Horizontal and Vertical Polarization: Task-Specific Technological Change in a Multi-Sector Economy*

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Abstract

We analyze the effect of technological change on inequality using a novel framework that integrates an economy's skill distribution with its occupational and industrial structure. Individuals become a manager or a worker based on their managerial vs. worker skills, and workers further sort into a continuum of tasks (occupations) ranked by skill content. Our theory dictates that faster technological progress for middle-skill tasks not only raises the employment shares and relative wages of lower- and higher-skill occupations (*horizontal* polarization among workers), but also raises those of managers over workers as a whole (*vertical* polarization). Both dimensions of polarization are faster within sectors that depend more on middle-skill tasks and less on managers. This endogenously leads to faster TFP growth among such sectors, whose employment and value-added shares shrink if sectoral goods are complementary (structural change). We present several novel facts that support our model, followed by a quantitative analysis showing that task-specific technological progress—which was fastest for occupations embodying routine-manual tasks but not interpersonal skills—is important for understanding changes in the sectoral, occupational, and organizational structure of the U.S. economy since 1980, much more so than skill-biased and sector-specific technological changes.

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1 Introduction

We develop a novel framework that integrates an economy's distribution of individual skills with its occupation and industrial structure. It enables an analysis of how changes in wage and employment shares across occupations and industrial sectors are interrelated, providing a comprehensive view on the economic forces that shape the occupational, sectoral, and organizational structure of an economy.

In our model, individuals are heterogeneous in two-dimensional skills—managerial talent and worker human capital—based on which they become a manager or a worker. Workers then select into a continuum of tasks (or occupations) based on their human capital.[1] Managers organize the workers' tasks, in addition to their own, to produce sector-specific goods. Sectors differ in how intensively different tasks are used in production. Individual skills are sector-neutral, so they only care about their occupation and are indifferent about which sector they work in. The equilibrium assignment is fully characterized, which is a theoretical contribution given the new, multi-layered aspects of our model.

We show that if different tasks are complementary in production, faster technological progress for middle-skill tasks relative to the others leads to: (i) higher employment shares and wages for low- and high-skill occupations relative to middle-skill occupations, i.e. job and wage polarization among workers; (ii) a higher employment share and wages for managers relative to workers as a whole, which we call *vertical* polarization to distinguish from the *horizontal* polarization across workers; (iii) faster horizontal and vertical polarization within sectors that depend more on middle-skill tasks and less on managers; and (iv) faster endogenous total factor productivity (TFP) growth of such sectors, shrinking their employment and value-added shares if sectoral goods are complementary (i.e., structural change).

The last theoretical result merits further discussion. First, because sector-level TFP in our model is endogenously determined by equilibrium occupational choices, task-specific technological progress—which is sector-neutral—has differential impact across sectors, causing structural change. Second, as the employment share of sectors that rely less on middle-skill workers and more on managers rise, the overall degree of (both horizontal and vertical) polarization is reinforced. Third, if all structural change is driven by task-specific technological progress, in the asymptotic balanced growth path only occupations with slow technological progress remain and all others vanish,

[1] Technically, a task is the technology used by a certain occupation. Throughout the paper, we will use task and occupation interchangeably.

but all sectors exist. This is in contrast to many theories of structural change that rely on sector-specific forces: In those models, the shift of production factors from one sector to another continues as long as those forces exist, so that shrinking sectors vanish in the limit. Because the task-specific technological progress in our model is sector-neutral, once the employment shares of the occupations with faster progress become negligible, structural change ceases.

Predictions (i) through (iv) are salient features of the U.S. economy since 1980: (i) job and wage polarization is well-documented in the literature, e.g., Autor and Dorn (2013), which we refer to as horizontal polarization; (ii) in the same data, we highlight vertical polarization; and (iii) we verify that manufacturing is more reliant on middle-skill workers and less on managers than services, and provide evidence that both dimensions of polarization are indeed faster within manufacturing than in services.[2] Finally, (iv) it has long been understood that the faster growth of manufacturing TFP is an important driver of structural change from manufacturing to services. Our model shows that all of the above empirical facts have a common cause: faster technological progress for middle-skill tasks than the rest.

Our theoretical model has one managerial task and a continuum of worker tasks. To quantify the model, we discretize the latter into 10 broadly-defined occupation categories in the data. Our quantitative analysis confirms that task-specific technological progress alone—without any exogenous change to sector-specific TFP growth—can account for almost all of the observed growth in sector-level TFP's. More broadly, task-specific technological progress, especially for middle-skill tasks, is important for understanding the changes in the sectoral, occupational, and organizational structure in the U.S. economy over the last 35 years.

The natural next question is what can explain such differential productivity improvements across tasks. To explain what we call horizontal polarization, Autor and Dorn (2013), Goos, Manning, and Salomons (2014) and others hypothesized that "routinization"—i.e., faster technological advancement for tasks that are more routine in nature (which tend to be middle-skill tasks in the data)—reduced the demand for middle-skill occupations. They test this empirically by constructing a "routine-task intensity" (RTI) index for each occupation, which is constructed by aggregating various information available from the Dictionary of Occupation Titles (DOT) and its

[2]In addition, we provide evidence from establishment-level data that corroborates faster vertical polarization in manufacturing: manufacturing establishments shrank faster in terms of employment and grew faster in terms of value-added than those in services, which is predicted by our model. This particular calculation assumes that the number of managers per establishment was stable over time.

successor O*NET. When we consider more detailed characteristics of occupations, we find that the task-specific technological progress we quantify is much more strongly correlated with the routine-manual index and the inverse of the manual-interpersonal index, which are less aggregated than the commonly-used RTI. In other words, technological progress in the last 35 years are heavily embodied in those manual tasks that are repetitive in nature and require few interpersonal skills. By contrast, the level or growth of the fraction of workers in an occupation that are college-educated—a popular proxy for skill-biased technological change—is not meaningfully correlated with task-specific productivities.

Related Literature The model we consider is of particular relevance for the U.S. and other advanced economies. The 1980s marks a starting point of rising labor market inequality, of which polarization is a salient feature. It coincided with the rise of low-skill service jobs (Autor and Dorn, 2013) and also a clear rise in manufacturing productivity (Herrendorf, Rogerson, and Valentinyi, 2014). Our main finding in this regard is that heterogeneous task-specific technological progress can be of first-order importance for understanding the observed changes not only across occupations but also in the industrial structure of the economy.

Costinot and Vogel (2010) presents a task-based model in which workers with a continuum of (one-dimensional) skill sort into a continuum of tasks. The worker side of our model is similar to theirs (except that we include capital), but we gain new insights by incorporating two dimensions of skills (managerial talent and worker human capital) and multiple sectors.

The only other paper we know of with a structure in which individuals with different skills sort themselves into occupations, which are then used as production inputs in multiple sectors, is Stokey (2016). The within-sector side of the model in it can be described as a version of Costinot and Vogel (2010) in which skills are continuous but tasks are discrete. The latter assumption enables an analytic characterization of the effect of task-specific technological change, which is in turn used for demonstrating the broad range of phenomena that can be explained by such a model.

We take a similar approach in our quantitative section (i.e., tasks are discretized), and use the U.S. data to quantify how relevant our model is for the employment and relative wage trends across occupations and sectors between 1980 and 2010. In particular, we explicitly relate polarization to structural change across sectors, and since we treat managers as an occupation that is qualitatively different from workers, the model also has implications for how production is organized and how it has changed

over time across different sectors. We document some new facts along this dimension.

The manager-level technology in our model extends the span-of-control model of Lucas (1978), in which managers hire workers for production. Unlike all existing variants of the span-of-control model, our managers organize tasks instead of workers. That is, instead of deciding how many workers to hire, they decide on the quantities of each task to use in production, and for each task, how much skill to hire. Moreover, we assume a CES technology between managerial talent and (worker) tasks.[3]

Goos et al. (2014) estimates that relative price changes in task-specific capital have driven employment polarization in Europe. It empirically decomposes employment polarization into within- and between-industry components, but abstracts from wage shifts and aggregate quantities. Dürnecker and Herrendorf (2017) also considers occupations and industries, showing that most of structural change across sectors can be accounted for by shifts at the occupation level in many countries. Its conclusion is based on classifying occupations in the data as (mutually-exclusive) manufacturing or service jobs. In contrast, we keep occupations and sectors separate, and analyze the effect of task-specific vs. sector-specific technologies for employment and wage inequalities across the skill distribution.

There is a rapidly growing literature in international trade that uses assignment models to explain inequality between occupations and/or industries (Burstein, Morales, and Vogel, 2015; Lee, 2015). The majority of such models follow the tradition of Roy: Workers have as many dimensions of skills as there are available industry-occupation combinations, and select themselves into the job in which they have a comparative advantage. To make the model tractable, a Fréchet distribution is utilized to collapse it into an empirically testable set of equations for each industry and/or occupation pair. The manager-worker division in our model is also due to Roy-selection (managerial talent vs. worker human capital), but workers sort into a continuum of tasks based on a one-dimensional skill. Having only two skill dimensions facilitates mapping them to individual characteristics in the data, so we can explore (as an extension) the endogenous skill formation of workers' human capital using traditional labor and macroeconomic models of human capital.

Structural change in our model occurs because sectoral productivities evolve differentially over time, an insight we borrow from Ngai and Pissarides (2007) and most

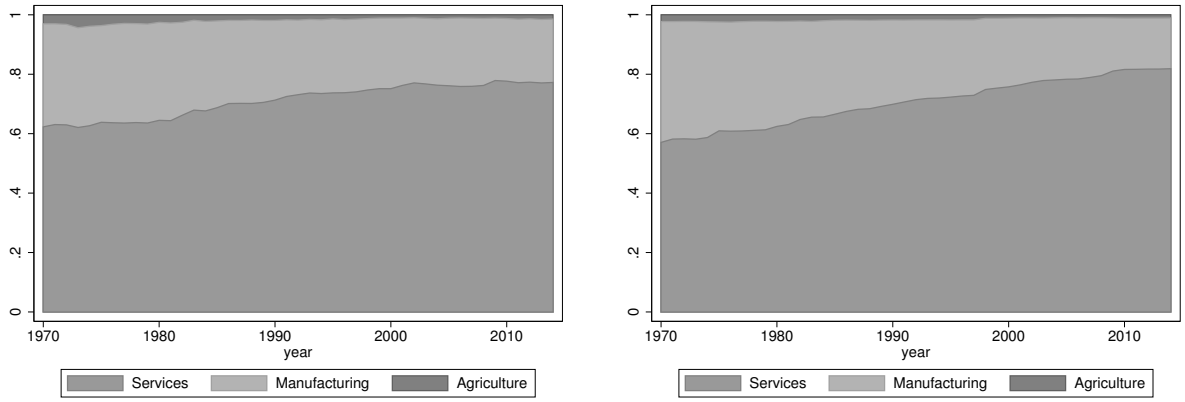
[3]While Lucas's original model is based on a generic HD1 technology, virtually all papers that followed assume a Cobb-Douglas technology. We incorporate (i) non-unitary elasticity between managers and workers, (ii) heterogeneity in worker productivity as well as in managerial productivity, (iii) multiple worker tasks (or occupations), and (iv) multiple sectors.

other production-driven models of structural change. What we add to this literature is a new mechanism for sectoral productivities to evolve *endogenously*: the changing selection on skills across tasks due to task-specific technological progress. Also related is Acemoglu and Guerrieri (2008), in which the capital-intensive sector vanishes in the limiting balanced growth path and all capital is used by the labor-intensive sector. In comparison, sectors in our model differ in how intensively they use different tasks. By contrasting different types of labor, rather than capital and labor, we can connect structural change across sectors to labor market inequality across occupations and skills. Moreover, unlike many existing explanations of structural change, our model implies that it is certain occupations rather than broadly-defined sectors that may vanish in the limit.

Finally, there exist a few recent papers that consider skill and structural change. Buera and Kaboski (2012) and Buera, Kaboski, and Rogerson (2015) feature heterogeneous worker types as different inputs of production. Similarly, Bárány and Siegel (2017) argues that polarization may be explained by structural change, in a model where skills are occupation-specific and occupations are sector-specific. In these models, task-specific technology is ruled out by assumption, so all change must be either skill- or sector-specific. The addition of the task dimension in our model separates skills from the occupation in which they are used, thereby permitting technological changes specific to a task or occupation. This extra dimension of heterogeneity allows us to exploit data on occupational employment and wages *within* sectors as well as across sectors.[4] In addition, as mentioned above, the sectoral TFPs in our model are endogenously determined by equilibrium occupational choices.

The rest of the paper is organized as follows. In Section 2, we summarize the most relevant empirical facts: horizontal and vertical polarization in the overall economy, the faster speed of polarization within manufacturing than in services, and structural change. In Section 3, we present the model and solve for its equilibrium. In Section 4, we perform comparative statics showing that faster technological progress for middle-skill tasks leads to horizontal and vertical polarization, and to structural change. In Section 5 we calibrate a discrete-occupation version of the model to data from 1980 to 2010, and in Section 6 we quantify the importance of task-specific technological progress and maps it to empirical measures of task characteristics. Section 7 concludes.

[4] Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011) and Lee, Shin, and Lee (2015) show that residual wage inequality controlling for education groups is much larger and has risen more since 1980 than between-group inequality.



(a) GDP, Value-Added

(b) Aggregate Employment Shares

Fig. 1: Structural Change, 1970-2013.
From the BEA NIPA accounts. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Employment is based on full-time equivalent number of persons in production in NIPA Table 6. Further details in Section 5.2.

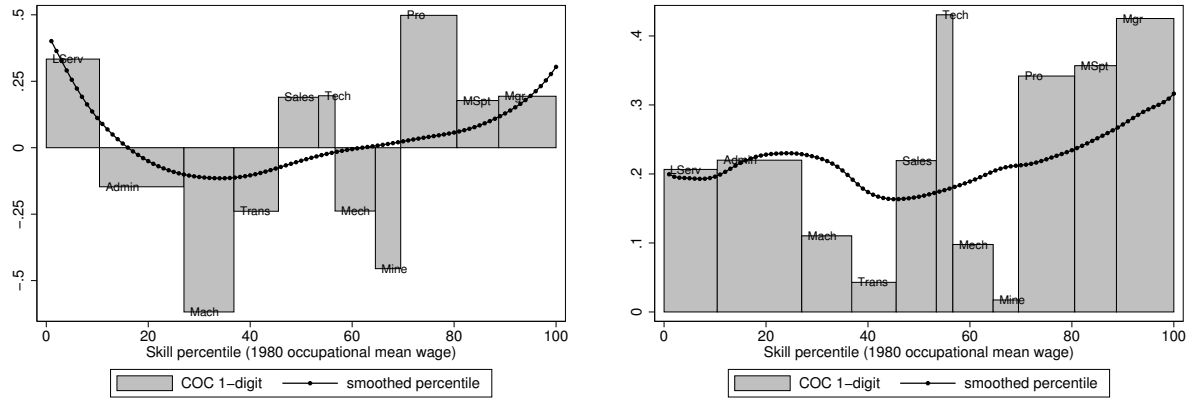
2 Facts

In this section we summarize known facts related to long-run trends in structural change and polarization, and present novel evidence on how the two may be related. We also provide a new way of looking at managerial occupations by considering them as qualtitatively different from all other occupations, while also relating them to establishments.

Structural change Figure 1 shows the (real) value-added output and employment share trends of three broadly defined sectors: agriculture, manufacturing and services, from 1970 to 2013. Following convention, e.g. Herrendorf et al. (2014), "manufacturing" is the aggregation of the manufacturing, mining and construction sectors and "services" the sum of all service and government sectors. The data are from the National Accounts published by the Bureau of Economic Analysis (BEA). In particular, employment is based on National Income and Product Accounts (NIPA) Table 6 (persons involved in production), counted in terms of full-time equivalent workers.[5]

Two facts are well documented in the literature. First, starting from even before 1970, agriculture was already a negligible share of the U.S. economy. While it shrank from about 4% of GDP to 2% in the 1990s, its share has stayed at this level both in terms of output and employment. Thus for the remainder of this paper, we will drop

[5]Computing employment shares from the decennial censuses yields more or less the same trend.



(a) Employment Polarization

(b) Wage Polarization

Fig. 2: Job and Wage Polarization, 1980–2010.
Source: U.S. Census (5%), extends Autor and Dorn (2013), which ends in 2005. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The x-axis units are in percentage share of 1980 aggregate employment. The y-axis measures changes the 30 year change, of which units are in percentage points in panel (a). Further details in text and Appendix A.

the agricultural sector and only consider the manufacturing and service sectors, broadly defined. Accordingly, all moments will be computed as if the aggregate economy consists only of these two sectors (e.g. aggregate employment is the sum of manufacturing and service employment, the manufacturing and service shares sum up to unity, etc).

Second, structural change—the shifting of GDP and employment from manufacturing to services—exhibits a smooth trend starting from at least the 1970s. Moreover, GDP and employment shares are almost identical both in terms of levels and trends. This implies a close to constant input share of labor across the two sectors, which we will assume in our theoretical model.

Job and Wage Polarization Most of the rest of our empirical analysis is based on the decennial U.S. Censuses 1980–2010, for which we closely follow Autor and Dorn (2013). We restrict our sample to 16–65 year-old non-farm employment. Figure 2 plots employment and wage changes by occupation from 1980 to 2010, extending Figure 1 in Autor and Dorn (2013) who considered changes up to 2005. Occupations are sorted into employment share percentiles by skill along the x-axis, where skill is proxied by the mean (log) hourly wage of each occupation. We follow the 3-digit `occ1990dd` occupation coding convention in Dorn (2009), which harmonizes the `occ1990` convention in Meyer and Osborne (2005). This results in 322 occupation categories for which employment is positive for all 4 censuses. Employment is defined as the product of

weeks worked times usual weekly hours.

The data is presented in two ways. First, following Autor and Dorn (2013), each dot in Figure 2 represents one percent of employment in 1980. The y-axis in Panel (a) measures each skill percentile's employment change from 1980 to 2010 in percentage points, and in Panel (b) the change in its mean wage. The changes are smoothed into percentiles across neighboring occupations using a locally weighted smoothing regression. Despite the Great Recession happening between 2005 and 2010, the long-run patterns are virtually the same as in their study: employment has shifted from the middle toward both lower and higher skill jobs. Likewise, wages have risen the least in the middle, and much more toward the top.

Second, we group all occupations into 11 broad categories, vaguely corresponding to the 1-digit Census Occupation Codes (COC). These groups are ordered by the mean wage of each broadly defined group. To represent the groups in skill percentiles, the length of each group along the x-axis is set equal to its 1980 employment share.[6] In Figure 2(a), the percentage point change of a group's employment share is represented by the area of the bar,[7] while the height of each bar in Panel (b) measures the change in its mean wage. Except for sales and (more so for) technician occupations, the patterns of polarization emphasized by Acemoglu and Autor (2011); Autor and Dorn (2013) are still intact.[8]

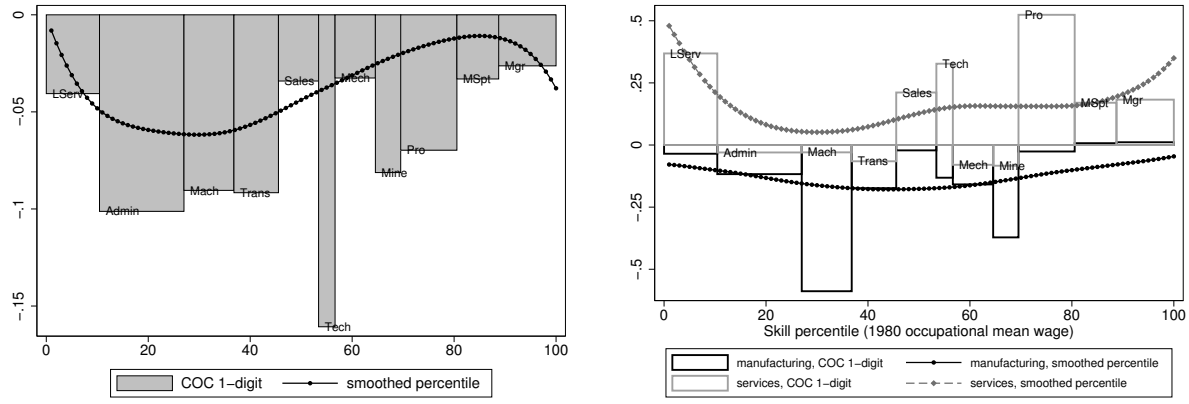
Polarization and Structural Change In Figure 3, we plot the same data but along two different dimensions. In Panel (a), occupations are ordered along the x-axis in the same way as we did in Figure 2. For each occupation, we compute the employment share of manufacturing in the two years 1980 and 2010, and plot the difference.[9] The bars measure the percentage point change in the share of manufacturing employment for each COC occupation group, and the dots the smoothed percentage point change for each skill percentile. The entire plot is negative, which merely indicates structural change—manufacturing shares went down in all occupations. More important, manufacturing shrunk the most in the middle (again, except for the COC

[6] See Appendix A for more details on wage, employment, and occupation definitions. Note that the 3-digit occupations in each group does not necessarily correspond to those used to generate the smooth graphs by percentile.

[7] So by definition, the area of all bars must add up to 0. The smoothed graph should also integrate to in theory, but does not due to the locally weighted regression errors.

[8] The exact numbers behind these graphs are summarized in Appendix A Table 5, which also form the basis for our calibration in Section 5.

[9] Appendix Figure 19(a) shows the share of manufacturing employment for each occupation in 1980, which shows more reliance on middle-skill jobs in manufacturing.



(a) Manufacturing Employment

(b) Employment Polarization by Sector

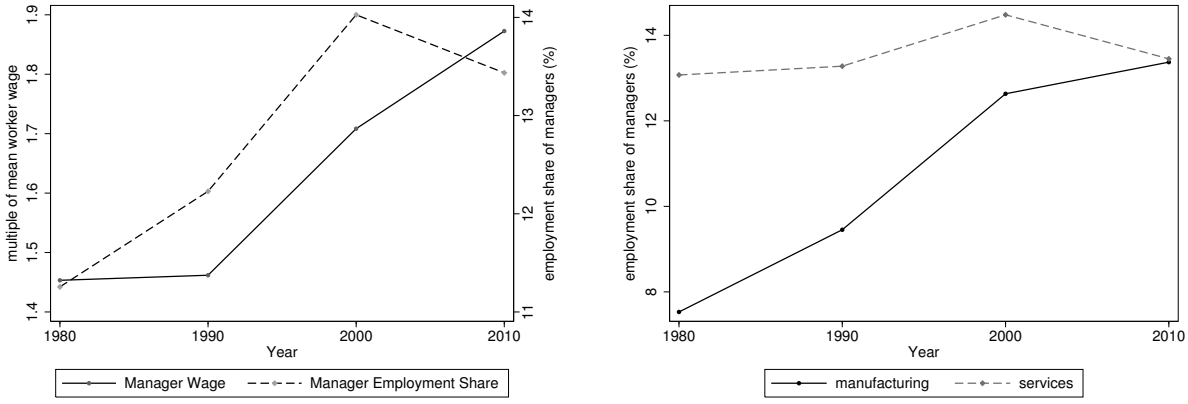
Fig. 3: Polarization and Structural Change, 1980–2010.
Source: U.S. Census (5%). Left: percentage point change in manufacturing employment by occupation-skill percentile. Right: percentage point change in occupation employment by sector. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. The x-axis units are in percentage share of 1980 aggregate employment. The y-axis measures changes 30 year change, of which units are in percentage points in Panel (b). Further details in text and Appendix A.

technicians group).[10] This suggests that structural change and polarization not only happened simultaneously, but may in fact be interrelated: Structural change tended to be faster among middle-skill occupations, which sectoral forces alone may have a hard time explaining.

To check the presence of task-specific effects, in Panel (b) we plot the same changes as in Figure 2(a), but separate manufacturing (in dark) and services (in light).[11] Manufacturing lost employment across all jobs, which again is just structural change. Note that this is mostly pronounced in the middle, especially among machine operators and miners. In contrast, services gained employment in all jobs, but mostly among extreme occupation-skill percentiles, particularly among low-skill services and professionals. What is important is that polarization is observed in both sectors. Hence it is unlikely that sectoral forces alone could have caused polarization: If so, we would expect employment shifts across occupations to be flatter either in Panel (a), which

[10]Technicians include software engineers and programmers, paralegals, and health technicians, which grew rapidly during this time period along with the service sector. Indeed, many of the smooth graphs are flatten due to occupations in this group. However, they comprise a very small fraction of the U.S. economy throughout the observed period.

[11]So the area of all bars for one sector admits structural change, while adding them across both sums to zero. Similarly the integrals of the smoothed graphs should sum to zero, subject to the locally weighted regression errors.



(a) Relative Employment and Wages

(b) Manager Employment Share

Fig. 4: Managers vs Workers

Source: U.S. Census (5%). Left: relative wage and employment share of managers in aggregate. Right: employment share of managers within manufacturing and services. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for how we define management in the census and Figure 18 for a detailed breakdown of the manager group.

plots structural change in levels, or in Panel (b), which plots it in ratios. The fact that the U-shape persists in all cases suggests task-specific forces that affect both sectors.

Vertical Polarization In our model and quantitative analysis, we treat managers as a special occupation that organizes all other occupations. While many studies emphasize the organize of production (Garicano and Rossi-Hansberg, 2006), most focused on top CEO's of publicly traded companies (Tervio, 2008; Gabaix and Landier, 2008) or certain industries (Caliendo, Monte, and Rossi-Hansberg, 2015). We treat managers as a broader group including the self-employed, and also relate it to aggregate establishment-size distributions by sector. Previous papers have shown top CEO wages rising astronomically compared to the median worker, and Figure 4(a) shows that even with our broader definition, mean manager wages have risen as much as 50% relative to all other workers. At the same time, the employment share of managers has also risen by 2–3 percentage points, although there is a small drop from 2000 to 2010.[12] We refer to this phenomena throughout the paper as "vertical polarization," to distinguish from the horizontal polarization discussed above.

More important for us, Figure 4 reveals that the rise in managers' employment

[12] A separate analysis of the American Community Survey, not shown here, shows that managers' employment share continued to rise up to 2005, but then dropped by more than a percentage point, especially since the Great Recession. Since we only focus on long-run trends, we do not specifically address such high-frequency movements.

shares rose mostly in manufacturing and barely at all in services; Appendix Figure 20 shows that managers' mean wages relative to workers' also grew much more quickly within manufacturing than in services. This again suggests task- or occupation-specific forces, since sector-specific forces would not create such differences across sectors.[13]

We now present a model which explains all these phenomena by a single force, task-specific technological progress among middle-skill worker occupations, followed by a quantitative assessment and empirical investigation of where this progress stems from.

3 Model

There are a continuum of individuals each endowed with two types of skill, $(h, z) \in \mathcal{H} \times \mathcal{Z} \subset \mathbb{R}^+ \times \mathbb{R}^+$. Worker human capital, h, is used to produced worker tasks. Managerial talent, z, is a skill for organizing tasks. Without loss of generality, we assume that the mass of individuals is 1, with associated cumulative distribution function $\mu(h, z)$.

There are 2 sectors $i \in \{m, s\}$.[14] In each sector, goods are produced by teams. A team is led by a manager who uses his managerial skill and physical capital to organize a continuum of worker tasks $j \in \mathcal{J} = [0, J]$, where J is finite.

We will refer to the managerial task as "task z," which is vertically differentiated (in a hierarchical sense) from tasks $j \in \mathcal{J}$, which are horizontally differentiated among workers. Each worker task requires both physical and worker human capital, and their allocations are decided by the manager. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M(z) k_{iz}(z)^{\alpha} z^{1-\alpha}, \qquad x_h(z) = \left[\int_{j=0}^{J} \nu_i(j)^{\frac{1}{\sigma}} \tau_i(j;z)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{1b}$$

$$\tau_i(j;z) = M(j) k_{ih}(j;z)^{\alpha} \left[\int_{\mathrm{h}_i(j;z)} b(h,j) d\mu\right]^{1-\alpha}, \tag{1c}$$

with $\{\nu_i(j), \eta_i\} \in (0,1)$ for all $i \in \{m, s\}$ and $j \in \mathcal{J}^z \equiv \mathcal{J} \cup \{z\}$, and $\int_j \nu_i(j) dj = 1$. The quantity $\tau_i(j;z)$ is the amount of task j output produced for a manager of skill z in sector i. This manager uses physical capital $k_{iz}(z)$ for himself, and allocates $k_{ih}(j;z)$

[13] In Appendix Figure 21(a), we instead plot the manufacturing employment share among managers and workers, which shows that structural change was much more prevalent among workers than managers. This is further evidence against sector-specific forces.

[14] In our application, the two sectors stand in for "manufacturing" and "services," respectively. However, the analysis can be extended to any finite number N of sectors. We use the subscripts m and s to avoid confusing them with the subscripts for tasks.

capital and a set of workers $\mathrm{h}_i(j;z)$ to task j. The function $b(h,j)$ is the productivity of a worker with human capital h assigned to task j.

Assumption 1 (Log-supermodularity) *The function $b : \mathcal{H} \times \mathcal{J} \mapsto \mathbb{R}^+$ is strictly positive and twice-differentiable, and is log-supermodular. That is, for all $h' > h$ and $j' > j$:*

$$\log b(h',j') + \log b(h,j) > \log b(h',j) + \log b(h,j'). \tag{2}$$

Assumption 1 will ensure that high-h workers sort into high-j tasks in any equilibrium. Integrating $b(h,j)$ over h of the workers in the set $\mathrm{h}_i(j;z)$ yields the total productivity of all workers assigned to task j by a manager of skill z in sector i.

Substitutability between tasks is captured by the elasticity parameter σ, while ω captures the elasticity between workers and managers (or, between the composite worker task x_h and the managerial task x_z). The $M(j)$'s, $j \in \mathcal{J}^z$, are task-specific TFP's, which are sector-neutral.

Let $\mathcal{Z}_i$ denote the set of individuals working as managers in sector i. Aggregating over the output from all managers in sector i yields sectoral output

$$Y_i = A_i \int_{\mathcal{Z}_i} y_i(z) d\mu, \quad i \in \{m,s\}, \tag{3}$$

where A_i is an exogenous, sector-level productivity parameter. Final goods are produced by combining output from both sectors according to a CES aggregator:

$$Y = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\gamma_m + \gamma_s = 1$. We will assume $\epsilon < 1$.[15]

Except Costinot and Vogel (2010), most task-based models assume that either tasks or skills are a continuum, but not both (Acemoglu and Autor, 2011; Stokey, 2016). The worker side of our model is similar to theirs, but includes capital as a production factor and is extended to multiple sectors. In contrast to all existing models, we also add 2-dimensional skills and consider managers as a special occupation, which generates additional insights both theoretically and quantitatively.

3.1 Planner's Problem

We start by solving a static planner's problem. A planner allocates aggregate capital K and all individuals into sectors $i \in \{m,s\}$ and tasks $j \in \mathcal{J}^z$. Formally, define $h_i(j;z)$

[15] The estimated ϵ between the manufacturing and the service sectors (broadly defined) is close to zero, as we show in section 5.2.

as the amount of human capital the planner allocates to task j in sector i under a manager with z. Also define $l_{ih}(s,j)$ as the number of individuals with skill $s=(h,z)$ that the planner assigns to task j, and $l_{iz}(s)$ the number of individuals with skill s the planner assigns as managers, in sector i. Then the planner's problem is to choose factor allocation rules $\{k_{iz}(z), k_{ih}(j;z), h_i(j;z)\}$ and assignment rules $\{l_{ih}(s,j), l_{iz}(s)\}$ to maximize current output (4) subject to

$$\begin{aligned}
&Y_i = A_i \int y_i(z) l_{iz}(s) ds, \quad \forall i \in \{m,s\},\\
&K = K_m + K_s \equiv \sum_{i\in\{m,s\}} \int \left\{ \left[k_{iz}(z) + \int_j k_{ih}(j;z) dj \right] \cdot l_{iz}(s) \right\} ds\\
&H_i(j) \equiv \int b(h,j) l_{ih}(s,j) ds = \int \left[\int_{\mathrm{h}_i(j;z)} b(h,j) d\mu \right] \cdot l_{iz}(s) ds \quad \forall i \text{ and } \forall j \in \mathcal{J},\\
&d\mu(s) = \sum_{i\in\{m,s\}} \left[\int l_{ih}(s,j) dj + l_{iz}(s) \right] ds, \quad \forall s \in \mathcal{H}\times\mathcal{Z},
\end{aligned} \tag{5}$$

where K_i is the capital allocated to sector i, and $H_i(j)$ the total productivity of workers allocated to task j in sector i.

For existence of a solution, we assume that

Assumption 2 *There exists a strictly positive mass of jobs such that $b(0,j)>0$ and individuals such that $b(h,J)>0$.*

The following assumption is needed for uniqueness:

Assumption 3 *The domain of skills $\mathcal{H}\times\mathcal{Z}=[0,h_M]\times[0,\infty)$, where $h_M<\infty$ is the upper bound of h. The measure $\mu(h,z)$ is differentiable and $d\mu(h,z)>0$ is continuous on $\mathcal{H}\times\mathcal{Z}$.*

Assumption 3 implies that we can write

$$\mu(\tilde{h},\tilde{z}) = \int^{\tilde{h}} \int^{\tilde{z}} dF(z|h) dG(h) = \int^{\tilde{h}} \left[\int^{\tilde{z}} f(z|h) dz \right] g(h) dh,$$

where (G,g), the marginal c.d.f. and p.d.f. of h, and (F,f), the c.d.f and p.d.f. of z conditional on h, are continuous.

The optimal factor allocation rules across managers, $\{k_{iz}(z), k_{ih}(j;z), h_i(j;z)\}$, are straightforward: They must equalize marginal products across managers with different z's. Since we assume a constant returns to scale technology at the level of managers, we can aggregate over managers in (1) to write sectoral output as

$$Y_i = A_i \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{6a}$$

$$X_{iz} = M(z)K_{iz}^{\alpha}Z_i^{1-\alpha}, \qquad X_{ih} = \left[\int_j \nu_i(j)^{\frac{1}{\sigma}} T_i(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{6b}$$

where K_{iz} is the total amount of capital allocated to managers and $Z_i \equiv \int z l_{iz}(s) ds$. Similarly, the sectoral task composite X_{ih} combines sectoral task aggregates

$$T_i(j) = M(j)K_{ih}(j)^{\alpha} H_i(j)^{1-\alpha}, \tag{7}$$

where $K_{ih}(j)$ is the total amount of capital allocated to task j in sector i.

In the remainder of this section, we characterize the solution to the planner's problem in the following order:

1. Optimal physical capital allocations across tasks within a sector.
2. Optimal worker assignment across tasks within a sector.
3. Optimal allocation of managers vs. workers within a sector.

We then solve for the within-sector solution in Section 3.2, which allows us to express the sectoral production function (3) only in terms of the optimal assignment rules and sectoral endowments. Given this, we show in Section 3.3 that the two-sector equilibrium is unique, which we use to conduct comparative statics in Section 4. In what follows, most algebraic derivations are relegated to the Appendix.

Capital allocation within sectors For any level of sectoral capital K_i, the planner equalizes the marginal product of capital across tasks. Given the technologies assumed in (6)-(7), this means that all capital decisions can be expressed as a linear function of the capital used in task 0. Specifically, capital input ratios across worker tasks must satisfy

$$\pi_{ih}(j) \equiv \frac{K_{ih}(j)}{K_{ih}(0)} = \left[\frac{\nu_i(j)}{\nu_i(0)}\right]^{\frac{1}{\sigma}} \cdot \left[\frac{T_i(j)}{T_i(0)}\right]^{\frac{\sigma-1}{\sigma}}, \tag{8}$$

with which we can express the worker task composite X_{ih} in (7) as

$$X_{ih} = \nu_i(0)^{\frac{1}{\sigma-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} T_{i0}, \quad \text{where } \Pi_{ih} \equiv \int_j \pi_{ih}(j) dj. \tag{9}$$

Of course, marginal products must also be equalized across the managerial task and the rest. So using (9) we can define

$$\pi_{iz} \equiv \frac{K_{iz}}{K_{ih}(0)} = \left(\frac{\eta_i}{1-\eta_i}\right)^{\frac{1}{\omega}} \cdot \left(\frac{X_{iz}}{X_{ih}}\right)^{\frac{\omega-1}{\omega}} \cdot \Pi_{ih}, \tag{10}$$

and note that π_{iz} does not vary with j. Equations (8) and (10) subsume the capital allocation decisions into the labor allocation rules through $[\pi_{ih}(j), \pi_{iz}]$.

Sorting workers across tasks within sectors Since we assume $b(h, j)$ is strictly log-supermodular, Assumptions 1-3 imply that there exists a continuous assignment function $\hat{j} : [0, h_M] \mapsto \mathcal{J}$ s.t. $\hat{j}'(h) > 0$, and $\hat{j}(0) = 0$, $\hat{j}(h_M) = J$.[16] That is, there is positive sorting of workers into tasks, and workers of skill h are assigned to job $\hat{j}(h)$. Since $\hat{j}'(h) > 0$, we can also define its inverse $\hat{h} : \mathcal{J} \mapsto [0, h_M]$. This inverse assignment function is the solution to the differential equation we will present in (35a), which will be needed to characterize and solve for the full equilibrium.

It should be clear that $[\hat{j}(h), \hat{h}(j)]$ are identical across sectors, and hence not indexed by i. Otherwise, the planner would be able to reallocate h across sectors and increase output. So the planner's problem of choosing $l_{ih}(s, j)$ has two parts: One of choosing $l_h(s, j)$, the number of individuals with skill s the planner assigns to task j *regardless of sector*, and the other of choosing $q_{ih}(j)$, the fraction of task j workers that are allocated to sector i, which satisfies $\sum_{i \in \{m,s\}} q_{ih}(j) = 1$. That is, we can write

$$l_{ih}(s, j) = q_{ih}(j) \cdot l_h(s, j).$$

Sorting managers vs. workers within a sector Since individuals are heterogeneous in 2 dimensions, standard Roy selection implies a cutoff rule $\tilde{z}(h)$ s.t. for every h, individuals with z above $\tilde{z}(h)$ become managers and the rest become workers. Since we know that workers sort positively into j, we can also define $\hat{z}(j) = \tilde{z}(\hat{h}(j))$. Then the planner's problem is to get this implied rule $\hat{z}(j)$ for all $j \in \mathcal{J}$. As was the case with workers, the manager selection rule must also be identical across sectors.

Similarly as in Lucas (1978), $\hat{z}(j)$ is chosen so that the marginal product of the threshold manager is equalized between tasks z and task j, so

$$\hat{z}(j)/b(\hat{h}(j), j) = \pi_{ih}(j) Z_i / \pi_{iz} H_i(j), \quad \forall j \in \mathcal{J}. \tag{11}$$

Without loss of generality, we normalize $b(0, 0) = 1$ to obtain

$$\hat{z} \equiv \hat{z}(0) = \tilde{z}(0) = Z_i / \pi_{iz} H_i(0) \tag{12}$$

which is the worker counterpart of (10): the total productivity of managers in sector i is normalized in terms of task 0's productivity, $H_i(0)$. In the next subsection, we normalize all other workers' productivities in terms of task 0's productivity a well.

[16] For a more formal proof, refer to Lemma 1 in Costinot and Vogel (2010).

3.2 Within-Sector Solution

First consider the rule $l_{iz}(s)$. Since the rule $\tilde{z}(h) = \hat{z}(\hat{j}(h))$ does not depend on sector, we have

$$\int l_{iz}(s)ds = q_{iz} \int [1 - F(\tilde{z}(h)|h)]\, dG(h), \tag{13}$$

where q_{iz} is a sectoral weight that satisfies $\sum_{i\in\{m,s\}} q_{iz} = 1$. Note that any solution that satisfies (13) such that $l_{iz}(s) = 0$ iff $z \leq \hat{z}(j)$ is optimal. Hence, the planner's choices of $\hat{z}(j)$ and q_{iz} are unique, but not the rule $l_{iz}(s)$: The planner does not care how managers are allocated between sectors for any particular $s \in \mathcal{S}$, and only about how the total Z is divided between sectors, where

$$Z \equiv \int \int_{\tilde{z}(h)}^{\infty} z dF(z|h) dG(h) = \sum_{i\in\{m,s\}} q_{iz} Z_i.$$

Next consider the planner's choice of $l_h(s, j)$. The characterization is similar to Costinot and Vogel (2010) and summarized in Lemma 1.

Lemma 1 *Define*

$$B_j(j; \hat{h}) \equiv \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj'\right]. \tag{14}$$

At the planner's solution, the productivity of all workers assigned to task j can be expressed as

$$H(j) = b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j)) \cdot \hat{h}'(j), \quad \forall j \in \mathcal{J}, \tag{15}$$

and their ratios across tasks in sector i must satisfy

$$q_{ih}(j)H(j) = H_i(j) = \pi_{ih}(j)H_i(0)B_j(j; \hat{h}), \tag{16}$$

Proof See Appendix B.1. ∎

The lemma expresses all other worker allocation decisions in terms of $H_i(0)$, similarly as we could normalize all other capital allocation decisions in terms of $K_i(0)$ in (8) and (10). Equation (15) simply states that total worker productivity in task j is the product of the infinitesimal mass of individuals assigned to task j, times their effective productivity. Equation (16) is the counterpart of (8), and means that the marginal products of labor are equated at every point along $\mathcal{J}$. Similarly, all manager allocations can also be normalized in terms of $\hat{z}$.

Corollary 1 *Define the counterpart of B_j in (14):*

$$B_h(h;\hat{j}) \equiv \exp\left[\int_0^h \frac{\partial \log b(h',\hat{j}(h'))}{\partial h'} dh'\right]. \tag{17}$$

At the planner's solution, the productivity of the cutoff rules $[\hat{z}(j), \tilde{z}(h)]$ can be expressed as

$$\hat{z}(j) = \hat{z} \cdot b(\hat{h}(j), j)/B_j(j;\hat{h}) = \hat{z} \cdot B_h(\hat{h}(j);\hat{j}) \quad \Leftrightarrow \quad \tilde{z}(h) = \hat{z} \cdot B_h(h;\hat{j}). \tag{18}$$

Proof See Appendix B.1. ∎

The corollary makes all manager cutoff rules except $\hat{z}$ (the rule at $h = 0$) redundant, since they can be expressed only in terms of $\hat{z}$ in (12) and $\hat{h}(j)$. In what follows, we suppress the dependence of (B_j, B_h) on $(\hat{h}, \hat{j})$ unless necessary.

So far we have normalized all allocations only in terms of task 0's capital and worker input, $[K_i(0), H_i(0)]$. Now we show that given a sectoral allocation rule for each task, $[q_{ih}(j), q_{iz}]$, the within-sector equilibrium is unique and completely independent of the aggregate level of capital and labor within a sector. This admits a sectoral production function in which sectoral TFP is solely determined by the optimal allocation rules.

Proposition 1 *Suppose $[q_{ih}(j), q_{iz}]$ are given. Under Assumptions 1-3, the within-sector solution to the planner's problem, $\left\{[\hat{h}(j)]_{j=0}^J, \hat{z}\right\}$, exists and is unique, and is independent of sectoral aggregates (K_i, L_i).*

Proof See Appendix B.2. ∎

Corollary 2 *At the planner's optimum, sectoral output can be expressed as*

$$Y_i = \Phi_i \cdot K_i^{\alpha} L_i^{1-\alpha}, \quad \text{where } \Phi_i \equiv M(0) \cdot \psi_i \cdot \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1} \tag{19}$$

is the sectoral TFP, endogenously determined by the optimal allocation rules. Sectoral TFP can be decomposed into 3 parts:

1. *$M(0)$, which is common across both sectors and exogenous;*
2. *$\psi_i \equiv A_i(1-\eta_i)^{\frac{1}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}}$, which is also exogenous but sector-specific;*
3. *the part determined by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by the allocation rules $[\hat{h}(j), \hat{z}]$, where*

$$\Pi_{K_i} \equiv \Pi_{ih} + \pi_{iz} = K_i/K_i(0), \quad \Pi_{L_i} \equiv \Pi_{il} + (\hat{z}/\bar{z})\pi_{iz} = L_i/H_i(0), \tag{20}$$

are the total amounts of capital and labor in sector i in units of task 0 capital and labor allocated to sector i, respectively, and $\Pi_{il} \equiv \int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj$.

Proof See Appendix B.2. ∎

Sectoral TFP's are independent of sectoral capital and labor shares because the rules $[\hat{h}(j), \hat{z}]$ depend only on the relative masses of individuals across tasks *within* a sector, and not on the employment shares *across* sectors (nor capital). In fact, it is the sectoral TFP's that determine sectoral input shares. Since sectors only differ in how intensively they use each task, employment shares are determined so that the marginal products of capital and labor are equalized across sectors:

$$\kappa \equiv \frac{K_s}{K_m} = \frac{L_s}{L_m} = \left(\frac{\gamma_s}{\gamma_m}\right)^{\frac{1}{\epsilon}} \left(\frac{Y_s}{Y_m}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\Phi_s}{\Phi_m}\right)^{\epsilon-1} \tag{21}$$

where κ is the capital input ratio between sector s and m. Hence relative employment between sectors is completely determined by the relative endogenous TFP ratio between the two sectors. And since the Φ_i's are just functions of $[\hat{h}(j), \hat{z}]$, so is κ and also sectoral employment shares L_i:

$$L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa). \tag{22}$$

Consequently, the aggregate levels of K or L (which we normalize to 1) have no impact whatsoever on the assignment rules nor employment shares either.

3.3 Existence and Uniqueness of Full Solution

A solution to the planner's problem coincides with an equilibrium in our economy, so existence and uniqueness of an equilibrium is equivalent to a unique solution to the full planner's problem. As a final step, the planner needs to ensure that the within-sector allocations are consistent with the between sector allocations. That is, $[q_{ih}(j), q_{iz}]$, the weights used to split the distribution μ between sectors, must be consistent with (21). In equilibrium, these are equivalent to the the labor market clearing conditions.

For ease of notation, let $q_h(j)$ and q_z denote the service share of employment in tasks j and z, respectively; so $q_{mh}(j) = 1 - q_h(j)$ and $q_{mz} = 1 - q_z$. Now since $[\hat{h}(j), \hat{z}]$ must be equal across sectors, we can use the within-sector solutions from Proposition 1 to find the $[q_h(j), q_z]$ that ensure this. The proposition already showed that the within-sector solution is unique, but for uniqueness of the full solution we need additional assumptions on the distribution μ and the worker productivity function $b(h, j)$ that will serve as sufficient conditions for a unique equilibrium: Assumption 4.1 will guarantee that the within-sector equilibrium is unique, and Assumptions 4.2-4.3 and 5.2 that the between-sector equilibrium is unique.

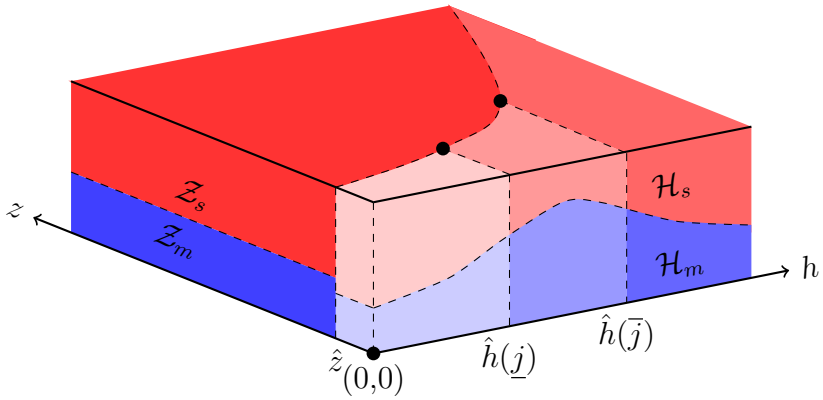


Fig. 5: Equilibrium

Assumption 4 *For all* $(h, z) \in \mathcal{H} \times \mathcal{Z}$,

1. $g'(h) \leq 0$ *and* $f'(z|h) \leq 0$,
2. $F(z|h)/\left[1 - F(z|h)\right] \leq z/\left[\int_z^\infty z' f(z'|h) dz'\right]$, *and*
3. $zf(z|h)/F(z|h) \geq (1-\alpha)(1-\omega)$.

Assumption 4.1 means that there are less people at higher levels of skill (the p.d.f.'s of h and z are declining everywhere), which is a common assumption and also consistent with empirical evidence. Assumptions 4.2-4.3 mean that the conditional distribution of z is declining but not too much, in the sense that it still has fat tails beyond any value of h.

Assumption 5 *For all* $(h, j) \in \mathcal{H} \times \mathcal{J}$,

1. $\partial b(h, j)/\partial h > 0$
2. *for all* $\varepsilon > 0$, $0 < \partial^2 \log b(h, j)/\partial h \partial j < \varepsilon$.

Assumption 5.1 captures the notion that higher-h workers perform better in any task; in particular, under this assumption, $\tilde{z}(h)$ in (18) is a strictly increasing function. That the cross partial is larger than 0 is already in Assumption 1. Assumption 5.2 means that there is just enough log-supermodularity so that workers positively sort into tasks.

Theorem 1 *Under Assumptions 1-5, the solution to the planner's problem,* $[\hat{h}(j), q_h(j)]_{j=0}^J$ *and* $(\hat{z}, q_z)$, *exists and is unique.*

Proof See Appendix B.3. ∎

For illustrative purposes, the equilibrium skill allocation with a uniform $\mu(z,h)$ is depicted in Figure 5. Those in $\mathcal{Z}$ are managers, and those in $\mathcal{H}$ are workers, where the subscripts s and m denote services and manufacturing. Workers sort into tasks indexed by j according to $\hat{h}(j)$. The different masses of sectoral employment across tasks are due to the task intensity parameters $\nu_i(j)$ and η_i.

3.4 Equilibrium wages and prices

The solution $[\hat{h}(j), \hat{z}]$ gives all the information needed to derive equilibrium prices (which are unique). The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = \left[Y_i/\gamma_i Y\right]^{-\frac{1}{\epsilon}}$$

and sectoral output prices p_i can be obtained by plugging in the sectoral production function (19).

Let R denote the rental rate of capital and $w_h(h)$ the wage of a worker with skill h. The rental rate R is given either by the dynamic law of motion for aggregate capital, or fixed in a small open economy. Since capital and labor input ratios are equalized across sectors, $w_h(0)$ can be found from either sector:

$$w_h(0) = \frac{1-\alpha}{\alpha} \cdot \frac{RK_i(0)}{H_i(0) \cdot b(0,0)} = \frac{1-\alpha}{\alpha} \cdot \frac{\Pi_{L_i}}{\Pi_{K_i}} \cdot RK,$$

where the second equality follows from (19)-(21), and since we normalized both $b(0,0)$ and the population size to 1. Similarly, all workers earn their marginal product, so we can write

$$w_h(h) = w_h(0) \cdot B_h(h). \tag{23}$$

Assumption 5.1 implies that $w(h)$ is strictly increasing in h.

For all $h \in \mathcal{H}$, threshold managers with skill $z = \tilde{z}(h)$ are indifferent between becoming a worker or manager, so we can determine a managerial wage rate or rental rate of z (w_z) that satisfies

$$w_z \tilde{z}(h) = w_h(h) \quad \Rightarrow \quad w_z = w_h(0)/\hat{z}, \tag{24}$$

and (18) and (23) guarantee that the first equality holds for all h.

4 Comparative Statics

The sectoral technology representation (19) implies that this model has similar implications as models of structural change, for example Ngai and Pissarides (2007). The major difference is that these sectoral TFP's are endogenous in our model.

What is more interesting is the implications of growth in task-specific TFP's.[17] In particular, we are interested in the effect of routinization, which we model as an increase in $M(j)$ for all $j \in \mathcal{J}_1 \equiv [\underline{j}, \overline{j}]$, where $0 < \underline{j} < \overline{j} < J$; that is, as an increase in the TFP of middle-skill tasks. We refer to this group of tasks as "routine jobs." The impact of such task-specific technological progress is illustrated in a series of comparative statics in this section, which is possible since the equilibrium is unique and skill distribution is assumed to be continuous.

4.1 Wage and Job Polarization

First, we will focus on the comparative statics of $[\hat{h}(j), \hat{z}]$ within a sector i. Our exercise assumes that there is an increase in the exogenous productivity, $M(j)$, of middle-skill j-tasks. Since $M(j)$'s are separate from the worker's human capital h, this can be interpreted as a rise in capital-augmented TFP for middle-skill jobs, or "routinization."[18]

Proposition 2 (Routinization and Polarization) *Let $\mathcal{J}^1 \equiv (\underline{j}, \overline{j}) \subset \mathcal{J}$, where $0 < \underline{j} < \overline{j} < J$. Suppose $q_h(j)$ and q_z are held constant, and that $M(j)$ uniformly grows to $M^1(j) = M(j)e^m$ for all $j \in \mathcal{J}^1$, where $m > 0$. Then under Assumptions 1-4,*

1. *if $\sigma < 1$, there exists $j^* \in \mathcal{J}^1$ such that $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (0, j^*)$ and $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j^*, J)$, and*
2. *if $\omega < \sigma < 1$ and Assumption 5.2 holds, there exists some $\varepsilon > 0$ s.t. $\tilde{z}^1(h) < \tilde{z}(h)$ for all $0 < m < \varepsilon$ and $h \in \mathcal{H}$.*

Proof See Appendix B.4. ∎

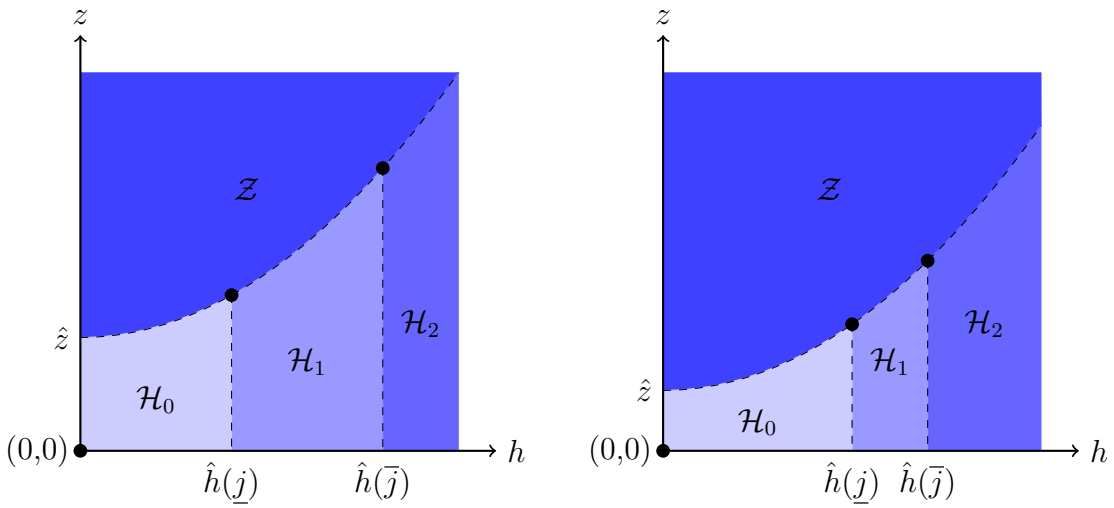
Part 1 implies that among worker tasks, capital and labor flow out from middle-skill tasks into the extremes (horizontal job polarization). The relative wages of the middle-skill tasks decline (horizontal wage polarization), since from (23),

$$\log\left[\frac{w^1(h)}{w^1(h^*)}\right] - \log\left[\frac{w(h)}{w(h^*)}\right] = \int_{h^*}^{h}\left[\frac{\partial \log b(h, \hat{j}^1(h))}{\partial h} - \frac{\partial \log b(h, \hat{j}(h))}{\partial h}\right] dh$$

is positive for all $j \neq j^* \in \mathcal{J}^1$. Part 2 implies that capital and labor flow into management from *all* worker tasks, and (24) means that each manager earns a higher

[17] Alternatively, this could have been modeled as the price of task-specific capital falling as in Goos et al. (2014), with few changes in model implications.

[18] This *within-sector* exercise is similar to Lemma 6 in Costinot and Vogel (2010), except that we have capital and, more important, the vertically differentiated manager.



(a) Equilibrium wt low M_1

(b) Equilibrium wt high M_1

Fig. 6: Comparative Static, Within-Sector

wage per managerial skill (vertical polarization). The within-sector comparative static for employment shares is depicted in Figure 6, and are consistent with the data we saw in Figures 2 and 4(a).

The mechanism for part 1 is the same as in Goos et al. (2014): when $\sigma < 1$, the exogenous rise in productivity causes factors to flow out to other tasks since tasks are complementary, and we get employment polarization. As in Costinot and Vogel (2010), this also leads to wage polarization in the presence of positive sorting. What is new in our model is that this happens even in the presence of the vertically differentiated management task, and that with stronger complementarity between workers and managers than among workers (i.e., $\omega < \sigma < 1$), similar forces drive vertical polarization in terms of both wages and employment.[19] Another novel feature is the impact of this task-specific technological progress on sectoral allocation, which we now explain.

4.2 Structural change

Previous models of structural change either rely on a non-homogeneous form of demand (rise in income shifting demand for service products) or relative technology differences across sectors (rise in manufacturing productivity relative to services, combined with

[19] A change in factor neutral TFP, $M(j)$, is different from an increase in the amount of skill working in any given task. Since we are modeling worker skill as human capital, the qualitative effect of a change in TFP is the same as if it were only capital-augmenting. Changes in the skill distribution could have different effects, which we do not pursue here.

complementarity between the two types of goods, shifting production to services). Our model is also technology driven, but structural change arises from a skill- and sector-neutral increase in task productivities. Most important, in contrast to recent papers arguing that sectoral productivity differences can generate broadly-measured skill premia or polarization (Buera et al., 2015; Bárány and Siegel, 2017), we argue exactly the opposite—that routinization explains not only job and wage polarization but also sectoral productivity differences and structural change.

Decomposing Polarization Define the "unnormalized" total worker productivity

$$V_{L_i} = (1-\eta_i)\nu_i(0)M(0)^{\sigma-1}\Pi_{L_i} = V_{il} + V_i(z), \quad \text{where } V_{il} = \int V_i(j)dj \tag{25}$$

and the weights $V_i(j)$ for each task $j \in \mathcal{J}^z$ are

$$V_i(j) = (1-\eta_i)\nu_i(j)\left[M_h(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}/B_h(\hat{h}(j)) \tag{26a}$$

$$V_i(z) = \eta_i M_z^{\omega-1}\cdot V_{ih}^{\frac{\sigma-\omega}{\sigma-1}}\cdot \hat{z}^{\alpha+\omega(1-\alpha)}/\bar{z}, \tag{26b}$$

and

$$V_{ih} \equiv \nu_i(0)M(0)^{\sigma-1}\Pi_{ih} = \int\left\{\nu_i(j)\cdot\left[\tilde{M}(j)\cdot B_j(j)^{1-\alpha}\right]^{\sigma-1}\right\}dj. \tag{27}$$

These are simply the marginal products of task j unnormalized by $H_i(0)$, so we know that taking the ratio of any pair yields the labor input ratio between the two tasks; (27) is the unnormalized counterpart of Π_{ih} in (9). So the definition of Π_{L_i} in (20) implies that the amount of labor (or worker human capital) in each task across both sectors can be expressed as

$$L(j) = \sum_{i\in\{m,s\}}\frac{L_i(j)}{L_i}\cdot L_i = \sum_{i\in\{m,s\}}\frac{V_i(j)}{V_{L_i}}\cdot L_i.$$

We consider the same exercise as in Proposition 2, that $M(j)$ grows to $M^1(j) = \exp\{\tilde{M}\}M(j)$ for $\tilde{M}>0$ and all $j \in \mathcal{J}^1 \equiv (\underline{j},\bar{j})$. Let Δ_X denote the log-derivative of X w.r.t. $\tilde{M}$, then

$$\begin{aligned}\Delta_{L(j)} &= \sum_{i\in\{m,s\}}\frac{L_i(j)}{L(j)}\cdot\left[\Delta_{V_i(j)} - \Delta_{V_{L_i}} + \Delta_{L_i}\right]\\ &= \sum_{i\in\{m,s\}}\frac{L_i(j)}{L(j)}\cdot\Bigg\{\underbrace{\Delta_{V_i(j)} - \int_{\mathcal{J}^z}\left[\frac{V_i(j')}{V_{L_i}}\cdot\Delta_{V_i(j')}\right]dj'}_{W_{ij}} + \Delta_{L_i}\Bigg\}.\end{aligned} \tag{28}$$

A change in the $V_i(j)$'s occurs even holding L_i's constant, shifting the term W_{ij}. This leads to "within-sector polarization," as we saw in Proposition 2 that intermediate

tasks are shrinking and management is growing. To compare the sectoral differences in its impact, we compare the $\Delta_{V_{il}}$ and $\Delta_{V_i(z)}$ across the two sectors, which represent, respectively, the change in workers and managers. Then we can sign $\Delta_{V_{L_i}}$, the change in sectoral employment shares which determines structural change.

Lemma 2 *Suppose $\omega < \sigma < 1$ in Proposition 2, so that we get both horizontal and vertical polarization within sectors. Then both horizontal and vertical polarization is faster in manufacturing in the sense that $\Delta_{V_{ml}} < \Delta_{V_{sl}} < 0$ and $\Delta_{V_m(z)} > \Delta_{V_s(z)} > 0$ if $L_m(j)/L_m > L_s(j)/L_s$.*

Proof See Appendix B.5. ∎

When the lemma holds, both horizontal and vertical polarization is faster in manufacturing, as we saw in the data in Figures 3 and 4. The assumptions in the lemma mean that the manufacturing sector has a larger share of middle-skill or routine jobs ($j \in \mathcal{J}^1$), and services a larger share of managers, which is also evident in the data as was shown in Figures 4(b) and 19(a). Of course, these are assumptions on endogenous variables: Because we do not know the value of task productivities $[M(j), \hat{h}(j), \hat{z}]$, this holds in general if there exists $\bar{\nu} \in (0,1)$ such that $\nu_m(j) - \nu_s(j) \geq \bar{\nu}$ for all $j \in \mathcal{J}^1$.

Note that the lemma holds holds regardless of the value of $q_h(0)$, which determines the between-sector equilibrium. So W_{ij} in (28) gives the equilibrium change in within-sector employment shares coming only from a change in the selection rules $[\hat{h}(j), \hat{z}]$. Clearly, a change in the rules will also alter the last term, Δ_{L_i}, which captures between-sector reallocation, or structural change. Lemma 3 summarizes when structural change is in the direction of shifting capital and labor from manufacturing to services.

Lemma 3 (Structural Change) *Suppose $\omega < \sigma < 1$ in Proposition 2, so that routinization causes both horizontal and vertical polarization within sectors. Further suppose that $\epsilon < 1$. Then there exists $(\bar{\nu}, \bar{\eta}) \in (0,1)$ such that for all $\nu_m(j) - \nu_s(j) \geq \bar{\nu}$, $j \in \mathcal{J}^1$, and $\eta_s - \eta_m \geq \bar{\eta}$,*

$$(\Delta_{V_{L_s}} = \Delta_{\Pi_{L_s}}) > (\Delta_{V_{L_m}} = \Delta_{\Pi_{L_m}}), \quad \Delta_{\Pi_{K_s}} > \Delta_{\Pi_{K_m}}, \quad \text{and } \Delta_{L_s} > 0$$

where the equalities follow from (25).

Proof See Appendix B.5. ∎

The additional assumption on η_i ensures that Lemma 2 holds even as the within-sector share of managers increase and routine jobs decrease, as the sectoral employment share of manufacturing declines.

There is a subtlety here, because structural change in fact has two parts. As a task becomes more productive than others, selection on skills ensures that less resources are allocated to it when we have complementarity across tasks (Proposition 2). If one sector uses the task that has become more productive more intensively, resources reallocate across sectors even in the absence of a change in the sectoral allocation rule (Lemma 2). This is the first part.

The second part is that, as the manufacturing sector becomes more productive—endogenously because it uses the task that has become more productive more intensively than services—the equilibrium price of its output falls relative to services. The strength of this force is governed by the elasticity between manufacturing and service outputs, and with complementarity, i.e. $\epsilon < 1$, more resources are allocated toward services, as in Ngai and Pissarides (2007). These two forces are formalized in the Appendix B.5.

Also formalized in the appendix is that the dependence of structural change depends differently on the productivity of capital and labor, as is apparent from (19)-(20). In contrast to both Ngai and Pissarides (2007) and Goos et al. (2014), capital is homogenous in our model but labor is not, which is measured in two different types of skill units. Since labor productivity is determined by the sorting of individuals on skill, how task-specific TFP changes sectoral capital and labor input ratios differs not only according to differences in the reliance of sectors on certain tasks, but also according to changes in the selection rules.

Of course from (28), it is clear that structural change (change in Δ_{L_i}) also contributes to polarization. To see this more precisely, rewrite (28) using (22) as

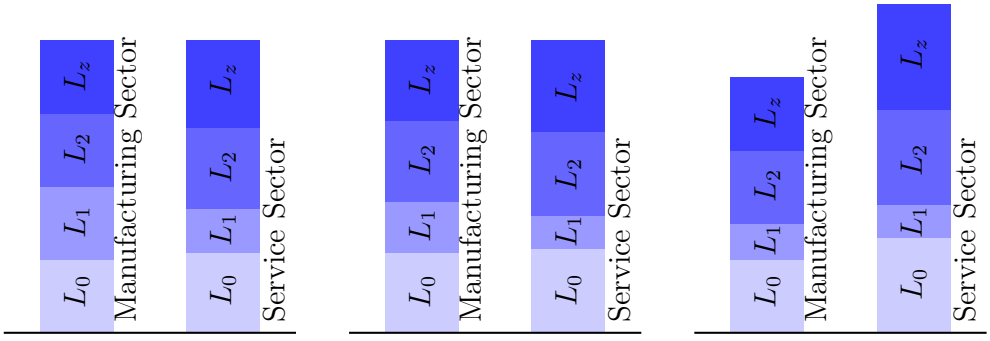
$$\Delta_{L(j)} = \Delta_{V_i(j)} - \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)} \cdot \Delta_{V_{L_i}} + \left[\frac{L_s(j)}{L(j)} \cdot L_m - \frac{L_m(j)}{L(j)} \cdot L_s\right] \Delta_\kappa \quad (29)$$

$$\Rightarrow \quad L(j)\left(\Delta_{L(j)} - \Delta_{V_i(j)}\right) = -\sum_{i\in\{m,s\}} L_i(j)\Delta_{V_{L_i}} + \left[\frac{V_s(j)}{V_s} - \frac{V_m(j)}{V_m}\right] L_m L_s \Delta_\kappa.$$

Lemma 4 *Suppose Lemma 2 holds. Then structural change also contributes to polarization.*

Proof Under Lemma 3, the term in the square brackets in (29) is negative for $j \in \mathcal{J}^1$, and positive for $j = z$. ∎

This is intuitive. If manufacturing is more reliant on middle-skill tasks (that is, if it is more routine-intense), and shrinks as a result of polarization, this leads to even more



(a) Low M_1 (b) High M_1, κ fixed (c) High M_1

Fig. 7: Comparative Statics, Across Sectors

horizontal polarization overall. The fact that manufacturing is less reliant on managers at the same time implies even more vertical polarization.

Lemmas 2-4 are depicted in the first 3 subplots in Figure 7. In Figure (a), manufacturing is depicted as having a higher share in intermediate tasks, and services in managers. As we move from (a) to (b), sectoral employment shares are held fixed, and intermediate tasks shrink in both sectors. The change in employment shares is larger in manufacturing due to Lemma 2. This leads to structural change in (c), according to Lemma 3. Because manufacturing uses intermediate tasks more intensively and managerial tasks less intensively than services, shrinking its size contributes to the horizontal and vertical polarization for the overall economy (not separately depicted).

In the model, task-specific technological progress (changes in $M(j)$) shifts relative employment shares as if the weights $[\nu_i(j), \eta_i]$ were changing, so the two are not separately identified in our comparative statics. But since the model is constructed so that the time-invariant weights capture an initial distribution of employment shares, while task-specific TFP changes drive employment share changes over time, the assumptions we made in the lemmas are valid insofar as they hold throughout our observation period in the data.[20]

[20]When we calibrate the model to the 1980 data—for which we assume that $M(j) = M$—the calibration naturally admits that $\eta_m < \eta_s$ and $\nu_m(j) > \nu_s(j)$ for a wide range of middle-skill jobs. Furthermore, since occupational employment ratios between sectors are never flipped for most occupations up to 2010, the quantitative analysis is robust to the year of normalization.

4.3 Polarization or Structural Change?

One may wonder if it is not task-specific productivities that lead to structural change, but advances in sector-specific productivities that lead to polarization, considering Lemma 4 in isolation. While it is most likely in reality that both forces are in play, in the context of our model, sector-specific productivity shifts does not lead to polarization *within* sectors.

To see this, consider an exogenous change in the manufacturing sector's exogenous productivity, A_m. As in Ngai and Pissarides (2007), a rise in A_m changes κ at a rate of $1-\epsilon > 0$; that is, manufacturing shrinks. It can easily be seen that none of the thresholds change, and hence neither do the Φ_i's (the endogenous sectoral TFP's). So polarization can only arise by the reallocation of labor across sectors *but while preserving their ratios within each sector*. To be precise, from (28),

$$\frac{d\log L(j)}{d\log A_m} = (1-\epsilon)\cdot\frac{d\log L(j)}{d\log\kappa} = (1-\epsilon)\left[\frac{L_s(j)}{L(j)}L_m - \frac{L_m(j)}{L(j)}L_s\right] < 0. \tag{30}$$

Note that $d\log L(j)/d\log\kappa$ is equal to the term in square brackets in (29), and negative for $j \in \mathcal{J}^1$. Hence, polarization only occurs because manufacturing shrinks, without within-sector polarization. The reason is that in our micro-founded model, tasks are aggregated up into sectoral output, not the other way around.

Even if one were to only consider sectoral aggregates, our model—specifically (30)—provides an upper bound on how much job polarization can be accounted for by structural change. For example, in the data, the manufacturing employment share fell from 33 percent to 19 percent between 1980 and 2010. If this were solely due to an exogenous change in A_m, this means, denoting empirical values with hats:

$$\frac{d\log\hat{\kappa}}{d\log A_m} \approx \frac{0.14}{0.67} + \frac{0.14}{0.33} \approx 0.63.$$

Now denote all routine jobs as $j=1$, then we can approximate

$$\frac{d\hat{L}_1}{d\log A_m} \approx 0.63\cdot\left[\hat{L}_{s1}\hat{L}_m - \hat{L}_{m1}\hat{L}_s\right] = 0.63\cdot\left[\frac{\hat{L}_{s1}}{\hat{L}_1}\cdot 0.33 - \frac{\hat{L}_{m1}}{\hat{L}_1}\cdot 0.67\right].$$

In Appendix Table 5, we measure the employment share of routine, manufacturing jobs and routine, service jobs as a share of total employment—that is, $\hat{L}_{m1}$ and $\hat{L}_{s1}$—in 1980 to be 26 and 33 percent, respectively. So

$$\frac{d\hat{L}_1}{d\log A_m} \approx 0.63\left[0.33\cdot 0.33 - 0.26\cdot 0.67\right] = -0.04,$$

which means that a change in A_m alone would imply a 4-percentage-point drop in routine jobs from 1980 to 2010 in the overall economy. As shown in Table 5, the

actual drop is 13 percentage points. In other words, an exogenous structural change can explain at best 30 percent of the polarization overall—and none whatsoever within sectors.

5 Calibration

The goal of our quantitative analysis is to quantify how much of the observed changes in employment and wage shares from 1980 to 2010 can be explained by task-level productivity growth, and relate such productivity growth to empirically measurable sources. Whenever possible, we fix parameters to their empirical counterparts, and separately estimate the aggregate technology (4) from time series data on sectoral price and output ratios. Then we choose most model parameters to fit the 1980 data exactly, including a parametric skill distribution of (h, z). The rest of the model parameters, which includes the elasticity parameters (σ, ω), are calibrated to empirical time trends from 1980 to 2010, but excluding any sector-specific moments.

5.1 Parametrization

Discrete Log-supermodularity In the quantitative analysis, we collapse the continuum of (horizontally differentiated) worker tasks into 10 groups, corresponding to the 1-digit occupation groups in the census we summarized in Section 2 and Appendix A Table 5. (There is still only one management task.) The 10 worker occupation groups are further broadly grouped into low/medium/high skill tasks, or manual/routine/abstract jobs, according to the mean wages of each occupation group. To facilitate visualization, in our analysis we will present moments based on these 3 occupation groups, plus the managers.

To discretize the model, we index occupations by $j = 0, \ldots, 9$ and assume the following log-supermodular technology:

$$b(h, j) = \begin{cases} \bar{h} = 1 & \text{for } j = 0, \\ h - \chi_j & \text{for } j \in \{1, \ldots, 9\}, \end{cases} \quad 0 = \chi_1 < \chi_2 < \ldots < \chi_9.$$

The characterization of the equilibrium is exactly the same, with the only difference being that we can obtain closed-form solutions. This technology implies that to work in the lowest-skill task 0, the worker's skill do not matter and everyone performs the task with equal efficiency. All skills are used in task 1. For tasks $j \in \{2, \ldots, 9\}$, there is a "skill loss," which increases with higher order tasks.[21] With 10 discrete tasks,

[21] This can be interpreted as lower-order skills not being used in higher-order tasks, or higher-order tasks

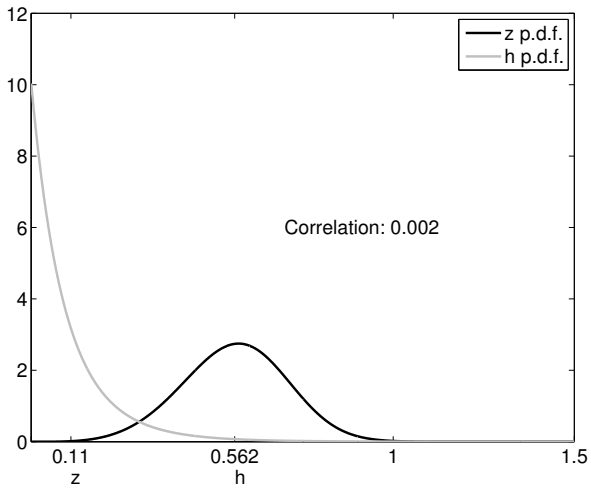


Fig. 8: Calibrated Skill Distribution

We use a type IV bivariate Pareto distribution to model the distribution over worker and manager skills (h, z). The figure depicts the marginal distributions of each skill, and also their mean values below the x-axis. The implied Pearson correlation coefficient between the two skills is low, at 0.002.

we only need to solve for 10 thresholds $\hat{h}_j$ at which workers are indifferent between the two neighboring sectors, as opposed to Proposition 1 in which we have to solve a differential equation (equation (35) in the appendix).[22]

Bidimensional Skill Distribution For the quantitative analysis, we assume a parametric skill distribution that is type IV bivariate Pareto (Arnold, 2014). Specifically, the c.d.f. we assume is

$$\mu(h, z) = 1 - \left[1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right]^{-a}.$$

We normalize $\gamma_z = 1$, since we cannot separately identify both skills from the skill-specific TFP's. This is consistent with an establishment size distribution that is Pareto, and a wage distribution that is hump-shaped with a thin tail, as depicted in Figure 8 which shows the marginal distribution of (h, z).

5.2 Aggregate Production Function

The aggregate production function is estimated outside of the model. For the estimation, we only look at the manufacturing and service sectors, where manufacturing

requiring a fixed cost of preparation to perform the task, resulting in less efficiency units of skills used. By assuming task 0 productivity to be constant, we can normalize all other tasks in terms of task 0, as we did for the continuous model.

[22] Characterization of the discrete model is summarized in Appendix C.

	(1)	(2)	(3)
γ_m	0.371**	0.346**	0.258**
	(0.003)	(0.005)	(0.004)
ϵ	0.003**	0.002**	0.003
	(0.000)	(0.000)	(0.004)
AIC	-550.175	-551.264	-550.866
RMSE_1	0.106	0.106	0.106
RMSE_2	0.039	0.039	0.039

Standard errors in parentheses
† $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table 1: Aggregate Production Function

The manufacturing share parameter γ_m and elasticity parameter between manufacturing and services, ϵ, are estimated off the time series of output and price ratios from 1947 to 2013, which are available from the National Industry Accounts. The service share parameter γ_s is assumed to equal $1 - \gamma_m$. For details of the estimation, we closely follow Herrendorf et al. (2014).

includes mining and construction, and government is included in services. We estimate the parameters (γ_m, ϵ) from the system of equations

$$\log\left(\frac{p_m Y_m}{PY}\right) = \log\gamma_m + (1-\epsilon)\log p_m - \log\left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right] + u_1$$
$$\log(Y) = c + \frac{\epsilon}{\epsilon-1}\log\left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2$$

where $\gamma_s \equiv 1 - \gamma_m$, using non-linear SUR (seemingly unrelated regression), on all years of real and nominal sectoral output observed in the BEA Industry Accounts. Real production by sector is computed by a cyclical expansion procedure as in (Herrendorf et al., 2014) using production value-added to merge lower-digit industries (as opposed to consumption value-added, as in their analysis).[23]

Sectoral prices are implied from nominal versus real sectoral quantities, which may change following the choice of base year. For robustness, we check results using three different base years, corresponding to columns (1)-(3) in Table 1. For each column, respectively, 1947 is the first year the required data is available, 1980 is the first year in our model, and 2005 is chosen as a year close to present but before the Great Recession. As shown there, the values are in a similar range as in Herrendorf et al. (2014), although ϵ is not significant with 2005 as a base year. For the calibration, we use the values of (γ_m, ϵ) in column (1).

The capital income share α is computed as the average of 1-(labor income/total income), and fixed at 0.360. Total income is GDI net of Mixed Income and Value-

[23]The constant c is included since it is not levels, but relative changes that identify ϵ.

	Parameter	Value	Target
Fixed from data	K_{1980}	2.895	Computed from BEA NIPA data
	K_{2010}	4.235	
	α	0.361	
	γ	0.371	Estimated in section 5.2
	ϵ	0.003	
Fit to 1980	$M_j \equiv M$	0.985	Output per worker, normalization
	A_m	1.112	Manufacturing employment share
	A_s	1.000	Normalization
	ν_{ij} (18)	Table 3	Witin-sector employment shares by occupation
	$\eta_i(2)$		Within-sector manager share
	χ_j (8)	Table 3	Relative wages by occupation
	a	10.087	
	γ_h	0.216	
	γ_z	1.000	Normalizations;
	$\bar{h}$	1.000	Not separately identified from M_j
Fit to 2010	σ	0.704	Witin-sector employment shares by occupation
	ω	0.341	
	m_j	Table 3	Output per worker growth and within-sector employment shares by occupation

Table 2: Parameters

We normalize population size, so K_t denotes capital per capita. Parameters valued 1 are normalizations. All employment share and relative wage targets are from the census, tabulated in Appendix A Table 5. See text for more details.

Added Tax from NIPA and Industry Accounts, and labor income from NIPA. Since we do not model investment, for the calibration we also need total capital stock (for manufacturing and services) for each decade, which we take from the Fixed Assets Account Table 3 and directly plug into the model.[24] Since we do not model population growth, in practice we we normalize output per worker in 1980, y_{1980} to unity, and plug in $K_t = k_t/y_{1980}$ for $t \in \{1980, 1990, 2000, 2010\}$, where k_t is capital per worker in year t.

5.3 Setting Parameters

All parameters are summarized in Table 2, except for the skill loss parameters χ_j, task intensity parameters (η_i, ν_{ij}), and task-TFP growth rates m_j, which are tabulated in Table 3. Below, we explain in detail how these parameters are recovered.

[24] Real capital stock is aggregated using the same cyclical expansion procedure as for value-added.

Calibrating the distribution For any guess of $(\gamma_h, a, \{\chi_j\}_{j=2}^9)$, we can numerically compute the thresholds $(\{\hat{h}_j\}_{j=1}^9, \hat{z})$ that exactly match observed employment shares by occupation in 1980, by integrating over the guessed skill distribution. Given these thresholds, we can compute model-implied relative wages using the discrete version of (23), which is (54) in Appendix C:

$$\frac{w_1\bar{h}_1}{w_0} = \frac{\bar{h}_1}{\hat{h}_1}, \quad \frac{w_z\bar{z}}{w_0} = \frac{\bar{z}}{\hat{z}}, \quad \text{and} \quad \frac{w_{j+1}(\bar{h}_{j+1} - \chi_{j+1})}{w_j(\bar{h}_j - \chi_j)} = \frac{\bar{h}_{j+1} - \chi_{j+1}}{\bar{h}_j(1 - \chi_{j+1}/\hat{h}_{j+1})}.$$

Here, $\bar{h}_j$ and $\bar{z}$ denote the mean skills in each task. The LHS is the ratio of mean wages by occupation, which we observe from the data. The RHS is a function only of the thresholds, which themselves are functions of (γ_h, a, χ_j). Hence, we iterate over $(\gamma_h, a, \{\chi_j\}_{j=2}^9)$ so that the model-implied ratios match observed mean wage ratios exactly, while at the same time computing the implied thresholds $(\{\hat{h}_j\}_{j=1}^9, \hat{z})$ that exactly fit 1980 employment shares by occupation.[25]

Similarly, once the skill distribution is fixed, we can compute the implied thresholds that fit 2010 employment shares by occupation. Denote these two sets of thresholds as $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$, respectively. Note that these thresholds are determined solely by the exogenously assumed skill distribution and the data, independently of our model equilibria, so these are calibrated once and then fixed for the rest of the calibration. We then calibrate the other parameters so that the implied thresholds $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$ are consistent with the 1980 and 2010 equilibria, respectively.

Calibrated within the model We have already normalized $(\gamma_z, \bar{h}) = 1$ and $\chi_1 = 0$. We also normalize $A_s = 1$, since the model only implies a relative TFP between sectors, and $M_j \equiv M$ for all $j \in \{0, 1, \ldots, z\}$ for 1980, since it is not separately identified from (η_i, ν_{ij}) in a static equilibrium. This is implied by the production technology we assume in (6)-(7). For notational convenience, we will denote the 1980 levels of the TFP's by (M, A_i) and denote their 2010 levels by multiplying them by their respective growth rates. For example, the manager-task TFP in 2010 is $M(1+m_z)^{30}$ and similarly sector i's TFP in 2010 is $A_i(1 + a_i)^{30}$.

This leaves us with 35 parameters to be calibrated: the elasticity parameters (σ, ω), TFP parameters (M, A_m), task intensities $(\eta_i, \{\nu_{ij}\}_{j=1}^9)$ for $i \in \{m, s\}$, and the task-TFP growth rates $\{m_j\}_{j=z,0}^9$.[26] Since we can solve for the discrete version of the model equilibrium in closed form, most parameters are chosen so that our 1980 equilibrium

[25] Interestingly, the calibration yields a near linear increase in the skill loss parameters χ_j with j.

[26] There are only 9 horizonatal intensity parameters to calibrate per sector, since we assume $\sum_{j=0}^9 \nu_{ij} = 1$.

Ranked by mean wage (except management)	χ_j	Emp Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j	RTI
Low Skill Services	**-**	**0.016**	**0.136**	**-0.731**	**-0.211**
Middle Skill		**0.816**	**0.524**		
Administrative Support	-	0.088	0.173	2.930	2.445
Machine Operators	0.001	0.256	0.015	9.122	0.602
Transportation	0.002	0.119	0.081	4.348	-0.576
Sales	0.003	0.026	0.123	0.012	0.483
Technicians	0.005	0.034	0.040	-1.144	-0.269
Mechanics & Construction	0.006	0.159	0.065	2.315	-0.630
Miners & Precision Workers	0.007	0.134	0.027	6.328	0.639
High Skill		**0.168**	**0.340**		
Professionals	0.009	0.070	0.195	-2.248	-0.725
Management Support	0.010	0.098	0.146	-0.489	-0.655
Management	**-**	**0.076**	**0.130**	**-0.017**	**-1.103**

Table 3: Calibrated Employment Weights and Growth Rates

exactly fits the 1980 data moments in Appendix Table 5 exactly.[27] Then except for capital per worker, which we plug in from the data, all other parameters are held fixed and only M_j, the exogenous task-TFP's, grow from 1980 to 2010 at rate m_j. In particular, our benchmark scenario assumes that $a_m = a_s = 0$.

The 11 constant task-TFP growth rates $\{m_j\}_{j=z,0}^{9}$ and elasticity parameters (σ, ω) are chosen to fit the time trends of aggregate output per worker growth and employment shares *within sectors* from 1980 to 2010 (13 parameter, 21 moments).[28] All resulting parameters are tabulated in Tables 2-3.

Discussion As implied by the data in Figure 19(a) and Appendix Table 5, the manufacturing sector has higher intensity parameters among middle skill jobs and a lower intensity in managers.[29] Then since the estimated elasticity between manufacturing and services $\epsilon \approx 0$, for structural change to occur as we observe in the data, productivity needs to rise relatively more in those occupations used more in manufacturing, which are the middle-skill jobs (m_j's must be higher in the middle). This is evident from the second-to-last column in Table 3, and Figure 16(a) shows that there is almost a perfectly negative correlation between our calibrated productivity growth rates and

[27] See Appendix C for details.

[28] We target the linear trend from 1980 to 2010 rather than their exact values. However, since most trends are in fact linear, using the exact values barely change any of our results.

[29] Since we normalize $M_j \equiv M$ in 1980, the parameters directly correspond to skill-adjusted employment weights.

employment share changes by occupation.[30]

The calibrated values for $\omega < \sigma < 1$ are important both for Section 4 and our quantitative results to follow. The only other paper we know that recovers a value of elasticities across tasks is Goos et al. (2014). Their point estimate for σ is around 0.9, which is much closer to 1 than our values for (σ, ω).[31] However, recall that theirs is an empirical framework that does not take into account general equilibrium effects. Both in their model and ours, the employment share change of occupation j is determined by $(1-\sigma)m_j$ (Appendix B.4).[32] Intuitively then, if we were to set $\sigma = \omega = 0.9$, we would recover much higher values for m_j in order to explain the employment share changes in the data. But this results in sectoral and aggregate TFP growth rates being counterfactually high compared to macro data.

Proposition 2 implies that if $\omega < \sigma < 1$, the model generates horizontal and vertical polarization as we observe in the data. While these were sufficient conditions, the calibrated elasticities are consistent with its predictions. And given these values, Lemmas 2-4 all apply, so we as long as employment shares do not flip over time, polarization will be faster in manufacturing and structural change will occur.

These are the main predictions of the theoretical model that we now quantify. Note that in addition to not allowing any exogenous sector-specific changes ($a_i = 0$), we did not target the change in sectoral employment shares either. The reason for doing so is that within-shares would remain constant in a model with only sector-specific forces, and is a unique prediction of our model. And since the model is under-identified, how well the model performs in replicating the speeds of polarization across sectors, and its implications for structural change, are the major tests of the quantitative model.

6 Quantitative Analysis

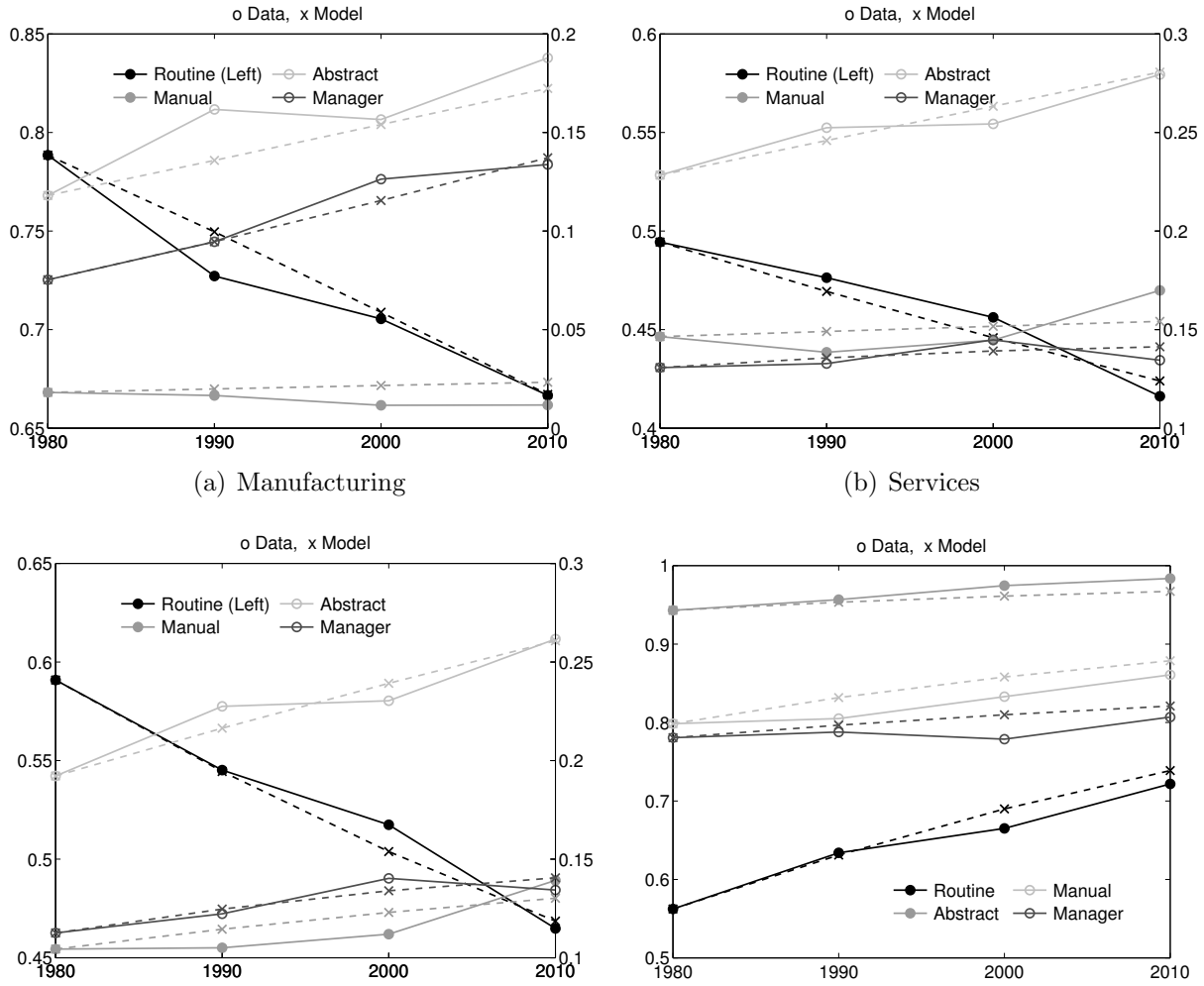
6.1 Model Fit

Employment Shares Figure 9 plots the model implied trends in employment shares across tasks, in aggregate and by sector, against the data. When computing the simulated paths for 1990 and 2000, we plug in the empirical values of $K_t = k_t/y_{1980}$

[30] The last column in Table 3 shows the empirical RTI indices constructed from Autor and Dorn (2013) for comparison.

[31] Since they do not model a vertical structure, there estimate stands in for both elasticities in our model.

[32] Of course, they use an empirical measure of m_j and estimate its effect on employment shares, while we calibrate them to fit employment shares.



(a) Manufacturing

(b) Services

(c) Aggregate

(d) Services Share By Task

Fig. 9: Data vs. Model, Employment Shares by Task

Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups, and regrouped into 4 broader categories as shown in Appendix A Table 5. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Further details in Section 2 and Appendix A.

and the level of task-TFP's implied by the calibrated growth rates, and compute the respective equilbria allocations.

At first glance, it may not be so surprising that we obtain a more or less exactly fit as seen in Figure 9(c)-(d), since the discrete model equilibrium can be solved in closed form in any given year, as we explain in Appendix C. However, while we target the starting points for all the shares (services employment share, and within-sector employment shares by task), we calibrated 21 trends using only 13 parameters (the 2 elasticity parameters (σ, ω) and 11 task-specific growth rates). Despite this, the

evolution of within-sector employment shares implied by the model are almost exact for both manufacturing and services, meaning that the model's mechanisms are sufficient for making up for the lack of parameters. The quantitative model picks up a slightly steeper polarization in the manufacturing sector as in the data, and as implied by the theory.

Furthermore, we did not target any aggregate or sector-specific employment shares, so the fact that aggregate occupation shares and structural change by occupation are almost exactly replicated, as seen in Figure 9(c)-(d), are also successes of the quantitative model. Taken together, this gives us a hint that the consequences of task-TFP growth implied by Lemmas 2-4 may be sufficient for explaining structural change.

Relative Wages Figure 10 plots relative mean wages in aggregate and by sector. The only targeted moments were the 1980 average wage ratios in Figure 10(a); all other moments were not. Manual and abstract wages are relative to routine jobs, and manager wages are relative to all workers. While the model trends are qualitatively consistent with the data (which is not surprising given Proposition 2), the quantitative fit is less satisfactory.

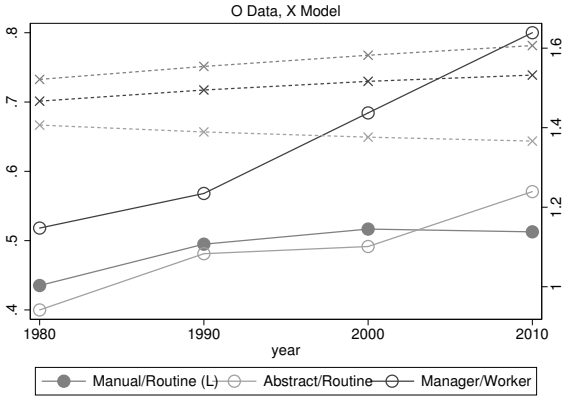
First, compared to the data visualized in Appendix Figure 21(b), all model-implied relative wages are too low in manufacturing, and too high in services. In the quantitative model, efficiency wages $(w_z, w_1, \ldots, w_9)$ are equal across sectors since we assume that individuals are indifferent between sectors, and we constrain our attention to equilibria in which mean skill levels within occupations were equal across sectors. Hence, relative wages are equal across sectors for any given occupation; the only reason they differ in Figures 10(b)-(c) is because we aggregate occupation groups into three broader categories; and for managers since they are compared against all workers. So by construction, our model lacks a cause for relative wages to be lower in services. Nonetheless, average wages in both sectors are more or less constant throughout the observation period, so we consider the indifference assumption to be valid up to a time-invariant constant.

Second, given the complementaries between occupations and sectors ($[\omega, \sigma, \epsilon] < 1$), Proposition 2 implies that the calibrated growth rates m_j are smaller for those occupations that are growing (Table 3). Then for all occupations in the middle ($j = 1, \ldots, 9$), whether their relative wages grow or shrink depends on the magnitude of negative selection (that comes from having more or less low-skill workers from the left-side of the h-skill distribution) and positive selection (that comes from having more



(a) Aggregate

(b) Manufacturing



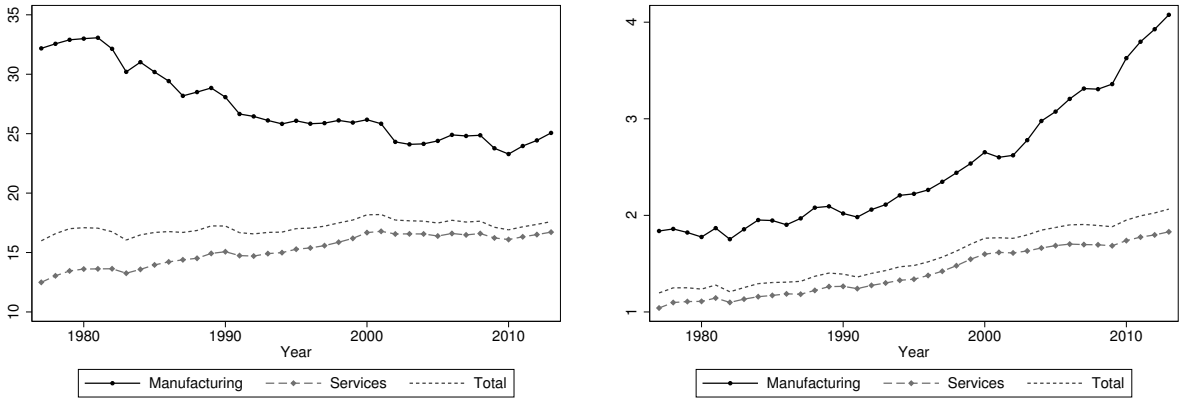
(c) Services

Fig. 10: Data vs. Model, Relative Wages by Task

Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups, and regrouped into 4 broader categories as shown in Appendix A Table 5. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Further details in Section 2 and Appendix A.

or less high-skill workers from the right-side of the h-skill distribution), since whether the employment share grows or shrinks, it will either absorb or lose employment from both sides of the distribution. This is the discrete counterpart of Proposition 2 and in Figure 6, can be seen from $\hat{h}(\underline{j})$ increasing and $\hat{h}(\overline{j})$ decreasing following a rise in the TFP among routine jobs.

For manual jobs, average skill is assumed to be constant ($\bar{h} = 1$), and only those workers with the lowest h-skill work in this job ($j = 0$). It turns out that routine jobs as a whole display enough negative selection so that the wages of manual workers relative to routine workers rise, although only slightly. This is in fact consistent with the data in aggregate and in the services sector, although in the manufacturing sector,



(a) Average No. of Workers

(b) Average Value-added Output

Fig. 11: Establishment Size by Sector

Source: BDS and NIPA. Left: average number of workers per establishment, in aggregate and by sector. Right: (real) value-added output by establishment (millions of 2010 USD), in aggregate and by sector. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government.

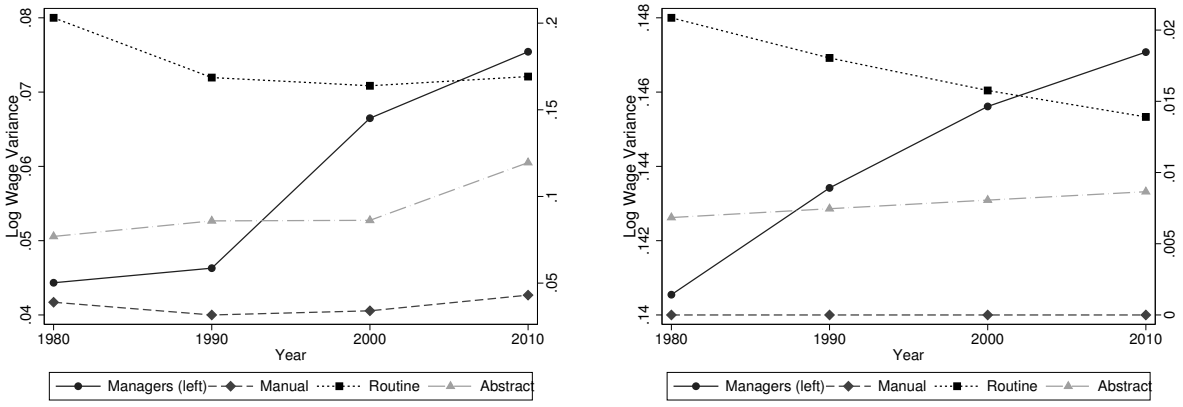
manual wages slightly dropped.

We also proved in Proposition 2 that as long as $\omega < \sigma$, which it is (Table 2), managers' relative wages would rise relative to workers as long as their task-TFP grew slower than workers'. However, the quantitative magnitude of this rise is small compared to the data. This is because as the employment share of managers grow, there is a negative selection along z-skills. In Figure 6, this can be seen from $\hat{z}$ decreasing to increase the mass of managers.

According to our model assumptions, all workers with the highest h-skill work in the highest-skill worker occupation ("professionals" in the data). Since their task-specific TFP grew relatively less, the average skill of workers in this occupation necessarily becomes lower, since employment growth can only lead to negative selection (there can be no inflow of workers from the right-side of the h-skill distribution. Consequently, both because of lower TFP growth and negative selection, relative wages decline for abstract jobs, in contrast to the data.

6.2 Other Moments

Establishment-size Distribution In our model, an establishment consists of one manager organizing a team of workers. Since manager employment grew more manufacturing while barely changing in services (Figure 4), our model would imply that establishment sizes shrinking primarily in manufacturing. This is confirmed in



(a) Data (b) Model

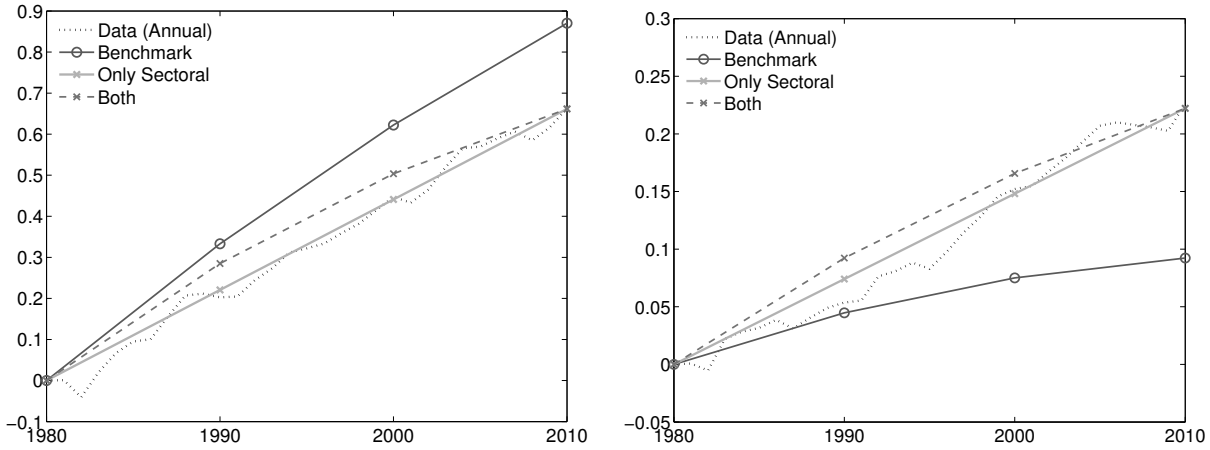
Fig. 12: Within Task Wage Inequality

Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups, and regrouped into 4 broader categories as shown in Appendix A Table 5. Further details in Section 2 and Appendix A.

Figure 11(a), in which we plot the average number of workers per establishment in the Business Dynamics Statistics (BDS) from the U.S. Census Bureau. Furthermore, since the model generates this by complementarity between manager and worker occupations, this should coincide with higher productivity and output growth among manufacturing establishments. In Figure 11(b), we divide value-added output in the NIPA by establishment size in the BDS, which again confirms the model mechanism.

An inconsistency with our definition of an establishment is that while the employment share of managers have grown in the data, implying smaller establishment sizes, average establishment sizes in aggregate have stayed more or less constant throughout the observation period. However, while there are more top managers today than in 1980, Figure 18 in the appendix shows that most of the manager employment growth comes from middle-managers (all managers excluding top managers). While we do not model the hierarchies of management as in Garicano and Rossi-Hansberg (2006) and others, the data can be reconciled with our model if establishment are split into more teams today than they were in 1980, with the teams being run by middle-management.

Within-Group Wage Inequality Since we assume heterogeneous skills, our model also has implications for wage inequality *within* occupation groups, and not only between. As shown in Figure 12, log-wage variances rose substantially among managers, slightly among abstract workers, dropped for routine workers and remained more or less constant for routine workers.



(a) Log TFP per Worker, Manufacturing

(b) Log TFP per Worker, Services

Fig. 13: Benchmark vs. Counterfactuals, TFP

Data: NIPA. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. 1980 levels are normalized to 0, so the slope of the lines are the growth rates.

These qualitative trends are replicated in our calibration, but the magnitudes of the changes are too small compared to the data. This is because we fixed held the wage distribution fixed to the bivariate Pareto calibrated in Section 5.3 for all periods. Thus most of the change in wage inequality is explained by between-group (the task TFP's and selection) rather than within-group (the distance between thresholds) inequality. To avoid this, we would need to increase the variance of the underlying skill distribution over time, which we have opted not to in order to isolate task specific forces—if the distribution of skills changed, we would be unable to separate the change in skills that is task-specific as opposed to task-neutral, without further assumptions on how skill is accumulated.

Some shortcomings notwithstanding, the model targeted only to within-sector moments delivers a good fit by task even across sectors in terms of employment shares, but less in terms of relative wages. Other non-targeted moments such as establishment size and wage inequality within occupation groups are also qualitatively consistent with the data. In what follows, we focus only on employment shares and investigate how much each of these trends is explained by task-specific TFP's, and its implications for other outcomes such as sectoral TFP's.

6.3 Counterfactuals

In this subsection, we analyze the role of task-specific TFP's on structural change compared to two counterfactuals:

(1) Set all task-specific TFP growth to be equal: $m_j = m$, and instead let both (A_m, A_s) change at rates (a_m, a_s). We jointly recalibrate (m, a_m, a_s) to match the empirical growth rate of TFP in aggregate, and also in manufacturing and services (i.e., the Solow residuals), from 1980 to 2010. This is a model only with exogenous sector-specific TFP growth, in the absence of any exogenous, task-specific TFP growth.[33]

(2) Allow both exogenous task- and sector-specific TFP growth, and recalibrate $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ to match the change in employment shares *and* the empirical growth rates of TFP in aggregate, manufacturing and services, from 1980 to 2010. This gives the model the best chance to explain the data.

For both counterfactuals, we keep all other parameters at their benchmark values in Tables 2-3, and only recalibrate the growth rates.

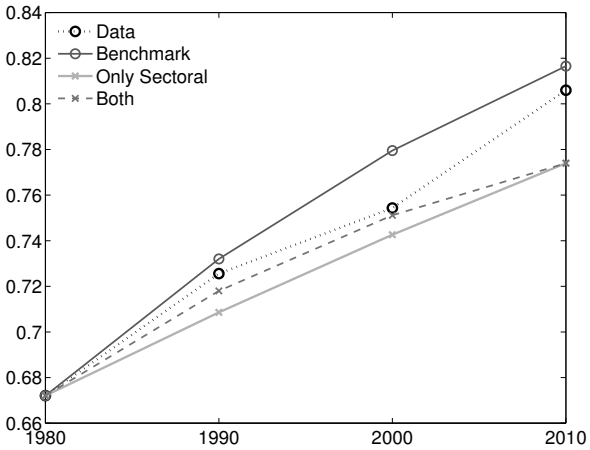
We focus on sectoral TFP's since in our model, structural change only results from the differential TFP growth across sectors—expressed in closed form in (19)—whether it is exogenous (caused by a_m and/or a_s) or endogenous (as in Section 4.2). The recalibrated parameters for the counterfactual scenarios are summarized in Appendix Table 6.

TFP and output growth In Figure 13, we compare the path of log sectoral TFP in the data, in our benchmark calibration, and two counterfactual scenarios. All scenarios match aggregate TFP and GDP growth from 1980-2010 in the calibration as shown in Appendix Figure 22, so we do not discuss them here.[34] While Lemma 3 tells us that given $\epsilon < 1$, structural change occurs due to differential changes of the endogenous TFP's between sectors, recall that these were no targeted by the calibration.

In our benchmark, we overshoot the growth rate of manufacturing TFP by about half a percentage point, while undershooting the services TFP growth rate by about half a percentage point. However, note that when we look at the growth rates of

[33]Sectoral TFP is computed from the NIPA accounts in a similar was as the capital income share and aggregate production function. Specifically, both real value-added and capital are computed via cyclical expansion from the industry accounts, labor is computed from full-time equivalent persons in production in NIPA Table 6, and TFP is the Solow residual by sector.

[34]Denote aggregate TFP as Z_t. Since $Y_t = Z_t K_t^{\alpha}$ (labor is normalized to one), and we plug in the empirical values of K_t for all calibrations, it is the same whether we matching aggregate TFP or GDP.



(a) Services Employment Share, All Jobs

Fig. 14: Benchmark vs. Counterfactuals, Structural Change

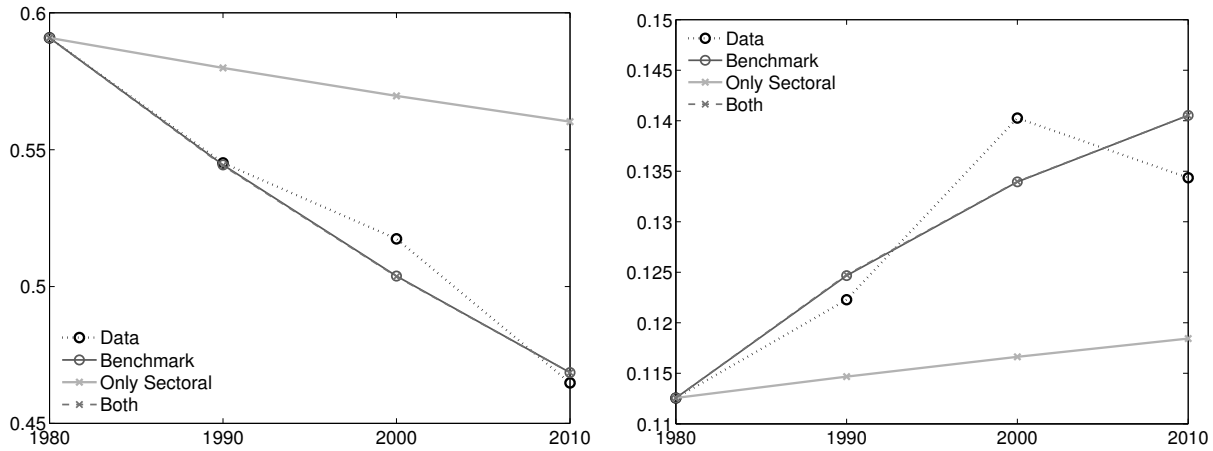
Data: U.S. Census (5%). "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Further details in Section 2 and Appendix A.

sectoral output, in Figure 23, this gap almost disappears. This is because while the model assumes that the sectoral capital input shares are equal to labor input shares, as shown in (21) earlier, in the data they are not. In fact, counterfactuals (1) and (2) in Appendix Figure 23 show that when sectoral TFP growth is matched exactly, manufacturing output grows more slowly, and services output more quickly, compared to the data. This implies that capital input ratio between manufacturing and services grew slower than the labor input ratio, although the differences are small.[35]

Structural Change and Polarization Since structural change is solely determined by output ratios (Lemma 3), the fact that sectoral output growth nearly tracks the data implies that the the benchmark model more or less fully explains structural change (in terms of employment), as shown in Figure 14(c). Both counterfactuals (1) and (2) undershoot the full extent of structural change, since sectoral output growth is too low and high in manufacturing and services, respectively. Moreover, Appendix Figure 24 shows that when we look at structural change within occupation categories, the benchmark model outperforms both counterfactuals (1) and (2), especially among managers.

Recall that exogenous changes in sectoral TFP's alone would not cause any *within* sector polarization. Nonetheless, we investigate its affect on the change in aggregate employment shares by occupation (Lemma 4). In Figure 15, we see that sectoral

[35] If sectoral production functions remain Cobb-Douglas, this means that capital intensity is higher in manufacturing, as analyzed in Acemoglu and Guerrieri (2008).



(a) Routine Employment Share

(b) Manager Employment Share

Fig. 15: Benchmark vs. Counterfactuals, Polarization

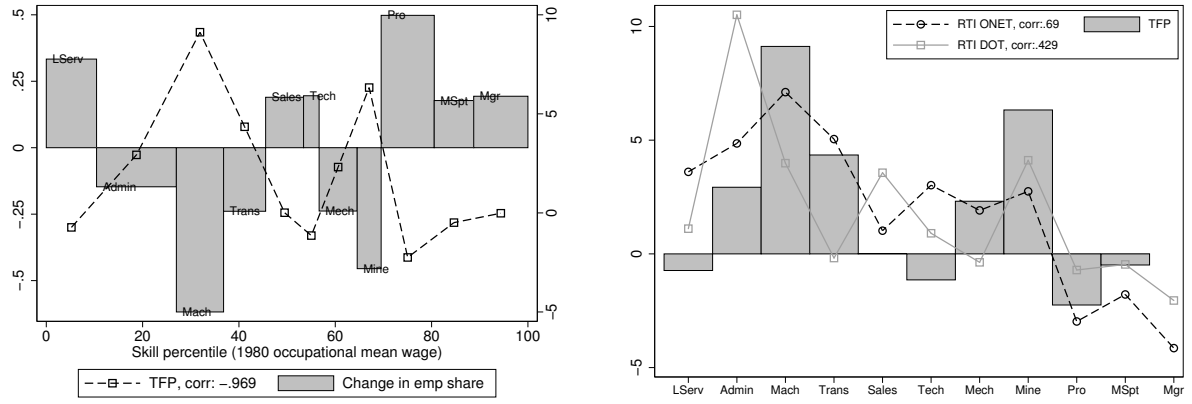
Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups, and regrouped into 4 broader categories as shown in Appendix A Table 5. Further details in Section 2 and Appendix A.

forces alone can account for about 15-20 percent of horizontal and vertical polarization. This is in the ball-park but slightly smaller than predicted by the back-of-the-envelope calculation we did in Section 4.3. As explained there, the effects of sector-TFP's on occupation shares are dampened since both sectors use routine employment.

In contrast, task-TFP's can account for almost all of the changes in both occupation and sectoral employment shares. At first glance, it may seem that the effect of task-TFP growth on relative sectoral TFP's—which drives structural change—should also be dampened, since services TFP is also affected the faster TFP growth of middle-skill tasks, and manufacturing by the slower TFP growth of other tasks. However, differential task-TFP growth is accompanied by a reallocation of skills across occupations, which endogenously reinforces the exogenous shifts.[36]

In sum, task-specific TFP growth can more or less fully account for sectoral output growth, and consequently for the observed level of structural change from 1980 to 2010. Due to the vertical and horizontal polarization induced by changes in task-specific TFP's, employment shifts to the sector that uses the routine task less and management more intensively. Conversely, sector-specific productivities can only account for 15-20 percent of polarization, and furthermore we have shown, both analytically and

[36]In Lemma 3, this is shown as the faster growth rates of the endogenous TFP components (Π_{K_i}, Π_{L_i}) *in services*, which implies faster TFP growth *manufacturing*, given the expression for TFP (19) and assumed values of $[\omega, \sigma, \epsilon]$. Details in Appendix B.5.



(a) Task-TFP growth and Employment Shares

(b) Task-TFP growth and RTI

Fig. 16: Employment Shares, Task TFP growth and the Routinization Index
Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups as shown in Appendix A Table 5. Left: x-axis units in percentage share of 1980 aggregate employment. Right: RTI indices Acemoglu and Autor (2011); Autor and Dorn (2013). Further details in Section 2 and Appendix A.

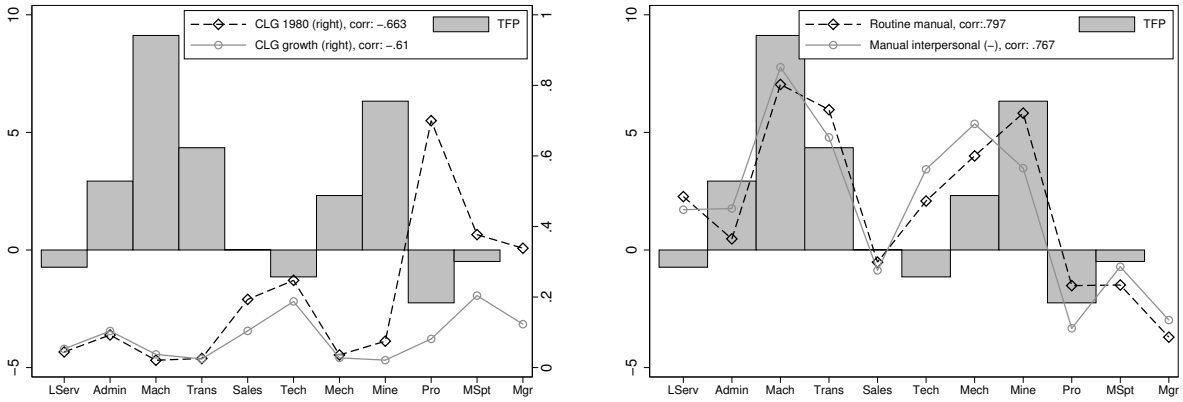
quantitatively, that it does not cause any polarization within sectors, contrary to the data.

6.4 What Are Task-Specific Productivities?

Despite having skill selection, horizontally and vertically differentiated jobs, and multiple sectors, Figure 16(a) shows that the bulk of the changes in occupational employment shares are still directly accounted for by task-specific TFP's, with a (negative) correlation of 0.97. This is also confirmed from the simple regression shown in the top panel of Appendix Table 7. This leads us to conclude that in order to understand changes in the employment structure, it is important to identify what these task-TFP's are.

How much of the variation in the TFP growth rates can be explained by routinization? As a first pass, in Figure 16(b) we correlate the TFP growth rates with the RTI index used in Autor and Dorn (2013), which aggregates indices used in Autor, Levy, and Murnane (2003), which in turn was constructed by aggregating over numerous task requirements for specific jobs recorded in the DOT.[37] We also correlate them with the RTI index from Acemoglu and Autor (2011), which was constructed in a similar way but instead using O*NET, the successor of DOT.

[37]The numerical values for the DOT RTI are summarized in the last column of Table 3. Figure 19(b), replicated from Autor and Dorn (2013) shows where the top employment-weighted third of occupations in terms of RTI like along the skill percentile. Because most routine jobs are found in the middle, they take it as preliminary evidence of routinization affecting job polarization (which is then formally tested).



(a) Task-specific TFP Growth and College

(b) Task-specific TFP Growth and O*NET

Fig. 17: Task TFP growth, College Shares and O*NET-based indices
Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups as shown in Appendix A Table 5. Further details in Section 2 and Appendix A.

While the TFP growth rates are positively correlated with both RTI indices, and more strongly with the latter, it is visually clear that there is much left to be explained. More precisely, both the correlation and R^2's are still quite low, as can be seen in Appendix Table 7.

What about college? Skill-biased technological change (SBTC) has been a usual suspect for changes in the employment structure since since Katz and Murphy (1992), e.g. Krusell, Ohanian, Ríos-Rull, and Violante (2000); Buera et al. (2015). In the SBTC literature, "skill" is usually a stand-in for whether or not an individual went to college, or obtained a college degree. However, as is evident from Figure 17(a), neither the fraction of college graduates within each occupation in 1980, nor the change in the fraction of graduates from 1980 to 2010, have much of a relationship with the task-specific TFP's calibrated from our model. Since the TFP's more or less entirely explain the employment shifts observed in the data, this means college cannot explain occupational employment shifts. Moreover, as is clearer in Appendix Table 7, the correlation between task-specific productivity growth and college measures is negative; that is, those occupations with more college graduates, or in which the college graduate share grew the fastest, in fact became relatively less productive. This is the opposite of many propositions in the SBTC literature.

What we do find, however, is that the TFP growth rates correlate strongly with specific raw indices recorded in O*NET, rather than the RTI index which aggregates over them. In particular, as shown in Figure 17(b), the correlation of the TFP growth rates with routine-manual and interpersonal skills indices, the latter of which is not

used for constructing RTI, is as high as 0.77–0.80. Appendix Table 7 shows that the R^2 for both are also high at 0.59–0.64. This means that those occupations with a higher share of routine-manual tasks have shrunk, while those with a higher share of interpersonal tasks have grown.

We conclude that productivity growth has been high in routine-manual tasks and low among interpersonal tasks, and that this can explain a significant part of shifts in occupational employment, polarization, and consequently structural change. Of course, it is evident from the regressions that this is not the end of the story. The unexplained part of task-specific TFP growth may also come from endogenous changes in the distribution of skill, heterogeneous degrees of capital-labor productivity across tasks, and in an open economy setting from off-shoring, all of which we have abstracted from.[38]

7 Conclusion

We presented a multi-sector model in which individuals select themselves into becoming managers or workers, and workers further sort positively into horizontally differentiated tasks. We showed analytically and quantitatively that the model can be a useful tool for analyzing the occupational, industrial and organizational structure of an economy.

Theoretically, we fully characterize the equilibrium and prove comparative statics to show that task-specific technological progress among middle-skill jobs alone can lead to polarization among workers, polarization between managers and workers, and structural change. Quantitatively, we show that task-specific technological progress alone is sufficient to capture all of the above phenomena, unaided by sector- or skill-specific technological changes. Consistently with the model, we document empirically that polarization is prevalent even within sectors (which suggests that trade may not have been the main driver), and establish that managers have been gaining against workers especially in the manufacturing sector, which coincided with manufacturing establishment sizes—measured by employment—becoming smaller relative to services. We also show that the task-specific changes since 1980 are most pronounced in occupations that intensively use routine-manual tasks but not interpersonal skills, a pattern that is obscured when more aggregate measures of occupation characteristics are used.

Our model admits many useful extensions that have not been pursued here: for instance, one could embed individual skill dynamics to separate task productivity growth

[38] In an open economy setting, cheaper foreign labor would be observationally equivalent to higher productivity.

from human capital accumulation, or include multiple layers of management for a more in-depth study of the organization of labor by sector. The latter will in particular be useful for understanding assortative matching between managers and workers, as well as for studying between- and within-firm inequality. A quantitative analysis with more than two sectors would facilitate a sharper decomposition of occupation- and industry-specific changes, and so would an empirical analysis at a higher frequency. We are actively exploring some of these exciting topics.

Appendices

A Census Employment/Wages/Occupations

Occupation Group		occ1990dd
Managers	self-employment+	4– 19
Management Support		22– 37
Professionals		43–199
Technicians		203–235
Sales		243–283
Administrative Support		303–389
Low Skill Services		405–472
Mechanics and Construction		503–599
Miners and Precision		614–699
Machine Operators		703–799
Transportation Workers		803–899

Table 4: Census Occupation Groups

322 non-farm occupations according `occ1990dd` (Dorn, 2009), itself harmonized from `occ1990` (Meyer and Osborne, 2005), are grouped into 11 occupation groups in order of their `occ1990dd` code. All self-employed workers are classified as managers. All other occupation groups correspond to their 1-digit census occupation group except for management support, technicians and sales. Groups are presented in their (contiguous), ascending order of their codes, excluding agricultural occupations 473–498 which are dropped. In the main text, occupation groups are presented in ascending order of skill (mean hourly wage).

We use the 5% census samples from IPUMS USA. We drop military, unpaid family workers, and individuals who were in correctional or mental facilities. We also drop workers who work either in an agricultural occupation or industry.

For each individual, (annual) employment is defined as the product of weeks worked times usual weekly hours, weighted by census sampling weights. Missing usual weekly hours are imputed by hours worked last week when possible. Missing observations are imputed from workers in the same year-occupation-education cell with 322 occupations×6 hierarchical education categories: less than high school, some high school, high school, some college, college, and more than college.

Hourly wages are computed as annual labor income divided by annual employment at the individual level. Hence while employment shares include the self-employed, hourly wages do not include self-employment income.[39] We correct for top-coded

[39]While we have only considered labor income in the paper, we have conducted robustness checks by including business income as well. Hourly business income is defined similarly as hourly wages. We also separately corrected for top-coding (the top-codes for labor and business income differ) and bottom-coded in a similar fashion.

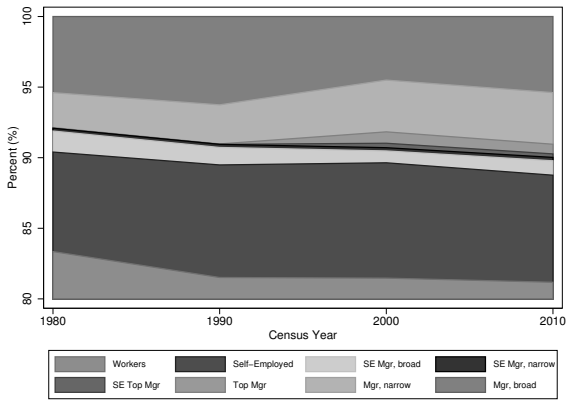


Fig. 18: Managers in the Census

Source: U.S. Census (5%). Top managers are coded 4 in `occ1990dd` while broad managers include code 22 which are not-elsewhere-classified managers, or manager occupations that do not exist across all 4 censuses.

incomes by multiplying them by 1.5, and hourly wages are set to not exceed this value divided by 50 weeks × 35 hours (full-time, full-year work). Low incomes are bottom-coded to first percentile of each year's wage distribution.

For the line graphs in Figures 2–3, we ranked occupations by their hourly wages defined as above, and smoothed across skill percentiles using a bandwidth of 0.75 for employment and 0.4 for wages; these are the same values used in Autor and Dorn (2013). For the bar graphs in Figures 2–3, 16–17 and 19, we grouped the 322 occupations vaguely up to their 1-digit Census Occupation Codes, resulting with the 11 categories summarized in Table 4 and used for our quantitative analysis. In the figures and in Tables 5–6, these groups are then ranked by the mean wage of the entire group. In particular, in Figures 2–3, 16(a) and 19, the horizontal length of a bar is set to equal the corresponding group's 1980 employment share, which does not necessarily coincide with the 3-digit occupations used to generate the smooth graphs by percentile.

Throughout the paper, we subsume all self-employed workers into the manager group. While the size of the group varies excluding them does not affect any of our results qualitatively because while the employment share of non-managerial self-employed workers was more or less constant throughout the observation period, as shown in Figure 18. There, we decompose managers into 9 subgroups. Our benchmark definition includes all 3 self-employed groups, top managers and narrow managers, but excludes broad managers. Top managers are coded 4 according to `occ1990dd` and includes CEO's, public administrators and legislators. Broad managers are coded 22 and are either not-elsewhere-classified or manager occupations that do not exist across all 4 censuses.

B Proofs

B.1 Proof of Lemma 1 and Corollary 1

The feasibility constraint (5) and the existence of $\hat{h}(j)$ and $\hat{z}(j)$ imply that the number of people with skill s assigned to task j is

$$l_h(s,j)ds = \delta(j - \hat{j}(h)) \cdot \mathcal{I}\left[z \leq \tilde{z}(h)\right] d\mu$$

where $\delta(\cdot)$ is the Dirac delta function and $\mathcal{I}$ the indicator function. Hence the allocation rule is completely determined by the assignment functions $\hat{h}(j)$ and $\hat{z}(j)$, and the productivity of all workers assigned to task $j = \hat{j}(h)$ is

$$H(j) = \int b(h, \hat{j}(h')) \cdot \delta(j - \hat{j}(h')) \cdot F(\tilde{z}(h')|h')dG(h').$$

With the change of variables $j' = \hat{j}(h')$, we can instead integrate over j':

$$\begin{aligned} H(j) &= \int b(\hat{h}(j'), j') \cdot \delta(j - j') \cdot F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j')) \cdot \hat{h}'(j')dj' \\ &= b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j)) \cdot \hat{h}'(j), \end{aligned}$$

which is (15).

For the optimal allocation, there can be no marginal gain from switching any worker's assignment. So for any $j' = j + dj$,

$$\begin{aligned} \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j), j) \geq & \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j), j'), \\ \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j'), j') \geq & \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j'), j), \end{aligned}$$

with equality if $|dj| = 0$. Substituting for $H_i(j) = H(j)/q_{ih}(j)$ using (15), we obtain

$$\frac{b(\hat{h}(j'), j')}{b(\hat{h}(j), j')} \geq \frac{\pi_{ih}(j')}{\pi_{ih}(j)} \cdot \frac{q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j)}{q_{ih}(j')F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j'))\hat{h}'(j')} \geq \frac{b(\hat{h}(j'), j)}{b(\hat{h}(j), j)},$$

and as $|dj| \to 0$,

$$\left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) = d \log \left\{ \pi_{ih}(j) \Big/ \left[q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j) \right] \right\} \Big/ dj.$$

Now using the total derivative of $b(\hat{h}(j), j)$:

$$d \log b(\hat{h}(j), j)/dj = \left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) + \partial \log b(\hat{h}(j), j)/\partial j, \tag{31}$$

and applying $\pi_{ih}(0) = 1$, we obtain (16):

$$H_i(j)/\pi_{ih}(j)H_i(0) = \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \equiv B_j(j; \hat{h}). \tag{32}$$

Plugging (12) and (16) into (11) yields the first equality in (18) in the corollary, and note that (31) implies that $b(h, \hat{j}(h)) = B_h(h; \hat{j}) \cdot B_j(\hat{j}(h); \hat{h})$, which yields the second equality.

B.2 Proof of Proposition 1 and Corollary 2

First, we re-express all capital input ratios only in terms of the thresholds $[\hat{h}(j), \hat{z}]$. Plugging (16) into (8), and applying the task production function (7) we obtain

$$\pi_{ih}(j) = v_{ih}(j) \Big/ \left[\tilde{M}(j) B_j(j;\hat{h})^{1-\alpha}\right]^{1-\sigma}, \quad \text{where } v_{ih}(j) \equiv \frac{\nu_i(j)}{\nu_i(0)} \tag{33}$$

and $\tilde{M}(j) \equiv M(j)/M(0)$. Similarly, plugging (9) and (12) in (10) we obtain

$$\pi_{iz} = v_{iz} \cdot \left(\tilde{M}(z) \cdot \hat{z}^{1-\alpha}\right)^{\omega-1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}}, \quad \text{where } v_{iz} \equiv \frac{\eta_i \nu_i(0)^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i} \tag{34}$$

and $\tilde{M}(z) \equiv M(z)/M(0)$.

Now given a between-sector allocation rule $[q_{ih}(j), q_{iz}]$, the optimal within-sector allocation is described by $[\hat{h}(j)]_{j=0}^{J}$ that solves a fixed point defined by (15)-(16) in Lemma 1, and $\hat{z}$ that solves the fixed point defined by (12) and (34):

$$\hat{h}'(j) = \frac{H_i(0) \cdot v_{ih}(j)}{q_{ih}(j)} \Big/ \left\{[\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))\right\} \tag{35a}$$

$$\hat{z}^{\alpha+\omega(1-\alpha)} = \frac{q_{iz}}{H_i(0) \cdot v_{iz}} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{1-\sigma}} \cdot \tilde{M}(z)^{1-\omega} \cdot Z \tag{35b}$$

where the boundary conditions for the ODE in (35a) are $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$, which implies

$$H_i(0) \cdot \int v_{ih}(j) \left\{q_{ih}(j) \cdot [\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))\right\}^{-1} dj = h_M. \tag{35c}$$

The functions $[B_j(j), B_h(h), \hat{z}(j), \tilde{z}(h)]$, which represent relative wages in equilibrium, are defined in (14), (17) and (18); in particular, the first two are functions of $[\hat{h}(j), \hat{j}(h)]$ only. That is, system (35) is a fixed point only in terms of the thresholds, so their determination is independent of the total amount of physical capital and labor in either sector. All that matters is relative masses *across tasks*.

Existence of a fixed point is straightforward. For an arbitrary guess of $\hat{z}(j)$, Assumptions 1-3 imply existence of a solution to the differential equation (35a) by Picard-Lindelöf's existence theorem. Similarly, a solution to (35b) exists by Brouwer's fixed point theorem once we apply a minimum value for $\hat{z} \geq \underline{z} > 0$ such that the denominator does not converge to zero.

To show that the within-sector solution is unique, we need the following lemma:

Lemma 5 *Suppose* $[q_h(j), q_z]$ *are fixed and that* $[\hat{h}(j), \hat{z}]$ *and* $[\hat{h}^1(j), \hat{z}^1]$ *are both an equilibrium for one sector. For any connected subset* $\mathcal{J}^1 \subseteq \mathcal{J}$, $\hat{h}$ *and* $\hat{h}^1$ *can never coincide more than once on* $\mathcal{J}^1$.

Proof We proceed by contradiction as in Lemmas 3-6 in Costinot and Vogel (2010). Suppose **(i)** $\hat{h}(j_a) = \hat{h}^1(j_a)$ and $\hat{h}(j_b) = \hat{h}^1(j_b)$ such that both $(j_a, j_b) \in \mathcal{J}^1$. Without loss of generality, we assume that $j_a < j_b$ are two adjacent crossing points. Then, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that

1. **(ii)** $\hat{h}^{1\prime}(j_a) \geq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \leq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j_a, j_b)$;
   or
2. **(ii)** $\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \geq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j_a, j_b)$.

Consider case **1**. Condition **(ii)** implies

$$\hat{h}^{1\prime}(j_b)/\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_b)/\hat{h}'(j_a)$$

so using (31)-(32) and (35a), and applying $\hat{h}^1(j) = \hat{h}(j)$ for $j \in \{j_a, j_b\}$ we obtain

$$\begin{aligned} 0 &< [\alpha + \sigma(1-\alpha)] \cdot \left[\int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}^1(j'), j')}{\partial j'} dj' - \int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \\ &\leq \log \left[F(\hat{z}^1(j_b)|\hat{h}(j_b))/F(\hat{z}(j_b)|\hat{h}(j_b)) \right] - \log \left[F(\hat{z}^1(j_a)|\hat{h}(j_a))/F(\hat{z}(j_a)|\hat{h}(j_a)) \right] \end{aligned} \tag{36}$$

where the first inequality follows since (2), the log-supermodularity of b, implies

$$\partial \log b(h^1, j)/\partial j > \partial \log b(h, j)/\partial j \qquad \forall h^1 > h, \tag{37}$$

and applying **(iii)**. Next, since (18) and Assumption 5.1 implies that $\hat{z}'(j) = \tilde{z}'(h)\hat{h}'(j) > 0$, Assumption 4.1 implies that the strict inequality in (36) holds only if

$$\hat{z}^1(j_b)/\hat{z}(j_b) > \hat{z}^1(j_a)/\hat{z}(j_a) \quad \Leftrightarrow \quad \log \left[\tilde{z}^1(h_b)/\tilde{z}^1(h_a) \right] > \log \left[\tilde{z}(h_b)/\tilde{z}(h_a) \right]$$

where we have written $h_x \equiv \hat{h}(j_x)$ for $x \in \{a, b\}$. Plugging in for $\tilde{z}(\cdot)$ using (18) we obtain

$$\int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' > \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

and since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(iii)** implies that $\hat{j}^1(h) < \hat{j}(h)$ for all $h \in (h_a, h_b)$. But (2), the log-supermodularity of b, implies

$$\partial \log b(h, j^1)/\partial h < \partial \log b(h, j)/\partial h, \qquad \forall j^1 < j. \tag{38}$$

a contradiction. Case **2** is symmetric. ∎

Lemma 5 implies, in particular, that any within-sector equilibria must have identical $\hat{h}(j)$, since $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$ in all equilibria. Moreover, the lemma also implies that $\hat{h}(j)$ is determined independently of $\hat{z}$, which is uniquely determined by $\hat{h}(j)$ given (35). Hence, the within-sector equilibrium is unique.

Sectoral production function The corollary expresses sectoral output only in terms of sectoral capital and labor, and the optimal assignment rules. To derive this, first note that using (8)-(10), sectoral capital can be written as $K_i = K_i(0)\Pi_{K_i}$, which is the first equation in (20). Next, from (12), we know that Z_i is linear in $H_i(0)$:

$$Z_i = q_{iz} L_z \bar{z} = L_{iz} \bar{z} = H_i(0) \cdot \hat{z} \pi_{iz}, \quad \text{where} \quad L_z \equiv \int_{z > \tilde{z}(h)} d\mu, \quad \bar{z} = Z / L_z,$$

and using Lemma 1 and (33), so is total worker productivity:

$$\begin{aligned} \int \left[H_i(j) \Big/ b(\hat{h}(j), j) \right] dj &= \int q_{ih}(\hat{j}(h)) F(\tilde{z}(h)|h) g(h) dh \\ &= H_i(0) \cdot \int \left[\pi_{ih}(j) / B_h(\hat{h}(j)) \right] dj \quad = L_i - L_{iz}. \end{aligned}$$

So rearranging, we can represent sectoral labor input as $L_i = H_i(0)\Pi_{L_i}$, which is the second equation in 20. Finally, use (7)-(10) to rewrite (6) as

$$Y_i = \psi_i \cdot \Pi_{K_i}^{\frac{\omega}{\omega - 1}} \Pi_{ih}^{\frac{\sigma - \omega}{(\sigma - 1)(1 - \omega)}} M(0) K_i(0)^{\alpha} H_i(0)^{1 - \alpha},$$

and replacing $[K_i(0), H_i(0)]$ with the expressions in (20) yields (19).

B.3 Proof of Theorem 1

Since Proposition 1 showed that the within-sector solution (and hence equilibrium) is unique, we only need to show that the sectoral allocation rules $\{[q_h(j)]_{j=0}^{J}, q_z\}$ are unique. In equilibrium, the allocation rules $[\hat{h}(j), \hat{z}]$ must be equal across sectors. Applying this to (35a) yields

$$q_h(j) = 1 \Big/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\nu_m(j)}{\nu_s(j)} \right] \tag{39}$$

and $q_h(0)$ must solve (35c), so the dependence of the between-sector allocation rule on the within-sector rule comes only through $q_h(0)$. Likewise, the rule for splitting individuals between managers and workers, (35b), implies

$$q_z = 1 \Big/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1 - \eta_s)}{(1 - \eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}} \right)^{\frac{\sigma - \omega}{1 - \sigma}} \right] \tag{40}$$

where V_{ih} is defined in (27) and depends on the within-sector allocation rule through B_j. But note that given $q_h(0)$, the other $q_h(j)$ only depend on the task intensity parameters $\nu_i(j)$ and are uniquely fixed by (39). Then we know from Proposition 1 that all $\hat{h}(j)$ are uniquely determined, as well as $\hat{z}$. Hence, q_z also only depends on the manager intensity parameters η_i, and are uniquely determined by (40) given $q_h(0)$.

So in equilibrium, $q_h(0)$ alone must solve the implied sectoral shares in (21) given (19):

$$\begin{aligned}\frac{q_h(0)}{1-q_h(0)} &\equiv Q(q_h(0)) \qquad (41)\\ =&\frac{\gamma_s}{\gamma_m}\cdot\left(\frac{\psi_s}{\psi_m}\right)^{\epsilon-1}\cdot\left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{(\sigma-\omega)(1-\epsilon)}{(1-\sigma)(1-\omega)}}\cdot\left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\left(\alpha+\frac{\omega}{1-\omega}\right)(1-\epsilon)}\cdot\left(\frac{\Pi_{L_s}}{\Pi_{L_m}}\right)^{-[\alpha+\epsilon(1-\alpha)]}.\end{aligned}$$

Existence of a solution is straightforward, since the LHS of (41) increases smoothly from 0 to ∞ as $q_h(0)$ varies from 0 to 1, while the RHS is always positive and strictly bounded regardless of the value of $q_h(0)$. To show uniqueness then, it suffices to show that the RHS cannot cross LHS more than once. We will consider the log derivatives of the RHS of (41) term by term.

Let Δ_x denote the log-derivative of x w.r.t. $q_h(0)$. Since Assumption 5.2 implies that

$$\Delta_{B_j(j)} = \int_0^j \frac{\partial^2 \log b(\hat{h}(j'), j')}{\partial h \partial j'} \cdot \frac{d\hat{h}(j')}{dj'} \cdot dj' < \varepsilon \qquad (42)$$

for all $\varepsilon > 0$, we obtain from (33) that

$$\Delta_{\pi_{ih}} = (1-\alpha)(\sigma-1)\cdot\Delta_{B_j(j)} \approx 0$$

so $\Delta_{\Pi_{ih}} \approx 0$. Likewise, Assumption 5.2 also implies that

$$\Delta_{B_h(h)} = \int_0^h \frac{\partial^2 \log b(h', \hat{j}(h'))}{\partial h' \partial j} \cdot \frac{d\hat{j}(h')}{dh'} \cdot dh' < \varepsilon \qquad (43)$$

for all $\varepsilon > 0$. This implies that $\hat{h}(j)$ is not affected by the choice of $q_h(0)$, and it is independent of the determination of $\hat{z}$ by Lemma 5. Intuitively, Assumption 5.2 makes the model behave as if there were no log-supermodularity. Then since we assume a constant returns technology, all worker allocations approach constant multiples of H_0 and does not depend on its particular value. So $\Delta_{\Pi_{ih}} \approx 0$, and $\Delta_{\Pi_{K_i}}$ only depends on $\Delta_{\hat{z}}$ since from the definition of Π_{K_i} in (20) and (34),

$$\Delta_{\pi_{iz}} = (1-\alpha)(\omega-1)\Delta_{\hat{z}} \quad \Rightarrow \quad \Delta_{\Pi_{K_i}}\Pi_{K_i} = \pi_{iz}\cdot(1-\alpha)(\omega-1)\Delta_{\hat{z}},$$

Similarly, $\Delta_{\Pi_{L_i}}$ only depends on $\Delta_{\hat{z}}$ as well, since from (18) and (43) we obtain

$$\Delta_{\tilde{z}(h)} = \Delta_{\hat{z}} + \Delta_{B_h(h)} \approx \Delta_{\hat{z}}. \tag{44}$$

so using Leibniz' rule,

$$\Delta_Z \cdot Z = -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \tag{45}$$

$$\Delta_{L_z} \cdot L_z = -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h) \cdot f(\tilde{z}(h)|h)\right] g(h)dh,$$

$$\Rightarrow \quad \Delta_{\bar{z}} = \Delta_Z - \Delta_{L_z} = \Delta_{\hat{z}} \cdot \underbrace{\int \left\{\tilde{z}(h)\left[1/L_z - \tilde{z}(h)/Z\right] \cdot f(\tilde{z}(h)|h)\right\} g(h)dh}_{\equiv \Lambda \in (0,1)}$$

where the inequality follows from selection and Assumption 4.2, so using this and (43), from the definition of Π_{L_i} in (20) we obtain

$$\Delta_{\Pi_{L_i}} \Pi_{L_i} = (\hat{z}/\bar{z})\pi_{iz} \cdot \left[\alpha + \omega(1-\alpha) - \Lambda\right] \Delta_{\hat{z}}.$$

Now rearranging (35b), plugging in (45), and using (35a) at $j = 0$ we obtain

$$\left\{\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh\right\} \Delta_{\hat{z}}$$
$$= \Delta_{q_z} - 1 \equiv \Gamma(X),$$

since $H_s(0) = q_h(0)H(0)$, $\Delta_{\hat{h}'(0)} = 0$ as it does not vary with $q_h(0)$, and $\Gamma(X)$ is defined from (40):

$$\Gamma(X) = q_h(0)(X-1)/\left[q_h(0) + (1-q_h(0))X\right],$$
$$\text{where} \quad X \equiv \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1-\eta_s)}{(1-\eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}.$$

So it follows that the log-slope of the RHS in (41) is

$$-\Bigg\{(1-\epsilon)(1-\alpha)[\alpha + \omega(1-\alpha)] \cdot \left[\frac{\pi_{sz}}{\Pi_{K_s}} - \frac{\pi_{mz}}{\Pi_{K_m}}\right]$$
$$+ (\hat{z}/\bar{z})[\alpha + \epsilon(1-\alpha)][\alpha + \omega(1-\alpha) - \Lambda] \cdot \left[\frac{\pi_{sz}}{\Pi_{L_s}} - \frac{\pi_{mz}}{\Pi_{L_m}}\right]\Bigg\}$$
$$\times \frac{\Gamma(X)}{\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh}.$$

The log-slope of the LHS in (41) is $1/\left[1 - q_h(0)\right]$, which increases from 1 to ∞ as $q_h(0)$ increases from 0 to 1, and is larger than $\Gamma(X)$ for all $X > 0$. Hence it suffices to show that the absolute value of all terms multiplying $\Gamma(X)$ are less than 1, which is true in particular due to Assumption 4.3.

Intuitively, what the planner cares about is the marginal products of Z and H in total. So when the distribution of z has a fat tail, the response of $\hat{z}$ to the choice of $q_h(0)$ is minimal as it changes Z smoothly along its entire support.

B.4 Proof of Proposition 2

Part 1. By Lemma 5, we know that no crossing can occur on $(0, \underline{j})$ or $(\bar{j}, J)$, since $\hat{h}$ and $\hat{h}^1$ already coincide at the boundaries 0 and J. Similarly, we also know from Theorem 1 that it can never be the case that there is no crossing ($\hat{h}^1(j) > \hat{h}(j)$ or $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in \mathcal{J} \setminus \{0, J\}$). Hence, there must be a single crossing in $\mathcal{J}^1$ since Lemma 5 also rules out multiple crossings.

At this point, the only possibility for j^* not to exist is if instead, there exists a single crossing j^{**} such that **(i)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (0, j^{**})$ and **(ii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j^{**}, J)$. If so, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that $\hat{h}^{1\prime}(0) < \hat{h}'(0)$, $\hat{h}^{1\prime}(j^{**}) > \hat{h}'(j^{**})$ and $\hat{h}^{1\prime}(J) < \hat{h}'(J)$. This implies

$$\hat{h}^{1\prime}(j^{**})/\hat{h}^{1\prime}(0) \geq \hat{h}'(j^{**})/\hat{h}'(0), \qquad \hat{h}^{1\prime}(J)/\hat{h}^{1\prime}(j^{**}) \leq \hat{h}'(J)/\hat{h}'(j^{**}). \tag{46}$$

Let us focus on the first inequality. Using (32) and (35a) we obtain

$$\begin{aligned} 0 >& [\alpha + \sigma(1-\alpha)] \cdot \left[\int_0^{j^{**}} \frac{\partial \log b(\hat{h}^1(j), j)}{\partial j} dj - \int_0^{j^{**}} \frac{\partial \log b(\hat{h}(j), j)}{\partial j} dj \right] \qquad (47) \\ \geq& (1-\sigma)m + \log \left[F(\hat{z}^1(j^{**})|\hat{h}(j^{**}))/F(\hat{z}(j^{**})|\hat{h}(j^{**})) \right] - \log \left[F(\hat{z}^1(0)|\hat{h}(0))/F(\hat{z}(0)|\hat{h}(0)) \right]. \end{aligned}$$

where the first inequality follows from (37), and applying **(i)**. Since $m > 0$, if $\sigma \in (0, 1)$, Assumptions 4.1 and 5.1 imply that the strict inequality in (47) holds only if

$$\int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' < \int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

where we have written $h^{**} \equiv \hat{h}(j^{**})$. And since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(i)** implies that $\hat{j}^1(h) > \hat{j}(h)$ for all $h \in (0, h^{**})$. But this violates (38), the log-supermodularity of b. The case for the second inequality in (46) is symmetric.

Part 2. Let Δ_x denote the log-derivative of x w.r.t. m. Applying (33) into the definition of Π_{ih} in (9), we obtain

$$\begin{aligned} \Delta_{\Pi_{ih}} \cdot \Pi_{ih} &= (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj + \int \left\{ \pi_{ih}(j) \cdot (1-\alpha)(\sigma - 1) \cdot \Delta_{B_j(j)} \right\} dj \\ &\approx (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj \qquad (48) \end{aligned}$$

where the approximation follows from Assumption 5.2 and (42). Hence $\Delta_{\Pi_{ih}} < 0$ if $\sigma < 1$. Rearranging (35b) and using (35a) at $j = 0$ we obtain

$$0 > \frac{\sigma - \omega}{1 - \sigma} \cdot \Delta_{\Pi_{ih}} - \Delta_{\hat{h}'(0)} = \left[\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) \right] \Delta_{\hat{z}} - \Delta_Z \tag{49}$$

where the inequality holds if $\omega < \sigma < 1$, and since we know from part 1 that $\Delta_{\hat{h}'(0)} \geq 0$. Now suppose $\Delta_{\hat{z}} \geq 0$. Then for (49) to hold it must be the case that $\Delta_Z > 0$, but from (45), $\Delta_Z \leq 0$ if $\Delta_{\hat{z}} \geq 0$, a contradiction. Hence, $\hat{z}^1 < \hat{z}$, and $\tilde{z}^1(h) < \tilde{z}(h)$ for all h by (44).

B.5 Proof of Lemmas 2 and 3

From (26), the $\Delta_{V_i(j)}$'s are sector-neutral and common across sectors, except for $\Delta_{V_i(z)}$. Under Assumption 5.2, (42)-(43) imply

$$\Delta_{V_i(j)} = \sigma - 1 < 0 \quad \forall j \in \mathcal{J}^1 \quad \text{and 0 otherwise.} \tag{50a}$$

So for workers, any difference in how the share of task j employment evolves differentially across sectors depends only on $\Delta_{V_{L_i}}$, the sum of within-sector employment shifts, weighted by the employment shares of all tasks within a sector $V_i(j)/V_{L_i} = L_i(j)/L_i$. Since we know that intermediate jobs are the ones that are declining, from the definition of Π_{L_i} in (20) a measure of the speed of polarization among workers is the total change in their employment:

$$\Delta_{V_{il}} V_{il} = \int_{\mathcal{J}} V_i(j) \cdot \Delta_{V_i(j)} dj = (\sigma - 1) \cdot \int_{\underline{j}}^{\bar{j}} V_i(j) dj$$

and we have used (50a). So we can compare the speeds of polarization across the two sectors from

$$\Delta_{V_{ml}} - \Delta_{V_{sl}} = (\sigma - 1) \cdot \int_{\underline{j}}^{\bar{j}} \left\{ \left[\frac{\nu_m(j)}{V_{ml}} - \frac{\nu_s(j)}{V_{sl}} \right] \cdot \left[M(j) B_j(j)^{1-\alpha} \right]^{\sigma-1} \Big/ B_h(\hat{h}(j)) \right\} dj. \tag{50b}$$

Manager employment has sector-differential effects through V_{ih}: Under Assumption 5.2 and using (48), we obtain

$$\Delta_{V_m(z)} - \Delta_{V_s(z)} = (\sigma - \omega) \cdot \int_{\underline{j}}^{\bar{j}} \left\{ \left[\frac{\nu_m(j)}{V_{mh}} - \frac{\nu_s(j)}{V_{sh}} \right] \cdot \left[M(j) B_j(j)^{1-\alpha} \right]^{\sigma-1} \right\} dj. \tag{50c}$$

Equations (50b) and (50c) imply that a sufficient condition for both horizontal and vertical polarization to be faster in manufacturing, as in the data, is $\omega < \sigma < 1$ and $\nu_{mh}(j) \gg \nu_{sh}(j)$ for all $j \in \mathcal{J}^1$, which is Lemma 2.

Structural change From (22) and (41) we obtain

$$\Delta_{L_s} = L_m \cdot \left\{ \Delta_{V_{L_s}} - \Delta_{V_{L_m}} + \Delta_Q \right\}. \tag{51a}$$

The term $\Delta_{L_s} - \Delta_{L_m}$ is the first-order force of structural change that comes only from the change in selection rules. However, since this takes us off the between-sector equilibrium, $q_h(0)$ must shift to satisfy the equilibrium condition (41). The net amount of structural change will depend on whether the selection effect is overturned or reinforced by the change in $q_h(0)$.

Since $Q(q_h(0))$ in (41) changes monotonically from 0 to ∞ in $q_h(0)$, we only need to consider the direction of the change of the RHS off equilibrium. Using (50), the log-derivative of the RHS of (41) can be written as

$$(1-\epsilon)\left\{\frac{\sigma-\omega}{(1-\sigma)(1-\omega)}\cdot\left(\Delta_{V_{sh}}-\Delta_{V_{mh}}\right)+\left(\alpha+\frac{\omega}{1-\omega}\right)\left(\Delta_{\Pi_{K_s}}-\Delta_{\Pi_{K_m}}\right)\right\}$$
$$-\left[\alpha+\epsilon(1-\alpha)\right]\left(\Delta_{V_{L_s}}-\Delta_{V_{L_m}}\right). \tag{51b}$$

Under Lemma 2, the part with $\Delta_{V_{ih}}$'s is positive from (50b). The part with $\Delta_{\Pi_{K_i}}$ is determined by

$$\Delta_{\Pi_{K_i}}\Pi_{K_i} = \Pi_{ih}\Delta_{V_{ih}} + \pi_{iz}\Delta_{\pi_{iz}}, \tag{51c}$$
$$\Delta_{\pi_{sz}} - \Delta_{\pi_{mz}} = \frac{\sigma-\omega}{1-\sigma}\left(\Delta_{V_{sh}}-\Delta_{V_{mh}}\right). \tag{51d}$$

Clearly, capital polarizes along with labor, both horizontally and vertically; and the speed is faster in manufacturing if the assumptions in Lemma 3 holds.

Why structural change cannot be overturned, as explained in the text, is also formalized here: Even if there is a decline in $q_h(0)$ due to the negative effect coming from last term in (51b) dominating the positive effect from the first two terms, it can never overturn the direction of structural change in (51a) as long as $\epsilon < 1$. Equations (50)-(51) also make it clear that structural change depends differently on the productivities of capital and labor.

C Quantitative Model and Numerical Details

With discrete tasks, it must be that the marginal product of the threshold worker is equalized between tasks:

$$MPT_{i0}\cdot\frac{(1-\alpha)T_{i0}}{L_{i0}} = MPT_{i1}\cdot\frac{(1-\alpha)T_{i1}}{\bar{h}_1 L_{i1}}\cdot\hat{h}_1,$$
$$MPT_{ij}\cdot\frac{(1-\alpha)T_{ij}}{(\bar{h}_j-\chi_j)L_{ij}}\cdot(\hat{h}_{j+1}-\chi_j) = MPT_{ij+1}\cdot\frac{(1-\alpha)T_{i,j+1}}{(\bar{h}_{j+1}-\chi_{j+1})L_{i,j+1}}\cdot(\hat{h}_{j+1}-\chi_{j+1})$$

using Assumption 3, and L_{ij} is the measure of workers in sector i, task j and $\bar{h}_j \equiv H_{ij}/L_{ij}$. Thus, we are assuming that the means of skills in task j are equal across

sectors $i \in \{m, s\}$, which is true when tasks are a continuum. Then

$$\hat{h}_1 = \frac{\bar{h}_1 L_{i1}}{\pi_{i1} L_{i0}}, \qquad \frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j} = \frac{\pi_{ij}(\bar{h}_{j+1} - \chi_{j+1}) L_{i2}}{\pi_{i,j+1}(\bar{h}_j - \chi_j) L_{i1}}, \tag{52}$$

where π_{ij} is the discrete version of (8), and can be expressed using (52) as

$$\pi_{i1} = \frac{\nu_{i1}}{\nu_{i0}} \cdot \left(\frac{M_1}{M_0} \cdot \hat{h}_1^{1-\alpha} \right)^{\sigma-1}, \quad \frac{\pi_{i,j+1}}{\pi_{ij}} = \frac{\nu_{i2}}{\nu_{i1}} \cdot \left[\frac{M_{j+1}}{M_j} \left(\frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j} \right)^{1-\alpha} \right]^{\sigma-1}. \tag{53}$$

In equilibrium, indifference across tasks for threshold workers imply

$$\begin{aligned} & w_0 = w_z \hat{z} = w_1 \hat{h}_1, \quad w_j(\hat{h}_{j+1} - \chi_j) = w_{j+1}(\hat{h}_{j+1} - \chi_{j+1}) \\ \Rightarrow \quad & w_z / w_0 = 1/\hat{z}, \quad w_1 / w_0 = 1/\hat{h}_1, \quad w_{j+1}/w_j = \frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j}. \end{aligned} \tag{54}$$

which is used to calibrate the distribution of skills in Section 5.3. The rest of the parameters are calibrated as follows:

1. Guess (σ, ω).
2. Given elasticities, first fit 1980 moments:
   (a) Guess (M, A_m).
   (b) Plug in the threshold values $\mathbf{x}_{1980}$ implied by the skill distribution, along with the empirical values of $(L_{iz}, L_{i0}, \ldots, L_{i9})$, the employment shares of each occupation in sector $i \in \{m, s\}$ from Table 5, into (12) and (52). Then we recover all the ν_{ij}'s from (52)-(53), and the η_i's from (12) and (34) in closed form (since $M_j = M$ are assumed to be equal for all j). This ensures that the 1980 equilibrium exactly fits within-sector employment shares by occupation (20 parameters, 20 moments).
   (c) Repeat from (a) until we exactly fit the manufacturing employment share in 1980, and output per worker of 1.[40] Since (19) and (21) are monotone in (M, A_m), the solution is unique (2 parameters, 2 moments).
3. Given elasticities and all parameters, calibrate growth rates to 2010 moments:
   (a) Guess m_0.
   (b) Guess $\{m_j\}_{j=z,1}^{9}$. Plug in threshold values $\mathbf{x}_{2010}$ and new TFP's into (12) and (52), which yields equilibrium employment shares by occupation, within each sector. Then use (19)-(21) to solve for the 2010 equilibrium, which yields equilibrium employment shares between sectors.

[40] The latter must be matched since the value of K_{1980} we plug in from the data was normalized by 1980's output.

(c) Repeat from (a) until we exactly exactly fit aggregate GDP (or equivalently TFP) in 2010. (1 parameter, 1 moment).

4. Repeat from 1. to minimize the distance between the within-sector employment shares by occupation (but not necessarily by sector) implied by the 2010 model equilibrium and the data (13 parameters, 21 moments).

For (σ, ω), we first search globally by setting a 100×100 grid that covers the box $[0, 2] \times [0, 2]$, then locally search from the best point using a Nelder-Meade simplex algorithm.

D Tables and Figures Not in Text

Ranked by mean wage (except management)	COC Group	Employment Shares 1980	Employment Shares 2010	Manufacturing	Manufacturing	Rel. Wages 1980	Rel. Wages 2010
Low Skill Services	400	**10.44**	**13.92**	**0.59**	**0.23**	**0.65**	**0.55**
Middle Skill		**59.09**	**46.48**	**25.86**	**12.93**	**0.90**	**0.77**
Administrative Support	300	16.57	14.13	3.47	1.53	0.78	0.68
Machine Operators	700	9.81	3.75	8.79	3.02	0.84	0.64
Transportation	800	8.73	6.64	3.80	2.28	0.89	0.63
Sales	240	7.87	9.37	0.79	0.62	0.94	0.90
Technicians	200	3.23	3.86	1.00	0.57	1.04	1.12
Mechanics & Construction	500	7.91	6.02	4.44	3.19	1.06	0.81
Miners & Precision Workers	600	4.97	2.71	3.58	1.73	1.09	0.77
High Skill		**19.22**	**26.16**	**3.87**	**3.64**	**1.26**	**1.30**
Professionals	40	11.02	16.51	1.73	1.45	1.21	1.26
Management Support	20	8.20	9.65	2.14	2.20	1.32	1.37
Management	1	**11.26**	**13.44**	**2.47**	**2.59**	**0.00**	**0.00**

Table 5: Occupation×Sector Employment and Relative Wages

Source: US Census (5%), 1980 and 2010. All employment shares are in percentage of aggregate employment. The first two columns show the employment share of each occupation for each year. The "Manufacturing" columns show manufacturing employment of each occupation for each year (so the sum across all occupations is the manufacturing employment share). Relative wages are normalized so that the mean wage across all occupations is 1.

Ranked by mean wage (except management)	(1) -	(2) m_j	BM m_j	Data
Low Skill Services	**1.973**	**-2.726**	**-0.731**	
Middle Skill				
Administrative Support	1.973	1.252	2.930	
Machine Operators	1.973	10.018	9.122	
Transportation	1.973	3.326	4.348	
Sales	1.973	-1.895	0.012	
Technicians	1.973	-2.484	-1.144	
Mechanics & Construction	1.973	1.742	2.315	
Miners & Precision Workers	1.973	6.367	6.328	
High Skill				
Professionals	1.973	-3.973	-2.248	
Management Support	1.973	-1.973	-0.489	
Management	**1.973**	**-1.438**	**-0.017**	
Aggregate TFP growth	**1.030**	**1.030**	**1.030**	**1.030**
a_m (Manu TFP growth)	0.252	0.252	2.943	2.229
a_s (Serv TFP growth)	-1.205	2.021	0.308	0.743

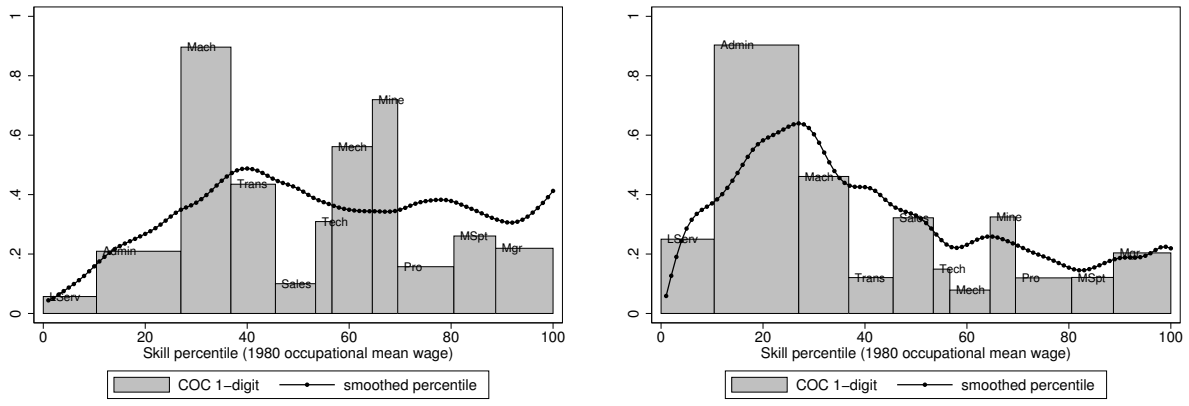
Table 6: Recalibrated TFP Growth Rates for Counterfactuals

Column (1) stands for the counterfactual in which we set $m_j = m$ and calibrate (a_m, a_s) to match sectoral TFP's, and (2) for when we let $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ all vary simultaneously. "BM" stands for the benchmark calibration. For all scenarios, aggregate GDP growth (and consequently TFP growth) is matched exactly, shown in the first row of the bottom panel. For the "BM" and "Data" columns, the a_m and a_s rows show the empirical growth rates of the manufacturing and services sectors' TFP's, respectively.

			ΔL_j				
		TFP	- 9.584 ***				
		R^2	0.939				
RTI (DOT)	0.429						
	(0.268)						
Routine manual		0.797**		0.618			
		(0.206)		(0.527)			
Manual interpersonal			-0.767**	-0.192			
			(0.192)	(0.549)			
College share 1980					-11.142*		-7.994**
					(3.599)		(2.269)
ΔCollege share 1980-2010						-33.673*	-20.295*
						(17.410)	(13.547)
Constant	1.061	0.377	3.281**	1.065	4.035*	4.818*	5.204*
	(0.941)	(0.738)	(0.970)	(2.339)	(1.401)	(1.674)	(1.759)
R^2	0.184	0.635	0.588	0.640	0.439	0.372	0.539

Standard errors in parentheses, $^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$

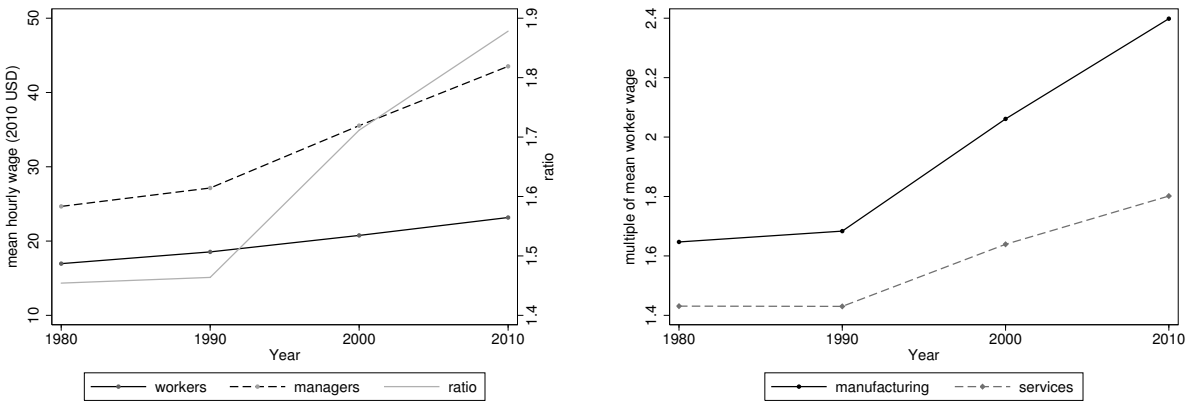
Table 7: Task-Specific TFP Growth, Employment, and Empirical Measures
The first panel shows the results from regressing employment share changes on the calibrated task-specific TFP growth rates, m_j. The second panel shows the results from regressing the TFP growth rates on various empirical measures.



(a) Manufacturing

(b) Routinization

Fig. 19: Manufacturing employment shares across skill percentiles.
Source: U.S. Census (5%). Left: Manufacturing employment share by occupation-skill percentile in 1980. Right: Share of top employment-weighted third of occupations in terms of RTI by skill percentile, replicates Autor and Dorn (2013) who construct RTI from detailed task requirements by occupation in DOT. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The x-axis units are in percentage share of employment. The y-axis measures changes from 1980 to 2010, of which units are in percentage points in Panel (a). Further details in text and Appendix A.

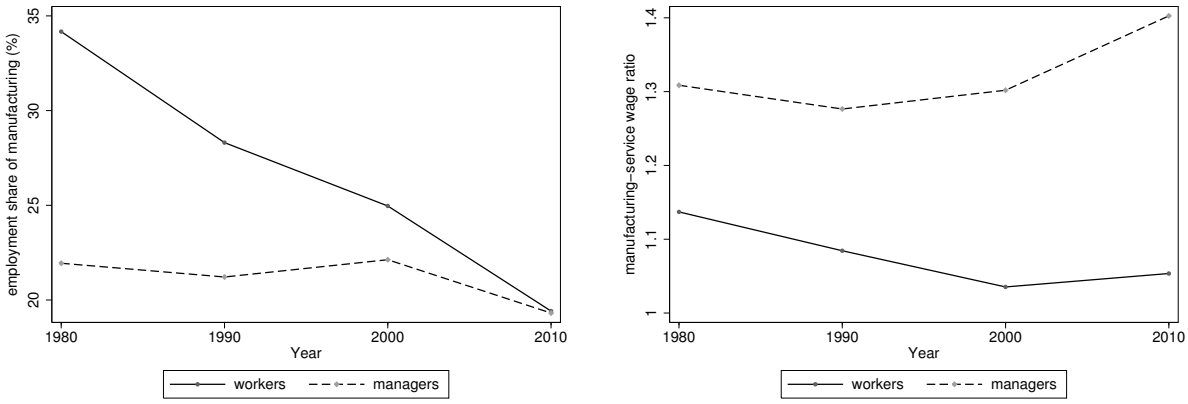


(a) Aggregate Wage Levels and Ratio

(b) Manager Wage by Sector

Fig. 20: Relative Manager Wages

Source: U.S. Census (5%). Left: levels and ratio of mean wages or managers and all other workers in aggregate. Right: relative mean wage of managers over all other workers within manufacturing and services. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for how we define management in the census and Figure 18 for a detailed breakdown of the manager group.

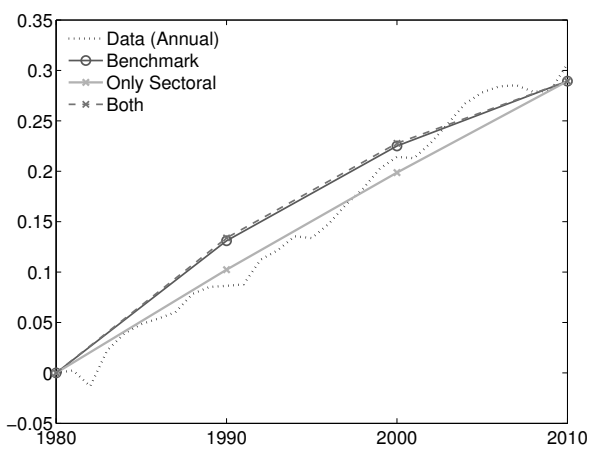


(a) Manufacturing Employment Share

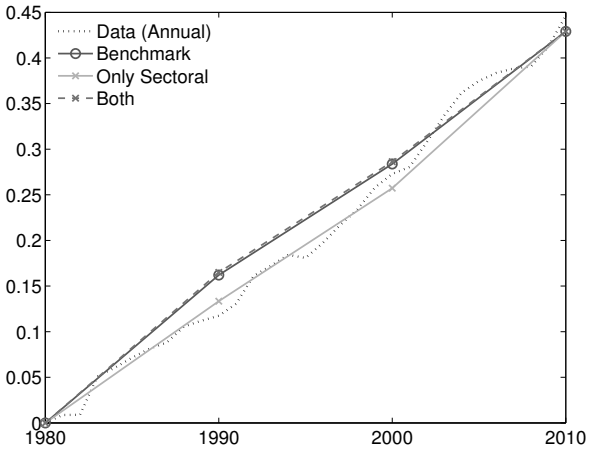
(b) Manufacturing-Services Average Wage Ratio

Fig. 21: Manufacturing vs. Services by Occuaption

Source: U.S. Census (5%). Left: manufacturing employment share within the manager occupation group and all other workers. Right: mean wage of manufacturing employment relative to services employment within the manager occupation group and all other workers. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for how we define management in the census and Figure 18 for a detailed breakdown of the manager group.



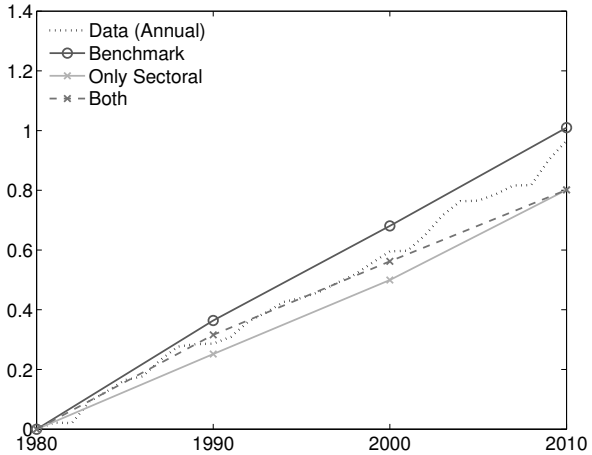
(a) Log TFP Growth



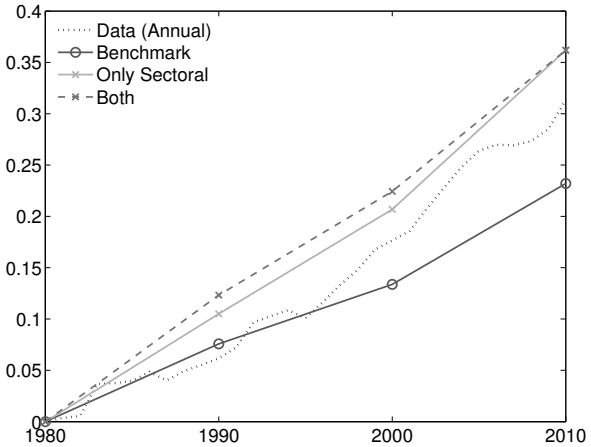
(b) Log GDP per Worker Growth

Fig. 22: Aggregate Output and TFP Growth

Data: NIPA. 1980 levels are normalized to 0, so the slope of the lines are the growth rates.



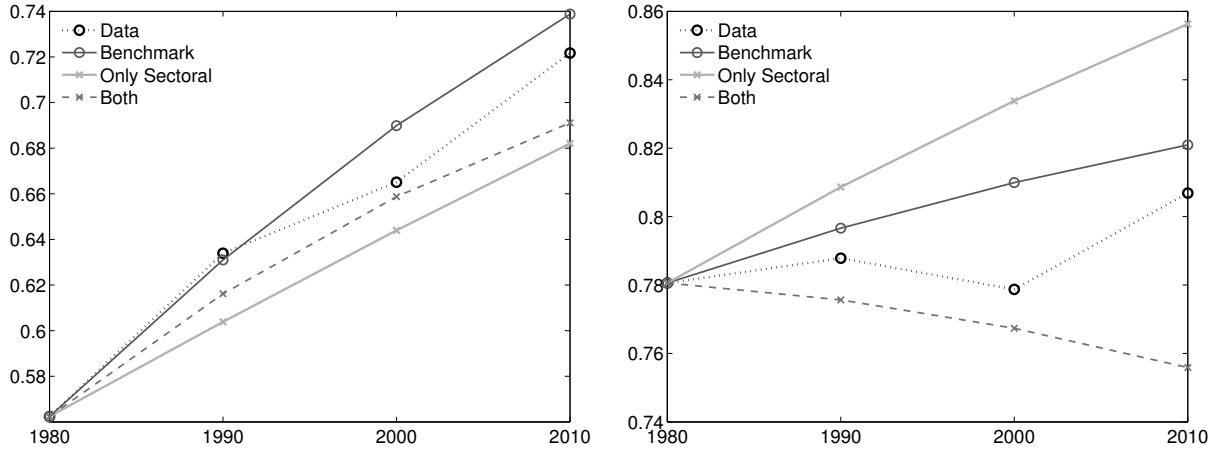
(a) Log GDP per Worker, Manufacturing



(b) Log GDP per Worker, Services

Fig. 23: Benchmark vs. Counterfactuals, GDP per Worker

Data: NIPA. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. 1980 levels are normalized to 0, so the slope of the lines are the growth rates.



(a) Services Employment Share, Routine Jobs

(b) Services Employment Share, Managers

Fig. 24: Benchmark vs. Counterfactuals, Structural Change

Data: U.S. Census (5%). Occupations are ranked by their 1980 mean wage for 11 one-digit groups, and regrouped into 4 broader categories as shown in Appendix A Table 5. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Further details in Section 2 and Appendix A.

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