# Some Unpleasant Central Bank Balance Sheet Arithmetic *

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**Abstract**

I analyze policy implications of maturity and currency mismatches in the central bank balance sheet. In the model, the central bank holds long-term domestic or short-term foreign currency assets while it issues short-term domestic currency liabilities. Following Sargent and Wallace (1981), I consider a central bank that is constrained in its remittances to the Treasury. Simple balance sheet arithmetic shows that such a central bank loses freedom in its policy actions. The expected future change of the short-term nominal interest rate or the nominal exchange rate get constrained by balance sheet considerations: if they increase today, they have to decrease in future. I embed such a central bank in a non-linear dynamic general equilibrium model. Following a positive short-term nominal interest rate shock, central bank balance sheet considerations lead, dynamically, to a drop in the short-term interest rate and a positive correlation between it and inflation, even with sticky prices. Additionally, following such a shock, inflation in fact decreases more dynamically when prices are more sticky. Whether the central bank is balance sheet constrained or not thus drastically alters the equilibrium dynamics and comovement of interest rate, exchange rate, and inflation.

*Keywords*: Central Bank Balance Sheet, Central Bank Balance Sheet Constraint, Central Bank Balance Sheet Mismatch, Central Bank Remittances to Treasury

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# 1 Introduction

The size and composition of many major central bank balance sheets have changed dramatically recently, primarily as a result of unconventional monetary policy that was undertaken to combat the financial crisis. In particular, maturity and currency mismatches between assets and liabilities have appeared in central bank balance sheets. Many advanced economy central banks now hold large amounts of long-term domestic currency assets or foreign currency assets while they have short-term domestic currency liabilities. Such mismatches, combined with large gross asset and liability positions, open up the possibility of non-trivial fluctuations in the net interest income of the central bank. These fluctuations can arise due to interest rate changes that affect returns on assets and liabilities of different maturities differentially, or due to exchange rate changes that affect returns on assets and liabilities of different currency denominations differentially, or simply due to interest rate expense on repurchases agreements. These movements then in turn can lead to volatility in the central bank remittances to the government.

For more context, in Figures 1-3, I present descriptive data on evolution of assets and liabilities as well as net income and transfers to the government of a few major central banks. As is obvious, all these central bank balance sheets have expanded substantially, increasing three to four times their pre-2008 levels. Panel (a) of Figure 1 shows clearly that for the U.S., the maturity of assets has increased substantially, which was funded mostly by issuing interest bearing short-term reserves. The maturity mismatch on the Federal Reserve balance sheet is quite substantial. Panel (b) of Figure 1 shows that until now, the Federal Reserve has been making profits and in fact remitting large amounts to the Treasury. This is in spite of interest expense related to payment on reserves as well as those due to repurchases agreements. In future however, as short-term interest rates rise, and interest expense on liabilities increase, this situation can change.

In fact, some other prominent central banks have already made losses and transfers to the government have already gone close to or even exactly to zero at times. Panel (a) of Figure 2 shows that for Japan, assets are predominantly domestic government bonds or foreign currency denominated assets. Moreover, the government bonds held by Bank of Japan are primarily long-term, with average maturity of around 7 years. These asset holdings are financed mostly by reserves and some by repurchase agreements. Thus, the Bank of Japan's balance sheet features both maturity and currency mismatch. Panel (b) of Figure 2 shows that there have been major fluctuations in income of Bank of Japan, and in fact, there have been years with losses due to exchange rate fluctuations. Finally, panel (a) of Figure 3 shows that for Switzerland, the major component of assets are foreign currency investments, which have been funded mostly by reserves. These reserves are domestic currency denominated. This massive currency mismatch has led to rapid fluctuations in the income of Swiss National Bank, with net losses made in several years recently. For both the Bank of Japan and the Swiss National Bank, transfers to the government have in turn varied, moving with net income, and in some years, declined close to or exactly to zero.[1]

In this paper, I first present simple closed and open economy models that feature a central bank with maturity and currency mismatch between assets and liabilities on its balance sheet. In particular, motivated by the stylized facts presented above, in the model, the central bank holds long-term domestic or short-term foreign currency assets while it issues short-term domestic currency liabilities. Using the flow budget constraint of the central bank together with just equilibrium asset pricing conditions, and a first-order approximation, allows me to undertake a present value analysis in the spirit of Sargent and Wallace (1981).

[1] Concerns about the central bank making large losses and remitting nothing to the Treasury have previously been discussed mostly for emerging markets and developing countries. See for instance, Sims (2004), Stella (2005), and Berriel and Bhattarai (2009) for some empirical facts and pre-crisis policy discussions of how central banks are worried about the state of their balance sheet for political economy/independence reasons.

In particular, I show with such balance sheet arithmetic that the outstanding net asset position of the central bank is equal, in equilibrium, to the difference between the present value of its transfers to the Treasury and net interest income of the central bank.[2] Then, consider a negative initial net asset position of the central bank. This can be financed, in a present value sense, either through lower transfers to the Treasury or through higher net interest income of the central bank. One key result of the simple model is that the net interest income is given by essentially the expected change in the short-term nominal interest rate in the closed economy model and the expected change in the nominal exchange rate in the open economy model.

I then consider a *balance sheet constrained central bank*, one which faces a constraint in the path of its remittances to the Treasury. This is similar in spirit to the central bank under fiscal dominance in Sargent and Wallace (1981). For simplicity, consider a case where remittances to the Treasury are fixed. Then, the present value analysis implies that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*.[3] Critically, since the interest income depends on the change in the short-term nominal interest rate or the nominal exchange rate, this means that *interest rate policy* or *exchange rate policy* get constrained in this case. Thus, the central bank cannot freely choose the path of its policy instruments in future as it is bound by balance sheet considerations. Precisely this constitutes *unpleasant central bank balance sheet arithmetic.*

In contrast, a *balance sheet unconstrained central bank* does not face a constraint in the path of its remittances to the Treasury. In particular, it can freely choose the path of its instrument, the short-term nominal interest rate of the nominal exchange rate, as remittances to the Treasury adjust endogenously to the state of its balance sheet. In such a case, the present value analysis implies that an increase (decrease) in transfers to the Treasury *today* must lead to a decrease (increase) in *future*.

The analysis of the simple model implies some other related important differences between the models with a balance sheet constrained and an unconstrained central bank. When the central bank is not balance sheet constrained, then changes in outstanding net asset position do not affect interest rate or exchange rate dynamics at all. Moreover, innovations to the transfers to the Treasury similarly have no effects on the interest rate or exchange rate. In sharp contrast, when the central bank is balance sheet constrained, both changes in outstanding net asset position or innovations to the transfers to the Treasury necessitate an adjustment of the central bank's policy instrument. This is because the net interest income of the central bank has to adjust for the present value budget constraint to hold in equilibrium. For example, consider transition dynamics associated with a negative initial net asset position of the central bank. This leads to the net interest income of the central bank increasing on impact. As the net asset position transitions back to steady-state, so does net interest income. Importantly, to be consistent with this dynamics of net interest income, the short-term nominal interest rate falls or the nominal exchange rate depreciates on impact. They then rise along the transition back to steady-state. The path of these policy variables thus clearly gets affected by balance sheet considerations of the central bank.

Finally, I embed the maturity mismatched central bank balance sheet case in a fully specified non-linear equilibrium model with nominal rigidities.[4] This allows me to assess the generality of the results from the simple model, such as precisely how central bank behavior gets constrained, as well as study the effects on endogenously determined inflation dynamics. The model features policy rules for the determination of

[2] Net interest income is the one-period interest on the net asset position of the central bank.

[3] In Sargent and Wallace (1981), the income of the central bank consists of classical seigniorage revenues, while in my model, it consists of net interest income from the portfolio composition of the central bank balance sheet. In Sargent and Wallace (1981), under fiscal dominance, an increase (decrease) in seigniorage today must lead to a decrease (increase) in future.

[4] Due to space constraints, I only consider a non-linear general equilibrium closed economy model. The extension to an open economy would be straightforward however.

the short-term nominal interest rate and for transfers to the Treasury. The key analysis I consider is the effect of a transitory positive innovation to the interest rate rule of the central bank. For the balance sheet unconstrained central bank, the dynamics following such a shock are standard: nominal interest rate increases on impact, inflation falls on impact, and the two are negatively correlated. Given the transitory shock and no state variable, the dynamics, including the negative effect on output, are completely transitory as well.

In sharp contrast, the model with balance sheet constrained central bank leads to very different dynamics of the short-term interest rate and its correlation with inflation. On impact, the short-term nominal interest rate still rises. The net interest income falls on impact, as is to be expected given a direct rise in interest paid on central bank liabilities. After the first period and dynamically however, the short-term nominal interest rate falls. This is essential because of balance sheet considerations that require the net interest income to rise in future. Thus, after falling in the second period, the short-rate transitions back to steady-state, rising along the transition. Inflation throughout is lower and slowly transitions back to steady-state. This means that after the initial period, the short-term interest rate and inflation are positively correlated, even though the model has sticky prices.

One key intuition for the dynamic correlation between inflation and short-term interest rate is that when the central bank is balance sheet constrained, it has to forego following the Taylor principle as otherwise the model does not have a stationary solution. In its interest rate rule, while the short-rate responds positively to inflation, it does so by less than one-for-one. This positive response in the policy rule then leads to a positive correlation over time between inflation and the short-term interest rate. These persistent dynamics arise, even though the shock is transitory, as central bank balance sheet variables are now endogenous state variables that affect the dynamics of interest rates and inflation. This also implies that when the central bank is balance sheet constrained, output declines persistently following a one-time contractionary monetary policy shock.

To further highlight these differences, I consider a comparative static exercise with respect to the degree of price stickiness, that is the cost of price adjustment. For the unconstrained central bank, as is to be expected, higher price stickiness leads to a smaller equilibrium response of inflation when a short-term interest rate shock hits the economy. This also implies a larger response of the short-term interest rate in equilibrium. Thus, for a positive innovation to the short-term interest rate, inflation decreases by less while the short-term interest rate increases by more. In sharp contrast, for the constrained central bank, higher price stickiness leads to a larger equilibrium response of inflation when an interest rate shock hits the economy. What drives this result? When prices are more sticky, in response to a positive innovation to the short-term interest rate, the short-term interest rate rise by more on impact, as is the case for the unconstrained central bank case. Thus, just on impact, inflation drops less for more sticky prices. But following the impact period, the short-term interest rate falls more with stickier prices, throughout the rest of the transition back to steady-state. Since short-term interest rate and inflation are positively correlated along the transition in this model, inflation also responds more strongly with greater degree of price stickiness, dropping more dynamically.

This paper is related to several strands of the literature. My analysis is clearly close to Sargent and Wallace (1981), whose focus was on the dynamics of the classical seigniorage revenues of the central bank and how the central bank might lose freedom in its path of inflation/seigniorage when its remittances to Treasury are constrained. I focus here on the dynamics of the net interest income of the central bank that arises from the portfolio composition of, and mismatch on, its balance sheet, a situation that has become relevant recently for central banks of even advanced countries. Their case of unpleasant monetarist

arithmetic when a central bank is under fiscal dominance motivates my analysis of unpleasant central bank balance sheet arithmetic when a central bank is balance sheet constrained.[5] On emphasizing the connections between fiscal and monetary policy, and in highlighting that inflation determination depends on the full general equilibrium of the model and requires a complete specification of policy rules, my paper also shares the theme of the fiscal theory of the price level, for example, as developed in Sims (1994). I use policy rules in the general equilibrium model that are motivated in their formulation by this literature. One difference regarding mechanisms is that the revaluation of government liabilities through inflation does not play a role in my model. Additionally, unlike in my model when the central bank is balance sheet constrained, stronger nominal rigidities do lead to a lower response of inflation for a monetary shock in the fiscal theory of the price level.[6]

My paper is also clearly related to recent work that has assessed how central bank balance sheet dynamics might affect monetary policy. Prior to the crisis and unconventional policy by central banks, Sims (2004), Sims (2005), Zhu (2004), and Berriel and Bhattarai (2009) analyze various implications of imposing a central bank budget constraint on the conduct of monetary policy. Recently, Hall and Reis (2013), Reis (2013), Bassetto and Messer (2013), and Del Negro and Sims (2015) have provided several important insights related to consequences of central banks issuing interest bearing liabilities, the reliability of stable money demand at high inflation, and whether the central bank might lose control of inflation because of solvency concerns.[7] My relative focus is on explaining precisely how the central bank behavior might get affected, in terms of dynamics of net interest income that constrains interest rate and exchange rate policy, under maturity and currency mismatches in its balance sheet. Additionally, I use a complete non-linear dynamic equilibrium sticky price model to assess the differences in the effects of a monetary policy shock, and the comovement of inflation and interest rates following such a shock, when a central bank is balance sheet constrained. This environment also allow me to do a comparative static exercise on the degree of price rigidity, which highlights further the effects on interest rate and inflation dynamics when the central bank is balance sheet constrained.

My paper is concerned with positive issues as I explain how central bank policy can get constrained by balance sheet considerations while taking the mismatches in the central bank balance sheet as given. It is however related to recent normative work that breaks the neutrality of open market operations of Wallace (1981) when the central bank cannot commit and cares about its balance sheet. In particular, in a specific zero lower bound situation, the constraints that I highlight in terms of interest rate and exchange rate policy could in fact be beneficial when the central bank cannot credibly commit to future interest rate policy. Bhattarai, Eggertsson, and Gafarov (2016) show this in a closed economy model with a maturity mismatch while Jeanne and Svensson (2007) show this in an open economy model with currency mismatch.[8] My results here show that generally, such balance sheet constraints can lead to a non standard transmission of monetary policy shock to interest rates. Moreover, in such a case, innovations to the balance sheet variables transmit to the rest of the macroeconomy by inducing responses from monetary policy.

Finally, my result on the transmission of the shock to the interest rate rule when the central bank is balance sheet constrained is related to some recent results in the fiscal theory of the price level and the Neo Fisherism literature. Under the fiscal theory of the price level, in a sticky price model with one-period

[5] I do set up a general model with cash, but eventually abstract from seigniorage considerations for sharp focus. Bhattarai, Lee, and Park (2014) sets up the fiscal dominance regime in an equilibrium model with nominal rigidities and cash.

[6] Canzoneri, Cumby, and Diba (2011) is a recent survey of both the unpleasant monetarist arithmetic and the fiscal theory of the price level literature. See also Sims (2013).

[7] See also Ennis and Wolman (2010) for a discussion of possible implications of a large amount of interest bearing excess reserves. Carpenter et al (2013) empirically assess how income of the Federal Reserve and its transfers to the Treasury might get affected in future, depending on increases in short-term interest rates.

[8] See also Berriel and Mendes (2015) and Benigno and Nistico (2016).

government debt, Bhattarai, Lee, and Park (2014) show analytically that a positive inflation target shock leads to a decrease in inflation while in a sticky price model with long-term government debt, Sims (2011) shows that a positive interest rate shock increases inflation after a delay. These results arise because of the wealth effect arising from government debt holdings that is inherent in the transmission mechanism of monetary policy in this theory. Bhattarai, Lee, and Park (2014) moreover emphasize that in this case the inflation target and inflation are negatively correlated, in sharp contrast to the standard case where they are positively correlated.[9] Additionally, under Neo Fisherism, for instance as described in Williamson (2016), raising interest rates raises inflation. In my model as well, when the central bank is balance sheet constrained, in contrast to the traditional text book case, interest rates and inflation are indeed dynamically positively correlated. At the same time however, a positive interest rate shock does lead to an increase in the short-term interest rate on impact, and more importantly, a negative response of inflation throughout.

# 2 Simple Model

In this section, I keep the model deliberately simple with the goal of focusing simply on the central bank balance sheet. I consider two important, and recently empirically relevant, mismatches between assets and liabilities of the central bank: maturity mismatch and currency mismatch. I model maturity mismatch in a closed economy and currency mismatch in an open economy separately to clarify the main results. The dynamics of the central bank balance sheet, together with some standard asset pricing conditions only, allows me to derive several insights on when and how monetary policy actions might get constrained by balance sheet considerations. Moreover, the analysis here is also deliberately in the spirit of Sargent and Wallace (1981)'s analysis of the implications of the present-value government budget constraint.

## 2.1 Maturity mismatch in a closed economy

I first consider a general, closed economy central bank balance sheet with nominal assets and liabilities. The central bank issues non-interest bearing and interest bearing short-term liabilities while it holds long-term assets. Without loss of generality, the short-term liabilities are one-period bonds while the long-term assets are two-period bonds. The model is thus one of maturity mismatch.

### 2.1.1 Unpleasant central bank balance sheet arithmetic

The nominal flow budget constraint of the central bank is given by

$$Q_{1t}B_{t-1} - L_{t-1} - M_{t-1} - P_tT_t = Q_{2t}B_t - Q_{1t}L_t - M_t$$

where $M_t$ is non-interest bearing liabilities (e.g. cash), $L_t$ is one-period liabilities (e.g. interest-bearing reserves) with $Q_{1t}$ its price, and $B_t$ is two-period assets with $Q_{2t}$ its price. Moreover, $P_t$ is the nominal price level and $T_t$ is the central bank transfer to the Treasury.[10] The flow budget constraint can be written in real terms as

[9] Bhattarai, Lee, and Park (2016) show this result in an estimated model as well.

[10] Through out the paper, I write flow budget constraints in terms of market values and define flow transfers to the Treasury accordingly. The transfers here can be interpreted as net of some operations costs of the central bank. For discussion of use of market prices compared to hold to maturity, see Bassetto and Messer (2013). Hall and Reis (2013) and Del Negro and Sims (2015) discuss issues related to the accounting standards that are used in determining central bank transfers to the Treasury. In terms of timing convention for stock variables, I use $X_t$ to represent the value of $X$ in the beginning of period $t+1$.

$$Q_{1t}b_{t-1}\Pi_t^{-1} - l_{t-1}\Pi_t^{-1} - m_{t-1}\Pi_t^{-1} - T_t = Q_{2t}b_t - Q_{1t}l_t - m_t \tag{1}$$

where $b_t = \frac{B_t}{P_t}$, $l_t = \frac{L_t}{P_t}$, $m_t = \frac{M_t}{P_t}$, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. Finally, under complete markets, the two bond prices are determined by standard asset-pricing conditions

$$Q_{1t} = E_t\left[\zeta_{t,t+1}\Pi_{t+1}^{-1}\right],\ Q_{2t} = E_t\left[\zeta_{t,t+1}Q_{1t+1}\Pi_{t+1}^{-1}\right] \tag{2}$$

where $E_t$ is the mathematical conditional expectation operator and $\zeta_{t,t+1}$ is the unique stochastic discount factor.[11]

To get insights on how the maturity mismatch between assets and liabilities can constrain the central bank, it is useful to approximate the central bank budget constraint around a non-stochastic steady-state. The analysis in this case will be very similar to the one in Sargent and Wallace (1981). Log-linearizing (1) gives

$$b_{N,t} = \frac{1}{\beta}b_{N,t-1} + \frac{1}{\beta}s_t - \frac{1}{\beta}\hat{T}_t + b\left[\hat{Q}_{1t} - \beta E_t\hat{Q}_{1t+1}\right] \tag{3}$$

where $b_{N,t} \equiv \beta\hat{b}_t - \hat{l}_t$ is the (real) net asset position of the central bank in terms of difference in the stock of interest bearing assets and liabilities, $s_t \equiv m[\hat{m}_t + \pi_t - \hat{m}_{t-1}]$ is revenue from issuing non-interest bearing liabilities or seigniorage, and $\beta$ is the discount factor.[12]

There are several aspects of (3) that require some discussion. First, while deriving it, (2) log-linearized has been imposed, which is a no-arbitrage condition that illustrates the expectation hypothesis

$$\hat{Q}_{2t} = \hat{Q}_{1t} + E_t\hat{Q}_{1t+1} \tag{4}$$

and links the two-period bond price with the current and expected future one-period bond prices. Second, other than through the seigniorage term, inflation does not appear directly as both assets and liabilities are nominal.[13] Third, in this approximated budget constraint, only the net asset position, and not the separate gross asset and liability positions, appears.[14]

Fourth, and perhaps more important, the dynamics of net asset of the central bank get affected by the net interest income on its position, $\hat{Q}_{1t} - \beta E_t\hat{Q}_{1t+1}$. Why is this the flow net interest income of the central bank? It is useful to go back to the original budget constraint (1). On its one-period liability, the central bank pays the one-period interest rate $\frac{1}{Q_{1t}}$. On its two-period asset, the effective one-period interest rate earned is $\frac{Q_{1t}}{Q_{2t}}$. With a first-order approximation, the difference between the two, after accounting for discounting in the definition of the net asset position, is given by $\hat{Q}_{1t} - \beta E_t\hat{Q}_{1t+1}$.[15] In particular, note that this is not

[11] $\zeta_{t,t+1}$ in a completely specified general equilibrium model, such as the one presented in the next section, will be given by $\beta\frac{U_{c,t+1}}{U_{c,t}}$ where $\beta$ is the discount factor and $U_c$ is the marginal utility of the representative household.

[12] In terms of approximation, I log-linearize all variables, except for asset and liability positions and transfers to the Treasury, which are expressed in terms of deviations from steady-state. The steady-state is a zero inflation steady-state where the net position is zero. In terms of notation, an approximated variable is represented by a circumflex above the variable while the variable in steady-state is represented without a time subscript.

[13] In addition, for this result, the steady-state values of assets and liabilities have to be equal, as is the assumption for simplicity here.

[14] This is a feature of log-linearization and in the general non-linear model presented in the next section, gross positions will need to be determined separately. Moreover, this is not a feature specific to my model and is a general property of any model with different assets. For instance, it is feature of international finance models that have portfolio choice, such as Berriel and Bhattarai (2013).

[15] Alternatively for intuition, note that under risk-neutrality, as $Q_{2,t} = Q_{1,t}E_t\left[Q_{1,t+1}\right]$, the difference between the two would be $\frac{1}{E_t[Q_{1,t+1}]} - \frac{1}{Q_{1,t}}$.

equal to the interest rate spread, the difference between the interest rate on the two-period and one-period bonds. Finally, the effect of the net interest income on the dynamics of net asset position depends on the steady-state gross position of the central bank balance sheet. In this sense, a large size of the balance sheet can magnify the effects of fluctuations in interest rates.

To get implications similar to Sargent and Wallace (1981) while focusing on the maturity mismatch, I will now simplify by assuming that seigniorage revenues are negligible. This will help make the new implications of this paper distinct.[16] Then, (3) is given by

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} - \frac{1}{\beta} \hat{T}_t + bR_{N,t} \tag{5}$$

where $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$ is the one-period net interest income of the central bank discussed above. The flow budget constraint can then be expressed in a present value form after imposing a terminal condition on net asset of the central bank as

$$b_{N,t-1} = \sum_{t=0}^{\infty} \beta^t \hat{T}_t - \beta \sum_{t=0}^{\infty} \beta^t bR_{N,t} \tag{6}$$

where I will take the initial net position $b_{N,t-1}$ as given.[17] Now, using (6), consider how any outstanding/initial net position of the central bank needs to be financed. In particular, if $b_{N,t-1} < 0$, then adjustment must come, in a present value sense, either from lower central bank transfers to the Treasury $\left(\hat{T}_t\right)$ or from higher net interest income of the central bank $(R_{N,t})$.[18]

Even more importantly, consider a case similar to Fiscal dominance in Sargent and Wallace (1981) where now the present value of central bank transfers to the Treasury is fixed (or exogenous generally). This describes the *closed economy balance sheet constrained central bank*. Then it is clear that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*. In particular, since the interest income depends on the one-period interest rate, the policy instrument of the central bank, this means that the central bank's *interest rate policy* gets constrained in this case. Thus for example, it cannot freely choose the path of its policy instrument in future. This constitutes *unpleasant central bank balance sheet arithmetic* in a closed economy with a maturity mismatch in the central bank balance sheet.[19]

Now consider again the case of $b_{N,t-1} < 0$ for the closed economy balance sheet constrained central bank. In this case, as adjustment does not come from central bank transfers to the Treasury (for intuition, we can think of there being no response of transfers at all), it must be the case that adjustment comes from higher net interest income of the central bank. Additionally, this unpleasant balance sheet arithmetic implies that if transfers to the Treasury increase (decrease) today with no future off-setting decreases (increases), then net interest income must increase (decrease) in future, in order to satisfy (6). Again, in both these cases, the policy instrument of the central bank must adjust and respond appropriately to support the required adjustment in net interest income. How exactly does the policy instrument respond? In both cases, the nominal short-term interest rate must fall, thereby ensuring a direct fall in interest paid on (short-term) liabilities, which in turn in equilibrium leads to an increase in the net interest income.[20]

[16] See Hall and Reis (2013), Reis (2013), and Del Negro and Sims (2015) for important insights on the role played by seigniorage revenues.

[17] One can focus here on a perfect foresight environment without loss of generality.

[18] This is similar to the Sargent and Wallace (1981) analysis on how outstanding debt of the government needs to be financed.

[19] This is an analogue to the unpleasant monetarist arithmetic in Sargent and Wallace (1981) where with the present value of tax revenues fixed, it implied that if seigniorage increased today, it must decrease in future.

[20] I illustrate below using an equilibrium analysis the full dynamic path through which the policy instrument responds to these two cases. There will be dynamics because of the state variable, the net asset position of the central bank, that matters for the evolution of the nominal short interest rate.

If the present value of central bank transfers to the Treasury are not fixed, and in particular, if it adjusts to the state of the balance sheet to satisfy (6), then that describes the *closed economy balance sheet unconstrained central bank*. For such a central bank, as the path of net interest income, and thereby its policy instrument, is not constrained, it can freely pursue its objectives regarding the dynamics of the one-period interest rate. On the flip side, this then implies that if transfers to the Treasury increase (decrease) today, they must decrease (increase) in future, in order to satisfy (6).

### 2.1.2 Policy rules, transition dynamics, and responses to shocks

To get further insights and to make the mechanisms described above even more concrete, I now consider explicit monetary and balance sheet/transfer policy rules that can be used to model the two types of central banks. A complete equilibrium analysis of the transition dynamics associated with $b_{N,t-1} < 0$ can now be undertaken in a straightforward way. Moreover, I introduce exogenous shocks to the interest rate and transfers to the Treasury in order to trace out the transmission of policy changes to model variables in a standard impulse response function analysis. This framework also allows a clear illustration of the basic logic discussed above regarding which term in (6) adjusts and which variable is set freely by the central bank.

**Policy rules for the two central banks** For the *closed economy balance sheet unconstrained central bank*, as it is free to set the path of the short-term interest rate ($i_t \equiv$ -$\hat{Q}_{1t}$) and transfers to the Treasury can respond appropriately to the state of its balance sheet, the rules are given by

$$\hat{Q}_{1t} = \varepsilon_t,\ \hat{T}_t = \psi_T b_{N,t-1} + \xi_t \tag{7}$$

where $\psi_T > 1 - \beta$, which ensures that the central bank does not have to use its policy instrument to ensure that it satisfies (6). That is, in this case, transfers to the Treasury will co-move sufficiently positively with the net asset of the central bank. Additionally, $\varepsilon_t$ in this simple model is simply an *iid* shock and just illustrates that the central bank is free to set the path of the short-term interest rate while $\xi_t$ is an *iid* shock in the transfer rule.[21]

For the *closed economy balance sheet constrained central bank*, the path of its transfers to the Treasury is constrained (or fixed/exogenous) and as a result the net interest income will have to respond to the state of its balance sheet. This implies that it cannot freely set the path of its policy instrument. The rules are then given by

$$R_{N,t} = \phi_R b_{N,t-1} + \varepsilon_t,\ \hat{T}_t = \xi_t \tag{8}$$

where $\phi_R < b\left(1 - \beta^{-1}\right)$, which ensures that (6) is satisfied in the face of exogenous transfers to the Treasury. That is, in this case, net interest income of the central bank will co-move sufficiently negatively with the net asset of the central bank.[22] Here, $\xi_t$ is an *iid* shock in the transfer rule, which just illustrates that the path

[21] Because the model here is very simple, I consider a somewhat unconventional interest rate rule to clarify the main point. In the fully specified equilibrium model in the next section, I will consider a standard interest rate feedback rule that responds to inflation. As I noted above, here, inflation does not appear in the approximated central bank budget constraint and specifying a rule like this is useful both to keep the model very parsimonious, as well as, in clarifying the central mechanism of the paper. Moreover, I use simple feedback rules for transfers to Treasury, as opposed to some other alternatives, such as lower bound on it, as that leads to a very straight forward analysis and makes clear the mechanisms. See Hall and Reis (2013) and Del Negro and Sims (2015) for using bounds on transfers to the Treasury.

[22] Note again that the net interest income is given by $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$, which is a quasi first-difference of the short-term interest rate. Here, it is instructive to directly specify a rule for net interest income as that is the main object of analysis. Bhattarai, Lee, and Park (2014), to illustrate the original Sargent and Wallace (1981) mechanism, use directly a rule for seigniorage for a central bank under fiscal dominance. In the unconstrained central bank case as well, I can use a rule for net interest income without any loss of generality for the iid shock case. Because in that case, since as we will see later, interest

of transfers is exogenous.

**Transition dynamics and responses to shocks** How do the variables respond to the two shocks, to the interest rate/interest income and transfers to the Treasury? How does the transition dynamics associated with $b_{N,t-1} < 0$ look like?

**Closed economy balance sheet unconstrained central bank** First, consider the *closed economy balance sheet unconstrained* central bank. The model here simply consists of (5) and (7). In this case, the solution of the model is similar to conventional text-book models, at least for interest rate shock and resulting dynamics. When there is an *iid* shock to the interest rate rule, say a shock that increases the short-term interest rate, then the short-term will increase on impact and then go back to steady-state next period. The central bank thus has complete control of its policy instrument, as in conventional models. Because of a temporary, one-period increase in the short-term interest rate, the net interest income of the central bank decreases by the same amount on impact. This means that the net asset position of the central bank and after a period, transfers to the Treasury, also decrease. The balance sheet variables (and not interest rate and central bank net interest income) have endogenous dynamics as the variables evolve over time, as given clearly in (5). The result for the interest rate shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (a) of Figure 4.[23]

Now I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, all the adjustment comes from transfers to the Treasury, which must decline. Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does transfers to the Treasury. The short-term nominal interest rate or net interest income of the central bank is not affected at all. The results for the transfer shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 4.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Following the present-value logic described above however, in order to satisfy (6), transfers will decrease in future. The net asset position of the central bank will decrease on impact, as transfers have increased, and then slowly transition back to steady-state. This shock does not affect interest rates or the net interest income of the central bank. Thus, other than in terms of implications for balance sheet variables, there is no effect of this shock on other variables in the model. The results for the transfer shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (c) of Figure 4.

**Closed economy balance sheet constrained central bank** In sharp contrast, next, consider the *closed economy balance sheet constrained* central bank. The model here simply consists of (5) and (8). In this case, the solution of the model is quite different from the case above that was more in line with conventional text-book models. When there is an *iid* shock to the net interest income, say a shock that decreases it, then on impact it will decrease. But, by the logic presented above, in this case, the net interest income in future must adjust as transfers do not. In particular, net interest income must increase in future in order to satisfy (6). Thus next period, it increases and then slowly transitions back to steady-state. What is the resulting

rates do not depend on central bank balance sheet variables, the response of the interest rate is completely transitory, thereby implying a one-to-one negative correspondence between interest income and the short-term interest rate. Finally, the analytical boundaries for the two types of central bank regimes are quite straightforward to derive, based on ruling out explosive dynamics of net assets of the central bank. The policy rules (where shocks can be ignored) can simply be plugged into (5) and then the derivation requires that the coefficient on lagged net asset position is less than 1. The signs of the policy rule coefficients are particularly intuitive as they appropriately ensure that (6) is satisfied.

[23] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b = 1$, and $\psi_T = 0.011$.

equilibrium dynamics of the short-term interest rate? The short-term interest rate does not move initially, by construction, and then it falls, before rising back to steady-state, as the net interest income is essentially the first-difference of the short-term interest rate. The result for the net interest income shock for the *closed economy balance sheet constrained* central bank is illustrated in panel (a) of Figure 5.[24] The dynamics of policy rate in this case are thus clearly determined by central bank balance sheet considerations and are in sharp contrast to those in panel (a) of Figure 4.

Next, I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, again, all the adjustment comes from net interest income of the central bank and none from central bank transfers to the Treasury. Thus, net interest income must increase to satisfy (6). Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does net interest income of the central bank. The short-term nominal interest rate is now clearly affected as it has to be consistent with the dynamics of the net interest income. Specifically, the short-term nominal interest rate must fall initially, so that there is a direct reduction in the interest paid on the (short-term) liabilities, and then it reverts back to steady-state, increasing along the transition (as is consistent with the net interest income, which is essentially the slope of this transition path, being positive). The results for the transition dynamics for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 5 and they are in sharp contrast to those in panel (b) of Figure 4.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Since this is a one-time shock, transfers go back to steady-state next period. In contrast to the *closed economy balance sheet unconstrained* central bank, now interest rate dynamics get affected as the net interest income of the central bank must respond in equilibrium. In particular, here, the net interest income must increase in a present value sense. Initially however, because of the rule in (8), as initial outstanding net asset position has not changed (making this different in this respect from the transition dynamics experiment above), net interest income does not change either. In future however, net asset decreases, and consistently, net interest income will increase. The equilibrium short-term interest rate path implies that first it decreases, again, to ensure a direct fall in the interest paid on (short-term) liabilities. It also decreases further next period, with an overshooting, before rising and transitioning back to steady-state.[25] Why does the short-term interest rate decrease further for one period? The key reason is that the net interest income in the first-period does not change. For this to be consistent with interest rate dynamics, it must be that the next period interest rate is slightly more negative as $\beta < 1$ (note that $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$). The results for the transfer shock for the *closed economy balance sheet constrained* central bank is illustrated in panel (c) of Figure 5. These results are in sharp contrast to those in panel (c) of Figure 4, where the shock to the transfer rule for instance did not affect interest rate or the net interest income of the central bank at all.

## 2.2 Currency mismatch in an open economy

I consider next a general, open economy central bank balance sheet with nominal assets and liabilities. The central bank issues non-interest bearing and interest bearing domestic currency liabilities while it holds foreign currency assets. To focus on the implications of the currency mismatch as opposed to the maturity mismatch, I here consider only one-period assets and liabilities. The model is thus one of currency mismatch.

[24] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b = 1$, and $\phi_R = -0.011$.

[25] Note that the short-term interest rate cannot rise on impact. There will be no stationary dynamics in that case.

### 2.2.1 Unpleasant central bank balance sheet arithmetic

The nominal flow budget constraint of the central bank is then given by

$$S_t B^*_{t-1} - L_{t-1} - M_{t-1} - P_t T_t = S_t Q_{B^*t} B^*_t - Q_{Lt} L_t - M_t$$

where $M_t$ is non-interest bearing domestic currency liabilities, $L_t$ is one-period domestic currency liabilities with $Q_{L,t}$ its price, and $B^*_t$ is one-period foreign currency assets with $Q_{B^*t}$ its price. Moreover, $P_t$ is the home nominal price level, $T_t$ is the central bank transfer to the Treasury, and $S_t$ is the nominal exchange, that is the price of the foreign currency in terms of home currency.[26] The flow budget constraint can be written in real terms as

$$\varsigma_t b^*_{t-1} \Pi^{*-1}_t - l_{t-1} \Pi^{-1}_t - m_{t-1} \Pi^{-1}_t - T_t = \varsigma_t Q_{B^*t} b^*_t - Q_{Lt} l_t - m_t \tag{9}$$

where $b^*_t = \frac{B^*_t}{P^*_t}$, $l_t = \frac{L_t}{P_t}$, $m_t = \frac{M_t}{P_t}$, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross home inflation while $\Pi^*_t = \frac{P^*_t}{P^*_{t-1}}$ is gross foreign inflation. Moreover, distinctly in this open economy environment, $\varsigma_t$ is the real exchange rate, defined as $\varsigma_t \equiv S_t \frac{P^*_t}{P_t}$, that is, the price of the foreign good in terms of the home good.

Under complete markets, the two bond prices are determined by standard open economy asset-pricing conditions

$$Q_{Lt} = \beta E_t \left[ \zeta_{t,t+1} \Pi^{-1}_{t+1} \right], \ \varsigma_t Q_{B^*t} = \beta E_t \left[ \zeta_{t,t+1} \varsigma_{t+1} \Pi^{*-1}_{t+1} \right] \tag{10}$$

where $E_t$ is the mathematical conditional expectation operator and $\zeta_{t,t+1}$ is the unique stochastic discount factor. Finally, while this is not necessary, for simplicity of exposition, I will assume that purchasing power parity holds in the model, so that the real exchange rate is unity

$$\varsigma_t = S_t \frac{P^*_t}{P_t} = 1. \tag{11}$$

To get insights on how the currency mismatch between assets and liabilities can constrain the central bank, it is again useful to approximate the central bank budget constraint around a non-stochastic steady-state. The analysis in this case will again be very similar to the one in Sargent and Wallace (1981). Then, (9) log-linearized gives

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} + \frac{1}{\beta} s_t - \frac{1}{\beta} \hat{T}_t + b^* \left[ \hat{S}_t - \beta E_t \hat{S}_{t+1} \right] \tag{12}$$

where $b_{N,t} \equiv \hat{b}^*_t - \hat{l}_t - b^* \hat{S}_t$ is the net asset position of the central bank in terms of difference in the stock of assets and liabilities after appropriately accounting for currency differences, $s_t \equiv m[\hat{m}_t + \pi_t - \hat{m}_{t-1}]$ is revenue from issuing non-interest bearing liabilities or seigniorage, and $\beta$ is the discount factor.[27]

There are several aspects of (12) that require some discussion. First, while deriving it, (10) log-linearized has been imposed, which is a no-arbitrage condition that illustrates uncovered nominal interest rate parity

$$\hat{Q}_{B^*t} - \hat{Q}_{Lt} = E_t \left[ \hat{S}_{t+1} - \hat{S}_t \right] \tag{13}$$

[26] An increase in $S_t$ is therefore a depreciation of the home currency.

[27] In terms of approximation, I again log-linearize all variables, except for asset and liability positions and transfers to the Treasury, which are expressed in terms of deviations from steady-state. The steady-state is a zero inflation, unit real exchange rate steady-state where net position is zero.

and links the home bond price with the foreign bond price and expected change in the nominal exchange rate. Moreover, (11) log-linearized has also been imposed as $\pi_t - \pi_t^* = \hat{S}_t - \hat{S}_{t-1}$. These manipulations have the advantage of focusing attention directly on the nominal exchange rate, a key variable in an open economy, and which in fact, is at the source of balance sheet mismatch. Second, other than through the seigniorage term, home inflation does not appear directly, as purchasing power parity has been imposed. Third, as in the closed-economy model, in this approximated budget constraint only the net asset position, and not the separate gross asset and liability positions, appear.

Fourth, and perhaps more important, the dynamics of net asset of the central bank get affected by the net interest income on its position, $\hat{S}_t - \beta E_t \hat{S}_{t+1}$. Why is this the flow net interest income of the central bank? It is useful to go back to the original budget constraint (9) and for simplicity, impose purchasing power parity. On its home-currency liability, the central bank pays the home interest rate $\frac{1}{\Pi_t Q_{Lt}}$. On its foreign currency asset, the interest rate earned is $\frac{1}{\Pi_t^* Q_{Bt}^*}$. With a first-order approximation, the difference between the two, accounting for discounting in the definition of the net asset position, is given by $\hat{S}_t - \beta E_t \hat{S}_{t+1}$. Finally, the effect of the net interest income on the dynamics of net asset position depends on the steady-state gross position of the central bank balance sheet. In this sense, a large size of the balance sheet can magnify the effects of fluctuations in exchange rates.

Like before, to get implications similar to Sargent and Wallace (1981) while focusing on the currency mismatch, I will now simplify by assuming that seigniorage revenues are negligible. Then, (12) is given by

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} - \frac{1}{\beta} \hat{T}_t + b^* R_{N,t} \tag{14}$$

where $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$ is the one-period net interest income of the central bank described above. The flow budget constraint can then again be expressed in a present value form after imposing a terminal condition on net asset of the central bank as

$$b_{N,t-1} = \sum_{t=0}^{\infty} \beta^t \hat{T}_t - \beta \sum_{t=0}^{\infty} \beta^t b^* R_{N,t} \tag{15}$$

where I will take the initial net position $b_{N,t-1}$ as given. Now, using (15), consider how the any outstanding/initial net position of the central bank needs to be financed. In particular, if $b_{N,t-1} < 0$, then adjustment must come, in a present value sense, either from lower central bank transfers to the Treasury $\left(\hat{T}_t\right)$ or from higher net interest income of the central bank $(R_{N,t})$.

Even more importantly, consider a case similar to Fiscal dominance in Sargent and Wallace (1981) where now the present value of central bank transfers to the Treasury is fixed (or exogenous generally). This describes the *open economy balance sheet constrained central bank*. Then it is clear that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*. In particular, since the interest income depends on the nominal exchange rate, here directly considered as the policy instrument of the central bank, this means that the central bank's *exchange rate policy* gets constrained in this case. Thus for example, it cannot freely choose the path of the nominal exchange rate in future. This constitutes *unpleasant central bank balance sheet arithmetic* in an open economy with a currency mismatch in the central bank balance sheet.

Now consider again the case of $b_{N,t-1} < 0$ for the *open economy balance sheet constrained central bank*. In this case, as adjustment does not come from central bank transfers to the Treasury (for intuition, can think of there being no response of transfers at all), it must be the case that adjustment comes from higher

net interest incomes of the central bank. Additionally, this balance sheet arithmetic implies that if transfers to the Treasury increase (decrease) today with no future off-setting decreases (increases), then net interest income must increase (decrease) in future, in order to satisfy (15). Again, in both these cases, the nominal exchange rate, the effective policy instrument here, must adjust and respond appropriately to support the required adjustment in net interest income. How exactly does the policy instrument respond? In both cases, the nominal exchange rate must depreciate, thereby ensuring a direct fall in the return on (domestic currency) liabilities, which in turn in equilibrium leads to an increase in the net interest income.[28]

If the present value of central bank transfers to the Treasury is not fixed, and in particular, if it adjusts to the state of the balance sheet to satisfy (15), then that describes the *open economy balance sheet unconstrained central bank*. For such a central bank, as the path of net interest income, and thereby its policy instrument, is not constrained, it can freely pursue its objectives regarding the dynamics of the nominal exchange rate. On the flip side, this then implies that if transfers to the Treasury increase (decrease) today, they must decrease (increase) in future, in order to satisfy (15).

### 2.2.2 Policy rules and dynamic responses

To get further insights and to make the mechanisms described above even more concrete, like in the previous section, I now consider explicit monetary and balance sheet policy rules that can be used to model the two types of central banks. A complete equilibrium analysis of the transition dynamics associated with $b_{N,t-1} < 0$ can be undertaken in a straightforward way. Moreover, I introduce exogenous shocks to the nominal exchange rate and transfers to the Treasury in order to trace out the transmission of policy changes to model variables in a standard impulse response function analysis. This framework also allows a clear illustration of the basic logic discussed above regarding which term in (15) adjusts and which variable is set freely by the central bank.

**Policy rules for the two central banks** For the *open economy economy balance sheet unconstrained central bank*, as it is free to set the path of the nominal exchange rate and transfers to the Treasury can respond appropriately to the state of its balance sheet, the rules are given by

$$\hat{S}_t = \varepsilon_t, \ \hat{T}_t = \psi_T b_{N,t-1} + \xi_t \tag{16}$$

where $\psi_T > 1 - \beta$, which ensures that the central bank does not have to vary the nominal exchange rate to ensure that it satisfies (15). That is, in this case, transfers to the Treasury will co-move sufficiently positively with the net asset of the central bank. Additionally, $\varepsilon_t$ in this simple model is simply an *iid* shock and just illustrates that the central bank is free to set the path of the nominal exchange rate, while $\xi_t$ is an *iid* shock in the transfer rule.

For the *open economy balance sheet constrained central bank*, the path of its transfers to the Treasury is constrained (or fixed/exogenous) and as a result the net interest income will have to respond to the state of its balance sheet. This implies that it cannot freely set the path of the nominal exchange rate. The rules are then given by

$$R_{N,t} = \phi_R b_{N,t-1} + \varepsilon_t, \hat{T}_t = \xi_t \tag{17}$$

[28] I illustrate below using an equilibrium analysis the full dynamic path through which the nominal exchange rate responds to these two cases. There will be dynamics because of the state variable, the net asset position of the central bank, that matters for the evolution of the nominal exchange rate.

where $\phi_R < b^* \left(1 - \beta^{-1}\right)$, which ensures that (15) is satisfied in the face of exogenous transfers to the Treasury. That is, in this case, net interest income of the central bank will co-move sufficiently negatively with the net asset of the central bank.[29] Here, $\xi_t$ is an *iid* shock in the transfer rule, which just illustrates that the path of transfers is exogenous.[30]

**Transition dynamics and responses to shocks** How do the variables respond to the two shocks, to the nominal exchange rate/interest income and transfers to the Treasury? How does the transition dynamics associated with $b_{N,t-1} < 0$ look like?

**Open economy balance sheet unconstrained central bank** First, consider the *open economy balance sheet unconstrained* central bank. The model here simply consists of (14) and (16). In this case, the solution of the model is similar to conventional text-book models, at least for the nominal exchange rate shock and resulting dynamics. When there is an *iid* shock to the exchange rate rule, say a shock that appreciates the nominal exchange rate, then $\hat{S}_t$ will decrease on impact and then go back to steady-state next period. The central bank thus has complete control of the nominal exchange rate, as in conventional open economy models. Because of a temporary, one-period appreciation of the nominal exchange rate, the net interest income of the central bank decreases by the same amount on impact. This means that the net asset position of the central bank and after a period, transfers to the Treasury, also decrease. The balance sheet variables (and not exchange rate and net interest income) have endogenous dynamics as the variables evolve over time, as given clearly in (14). The result for the exchange rate shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (a) of Figure 6.[31]

Now I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, all the adjustment comes from transfers to the Treasury, which must decline. Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does transfers to the Treasury. The nominal exchange rate or net interest income of the central bank is not affected at all. The results for the transfer shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 6.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Following the present-value logic described above however, in order to satisfy (15), transfers will decrease in future. The net asset position of the central bank will decrease on impact, as transfers have increased, and then slowly transition back to steady-state. This shock does not affect nominal exchange rate or the net interest income of the central bank. Thus, other than in terms of implications for balance sheet variables, there is no effect of this shock on other variables in the model. The results for the transfer shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (c) of Figure 6.

**Open economy balance sheet constrained central bank** In contrast, next, consider the *open economy balance sheet constrained* central bank. The model here simply consists of (14) and (17). In this case, the solution of the model is quite different from the case above that was more in line with conventional open economy text-book models. When there is an *iid* shock to the net interest income, say a shock that decreases it, then on impact it will decrease. But, by the logic presented above, in this case, the net interest

[29] Note again that the net interest income is given by $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$, which is a quasi first-difference of the nominal exchange rate.

[30] These analytical boundaries for the two types of central bank regimes are again quite straightforward to derive, based on ruling out explosive dynamics of net assets of the central bank, as in the closed economy model.

[31] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b^* = 1$, and $\psi_T = 0.011$.

income in future must adjust as transfers to the Treasury do not. In particular, net interest income must increase in future in order to satisfy (15). Thus next period, it increases and then slowly transitions back to steady-state. What is the resulting equilibrium dynamics of the nominal exchange rate? The nominal exchange rate does not move initially, by construction, and then it depreciates, before transitioning back to steady-state with an appreciation. This is the equilibrium dynamics as the net interest income is essentially the first-difference of the nominal exchange rate. The result for the net interest income shock for the *open economy balance sheet constrained* central bank is illustrated in panel (a) of Figure 7.[32] The dynamics of the nominal exchange rate in this case are thus clearly determined by central bank balance sheet considerations and are in sharp contrast to those in panel (a) of Figure 6.

Now, I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, again, all the adjustment comes from net interest income of the central bank and none from central bank transfers to the Treasury. Thus, net interest income must increase to satisfy (15). Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does net interest income of the central bank. The nominal exchange rate rate is now clearly affected as it has to be consistent with the dynamics of the net interest income. Specifically, the nominal exchange rate must depreciate initially, so that there is a direct reduction in the return on (domestic currency) liabilities, and then it reverts back to steady-state, appreciating along the transition (as is consistent with the net interest income, which is essentially the slope of this transition path, being positive). The results for the transition dynamics for the *open economy balance sheet constrained* central bank is illustrated in panel (b) of Figure 7 and they are in sharp contrast to those in panel (b) of Figure 6.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Since this is a one-time shock, transfers go back to steady-state next period. In sharp contrast to the *open economy balance sheet unconstrained* central bank, now exchange rate dynamics get affected as the net interest income of the central bank must respond in equilibrium. In particular, here, the net interest income must increase in a present value sense. Initially however, because of the rule in (17), as initial outstanding net asset position has not changed (making this different in this respect from the transition dynamics experiment above), net interest income does not change either. In future however, net asset decreases, and consistently, net interest income will increase. The equilibrium nominal exchange rate path implies that first it depreciates, again, to ensure a direct fall in the interest paid on (domestic currency) liabilities. It also depreciates further next period, with an overshooting, before appreciating and transitioning back to steady-state.[33] Why does the nominal exchange rate depreciate further for one period? The key reason is that the net interest income in the first-period does not change. For this to be consistent with nominal exchange rate dynamics, it must be that the next period interest rate is slightly more negative as $\beta < 1$ (note that $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$). The results for the transfer shock for the *open economy balance sheet constrained* central bank is illustrated in panel (c) of Figure 7. These results are in sharp contrast to those in panel (c) of Figure 6, where the shock to the transfer rule for instance did not affect the nominal exchange rate or the net interest income of the central bank at all.

[32] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b^* = 1$, and $\phi_R = -0.011$.

[33] Note that the nominal exchange rate cannot appreciate on impact. There will be no stationary dynamics in that case.

# 3 Non-linear General Equilibrium Model

I now present a fully specified non-linear general equilibrium closed economy model to illustrate the mechanisms highlighted above using the simple model.[34] In this set-up I use standard interest rate policy rule specifications and will evaluate inflation dynamics in equilibrium. Moreover, I specify clearly both the central bank and Treasury balance sheet policies and budget constraints. Relatedly, as the model is non-linear, the central bank balance sheet rules that I specify will need to make clear how gross, and not just net, positions are determined. The non-linear model also helps make the point that the results in the previous section are not specific to a first-order approximated model, and in particular, one where the central bank budget constrained is linearized and the asset pricing conditions imply the expectation hypothesis.[35]

## 3.1 Private sector

I first describe the environment and the maximization problems faced by households and firms.

### 3.1.1 Households

A representative household maximizes expected discounted utility over the infinite horizon

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right) \tag{18}$$

where $0 < \beta < 1$ is the discount factor, $c_t$ is household consumption of the composite good, and $h_t$ is hours supplied by the household. $E_0$ is the mathematical expectation operator conditional on period-0 information and $U\left(c_t, h_t\right)$, the period utility, is increasing in $c_t$ and decreasing in $h_t$. The composite consumption good $c_t$ is an aggregate of a continuum of differentiated varieties indexed by $i \in [0,1]$. The consumption good is defined as $c_t = \left[\int_0^1 c_t(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$, where $\nu > 1$ is the elasticity of substitution among the varieties.

Before presenting the flow budget constraint, it is useful to discuss the static expenditure minimization problem faced by households. Let $P_t$ be the nominal price of the aggregate consumption good and $P_t(i)$ the nominal price of the varieties. Then, formally, the household chooses $\{c_t(i)\}_{t=0}^{\infty}$ to minimize

$$\int_0^1 c_t(i) P_t(i) di$$

subject to

$$\left[\int_0^1 c_t(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \geq c_t \tag{19}$$

while taking as exogenously given $\{P_t(i)\}_{t=0}^{\infty}$. Then, the shadow price on (19) is equal to $P_t$, the nominal price of the home consumption good.[36] The demand function for the variety of goods takes the standard downward-sloping form and is obtained from the optimality condition of the problem above.

The household's flow budget constraint (in real terms) is then given by

[34] In the interest of space, I do not separately present an open economy model. The insights from the simple open economy model however will go through in a fully specified general equilibrium model. The small open economy non-linear monetary model in Bhattarai and Egorov (2016) for instance can be used to show those results.

[35] The baseline results I present are based on a fully non-linear deterministic solution method. I also present results in an extension based on a stochastic, third-order approximation solution method.

[36] $P_t$ is thus the minimum-expenditure price index.

$$l_{t-1}^H \Pi_t^{-1} - Q_{1t} b_{t-1}^H \Pi_t^{-1} = Q_{1t} l_t^H - Q_{2t} b_t^H + c_t - w_t h_t - \varphi_t + \tau_t \tag{20}$$

where $b_t^H = \frac{B_t^H}{P_t}$ with $B_t^H$ the nominal two-period debt of the household and $l_t^H = \frac{L_t^H}{P_t}$ with $L_t^H$ the nominal one-period asset held by the household. Here, $\Pi_t = \frac{P_{t-1}}{P_t}$ is gross inflation, $Q_{1t}$ is the price of the one-period asset, and $Q_{2t}$ is the price of the two-period debt. Moreover, $w_t$ is real wages, $\varphi_t$ is profits from firms, and $\tau_t$ is net lump-sum taxes from the Treasury.[37]

Given the first-stage, static expenditure minimization problem discussed above, the problem of the home household then is to choose $\{c_t,\, h_t,\, l_t^H,\, b_t^H\}_{t=0}^{\infty}$ to maximize (18) subject to a sequence of budget constraints (20), while taking as exogenously given initial wealth and $\{\Pi_t,\, \varphi_t,\, w_t,\, Q_{1t},\, Q_{2t},\, \tau_t\}_{t=0}^{\infty}$.

### 3.1.2 Firms

There is a continuum of monopolistically competitive firms that produce differentiated varieties. The firms are of measure 1 and indexed by $i \in [0,1]$. Firm $i$ produces output $y_t$ using labor as input

$$y_t(i) = F(h_t(i)) \tag{21}$$

where the production function $F(h_t(i))$ is increasing in $h_t\,(i)$. Firms hire labor in a common, perfectly competitive factor market.

Firm $i$ sets prices $P_t(i)$ for its good. I introduce nominal rigidities following Rotemberg (1982). Thus, firms face a cost of adjusting prices given by $d\,(P_t(i)/P_{t-1}(i))$ where $d\,(.)$ is a convex function. Moreover, the demand function for variety $i$ is derived from the cost-minimization problem of the household over differentiated varieties discussed above and given by

$$\frac{y_t(i)}{y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\nu} \tag{22}$$

where $y_t$ is aggregate demand that is taken as given by the firms.

Firms maximize expected discounted profits over the infinite horizon

$$E_0 \sum_{t=0}^{\infty} \zeta_{0,t} \varphi_t\,(i) \tag{23}$$

where they use the stochastic discount factor of the household $\zeta_{0,t} = \beta^t \frac{U_c(c_t,h_t)}{U_c(c_0,h_0)}$ to discount future profits and where (real) flow profits $\varphi_t(i)$ are given by

$$\varphi_t(i) = F(h(i)) - d\left(\frac{P_t(i)}{P_{t-1}(i)}\right) - w_t h_t(i).$$

The problem of firm $i$ is then to choose $\{h_t(i),\, P_t(i)\}_{t=0}^{\infty}$ to maximize (23) subject to a sequence of production functions (21) and demand functions (22), while taking as exogenously given $\{P_t,\, y_t,\, \zeta_{0,t},\, w_t\}_{t=0}^{\infty}$. As is standard, I will focus on a symmetric equilibrium where all firms choose the same price and produce the same amount of output.

[37] Note that the flow budget constraint is written in terms of real values, where the deflator is the common price level of the aggregate consumption good. Also, while I make this clear later, I use debt and asset holdings of the household in terms of terminology to pre-empt the market clearing conditions on the two-types of assets and the assumptions on monetary and fiscal policy. The household is subject to a no-Ponzi game condition.

## 3.2 Government

I now describe how monetary and fiscal policy is determined and what are the balance sheets of the central bank and the Treasury. This is a key aspect of the analysis. In a similar manner to the simple model in the previous section, I also define under what policy rule configurations the central bank is balance sheet constrained.

### 3.2.1 Monetary policy

The central bank faces the flow budget constraint written in real terms as

$$Q_{1t}b_{t-1}\Pi_t^{-1} - l_{t-1}\Pi_t^{-1} - T_t + K = Q_{2t}b_t - Q_{1t}l_t \tag{24}$$

where $b_t = \frac{B_t}{P_t}$ with $B_t$ the nominal two-period asset of the central bank and $l_t = \frac{L_t}{P_t}$ with $L_t$ the nominal one-period liability of the central bank. Moreover, $T_t$ is the central bank transfer to the Treasury and $K$ its any other revenue net of operations cost (constant for simplicity). The budget constraint takes the same form as in the simple model of the previous section.[38]

Central bank's short-term interest rate policy is determined according to a standard and simple feedback rule

$$\beta Q_{1t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(\varepsilon_{It}) \tag{25}$$

where $\phi_\pi \geq 0$ is the feedback parameter, $\Pi$ is the steady-state value of gross inflation, and $\varepsilon_{It}$ is an interest rate shock that follow a stationary process.

Next, I have to specify the central bank's policy that determines the evolution of its balance sheet. Unlike in the simple linearized model, it is not enough to posit a rule that only determines the evolution of the net asset position of the central bank. Here, I will need to model separate policy rules for both asset and liability positions (that is, we also need to determine the gross positions, in order to complete the model). I assume that the central bank uses the following rules for determining transfers to the Treasury

$$T_t - T = \psi_{bT}\beta\left(b_{t-1} - b\right), T_t - T = -\psi_{lT}\left(l_{t-1} - l\right) \tag{26}$$

where $\psi_{bT} \geq 0$, $\psi_{lT} \geq 0$ are the feedback parameters, $b$ is the steady-state value of central-bank assets, $l$ is the steady-state value of central-bank liabilities, and $T$ is the steady-state value of transfers.[39] Notice that generally, I need not restrict the feedback parameters to be the same. But in that specific case, when $\psi_{bT} = \psi_{lT} = \psi$, there is a natural interpretation in terms of the response of the central bank transfers to the Treasury as being determined by the net asset position

$$T_t - T = \frac{\psi}{2}\left[\beta\left(b_{t-1} - b\right) - \left(l_{t-1} - l\right)\right]$$

just as in the simple linearized model of the previous section. In this case, the gross positions will then always effectively be in steady-state.[40]

[38] In this model since I directly abstract from cash, I interpret this revenue as coming from seigniorage, which net of operations cost, I assume to be constant.

[39] Like in the simple model, I thus use a feedback rule for transfers to the Treasury. Again, another alternative would be use some lower bound for such transfers. My approach leads to a more straightforward analysis.

[40] For conciseness, in this general equilibrium model, I only study the implications of an interest rate shock. It is however, straightforward to also introduce shocks in the central bank balance sheet, as in the simple model, here as well.

### 3.2.2 Fiscal policy

I keep fiscal policy deliberately very simple to focus the analysis on monetary policy. In particular, I assume that the Treasury follows essentially a balanced budget policy. For simplicity, there is no government spending, and the Treasury turns over all remittances from the central bank to the household. Moreover, for simplicity, it pursues a zero holdings policy of the two nominal bonds. Thus, formally

$$T_t = \tau_t \tag{27}$$

and

$$L_t^T = 0,\ B_t^T = 0 \tag{28}$$

where $L_t^T$ is the Treasury holding of central bank issued one-period nominal liabilities and $B_t^T$ is the Treasury's supply of two-period nominal bonds. Note that the assumption of zero holdings from the Treasury is for simplification and generally, it can be any fixed amount.[41]

### 3.2.3 Central bank balance sheet constraints

Given the description of the balance sheet and policy rules of the central banks and Treasury, I now define, in a manner similar to the simple model, a balance sheet unconstrained and a balance sheet constrained central bank. Generally, a balance sheet unconstrained central bank is free to adjust interest rates to focus on inflation stabilization without being constrained by balance sheet considerations and in particular, the path of transfers to the Treasury. A stable dynamics of its assets and liability positions is thus assured through appropriate changes in the path of its transfers to the Treasury.

A balance sheet constrained central bank has to adjust its interest rate path, in equilibrium, as the path of its transfers to the Treasury is constrained. Only doing so will ensure that the dynamics of its asset and liability positions are stable/stationary. Again, a balance sheet constrained central bank in my model is similar in spirit to the central bank under fiscal dominance in the Sargent and Wallace (1981) model as its path of remittances to the Treasury is constrained. One can thus interpret the constraint on the central bank, in terms of its remittances to the Treasury, as coming from fiscal dominance.[42]

Specifically, here, with the policy rules I use, a balance sheet unconstrained central bank will follow the Taylor principle in setting interest rate policy ($\phi_\pi > 1$) and whose path of transfers to the Treasury respond strongly enough to asset and liability positions (high enough $\psi_{bT}$ and $\psi_{lT}$) such that a stable path of asset and liability dynamics is assured. With the policy rules I use, a balance sheet unconstrained central bank in contrast will not follow the Taylor principle in setting interest rate policy ($\phi_\pi < 1$). Moreover, its path of transfers to the Treasury do not respond strongly enough to asset and liability positions (low $\psi_{bT}$ and $\psi_{lT}$) such that a non-explosive path of asset and liability dynamics is not assured through adjustment in the transfers automatically. These are the two policy parameter configurations that will lead to a unique stationary/stable equilibrium in the model.[43]

[41] I avoid more completely endogenizing the spending and debt policies of the Treasury in order to keep the focus of the paper sharply on the balance sheet considerations of the central bank in this general equilibrium model. My goal here is to keep the model relatively simple to bring out the new results as transparently as possible.

[42] In fact, given the simple model of fiscal policy I use, where there are no other sources of taxes and the Treasury has zero debt/asset holdings, if one consolidates the Treasury and central bank budget constraints into one, the interpenetration will be exactly the same as in Sargent and Wallace (1981).

[43] Del Negro and Sims (2015) study the possibility of self-fulfilling multiple equilibria due to central bank solvency concerns. I abstract from solutions that feature explosive inflation as well. Moreover, these policy parameter configurations are based on the convention and insights from models that feature the fiscal theory of the price level such as Sims (1994). I note however

### 3.3 Market clearing and exogenous processes

The goods, factor, and bonds markets clear in equilibrium.[44] In particular, the social resource constraint, at the variety level, is given by

$$y_t(i) = c_t(i) + d\left(\frac{P_t(i)}{P_{t-1}(i)}\right)$$

where I incorporate the resource cost of adjusting prices. Given the assumptions on Treasury's debt policy, the bond market clearing conditions are then given by

$$b_t - b_t^H = b_t^T = 0,\ l_t^H - l_t = l_t^T = 0. \tag{29}$$

Finally, I assume that the interest rate shock $\varepsilon_{It}$ follows an AR(1) process

$$\varepsilon_{It} = \rho\varepsilon_{It-1} + \exp(\sigma)\epsilon_t \tag{30}$$

where the innovation $\epsilon_t \sim N(0,1)$, $\rho \in [0,1)$, and $\exp(\sigma) > 0$.

### 3.4 Equilibrium

I now define the equilibrium of the model and discuss the policy rules and aggregate optimality and feasibility conditions that characterize it.

#### 3.4.1 Definition

An equilibrium is a collection of allocations (of goods varieties and aggregates) for the household, $\{c_t(i),\ c_t,\ h_t,\ l_t^H,\ b_t^H\}_{t=0}^{\infty}$, allocations and goods prices for the firms $\{h(i),\ p_{H,t}(i)\}_{t=0}^{\infty}$, a sequence of aggregate prices $\{\Pi_t,\ Q_{2t},\ w_t,\ \rho_{0,t}\}_{t=0}^{\infty}$ and output $\{y_t\}_{t=0}^{\infty}$, and monetary and fiscal policy instruments $\{Q_{1t}^{-1}, b_t,\ l_t,\ T_t,\ \tau_t,\ l_t^T,\ b_t^T\}_{t=0}^{\infty}$ such that

(i) Given prices and monetary and fiscal policy, the allocations are such that they satisfy the maximization problems of the household,

(ii) Given aggregate prices, aggregate output, and monetary and fiscal policy, the goods prices and allocations are such that they satisfy the maximization problem of the firms,

(iii) The allocations and goods prices across firms are symmetric,

(iv) The nominal interest rate is determined by the interest rate policy rule and central bank assets and liabilities by the central bank balance sheet policy rule,

(v) Household taxes are determined by the Treasury tax rule and Treasury debt and asset holdings by the Treasury debt policy rule, and

(vi) Goods, factor, and bonds markets clear,

given the initial household and central bank balance sheet positions and an exogenous process for $\{\varepsilon_{It}\}_{t=0}^{\infty}$. As I discussed above while defining a balance sheet constrained and unconstrained central bank, I will focus on two types of stationary equilibria, depending on the configurations of the central bank policy parameters.

that an exogenous process, such as a peg, for the nominal interest rate will not lead to a determinate equilibrium in my model, unlike the fiscal theory of the price level.

[44] The notation on hours/labor already imposes that factor markets clear in equilibrium.

### 3.4.2 Equilibrium conditions

The aggregate non-linear equilibrium conditions of the model are fairly standard. To simplify even further, I assume log-linear specifications for preferences and technology

$$U(c_t, h_t) = \log C_t - Ah_t,\ y_t = F(h_t) = h_t$$

and a standard quadratic price-adjustment cost function

$$d(\Pi_t) = \frac{\kappa}{2}(\Pi_t - 1)^2.$$

Given these specifications, the aggregate private sector equilibrium conditions are given by

$$Q_{1,t} = \beta E_t\left[\frac{C_t}{C_{t+1}}\Pi_{t+1}^{-1}\right],\ Q_{2,t} = \beta E_t\left[\frac{C_t}{C_{t+1}}Q_{1,t+1}\Pi_{t+1}^{-1}\right] \tag{31}$$

$$\left[\left(\frac{\nu-1}{\nu}\right) - AC_t\right]\nu Y_t + \kappa(\Pi_t - 1)\Pi_t = \beta E_t\left[\frac{C_t}{C_{t+1}}\kappa(\Pi_{t+1} - 1)\Pi_{t+1}\right] \tag{32}$$

$$C_t + \frac{\kappa}{2}(\Pi_t - 1)^2 = Y_t \tag{33}$$

where (31) are the two asset pricing conditions, (32) is the Phillips curve, and (33) is the resource constraint.[45] Then, the policy block is given by the central bank budget constraints and the rules for setting monetary policy and balance sheet variables

$$Q_{1t}b_{t-1}\Pi_t^{-1} - l_{t-1}\Pi_t^{-1} - T_t + K = Q_{2t}b_t - Q_{1t}l_t$$

$$\beta Q_{1,t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\exp(\varepsilon_{It})$$

$$T_t - T = \psi_{bT}\beta(b_{t-1} - b),\ T_t - T = -\psi_{lT}(l_{t-1} - l).$$

Note that the above eight equations determine the dynamics of the eight endogenous variables $\{Q_{1,t}, Q_{2,t}, C_t, Y_t, \Pi_t, T_t, b_t, l_t\}$ given initial conditions, exogenous processes, and appropriate restrictions on the policy parameters that ensure stationary dynamics of the endogenous variables above.[46]

In addition to these variables, for comparison with the simple linear model presented in the last section, I will define the net interest income $R_{N,t}$ and net asset position $b_{N,t}$ of the central bank in a similar manner in this non-linear model as well

$$R_{N,t} \equiv \frac{Q_{1t}}{Q_{2t}} - \frac{1}{Q_{1t}}$$

$$b_{N,t} \equiv [\beta(b_{t-1} - b) - (l_{t-1} - l)].$$

While in this non-linear model, these variables are not necessarily always sufficient statistics to ascertain the behavior of the balance sheet constrained central bank, I will show in the next section that they continue to

[45] Note that the specific functional forms for preferences and technology are useful to reduce the number of parameters in the model, but the private sector equilibrium conditions can still be reduced to such three conditions generally, even without the assumptions. The detailed derivations of all the optimality conditions in the model are very standard and are omitted for brevity.

[46] Note that we do not need to keep track of either the household budget constraint (due to Walras law) or the Treasury policy rules separately as well (as they have essentially already been imposed above in terms of market clearing conditions).

be very helpful in interpreting the results.[47]

## 3.5 Results

I solve the model non-linearly, where in period 0, an unexpected shock to the interest rate, $\varepsilon_{It}$, hits the economy, and then the economy evolves deterministically thereafter. In the initial period, the economy is assumed to be in the non stochastic steady-state.[48] Then, I solve for a stationary equilibrium such that after the shock, in the long-run, variables transition back to the same non-stochastic steady-state.[49] As I noted above, I will consider two different types of central bank regimes under which such stationary equilibria exist.

### 3.5.1 Parameterization

As the main objective of the model simulation results is to show qualitative properties, I use parameter values that are common in the literature and similar to the simple model. The parameter values used in the numerical analysis are presented in Table 1. The assumption on preferences and technology described above already imposes several parameter values. Like the simple model, I then use $\beta = 0.99$ and $b = 1$. For the steady-state central bank transfers to the Treasury, I use a value of 0.1. For the price stickiness parameter $\kappa$ I use the value of 50 used in Ireland (2000) and a standard value of 7 for the elasticity of substitution across varieties of goods.

In terms of the policy parameters, for the baseline results here, I impose the same values on the two balance sheet feedback policy parameters and an *iid* process for the interest rate policy shock. The transitory nature of the shock is very useful to illustrate clearly when central bank balance sheet variables are endogenous state variables for inflation. For the two types of central banks, I consider feedback parameters that are useful to illustrate the differences between the two cases. For the balance sheet unconstrained central bank, I use a standard Taylor principle value of the interest rate feedback parameter ($\phi_\pi$=1.5) and a sufficiently large value for the transfer feedback parameter ($\psi_T = 0.02$) while for the balance sheet unconstrained central bank, I use a value for the interest rate feedback parameter that is lower than 1 ($\phi_\pi$=0.5) combined with a low value for the transfer feedback parameter ($\psi_T = 0.009$).

### 3.5.2 Balance sheet unconstrained central bank

Equilibrium responses of variables to a positive interest rate shock (of 1%) for the balance sheet unconstrained central bank are shown in Figure 8.[50] The responses of the short-term interest rate, inflation, and output are standard and follow textbook analysis. With a rise in the short-term interest rate, and with the interest rate rule satisfying the Taylor principle, inflation decreases. Moreover, in this model with nominal rigidities, the fall in inflation leads to a fall in output as well. Since the shock is transitory and the private sector equilibrium conditions are all purely forward looking, all three of these variables go back to steady state next period. The balance sheet variables do not affect the dynamics of these standard macro variables at all, as

[47] These variables will be sufficient as long as a linear approximation is a good enough approximation, which is the case here with small shocks.

[48] The non-stochastic steady-state is a standard one where I assume zero net inflation and equal asset and liability positions of the central bank (such that Transfers to the Treasury are equal to the revenue net of operations cost of the central bank). Moreover, I normalize steady-state output to be 1 by picking a value for $A$ appropriately.

[49] I use a non-linear solver to compute this perfect foresight solution. In particular, I use the solver csolve written by Chris Sims. In an extension, I consider a stochastic solution based on a third-order approximation.

[50] The responses are shown in terms of deviations from the non-stochastic steady-state.

in the simple model.[51]

For the balance sheet variables, a transitory increase in the short-term nominal interest rate leads to a decline in the net interest income by essentially the same amount. As a result, net asset position falls next period. Since this is the balance sheet unconstrained model, transfers to the Treasury adjust by enough to ensure that in the long-run, balance sheet variables revert back to steady-state. Here, as in the simple model, transfers to the Treasury decline as net asset position of the central bank deteriorates.

### 3.5.3 Balance sheet constrained central bank

Equilibrium responses of variables to a positive interest rate shock (of 1%) for the balance sheet constrained central bank are shown in Figure 9. The responses of the short-term interest rate, inflation, and output are now very different from the standard case and are determined by balance sheet considerations of the central bank.[52] First, note that on impact, the short-term interest rate increases. Moreover, like for the unconstrained central bank case, net interest income of the central bank decreases on impact, as is to be expected given an increase on interest paid on liabilities. This however now has important consequences for the determination of future short-term interest rate. Because the central bank here is balance sheet constrained, while transfers to Treasury do decrease as the net asset position of the central bank falls, they do not decrease by enough to ensure stationary dynamics.[53] Thus, net interest income will have to increase in future. This is exactly what we see in Figure 9 and in fact the dynamics is qualitatively the same as in the simple model. Thus, the intuition can be obtained again from a present value analysis of the central bank budget constraint.

What are the implications of this equilibrium dynamics of net interest income for the short-term interest rate? For interest rate dynamics to be consistent with the net interest income behavior, it must be that in future, the short-term interest rate falls, and then it transitions back to steady-state, rising along the transition.[54] Now I can also assess the implications for inflation and output as they are both endogenously determined in this model and depend on monetary policy in a non-trivial way. Because of the positive relationship between the short-term interest rate and inflation implied by the monetary policy rule (25), inflation then follows the same path as the short-term interest rate.[55] Moreover, on impact as well, inflation must fall, because of forward looking dynamics implied by the Phillips curve in the model. Finally, given nominal rigidities, output also falls along the entire path, basically following the path of inflation. These persistent dynamics of interest rates, inflation, and output arise, in spite of a transitory shock, as now central bank balance sheet variables are endogenous state variables in the model.

There are thus critical differences between the balance sheet unconstrained and constrained central bank in terms of the central bank's control of the monetary policy instrument. Unlike the balance sheet unconstrained central bank, the constrained one cannot freely set the path of the its policy instrument. In fact, here, while a positive interest rate shock today increases the short-term interest rate on impact, starting

[51] That is, the model is block-recursive, and balance sheet variables are not state variables for inflation, interest rate, and output.

[52] Thus, the model is no longer block recursive and the central bank balance sheet variables are state variables for inflation, interest rate, and output.

[53] One way to make this even starker would have been, like in the simple model, to make transfers to Treasury completely unresponsive to the state of the central bank balance sheet. I take a more reasonable modeling of the central bank policy here.

[54] Because the short-term interest rate needs to follow this path as a function of the state variables, in my model, unlike in the fiscal theory of the price level, a purely exogenous path for the interest rate does not lead to a determinate equilibrium. This is quite straightforward to prove in the simple linear model.

[55] Of course, the coefficient on the policy rule is positive even for the balance sheet unconstrained central bank. In that case however, the coefficient is greater than 1. Price level determination then happens through the Taylor Principle channel, which leads to inflation falling and interest rates rising when a positive interest rate shock hits the economy.

with next period, the short-term interest rate falls, and then slowly rises back to steady-state. This path is the only one consistent with the required equilibrium path of net interest income of the central bank. This is thus the clearest illustration of unpleasant central bank balance sheet arithmetic in this fully general equilibrium model.

Finally, in this case, dynamically, the short-term interest rate and inflation are positively correlated. This is in sharp contrast to the standard case, where they are negatively correlated. Thus, in this respect the model has so called Neo Fisherian properties. At the same time however, a positive interest rate shock still leads to an increase in the short-term interest rate on impact, and more importantly, to a consistent decline in inflation. Thus, it does not follow from this model that a positive short-term interest rate shock actually leads to higher inflation.

### 3.5.4 Price stickiness comparative statics

I now consider a comparative static experiment to further highlight the differences between the balance sheet unconstrained and constrained central bank cases in this general equilibrium model. Additionally, this exercise also serves to highlight the robustness of the results described above for various parameter values, especially for the balance sheet constrained central bank.

In particular, I consider different degrees of price stickiness, that is various values for the price adjustment cost parameter. For the unconstrained central bank, as shown in Figure 10, and as is to be expected given the Taylor rule used by the central bank, higher price stickiness (that is, a greater value of $\kappa$) leads to a smaller equilibrium response of inflation when an interest rate shock hits the economy. This also implies a larger response of the short-term interest rate in equilibrium. For a positive interest rate shock, inflation drops by less when prices are more sticky.

In sharp contrast, for the constrained central bank, as shown in Figure 11, higher price stickiness leads to a larger equilibrium response of inflation when an interest rate shock hits the economy. What drives this result? When prices are more sticky, in response to the interest rate policy shock, the short-term interest rate rises by more on impact, as is the case for the unconstrained central bank case. Thus, just on impact, inflation drops less for more sticky prices. But following the impact period, the short-term interest rate falls more, throughout the rest of the transition back to steady-state. Since short-term interest rate and inflation are positively correlated along the transition in this model, inflation also responds more strongly with greater degree of price stickiness. This stronger response of the inflation when prices are more sticky is a unique feature of my model, one that is driven by the restriction imposed on future interest rate policy due to central bank balance sheet considerations. For instance, in the fiscal theory of the price level model, when prices are more sticky, inflation responds by less (not that a positive interest rate shock in that model raises inflation).[56]

[56] Additionally, here output responds more negatively when prices are more sticky for both cases. The model and mechanism is thus completely different from the standard sticky price model in Bhattarai, Eggertsson, and Schoenle (2014), where output and inflation both can respond less negatively when prices are more sticky, even with the Taylor principle holding. This happens in the standard sticky price model if the weight on output in the Taylor rule is high enough to satisfy the Taylor principle, while the weight on inflation is less than the persistence of the monetary policy shock, and thus necessarily less than one. Bhattarai, Eggertsson, and Schoenle (2014) show that in such a case, the aggregate demand curve becomes upward sloping, thereby leading to a smaller response of both output and inflation for a monetary policy shock when the aggregate supply curve becomes steeper. Here, in the Taylor rule, there is only a response to inflation in both the balance sheet constrained and unconstrained cases and the Taylor principle does not hold in the balance sheet constrained central bank case.

### 3.6 Robustness and Extensions

While the purpose of the numerical results above is to highlight the qualitative aspects of the model, it is nevertheless important to assess the robustness of the results to various parameter values. I consider now several values for the policy feedback parameters, both in the interest rate rule and transfer to Treasury rule. Since the balance sheet unconstrained central bank model is well understood, for concreteness, I focus on the balance sheet constrained case. Figures 12 and 13 show the results for various values of the two policy feedback parameters for the balance sheet constrained central bank, where I include the baseline parameterization for straightforward comparison. It is clear that the shape of the responses, including critically, the dynamics of the net interest income of the central bank, are robust.

I next consider two extensions to the baseline analysis where I focus again on showing results for the balance sheet constrained central bank. First, I allow the two balance sheet feedback rule parameters to be different. Thus, I now use $\psi_{bT}$ =0.009 and $\psi_{lT}$=0.005, whereas in the baseline they were both equal to 0.009. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different model and calibration are shown in Figure 14. It is clear that the results are robust to this extension.

Second, I use a different non-linear solution method, in particular, a stochastic third-order perturbation solution method. Compared to the baseline non-linear deterministic solution method, this method in principle incorporates stochastic aspects such as a term premium. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different solution method are shown in Figure 15. It is again clear that the results are robust to this alternate solution method.

## 4 Conclusion

As a result of unconventional monetary policy, maturity and/or currency mismatches have appeared in many central bank balance sheets. I analyze the implications of such developments in the central bank balance sheet in an equilibrium model. In the model, the central bank holds long-term domestic or short-term foreign currency assets while it issues short-term domestic currency liabilities. Following Sargent and Wallace (1981), I consider a *balance sheet constrained central bank*, one which faces a constraint in the path of its remittances to the Treasury. Overall, my results suggest that whether a central bank is affected by balance sheet constraints or not can drastically affect the equilibrium dynamics and comovement of interest rates, exchange rates, and inflation.

In particular, I show using simple balance sheet arithmetic based on a present value analysis that such a central bank loses freedom in its policy actions. Because of the constraint on the path of its remittances to the Treasury, the net interest income of the central bank has to adjust in a present value sense. In particular, *interest rate policy* or *exchange rate policy* get constrained by balance sheet considerations. If the expected future change of the short-term nominal interest rate or the nominal exchange rate increase today, they have to decrease in future. This constitutes *unpleasant central bank balance sheet arithmetic*.

I embed such a central bank in a non-linear dynamic general equilibrium model and consider equilibrium dynamics following a transitory positive shock to the interest rate rule of the central bank. In sharp contrast to the standard case, the model with balance sheet constrained central bank leads to very different dynamics of the short-term interest rates and its correlation with inflation. On impact, the short-term nominal interest rate rises and inflation drops, as is the case of the balance sheet unconstrained central bank. The net interest income falls as well, because of a direct increase in the interest paid on the central bank liabilities. After the

first period and dynamically however, the short-term nominal interest rate falls. This is essential because of balance sheet considerations that require the net interest income to rise in future periods. This means that after the initial period, the short-term interest rate and inflation are positively correlated, even though the model has sticky prices. Moreover, when prices are stickier, inflation drops more.

In future work, it will be interesting to fully assess the quantitative significance of these results, including when the zero lower bound can be a concern. Moreover, since the equilibrium dynamics of inflation, interest rates, and exchange rates can be very different when the central bank is balance sheet constrained compared to when it is not, it might be worthwhile to empirically assess whether some historically puzzling behavior of these variables, either for advanced or emerging market countries, can be explained in terms of the model presented in this paper.

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# 5 Tables and figures

Table 1: Parameterization of the model

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $b$ | 1 | $\phi_\pi$ | 1.5; 0.5 |
| $\kappa$ | 50 | $\psi_T$ | 0.02; 0.009 |
| $T$ | 0.1 | $\nu$ | 7 |
| $\beta$ | 0.99 | $\rho$ | 0 |

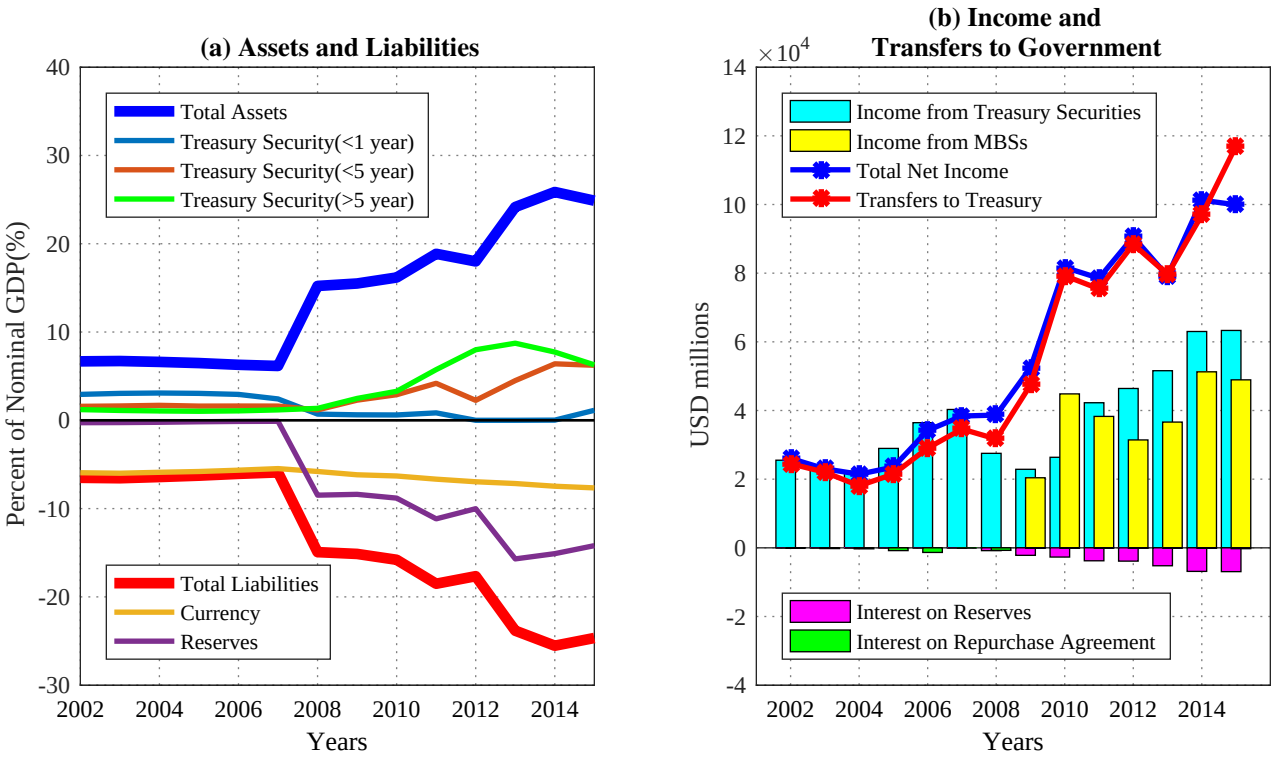



Figure 1: Assets, Liabilities, and Income of the Federal Reserve

*Notes*: Source of data is Federal Reserve Board. For assets, other than Treasuries, the rest of holdings are primarily Agency debt securities and MBSs, which are also long-term.

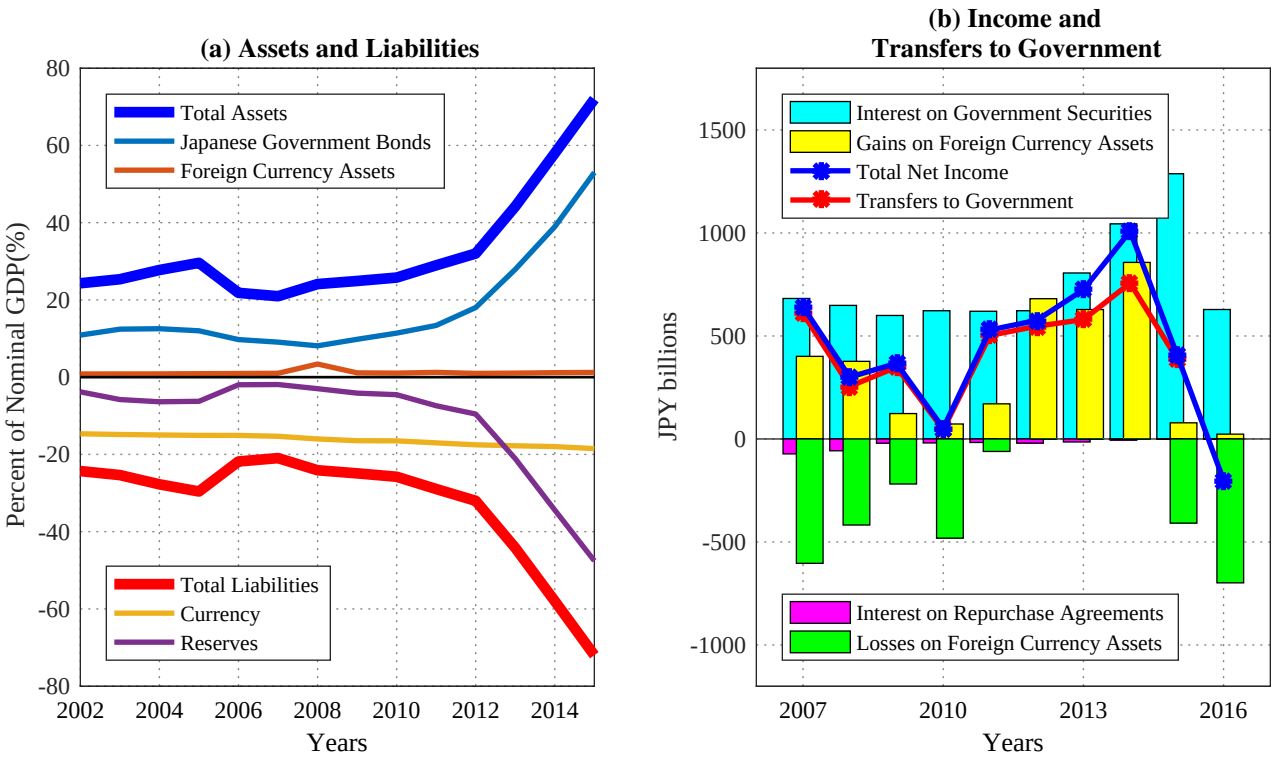



Figure 2: Assets, Liabilities, and Income of the Bank of Japan

*Notes*: Source of data is Bank of Japan. For assets, the other two important categories are Treasury discount bills and loans. The total net income measure is the sum of net operational and special income.

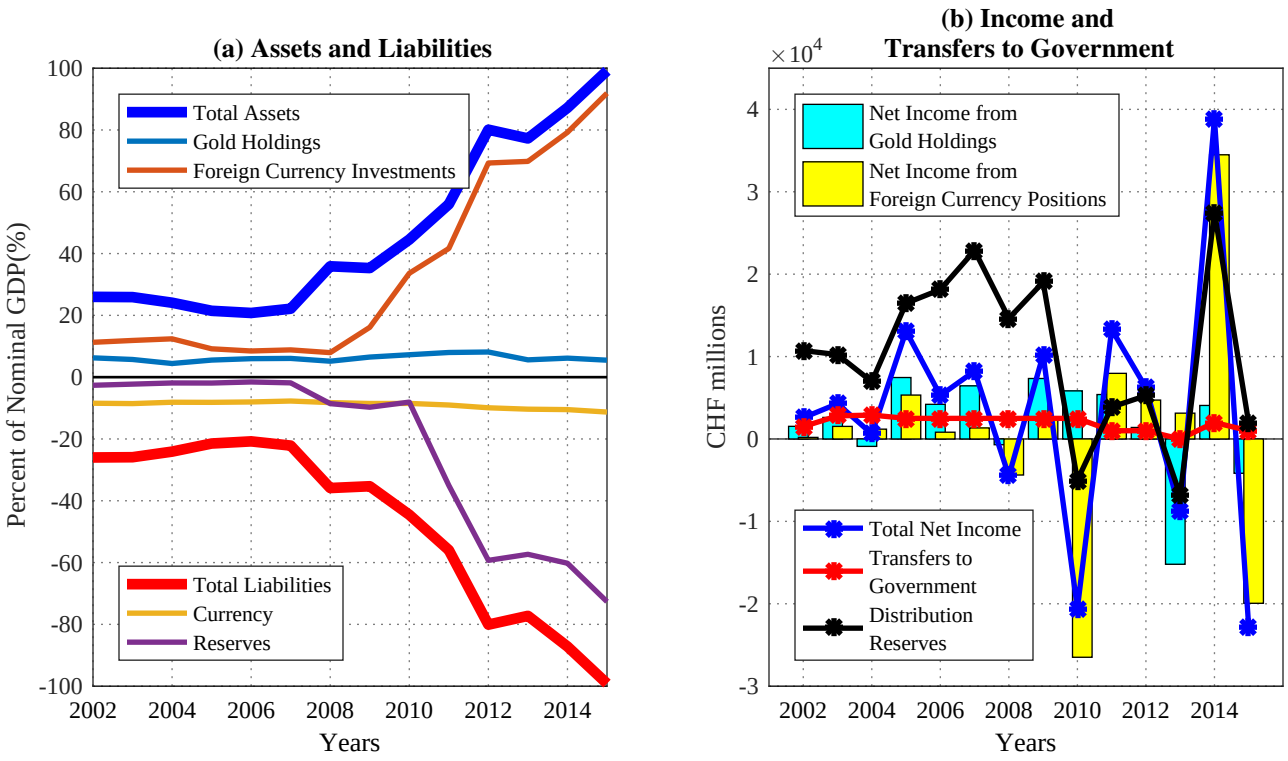



Figure 3: Assets, Liabilities, and Income of the Swiss National Bank

*Notes*: Source of data is Swiss National Bank. The transfers to government for SNB is fixed over the medium term using projections with an aim of smoothing the flow. SNB has a distribution reserve which is shown in the figure as well.

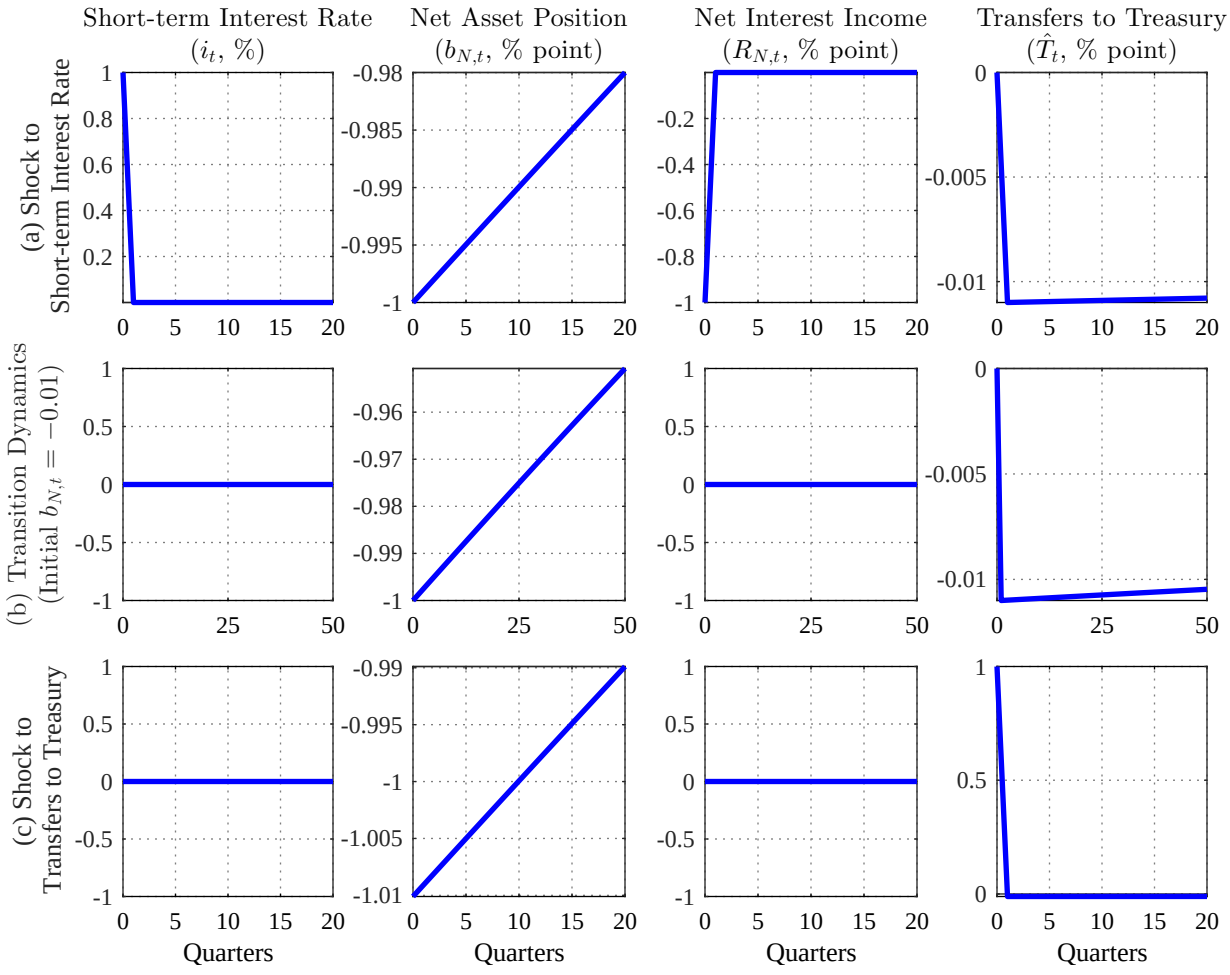



Figure 4: Impulse responses and transition dynamics for the closed economy balance sheet unconstrained central bank model

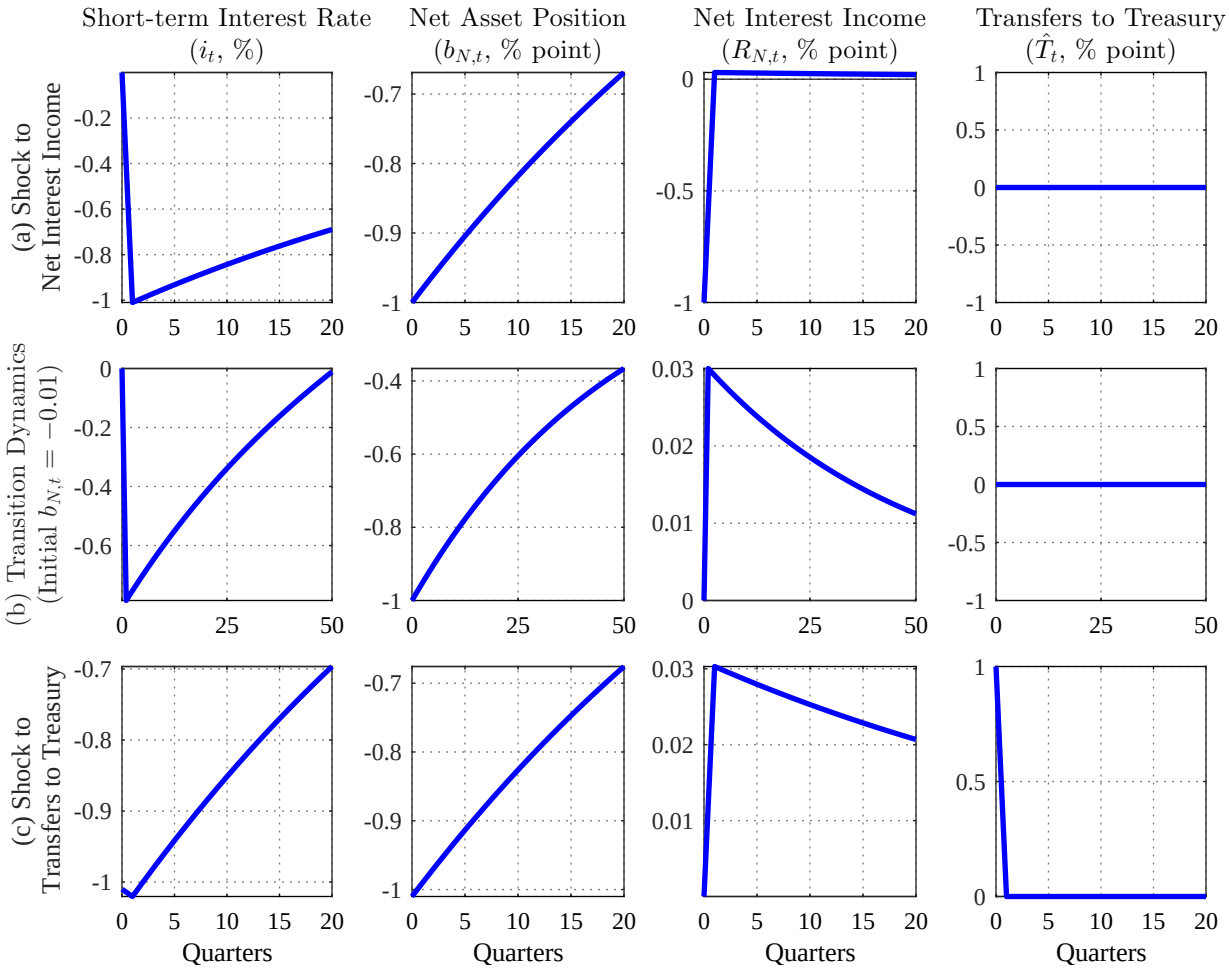



Figure 5: Impulse responses and transition dynamics for the closed economy balance sheet constrained central bank model

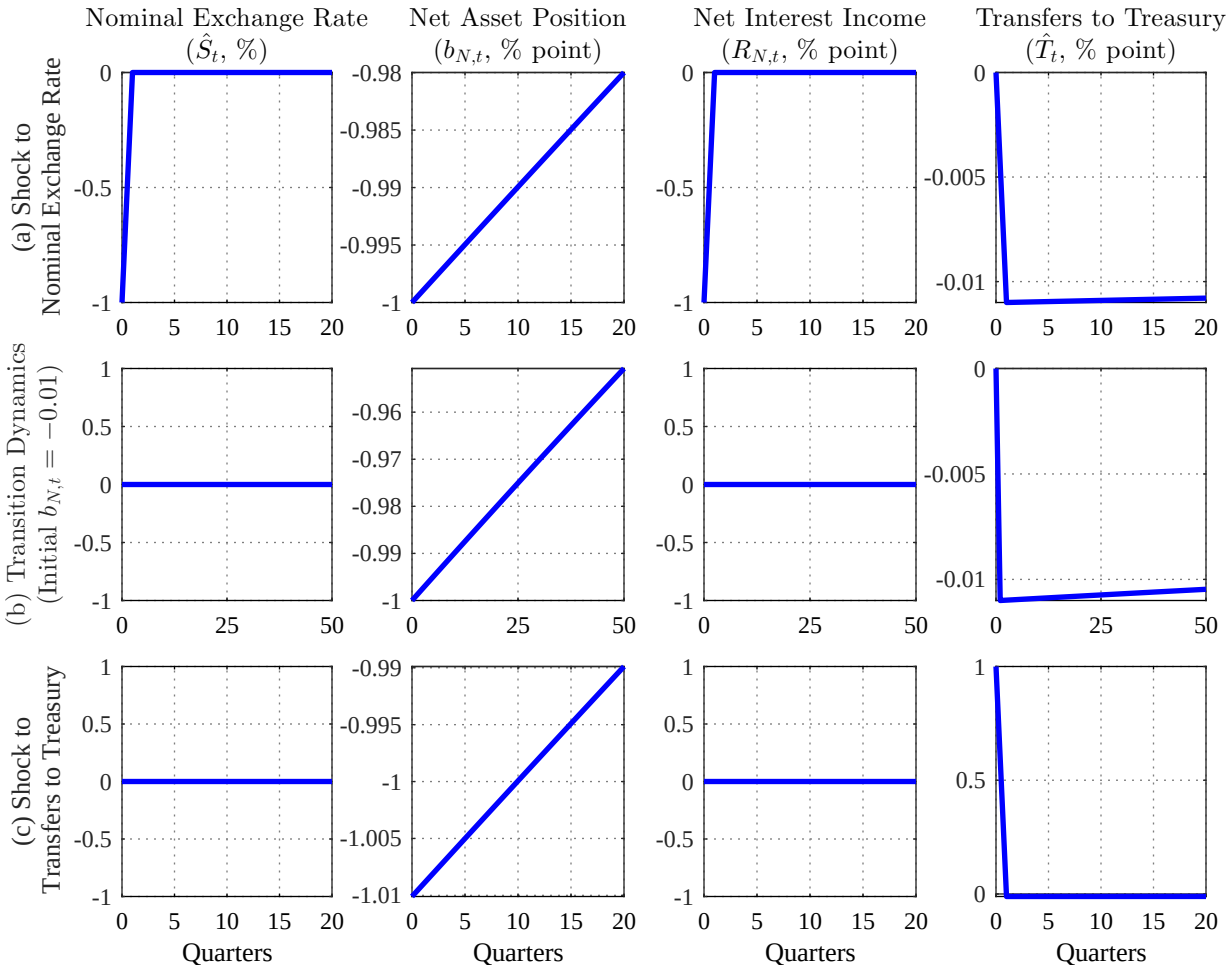



Figure 6: Impulse responses and transition dynamics for the open economy balance sheet unconstrained central bank model

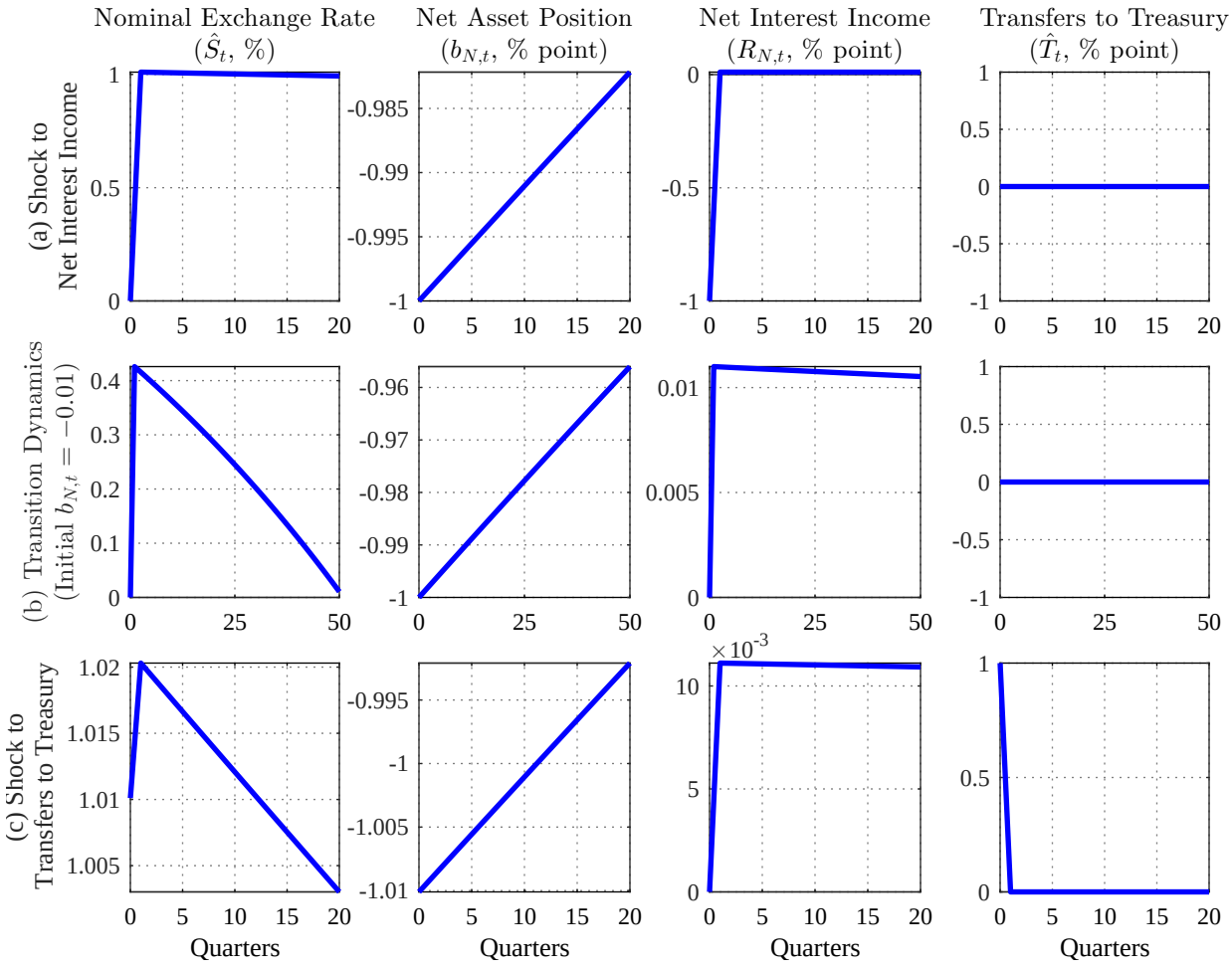



Figure 7: Impulse responses and transition dynamics for the open economy balance sheet constrained central bank model

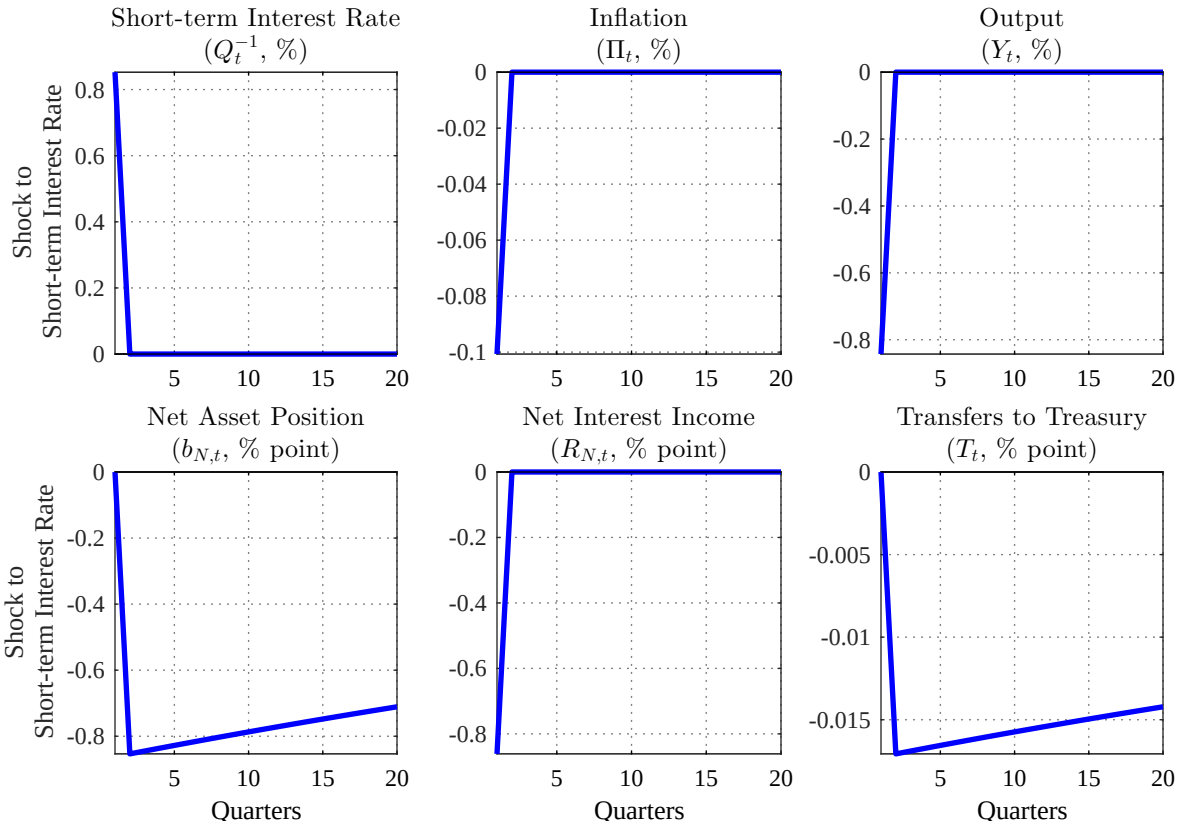



Figure 8: Non-linear impulse responses for the closed economy general equilibrium balance sheet unconstrained central bank model

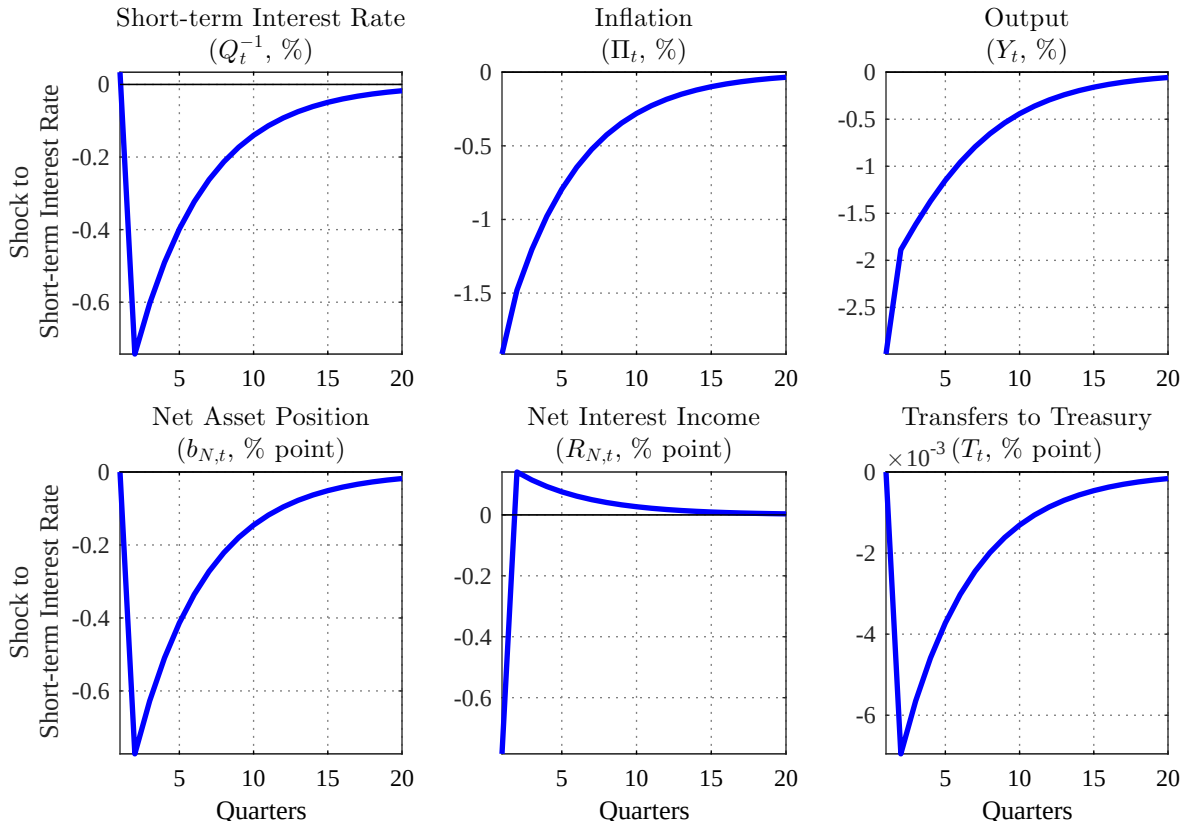



Figure 9: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model

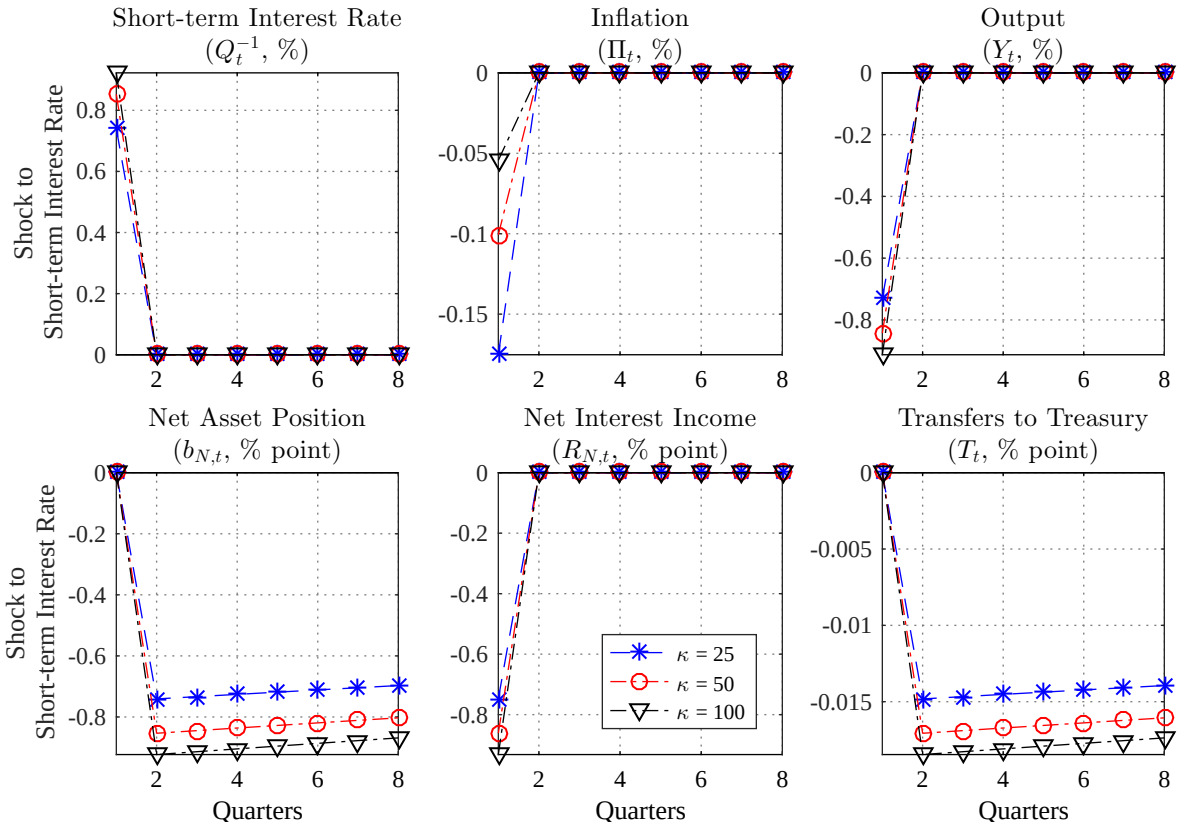



Figure 10: Non-linear impulse responses for the closed economy general equilibrium balance sheet unconstrained central bank model (different degrees of price stickiness)

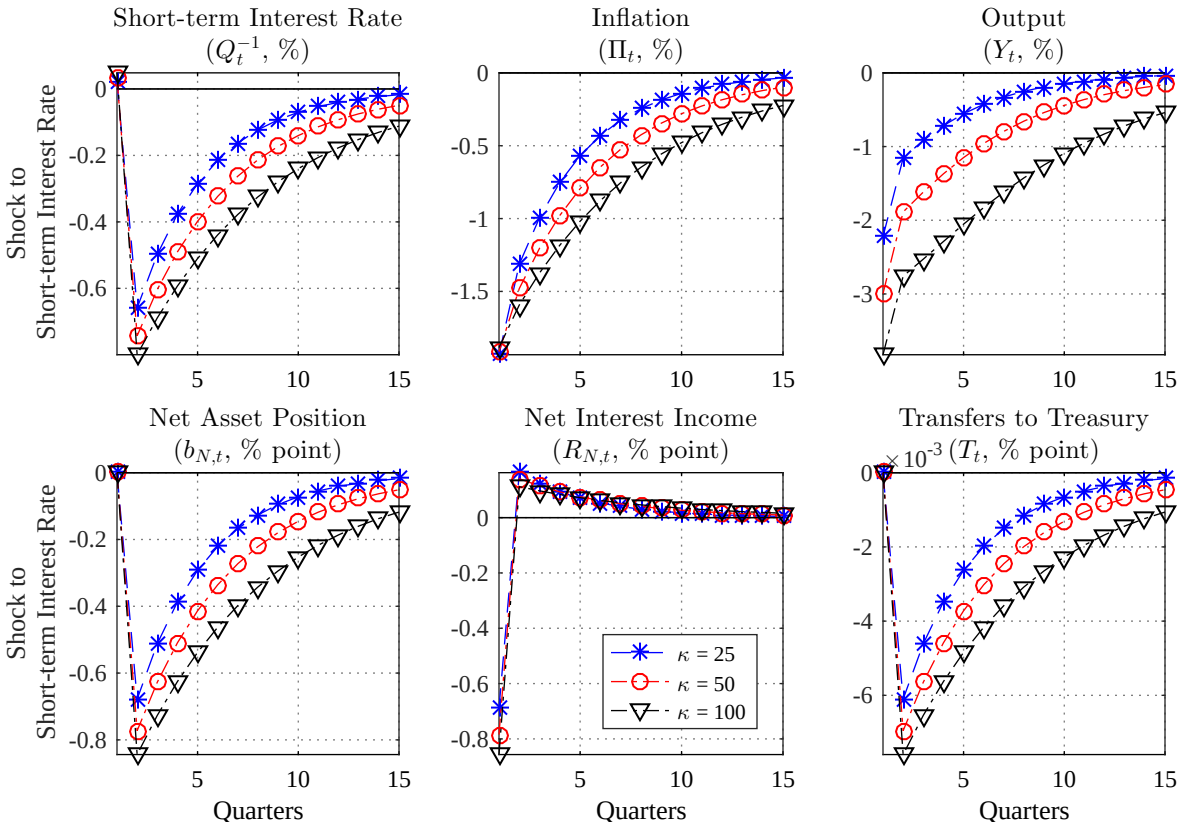



Figure 11: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model (different degrees of price stickiness)

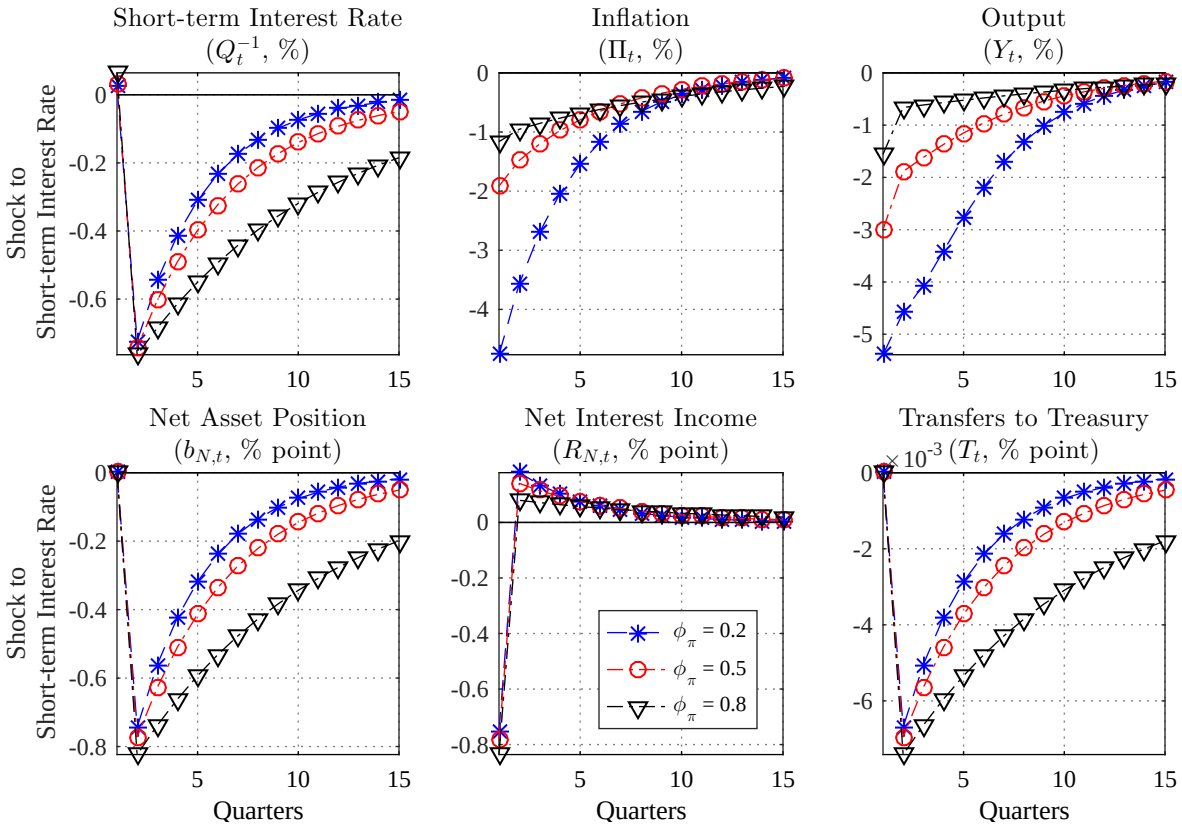



Figure 12: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model (different interest rate rule parameter)

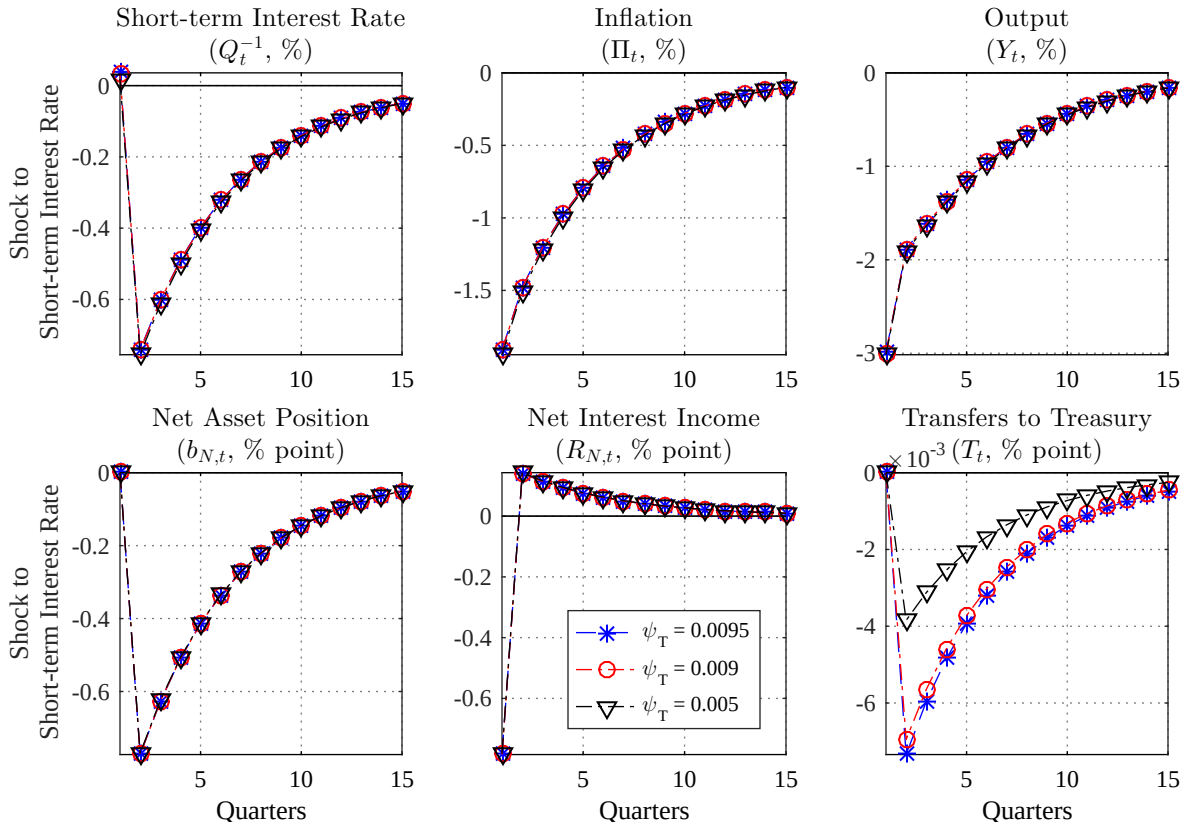



Figure 13: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model (different interest rate rule parameter)

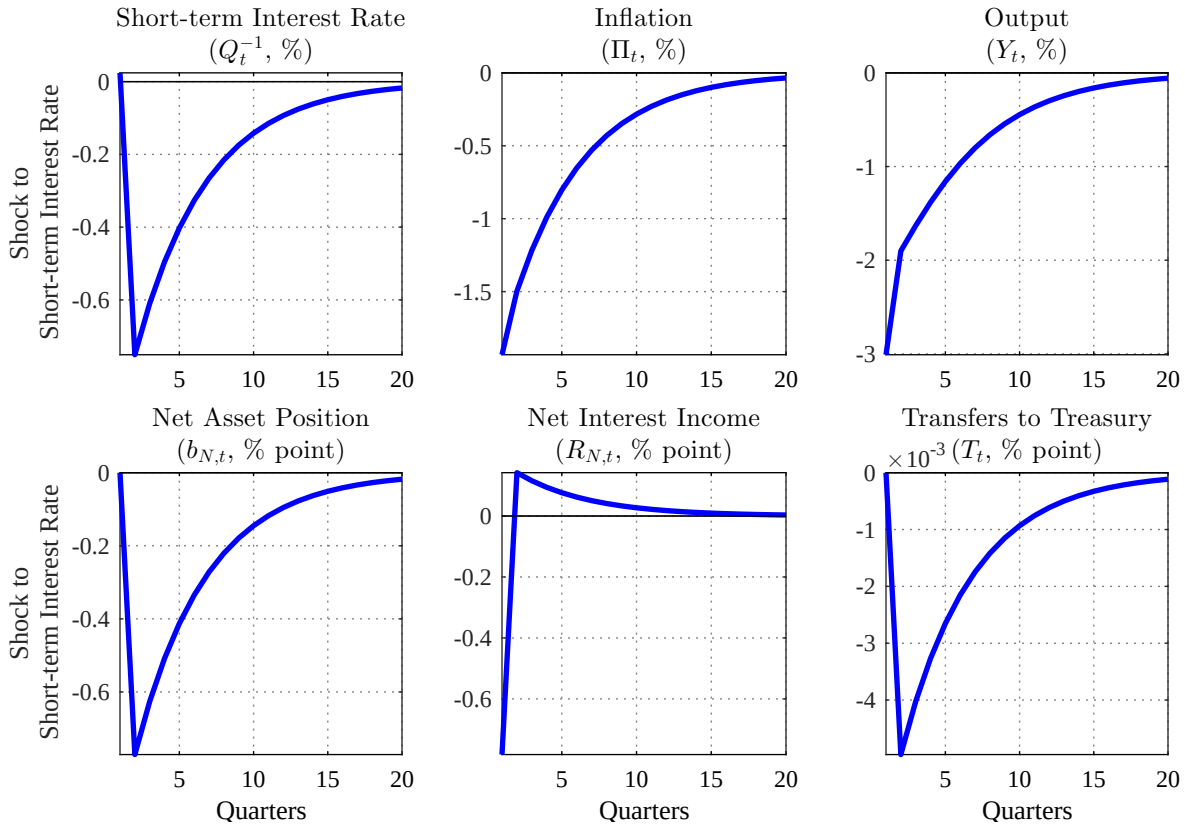



Figure 14: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model (alternative policy rule)

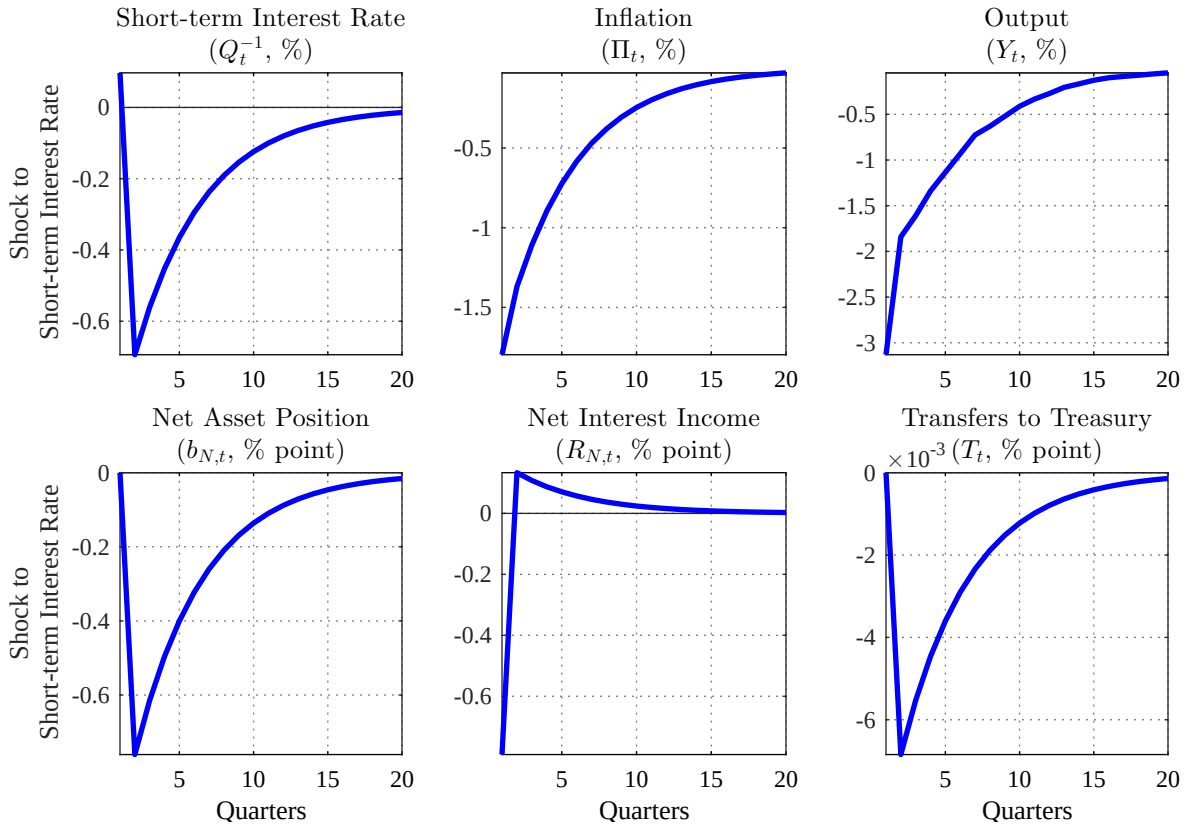



Figure 15: Non-linear impulse responses for the closed economy general equilibrium balance sheet constrained central bank model (third-order approximation)