Tunneling via Fairly Priced Equity Issuance

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Part I
Overview

Preemptive rights allow a corporation's existing stockholders to participate pro rata in any new stock issue. They are supposed to protect shareholders against dilutive stock issues. In particular, they are supposed to protect minority shareholders against issues priced below the stock's intrinsic value to a controlling stockholder or her affiliates. At first sight, it seems that this protection works, at least absent liquidity constraints or assymetric information. If the corporation issues shares below intrinsic value, preemptive rights provide minority stockholders the opportunity to participate pro rata, maintaining their fraction of the firm and its value. And if the corporation were to issue shares above intrinsic value, nobody would want to buy, or if someone did, the minority would welcome it because the value of the minority's shares would increase.

This paper shows, however, that preemptive rights' protection fails if and because the controller extracts from the firm private benefits that increase with firm size. In this case, the controller has a reason to favor an equity issue even if its effect on existing shareholders *net of* private benefits is negative. This reason is that the issue increases firm size and hence the controller's private benefits. At the same time, the private benefits act as a tax on the buyers of the new stock. In compensation for this tax, the buyers will demand a concomitantly larger share of the corporation, diluting the old shareholders. Consequently, the issue can hurt the existing (minority) shareholders even if it is competitively priced. (Whether it does so depends on how profitably the corporation invests the issue proceeds.) The controller compensates herself with an increase in private benefits. The controller effectively sells the buyers some of the minority's stake in exchange for increased private benefits for herself. Preemptive rights cannot prevent this, even though they do prevent outright expropriation.

Our paper can explain why sophisticated private parties do not rely on preemptive rights for protection against dilution. Instead, venture capitalists and others insist on veto rights. The mechanism of our model would also provide an explanation why share prices tend to drop upon announcement of a seasoned equity offering.

The problem we describe arises with any form of private benefits. It does not matter if the controller "steals" from the minority or simply derives a psychic benefit from a larger firm etc. The key mechanism is that private benefits drive a wedge between the value of the issue to the controller, who approves the

issue and increases her private benefits, and the value to the minority, who pay for these private benefits through dilution of their shares, whether or not they decide to participate.

It is also irrelevant whether the purchasers are existing shareholders or new ones. At a price equal to intrinsic value, everyone is indifferent between buying or not.[1] Below intrinsic value, everyone would like to buy. Above intrinsic value, only the controller would like to buy (or to be more exact, for the issue to proceed). Put differently, the reason why minority shareholders would buy the issue is never to "protect their investment," which is impossible, but merely to seize a good investment opportunity like any other. As to the controller, at price equal to intrinsic value she is indifferent between buying the stock herself or having outside investors buy it – either way, the issue increases her private benefits.

It is true that the minority would prefer that the issue not happen at all. But if it does happen, then at intrinsic value they are financially indifferent between participating or not. If nobody but the existing minority is allowed to participate, then dispersed minority shareholders may face a prisoners' dilemma where collectively they would be better off not to participate, but every individual minority shareholder is better of participating.

The protection of preemptive rights breaks down completely if new issues are costless to the controller and the firm is unlimited in its capacity to costlessly store cash. In this case, the controller can extract all of the minority's value. In reality, the controller may have to pay administrative, legal, or reputational costs for the issue, costless storage may not be compatible with private benefits (a checking account is too transparent), and investment may exhibit diminishing returns. In that case, preemptive rights limit, but do not eliminate, the controller's ability to dilute the value of the minority.

It is worth emphasizing that all of the above happens even with symetric information, which is the only environment considered here. If the controller has superior information about the value of the firm, as in reality, things will only get worse for the minority, as we discuss extensively elsewhere. Outside shareholders will then have difficulty deciding whether they should exercise their preemptive rights. Without private benefits, they might simply go along with whatever the controller is doing (assuming they know what the controller is doing). But with private benefits, they will not be able to tell apart whether what is an acceptable price for the controller is also an acceptable price to them.

The rest of this paper is structured as follows. Part II isolates the effect in the simplest possible model. Part III embeds this effect in a richer model with varying investment opportunities.

Part II

[1] The controller is not indifferent between somebody buying and nobody buying. But she indifferent between buying herself and having somebody else buy.

Simple Model

This part exposits the effect described above in the simplest possible model: a model where a dollar inside the firm is worth exactly as much as outside the firm. That is, all the firm's projects are zero NPV. We defer to part III consideration of the general case of (possibly) positive NPV investments with diminishing returns (which may include variable costs of the issue itself).

In section 1, we begin by considering a single mid-stream issue of new stock. We show that the impact on minority stockholders is devastating. Unless some exogenous cost of issuance constrains the controller, she can appropriate *all* of the value of the minority's stock. Anticipating this, no rational investor would purchase a minority stake in the first place. Even the controller would therefore be better off *ex ante* if mid-stream issues were costly or better yet impossible. We consider these dynamic consequences in section 2. We conclude this part with some notes on the generality of the model in section 3.

1 Single Mid-Stream Issue

1.1 Setup

Consider a firm with two (groups of) stockholders: a controlling stockholder and dispersed minority stockholders, whom we will initially treat as a monolithic block. The value of the firm's initial assets is normalized to 1. There is one share divided into infinitesimally small increments; minority stockholders initially own fraction $\alpha \in (0, 1)$. The controller receives a fraction $\delta \in [0, 1)$ of the firm's value in private benefits. So the initial value of the minority's stock is

$$V_0 \equiv \alpha (1 - \delta) > 0.$$

The firm issues a quantity $q > 0$ of new stock at price $p > 0$ per share. The controller chooses q and p.

For ease of exposition, we initially simply assert that the issue will be fully subscribed. In the final subsection 1.5, we will show formally that this assertion will hold true in equilibrium regardless of the sequencing and disclosure of purchase decisions.

As mentioned, we assume in this part that the firm invests the proceeds in a zero NPV project. That is, the social value of the firm's use of the issue proceeds is exactly equal to pq. For example, the firm might just invest the receipts in treasury securities or some diversified portfolio. The firm neither dissipates nor increases the *social* value of the money invested.

1.2 Effect on Minority's Wealth

If the minority stockholders do not participate (but the controller buys up the full issue), their wealth will be

$$\begin{aligned} V_N &\equiv \frac{\alpha}{1+q}(1-\delta)(1+pq) \\ &= V_0 \frac{1+pq}{1+q}. \end{aligned}$$

This is less than what they had before if and only if the issue price $p < 1$, i.e., if the price is less than the pre-issue firm-value per share (which will be higher than the pre-issue value to the minority if $\delta > 0$).

If minority shareholders do participate, their wealth will be

$$\begin{aligned} V_Y &\equiv \frac{\alpha(1+q)}{1+q}(1-\delta)(1+pq) - \alpha pq \\ &= \alpha(1-\delta)(1+pq) - \alpha pq \\ &= V_0 - \alpha pq\delta. \end{aligned}$$

It follows that a new issue is *always* bad for participating minority shareholders if the controller receives any private benefits. The reason is that some fraction of what the minority contributes will be consumed by the controller (recall that we are dealing with a special case of zero NPV investment).

The bottom line is that all stock issues are bad for minority stockholders unless the issue is very expensive, i.e., priced above pre-issue pro-rata firm value.

1.3 Minority Stockholders' Choice to Participate

A separate question is whether given an issue, it is better for minority shareholders to participate or not. We focus here on the minority's collective payout, and show in section 1.5 below that considering individual decisions does not make a difference. Participating will be collectively better only if

$$\begin{aligned} \alpha\left[(1-\delta)(1+pq) - pq\right] &> \alpha(1-\delta)\frac{1+pq}{1+q} \\ \Leftrightarrow 1-p &> \delta(1+pq) \end{aligned}$$

i.e., if the issue discount $1-p$ (relative to firm value) is larger than the total amount of private benefits. We are assuming here that any shares not bought by the minority will be bought by the controller or new investors, as explained in the setup of the model.

1.4 Controller's Optimal Quantity and Price

1.4.1 If the issue is costless

The easiest way for the controller to expropriate the minority through dilution would be to issue stock to only herself at price zero. Preemptive rights do not allow this (and in this sense they are effective).

With preemptive rights, the controller optimally issues stock at a price p^* equal to the stock's intrinsic value, i.e., the stock's expected cash flows net of private benefits:[2]

$$\begin{aligned} p^* &= \frac{(1-\delta)(1+p^*q)}{1+q} \\ \iff p^*(q) &= \frac{1-\delta}{1+\delta q} < 1-\delta. \end{aligned}$$

A higher price would be worse for the controller because she would be the only one willing to buy, in which case setting a higher price is equivalent to giving herself less stock for a given level of investment. A lower price would also be worse for the controller: it would reduce issue proceeds and hence her private benefits while not increasing her share of the equity, assuming that at any price below p^*, rational minority shareholders will exercise their preemptive rights.[3]

By definition, at p^* everyone is indifferent between buying or not buying the new stock (abstracting from the question whether the controller needs to

[2]Formally, the controller's pre-issue payoff is $V_0^K \equiv 1-\alpha(1-\delta)$.

The controller's payoff from an issue fully subscribed by the controller is

$$\begin{aligned} V_{FS}^K &\equiv \frac{1+q-\alpha}{1+q}(1+qp)(1-\delta)+\delta(1+qp)-qp \\ &= 1-\frac{\alpha(1-\delta)(1+qp)}{1+q} \\ &= V_0^K+\alpha(1-\delta)\left[1-\frac{1+qp}{1+q}\right] \end{aligned}$$

This is decreasing in p. If p is less than p^*, however, minority shareholders will exercise their preemptive rights. Hence the best price at which the controller can fully subscribe the issue is p^*, for a payoff $V_{FS}^K(p^*) = 1-\alpha p^*$.

Conversely, if outside shareholders buy the full issue, the controller's payoff is

$$\begin{aligned} V_N^K &\equiv \frac{1-\alpha}{1+q}(1+qp)(1-\delta)+\delta(1+qp) \\ &= (1+qp)\left(V_0^K+\delta q\right) \end{aligned}$$

This is increasing in p. But the maximal p that the controller can choose and still entice outsiders to buy is p^*. At that price, again $V_N^K(p^*) = 1-\alpha p^* = V_{FS}^K(p^*)$.

Finally, if outsiders and the controller subscribe pro rata, the controller's payoff is

$$\begin{aligned} V_{PS}^K &\equiv (1-\alpha)(1+qp)(1-\delta)+\delta(1+qp)-(1-\alpha)qp \\ &= 1-\alpha(1-\delta-\delta qp) \\ &= V_0^K+\alpha\delta qp. \end{aligned}$$

This is again increasing in p, but again the largest p at which outsiders would buy is p^*. So the maximal payoff for a given q is once again $V_{PS}^K(p^*) = 1-\alpha p^* = V_N^K(p^*) = V_{FS}^K(p^*)$.

Note that the controller's post-issue payoff is (1) unambiguously larger than the pre-issue payoff (since $\alpha p^* < \alpha 1-\delta$) and (2) independent of who buys the stock.

[3]If minority stockholders abstained from buying regardless of price, perhaps because of inattention, the controller might prefer a lower price. For example, assume that a fraction ϕ of minority shareholders does not exercise their rights regardless of price, and the controller is able to buy the corresponding shares. Then the controller's payoff is

maintain her stake to maintain control).[4] This includes the existing minority: the expression for p^* is just a rearrangement of the minority's participation condition (changed to an equality). Formally, $V_N(p^*) = \frac{V_0}{1+\delta q} = V_Y(p^*)$. In this sense, preemptive rights are irrelevant at this point.

p^* is declining in q because as q gets larger, the value of the original equity is spread ever thinner across all stock now outstanding. Recall that, with zero NPV investment, the only reason a sane outside investor would buy the issue is that it gives a claim on the existing equity. Other than that, the issue would be mechanically overpriced, as any dollar invested is immediately reduced to $1-\delta < 1$ dollar from the perspective of outside investors.

By the same token, the controller's payoff is increasing in q. The controller thus maximizes her payoff by setting q as large as possible.[5] Intuitively, the controller maximizes her payoff by "selling" all of the value of the original stock, which requires diluting it to zero, which she achieves by selling an infinite amount of new stock. That is, in the limit the controller appropriates the entire original value of the firm, grabbing as her gain the $(1-\delta)\alpha$ she did not already own before.[6] Formally, this manifests in $\lim_{q\to\infty} V_\bullet(p^*) = 0$.

1.4.2 If the issue is costly to the controller

The aforementioned scenario may appear extreme. In reality, controllers do not issue infinite amounts of stock, and the value of the existing stock does not drop to zero. One imagines that such extreme results carry reputational or other penalties for the controller.

For example, if equity issues entail a fixed cost to the controller, then the controller will not undertake the issue unless her maximal gain $(1-\delta)\alpha$ exceeds the fixed cost.

Alternatively, the controller may bear an increasing penalty the more the value of the stock drops below the initial value $1-\delta$. For example, consider a simple quadratic cost $\frac{c}{2}(1-\delta-p)^2$ for some $c > 0$. The controller will still set

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$$\begin{aligned} V_\phi^K &\equiv V_{PS}^K + \alpha\phi q\left(\frac{1+pq}{1+q} - p\right) \\ &= V_{PS}^K + \alpha\phi q\frac{1-p}{1+q} \\ &= V_0^K + \alpha q\left(\delta p + \phi\frac{1-p}{1+q}\right). \end{aligned}$$

This is decreasing in p if and only if $\delta(1+q) < \phi$.

[4] For a formal derivation of this point with respect to the controller, see the end of footnote 2.

[5] Formally, regardless of who buys the stock, the limit of the controller's payoff (cf. footnote 2) as $q \to \infty$ is:

$$\lim_{q\to\infty} V_N^K(p^*) = \lim_{q\to\infty} V_{FS}^K(p^*) = \lim_{q\to\infty} V_{PS}^K(p^*) = 1.$$

[6] The overall cash inflow into the corporation thus tends to $\frac{1-\delta}{\delta}$ at the controller's theoretical optimum. Formally, $\lim_{q\to\infty} qp^*(q) = \lim_{q\to\infty} q\frac{1-\delta}{1+\delta q} = \frac{1-\delta}{\delta}$.

the issue price at p^* because the penalty attaches directly to the low value of the stock rather than the issue price. At p^*, the controller's payoff is $1-\alpha p^* - \frac{c}{2}\left(1-\delta-p^*\right)^2$, regardless of who ends up buying the stock.[7] This payoff is maximized at $p^* = \max\left\{0, 1-\delta-\frac{\alpha}{c}\right\}$, or equivalently and assuming $c > \frac{\alpha}{1-\delta}$, at $q = \frac{\alpha}{\delta[(1-\delta)c-\alpha]}$. In words, the optimal issue size for the controller is finite and decreasing in c if the controller bears a sufficiently large and increasing cost of devaluing the minority's stake through dilution.

1.5 Formal Equilibrium Analysis: Proof that the Offer will be Fully Subscribed

We now show formally that our initial assertion is true: the offer will be fully subscribed in any possible equilibrium regardless of the sequencing and disclosure of participation decisions. We offer three sufficient conditions.

First, full subscription will trivially obtain if the controller is not liquidity constrained. In this case, if there were any possibility that the offer is not fully subscribed, the only rational strategy for the controller would be to submit a bid for the maximum amount of stock available to her, which would lead to full subscription after all. The reason is that, absent liquidity constraints, it is always strictly optimal for the controller to buy up any stock available at any price $p < 1$.[8]

Second, full subscription of the offer q obtains if minority shareholders are atomistic and $p \leq p^*(q)$. At p^*, atomistic minority shareholders are indifferent between buying or not if the offer is fully subscribed, and strictly prefer to buy if it is not fully subscribed.[9] In other words, participation is a weakly dominant

[7] See footnote 2 above. By definition, p^* is equal to the post-issue stock price.

[8] Let the quantity purchased by the controller and the minority be q_c and q_m, respectively. The controller's payoff is

$$\begin{aligned} V^K(q_c; q_m) &\equiv \left[(1-\delta)\frac{1-\alpha+q_c}{1+q_m+q_c}+\delta\right][1+(q_m+q_c)p] - q_c p \\ &= \frac{1-\alpha+q_c+\delta(\alpha+q_m)}{1+q_m+q_c}[1+(q_m+q_c)p] - q_c p \end{aligned}$$

The derivative w.r.t. q_c is

$$\frac{\partial V^K}{\partial q_c} = \frac{(1-\delta)(\alpha+q_m)(1-p)}{(1+q_m+q_c)^2},$$

which is positive provided $p < 1$.

Intuitively, unlike other shareholders, the controller gets back any dollar invested as a mix of shared benefits *and* private benefits.

[9] Formally, the payoff for an outside investor owning $\beta < \alpha$ shares and buying Δ additional shares conditional on q shares being issued to other investors is

$$\frac{\beta+\Delta}{1+q+\Delta}(1-\delta)[1+p(q+\Delta)] - \Delta p.$$

The limit of this payoff's derivative with respect to Δ as $\beta, \Delta \to 0$ is

$$(1-\delta)\frac{1+pq}{1+q} - p.$$

strategy for the minority stockholders, and it is an equilibrium that they all do. On the other hand, it would be strictly optimal for any minority shareholder to switch to always participating if the offer were undersubscribed with positive probability. This in turn would ensure full participation after all. Anything less than certain full subscription can thus not be part of any equilibrium.

Third, the preceding argument holds for any atomistic outside buyer, not just existing minority shareholders. Full subscription thus obtains even if the existing minority shareholders are not atomistic, provided enough atomistic outside buyers can access to the SEO.

Technically, if the controller is liquidity constrained, it is no longer guaranteed that prices above $p^*(q)$ are not optimal for the controller given issue size q. For example, for $q^+ > q$, it would be an equilibrium for the controller to set $(q^+, p^*(q))$ but for minority shareholders collectively to purchase only q (for example by deterministically exercising only q/q^+ of their preemptive rights, or by fully exercising with probability q/q^+). This particular equilibrium would lead to the same ultimate investment and to the same payoffs for the controller and (at least in expectation and collectively) the minority. We conjecture that this will be a general feature of all possible equilibria because the controller can hold outside investors to a zero payoff by setting $p = p^*$; the controller cannot squeeze more value out of anyone by setting a different price. It is certainly not incentive compatible for minority or other outside investors predictably to pay more than p^* in expectation. We do not pursue this point, however, because we believe it adds little to the economic insight of the model.

2 Dynamics: Sequential Issues

The analysis thus far is incomplete because it only considered one round of stock issue. After the first round, the controller can in principle do it all over again, now diluting out the shares purchased in the first round. In anticipation of this, outside investors will reduce the amount they are willing to pay for the shares in the first round.

In fact, outside investors will not be willing to pay anything for the shares if subsequent issues are costless to the controller. The reason is that in this first scenario considered above, we have shown that the controller can costlessly appropriate the entire value of the outsiders' investment. Consequently, no rational outsider will invest in an SEO at any positive price. By the same token, no outside investors would be willing to invest at the IPO stage. That is, external equity financing is impossible unless the corporation adopts specific protections against dilutive SEOs such as minority veto rights, or there are other constraints on dilutive SEOs, such as reputation or government interventions.

Importantly, outside investors may be deterred from investing even if the controller is permanently liquidity constrained. The reason is that the controller

By the definition of p^*, this expression is zero if and only if $p = p^*(q)$. Since the expression is decreasing in q (provided $p < 1$), it is strictly negative (positive) at $(q', p^*(q))$ for any $q' > q$ ($q' < q$).

can expropriate the existing minority by issuing stock to outside stockholders who are not liquidity constrained. As shown above, the controller is financially indifferent between buying the issue herself or having outsiders buy it (provided she can set the price at p^*). Technically, one can construct a subgame perfect equilibrium in which outsiders invest in the IPO but not any subsequent SEO because everyone's strategy is to invest (only) in all uneven rounds. But of course there is a similar equilibrium in which outsiders are willing to invest only in even rounds (and hence not in the IPO).[10] A more practical limitation is that the controller may need to invest in the SEO to preserve her controlling position, in which case her liquidity constraints would be relevant after all.

Against this backdrop, costs of an SEO can serve as a useful commitment device. For example, if the controller incurred a fixed cost K for an SEO, up to a share $\bar{\alpha} \equiv K/IR(1-\delta)$ of the initial firm could be financed by external equity, where I is the total amount invested and R the gross rate of return (such that IR is the post-IPO firm size – recall that the model normalized firm size, which we have to account for when considering fixed costs). The reason is that with such limited outside investment, the controller's benefit $\bar{\alpha}(1-\delta)IR$ from expropriating the minority would not exceed her cost K of doing so. The larger K, the more outside investment is possible. Specifically, in this simple model, the maximum amount of outside investment is simply K.[11]

3 Notes on the Generality of the Model

3.1 Rights Issue

If the SEO were structured as a rights issue, existing minority shareholders could theoretically prevent any stock issue other than pro rata to the controller by holding on to, but not exercising, their rights. This does not change the ultimate outcome, however.

First off, the minority stockholders will not be able to coordinate on a boycot of the rights issue. They face a prisoner's dilemma. The would all be collectively better off if no new stock were issued, but each individual stockholder is better

[10] This is so because with this strategy profile, (1) an investor in an uneven round (in particular, in the IPO) need not fear expropriation if nobody will invest in the next, even round (including the controller, because we have here assumed her to be permanently and totally liquidity constrained); and (2) an investor in an even round (in particular, the first SEO after the IPO) would have to assume complete expropriation in the subsequent, uneven round (e.g., the second SEO after the IPO). Thus, every investor's actions are optimal given everybody else's strategy, and thus the strategy profile is a (subgame perfect) Nash equilibrium.

[11] Intuitively, the reason is that the controller can expropriate all of the minority's investment, and will rationally do so unless that amount is less than K. Doing so may require a large SEO, but in the simple model with unlimited zero NPV investment nothing stops a large SEO.

Formally, denote outsiders' external investment by O. Assuming a dilutive SEO is not possible, outside investors will accept any $\alpha \geq \frac{O}{IR(1-\delta)}$. The controller will prefer not to do the dilutive SEO if and only if $\alpha \leq \frac{K}{IR(1-\delta)}$. Both inequalities can hold simultaneously if and only if $K \geq O$.

off participating if less than q is issued at $p^*(q)$. See section 1.5 above.

In any event, the controller does not need the minority's participation to effect any desired issue $(q, p^*(q))$. She can implement any desired issue by herself even if it is structured as a rights issue. All she needs to do is to raise the total number of rights issued to $q'' = q/\alpha$. Exercising her share α of these rights then leads to an issue of q new shares. If she is liquidity constrained, she can sell some of her rights to new outside investors. Minority stockholders would not now change their strategy and exercise. By definition of p^*, the minority is indifferent on the margin between exercising if q new shares are issued; they would be strictly worse off now that their investment would lead to an overall issue larger than q.

3.2 Type of private benefits

This model nests two interpretations of private benefits. One interpretation is that the private benefits are money that the controller "tunnels" from the firm, through transfer pricing, outright theft, or some other scheme. This need not happen with certainty; δ can be interpreted as the expected value of the tunneling. An example of framing the SEO in this way is that the firm invests all the issue proceeds into zero-NPV treasuries but with probability δ the controller absconds with the funds.

The other interpretation is that the private benefits are psychic or some other form of benefits that the minority is not able to receive in the first place, valued in the money-equivalent value to the controller. An example of framing the SEO in this way is that the firm invests the proceeds in a sports stadium that will generate only $1-\delta$ times the amount invested in terms of (discounted) cash-flows, but that yields a psychic benefit of δ times the amount invested to the controller, who is passionate about sports and loves the idea of owning a stadium.

Part III
General case: finance investment I to produce $F(I)$

We now consider a more general case of investment opportunities for the firm. The basic mechanism exposited above is still present here. But there are two forces that may ameliorate the minority's situation. First, their dilution may be offset by a positive NPVgenerated with the issue proceeds. If the firm's additional investment is profitable enough, the issue's net effect for the minority can be positive. Second, diminishing returns may lead to a natural stopping point. If the firm's additional investment is too wasteful, even the controller may be worse off.

As in the special case above, we begin by considering a single mid-stream issue before moving on to sequential issues and other complications.

4 Single Mid-Stream Issue

4.1 Setup

Same as before, but imagine the firm invests the issue proceeds $I = pq$ to produce $F(I)$.[12] We assume a generic weakly concave and increasing production function, i.e., $F(0) = 0, F' \geq 0, F'' \leq 0 \, \forall I \geq 0$, and $\lim_{I \to \infty} F'(I) \leq 1$.[13]

Interpretation: The general case combines the exploitative/dilutive issue discussed in the special case with a run-of-the-mill capital raising for benign, productive purposes. As such, it shares attributes of both, and which dominates depends on the production function.

NB: the simple model above is the same as the general case with $\frac{F(I)}{I} = R = 1 \, \forall I \Rightarrow F'(I) = 1, F''(I) = 0 \, \forall I$.

4.2 Minority Stockholders' Payoffs

In this case, non-participating shareholders' wealth will be

$$\begin{aligned} V_N^R &= \frac{\alpha}{1+q}(1-\delta)(1+F(I)) \\ &= V_0 \frac{1+F(I).}{1+q}. \end{aligned}$$

This is less than what they had before if and only if $F(I) < q$, i.e., if the added value per new share is less than the value per share prior to the issue.

Participating shareholders' wealth will be

$$\begin{aligned} V_Y^R &\equiv \frac{\alpha(1+q)}{1+q}(1-\delta)(1+F(I)) - \alpha I \\ &= V_0 + \alpha[(1-\delta)F(I) - I] \end{aligned}$$

This is less than what they had before if and only if

$$(1-\delta)F(I) < I,$$

[12] One may also think of this in terms of the gross return $R \equiv \frac{F(I)}{I}$. In this case, one can replace all the $F(I)$ terms below by pqR and I by pq. But the investment/production formulation is more flexible. In particular, it will be able to take into account that the return R is endogenous to investment size, i.e., it will accomodate decreasing returns to scale.

[13] The last condition means the company cannot make infinite amounts of money.

One might think that $F'(I) \geq 1 \, \forall I$ because the corporation could always invest the money into treasuries. But treasuries do not generate private benefits. "Stealing" may require investing the money into business activity that obfuscates the money flows – simply withdrawing the money from an account might be too obvious. Similarly, pet projects and other projects generating psychic benefits will require real investment; treasuries do not deliver them.

that is, if production net of private benefits is less than the initial investment, or equivalently, if the gross return net of private benefits is less than one.

Summing up, stock issues are unambiguously bad for the minority if the gross return of the investment is sufficiently low:

$$\frac{F(I)}{I} < \min\left\{\frac{1}{1-\delta}, \frac{1}{p}\right\}.$$

Conversely, stock issues are good for the minority if the inequality is reversed.

4.3 Minority Stockholders' Choice to Participate

Now participating is better than not if

$$\begin{aligned} \alpha(1-\delta)(1+F(I)) - \alpha I &> \frac{\alpha}{1+q}(1-\delta)(1+F(I)) \\ q(1-\delta)(1+F(I)) &> (1+q)I \end{aligned}$$

Dividing by q, this leads to the condition

$$(1-\delta)(1+F(I)) > (1+q)p.$$

Notice that the left hand side is the value of the firm net of private benefits, while the right hand side is the valuation of the firm at the issue price. That is, participating is advantageous if the issue price under-values the equity net of private benefits. Intuitively, the minority is indifferent between participation only if the price equals intrinsic value (to the minority, i.e., net of private benefits).

4.4 Controller's Optimal Quantity and Price

As in the special case, the controller maximizes her payoff in the presence of preemptive rights by setting the issue price equal to intrinsic value (from outsiders' point of view):

$$\begin{aligned} p^{**} &= \frac{1-\delta}{1+q}(1+F(I)) \\ \Leftrightarrow p^{**} &= (1-\delta)(1+F(I)) - I. \end{aligned}$$

While p^{**} may initially increase in I, p^{**} will eventually equal zero at some $\bar{I} \in [1-\delta, \infty)$ and be negative beyond that.[14] $\bar{I}$ is the supremum of investment that outside investors would be willing to finance because the stock's value net of private benefits is non-positive beyond that. By the same token, $\bar{I}$ is the minimum investment necessary to dilute the value of the minority's stake down to zero even in the presence of preemptive rights (by selling a very large number

[14] Technically, $\bar{I} \geq 1-\delta$ because $F(0) = 0$ and $F' \geq 0$, while $\lim_{I\to\infty} \frac{dp^{**}}{dI} \leq -\delta < 0$ because $\lim_{I\to\infty} F'(I) \leq 1$, and hence $\lim_{I\to\infty} p^{**} = -\infty$.

of shares at a price that is low in absolute terms but higher than its value net of private benefits, i.e., above p^{**}). Note that $\bar{I}$ does not depend on α.

The marginal profitability of investment at $\bar{I}$ determines whether the controller will want to dilute out the minority completely:

- If further investment is still profitable at $\bar{I}$ (i.e., if $F'\left(\bar{I}\right) > 1$), the controller will fully dilute out the minority and, as sole owner, will invest the socially optimal amount I^{FB} (defined by $F'\left(I^{FB}\right) = 1$) for a gain of $F\left(I^{FB}\right) - I^{FB} + \alpha\left(1-\delta\right)$.[15]

- Inversely, if marginal investment at $\bar{I}$ is sufficiently unprofitable, the controller will prefer a lower level of investment. Concretely, if $F'\left(\bar{I}\right) < \frac{1-\alpha}{1-\alpha(1-\delta)}$, the controller is better off not completely expropriating the minority because her marginal gains from expropriation are outweighed by her marginal investment losses. Instead, she will have the corporation invest only $\hat{I}$ defined by $F'\left(\hat{I}\right) = \frac{1-\alpha}{1-\alpha(1-\delta)} \in (0,1)$, for a gain $(1-\alpha)\left[F\left(\hat{I}\right) - \hat{I}\right] + \alpha\delta F\left(\hat{I}\right)$.[16] Of course, the controller will prefer

[15] The controller would not be able to split this investment into two, luring outsiders to invest up to $\bar{I}$ in the first round and then diluting them further in a second round. Outsiders would rationally expect this if a second issue is possible, and hence not accept the price offered in the first round. On the other hand, if the controller could somehow commit only to conduct one issue, she would only make herself worse off by stopping short of I^{FB}. Cf. next subsection.

[16] This gain (and the controller-optimal investment) is independent of whether the controller buys all, none, or her pro rata share of the stock. The controller's gross payoff if she subscribes (only) her pro rata share of the issue is

$$\begin{aligned} V_{PS}^{K}(I) &= (1-\alpha)\left[1+F(I)\right] + \alpha\delta\left[1+F(I)\right] - (1-\alpha)I \\ &= \left(1-\alpha\left(1-\delta\right)\right)\left[1+F(I)\right] - (1-\alpha)I, \end{aligned}$$

which at p^{**} (and $q = I/p^{**}$) is easily seen to be identical to

$$\begin{aligned} V_{FS}^{K}(I) &= 1 + F(I) - I - \frac{\alpha\left(1-\delta\right)\left[1+F(I)\right]}{1+q} \\ &= 1 + F(I) - I - \alpha\left[\left(1-\delta\right)\left(1+F(I)\right) - I\right] \end{aligned}$$

or from selling all the stock to outsiders:

$$\begin{aligned} V_{N}^{K} &= \frac{1-\alpha}{1+q}\left[1+F(I)\right] + \frac{q+\alpha}{1+q}\delta\left[1+F(I)\right] \\ &= \left[1+q\delta-\alpha\left(1-\delta\right)\right]\frac{1+F(I)}{1+q} \\ &= \left[1+\frac{I}{\left(1-\delta\right)\left(1+F(I)\right)-I}\delta-\alpha\left(1-\delta\right)\right]\frac{\left(1-\delta\right)\left(1+F(I)\right)-I}{1-\delta} \\ &= 1+F(I)-I-\alpha\left[\left(1-\delta\right)\left[1+F(I)\right]-I\right]. \end{aligned}$$

The value of the controller's initial stake is of course always $1-\alpha\left(1-\delta\right)$.

The equation defining $\hat{I}$ is merely the first order condition from maximizing V^K. It is sufficient because F is concave, so that the maximization problem is also concave. If F and hence the maximization problem is merely weakly concave, the maximum is not unique but all points in the interval that fulfills the first order condition.

zero investment and earn nothing if even $F'(0) \leq \frac{1-\alpha}{1-\alpha(1-\delta)}$.[17]

- For intermediate marginal profitability at $\bar{I}$ (i.e., $F'\left(\bar{I}\right) \in \left[\frac{1-\alpha}{1-\alpha(1-\delta)}, 1\right]$), the controller optimally chooses exactly $\bar{I}$ – enough to expropriate the minority, but no more – and gains $\delta F\left(\bar{I}\right) - (1-\alpha)(1-\delta)$.

Thus the controller completely expropriates the minority or overinvests due to the private benefit wedge $\alpha\delta$, or both.

The following table summarizes the controller's preferred investment and corresponding gain as a function of $F'\left(\bar{I}\right)$:

$F'\left(\bar{I}\right)$	**Controller's preferred investment** I^*	**Controller's gain** Π^K
> 1	I^{FB}	$F\left(I^{FB}\right) - I^{FB} + \alpha(1-\delta)$
$\in \left[\frac{1-\alpha}{1-\alpha(1-\delta)}, 1\right]$	$\bar{I}$	$\delta F\left(\bar{I}\right) - (1-\alpha)(1-\delta)$
$< \frac{1-\alpha}{1-\alpha(1-\delta)}$	if $F'(0) > \frac{1-\alpha}{1-\alpha(1-\delta)}$: $\hat{I}$ if $F'(0) \leq \frac{1-\alpha}{1-\alpha(1-\delta)}$: 0	$(1-\alpha)\left[F\left(\hat{I}\right) - \hat{I}\right] + \alpha\delta F\left(\hat{I}\right)$ 0

Π^K is increasing in α whenever the production function allows the controller to completely expropriate the minority, i.e., if $I^* \in \left\{\bar{I}, I^{FB}\right\}$ (just like in the special case). Intuitively, a higher minority share α means there is more to expropriate, while the expropriation technology does not depend on the minority's share (i.e., $\bar{I}$, I^{FB} are not functions of α). By contrast, the effect of α is ambiguous if the controller stops short of full expropriation and chooses to invest only $\hat{I}$: if the add-on investment is sufficiently profitable ($(1-\delta) F\left(\hat{I}\right) > \hat{I}$), higher α hurts the controller because she captures less of this profit; if the add-on is not so profitable, her larger gain from diluting the minority outweighs this sharing effect.[18]

The effects of δ are almost exactly the reverse. Higher δ helps the controller at $\hat{I}$ because it accelerates expropriation of the minority.[19] By contrast, if the controller fully expropriates the minority anyway and chooses I^{FB} independently of the minority's involvement, higher δ reduces her gains simply because it means she already got most of the minority's stake in private benefits to begin

[17] As we have assumed $F'' \leq 0$, the inequality will hold for all $I > 0$ if it holds at $I = 0$.

[18] By the envelope theorem,

$$\frac{d\Pi^K\left(\hat{I}\right)}{d\alpha} = \hat{I} - (1-\delta) F\left(\hat{I}\right).$$

Notice that the range where higher α hurts the controller is one where the minority actually benefits from the stock issue. A necessary condition for this to happen is $F'(0) > \frac{1}{1-\delta}$.

[19] By the envelope theorem,

$$\frac{d\Pi^K\left(\hat{I}\right)}{d\delta} = \alpha F\left(\hat{I}\right) > 0.$$

with. At the intermediate $\bar{I}$, δ's effect is ambiguous.[20]

5 Founding Stage: Structuring Ownership to Prevent Expropriation

5.1 Dynamics: Multiple Rounds

The analysis thus far is incomplete because it only considered one round of stock issue. After the first round, the controller can in principle do it all over again, now diluting out the shares purchased in the first round. This does not affect the controller's payoffs and optimal strategies because at p^{**} (or p^* in the special case), she is indifferent between having outsiders invest or not, so she might as well buy the full issue herself if that turns out to be necessary (if she prices above p^{**}, she is the only investor anyway). Sometimes, however, outside investors may still invest pro rata with the controller. Specifically:

- If $I^* \in \left\{\bar{I}, I^{FB}\right\}$, outside investors would not invest in the first round even if no second round were possible. Nor would rational, fully informed outside investors be lured into investing in a slightly smaller issue if the controller could dilute them out in a follow-up issue. Thus the controller becomes the single owner in the first round, as in the special case.
- If $I^* = \hat{I}$, outsiders can invest alongside the controller *unless the controller subscribed less than her pro rata share*. This is because the optimality condition for $\hat{I}$ in the first round implies that the no-profit condition will hold (with equality) for the second round.[21] Intuitively, diminishing returns stop the controller from a larger stock issue in the first round, and

[20] Since $p^{**} = 0$ implicitly defines $\bar{I}$ as a function of δ, we have

$$\begin{aligned}\frac{d\Pi^K\left(\bar{I}\right)}{d\delta} &= \frac{\partial\Pi^K\left(\bar{I}\right)}{\partial\delta} + \frac{\partial\Pi^K\left(\bar{I}\right)}{\partial\bar{I}}\frac{\partial\bar{I}}{\partial\delta} \\ &= F\left(\bar{I}\right) + 1 - \alpha - \delta F'\left(\bar{I}\right)\frac{1+F\left(\bar{I}\right)}{1-(1-\delta)F'\left(\bar{I}\right)} \\ &= \left[1+F\left(\bar{I}\right)\right]\frac{1-F'\left(\bar{I}\right)}{1-(1-\delta)F'\left(\bar{I}\right)} - \alpha.\end{aligned}$$

(An envelope theorem does not apply because the value function is not derived from marginal maximization w.r.t. I but dictated by the maximum price outside stockholders would pay.)

So everything depends on the size of α. I believe the most we can say, using $F\left(\bar{I}\right) \in \left[\frac{1-\alpha}{1-\alpha(1-\delta)}, 1\right]$, is

$$\frac{d\Pi^K\left(\bar{I}\right)}{d\delta} \in \left[-\alpha, \alpha F\left(\bar{I}\right)\right].$$

If this is right, then the derivative could be of either sign.

[21] Technically, these conditions will be different in the second round because investment I and output F in the analysis are relative to, and normalized by, "initial" firm size, which is different in the first round. This is without consequence for present purposes, however, because it does not affect $F'(I)$. This derivative is always equal to the derivative of the overall production function $\Phi(X)$ with inputs X and output expressed in "natural" (arbitrary) units.

this does not change in a second round *unless the ownership structure changes*. If outsider shareholders subscribed more than a fraction α of the first round, the controller now owns a smaller share and thus can gain more from diluting the (new) minority, which in turn implies that she accepts less profitable investments.[22]

The bottom line is that dynamics do not affect if and how much the controller can expropriate the minority. But dynamics do rule out certain participations by outside stockholders that they would have found acceptable if they were guaranteed that no further issues will happen. Specifically, outsiders will not invest at all in the special case, and will only invest pro rata with the controller in the general case, and only if additional investment would be prohibitively unproductive even for the controller.

5.2 optimal IPO

The foregoing suggests the obvious strategy to rule out expropriation later: outside ownership α must be small enough, and additional investment gains small enough, that the controller's gains from dilution do not exceed her fixed costs of engineering it.

Without fixed costs, the only way to do so is to have the firm invest at an inefficient scale from the outset. Concretely, it must be that the firm's marginal productivity be at most equal to $\frac{1-\alpha}{1-\alpha(1-\delta)}$. Since this expression is decreasing in α, it means that the higher the minority's share, the more inefficiently the firm must invest. The controller may be better off not inviting outside investment at all.

For this reason, fixed costs of SEOs can actually be beneficial ex ante, because they rule out expropriation ex post at lower and hence more efficient investment levels. Still, the dilution mechanism described in the model rules out large minority investments because the controller's benefits from the SEO increase in the minority's stake α.

NEED TO FIRM THIS UP – AT $I^* = \hat{I}$ AND $(1-\delta) F\left(\hat{I}\right) > \hat{I}$, HIGHER α MAY MAKE SEO LESS ATTRACTIVE. BUT THAT IS ALSO A REGION WHERE EVEN THE MINORITY LIKES THE SEO, SO IF THE FIXED COST IS A REPUTATION COST, THE CONTROLLER MAY NOT NEED TO PAY IT.

In particular, for any initial investment X_0, we have

$$\begin{aligned} I &\equiv \frac{X - X_0}{\Phi(X_0)} \\ F(I) &\equiv \frac{\Phi(X) - \Phi(X_0)}{\Phi(X_0)}, \end{aligned}$$

which imply $F'(I) = \Phi'(X)$. It immediately follows that the optimality condition in the first round implies that the no-profit condition holds (with equality) in the second round.

[22] Technically, the right-hand side of first-order condition defining $\hat{I}$ and of the inequality defining the cut-off point where any positive investment is unprofitable, $\frac{1-\alpha}{1-\alpha(1-\delta)}$, is declining in α.