent with (25).

down the only budget-consistent long-run expectation, and hence the instantaneous level of the exchange rate e_t, according to (27).[22] Indeed, any deviation from these values of e_t and $\mathbb{E}_t e_\infty$ would shift the whole path of the exchange rate, and hence all trade surpluses on the right-hand side of (26), violating the intertemporal budget constraint. This general equilibrium discipline on the exchange rate determination is what distinguishes our solution from that in Engel and West (2005), as we discuss in detail in Appendix A.5.

An increase in foreign asset demand (i.e., $\varepsilon_t > 0$ in (27)) results in an instantaneous depreciation of the home currency, while also predicting an expected appreciation according to (25), akin to the celebrated overshooting mechanism of Dornbusch (1976). This exchange rate path ensures both equilibrium in the financial market (via expected appreciation) and balanced country budget (via instantaneous depreciation). Lastly, note that the equilibrium exchange rate process (27) is shaped by parameters ρ and β, and depends on the other parameters of the model only through the proportional volatility scaler $1/(1+\gamma\lambda_1)$. In particular, since $\lambda_1 > 0$, the exchange rate volatility increases as the economy becomes more closed to international trade, and it is maximized in the autarky limit.

Equation (25) suggests departures from the random walk behavior and implies predictability of the nominal exchange rate. We now explore how pronounced are these properties quantitatively and whether the equilibrium exchange rate process predicted by our model can be consistent with the empirical random-walk-like behavior.

Proposition 4 (Near-random-walk behavior) *The equilibrium exchange rate process* (27) *becomes indistinguishable from a random walk as* $\beta\rho \to 1$. *In particular, the following properties hold:*[23]

1. *The autocorrelation of exchange rate changes becomes arbitrary close to zero:*

$$\lim_{\beta\to 1} \frac{\operatorname{cov}(\Delta e_{t+1}, \Delta e_t)}{\operatorname{var}(\Delta e_t)} = -\frac{1-\rho}{2} \xrightarrow[\rho\to 1]{} 0.$$

2. *The contribution of the predictable component to the variance of* Δe_{t+1} *shrinks to zero, or equivalently the unpredictable component (innovation) fully dominates the variance of* Δe_{t+1}*:*

$$\lim_{\beta\to 1} \frac{\operatorname{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\operatorname{var}(\Delta e_{t+1})} = \frac{1+\rho}{2} \xrightarrow[\rho\to 1]{} 1.$$

3. *The volatility of the exchange rate innovation (surprise) becomes unboundedly large relative to the*

[22] Iterating (26) forward and applying the NPGC, we obtain $b_t^* + \gamma\lambda_2 \sum_{j=0}^{\infty} e_{t+j} = 0$, which *a fortiori* holds in expectations. Combining this with (28), which implies $\mathbb{E}_t e_{t+j} = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\lambda_1}\frac{1}{1-\rho}\rho^j \psi_t$, we obtain the solution for the level of exchange rate as a function of the shock ψ_t and the state variable b_t^* (the predetermined NFA position):

$$e_t = \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\psi_t - \frac{1}{\gamma\lambda_2}(1-\beta)b_t^*.$$

Lastly, combining this with (26), we express $(\Delta e_t, \Delta b_{t+1}^*)$ as a function of exogenous innovations to the financial shock ψ_t.

[23] Note that we first take the $\beta \to 1$ limit and then consider the $\rho \to 1$ limit. This is because for $\beta < 1$ and $\rho = 1$ the unconditional second moments are not well-defined for Δe_t as it becomes an integrated process. This sequential limit provides a good quantitative approximation to our baseline case in which $\rho = 0.97 < \beta = 0.99$. In the last two moments, we assume that $\{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$ is part of the information set at time t when constructing $\mathbb{E}_t \Delta e_{t+1}$, even though the MA root of the e_t process is non-invertible, offering a conservative lower bound for our results.

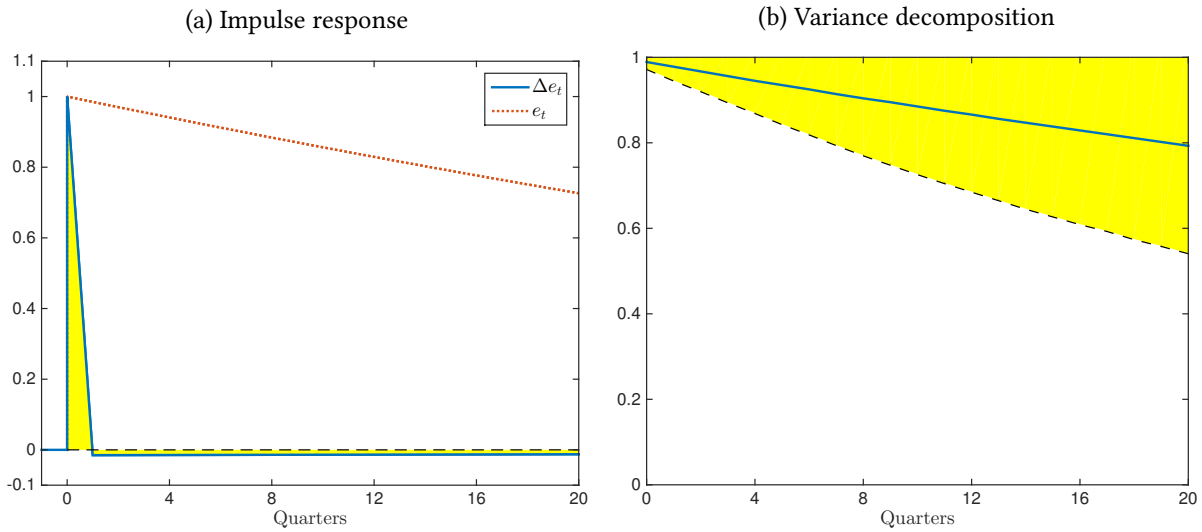


Figure 1: Impulse response and variance decomposition of Δe_t

Note: baseline model with $\rho = 0.97$ and $\beta = 0.99$. Left panel: impulse response to ε_0 of Δe_t (solid blue) and e_t (dashed red equal to the area under the blue curve). Right panel: the contribution of the unexpected component $(\Delta_k e_{t+k} - \mathbb{E}_{t-1}\Delta_k e_{t+k})$ to the variance of the exchange rate change $\Delta_k e_{t+k} \equiv e_{t+k} - e_{t-1}$ at various horizons $k \geq 0$ (the $k = 0$ case corresponds to the second moment in Proposition 4). The shaded area is the 95% bootstrap confidence interval in a sample of 120 quarters.

volatility of the financial shock:

$$\lim_{\beta \to 1} \frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2} \frac{1+\rho}{1-\rho} \xrightarrow[\rho \to 1]{} \infty.$$

The results in Proposition 4 derive directly from the exchange rate process in (27), and we provide the formal algebra behind them in Appendix A.4. A simple way to see why the exchange rate process in (27) approaches a random walk is to rewrite it using the lag operator:

$$(1 - \rho L)\Delta e_t = \frac{1}{1+\gamma\lambda_1} \frac{\beta}{1-\beta\rho} \left(1 - \frac{1}{\beta} L\right) \varepsilon_t.$$

As both β and ρ become close to 1, the lag operators on the two sides of the equation cancel out, and the process converges to a random walk (with unbounded volatility). Intuitively, a more persistent financial shock results in a more persistent expected appreciation with a larger effect on the cumulative discounted value of future net exports, especially when β is high. Therefore, when β and ρ are both large, a given shock ε_t requires a large contemporaneous jump in the exchange rate e_t to balance the intertemporal budget constraint of the country. In other words, the innovation to e_t is large relative to the innovation to $\mathbb{E}_t \Delta e_{t+1}$, and hence the unexpected component of the exchange rate changes dominates the expected component, making the process more like a random walk.

We now explore the extent to which Proposition 4 provides an accurate approximation of the exchange rate properties away from the $\beta\rho \to 1$ limit. In particular, Figure 1 plots the impulse response of the exchange rate (left panel) and the small sample variance decomposition of exchange rate changes

(right panel) for our baseline case with $\beta = 0.99$ and $\rho = 0.97$. The left panel illustrates that in response to a financial shock ε_t the model produces a large depreciation $\Delta e_t > 0$ on impact, followed immediately by small and persistent expected appreciations in all future periods $\mathbb{E}_t \Delta e_{t+j} < 0$ for $j \geq 1$, with Δe_t about 50 times larger than $\mathbb{E}_t \Delta e_{t+1}$ in absolute values. In the limit, the expected future appreciations become arbitrary small relative to the size of the devaluation on impact, and thus the impulse response converges to that of a white noise for Δe_t (or equivalently, random walk for e_t). The right panel of Figure 1 further shows that under our baseline parametrization the unexpected component dominates the variance of the exchange rate at all horizons. Furthermore, in a 30-year-long sample (typical for empirical studies of exchange rates), one cannot reject that the contribution of the expected component is negligible, as should be the case for a random walk.

To summarize, we find that with a conventional value of the discount factor β and the value of $\rho = 0.97$, which we chose to match the persistence of the interest rate differential, the model reproduces a volatile near-random-walk behavior for the nominal exchange rate, consistent with Meese and Rogoff (1983). Lastly, we point out that these near-random-walk behavior of the nominal exchange rate is a joint equilibrium outcome with the persistent evolution of the net foreign asset position b_t^* (the state variable), which follows an AR(1) in first differences (with AR root ρ). Nonetheless, the response of b_t^* to the financial shock is very muted quantitatively when the economy is relatively closed to international trade (i.e., when γ is low).[24]

4.3 Real exchange rate and the PPP puzzle

We next explore the equilibrium dynamics of the real exchange rate (RER), defined in (14), which we write in logs as:

$$q_t \equiv e_t + p_t^* - p_t. \tag{29}$$

Combining the definition of the price level (7) with the price setting equations for domestic and foreign goods ((12) and the foreign counterpart to (13)), we obtain (in the absence of the markup shocks):

$$p_t = \left(w_t - \frac{1}{1-\phi} a_t \right) + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t, \tag{30}$$

where $(w_t - \frac{1}{1-\phi} a_t)$ is the domestic nominal unit labor cost and $\frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t$ captures the effect of the cost of foreign goods (both final and intermediate) on the price of the domestic consumption bundle.[25] A long tradition in international macro literature models real exchange rate dynamics as a reflection of the relative price of non-tradables (e.g., the Balassa-Samuelson effect). This approach is, however, at odds with the empirical decomposition in Engel (1999), which finds a negligible role for this mechanism in the short and the medium run. We, therefore, do not model explicitly the relative productivity shocks

[24] In Appendix A.4, we show that NFA evolve according to $\Delta b_{t+1}^* = \frac{\gamma \lambda_2}{1+\gamma \lambda_1} \frac{1}{1-\beta\rho} \psi_t$, with volatility decreasing in γ. In Section 5.2, we consider an extension of the model with a financial sector that results in b_t^* following a stationary AR(2) process, which is however indistinguishable in the short run from the integrated process we obtain here.

[25] Note that the expression for p_t does not depend on the strategic complementarity elasticity α, which controls the extent of exchange rate pass-through. With firms symmetric in α, low pass-through into import prices is exactly offset by the domestic firm price adjustment (for further analysis see Burstein and Gopinath 2012, Amiti, Itskhoki, and Konings 2016).

for non-tradables. As a result, the price level in (30) depends on the overall degree of home bias γ, but *not* on how the domestic expenditure is split between tradable and non-tradable goods.

Combining (30) and its foreign counterpart for p_t^* with the definition of q_t in (29), we solve for the relationship between nominal and real exchange rates:

$$\left[1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right]q_t = e_t + \left((w_t^*-w_t)-\frac{1}{1-\phi}(a_t^*-a_t)\right). \tag{31}$$

Intuitively, when γ is small, real exchange rate approximately equals the differential unit labor costs in the two countries, and as the home bias disappears (γ increases towards 1/2), we have $q_t \to 0$ and the purchasing power parity holds in the limit. Recall that in our baseline model we switch off productivity shocks (that is, $a_t - a_t^* \equiv 0$) and assume that the monetary policy fully stabilizes wage inflation (namely, $\tilde{w}_t = w_t^* \equiv 0$), and therefore (31) implies the following result:

Proposition 5 (Real exchange rate) *In the baseline model, the relationship between the real exchange rates q_t and the nominal exchange rate e_t is given by:*

$$q_t = \frac{1}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}e_t. \tag{32}$$

Therefore, q_t is perfectly correlated with e_t, follows the same ARIMA process with a proportionally smaller innovation, and $\lim_{\gamma\to 0}(q_t - e_t) = 0$. The estimated AR(1) coefficient for q_t increases towards 1 with sample size, and hence the corresponding half-life estimate increases without bound.

Thus, in response to financial shocks ψ_t, the nominal and real exchange rates are perfectly correlated, equally persistent, and the real exchange rate is somewhat less volatile:

$$\frac{\mathrm{std}(\Delta q_t)}{\mathrm{std}(\Delta e_t)} = \frac{1}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} < 1,$$

with this gap vanishing as the economy becomes closed to international trade ($\gamma \to 0$). Quantitatively, under our baseline parameterization with $\gamma = 0.07$, the real exchange rate is three-quarters as volatile as the nominal exchange rate. We show in Section 5.1 that incorporation of sticky prices easily pushes this relative volatility close to 1, as in the data. However, we emphasize here that even the simple version of our model without any price or wage stickiness can account for the bulk of the empirical properties of the real exchange rate. In particular, if one were to fit an AR(1) process for the real exchange rate, as is conventionally done in the PPP puzzle literature surveyed in Rogoff (1996), one would be challenged to find evidence of mean reversion and would infer very long half-lives for the real exchange rate process. In Appendix Figure A1, we show that under our baseline parametrization with $\rho = 0.97$, the model reproduces the 3-to-5 year half-lives of the real exchange rate in finite (30-year-long) samples.[26]

In summary, our model resolves the PPP puzzle, which we broadly interpret as the close comovement between the nominal and the real exchange rates, and a volatile near-random-walk behavior for

[26] While real exchange rate follows an integrated ARIMA(1,1,1) process in the baseline model, Section 5.2 offers an extension with a financial sector, in which it follows a mean-reverting ARMA(2,1), yet the two are indistinguishable in finite samples.

both variables.[27] Furthermore, this is achieved without any reliance on nominal rigidities. A natural question then is why the PPP puzzle posed such a challenge to the literature? Equations (29)–(31) offer an explanation. The close comovement between q_t and e_t suggests that the price levels p_t and p_t^* should, in turn, move little with the nominal exchange rate e_t (see (29)). The PPP puzzle literature has largely focused on one conceptual possibility, namely that price levels move little due to nominal rigidities, while the main driver of nominal exchange rate are monetary shocks. The issue with this approach is that monetary shocks necessarily imply cointegration between relative nominal variables — $(w_t - w_t^*)$, $(p_t - p_t^*)$ and e_t — which results in mean reversion in the real exchange rate q_t. The speed of this mean reversion is directly controled by the duration of nominal price stickiness, which is empirically insufficient to generate long half lives of the real exchange rate.

We focus on the other conceptual possibility, namely that prices are largely disconnected from exchange rates, or in other words the low exchange rate pass-through into CPI inflation even in the long run, due to substantial home bias (small γ; see (30)), as is the case empirically. Importantly, this mechanism requires that the main drivers of the exchange rate cannot be productivity or monetary shocks, which drive a wedge between nominal and real exchange rates independently of the extent of home bias, as reflected by $(a_t - a_t^*)$ and $(w_t - w_t^*)$ terms in (31). Instead, the shock we focus on is the financial shock ψ_t, and it drives no wedge between nominal and real exchange rates, even in the long-run. Home bias is thus the only relevant part of the transmission mechanism, which does *not* rely on nominal rigidities, real rigidities (α), or the extent of expenditure switching (θ).[28] The mechanism does rely, however, on the monetary policy rule. We show in Section 5.1 that our results are robust to conventional Taylor rules, but are sensitive if the monetary authority pegs instead the nominal exchange rate. We also discusses there the related, yet distinct, Mussa puzzle, associated with the change in the monetary policy rule.

4.4 Exchange rates and the terms of trade

In this section we explore the joint properties of the equilibrium real exchange rate (CPI-based), terms of trade and producer prices. As emphasized by Atkeson and Burstein (2008), the conventional models imply a counterfactually volatile terms of trade and producer prices relative to consumer prices. In the data, consumer- and producer-based real exchange rates are equally volatile, while the terms of trade are substantially more stable — about three times less volatile than the real exchange rate.

The results in this section follow from two equilibrium relationships between real exchange rates

[27] The PPP puzzle can be restate as a combination of the large volatility and high persistence characteristic of the real exchange rate in the data. Large volatility is at odds with the productivity shocks, including shocks to the relative productivity of non-tradables, while high persistence (3–5 year half-lives) is at odds with monetary shocks under nominal rigidities (for further discussion and references see Rogoff 1996).

[28] A nominal devaluation in this case reduces the relative home unit labor cost and hence the relative home price level $(p_t - p_t^* - e_t)$, thus resulting in a nearly-proportional real devaluation when γ is low, as can be seen from (30) and (31).

(RER) and the terms of trade (ToT):

$$q_t = (1-\gamma)q_t^P - \gamma s_t, \tag{33}$$

$$s_t = q_t^P - 2\alpha q_t, \tag{34}$$

where $s_t = p_{Ft} - p_{Ht}^* - e_t$ is the log of the ToT in (21) and $q_t^P \equiv p_{Ft}^* + e_t - p_{Ht}$ is the log producer-price RER. Intuitively, (33) reflects that the relative consumer prices q_t differ from the relative producer prices q_t^P by the relative price of imports s_t. Equation (34), in turn, states that the terms of trade reflect the relative producer prices adjusted for the law of one price deviations of export prices from producer prices (recall from (14) that the log LOP deviation equals αq_t).

Conventional models, without strategic complementarities ($\alpha = 0$), and thus without LOP deviations, imply that the ToT equals producer RER, and both are more volatile than the consumer RER:

$$s_t = q_t^P = \frac{1}{1-2\gamma} q_t.$$

Intuitively, consumer prices are less volatile than producer prices as they smooth out the relative price fluctuations by combining home and foreign goods into the consumption bundle (i.e., a diversification argument). This is, however, empirically counterfactual, and as explained by Atkeson and Burstein (2008) is not necessarily the case in the models with *pricing to market* (PTM), arising, for example, from strategic complementarities in price setting ($\alpha > 0$). Combining (33) and (34) together, we arrive at:[29]

Proposition 6 (Real exchange rate and the terms of trade) *The real exchange rates and the terms of trade are linked by the following equilibrium relationships:*

$$q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t \qquad \textit{and} \qquad s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t. \tag{35}$$

The empirical stylized facts $\mathrm{std}(\Delta s_t) \ll \mathrm{std}(\Delta q_t) \approx \mathrm{std}(\Delta q_t^P)$ *and* $\mathrm{corr}(\Delta s_t, \Delta q_t) > 0$ *obtain when strategic complementarities in price setting are significant, but not too strong:*

$$\frac{\gamma}{1-\gamma} \ll \alpha < \frac{1}{2(1-\gamma)}. \tag{36}$$

When the left inequality in condition (36) is satisfied, we obtain the empirically relevant case, in which the RER q_t is considerably more volatile than ToT s_t. Pricing to market ($\alpha > 0$) smoothes out the response of ToT to changes in the producer prices q_t^P, and hence makes ToT less volatile, as export prices mimic the local competition. Very strong PTM ($\alpha \approx 1$), just like local currency pricing (LCP), can turn the positive correlation between ToT and RER into negative, which is empirically counterfactual (as emphasized by Obstfeld and Rogoff 2000, who documented a mild positive correlation). For the intermediate values of α (when both inequalities in (36) are satisfied), the performance of the model is good on both margins. Additionally, a low value of γ ensures that $\mathrm{std}(\Delta q_t) \approx \mathrm{std}(\Delta q_t^P)$, as is the case

[29] The relationships (35) in this proposition hold not just for ψ_t shocks, but for all shocks with the exception of the LOP deviation shocks η_t, as can be seen from the derivations in Appendix A.2.2.

empirically. Under our baseline parameterization with $\gamma = 0.07$ and $\alpha = 0.4$, the terms of trade are about a third as volatile as the real exchange rate, yet still positively correlated, exactly as in the data. We further explore these quantitative implications of the model in Appendix Figure A2.

4.5 Real exchange rate and consumption: the Backus-Smith puzzle

We now study the relationship between the real exchange rate and aggregate consumption. We start with the static optimality conditions characterizing equilibrium in the labor and product markets. First, we combine labor supply (3) with labor demand (10), and express out the real wage $w_t - p_t$ using (30) to arrive at:

$$\sigma\nu c_t + y_t = -\frac{\nu+\phi}{1-\phi}\frac{\gamma}{1-2\gamma}q_t. \tag{37}$$

This equation characterizes the locus of allocations consistent with equilibrium in the labor market. The right-hand side of this equation reflects the real wage $(w_t - p_t)$, which according to (30) is decreasing with real depreciation (high q_t), as the price level increases in the cost of foreign goods. Holding the real wage constant, output is high when consumption is low, reflecting the income effect on labor supply.

Second, we look at the product market clearing condition, which derives from (18)–(19), and upon log-linearization results in:[30]

$$y_t = (1-\phi)\left[(1-\gamma)c_t + \gamma c_t^*\right] + \phi\left[(1-\gamma)y_t + \gamma y_t^*\right] - \phi\gamma q_t + 2\theta(1-\alpha)\frac{\gamma(1-\gamma)}{1-2\gamma}q_t. \tag{38}$$

This equation characterizes the locus of allocations consistent with equilibrium in the product market. Note that in the closed economy ($\gamma = 0$), we simply have $y_t = c_t$, which holds in log deviations independently of the intermediate share ϕ. In the open economy, the home production y_t is split between final consumption at home and abroad, as well as the intermediate use at home and abroad, as reflected by the first two terms on the right-hand side of equation (38). The remaining terms in the real exchange rate q_t correspond respectively to the substitution away from the intermediate inputs towards local labor and to the expenditure switching effect between home and foreign goods. The *expenditure switching effect* acts to increase the demand for domestic output y_t when the home exchange rate is depreciated (q_t is high), and this effect is shaped by $\theta(1-\alpha)$ — the product of pass-through elasticity $(1-\alpha)$ of exchange rate into relative prices and the elasticity of substitution θ.

Now combining (37)–(38) together with their foreign counterparts, we can solve for the equilibrium relationship between the relative consumption $c_t - c_t^*$ and the real exchange rate q_t (see Appendix Figure **??** for a simple geometric solution):

Proposition 7 (Backus-Smith resolution) *In the baseline model with financial shocks ψ_t only, the*

[30] To derive (38), upon log-linearization of (18)–(19), we solve out the real wage and the relative prices as a function of the real exchange rate. In particular, in Appendix A.4 we show that the exchange rate pass-through into relative prices is determined by $(1-\alpha)$, as $p_{Ht} - p_t = -(1-\alpha)\frac{\gamma}{1-2\gamma}q_t$ and $p_{Ht}^* - p_t^* = -(1-\alpha)\frac{1-\gamma}{1-2\gamma}q_t$.

relative home consumption declines with real depreciation, according to the following equilibrium relation:

$$c_t - c_t^* = -\gamma \kappa_q q_t, \quad \textit{where} \quad \kappa_q \equiv \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu + \frac{\nu+\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}}{(1-\phi)\left[1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)\right]}\frac{2}{1-2\gamma} > 0. \tag{39}$$

The effect of the real exchange rate on the relative consumption vanishes as the economy becomes closed to international trade ($\gamma \to 0$). For $\gamma > 0$, the elasticity of the relative consumption with respect to real exchange rate increases (in absolute terms) in the strength of the expenditure switching effect, $\theta(1-\alpha)$.

It follows from Proposition 7 that our baseline model robustly reproduces a negative correlation between relative consumption and real exchange rate, both in levels and in growth rates. That is, our model predicts that consumption is low when prices are low (in relative terms), which violates the efficient international risk sharing (i.e., the celebrated Backus-Smith condition), yet is consistent with the empirical patterns (see estimates across countries in Benigno and Thoenissen 2008). This property of our model stands in stark contrast with both the productivity-shock-driven International RBC models and the monetary-shock-driven international New Keynesian (NOEM) models, even when those models assume incomplete markets. What is most striking is that we have derived (39) using solely the labor and goods market clearing conditions, which are completely ubiquitous in all international general equilibrium models. Indeed, the negative correlation between consumption and real exchange rate is a robust feature of the expenditure switching mechanism: Depreciated real exchange rate switches expenditure towards the home goods, and in order to clear the markets home output needs to rise and home consumption, in view of home bias, needs to fall.[31] A natural question then is what makes our model different?

There are two key features of our model that allow it to produce the empirically relevant correlation between consumption and the real exchange rate. First, we shift the determination of consumption from asset to product markets. Indeed, in complete market models, partial equilibrium in the asset market requires a positive correlation between relative consumption and real exchange rate: $\sigma(c_t - c_t^*) = q_t$, which is the outcome of the optimal international risk-sharing. Our model instead features incomplete markets and a risk-sharing shock, which implies the following relationship between consumption growth and real appreciation in expectations:

$$\mathbb{E}_t\left\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\right\} = \psi_t. \tag{40}$$

Furthermore, the equilibrium relationship between consumption and exchange rate (39) is fully determined in product market, without reference to asset market and risk sharing.[32]

Second, our model ensures that the key force shaping the comovement between consumption and

[31] Note also that an increase in consumption is inconsistent with labor market clearing (37), as then labor supply and output would decline due to both income effect (increase in consumption) and substitution effect (reduction in real wage).

[32] Of course, asset markets matter in general equilibrium, as both consumption and exchange rate dynamics needs to be consistent with asset market clearing and equilibrium interest rates (see Section 4.6). However, equation (39) is a static equilibrium condition, which determines the correlation between consumption and real exchange rate independently of their dynamic processes, as it holds state-by-state. This is possible because the financial shock ψ_t does not directly affect the labor and product markets, and thus is absent from (39).

the real exchange rate is expenditure switching, emphasized in equation (39). This is in contrast with the IRBC and NOEM models, where real depreciation is associated with increased supply of domestic output (either due to a high productivity shock or due to suppressed markups and prices in response to a monetary easing), while expenditure switching is only a byproduct (see discussion below) As a result, a real depreciation in these models is associated with an empirically counterfactual domestic consumption boom. The mechanism in our financial model is different. A real depreciation is not caused by increased supply of domestic goods, but instead by increased demand for foreign assets. Therefore, the only effect on the real economy is indirect, induced by expenditure switching arising from a real depreciation, which causes a decline in real wages and consumption and an increase in employment, output and net exports (see formal expressions in Appendix A.4).

While a negative correlation between consumption and real exchange rate emphasized in Proposition 7 is a desirable property of the model, we are also interested in its quantitative implications. A salient feature of the data is the much greater volatility of the exchange rate relative to the other macroeconomic variables, and in particular consumption (see Section 5.1 for the additional discussion of the weak correlation between RER and macro variables in an economy with multiple shocks). From (39), we know that the relative volatility of consumption tends to zero as the economy becomes closed to international trade ($\gamma \to 0$). Outside this limit, for our baseline parameterization $\text{std}(\Delta c_t)/\text{std}(\Delta q_t) = 0.15$. That is, consumption is about 6 times less volatile than the real exchange rate, in line with the empirical magnitudes (for example, Chari, Kehoe, and McGrattan (2002) target a ratio of 5). We further find that this quantitative performance of the model is not very sensitive to the specific values of the relative risk aversion σ and Frish elasticity of labor supply ν, but is mostly sensitive to trade openness γ and the combined expenditure switching elasticity $\theta(1-\alpha)$, as we illustrate in the Appendix Figure A2.[33] We conclude that the model is not only consistent with the qualitative negative correlation between consumption and real exchange rate, but also captures the quantitative extent of the stability of macro variables relative to exchange rates.

Alternative mechanisms in the literature Naturally, all explanations of the Backus-Smith puzzle must relax the straitjacket of the international risk-sharing condition $\sigma(c_t - c_t^*) = q_t$, either by assuming incomplete markets (e.g. Corsetti, Dedola, and Leduc 2008, Benigno and Thoenissen 2008), or departing from separable CRRA utility (e.g. Colacito and Croce 2013, Karabarbounis 2014). The analytical tractability of our model, coupled with the conventional product and labor market structure, allows us to shed light on the mechanisms in other papers in relationship to our mechanism. In particular, expression (A49) in Appendix A.2.2 extends the equilibrium condition for consumption (39) to the general framework of Section 3 with all potential shocks and wedges, which nests as special cases both IRBC and NOEM models.

Here for concreteness we focus on the model with productivity shocks only (as is the case in much

[33] When we simultaneously reduce σ from 2 to 1 and ν from 1 to 0.5, consumption still remains about 4 times less volatile then RER. In contrast, if we double γ from 0.07 to 0.14 or increase θ from 1.5 to 5, consumption becomes only about 2.75 times less volatile than RER, which is sizably more volatile than in the data. If $\theta \leq 3$, however, the model still maintains that consumption is 4 times less volatile than RER. See Appendix A.4.2 for a more systematic quantitative robustness analysis.

of the Backus-Smith puzzle literature), which results in the following equilibrium relationship:

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{41}$$

where $\kappa_a > 0$ (see appendix), while $\kappa_q > 0$ is as before in (39). This is the sense in which the same expenditure switching effect of the real exchange rate on consumption is still present in the models with other shocks, but these shocks also have a direct effect in product and labor markets. Importantly, κ_a does not go to zero with $\gamma \to 0$, and therefore, the direct effect always dominates the expenditure switching effect when economies are relatively closed to international trade. While we emphasize expenditure switching as the key (and only) source of the relationship between consumption and real exchange rate, other papers have it only as a feedback mechanism, typically not strong enough to overturn the direct effects of the product market shocks. This explains why Backus-Smith puzzle proved such a challenge in the literature, despite the natural expenditure switching forces.[34]

Equilibrium relationship (41) makes clear the two possible ways in which the Backus-Smith puzzle can be resolved. First, this occurs when exchange rate dynamics is shaped by a shock with a small direct effect on consumption relative to its indirect (expenditure switching) effect through exchange rate. The financial shocks emphasized in this paper, as well as the news shocks about future productivity (e.g., the long-run risk shocks in Colacito and Croce 2013) and labor-wedge shocks under non-separable utility (Karabarbounis 2014), all operate in this fashion. The alternative way to resolve the Backus-Smith puzzle, when the direct effect of the shock dominates and consumption increases with productivity, is to have real exchange rate appreciate in response to a positive productivity shock (that is, $\partial q_t/\partial a_t < 0$). This may arise due to Balassa-Samuelson forces (e.g., Benigno and Thoenissen 2008), persistent productivity growth rates or low elasticity of substitution between home and foreign goods ($\theta < 1$), as in Corsetti, Dedola, and Leduc (2008). The mechanisms that do not rely on financial shocks, however, are at odds with other exchange rate puzzles, including Meese-Rogoff and the PPP puzzle discussed above. See Appendix A.6 for two illustrations and a further discussion.

4.6 Exchange rate and interest rates: the UIP Puzzle

Lastly, we explore the equilibrium properties of the interest rates, and in particular the UIP puzzle. The home Euler equation (4), characterizing demand for domestic bond, yields upon log-linearization $i_t = \mathbb{E}_t\{\sigma\Delta c_{t+1} + \Delta p_{t+1}\}$. Combining it with the foreign counterpart for i_t^*, and using the solution for consumption (39) and prices (30), we obtain:

$$i_t - i_t^* = -\gamma\lambda_1 \mathbb{E}_t \Delta e_{t+1}, \tag{42}$$

in light of the relationship (32) between nominal and real exchange rates, and where $\lambda_1 > 0$ under a mild parameter restriction discussed in Appendix A.4.1.

[34] In contrast to the macro literature, the asset pricing models of international risk sharing often assume an exogenous process for consumption, so that relationship (41) does not hold and there is no expenditure switching. As a result, these models are usually unable to explain the Backus-Smith puzzle (e.g., compare Colacito and Croce 2011 versus 2013).

Combining (42) with the UIP condition (23) results both in the expression for the expected devaluation (25) used in Section 4.2 and in the following solution for the equilibrium interest rate differential:

$$i_t - i_t^* = \frac{\gamma\lambda_1}{1+\gamma\lambda_1}\psi_t. \tag{43}$$

As we already discussed, the demand shock for foreign bond $\psi_t > 0$ raises the interest rate differential $i_t - i_t^*$ to balance out the asset markets. The increase in the home interest rate supports the decline in consumption associated with the real devaluation on impact of the shock (the Backus-Smith puzzle). Equation (43) further implies that, like ψ_t, the interest rate differential follows an AR(1) process with persistence ρ, and in addition with volatility declining towards zero in the closed economy limit ($\gamma \to 0$). Recall that the empirically high persistence of the interest rate differentials was our calibration target in setting $\rho = 0.97$.

Next, making use of our characterization in Proposition 3, we study the joint properties of the interest rate differential and the nominal exchange rate (similar relationships also hold in real terms):

Proposition 8 (Exchange rate and interest rates) *The Fama regression, i.e. the projection of the exchange rate change Δe_{t+1} on the interest rate differential $(i_t - i_t^*)$, has a negative coefficient $\beta_F = -1/(\gamma\lambda_1) < 0$. Furthermore, around the $\beta\rho \to 1$ limit:*

(i) the R^2 in the Fama regression *becomes arbitrary small (goes to zero);*

(ii) the volatility of Δe_{t+1} relative to $(i_t - i_t^*)$ becomes arbitrary large (goes to infinity);

(iii) the persistence of Δe_{t+1} relative to $(i_t - i_t^*)$ becomes arbitrary small (goes to zero);

(iv) the Sharpe ratio of the Carry trade[35] *becomes arbitrary small (goes to zero).*

Proposition 8 suggests that our model provides a good approximation to the the observed empirical patterns. As in the data, positive interest rate differentials predict expected exchange rate appreciations — a pattern of the *UIP deviations* known as the *Forward Premium puzzle* (Fama 1984). This result follows directly from (42), as a $\psi_t > 0$ shock results both in a positive interest rate differential and an expected appreciation of the home currency.

At the same time, the predictive ability of the interest rate differentials for future devaluations is very weak in the data (e.g., see Table 1 in Valchev 2015), and our model captures this with a vanishingly small R^2 in the Fama regression as the ψ_t shocks become more persistent. Recall from Proposition 4 that this is the case in which the unexpected changes dominate the dynamics of the exchange rate, while the expected changes play a vanishingly small role. Under our baseline parameterization, the R^2 in the Fama regression is below 0.05, in line with the data. In addition, the interest rate differentials, unlike exchange rate changes, are very smooth and very persistent in the data — another disconnect pattern captured by our model. For example, under the baseline parametrization, the model produces interest rate differentials that are less than one-tenth as volatile as the exchange rate changes (see

[35] A carry trade is a zero-capital investment strategy, which shorts the low interest rate currency and longs the high interest rate currency. For concreteness, following Lustig and Verdelhan (2011) and Hassan and Mano (2014), we focus on a strategy with an intensity (size of the positions taken, x_t) proportional to the expected return, i.e. $x_t = i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \psi_t$. See Appendix A.4 for the derivation of the expected return and the Sharpe ratio of this strategy.

Appendix A.4.2 for detailed quantitative results). Lastly, our model does not imply counterfactually large returns on the Carry trade, predicting small Sharpe ratios. In Appendix Figure A3, we illustrate the small-sample properties of the carry trade returns, along with the properties of the Fama regression, and show that the associated Sharpe ratio varies between 0.15 and 0.3, in line with the empirical patters.[36]

To summarize the results of Section 4, our baseline model with a simple transmission mechanism and a single financial exchange rate shock is consistent, both qualitatively and quantitatively, with a rich set of moments describing the dynamic comovement between exchange rates and macro variables. Many of these moments correspond to the long-standing puzzles from the point of view of the conventional international-macro models, including the PPP puzzle and the Backus-Smith puzzle. The financial shock ψ_t admits a number of micro-foundations, yet is not directly observable in the data. When ψ_t is calibrated to follow a persistent AR(1) process with small innovations to match the properties of the interest rate differentials, our model reproduces both the empirical Meese-Rogoff exchange rate dynamics and the empirical comovement properties between exchange rates and interest rates, including the Forward Premium (UIP) puzzle.

5 Extensions

This section considers two main extensions to our analysis. Section 5.1 generalizes the baseline model to a full-fledged monetary model with nominal stickiness and a Taylor rule. In the context of this extension, we discuss the properties of the model with multiple shocks, including productivity and monetary shocks, as well as the Mussa puzzle associated with the switch in the monetary regimes. Section 5.2 extends the baseline model to feature an explicit financial sector with risk-averse intermediaries and noise traders to shed light on a number of issues including the recent Engel (2016) puzzle. The additional extensions of the baseline framework, including the model with capital, are considered in Appendix A.7.

5.1 A monetary model with nominal rigidities. Mussa puzzle

We now consider the robustness of our findings in Sections 3 and 4 in a fully-specified monetary model with nominal rigidities, which is arguably a salient feature of the world. First, we demonstrate that nominal shocks *per se* cannot reproduce the empirical exchange rate behavior, as suggested by Proposition 1. Second, we show that the financial shock ψ_t has similar quantitative properties in the monetary model, as in the real model of Section 4, despite a different transmission mechanism for interest rates. Finally, we discuss the robustness of the results to various Taylor rules, and the associated Mussa puzzle.

We introduce nominal rigidities as in the standard New Keynesian model (see e.g. Woodford 2003), while leaving the structure of international financial markets as in the baseline model. We focus on a cashless-limit economy and abstract from ZLB, commitment problems and multiplicity of equilibria. In

[36]The unconditional Sharpe ratio of the carry trade in the data is about 0.5, but at least half of it comes from the cross-sectional country fixed effects not modeled in our framework, which instead focuses on the time-series properties. Therefore, our empirical target for the Sharpe ratio is 0.2, corresponding to the *forward premium trade* in Hassan and Mano (2014).

particular, the nominal interest rate is set by a central bank according to the Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m) \left[\delta_\pi \pi_t + \delta_y \hat{y}_t\right] + \varepsilon_t^m, \tag{44}$$

where $(\rho_m, \delta_\pi, \delta_y)$ are parameters, π_t is the CPI inflation rate, $\hat{y}_t$ is the output gap, and ε_t^m is the source of exogenous monetary shocks.

Firms are subject to a Calvo price-setting friction with the probability of price adjustment equal to $(1 - \lambda_p)$. We maintain the assumption that the optimal static price depends both on own marginal cost and competitor prices, as in (12), with the exogenous markup shocks shut down. We further assume that exporters set prices in the local currency (LCP), as for example in Chari, Kehoe, and McGrattan (2002) and Devereux and Engel (2002). Households set wages and are also subject to the Calvo friction with the probability of wage adjustment equal to $(1 - \lambda_w)$, as described in Galí (2008). Appendix A.7 provides a full description of the model and the characterization of its solution, as well as several extensions with PCP price stickiness and alternative Taylor rules.

To calibrate the model, we keep the same values of the parameters as in the baseline case discussed in Section 4. The prices are assumed to adjust on average once a year, i.e. $\lambda = 0.75$ (Nakamura and Steinsson 2008). For wages, we set $\lambda_w = 0.85$ corresponding to a longer expected wage duration equal to 1.5 years. For the Taylor rule parameters, we use the estimates from Clarida, Gali, and Gertler (2000), setting the interest rate smoothness parameter $\rho_m = 0.8$, the inflation response elasticity $\delta_\pi = 2.15$ and the output gap elasticity $\delta_y = 0.25$, and assume that the monetary shocks ε_t^m follow an iid process, whenever present. SHOULD WE CONSIDER THE CASE WITH $\rho_m = 0.95$ AT LEAST FOR NOEM?

We summarize the results of our analysis in Table 2, where we contrast the moments in the data and in various versions of the model. Panel A considers various single-shock models. We first report the moments in the baseline model and in the monetary model, both subject exclusively to the financial shock ψ_t. We then report the moments in the New Keynesian NOEM model subject to the monetary shock ε_t^m only, i.e. with the ψ_t shock shut down. Lastly, for comparison, we also provide the moments from an IRBC model (without nominal rigidities, $\lambda_w = \lambda_p = 0$) with productivity shocks a_t alone, which follow a persistent AR(1) process with $\rho_a = 0.95$. EXPLAIN THAT RELATIVE SHOCKS ONLY Given our parameter choices described above, all these models require no further calibration.

Table 2: Quantitative properties and comparisons across models

Moment	Data	Panel A: Single-shock models				Panel B: Multi-shock models				
		Financial shock ψ		NOEM	IRBC	IRBC	NOEM	Mussa Puzzle		Financial
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\rho(\Delta e)$	0.00	−0.02 (0.09)	−0.03 (0.09)	−0.15 (0.08)	0.00 (0.09)	−0.02 (0.09)	−0.02 (0.09)	−0.10 (0.09)	−0.06 (0.09)	
$\rho(q)$	0.94	0.93 (0.04)	0.93 (0.04)	0.67 (0.07)	0.93 (0.04)	0.93 (0.04)	0.92 (0.04)	0.97 (0.02)	0.96 (0.03)	
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.75	0.98	0.99	1.13	0.99	0.97	1.11	1.04	
$\text{corr}(\Delta q, \Delta e)$	0.99	1	1	1	1	1.00	1.00	0.93	0.98	
$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	0.25	0.31	0.33	0.51	0.64	0.29	0.30	2.21	0.53	
$\text{corr}(\Delta c - \Delta c^*, \Delta q)$	−0.28	−1	−1	1	1	−0.21 (0.09)	−0.20 (0.09)	−0.77 (0.04)	−0.47 (0.07)	
$\sigma(\Delta nx)/\sigma(\Delta q)$	0.12	0.25	0.19	0.08	0.14	0.30	0.24	1.01	1.01	
$\text{corr}(\Delta nx, \Delta q)$	$\gtrsim 0$	1	1	1	1	−0.03 (0.09)	−0.07 (0.09)	−0.19 (0.09)	−0.80 (0.03)	
$\sigma(\Delta s)/\sigma(\Delta e)$	0.34	0.30	0.80	0.89	0.34	0.29	0.79	0.86	0.83	
$\text{corr}(\Delta s, \Delta e)$	0.30	1	−0.93	−0.99	1	1	−0.92	−0.91	−0.92	
Fama β	$\lesssim 0$	−8.1 (4.7)	−2.7 (1.5)	1.1 (0.3)	1.4 (0.6)	−0.2 (0.9)	0.0 (0.8)	−0.1 (0.1)	1.1 (0.3)	
Fama R^2	0.02	0.04 (0.02)	0.04 (0.02)	0.15 (0.04)	0.08 (0.02)	0.00 (0.01)	0.00 (0.01)	0.00 (0.01)	0.09 (0.04)	
$\sigma(i - i^*)/\sigma(\Delta e)$	0.10	0.03 (0.01)	0.03 (0.02)	0.36 (0.04)	0.19 (0.06)	0.11 (0.03)	0.10 (0.02)	0.55 (0.16)	0.27 (0.03)	
$\rho(i - i^*)$	0.80	0.95 (???)	0.97 (0.02)	0.67 (0.07)	0.93 (0.04)	0.93 (0.04)	0.81 (0.07)	0.89 (0.06)	0.60 (0.08)	
Carry SR	0.20	0.21 (0.04)	0.26 (0.04)	0	0	0.19 (0.06)	0.20 (0.07)	0.60 (0.13)	—	

Note:

The results in Panel A of Table 2 have a number of implications. First, as we already discussed in Section 4, the baseline model in column 1 matches closely most empirical moments. Second, the quantitative results for the monetary model with a financial shock in column 2 are remarkably close to the baseline model in column 1, even despite the differences in the transmission mechanism. Indeed, in the baseline model, the interest rates settle down at levels that clear the asset markets, while in the monetary model the path of the interest rates is chosen by the government according to the Taylor rule (44). In both models, a ψ_t shock results in a sharp nominal depreciation, which in turn leads to a mild home inflation as the prices of the foreign goods increase. In a monetary model, the central bank responds by raising the interest rate, and the households respond by cutting their current consumption expenditures, thus enabling the model to reproduce both the Backus-Smith puzzle and the UIP puzzle. SHOULD WE DISCUSS ROBUSTNESS TO TAYLOR RULES? Furthermore, nominal rigidities result in a dynamic price adjustment process, which acts to increase the volatility of the real exchange rate, bringing it closer to the empirical counterpart, without sacrificing its persistence, and hence improving the fit of the model.[37]

Lastly, columns 3 and 4 of Table 2 report the results from the conventional NOEM and IRBC models, without the financial shock. These models fail on a number of moments and reproduce the familiar exchange rate puzzles. In particular, both of these models cannot reproduce the comovement between interest rates and exchange rate (UIP puzzle) and consumption and exchange rate (Backus-Smith puzzle). Furthermore, the NOEM model is challenged to reproduce the persistence of the real exchange rate (PPP puzzle), while for the IRBC model matching the volatility of the nominal exchange rate requires incredibly large volatility of the relative TFP shocks across countries. HOW ABOUT MEESE-ROGOFF PREDICTABILITY?

Multiple shocks and variance decomposition A natural deficiency of any one-shock model is that it can only speak to the relative volatilities of variables, while implying counterfactual perfect correlations between them. We consider now two models with multiple shocks to study whether they can successfully reproduce the imperfect, and in general weak, empirical correlations between exchange rates and the macro variables. In particular, we consider an IRBC model with a productivity shock a_t and a NOEM model with a monetary (Taylor rule) shock ε_t^m. In both cases, we additionally include the financial shock ψ_t along with a demand shock for foreign goods ξ_t.[38] We use these calibrated multi-shock models to conduct variance decompositions in order to assess the relative importance of productivity, monetary and financial shocks for the exchange rate dynamics. The results of this analysis are reported in columns 5 and 6 of Table 2.

For the exchange rate moments we target, we only need to specify the relative magnitudes of these shocks across countries (that is, $\tilde{a}_t = (a_t - a_t^*)/2$, and similarly for other shocks). We keep the parameters of the model as in the baseline above, but we now need to calibrate the relative variances

[37] On the other hand, as pointed out by Obstfeld and Rogoff (2000), models with LCP imply a counterfactual negative correlation between the exchange rate and the terms-of-trade. This issue can be addressed with heterogeneity in price setting across firms (flexible, sticky LCP and PCP), as we discuss in Appendix A.7.

[38] We have also experimented with LOP-deviation shock η_t and a nearly-equivalent iceberg trade cost shock, but find them largely redundant as long as ξ_t is included, which has superior quantitative properties for the moments we have chosen.

of the shocks, with the overall level of variance kept to match the volatility of the nominal exchange rate. To do so, we target the relative standard deviations and the correlations of exchange rates with consumption, output and net exports. Table **??** reports the results.

The models with three shocks are, in general, successful at matching most moments, both relative volatilities and correlations. The resulting decomposition of the contribution of the shocks to the volatility of the exchange rate reveals in both cases that ψ_t plays the dominate role, while the contribution of the productivity and monetary shocks is minimal, as suggested by Proposition 1. The role of the ξ_t shock in both cases is also very important, offsetting the strong correlations between exchange rates and the macro variables arising from the ψ_t shock, without inducing counterfactually strong movements in consumption and other macro variables. [TO BE COMPLETED]

5.1.1 Mussa puzzle

The extension with nominal rigidities allows us to address the Mussa puzzle. Comparing empirical moments for several developed countries before and after the end of Bretton Woods system, the literature has emphasized the following stylized facts:

i. The volatility of the *real* exchange rate has increased almost as much as the volatility of the nominal exchange rate (Mussa 1986).

ii. There are almost no differences in the output or consumption volatilities between the two periods (Baxter and Stockman 1989, Flood and Rose 1995).

iii. The Backus-Smith risk-sharing condition and the UIP hold much better in the data during the Bretton Woods period Colacito and Croce (2013).

We check whether the model can reproduce these findings by comparing the benchmark floating exchange regime under the Taylor rule (44), with an alternative Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m)\, \delta_e e_t, \tag{45}$$

where parameter $\delta_e = 0.45$ is calibrated to match the empirical volatility of the nominal exchange rate.

We find (see Appendix A.7 for details) that the volatility of the real exchange rate falls about two times less than the volatility of the nominal exchange rate, which is in line with the estimates in Monacelli (2004). A decrease in the correlation between the nominal and the real exchange rates from 1.00 to 0.61 is also very close to the empirical estimate of 0.66. Furthermore, the volatility of the relative consumption remains almost unchanged. The model, however, cannot reproduce the third stylized fact, as it suggests that the Backus-Smith and the UIP puzzles remain unchanged across the nominal exchange rate regimes. This suggests that the financial exchange rate shock ψ_t might have been smaller (less volatile) during the Bretton Woods period. As was shown by Jeanne and Rose (2002), pegged exchange rate may render arbitrageurs more active in the currency markets, thereby offsetting a larger portion of the noise trading shock. Indeed, reducing the volatility of the ψ_t shock (and adjusting $\delta_e = 0.13$ to maintain the same equilibrium volatility of the nominal exchange rate) allows the model to be consistent with all empirical patterns, both under the float and under the peg.

5.2 Multiple foreign assets, global imbalances and valuation effects

Appendix A.8 [TO BE COMPLETED]

6 Conclusion

[TO BE COMPLETED]

A Appendix

A.1 Demand structure

Consider the general separable homothetic (Kimball 1995) demand aggregator C_t defined implicitly by:

$$\Omega_{Ht}\, g\left(\frac{C_{Ht}}{\Omega_{Ht}C_t}\right) + \Omega_{Ft}\, g\left(\frac{C_{Ft}}{\Omega_{Ft}C_t}\right) = 1, \tag{A1}$$

where $g' > 0$, $g'' < 0$ and $g(1) = g'(1) = 1$ (a normalization), and

$$\Omega_{Ht} \equiv (1-\gamma)e^{-\gamma\xi_t} \qquad \text{and} \qquad \Omega_{Ft} \equiv \gamma e^{(1-\gamma)\xi_t}$$

are the weights which the required properties. Then expenditure minimization results in the following demand schedules:

$$C_{Ht} = (1-\gamma)e^{-\gamma\xi_t} h\left(\frac{P_{Ht}}{D_t}\right) C_t \qquad \text{and} \qquad C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{D_t}\right) C_t, \tag{A2}$$

where $h(x) \equiv g'^{-1}(x)$, which implies $h' < 0$ and $h(1) = 1$, and where D_t is the Lagrange multiplier on the aggregator (A1). We can solve for D_t by substituting demand (A2) into the aggregator (A1):

$$(1-\gamma)e^{-\gamma\xi_t}\, g\left(h\left(\frac{P_{Ht}}{D_t}\right)\right) + \gamma e^{(1-\gamma)\xi_t}\, g\left(h\left(\frac{P_{Ft}}{D_t}\right)\right) = 1, \tag{A3}$$

and also define the price index in this economy from the expenditure per unit of C_t:

$$P_t = \frac{P_{Ht}C_{Ht} + P_{Ft}C_{Ft}}{C_t} = (1-\gamma)e^{-\gamma\xi_t} P_{Ht} h\left(\frac{P_{Ht}}{D_t}\right) + \gamma e^{(1-\gamma)\xi_t} P_{Ft} h\left(\frac{P_{Ft}}{D_t}\right). \tag{A4}$$

Proof of Lemma 1 It is immediate to check from (A3)–(A4), using $g(1) = h(1) = 1$, that when $\xi_t = 0$ and $P_{Ht} = P_{Ft}$, we have $P_t = D_t = P_{Ht} = P_{Ft}$, which corresponds to the symmetric steady state. Using demand (A2), we immediately have that the symmetric steady state foreign share is:

$$\left.\frac{P_{Ft}C_{Ft}}{P_{Ht}C_{Ht} + P_{Ft}C_{Ft}}\right|_{\xi_t=0,\ P_{Ht}=P_{Ft}} = \frac{\gamma C_t}{(1-\gamma)C_t + \gamma C_t} = \gamma,$$

where we again use $h(1) = 1$. We next use (A3)–(A4) to obtain the log-linear approximation for P_t and D_t around the symmetric steady state:[39]

$$p_t = d_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}, \tag{A5}$$

where p_t and d_t are the log deviations from the steady state values. Note that the taste shock $\xi_t \neq 0$ does not affect the first order approximation to the prices index (due to the way it enters the weights

[39] In the CES case, which obtains with $g(z) = \frac{1}{\theta-1}\left(\theta z^{\frac{\theta-1}{\theta}} - 1\right)$, we have $P_t = D_t$, while for a more general demand P_t and D_t different by a second order term around a symmetric steady state. Since our analysis relies on the first order approximation to the equilibrium system, we replace D_t with P_t in the demand equations (5) in the text.

Ω_{Ht} and Ω_{Ft}). Finally, log-linearizing (A2), we have:

$$c_{Ht} = -\gamma\xi_t - \theta(p_{Ht} - d_t) + c_t \qquad \text{and} \qquad c_{Ft} = (1-\gamma)\xi_t - \theta(p_{Ft} - d_t) + c_t,$$

where $\theta \equiv -\frac{h'(x)x}{x}\big|_{x=1} = -\frac{\partial \log h(x)}{\partial x}\big|_{x=1}$. Together with (A5), these expressions results in (6). Subtracting, we have $c_{Ft} - c_{Ht} = \xi_t - \theta(p_{Ft} - p_{Ht})$, which implies that the elasticity of substitution is indeed θ. ■

Monopolistic competition and price setting Consider now a unit continuum of symmetric domestic firms with marginal cost MC_t and a unit continuum of symmetric foreign firms with marginal cost $\mathcal{E}_t MC_t^*$ monopolistically competing in the domestic market. We generalize the consumption aggregator C_t to be defined in the following way:

$$\int_0^1 \Omega_{Ht}\, g\left(\frac{C_{Ht}(i)}{\Omega_{Ht} C_t}\right) \mathrm{d}i + \int_0^1 \Omega_{Ft}\, g\left(\frac{C_{Ft}(i)}{\Omega_{Ft} C_t}\right) \mathrm{d}i = 1, \tag{A6}$$

with taste shocks $(\Omega_{Ht}, \Omega_{Ft})$ determined as above by a common home bias parameter γ and a common demand shifter ξ_t for all varieties $i \in [0,1]$. The households choose $\{C_{Ht}(i), C_{Ft}(i)\}$ to maximize C_t given prices and total expenditure:

$$E_t = P_t C_t = \int_0^1 P_{Ht}(i) C_{Ht}(i) \mathrm{d}i + \int_0^1 P_{Ft}(i) C_{Ft}(i) \mathrm{d}i. \tag{A7}$$

This expenditure minimization results in individual firm demand as in (A2). A representative home firm takes (C_t, P_t, D_t) as given and sets its price to maximize profits from serving the domestic market:

$$P_{Ht}(i) = \arg\max_{P_{Ht}(i)} \left\{ (P_{Ht}(i) - MC_t)(1-\gamma) e^{-\gamma\xi_t} h\left(\frac{P_{Ht}(i)}{D_t}\right) C_t \right\},$$

which results in the standard markup pricing rule, with the markup $\mathcal{M}_t$ determined by the elasticity of the demand curve $h(\cdot)$. Since all domestic firms are symmetric, we have $C_{Ht} = C_{Ht}(i)$ and $P_{Ht} = P_{Ht}(i)$ for all $i \in [0,1]$. Similar price setting rule is used by symmetric foreign firms with marginal costs $\mathcal{E}_t MC_t^*$, and we also have $C_{Ft} = C_{Ft}(i)$ and $P_{Ft} = P_{Ft}(i)$ for all $i \in [0,1]$. Following the proof of Lemma 1, the elasticity of demand in a symmetric steady state equals θ, and therefore the steady state markup is given by $\mathcal{M} = \theta/(\theta - 1)$ for both home and foreign firms.

We next take a log-linear approximation to the optimal price P_{Ht} around the symmetric steady state:

$$p_{Ht} = -\Gamma(p_{Ht} - p_t) + mc_t,$$

where we use the approximation $d_t = p_t$ and Γ denotes the elasticity of the markup $\mathcal{M}_t$ with respect to the relative price of the firm, evaluated at the symmetric steady state. Note that this equation is the counterpart to (12) in the text with $\alpha \equiv \frac{\Gamma}{1+\Gamma}$ and $\mu_t = 0$.

Lastly, we provide further details about the primitive determinants of θ and α (see Amiti, Itskhoki, and Konings 2016, for a more indepth exposition). Define the demand elasticity function $\tilde{\theta}(x) \equiv -\frac{\partial \log h(x)}{\partial \log x}$, so that $\theta \equiv \tilde{\theta}(1)$. Then the markup function is given by $\tilde{\mathcal{M}}(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$, and the elas-

ticity of the markup is given by $\tilde{\Gamma}(x) \equiv -\frac{\partial \log \tilde{\mathcal{M}}(x)}{\partial x}$, with $\mathcal{M} = \tilde{\mathcal{M}}(1)$ and $\Gamma = \tilde{\Gamma}(1)$. Manipulating these definitions, we can represent

$$\tilde{\Gamma}(x) = \frac{\tilde{\epsilon}(x)}{\tilde{\theta}(x) - 1}, \qquad \text{where} \qquad \tilde{\varepsilon}(x) \equiv \frac{\partial \log \tilde{\theta}(x)}{\partial \log x}$$

is the elasticity of elasticity (or super-elasticity) of demand. Therefore, $\Gamma = \frac{\epsilon}{\theta - 1}$, where $\epsilon = \tilde{\epsilon}(1)$, and we further have:

$$\alpha = \frac{\Gamma}{1 + \Gamma} = \frac{\epsilon}{\epsilon + \theta - 1}.$$

To the extent ϵ and θ are controlled by independent parameters, we can decouple the elasticity of substitution θ from the strategic complementarity elasticity α. Indeed, θ is a characteristic of the slope (the first derivative) of demand h', while ϵ is a characteristic of the curvature (the second derivative) of demand h''. Formally, we have:

$$\theta = -\frac{h'(x)x}{h(x)}\Big|_{x=1} \qquad \text{and} \qquad \epsilon = \frac{\partial \log \tilde{\theta}(x)}{\partial \log x} = \left[1 - \frac{h'(x)x}{h(x)} + \frac{h''(x)x}{h'(x)}\right]\Big|_{x=1} = 1 + \theta + \frac{h''(x)x}{h'(x)}\Big|_{x=1}.$$

We assume that the demand schedule $h(\cdot)$ is log-concave, that is $\epsilon \geq 0$, and therefore $\alpha \in [0, 1)$, since $\theta > 1$ is the second order requirement for price setting optimality. An appropriate choice of ϵ produces any required value of α for any given value of θ. A suitable parametric example can be found in Klenow and Willis (2006) and Gopinath and Itskhoki (2010), where $h(x) = [1 - \epsilon \log(x)]^{\theta/\epsilon}$ for some elasticity parameter $\theta > 1$ and super-elasticity parameter $\epsilon > 0$.

A.2 Equilibrium system

We summarize here the equilibrium system of the general model from Section 3.1 by breaking it into blocks:

1. Labor supply (3) and its exact foreign counterpart.
2. Labor demand in (10), used together with the definition of the marginal cost (9), and its exact foreign counterpart.[40]
3. Demand for home and foreign goods:

$$Y_t = Y_{Ht} + Y_{Ht}^* \qquad \text{and} \qquad Y_t^* = Y_{Ft} + Y_{Ft}^*, \tag{A8}$$

where the sources of demand for home good are given in (18) and (19), and the counterpart sources of demand for foreign good are given by:

$$Y_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t}\right)\left[C_t + X_t + e^{g_t}\right], \tag{A9}$$

$$Y_{Ft}^* = (1-\gamma) e^{-\gamma \xi_t^*} h\left(\frac{P_{Ft}^*}{P_t^*}\right)\left[C_t^* + X_t^* + e^{g_t^*}\right], \tag{A10}$$

where X_t and X_t^* satisfy the intermediate good demand in (10) and its foreign counterpart.

[40] Note that the input demand equations (10) together with the marginal cost (9) imply the production function equation (8).

4. Supply of goods: given price setting (12)–(13) and their foreign counterparts given by:

$$P_{Ft} = e^{\mu_t^* + \eta_t^*} (MC_t^* \mathcal{E}_t)^{1-\alpha} P_t^{\alpha}, \tag{A11}$$

$$P_{Ft}^* = e^{\mu_t^*} MC_t^{*1-\alpha} P_t^{*\alpha}, \tag{A12}$$

output produced is determined by the demand equation (A8).

Given prices $(P_{Ht}, P_{Ht}^*, P_{Ft}, P_{Ft}^*)$, equation (7) defines the price level P_t as a log-linear approximation, and a similar equation defines P_t^*.[41]

5. Asset demand by home and foreign households (4) and (17), which can be rewritten as an international risk sharing condition and a no-arbitrage condition:

$$\mathbb{E}_t \left\{ e^{\psi_t} \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - \Theta_{t+1}^* \right\} = 0, \tag{A13}$$

$$\mathbb{E}_t \left\{ \Theta_{t+1} \left[e^{\psi_t} R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - R_t \right] \right\} = 0, \tag{A14}$$

with the stochastic discount factors Θ_{t+1} and Θ_{t+1}^* defined in the text.

6. Home-country flow budget constraint (20), with its foreign counterpart redundant by Walras Law.

A.2.1 Symmetric steady state

In a symmetric steady state, $B^* = B^{*F} = 0$, and the shocks (defined in Table 1) take the following values:

$$\psi = \xi = \xi^* = \eta = \eta^* = \chi = \chi^* = 0,$$

and we normalize $W = W^* = 1$ (corresponding to $w = w^* = 0$). We let the remaining shocks take arbitrary (zero or non-zero) symmetric values:

$$a = a^*, \qquad g = g^*, \qquad \kappa = \kappa^* \qquad \text{and} \qquad \mu = \mu^*.$$

We start with the equations for prices. In a symmetric steady state, exchange rates and terms of trade are equal to 1:

$$\mathcal{E} = \mathcal{Q} = \mathcal{S} = 1, \tag{A15}$$

and therefore we can evaluate the prices using the equilibrium conditions described above:

$$P = P^* = P_H = P_F^* = P_H^* = P_F = \left[\frac{e^{\frac{\mu}{1-\alpha} - a}}{(1-\phi)^{1-\phi} \phi^{\phi}} \right]^{\frac{1}{1-\phi}}, \tag{A16}$$

[41] Log-linear expression for p_t in (7) can be replaced with two non-linear expressions (A3)–(A4) defining (P_t, D_t), and P_t should be replaced with D_t in demand equations (18)–(19) and (A9)–(A10). The rest of the equilibrium system stays unchanged. However, these adjustments do not have first order consequences, as P_t and D_t are the same up to second order terms, and therefore the log-linearized system in Appendix A.2.2 is unchanged.

with the marginal costs given by $MC = MC^* = \frac{e^{-a}P^{\phi}}{(1-\phi)^{1-\phi}\phi^{\phi}} = \left[\frac{e^{\frac{\phi\mu}{1-\alpha}-a}}{(1-\phi)^{1-\phi}\phi^{\phi}}\right]^{\frac{1}{1-\phi}}$.

Next we use these expressions together with production function, labor demand and labor supply to obtain two relationships for (C, Y, L):

$$L = e^{\frac{\phi}{1-\phi}\frac{\mu}{1-\alpha}-\frac{a}{1-\phi}}\phi^{-\frac{\phi}{1-\phi}}Y, \tag{A17}$$

$$C^{\sigma}L^{1/\nu} = \frac{e^{-\kappa}}{P} = e^{\frac{a}{1-\phi}-\frac{\mu}{(1-\alpha)(1-\phi)}-\kappa}(1-\phi)\phi^{\frac{\phi}{1-\phi}}. \tag{A18}$$

Substituting prices (and using $h(1) = 1$) and intermediate good demand $X = \phi\frac{MC}{P}Y = e^{\frac{-\mu}{1-\alpha}}\phi Y$ into the goods market clearing, we obtain an additional relationship between C and Y:

$$C + e^{g} = \left[1 - e^{-\frac{\mu}{1-\alpha}}\phi\right]Y. \tag{A19}$$

We further have $Y = Y^*$, and $Y_H = Y_F^* = (1-\gamma)Y$ and $Y_H^* = Y_F = \gamma Y$.

The asset demand conditions imply that $R = R^* = 1/\beta$.

Lastly, we define the following useful ratios:

$$\zeta \equiv \frac{\text{GDP}}{\text{Output}} = \frac{P(C+G)}{P_H Y} = 1 - e^{-\frac{\mu}{1-\alpha}}\phi, \tag{A20}$$

$$\gamma \equiv \frac{\text{Import}}{\text{Expenditure}} = \frac{P_F Y_F}{P_H Y_H + P_F Y_F} = \frac{P_F Y_F}{P_H Y} = \gamma, \tag{A21}$$

$$\frac{\text{Import+Export}}{\text{GDP}} = \frac{\mathcal{E}P_H^* Y_H^* + P_F Y_F}{P(C+G)} = \frac{2\gamma}{\zeta}. \tag{A22}$$

A.2.2 Log-linearized system

We log-linearize the equilibrium system (summarized above in Appendix A.2) around the symmetric steady state (described in Appendix A.2.1). We split the equilibrium system into three blocks — prices, quantities and dynamic equations — and solve them sequentially, as the equilibrium system is block-recursive.

Exchange rates and prices The price block contains the definitions of the price index (7) and its foreign counterpart:

$$p_t^* = \gamma p_{Ht}^* + (1-\gamma)p_{Ft}^*, \tag{A23}$$

as well as the price setting equations (12)–(13) and (A11)–(A12), in which we substitute the marginal cost (9) and its foreign counterpart and log-linearize to obtain:

$$p_{Ht} = \mu_t - (1-\alpha)a_t + (1-\alpha)(1-\phi)(w_t - p_t) + p_t, \tag{A24}$$

$$p_{Ht}^* = \mu_t + \eta_t - (1-\alpha)a_t + (1-\alpha)(1-\phi)(w_t - p_t) + (1-\alpha)p_t + \alpha p_t^*, \tag{A25}$$

$$p_{Ft}^* = \mu_t^* - (1-\alpha)a_t^* + (1-\alpha)(1-\phi)(w_t^* - p_t^*) + p_t^*, \tag{A26}$$

$$p_{Ft} = \mu_t^* + \eta_t^* - (1-\alpha)a_t^* + (1-\alpha)(1-\phi)(w_t^* - p_t^*) + (1-\alpha)p_t^* + \alpha p_t. \tag{A27}$$

In addition, we use the logs of the definitions of the real exchange rate and terms of trade (14) and (21):

$$q_t = p_t^* + e_t - p_t, \tag{A28}$$

$$s_t = p_{Ft} - p_{Ht}^* - e_t. \tag{A29}$$

First, it is useful to define the log LOP deviations (as in equation (14) and in its foreign counterpart):

$$q_{Ht} \equiv p_{Ht}^* + e_t - p_{Ht} = \eta_t + \alpha q_t, \tag{A30}$$

$$q_{Ft} \equiv p_{Ft}^* + e_t - p_{Ft} = -\eta_t^* + \alpha q_t, \tag{A31}$$

where the expression on the right-hand side are obtained by using (A24)–(A27) together with (A28). Then, we combine (A28)–(A29) together with these expressions, to obtain:

$$s_t = q_t^P - 2\tilde{\eta}_t - 2\alpha q_t, \tag{A32}$$

$$q_t = (1-\gamma) q_t^P - \gamma s_t, \tag{A33}$$

where $q_t^P = p_{Ft}^* + e_t - p_{Ht}$ is the producer-price-based real exchange rate and we use the tilde notation $\tilde{x}_t \equiv (x_t - x_t^*)/2$ for any pair of variables (x_t, x_t^*). Lastly, we solve for q_t^P and s_t as function of q_t:

$$q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t - \frac{2\gamma}{1-2\gamma}\tilde{\eta}_t, \tag{A34}$$

$$s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t - \frac{2(1-\gamma)}{1-2\gamma}\tilde{\eta}_t. \tag{A35}$$

Next, we use these solutions together with the expressions for price indexes (7) and (A23), to solve for:[42]

$$p_{Ht} - p_t = -\frac{\gamma}{1-\gamma}(p_{Ft} - p_t) = \gamma(p_{Ht} - p_{Ft}) = -\frac{(1-\alpha)\gamma}{1-2\gamma} q_t - \frac{\gamma}{1-2\gamma}\eta_t^*, \tag{A36}$$

$$p_{Ft}^* - p_t^* = -\frac{\gamma}{1-\gamma}(p_{Ht}^* - p_t^*) = \gamma(p_{Ft}^* - p_{Ht}^*) = \frac{(1-\alpha)\gamma}{1-2\gamma} q_t - \frac{\gamma}{1-2\gamma}\eta_t. \tag{A37}$$

Combining these expression with (A24) and (A26), we can solve for the price levels:

$$p_t = w_t + \frac{1}{1-\phi}\left[\frac{\mu_t + \frac{\gamma}{1-2\gamma}\eta_t^*}{1-\alpha} - a_t + \frac{\gamma}{1-2\gamma} q_t\right], \tag{A38}$$

$$p_t^* = w_t^* + \frac{1}{1-\phi}\left[\frac{\mu_t^* + \frac{\gamma}{1-2\gamma}\eta_t}{1-\alpha} - a_t^* - \frac{\gamma}{1-2\gamma} q_t\right], \tag{A39}$$

[42] Note from (7) that $p_{Ht} - p_t = \gamma(p_{Ht} - p_{Ft})$, and we use the following steps to solve for:

$$p_{Ht} - p_{Ft} = -(p_{Ft} - p_{Ht}^* - e_t) - (p_{Ht}^* + e_t - p_{Ht}) = -(s_t + q_{Ht}) = -(s_t + \alpha q_t + \eta_t),$$

in which we then substitute (A35) to solve out s_t. Similarly, we solve for $p_{Ft}^* - p_t^*$.

which together allow to solve for the relationship between q_t and nominal exchange rate e_t:

$$\left[(1-\phi)+\frac{2\gamma}{1-2\gamma}\right]q_t=(1-\phi)e_t-(1-\phi)2\tilde{w}_t+2\tilde{a}_t-\frac{2\tilde{\mu}_t}{1-\alpha}+\frac{2\gamma}{1-2\gamma}\frac{\tilde{\eta}_t}{1-\alpha}. \tag{A40}$$

Real exchange rate and quantities The supply side is the combination of labor supply (3) and labor demand (10) (together with marginal cost (9)), which we log-linearize as:

$$\kappa_t+\sigma c_t+\tfrac{1}{\nu}\ell_t=w_t-p_t, \tag{A41}$$

$$\ell_t=-a_t-\phi(w_t-p_t)+y_t. \tag{A42}$$

Combining the two to solve out ℓ_t, and using (A38) to solve out (w_t-p_t), we obtain:[43]

$$\nu\sigma c_t+y_t=\frac{1+\nu}{1-\phi}a_t-\frac{\nu+\phi}{1-\phi}\left[\frac{\mu_t+\frac{\gamma}{1-2\gamma}\eta_t^*}{1-\alpha}+\frac{\gamma}{1-2\gamma}q_t\right]-\nu\kappa_t. \tag{A43}$$

Symmetrically, the same expression for foreign is:

$$\nu\sigma c_t^*+y_t^*=\frac{1+\nu}{1-\phi}a_t^*-\frac{\nu+\phi}{1-\phi}\left[\frac{\mu_t^*+\frac{\gamma}{1-2\gamma}\eta_t}{1-\alpha}-\frac{\gamma}{1-2\gamma}q_t\right]-\nu\kappa_t^*.$$

Adding and subtracting the two we obtain:

$$\nu\sigma\bar{c}_t+\bar{y}_t=\frac{1+\nu}{1-\phi}\bar{a}_t-\frac{\nu+\phi}{1-\phi}\frac{\bar{\mu}_t+\frac{\gamma}{1-2\gamma}\bar{\eta}_t}{1-\alpha}-\nu\bar{\kappa}_t, \tag{A44}$$

$$\nu\sigma\tilde{c}_t+\tilde{y}_t=\frac{1+\nu}{1-\phi}\tilde{a}_t-\frac{\nu+\phi}{1-\phi}\left[\frac{\tilde{\mu}_t-\frac{\gamma}{1-2\gamma}\tilde{\eta}_t}{1-\alpha}+\frac{\gamma}{1-2\gamma}q_t\right]-\nu\tilde{\kappa}_t, \tag{A45}$$

where $\bar{x}_t\equiv(x_t+x_t^*)/2$ and $\tilde{x}_t\equiv(x_t-x_t^*)/2$ for any pair of variables (x_t,x_t^*).

The demand side is the goods market clearing (A8) together with (18)–(19), which we log-linearize as:

$$\begin{aligned}
y_t&=(1-\gamma)y_{Ht}+\gamma y_{Ht}^*,\\
y_{Ht}&=-\gamma\xi_t-\theta(p_{Ht}-p_t)+\zeta[\varsigma c_t+(1-\varsigma)g_t]+(1-\zeta)\big[(1-\phi)(w_t-p_t)-a_t+y_t\big],\\
y_{Ht}^*&=(1-\gamma)\xi_t^*-\theta(p_{Ht}^*-p_t^*)+\zeta[\varsigma c_t^*+(1-\varsigma)g_t^*]+(1-\zeta)\big[(1-\phi)(w_t^*-p_t^*)-a_t^*+y_t^*\big],
\end{aligned}$$

where $\varsigma\equiv C/(C+G)$, $\zeta\equiv P(C+G)/(P_HY)$, and we used expression (10) and (9) to substitute for X_t (and correspondingly for X_t^*). Combining together, we derive:

$$\begin{aligned}
&y_t-(1-\zeta)[y_t-2\gamma\tilde{y}_t^*]-\zeta\varsigma[c_t-2\gamma\tilde{c}_t]=\gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma}-(1-\zeta)\right]q_t \qquad \text{(A46)}\\
&+\zeta(1-\varsigma)[g_t-2\gamma\tilde{g}_t]-\frac{1-\zeta}{1-\alpha}[\mu_t-2\gamma\tilde{\mu}_t]-\frac{1-\zeta}{1-\alpha}\frac{\gamma}{1-2\gamma}\eta_t^*-2\gamma\left[\theta-\gamma\frac{1-\zeta}{1-\alpha}\right]\tilde{\eta}_t-2\gamma(1-\gamma)\tilde{\xi}_t,
\end{aligned}$$

where we have slowed out (w_t-p_t) and $(w_t^*-p_t^*)$ using (A38)–(A39) and solved out $(p_{Ht}-p_t)$ and

[43] A useful interim step is: $\nu\sigma c_t+y_t=(\nu+\phi)(w_t-p_t)+a_t-\nu\kappa_t$.

$(p^*_{Ht} - p^*_t)$ using (A36)–(A37). Adding and subtracting the foreign counterpart, we obtain:[44]

$$[1-(1-2\gamma)(1-\zeta)]\bar{y}_t = (1-2\gamma)\zeta\varsigma\bar{c}_t + \zeta(1-\varsigma)\bar{g}_t - \frac{1-\zeta}{1-\alpha}\left(\bar{\mu}_t + \frac{\gamma}{1-2\gamma}\bar{\eta}_t\right), \tag{A47}$$

$$\begin{aligned}[1-(1-2\gamma)(1-\zeta)]\tilde{y}_t = (1-2\gamma)\zeta\varsigma\tilde{c}_t + (1-2\gamma)\left[\zeta(1-\varsigma)\tilde{g}_t - \frac{1-\zeta}{1-\alpha}\tilde{\mu}_t\right] \\ - 2\gamma\left[\theta - \gamma\frac{1-\zeta}{1-\alpha}\right]\tilde{\eta}_t - 2\gamma(1-\gamma)\tilde{\xi}_t + \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} - (1-\zeta)\right]q_t.\end{aligned} \tag{A48}$$

An immediate implication of (A44) and (A47) is that $(\bar{y}_t, \bar{c}_t)$ depends only on $(\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)$ and does not depend on the real exchange rate q_t. In particular, if $\bar{a}_t = \bar{g}_t = \bar{\kappa}_t = \bar{\mu}_t = \bar{\nu}_t = 0$, then $\bar{y}_t = \bar{c}_t = 0$. This is the case we focus on throughout the paper, since as we see below the variation in $(\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)$ does not affect q_t. Combining (A47) and (A47) we can solve for $\tilde{y}_t$ and $\tilde{c}_t$. For example, the expression for $\tilde{c}_t$ is:

$$\begin{aligned}\Big[(1-2\gamma)\zeta(\nu\sigma+\varsigma) + 2\gamma\nu\sigma\Big]\tilde{c}_t = [(1-2\gamma)\zeta + 2\gamma]\left[\frac{1+\nu}{1-\phi}\tilde{a}_t - \frac{\nu+\phi}{1-\phi}\frac{\tilde{\mu}_t - \frac{\gamma}{1-2\gamma}\tilde{\eta}_t}{1-\alpha} - \nu\tilde{\kappa}_t\right] - (1-2\gamma)\zeta(1-\varsigma)\tilde{g}_t \\ + (1-2\gamma)\frac{1-\zeta}{1-\alpha}\tilde{\mu}_t + 2\gamma\left[\theta - \gamma\frac{1-\zeta}{1-\alpha}\right]\tilde{\eta}_t + 2\gamma(1-\gamma)\tilde{\xi}_t - \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \frac{\nu+\phi}{1-\phi}\frac{1}{1-2\gamma} - \frac{1+\nu}{1-\phi}(1-\zeta)\right]q_t.\end{aligned} \tag{A49}$$

Lastly, we provide the linearized expression for net exports:

$$nx_t = \gamma\Big(y^*_{Ht} - y_{Ft} - s_t\Big),$$

where $nx_t = \frac{1}{P_H Y}NX_t$ is linear deviation of net exports from steady state $NX = 0$ relative to the total value of output. Substituting in the expressions for s_t, y^*_{Ht} and y_{Ft}, we obtain:

$$\begin{aligned}nx_t = \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \alpha - \frac{1-\alpha}{1-2\gamma} + \frac{2\gamma(1-\zeta)}{1-2\gamma}\right]q_t \\ + 2\gamma\left[(1-\theta)\tilde{\eta}_t + (1-\zeta)\frac{\tilde{\mu}_t - \frac{\gamma}{1-2\gamma}\tilde{\eta}_t}{1-\alpha} - (1-\gamma)\tilde{\xi}_t\right] - 2\gamma\Big[\zeta[\varsigma\tilde{c}_t + (1-\varsigma)\tilde{g}_t] + (1-\zeta)\tilde{y}_t\Big].\end{aligned} \tag{A50}$$

The log-linear approximation to the flow budget constraint (20) is given then by:

$$\beta b^*_{t+1} - b^*_t = nx_t, \tag{A51}$$

where $b^*_t = \frac{\mathcal{E}}{P_H Y}B^*_t$ is the linear deviation of the net foreign asset (NFA) position from its steady state value of $B^* = 0$ relative to the total value of output (both in foreign currency, using steady state exchange rate of $\mathcal{E} = 1$). Note that the dynamics of $\mathcal{E}_t$ and R^*_t has only second order effects on the returns on NFA (and hence drops out from the linearized system), as we approximate around a symmetric steady state with zero NFA position. Equations (A51) is part of the dynamic block.

[44] The foreign counterpart is obtained from combining together and rearranging:

$$\begin{aligned}y^*_t &= (1-\gamma)y^*_{Ft} + \gamma y_{Ft},\\ y^*_{Ft} &= -\gamma\xi^*_t - \theta(p^*_{Ft} - p^*_t) + \zeta[\varsigma c^*_t + (1-\varsigma)g^*_t] + (1-\zeta)\big[(1-\phi)(w^*_t - p^*_t) - a^*_t + y^*_t\big],\\ y_{Ft} &= (1-\gamma)\xi_t - \theta(p_{Ft} - p_t) + \zeta[\varsigma c_t + (1-\varsigma)g_t] + (1-\zeta)\big[(1-\phi)(w_t - p_t) - a_t + y_t\big].\end{aligned}$$

Exchange rate and interest rates It only remains now to log-linearize the asset demand conditions (4) and (17), which pins down the equilibrium interest rates, as well as provide an international risk sharing condition:

$$\begin{aligned} i_t &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1} + \Delta p_{t+1} - \Delta \chi_{t+1} \right\}, \\ i_t^* &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1} + \Delta p_{t+1} - \Delta e_{t+1} - \Delta \chi_{t+1} \right\} - \psi_t, \\ i_t^* &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1}^* + \Delta p_{t+1}^* - \Delta \chi_{t+1}^* \right\}, \end{aligned}$$

where $i_t \equiv \log R_t - \log R$ and $i_t^* \equiv \log R_t^* - \log R^*$. We combine the first two to obtain a no-arbitrage (UIP) condition, the last two to obtain a risk-sharing (Backus-Smith) condition, and the first with the third to solve for the interest rate differential:

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \psi_t, \tag{A52}$$

$$\mathbb{E}_t \left\{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \right\} = \psi_t + \mathbb{E}_t \left\{ 2 \Delta \tilde{\chi}_{t+1} \right\}, \tag{A53}$$

$$\tilde{i}_t \equiv \tfrac{1}{2}(i_t - i_t^*) = \mathbb{E}_t \left\{ \sigma \Delta \tilde{c}_{t+1} + \Delta \tilde{p}_{t+1} - \Delta \tilde{\chi}_{t+1} \right\}. \tag{A54}$$

Substituting out $\Delta c_{t+1} - \Delta c_{t+1}^* = 2 \Delta \tilde{c}_{t+1}$ in (A53) using (A49), we obtain an equation characterizing the expected real depreciation $\mathbb{E}_t \Delta q_{t+1}$ as a function of exogenous shocks. Together with (A51), in which we substitute (A50), it forms a system of two dynamic equations that describe the equilibrium dynamics of the real exchange rate given the exogenous dynamic processes for the shocks.

A.3 Autarky Limit and Proofs for Section 3.2

Proof of Propositions 1 The strategy of the proof is to evaluate the log deviations of the macro variables $z_t \equiv (w_t, p_t, c_t, \ell_t, y_t, i_t)$ from the deterministic steady state (described in Appendix A.2.1) in response to a shock $\varepsilon_t = \mathbf{V}' \Omega_t \neq 0$.[45] In particular, we explore under which circumstances $\lim_{\gamma \to 0} z_t = 0$. It is sufficient to consider the log-linearized equilibrium conditions described in Appendix A.2.2, as providing a counterexample is sufficient for the prove (hence, the focus on the small log deviations is without loss of generality). Furthermore, the proof does not rely on the international risk sharing conditions, and hence does not depend on the assumptions about the (in)completeness of the international asset markets.

To prove the propositions, consider any shock ε_t with the restriction that

$$\eta_t = \eta_t^* = \xi_t = \xi_t^* = \psi_t \equiv 0. \tag{A55}$$

We now go through the list of requirements imposed by the first part of the condition (22):

1. No wage response $\lim_{\gamma \to 0} w_t = 0$ implies $w_t = 0$, i.e. the unit of account shocks cannot lead to the exchange rate disconnect in the limit.

[45] We do not impose any restrictions on the process for shocks in Ω_t, with the exception of the mild requirement that any innovation in Ω_t has some contemporaneous effect on the value of shocks in Ω_t, i.e. we rule out pure news shocks. We discuss examples with specific time series processes for the shocks in the end of this appendix.

2. No price level response implies, using (A38) and (A55):

$$\lim_{\gamma\to 0} p_t = w_t + \frac{1}{1-\phi}\left[\frac{\mu_t}{1-\alpha} - a_t\right] = 0,$$

which in light of $w_t = 0$ requires $\mu_t = (1-\alpha)a_t$, i.e. the markup shocks must offset the productivity shocks to avoid variation in the price level.

When the same requirements are imposed for foreign, it ensures $\lim_{\gamma\to 0}\{q_t - e_t\} = 0$, as immediartely follows from the the definition of the real exchange rate $q_t = p_t^* + e_t - p_t$ (see also (A40)).

3. From the labor supply and labor demand conditions (A41)–(A42), no consumption, employment and output response require:

$$\lim_{\gamma\to 0}\left\{\sigma c_t + \tfrac{1}{\nu}\ell_t + p_t\right\} = w_t - \kappa_t = 0,$$
$$\lim_{\gamma\to 0}\left\{y_t - \ell_t + \phi p_t\right\} = a_t + \phi w_t = 0,$$

which then implies $a_t = \kappa_t = w_t \equiv 0$ and by consequence $\mu_t \equiv 0$ from the result above. That is, there cannot be productivity, markup or labor wedge shocks, if the price level, consumption, output and employment are not to respond in the autarky limit.

4. Rearranging the goods market clearing in the home market (A46), we have:

$$\lim_{\gamma\to 0}\left\{\zeta y_t - \zeta\varsigma c_t\right\} = \zeta(1-\varsigma)g_t - \frac{1-\zeta}{1-\alpha}\mu_t = 0,$$

which in light of the above results requires $g_t \equiv 0$.

5. Lastly, the home bond demand requires:

$$\lim_{\gamma\to 0}\left\{\sigma\Delta\mathbb{E}_t c_{t+1} + \Delta\mathbb{E}_t p_{t+1} - i_t\right\} = \mathbb{E}_t\Delta\chi_{t+1} = 0,$$

therefore there cannot be predictable changes in χ_t and unpredictable changes in χ_t do not affect allocations in a one-period bond economies, hence without loss of generality we impose $\chi_t \equiv 0$.

To summarize, the first condition in (22) (combined with the absence of η_t, ξ_t and ψ_t shocks) implies:

$$w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0,$$

i.e. no other shock can be consistent with $\lim_{\gamma\to 0} z_{t+j} = 0$ for all $j \geq 0$, however in the absence of shocks $\lim_{\gamma\to 0} e_{t+j} = 0$, violating the second condition in (22). A symmetric argument for foreign rules out the foreign counterparts of these shocks. This completes the proof. ■

Proof of Proposition 2 For the proof, we consider the equilibrium system in the autarky limit by only keeping the lowest order terms in γ for each shock or variable.[46] Throughout the proof we impose

[46] For example, consider equation (A40), which we now rewrite as:

$$q_t - e_t = 2\left[\frac{1}{1-\phi}\left(\tilde{a}_t - \frac{\tilde{\mu}_t}{1-\alpha}\right) - \tilde{w}_t\right] + 2\gamma\frac{1}{1-\phi}\frac{\tilde{\eta}_t}{1-\alpha}.$$

$w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0$, as well as for their foreign counterparts.

First, we consider our three moments of interest when ψ_t is the only shock, that is we set $\eta_t = \xi_t \equiv 0$. For this purpose, it is sufficient to consider the static equilibrium conditions only, as the effect of the ψ_t shock on the macro variables is exclusively indirect through q_t. Specifically:

1. Consider the near-autarky comovement between the terms of trade and the real exchange rate from (A35):
$$\lim_{\gamma\to 0} \frac{\text{cov}(\Delta s_{t+1}, \Delta q_{t+1})}{\text{var}(\Delta q_{t+1})} = (1-2\alpha) > 0 \quad \text{iff} \quad \alpha < \frac{1}{2},$$
since we have $\tilde{\eta}_t = 0$. $\alpha < 1/2$ is a necessary parameter requirement for this result, which is borne out in the data, as we discuss in Section 4.

2. Consider the near-autarky comovement between the relative consumption and the real exchange rate from (A49), which in the absence of all shocks but ψ_t simplifies to:
$$\Big[(1-2\gamma)\zeta(\nu\sigma+\varsigma) + 2\gamma\nu\sigma\Big]\tilde{c}_t = -\gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \frac{\nu+\phi}{1-\phi}\frac{1}{1-2\gamma} - \frac{1+\nu}{1-\phi}(1-\zeta)\right] q_t.$$
Hence, we have:
$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\text{cov}\left(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta q_{t+1}\right)}{\text{var}(\Delta q_{t+1})} = -\frac{2}{\zeta(\nu\sigma+\varsigma)}\left(2\theta(1-\alpha) + \zeta\frac{\nu+\phi}{1-\phi} - (1-\zeta)\right) < 0,$$
which is negative for all parameter values since
$$\zeta\frac{\nu+\phi}{1-\phi} - (1-\zeta) = \frac{\zeta\nu + \zeta - (1-\phi)}{1-\phi} > 0$$
as from (A20) $\zeta = 1 - e^{-\mu/(1-\alpha)}\phi > 1-\phi$.

3. Consider the near-autarky comovement between the nominal exchange rate and the nominal interest rate differential (the Fama coefficient) by using (A54), which we write in the limit as:
$$i_t - i^*_t = \mathbb{E}_t\{2\sigma\Delta\tilde{c}_{t+1} + 2\Delta\tilde{p}_{t+1}\} = -\frac{2\gamma\sigma}{\zeta(\nu\sigma+\varsigma)}\left[2\theta(1-\alpha) - 1 + \frac{\zeta(1-\varsigma/\sigma)}{1-\phi}\right]\mathbb{E}_t\Delta q_{t+1}.$$
where we used expression (A49) for $\tilde{c}_t$ and expressions (A38)–(A39) for p_t and p^*_t. Furthermore, (A40) and (A53) imply $\mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\Delta q_{t+1} = -\psi_t$ in the limit and with ψ_t shocks only. Therefore, the Fama regression coefficient in the limit is:[47]
$$\lim_{\gamma\to 0}\gamma\frac{\text{cov}\left(\mathbb{E}_t\Delta e_{t+1}, i_t - i^*_t\right)}{\text{var}\left(i_t - i^*_t\right)} = -\frac{\zeta(\nu\sigma+\varsigma)}{2\sigma}\frac{1}{2\theta(1-\alpha) - 1 + \frac{\zeta(1-\varsigma/\sigma)}{1-\phi}} < 0,$$
which is always negative under a mild additional requirement that $\theta > 1$ and $\sigma > 1$ (since $\varsigma \leq 1$ and $\alpha < 1/2$), with a necessary condition being substantially weaker.[48]

Note that the gap between q_t and e_t is zero-order in γ for shocks $(\tilde{a}_t, \tilde{\mu}_t, \tilde{w}_t)$ and first-order in γ for shock $\tilde{\eta}_t$.

[47] We make use of the fact that $\text{cov}\left(\Delta e_{t+1}, i_t - i^*_t\right) = \text{cov}\left(\mathbb{E}_t\Delta e_{t+1}, i_t - i^*_t\right)$ since $i_t - i^*_t$ is known at t.

[48] The Fama coefficient for the real interest rates is always negative without any further parameter restrictions, as it is proportional to the expression for the Backus-Smith correlation, since the real interest rate $r_t \equiv i_t - \mathbb{E}_t\Delta p_{t+1} = \sigma\mathbb{E}_t\Delta c_{t+1}$

This proves the first claim of the proposition that ψ_t robustly and simultaneously produces all three empirical regularities in the autarky limit.[49]

Second, recall that the uncovered interest rate parity (A52) implies that the Fama regression coefficient:

$$\beta_F \equiv \frac{\text{cov}(\Delta e_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = 1 \qquad \text{whenever} \qquad \psi_t \equiv 0.$$

Therefore, $(\eta_t, \eta_t^*, \xi_t, \xi_t^*)$ shocks that follow any joint process cannot resolve the forward premium puzzle. This is sufficient for the second claim of the proposition that the remaining shocks cannot deliver the empirical comovement for all three moments. Nonetheless, we explore the remaining two moments as well.

Third, in the remainder of the proof, we focus on the ξ_t and η_t shocks (setting all other shocks including ψ_t to zero), and impose specific time series process for these two types of shocks, which can be viewed as providing counterexamples sufficient for our argument in Proposition 2. Specifically, we focus on AR(1) processes for relative shocks:

$$\tilde{\xi}_t = \rho_\xi \tilde{\xi}_{t-1} + \sigma_\xi \varepsilon_t^\xi,$$
$$\tilde{\eta}_t = \rho_\eta \tilde{\eta}_{t-1} + \sigma_\eta \varepsilon_t^\eta,$$

with $\rho_\xi, \rho_\eta \in [0,1]$ and where $\varepsilon_t^\xi, \varepsilon_t^\eta \sim iid(0,1)$. We focus on the zero-order component of the exchange rate dynamics in γ, as this component is non-trivial for both $\tilde{\xi}_t$ and $\tilde{\eta}_t$ shocks. Therefore, we drop the first and higher order components in γ, so that we have $e_t = q_t$ from (A40) and $\mathbb{E}_t \Delta q_{t+1} = \mathbb{E}_t \Delta e_{t+1} = 0$ from (A53) together with (A49). Hence, the dynamics of the exchange rates is a random walk with jumps that satisfy the intertemporal budget constraints. The flow budget constraint (A51) (with net exports (A50), in which we substitute the solutions for $\tilde{c}_t$ and $\tilde{y}_t$ from (A45) and (A48)) up to first order terms in γ is given by:

$$\beta b_{t+1} - b_t = 2\gamma \left[\vartheta q_t - \tilde{\xi}_t - (\theta - 1)\tilde{\eta}_t \right],$$

where $\vartheta \equiv \theta(1-\alpha) - \frac{1-2\alpha}{2}$. Solving this equation forward and imposing $\lim_T \beta^T b_{T+1}^* = 0$, we obtain the solution for the equilibrium exchange rate:[50]

$$\Delta q_{t+1} = \frac{1}{\vartheta} \frac{1-\beta}{1-\beta\rho_\xi} \sigma_\xi \varepsilon_{t+1}^\xi + \frac{\theta - 1}{\vartheta} \frac{1-\beta}{1-\beta\rho_\eta} \sigma_\eta \varepsilon_{t+1}^\eta.$$

in the absence of χ_t shocks.

[49] It is also easy to verify that the dispersion of the (real and nominal) exchange rate is separated from zero in response to a ψ_t shock since from (A53) $\mathbb{E}_t \Delta q_{t+1} = -\psi_t$ and q_t needs to adjust in response to ψ_t to ensure intertemporal budget constraint with net exports following (A50). We show this formally in Appendix A.4 for ψ_t following an AR(1) process.

[50] We describe a rigorous solution method in Appendix A.4, while here we offer a heuristic argument: the net present value (using β as a discount factor) of any innovation to the right hand side of the flow budget constraint needs to be zero for intertemporal budget balance. Denote $\varepsilon_t \equiv \Delta q_t$ the (random walk) innovation of the exchange rate. The net present value of the innovation to the flow budget constraint is therefore $\sum_{j=0}^{\infty} \beta^j \left[\vartheta \varepsilon_t - \rho_\xi^j \sigma_\xi \varepsilon_t^\xi - (\theta - 1)\rho_\eta^j \sigma_\eta \varepsilon_t^\eta \right] = 0$, and solving for ε_t from this equation we obtain the expression in the proof.

We can now calculate the moments using static equilibrium conditions (A35) for s_t and (A49) for $\tilde{c}_t$:[51]

$$\lim_{\gamma\to 0}\frac{\text{cov}(\Delta s_{t+1},\Delta q_{t+1})}{\text{var}(\Delta q_{t+1})}=\begin{cases}(1-2\alpha)>0, & \text{for } \xi_t \text{ shock}\\ (1-2\alpha)-2\frac{\text{cov}(\Delta\eta_{t+1},\Delta q_{t+1})}{\text{var}(\Delta q_{t+1})} & \\ =(1-2\alpha)\left[1-\frac{\frac{2(1-\alpha)}{1-2\alpha}\theta-1}{\theta-1}\frac{1-\beta\rho_\eta}{1-\beta}\right]<0, & \text{for } \eta_t \text{ shock,}\end{cases},$$

$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\text{cov}\left(\Delta\tilde{c}_{t+1},\Delta q_{t+1}\right)}{\text{var}(\Delta q_{t+1})}=\begin{cases}\frac{2\theta(1-\alpha)-(1-2\alpha)}{\zeta(\nu\sigma+\varsigma)}\left[\frac{\beta(1-\rho_\xi)}{1-\beta}-\frac{(1-2\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)}{2\theta(1-\alpha)-(1-2\alpha)}\right]\gtrless 0, & \text{for } \xi_t,\\ \frac{1}{\zeta(\nu\sigma+\varsigma)}\frac{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}}{\frac{\theta-1}{\theta-\frac{1-2\alpha}{2(1-\alpha)}}\frac{1-\beta}{1-\beta\rho_\eta}}\left[1-\frac{\theta-1}{\theta-\frac{1-2\alpha}{2(1-\alpha)}}\frac{1-\beta}{1-\beta\rho_\eta}\frac{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)}{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}}\right]>0, & \text{for } \eta_t,\end{cases}$$

where for the first moment we maintain the assumption that $\alpha<1/2$ and to sign the second moment we use the fact that $\zeta>1-\phi$. To see that the Backus-Smith correlation under ξ_t shocks can take both signs, it is sufficient to consider the case with $\rho_\xi=1$ (when the correlation is negative) and the case with $\rho_\xi=0$ and $\beta\approx 1$ (when the correlation is positive). If $\beta\geq\rho_\xi$, under our parameterization it is sufficient to have the quarterly discount factor $\beta>0.75$ for the sign to be positive (with the calibrated value of $\beta=0.99$). This shows that the $\tilde{\eta}_t$ shock robustly generates counterfactual comovement with all three macro variables, while the $\tilde{\xi}_t$ shock does not robustly deliver empirically relevant comovement between exchange rates on one hand and interest rates and relative consumption on the other hand. ■

A.4 The Baseline Model with ψ_t Shock

Consider the log-linearized equilibrium system from Appendix A.2.2, in which we set $w_t=\mu_t=\eta_t=\xi_t=g_t=a_t=\kappa_t=\chi_t=0$ at Home, and equivalently in Foreign. The equilibrium system is block recursive, and we solve it in turn for prices, quantities and equilibrium dynamics, followed by the discussion of interest rates.

Price block We rewrite the solution from Appendix A.2.2 for this special case as follows:[52]

$$\begin{aligned}s_t&=\frac{1-2\alpha(1-\gamma)}{1-2\gamma}q_t \qquad \text{and} \qquad q_t^P=\frac{1-2\alpha\gamma}{1-2\gamma}q_t,\\ p_t&=\frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t \qquad \text{and} \qquad p_t^*=-\frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t,\end{aligned}\tag{A56}$$

and

$$q_t=\frac{1}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}e_t.\tag{A57}$$

[51] In our calculations, we use the interim results that in response to η_t shocks:

$$\text{var}(\Delta q_{t+1})=\left(\frac{\theta-1}{\theta(1-\alpha)-\frac{1-2\alpha}{2}}\frac{1-\beta}{1-\beta\rho_\eta}\right)^2\sigma_\eta^2 \qquad \text{and} \qquad \text{cov}(\Delta\eta_{t+1},\Delta q_{t+1})=\frac{\theta-1}{\theta(1-\alpha)-\frac{1-2\alpha}{2}}\frac{1-\beta}{1-\beta\rho_\eta}\sigma_\eta^2,$$

and similarly for the ξ_t shock.

[52] We also have $p_{Ht}-p_t=-\frac{\gamma}{1-\gamma}(p_{Ft}-p_t)=-\frac{(1-\alpha)\gamma}{1-2\gamma}q_t$, and symmetrically in the Foreign market.

Quantity block Again, rewriting the general solution (A45) and (A48) for this special case, we have:

$$\nu\sigma\tilde{c}_t + \tilde{y}_t = -\frac{\nu+\phi}{1-\phi}\frac{\gamma}{1-2\gamma}q_t, \tag{A58}$$

$$[1-(1-2\gamma)(1-\zeta)]\tilde{y}_t - (1-2\gamma)\zeta\tilde{c}_t = \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - (1-\zeta)\right]q_t, \tag{A59}$$

where we used the fact that now $\varsigma = \frac{C}{C+G} = 1$, and in what follows we also use $\zeta = 1-e^{-\frac{\bar{\mu}}{1-\alpha}}\phi = 1-\phi$. (Recall that $\tilde{c}_t \equiv \frac{1}{2}(c_t - c_t^*)$.)

We solve (A58)–(A59) for consumption and output explicitly:

$$c_t - c_t^* = -\gamma\kappa_q^c q_t, \qquad \kappa_q^c \equiv \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu + \frac{\nu+\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)}\frac{1}{1-\phi}\frac{2}{1-2\gamma} > 0, \tag{A60}$$

$$y_t - y_t^* = \gamma\kappa_q^y q_t, \qquad \kappa_q^y \equiv \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi}\frac{2}{1-2\gamma} = \frac{\sigma\nu\left(2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\right) - (\nu+\phi)}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)}\frac{1}{1-\phi}\frac{2}{1-2\gamma}.$$

Note that in the text in (39) we simply use κ_q for κ_q^c.

Using (A50), we can rewrite net exports in this case as:[53]

$$\begin{aligned} nx_t &= \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \alpha - \frac{1-\alpha}{1-2\gamma} + \frac{2\gamma}{1-2\gamma}\phi\right]q_t - 2\gamma((1-\phi)\tilde{c}_t + \phi\tilde{y}_t) \\ &= \gamma\kappa_q^{nx}q_t, \qquad \kappa_q^{nx} \equiv \left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + 2(1-\gamma)\alpha - 1 - \gamma\kappa_q^y\right]\frac{1}{1-2\gamma}. \end{aligned} \tag{A61}$$

Note that $\kappa_q^y < \kappa_q^c$ and may be negative (consistence with the lack of a robust correlation between output and RER in the data). Furthermore, $\kappa_q^{nx} > 0$ iff (after using κ_q^y and simplifying):

$$2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}\frac{1+\sigma\nu}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)} > 1 - 2(1-\gamma)\alpha - \frac{\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\left[\nu + (1+\sigma\nu)\phi\right]}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)}. \tag{A62}$$

Finally, note that we scaled the coefficient κ_q^c, κ_q^y and κ_q^{nx} so that they are zero-order in γ, that is the limits of these coefficients as $\gamma \to 0$ are separated from both 0 and ∞.

Dynamic block We combine together the international risk-sharing condition (A53) and the flow budget constrain (A51), and use the solution (A60)–(A61) above, to obtain:

$$\psi_t = \mathbb{E}_t\left\{\sigma\Delta(c_{t+1} - c_{t+1}^*) - \Delta q_{t+1}\right\} = -\left(1+\gamma\sigma\kappa_q^c\right)\mathbb{E}_t\Delta q_{t+1}, \tag{A63}$$

$$\beta b_{t+1}^* - b_t^* = nx_t = \gamma\kappa_q^{nx}q_t. \tag{A64}$$

Given the proportional relationship between real and nominal exchange rates (A57), we can equivalently rewrite the dynamic system above in terms of the nominal exchange rate e_t, corresponding

[53] Note that $\phi\tilde{y}_t + (1-\phi)\tilde{c}_t = \frac{1}{1-2\gamma}\tilde{y}_t - \left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\right]\frac{\gamma}{1-2\gamma}q_t$.

to (25)–(26) in the text, with:

$$\lambda_1 = \frac{\sigma\kappa_q^c - \frac{1}{1-\phi}\frac{2}{1-2\gamma}}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} = \frac{\frac{1}{1-\phi}\frac{2}{1-2\gamma}}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}\frac{\sigma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\frac{\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}\right]-1}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)}, \tag{A65}$$

$$\lambda_2 \equiv \frac{\kappa_q^{nx}}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}. \tag{A66}$$

Note that (A62) ensures $\kappa_q^{nx} > 0$, which also implies $\lambda_2 > 0$. In turn, $\lambda_1 > 0$ iff:

$$\sigma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\frac{\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}\right] > 1. \tag{A67}$$

We discuss sufficient conditions for (A62) and (A67) in Appendix A.4.1. Note, however, that the signs of λ_1 and λ_2 are inconsequential for the proofs of the main results.

Equations (A63)–(A64) define a dynamic system in (b_t, q_t), where we assume the exogenous shock ψ_t follows an AR(1) process:

$$\psi_t = \rho\psi_{t-1} + \varepsilon_t. \tag{A68}$$

This system can be solved for the equilibrium dynamics of q_t and b_t using the Blanchard and Kahn (1980) method.[54] We prove:

Lemma A1 *The unique non-explosive solution to the dynamic system* (A63)–(A68) *is given by:*

$$\Delta q_{t+1} = \rho\Delta q_t + \frac{1}{1+\gamma\sigma\kappa_q^c}\frac{\beta}{1-\beta\rho}\left(\varepsilon_{t+1} - \frac{1}{\beta}\varepsilon_t\right), \tag{A69}$$

$$\Delta b_{t+1}^* = \rho\Delta b_t^* + \frac{\gamma\kappa_q^{nx}}{1+\gamma\sigma\kappa_q^c}\frac{1}{1-\beta\rho}\varepsilon_t. \tag{A70}$$

Proof: We rewrite the dynamic system (A63)–(A64) in matrix form as:

$$\mathbb{E}_t\begin{pmatrix} q_{t+1} \\ \hat{b}_{t+1}\end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 1/\beta & 1/\beta\end{pmatrix}}_{\equiv A}\begin{pmatrix} q_t \\ \hat{b}_t\end{pmatrix} - \begin{pmatrix} \hat{\psi}_t \\ 0\end{pmatrix},$$

where we use the rescaled variables $\hat{b}_t = \frac{b_t}{\gamma\kappa_q^{nx}}$ and $\hat{\psi}_t = \frac{\psi_t}{1+\gamma\sigma\kappa_q^c}$ for convenience. Matrix A has two eigenvalues: 1 and $1/\beta > 1$. The left eigenvector of matrix A associated with eigenvalue $1/\beta$ is $v = (1,\ 1-\beta)$. Premultiplying the system by v from the left, we have:

$$\mathbb{E}_t z_{t+1} = \frac{1}{\beta}z_t - \hat{\psi}_t, \qquad \text{where} \qquad z_t \equiv v\begin{pmatrix} q_t \\ \hat{b}_t\end{pmatrix} = q_t + (1-\beta)\hat{b}_t.$$

[54] Alternatively, this system can be solved by the method of undetermined coefficients. Rewriting (A63) as $\mathbb{E}_t q_{t+1} = q_t - \hat{\psi}_t$, it must be that $q_t = \frac{1}{1-\rho}\hat{\psi}_t + m_t$, where m_t is a martingale with the innovation given by the only fundamental shock ε_t. That is, $m_{t+1} = m_t + \vartheta\varepsilon_{t+1}$, where ϑ is the undetermined coefficient. We then use (A64) to find the unique value of ϑ that results in a non-explosive path for b_t^*.

The unique non-explosive (forward) solution of this dynamic equation is:[55]

$$z_t = \beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{\psi}_{t+j} = \frac{\beta}{1-\beta\rho} \hat{\psi}_t.$$

This implies a cointegration relationship for the endogenous variables, $q_t + (1-\beta)\hat{b}_t = \frac{\beta}{1-\beta\rho}\hat{\psi}_t$, which can be used to solve out q_t in (A64), yielding:

$$\Delta \hat{b}_{t+1} = \frac{1}{1-\beta\rho} \hat{\psi}_t,$$

which implies (A70) given the definitions of $\hat{b}_t$ and $\hat{\psi}_t$, and using the fact that $(1-\rho L)\psi_t = \varepsilon_t$, where the lag operator $Lx_t = x_{t-1}$ for an arbitrary variable x_t.

Next we take the first difference of (A64):

$$\Delta q_t = \beta \Delta \hat{b}_{t+1} - \Delta \hat{b}_t = \frac{\beta}{1-\beta\rho}\hat{\psi}_t - \frac{1}{1-\beta\rho}\hat{\psi}_{t-1},$$

where the second equality substitutes in the solution for $\Delta \hat{b}_{t+1}$. Applying the $(1-\rho L)$ operator on both sides, we obtain equation (A69), since $(1-\rho L)\hat{\psi}_t = \hat{\psi}_t - \rho\hat{\psi}_{t-1} = \frac{1}{1+\gamma\sigma\kappa_q^c}\varepsilon_t$. ■

Lemma A1 implies that the unique (NPG-admissible) solution of the dynamic system results in an ARIMA(1,1,1) process for the exchange rate and an ARIMA(1,1,0) process of the NFA position of the country. Equivalently, the change in the exchange rate Δq_t follows an ARMA(1,1) with the AR root ρ and the MA root $1/\beta$, while the change in the NFA Δb^*_{t+1} follows an AR(1) process with root ρ. Also note that the smaller is γ, the larger is the response of q_t and the smaller is the response of b^*_{t+1} to the innovation ε_t of the financial shock ψ_t. In light of (A57) and (A65), Lemma A1 implies Proposition 3. ■

Proofs for Proposition 4 and 5 Next we discuss the properties of the equilibrium exchange rate dynamics, which in light of (A57) apply equally to both the nominal and the real exchange rate. Given the solution (A69), we can now characterize the statistical properties of the exchange rate process:

1. Unconditional variance of Δq_t can be calculated from (A69) as follows:

$$\sigma^2_{\Delta q} = \rho^2 \sigma^2_{\Delta q} + \frac{1}{(1+\gamma\sigma\kappa_q^c)^2}\frac{1+\beta^2}{(1-\beta\rho)^2}\sigma^2_\varepsilon - \frac{2\beta\rho}{(1+\gamma\sigma\kappa_q^c)^2}\frac{1}{(1-\beta\rho)^2}\sigma^2_\varepsilon,$$

where $\sigma^2_{\Delta q} = \mathrm{var}(\Delta q_{t+1})$. Solving for $\sigma^2_{\Delta q}$, and for $\sigma^2_{\Delta e}$ using (A57) and (A65), we have:

$$\sigma^2_{\Delta q} = \frac{1}{(1+\gamma\sigma\kappa_q^c)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2}\frac{\sigma^2_\varepsilon}{1-\rho^2} \qquad \text{and} \qquad \sigma^2_{\Delta e} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2}\frac{\sigma^2_\varepsilon}{1-\rho^2},$$

since we have $1+\gamma\lambda_1 = \frac{1+\gamma\sigma\kappa_c^q}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}$.

[55] The remaining explosive solutions feature $\lim_{j\to\infty}\beta^j\mathbb{E}_t z_{t+j+1} = \infty$, violating the No Ponzi Game Condition for b^*_t.

Noting that $\text{var}(\psi_{t+1}) = \sigma_\varepsilon^2/(1-\rho^2)$, we further have:

$$\frac{\text{var}(\Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2} \xrightarrow[\beta\to 1]{} \frac{1}{(1+\gamma\lambda_1)^2}\frac{2}{1-\rho},$$

which increases without bound as $\rho \to 1$.

2. Auto-covariance of Δq_t can be similarly calculated using (A69) as:

$$\text{cov}(\Delta q_{t+1}, \Delta q_t) = \rho\sigma^2_{\Delta q} - \frac{1}{(1+\gamma\sigma\kappa_q^c)^2}\frac{\beta}{(1-\beta\rho)^2}\sigma_\varepsilon^2 = -\frac{(\beta-\rho)(1-\beta\rho)}{1-2\beta\rho+\beta^2}\sigma^2_{\Delta q}.$$

The autocorrelation of the exchange rate changes (both nominal and real, in light of (A57)) is:

$$\rho_{\Delta e} = \rho_{\Delta q} = \frac{\text{cov}(\Delta q_{t+1}, \Delta q_t)}{\sigma^2_{\Delta q}} = -\frac{(\beta-\rho)(1-\beta\rho)}{1-2\beta\rho+\beta^2} \xrightarrow[\beta\to 1]{} -\frac{1-\rho}{2},$$

confirming claim 1 in Propositions 4.

3. The variance of innovation of Δq_{t+1} is:

$$\text{var}_t(\Delta q_{t+1}) = \text{var}(\Delta q_{t+1} - \mathbb{E}_t \Delta q_{t+1}) = \left(\frac{\beta}{1-\beta\rho}\right)^2 \frac{\sigma_\varepsilon^2}{(1+\gamma\sigma\kappa_q^c)^2},$$

where we assume that the information set at time t includes $\{q_t, q_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots\}$. Since Δe_t equals Δq_t scaled by a constant, we further have:

$$\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{\text{var}(\Delta q_{t+1} - \mathbb{E}_t \Delta q_{t+1})}{\text{var}(\Delta q_{t+1})} = \frac{\beta^2(1-\rho^2)}{1-2\beta\rho+\beta^2} \xrightarrow[\beta\to 1]{} \frac{1+\rho}{2},$$

confirming claim 2 in Propositions 4.

Combining with the result in point 1 above, we have:

$$\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{\beta^2(1-\rho^2)}{(1-\beta\rho)^2} \xrightarrow[\beta\to 1]{} \frac{1}{(1+\gamma\lambda_1)^2}\frac{1+\rho}{1-\rho},$$

which tends to infinity with $\rho \to 1$, confirming claim 3 in Propositions 4. ■

4. We now calculate the finite-sample autocorrelation of the real exchange rate in levels, that is the coefficient from a regression of q_t on q_{t-1} (with a constant) in a sample with $T+1$ observations. Even though the second moments are not well-defined in population, this finite sample correlation is well-defined. We have:

$$\hat{\rho}_q(T) = \frac{\frac{1}{T}\sum_{t=1}^T (q_t - \bar{q})(q_{t-1} - \bar{q})}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2} = 1 + \frac{\frac{1}{T}\sum_{t=1}^T \Delta q_t q_{t-1}}{\frac{1}{T}\sum_{t=1}^T (q_{t-1} - \bar{q})^2}.$$

Note that the denominator is positive and finite for any finite T, but diverges as $T \to \infty$, since q_t is an integrated process. The numerator, however, has a finite limit (assuming $\beta, \rho < 1$, which

ensures stationarity of Δq_t, and conditioning on the given initial value of the process q_0):

$$\underset{T\to\infty}{p\lim}\frac{1}{T}\sum_{t=1}^{T}\Delta q_t q_{t-1} = \text{cov}(\Delta q_t, q_{t-1}) = \sum_{j=1}^{\infty}\text{cov}(\Delta q_t, \Delta q_{t-j})$$
$$= \frac{\text{cov}(\Delta q_t, \Delta q_{t-1})}{1-\rho} = -\frac{\beta-\rho}{(1-\rho)(1-\beta\rho)}\frac{\sigma_\psi^2}{(1+\gamma\sigma\kappa_q^c)^2},$$

where we used the fact that for process (A69) $\text{cov}(\Delta q_t, \Delta q_{t-j}) = \rho^{j-1}\text{cov}(\Delta q_{t-j+1}, \Delta q_{t-j})$ for $j \geq 1$ and the expression for $\text{cov}(\Delta q_t, \Delta q_{t-1})$ obtained above.

To summarize, this analysis implies that the finite sample autocorrelation of q_t: (a) tends to 1 asymptotically as samples size increases; and (b) is smaller than 1 in large but finite samples, provided that $\rho < \beta$. This confirms the claims in Proposition 5. ■

Interest rates and Carry trades Finally, we turn to the properties of the interest rates, which are linked to consumption and prices by (A54). Substituting in the solution for consumption (A60) and prices (A56), we arrive at:

$$i_t - i_t^* = -\left(\sigma\gamma\kappa_q^c - \tfrac{1}{1-\phi}\tfrac{2\gamma}{1-2\gamma}\right)\mathbb{E}_t\Delta q_{t+1} = -\gamma\lambda_1\mathbb{E}_t\Delta e_{t+1},$$

corresponding to (42) and with λ_1 defined in (A65). In this analysis we assume that the parameter restriction (A67) is satisfied, and $\lambda_1 > 0$.[56] Combining the expression for interest rate differential with the UIP condition (A52), we obtain expression (43) in the text.

Lastly, we define a Carry trade. Consider a trade strategy that invests $x_t \equiv i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}$ in the home bond and sells short x_t units of foreign bond, including the case when $x_t < 0$ (i.e., shorting the home bond and investing in foreign bond in this case). We refer to this strategy as a *Carry trade*. Note that this trade requires zero capital at t and the intensity (exposure) of the trade is proportional to its expected return, which from (A52) equals $x_t = \psi_t$. The return on this trade and the corresponding (unconditional) Sharpe ratio are given by:

$$r_{t+1}^C = x_t(i_t - i_t^* - \Delta e_{t+1}) \qquad \text{and} \qquad SR^C = \frac{\mathbb{E}r_{t+1}^C}{\text{std}(r_{t+1}^C)}. \tag{A71}$$

Proof of Proposition 8 First, consider the Fama regression of Δe_{t+1} on $i_t - i_t^*$. From (27), it follows:

$$\Delta e_{t+1} = \mathbb{E}_t\Delta e_{t+1} + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\varepsilon_{t+1},$$

[56] There exists a parallel relationship in real terms (where $r_t \equiv i_t - \mathbb{E}_t\Delta p_{t+1}$ denotes the real interest rate):

$$r_t - r_t^* = \sigma\mathbb{E}_t\{\Delta c_{t+1} - \Delta c_{t+1}^*\} = -\gamma\sigma\kappa_q^c\,\mathbb{E}_t\Delta q_{t+1},$$

with the negative sign independently of the parameter values.

since ε_{t+1} is the only innovation relative to the information set of time t. Furthermore, from our derivations above we have $i_t - i_t^* = -\gamma\lambda_1 \mathbb{E}_t \Delta e_{t+1}$, and therefore, we can write the regression as:

$$\Delta e_{t+1} = -\frac{1}{\gamma\lambda_1}(i_t - i_t^*) + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\varepsilon_{t+1}.$$

Since ε_{t+1} is a regression residual (i.e., $\mathbb{E}\{\varepsilon_{t+1}|i_t - i_t^*\} = 0$), the Fama regression coefficient is given by $\beta_F \equiv -\frac{1}{\gamma\lambda_1} < 0$. The R^2 in this regression is given by the share of the predictable variation in Δe_{t+1}, since $i_t - i_t^*$ absorbes the entire predictable component $\mathbb{E}_t \Delta e_{t+1}$ (see Proposition 4 and its proof for derivation of the variance share of the unpredictable component):

$$R^2 = \frac{\text{var}(\mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\beta^2(1-\rho^2)}{1-2\beta\rho+\beta^2} = \frac{(1-\beta\rho)^2}{1-2\beta\rho+\beta^2}.$$

Note that $\lim_{\beta\to 1} R^2 = \frac{1-\rho}{2}$, which tends to zero with $\rho \to 1$. This proves claim (i).

Using the same arguments, we prove claim (ii):

$$\frac{\text{var}(\Delta e_{t+1})}{\text{var}(i_t - i_t^*)} = \frac{1}{(\gamma\lambda_1)^2}\frac{\text{var}(\Delta e_{t+1})}{\text{var}(\mathbb{E}_t \Delta e_{t+1})} = \frac{1}{(\gamma\lambda_1)^2}\frac{1}{R^2}.$$

Since $\gamma\lambda_1$ is separated from zero when $\gamma > 0$ and does not depend on β and ρ, the asymptotics of this relative variances is the same as that of $1/R^2$, which goes to infinity as $\beta, \rho \to 1$.

Claim (iii) follows form the fact that $\rho_{\Delta e} = \text{corr}(\Delta e_{t+1}, \Delta e_t) \to 0$ as $\beta, \rho \to 1$ (see Proposition 4), while the persistence of $i_t - i_t^* = \frac{\gamma\lambda_1}{1+\gamma\lambda_1}\psi_t$ equals $\rho \to 1$.

Lastly, we make use of the definition of the Carry trade return r_{t+1}^C and its Sharpe ratio SR^C in (A71), to prove claim (iv). In particular, we calculate the expected return and the variance of the returns (using the fact that from (A52) we have $i_t - i_t^* - \Delta e_{t+1} = \psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})$):

$$\begin{aligned}
\mathbb{E} r_{t+1}^C &= \mathbb{E}\big\{\psi_t \mathbb{E}_t\{i_t - i_t^* - \Delta e_{t+1}\}\big\} = \mathbb{E}\psi_t^2 = \text{var}(\psi_t) = \sigma_\psi^2, \\
\text{var}(r_{t+1}^C) &= \mathbb{E}(r_{t+1}^C)^2 - (\mathbb{E} r_{t+1}^C)^2 = \mathbb{E}\big\{\psi_t^2[\psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})]^2\big\} - \sigma_\psi^4 \\
&= \mathbb{E}\psi_t^4 + \mathbb{E}\left\{\psi_t^2 \text{var}_t(\Delta e_{t+1})\right\} - \sigma_\psi^4 = 2\sigma_\psi^4 + \text{var}_t(\Delta e_{t+1})\sigma_\psi^2,
\end{aligned}$$

where the last line uses the fact that $\text{var}_t(\Delta e_{t+1}) = \mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1}\}^2$ depends only on the parameters and does not depend on ψ_t (i.e., the unexpected component of Δe_{t+1} is homoskedastic; see the proof of Proposition 4), the fact that $\mathbb{E}\{\psi_t^3(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\} = \mathbb{E}\{\psi_t^3 \mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1}\}\} = 0$, and lastly that $\mathbb{E}\psi_t^4 = 3(\mathbb{E}\psi_t^2)^2 = 3\sigma_\psi^4$ under the additional assumption that ε_t is normally distribution (in which case ψ_t is also normal). With this, we calculate:

$$SR^C = \frac{\sigma_\psi^2}{\sqrt{2\sigma_\psi^4 + \text{var}_t(\Delta e_{t+1})\sigma_\psi^2}} = \left(2 + \frac{1}{(1+\gamma\lambda_1)^2}\frac{\beta^2(1-\rho^2)}{(1-\beta\rho)^2}\right)^{-1/2},$$

where we use the expression for $\text{var}_t(\Delta e_{t+1})/\sigma_\psi^2$ from the proof of Proposition 4. Note that $SR^C \to 0$ as $\beta, \rho \to 1$, as $\text{var}_t(\Delta e_{t+1})/\sigma_\psi^2 \to \infty$. ■

A.4.1 Parameter restrictions

The primitive parameters (defined in Table 1) can take the following values:

- $\beta \in (0,1)$ and $\rho \in [0,1]$
- $\sigma, \theta > 0$ and $\nu \geq 0$
- $\gamma \in [0, 1/2]$ with $\gamma = 0$ corresponding to autarky and $\gamma = 1/2$ corresponding to no home bias
- $\alpha \in [0,1)$ with $\alpha = 0$ corresponding to no strategic complementarities and complete pass-through
- $\phi \in [0,1)$ with $\phi = 0$ corresponding to no intermediate inputs.

In addition, we impose the following sufficient parameter restrictions needed for certain results:

Assumption A1: $\alpha < 1/2$.

A1 ensures positive correlation between RER and ToT (Proposition 6). A1 is consistent with the range of empirical estimates for the elasticity of strategic complementarities.

Assumption A2: $\theta > 1/\sigma$.

Together with A1, A2 implies $2\sigma\theta(1-\alpha) > 1$, which is a sufficient condition for (A67), ensuring that $\lambda_1 > 0$, i.e. that the nominal interest rate falls with expected deprecation (see (42); needed for Proposition 8, and useful but not necessary in discussion of Proposition 3).[57] Note that here $1/\sigma$ plays the role of IES (elasticity of intertemporal substitution), rather than the income effect in the labor supply (and therefore does not exclude GHH preferences). Empirically, $\theta > 1$ and $1/\sigma \in (1/2, 1)$, so this sufficient condition is met with ease, and the necessary condition (A67) is even further lax.

Assumption A3: $\theta > \frac{1}{2} + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}$.

A3 is a sufficient condition for (A62), a variant of the Marshall-Lerner condition in our general equilibrium model, which ensuring that $\lambda_2 > 0$ and $\kappa_q^{nx} > 0$, i.e. that net export improves in response to a devaluation (see (26); a useful but not necessary condition for discussion of Proposition 3). An alternative necessary condition for (A62) can be written as $\theta > \frac{1}{2}\frac{1}{1-\gamma}\left[1 + 2\gamma\frac{\phi}{1-\phi}\right]$. A necessary condition (A62) is noticeably weaker, and in particular is relaxed when $\alpha > 0$. In the limit of autarky ($\gamma \approx 0$), $\theta > 1/2$ is both necessary and sufficient, corresponding to the classical Marshall-Lerner condition. Since empirically $\theta > 1$ and $\phi \approx 1/2$, A3 would be easily satisfied even for countries that are a number of times more open than the United States.

[57] Condition A2 effectively ensures that nominal and real interest rates move in the same direction, i.e. the expected inflation response does not more than offset the movement in the real interest rate.

A.4.2 Quantitative properties and robustness

We now explore the robustness of our quantitative findings in the baseline model with ψ_t shocks only to the departures from the limiting cases ($\gamma \to 0$ or $\beta\rho \to 1$) and also from our baseline parameterization discussed in Section 4. The results are reported in Table A1, along with the benchmark empirical moments. The robustness analysis suggests that the model requires a high ρ in order to capture the dynamic properties of the exchange rates, as a lower ρ results in less persistent exchange rates and in more predictable exchange rate changes. The quantitative success of the model also relies on home bias (a low γ), as when γ is doubled (corresponding to a 60% trade to GDP ratio), the model predicts a considerably more volatile response of the real variables to the real exchange rate, in contrast with the data. Having moderately low θ and high α is also important for the fit of the model, while it is more robust with respect to variation in other parameters, including risk aversion σ and Frisch elasticity ν (not reported in the table).

Table A1: Baseline model: quantitative properties

		Data	Baseline	Robustness				
				$\theta = 2.5$	$\alpha = 0$	$\gamma = .15$	$\rho = 0.9$	$\sigma = 1$
1.	$\rho(\Delta e)$	0.00	-0.02 (0.09)				-0.05	
2.	$\rho(q)$	0.94	0.93 (0.04)				0.87	
	$HL(q)$	12.0	9.9 (6.4)				4.9	
	$\sigma(\Delta q)/\sigma(\Delta e)$	0.98	0.75			0.54		
3.	$\sigma(\Delta s)/\sigma(\Delta q)$	0.34	0.30		1.16	0.46		
	$\sigma(\Delta q^P)/\sigma(\Delta q)$	0.95	1.10		1.16	1.26		
4.	$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	0.25	0.31	0.42	0.42	0.81		0.48
	$\sigma(\Delta y - \Delta y^*)/\sigma(\Delta q)$	—	0.12	0.36	0.36	0.34		0.01
	$\sigma(\Delta gdp - \Delta gdp^*)/\sigma(\Delta q)$	0.25	0.19	0.44		0.49		0.07
	$\sigma(\Delta nx)/\sigma(\Delta q)$	0.12	0.25			0.65		
5.	Fama β_F	$\lesssim 0$	-8.1 (4.7)					
	Fama R^2	0.02	0.04 (0.02)				0.07	
	$\sigma(i - i^*)/\sigma(\Delta e)$	0.10	0.03 (0.01)					
	Carry SR	0.20	0.21 (0.04)				0.29	

Notes: Baseline parameters: $\gamma = 0.07$, $\alpha = 0.4$, $\theta = 1.5$, $\rho = 0.97$, $\sigma = 2$, $\nu = 1$, $\phi = 0.5$, $\mu = 0$, $\beta = 0.99$, and the results are robust to changing ν, ϕ, μ and β (not reported for brevity). The robustness panel of the table shows only the moments that are sensitive to the change in the parameter values.

Notation: $\sigma(\cdot)$ stands for standard deviation, $\rho(\cdot)$ for autocorrelation, and $HL(\cdot)$ for half-life.

The dynamic moments are calculated as the median off the in-sample estimates over 10,000 simulations with 30 years (120 quarters) with the standard deviation of estimates across simulations reported in brackets. The asymptotic values of the estimates are similar to the medians except for $\rho(q) \to 1$, $HL(q) \to \infty$ and $\beta_F \to -4.6$.

In response to ψ_t shock, c_t and c_t^* are perfectly negatively correlated so that $\Delta c - \Delta c^* = 2\Delta c$. Therefore, while $\Delta c - \Delta c^*$ is about three times less volatile than Δq, Δc alone is six times less volatile than Δq.

The in-sample Fama regression coefficient is -8 with huge variation covering zero within two standard deviations. When the ψ_t is combined with other shocks, the median coefficient becomes closer to zero, as in the data (see Section 5.1).

A.5 Complete Markets and the Relationship to Engel and West (2005)

A.6 Alternative Models of the Backus-Smith puzzle

General model has three equations:

$$\begin{aligned} 2\sigma\tilde{c}_t &= q_t + \zeta_t, \\ \tilde{c}_t &= \kappa_a \tilde{a}_t - \frac{1}{2}\gamma\kappa_q q_t, \\ \beta b_{t+1} - b_t = nx_t &= \lambda_q \theta q_t - \lambda_a \tilde{a}_t, \end{aligned}$$

where $\lambda_q \gtrless 0$ and (a_t, ζ_t) follow some joint stochastic process.

A.6.1 Integrated Productivity Shocks

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t,$$

$$\begin{aligned} \kappa_a &\equiv \frac{1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}{1 + \sigma\nu\left(1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)} \frac{1+\nu}{1-\phi}, \\ \kappa_q &\equiv \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu + \frac{\nu+\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}}{1 + \sigma\nu\left(1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)} \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma} \end{aligned}$$

Exchange rates and prices

$$\begin{aligned} s_t &= (1-2\alpha)q_t, \\ p_{Ht} - p_t &= \gamma(p_{Ht} - p_{Ft}) = -\gamma(1-\alpha)q_t, \\ p_{Ft}^* - p_t^* &= \gamma(p_{Ft}^* - p_{Ht}^*) = \gamma(1-\alpha)q_t, \\ p_t - w_t &= -\frac{1}{1-\phi}a_t + \gamma\frac{1}{1-\phi}q_t, \\ p_t^* - w_t^* &= -\frac{1}{1-\phi}a_t^* - \gamma\frac{1}{1-\phi}q_t, \\ q_t - e_t &= 2\frac{1}{1-\phi}\tilde{a}_t. \end{aligned}$$

Exchange rate and quantities

$$\begin{aligned} \tilde{y}_t + \nu\sigma\tilde{c}_t &= \frac{1+\nu}{1-\phi}\tilde{a}_t - \gamma\frac{\nu+\phi}{1-\phi}q_t, \\ \tilde{y}_t - \varsigma\tilde{c}_t &= \gamma\frac{1}{\zeta}\left(2\theta(1-\alpha) - (1-\zeta)\right)q_t. \end{aligned}$$

implies

$$
\begin{aligned}
(\nu\sigma+\varsigma)\tilde{c}_t &= \frac{1+\nu}{1-\phi}\tilde{a}_t - \gamma\frac{1}{\zeta}\left(2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)\right)q_t,\\
nx_t &= -2\gamma\left[\zeta\varsigma\tilde{c}_t+(1-\zeta)\tilde{y}_t\right]+2\gamma\left(\theta(1-\alpha)-\frac{1-2\alpha}{2}\right)q_t,\\
&= -2\gamma\frac{\varsigma}{\nu\sigma+\varsigma}\frac{1+\nu}{1-\phi}\tilde{a}_t+2\gamma\left(\theta(1-\alpha)-\frac{1-2\alpha}{2}\right)q_t.
\end{aligned}
$$

Exchange rate and interest rates

$$
\begin{aligned}
\mathbb{E}_t\Delta q_{t+1} &= \mathbb{E}_t 2\sigma\Delta\tilde{c}_{t+1},\\
i_t - i_t^* &= \mathbb{E}_t\Delta e_{t+1}.
\end{aligned}
$$

First equation implies in the limit:

$$
\begin{aligned}
\mathbb{E}_t\Delta q_{t+1} &= \sigma\kappa\mathbb{E}_t\Delta\tilde{a}_{t+1},\\
\beta\hat{b}_{t+1}-\hat{b}_t &= \vartheta q_t-\varsigma\kappa\tilde{a}_t,\\
\Delta\tilde{a}_{t+1} &= \rho\Delta\tilde{a}_t+\varepsilon_{t+1},
\end{aligned}
$$

where $\bar{b}_t \equiv b_t^*/\gamma$ and:

$$
\vartheta \equiv (2\theta(1-\alpha)-(1-2\alpha)) \qquad \text{and} \qquad \kappa \equiv \frac{2}{\nu\sigma+\varsigma}\frac{1+\nu}{1-\phi}.
$$

Solving with $x_t \equiv (q_t \;\; \hat{b}_t)'$:

$$
\mathbb{E}_t x_{t+1} = \begin{pmatrix} 1 & 0 \\ \vartheta/\beta & 1/\beta \end{pmatrix} x_t + \kappa \begin{pmatrix} \sigma\rho & 0 \\ 0 & -\varsigma/\beta \end{pmatrix} \begin{pmatrix} \Delta\tilde{a}_t \\ \tilde{a}_t \end{pmatrix},
$$

and the left eigenvector associated with eigenvalue $1/\beta$ is $v = \begin{pmatrix}\vartheta & (1-\beta)\end{pmatrix}$. Then $z_t \equiv vx_t$ satisfies:

$$
\begin{aligned}
\vartheta q_t + (1-\beta)\hat{b}_t = z_t &= \beta\mathbb{E}_t z_{t+1} + \kappa\left[\varsigma(1-\beta)\tilde{a}_t - \vartheta\sigma\beta\rho\Delta\tilde{a}_t\right]\\
&= \kappa\sum_{j=0}^{\infty}\beta^j\left[\varsigma(1-\beta)\mathbb{E}_t\tilde{a}_{t+j} - \vartheta\sigma\beta\rho\mathbb{E}_t\Delta\tilde{a}_{t+j}\right]\\
&= \kappa\left[\varsigma\tilde{a}_t - \frac{\beta\rho(\vartheta\sigma-\varsigma)}{1-\beta\rho}\Delta\tilde{a}_t\right],
\end{aligned}
$$

where we used the fact that:

$$
\mathbb{E}_t\tilde{a}_{t+j} = \tilde{a}_t + \sum_{k=1}^{j}\mathbb{E}_t\Delta\tilde{a}_{t+k} = \tilde{a}_t + \rho\sum_{k=0}^{j-1}\rho^k\Delta\tilde{a}_t = \tilde{a}_t + \rho\frac{1-\rho^j}{1-\rho}\Delta\tilde{a}_t.
$$

Therefore, we have:[58]

$$\beta\Delta\hat{b}_{t+1} = -\kappa(\vartheta\sigma - \varsigma)\frac{\beta\rho}{1-\beta\rho}\Delta\tilde{a}_t,$$
$$\Delta q_{t+1} = -\frac{\vartheta\sigma\beta\rho - \varsigma}{1-\beta\rho}\frac{\kappa}{\vartheta}\left[\Delta\tilde{a}_{t+1} - \frac{\rho(\vartheta\sigma-\varsigma)}{\vartheta\sigma\beta\rho-\varsigma}\Delta\tilde{a}_t\right].$$

The volatility of exchange rate in this case:

$$\mathrm{var}(\Delta q_{t+1}) = (\sigma\kappa)^2\left[\rho^2 + (1-\rho^2)\left(\frac{\beta\rho - \varsigma/(\sigma\vartheta)}{1-\beta\rho}\right)^2\right]\frac{\sigma_a^2}{1-\rho^2}.$$

Imagine the limit $\rho \to 1$ and $\sigma_a^2 \to 0$ such that $\sigma_a^2/(1-\rho^2)$ stays finite. Then $\mathrm{var}(\Delta q_{t+1})$ stays separated from zero, while the conditional variance $\mathrm{var}_t(\Delta c_{t+1})$ goes to zero (under the joint limit $\gamma \to 0$, $\rho \to 1$), and the correlations between RER and other variables are exactly as in the case of ψ_t shocks (only indirect effect of shocks on variables through q_t).

A.6.2 Low elasticity of substitution

[58] For comparison, if $a_t = \rho a_{t-1} + \varepsilon_t$, then:

$$\beta\Delta\hat{b}_{t+1} = \kappa(\vartheta\sigma - \varsigma)\frac{\beta(1-\rho)}{1-\beta\rho}\tilde{a}_t,$$
$$\Delta q_{t+1} = \beta\Delta\hat{b}_{t+2} - \Delta\hat{b}_{t+1} + \varsigma\kappa\Delta\tilde{a}_{t+1}$$
$$= \frac{\kappa}{\vartheta}\frac{\vartheta\sigma\beta(1-\rho) + \varsigma(1-\beta)}{1-\beta\rho}\left[\tilde{a}_{t+1} - \frac{\vartheta\sigma(1-\rho) + \varsigma\rho(1-\beta)}{\vartheta\sigma\beta(1-\rho) + \varsigma(1-\beta)}\tilde{a}_t\right]$$

and the two models converge to the same limit as $\rho \to 0$ in the first case and $\rho \to 1$ in the second case.

A.7 Monetary model with nominal rigidities

We outline the details of the monetary model, adopting a general enough setup to nest several extensions as special cases. In particular, we allow for both nominal wage and price rigidities. As before, we focus on Home and symmetric relationships hold in Foreign.

Households Consider a standard New Keynesian two country model in a cashless limit. The problem of Home household is

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{\kappa}{1+1/\nu}\int L_{it}^{1+1/\nu}di\right)$$

$$\text{s.t. } P_tC_t+\frac{B_{t+1}}{R_t}+\frac{B_{t+1}^*\mathcal{E}_t}{e^{\psi_t}R_t^*}\leq B_t+B_t^*\mathcal{E}_t+W_tL_t+\Pi_t+T_t$$

$$L_{it}=\left(\frac{W_{it}}{W_t}\right)^{-\epsilon}L_t,$$

where the last relation is demand for labor, $L_t=\left(\int L_{it}^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\epsilon/(\epsilon-1)}$, and the associate wage index is $W_t=\left(\int W_{it}^{1-\epsilon}\mathrm{d}i\right)^{1/(1-\epsilon)}$. Taking the first order conditions for asset holdings, log-linearizing them and expressing in terms of $\tilde{x}_t=\frac{1}{2}\left(x_t-x_t^*\right)$, we obtain the New Keynesian IS curve and the UIP condition:

$$\mathbb{E}_t\left[\sigma\triangle\tilde{c}_{t+1}+\triangle\tilde{p}_{t+1}\right]=\tilde{i}_t \tag{A72}$$

$$\mathbb{E}_t\triangle\tilde{e}_{t+1}=2\tilde{i}_t-\psi_t \tag{A73}$$

We also assume that the household sets wages a la Calvo and supplies as much labor as demanded at a given wage rate. The probability of changing wage in the next period is $1-\lambda_w$. Then the first order condition for wage setting is

$$\mathbb{E}_0\sum_{t=0}^{\infty}\left(\beta\lambda_w\right)^t\left(\kappa\frac{L_{it}^{1/\nu}}{W_t}-\frac{\epsilon-1}{\epsilon}\frac{C_t^{-\sigma}}{P_t}\right)L_{it}=0.$$

Log-linearizing, we obtain:

$$\hat{w}_{it}=\frac{1-\beta\lambda_w}{1+\epsilon/\nu}\left(\sigma c_t+\frac{1}{\nu}\ell_t+p_t+\frac{\epsilon}{\nu}w_t\right)+\beta\lambda_w\mathbb{E}_t\hat{w}_{it+1}.$$

Note that the wage inflation can be expressed as $\pi_t^w\equiv\Delta w_t=\left(1-\lambda_w\right)\left(\hat{w}_{it}-w_{t-1}\right)$. Also, notice that the price index can be written as $\tilde{p}_t=\left(1-\gamma\right)\tilde{p}_{Ht}-\gamma\tilde{p}_{Ht}^*$. Aggregate wages using these equalities and express the wage process in terms of cross-country differences to obtain the NKPC for wages:

$$-\beta\mathbb{E}_t\tilde{w}_{t+1}=\tilde{w}_{t-1}+k_w\left[\sigma\tilde{c}_t+\frac{1}{\nu}\tilde{\ell}_t+\left(1-\gamma\right)\tilde{p}_{Ht}-\gamma\tilde{p}_{Ht}^*\right]-\left[1+\beta+k_w\right]\tilde{w}_t, \tag{A74}$$

where $k_w=\frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)}$.

Firms Assume that firms set prices a la Calvo with probability of changing price next period equal $1-\lambda_p$. There are two Phillips curves, one for domestic sales $\tilde{p}_{Ht}$ and one for export $\tilde{p}^*_{Ht}$. The first order conditions for prices in log-linearized form are

$$\hat{p}_{Hit} = (1-\beta\lambda_p)\,\mathbb{E}_t \sum_{j=t}^{\infty} (\beta\lambda_p)^{j-t} \left[(1-\alpha)\left(-a_j + (1-\phi)\,w_j + \phi p_j\right) + \alpha p_j\right]$$

$$\hat{p}^*_{Hit} = (1-\beta\lambda_p)\,\mathbb{E}_t \sum_{j=t}^{\infty} (\beta\lambda_p)^{j-t} \left[(1-\alpha)\left(-a_j + (1-\phi)\,w_j + \phi p_j - e_j\right) + \alpha p^*_j\right].$$

Note these equations do not depend on whether firms use LCP or PCP. The law of motion for home prices and the resulting NKPC are then

$$\pi_{Ht} = (1-\lambda_p)\left(\hat{p}_{Hit} - p_{Ht-1}\right) = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}_{Hit} - p_{Ht}\right)$$

$$\begin{array}{c} -\beta\mathbb{E}_t\tilde{p}_{Ht+1} = \tilde{p}_{Ht-1} + k_p\,(1-\alpha)\left[-\tilde{a}_t + (1-\phi)\,\tilde{w}_t\right] - \\ -k_p\gamma\left[1-(1-\alpha)\,(1-\gamma)\right]\tilde{p}^*_{Ht} - \left[1+\beta+k_p\left(\gamma+(1-\alpha)\,(1-\gamma)\,(1-\phi)\right)\right]\tilde{p}_{Ht} \end{array}, \tag{A75}$$

where $k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$. On the other hand, the law of motion for export prices depends on currency of invoicing. Assuming LCP one obtains

$$\pi^*_{Ht} = (1-\lambda_p)\left(\hat{p}^*_{Hit} - p^*_{Ht-1}\right) = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}^*_{Hit} - p^*_{Ht}\right)$$

$$\begin{array}{c} -\beta\mathbb{E}_t\tilde{p}^*_{Ht+1} = \tilde{p}^*_{Ht-1} + k_p\,(1-\alpha)\left[-\tilde{a}_t + (1-\phi)\,\tilde{w}_t - e_t\right] - \\ -k_p\,(1-\gamma)\left[\alpha-(1-\alpha)\,\phi\right]\tilde{p}_{Ht} - \left[1+\beta+k_p\,(1-\alpha\gamma+\gamma\phi\,(1-\alpha))\right]\tilde{p}^*_{Ht}, \end{array} \tag{A76}$$

In case of PCP the law of motion of price index and NKPC are

$$\pi^*_{Ht} = (1-\lambda_p)\left(\hat{p}^*_{Hit} - p^*_{Ht-1}\right) + \lambda_p\triangle e_t = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}^*_{Hit} - p^*_{Ht}\right) + \triangle e_t$$

$$\begin{array}{c} -\beta\mathbb{E}_t\left(\tilde{p}^*_{Ht+1} + e_{t+1}\right) = \left(\tilde{p}^*_{Ht-1} + e_{t-1}\right) + k_p\,(1-\alpha)\left[-\tilde{a}_t + (1-\phi)\,\tilde{w}_t\right] - \left[1+\beta+k_p\,(1-\alpha)\right]e_t \\ -k_p\,(1-\gamma)\left[\alpha-(1-\alpha)\,\phi\right]\tilde{p}_{Ht} - \left[1+\beta+k_p\,(1-\alpha\gamma+\gamma\phi\,(1-\alpha))\right]\tilde{p}^*_{Ht} \end{array}$$

Government policy and shocks We assume that Central Bank conducts active monetary policy, while the government chooses the fiscal policy (taxes) passively to balance the budget. The monetary policy is represented by a standard Taylor rule:

$$\tilde{i}_t = \rho_m\tilde{i}_{t-1} + (1-\rho_m)\,\delta_\pi\left[(1-\gamma)\triangle\tilde{p}_{Ht} - \gamma\triangle\tilde{p}^*_{Ht}\right] + (1-\rho_m)\,\delta_y\tilde{gdp}_t + \epsilon^m_t, \tag{A77}$$

where $\tilde{gdp} = \tilde{c} + nx$. We allow persistence of the interest rate to be different from the autocorrelation of other shocks:

$$\psi_t = \rho\psi_{t-1} + \epsilon^\psi_t \tag{A78}$$

$$\tilde{\xi}_t = \rho\tilde{\xi}_{t-1} + \epsilon^\xi_t \tag{A79}$$

$$\tilde{a}_t = \rho \tilde{a}_{t-1} + \epsilon_t^a \tag{A80}$$

Market clearing The last dynamic equation is the country's budget constraint:

$$\beta b_{t+1} = b_t + 2\left(\tilde{p}_{Ht}^* + \tilde{y}_{Ht}^*\right) + e_t, \tag{A81}$$

where b_t is the net foreign asset position of the Home country. The static part of the model is represented by goods market equilibrium and labor demand condition:

$$\tilde{\ell}_t = \tilde{y}_t - \tilde{a}_t + \phi\left((1-\gamma)\,\tilde{p}_{Ht} - \gamma \tilde{p}_{Ht}^* - \tilde{w}_t\right) \tag{A82}$$

$$\tilde{y}_{Ht} = -\gamma\tilde{\xi}_t - \theta\gamma\left(\tilde{p}_{Ht} + \tilde{p}_{Ht}^*\right) + (1-\phi)\,\tilde{c}_t + \phi\left((1-\phi)\left(\tilde{w}_t - (1-\gamma)\,\tilde{p}_{Ht} + \gamma\tilde{p}_{Ht}^*\right) - \tilde{a}_t + \tilde{y}_t\right) \tag{A83}$$

$$\tilde{y}_{Ht}^* = -\tilde{y}_{Ht} - \tilde{\xi}_t - \theta\left(\tilde{p}_{Ht} + \tilde{p}_{Ht}^*\right) \tag{A84}$$

$$\tilde{y}_t = (1-\gamma)\,\tilde{y}_{Ht} + \gamma\tilde{y}_{Ht}^* \tag{A85}$$

The system (1)-(14) fully describes the equilibrium of the model.

Quantitative results Table A2 shows the results for alternative monetary models. Columns 1 and 2 reproduce the results from Table **??** for a monetary model with a single ψ-shock and m-shock respectively. Column 3 reproduces results from Table **??** for the monetary model with three shocks (ψ, ξ, m). In the remaining columns we keep the mix of the shocks the same (ψ, ξ, m) and consider alternative specifications:

- Columns 4-5: a model with Peg (Taylor rule (45)), with ψ-shocks shut down in column 5, as discussed in the main text.
- Column 6: More general Taylor rule from Clarida, Gali and Gertler (2000):

  $$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\delta_\pi \mathbb{E}_t \pi_{t+1} + \delta_y gdp_t\right) + \epsilon_t^m,$$

  where $\rho_r = 0.8$, $\delta_\pi = 2.15$, $\delta_y = 0.23$ and the policy depends on expected inflation.
- Column 7: A model with sticky nominal wages instead of sticky prices.
- Column 6: A monetary model with PCP and flexible wages.

Table A2: Alternative specifications of the monetary model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\frac{\text{std}(\triangle q)}{\text{std}(\triangle e)}$	1.00	0.94	1.00	2.12	1.55	0.99	0.77	0.86
	(0.00)	(0.00)	(0.00)	(0.22)	(0.16)	(0.01)	(0.01)	(0.01)
$\frac{\text{std}(\triangle s)}{\text{std}(\triangle e)}$	0.80	0.91	0.79	1.15	1.12	0.78	0.23	0.85
	(0.01)	(0.00)	(0.01)	(0.09)	(0.06)	(0.02)	(0.00)	(0.01)
$\frac{\text{std}(\triangle c - \triangle c^*)}{\text{std}(\triangle q)}$	0.19	0.50	0.18	1.76	0.98	0.26	0.52	0.36
	(0.00)	(0.00)	(0.02)	(0.17)	(0.10)	(0.02)	(0.04)	(0.03)
$\frac{\text{std}(\triangle nx)}{\text{std}(\triangle q)}$	0.20	0.00	0.28	1.00	1.14	0.25	0.38	0.36
	(0.00)	(0.00)	(0.03)	(0.12)	(0.10)	(0.02)	(0.03)	(0.03)
$\frac{\text{std}(\triangle gdp - \triangle gdp^*)}{\text{std}(\triangle q)}$	0.21	0.51	0.45	2.40	3.20	0.27	0.69	0.61
	(0.00)	(0.00)	(0.05)	(0.25)	(0.31)	(0.03)	(0.06)	(0.06)
cor $(\triangle e, \triangle q)$	1.00	0.99	1.00	0.63	0.61	0.99	1.00	1.00
	(0.00)	(0.00)	(0.00)	(0.06)	(0.07)	(0.01)	(0.00)	(0.00)
cor $(\triangle e, \triangle s)$	−0.93	−0.99	−0.92	−0.53	−0.82	−0.91	1.00	0.97
	(0.02)	(0.00)	(0.02)	(0.07)	(0.04)	(0.02)	(0.00)	(0.01)
cor $(\triangle c - \triangle c^*, \triangle q)$	−0.99	1.00	−0.42	−0.62	−0.03	−0.39	−0.68	−0.58
	(0.00)	(0.00)	(0.08)	(0.05)	(0.07)	(0.08)	(0.05)	(0.06)
cor $(\triangle nx, \triangle q)$	1.00	0.98	−0.02	0.13	−0.41	0.14	0.30	0.14
	(0.00)	(0.00)	(0.09)	(0.09)	(0.05)	(0.09)	(0.09)	(0.09)
cor $(\triangle gdp - \triangle gdp^*, \triangle q)$	1.00	1.00	−0.20	−0.35	−0.30	−0.12	−0.19	−0.19
	(0.00)	(0.00)	(0.09)	(0.06)	(0.06)	(0.09)	(0.09)	(0.09)
Fama coefficient	−2.0	1.1	−0.1	0.0	1.1	0.1	0.1	0.0
	(1.7)	(0.3)	(0.9)	(0.1)	(0.2)	(1.3)	(0.5)	(0.6)
R^2 in Fama regression	0.02	0.10	0.00	0.00	0.16	0.01	0.00	0.00
	(0.02)	(0.04)	(0.01)	(0.01)	(0.04)	(0.02)	(0.01)	(0.01)
$\frac{\text{std}(r - r^*)}{\text{std}(\triangle e)}$	0.08	0.29	0.09	0.88	0.38	0.08	0.18	0.15
	(0.03)	(0.03)	(0.03)	(0.28)	(0.04)	(0.03)	(0.03)	(0.03)
Sharpe Ratio	0.23	-	0.19	0.71	-	0.21	0.23	0.21
	(0.04)		(0.06)	(0.15)		(0.06)	(0.06)	(0.06)
acor $(r - r^*)$	0.98	0.65	0.88	0.93	0.66	0.96	0.80	0.81
	(0.01)	(0.07)	(0.06)	(0.04)	(0.07)	(0.03)	(0.07)	(0.07)
acor $(\triangle e)$	−0.04	−0.15	−0.03	−0.19	−0.15	−0.01	0.03	0.00
	(0.09)	(0.08)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)
acor (q)	0.92	0.65	0.93	0.98	0.98	0.92	0.95	0.94
	(0.05)	(0.07)	(0.04)	(0.01)	(0.01)	(0.04)	(0.03)	(0.03)
$\frac{\text{std}(\triangle \Pi / GDP)}{\text{std}(\triangle q)}$	0.29	0.46	0.37	5.54	3.84	0.34	0.09	0.32
	(0.00)	(0.00)	(0.02)	(0.45)	(0.37)	(0.01)	(0.00)	(0.03)
RER decomposition:								
ψ-shocks	100%	–	64%	59%	–	63%	71%	62%
m-shocks	–	100%	3%	8%	38%	1%	12%	10%
ξ-shocks	–	–	33%	33%	62%	36%	17%	28%

A.8 A model of the financial sector

Consider a financial sector in which three types of agents trade assets:

1. Home and foreign households trade their local-currency bonds, holding respectively NFA positions B_{t+1} and B^*_{t+1} from t to $t+1$.

2. N Noise traders that take a zero-capital position long $N^*_t > 0$ in foreign currency and short $\frac{N^*_t}{R^*_t}\mathcal{E}_t R_t$ in home currency, and vice versa when $N^*_t < 0$. We assume that the size of their position:
$$N^*_t = N\left(e^{\tilde{\psi}_t} - 1\right),$$
where $\tilde{\psi}_t$ is the noise-trader demand shock for foreign currency, which follows an exogenous stochastic process.

3. M arbitrageurs that take a zero-capital position D^*_t in foreign currency and short $\frac{D^*_t}{R^*_t}\mathcal{E}_t R_t$ in home currency, and vice versa when $D^*_t < 0$. The arbitrageurs maximize the CARA utility of profits:
$$\max_{D^*_t} \; -\frac{1}{\gamma}\mathbb{E}_t \exp\left\{\left(R^*_t - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)\frac{D^*_t}{R^*_t}\right\}.$$
Note that arbitrageurs maximize nominal one-period profits in foreign currency.[59] Each arbitrage has risk aversion γ, so the effective risk-aversion in the market is given by γ/M.

The profits and losses of arbitrageurs are transferred to the foreign households, and hence the budget constraints of the home and foreign households take the following form:

$$\begin{aligned}
\frac{B_{t+1}}{R_t} - B_t &= NX_t, \\
\frac{B^*_{t+1}}{R^*_t} - B^*_t &= NX^*_t + \left(R^*_{t-1} - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right)\frac{D^*_{t-1} + N^*_{t-1}}{R^*_{t-1}}.
\end{aligned}$$

The market clearing conditions in the bond market are given by:

$$\begin{aligned}
B^*_{t+1} + D^*_{t+1} + N^*_{t+1} &= 0, \\
B_{t+1} + \frac{D^*_{t+1} + N^*_{t+1}}{R^*_t}\mathcal{E}_t R_t &= 0.
\end{aligned}$$

Combining these two market clearing conditions, we can check that the home budget constraint implies the foreign budget constraint (after noting that $NX_t = -NX^*_t\mathcal{E}_t$), and therefore we can drop one of them from the equilibrium system (by Walras law).

We use the following approximation:

$$R^*_t - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \approx i^*_t - i_t + \Delta e_{t+1},$$

[59] This assumption can be motivated by OLG of arbitrageurs hired by households with an agency problem.

and guess that the innovation of the return (equal to the innovation of Δe_{t+1}) is normal. Then the optimal portfolio of arbitrageurs is given by:

$$\frac{D_t^*}{R_t^*} = \frac{i_t^* - i_t + \mathbb{E}_t \Delta e_{t+1}}{\gamma / M \cdot \sigma_{\Delta e}^2}.$$

Combining this with the other equations, we end up with two dynamic equilibrium conditions:

$$\begin{aligned} \frac{B_{t+1}}{R_t} - B_t &= NX_t, \\ \frac{B_{t+1}}{\mathcal{E}_t R_t} &= \frac{i_t^* - i_t + \mathbb{E}_t \Delta e_{t+1}}{\gamma / M \cdot \sigma_{\Delta e}^2} + \frac{N}{R_t^*} \left(e^{\tilde{\psi}_t} - 1 \right). \end{aligned}$$

We consider a steady state with $\tilde{\psi} = 0$, $\mathcal{E} = 1$, $B = B^* = NX = 0$ and $R = R^* = 1/\beta$, and linearization around this steady state results in:

$$\begin{aligned} \beta b_{t+1} - b_t &= n x_t, \\ GDP \cdot b_{t+1} &= -\frac{\beta M}{\gamma \sigma_{\Delta e}^2} (i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}) + N \tilde{\psi}_t, \end{aligned}$$

where GDP is steady state GDP used as normalization for net exports and NFA. We rewrite:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \delta \psi_t - \chi b_{t+1}.$$

where:

$$\delta \equiv \frac{N}{M}, \qquad \psi_t \equiv \frac{\gamma \sigma_{\Delta e}^2}{\beta} \tilde{\psi}_t \qquad \text{and} \qquad \chi \equiv \frac{\gamma \sigma_{\Delta e}^2 GDP}{\beta M}.$$

Our baseline model corresponds to the limit $N = M \to \infty$, while the case with finite N and M generalizes our model, and the exchange rate process in this case in equilibrium follows a stationary ARMA(2,1) instead of ARIMA(1,1,1).

A.9 Additional Figures and Tables

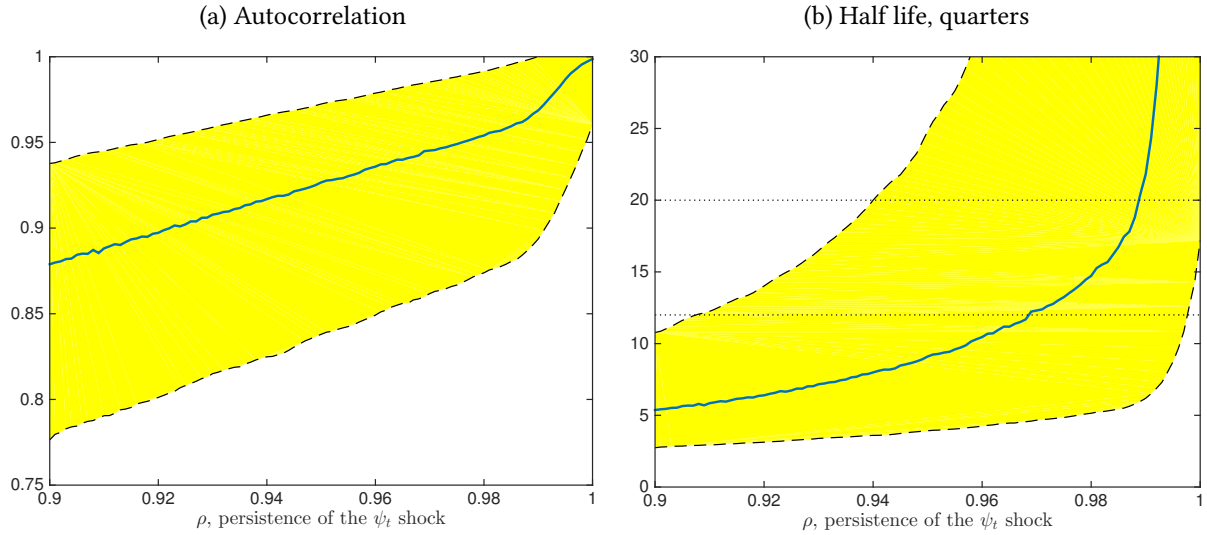


Figure A1: Persistence of the real exchange rate q_t

Note: Left panel: OLS estimated $\hat{\rho}_q$ from projection $q_t = \rho_q q_{t-1} + \epsilon_t^q$. Right panel: corresponding half-life of an AR(1) process with autorcorrelation $\hat{\rho}_q$, calculated according to $HL_q = -\log 2/\log \hat{\rho}_q$. Based on 10,000 simulations with 120 quarters (30 years) each, where the solid lines plot the median estimates and the areas are the 90% bootstrap sets. The dotted lines in the right panel indicate the conventional 3–5 year half life estimates in the data (see Rogoff 1996).

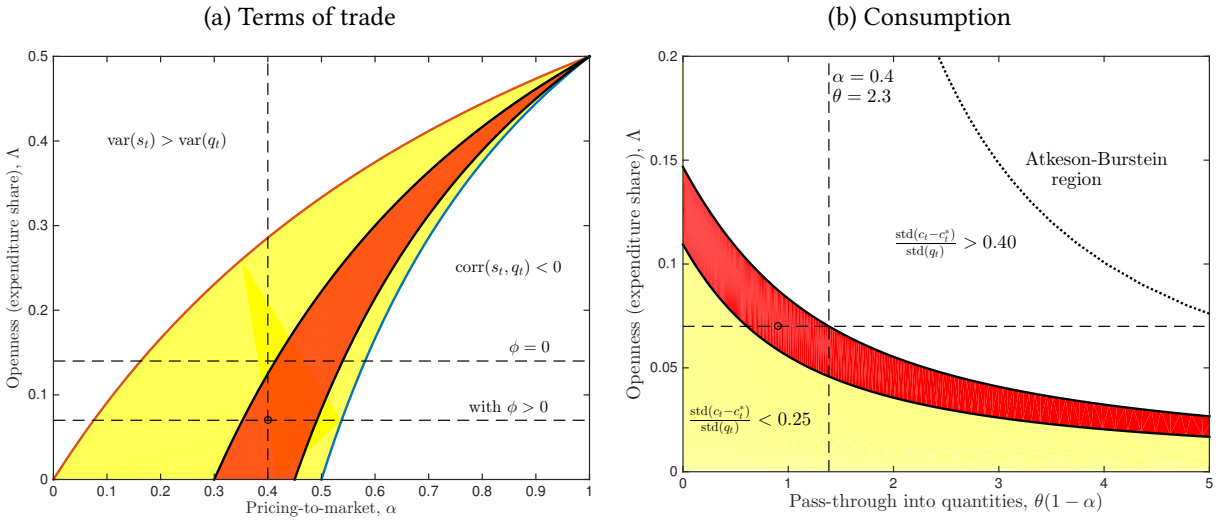


Figure A2: Relative volatility with the real exchange rate

Note: the yellow region satisfies joint requirement $\text{std}(s_t) < \text{std}(q_t)$ and $\text{corr}(s_t, q_t) > 0$, which are weak realistic restrictions imposed by the data. The red area imposes the tighter empirical restrictions that $\text{std}(s_t)/\text{std}(q_t) \in [0.1, 0.4]$. REDO THE FIGURE USING NEW NOTATION AND ADD FIGURE FOR PPI-RER...
Note: the yellow region satisfies $-\frac{\text{std}(c_t - c_t^*)}{\text{std}(q_t)} \in [-0.4, -0.25]$ and in the red region this relative volatility is in $[-0.25, 0]$.

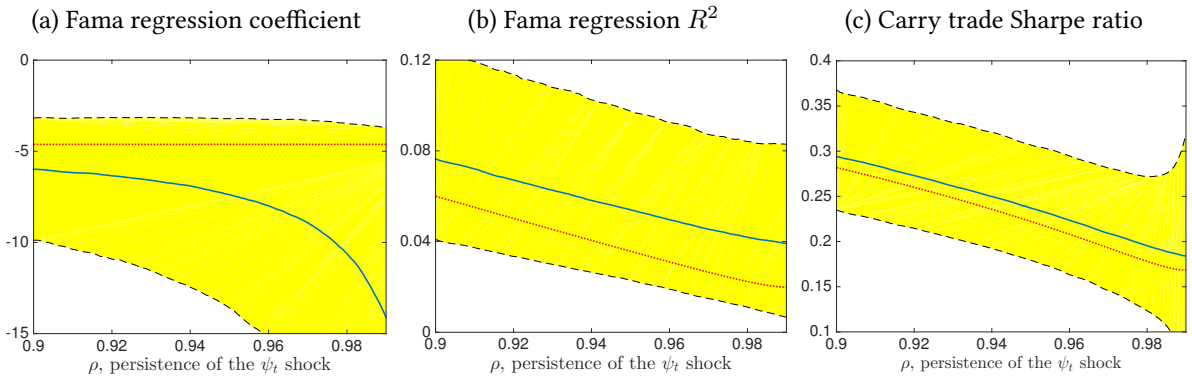


Figure A3: Deviations from the Uncovered Interest Rate Parity

Note: Monte Carlo study based on 10,000 simulations of the model with 120 quarters (30 years). The solid lines plot the median estimates across simulations, the areas represent 90% bootstrap sets, and the red dotted lines are the asymptotic values from the text. Panel (a) plots β_F from the Fama regression of Δe_{t+1} on $(i_t - i_t^*)$, while panel (b) plots the R^2 from these regressions. Panel (c) plots the within-sample Sharpe ratio calculated as the coefficient of variation for the carry return $\psi_t \cdot (i_t - i_t^* - \Delta e_{t+1})$. The parameter values for calibration are as described in footnote **??**.

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