Horizontal and Vertical Polarization: Task-Specific Technological Change in a Multi-Sector Economy*

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Abstract

We develop a novel framework that integrates an economy's skill distribution with its occupational and industrial structure, and analyze how technology shapes economic inequality. Individuals become a manager or a worker based on their managerial vs. worker skills, and workers further sort into a continuum of occupations ranked by skill content. Our theory dictates that faster technological progress for middle-skill worker tasks not only raises the employment shares and relative wages of lower- and higher-skill worker occupations (*horizontal* polarization), but also raises those of managers over workers as a whole (*vertical* polarization). Both dimensions of polarization are faster within sectors that depend more on middle-skill tasks. The TFP of such sectors grows faster endogenously, and, with complementarity across sectoral goods, their employment and value-added shares decrease. In the limiting growth path, middle-skill occupations vanish but all sectors coexist. Our quantitative analysis of the U.S. economy, utilizing several novel facts, shows that task-specific technological progress—which was fastest for occupations that embody routine-manual tasks but not interpersonal skills—is important for understanding the changes in its sectoral, occupational, and organizational structure since 1980. By comparison, skill-biased technological change and sector-specific technological progress played only a minor role.

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1 Introduction

We develop a novel framework that integrates the distribution of individual skills in an economy with its occupational and industrial structure. It enables us to analyze how changes in wage and employment shares across occupations and across industrial sectors are interrelated and reinforce each other, providing a comprehensive view on the economic forces that shape the sectoral, occupational, and organizational structure of an economy.

In our model, individuals are heterogeneous in two-dimensional skills, i.e., managerial talent and worker human capital, based on which they become a manager or a worker. Workers further select into a continuum of worker occupations of varying skill contents, based on their worker human capital. Managers organize tasks (i.e., occupation-level output) to produce sector-level output, and sectors differ in how intensively different tasks are used for production. We assume individual skills are sector-neutral. Our contribution on the theory side is the characterization of the equilibrium assignment in this new class of models.

If different tasks are complementary for production, our theoretical analysis shows that faster technological progress for middle-skill worker tasks relative to the others leads to: (i) rising employment shares and wages of low- and high-skill occupations relative to middle-skill occupations, commonly known as job and wage polarization; (ii) rising employment share and wage of managers relative to workers as a whole, which we call *vertical* polarization to distinguish from the *horizontal* polarization across workers; (iii) faster horizontal and vertical polarization within those sectors that depend more on middle-skill tasks; and (iv) faster endogenous growth of the total factor productivity (TFP) of such sectors, shrinking their employment and value-added shares if sectoral goods are complementary (i.e., structural change).

The last model prediction merits further discussion. First, because sector-level TFP is endogenously determined by equilibrium occupational choices, task-specific technological progress—which is sector-neutral—has differential impact across sectors. Second, when the employment shares of the sectors that rely less on middle-skill workers and more on managers rise, they reinforce the overall degree of horizontal and vertical polarization. Third, in the asymptotic dynamics of the structural change driven by task-specific technological progress, only occupations with slow technological progress remain and all others vanish, but all sectors exist. This is in contrast to many theories of structural change that rely on sector-specific forces: the shift of production factors from one sector to another continues as long as those forces exist, so that shrinking sectors vanish in the limit. In our model, task-specific technological progress is sector-neutral and only affects sectors indirectly through how they combine tasks. Once the

employment shares of the occupations with faster progress become negligible, structural change ceases even as productivity continues to grow differentially across occupations.

Empirically, predictions (i) through (iv) are salient features of the changes in the U.S. economy since 1980: (i) job and wage polarization is well-documented in the literature, e.g., Autor and Dorn (2013), which here we call horizontal polarization; (ii) using the same data, we highlight vertical polarization as new fact; and (iii) we verify that manufacturing is more reliant on middle-skill workers and less on managers than services, and also that both dimensions of polarization are indeed faster within manufacturing than in services. In addition, we provide evidence from establishment-level data that corroborates the faster vertical polarization in manufacturing: manufacturing establishments shrank faster when measured by employment and grew faster by value-added than those in services, which is consistent with the model prediction once we assume that the number of managers per establishment is stable over time. Finally, (iv) it has long been understood that the faster growth of manufacturing TFP is an important driver of structural change from manufacturing to services. Consequently, our model shows that all these empirical facts have a common cause: faster technological progress for middle-skill worker tasks than the rest.

In our quantitative analysis, we confirm that task-specific technological progress, rather than exogenous sector-specific TFP growth, accounts for the observed growth of sector-level TFP. More broadly, task-specific technological progress—especially for middle-skill tasks—is important for understanding the changes in the sectoral, occupational, and organizational structure in the United States over the last 35 years.

The natural next question is what explains such differential productivity improvements across tasks. To explain what we call horizontal polarization, Autor and Dorn (2013), Goos et al. (2014) and others hypothesized that "routinization"—i.e., faster technological advancement for tasks that are more routine in nature (which tend to be middle-skill tasks in the data)—reduced the demand for the middle-skill occupations. We provide a sharper answer. When we consider detailed characteristics of occupations, we find that the task-specific technological progress we quantify is much more strongly correlated with the routine-manual index and the inverse of the manual-interpersonal index across occupations than with the routinization index commonly used in the polarization literature. In other words, the technological progress of the last three or four decades heavily favored those manual tasks that are repetitive in nature and require few interpersonal skills.

Related Literature Our model is a first attempt to provide a framework that links the occupational structure of an economy to sectoral aggregates. In particular, we can use micro-estimates of occupational employment and wages, which have been

studied extensively in labor economics, to study how occupational choices aggregate up to macro-level sectoral shifts. This is of particular empirical relevance for the U.S. and other advanced economies. The 1980s marks a starting point of rising labor market inequality, which can partly be attributed to polarization. It was also the starting point of a clear rise in manufacturing productivity (Herrendorf et al., 2014) and the rise of low-skill service jobs (Autor and Dorn, 2013). The main finding in this regard is that task-specific productivity growth and micro-level elasticities in the labor market can be of first-order importance for understanding the sector-level and aggregate economic growth.

Task-based models are found in Acemoglu and Autor (2011) and Goos et al. (2014) to explain polarization in employment shares, but neither addresses wage polarization. Furthermore, neither relates polarization to structural change across industrial sectors, nor treat managers as an occupation that is qualitatively different from workers. In the same vein, if we remove sectors and managers from our model, it becomes similar to the one in Costinot and Vogel (2010), except that we have capital as well.

The manager-level technology in our model is an extension of the span-of-control model of Lucas (1978), in which managers hire workers to produce output. However, unlike all existing variants of the span-of-control model, in our model managers organize tasks instead of workers. That is, instead of deciding how many workers to hire, they decide on the quantities of each task to use in production, and for each task, how much skill to hire (rather than how many homogeneous workers). Moreover, we assume a CES technology between managerial talent and tasks.[1]

Our model is closely related to the rapidly growing literature in international trade that use assignment models to explain inequality between occupations and/or industries (Burstein et al., 2015; Lee, 2015). The majority of such models follow the tradition of Roy: all workers have as many types of skills as there are available industry/occupation combinations, and select themselves into the job in which they have a comparative advantage. To make the model tractable, they typically employ a Fréchet distribution which collapses the model into an empirically testable set of equations for each industry and occupation pair. While the manager-worker division in our model is also due to Roy-selection—that is, two-dimensional skill that is specific to being a manager vs. a worker—the horizontal sorting of workers into a continuum of tasks is qualitatively different. With only two skill dimensions, one can map them more clearly into observable individual characteristics in the data, and more transparently explore their endogenous formation.

[1]In this context, we add four dimensions to the standard span-of-control model: non-unitary elasticity between managers and workers, heterogeneity in worker productivity as well as in managerial productivity, multiple worker tasks or occupations, and multiple sectors.

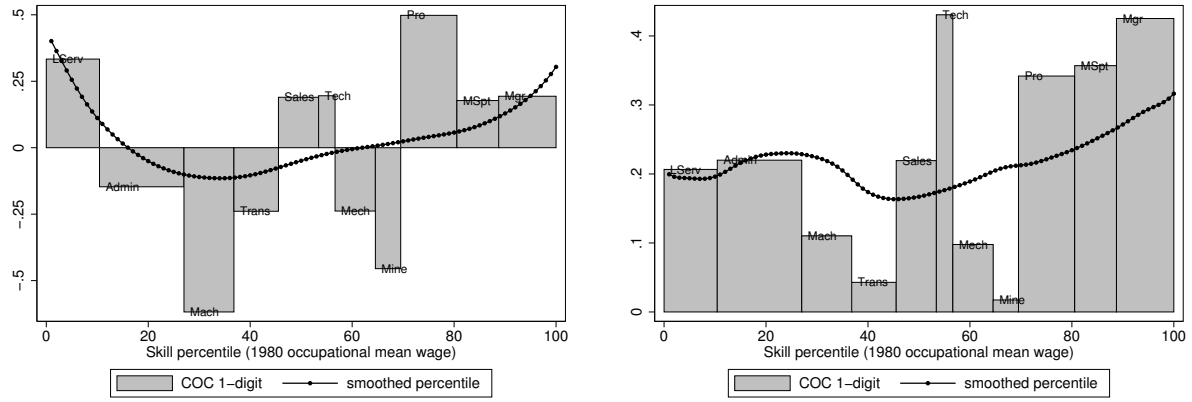
Since Ngai and Pissarides (2007), most production-driven models of structural change rely on exogenously evolving sectoral productivities. Closer to our model is Acemoglu and Guerrieri (2008), in which the capital-intensive sector (in the sense of having a larger capital share in a Cobb-Douglas technology) vanishes in the limiting balanced growth path. Sectors in our model differ in how intensively they use different tasks. By contrasting different types of labor, rather than capital and labor, we can connect structural change—which happens across sectors—to labor market inequality across occupations. Moreover, as mentioned above, unlike any other existing explanation of structural change, our model implies that it is certain occupations, not broadly-defined sectors, that may vanish in the limit.

While we are the first to build a model in which individuals with different skills sort themselves into different occupations, which in turn are used as production inputs in multiple sectors,[2] there have been recent attempts such as Buera and Kaboski (2012) and Buera et al. (2015), in which multiple sectors use different combinations of heterogeneous skills as production inputs. One important distinction is that we separate worker human capital and skill levels of a task or occupation. Our driving force (i.e., routinization) is specific to a task or occupation. This way, not only can we address broader dimensions of wage inequality and use micro-labor estimates to discipline our model,[3] but also represent sectoral TFPs endogenously by aggregating over equilibrium occupational choices, rather than relying on exogenously evolving productivities.

The rest of the paper is organized as follows. In section 2, we summarize the most relevant empirical facts: horizontal and vertical polarization in the overall economy, the faster speed of polarization within manufacturing than in services, and structural change. In section 3, we present the model and solve for its equilibrium. In section 4 we perform comparative statics demonstrating that faster technological progress for the middle-skill worker tasks leads to horizontal and vertical polarization, and ultimately to structural change. Section 5 characterizes the limiting behavior of the dynamic economy. Section 6 calibrates an expanded version of the theoretical model to data from 1980 to 2010, and quantifies the importance of task-specific technological progress. Section 8 concludes.

[2] Bárány and Siegel (2016) build a model in which occupations are tied to sectors.

[3] Autor et al. (2006); Acemoglu and Autor (2011) show that residual wage inequality controlling for education groups is much larger than between-group inequality.



(a) Employment Polarization

(b) Wage Polarization

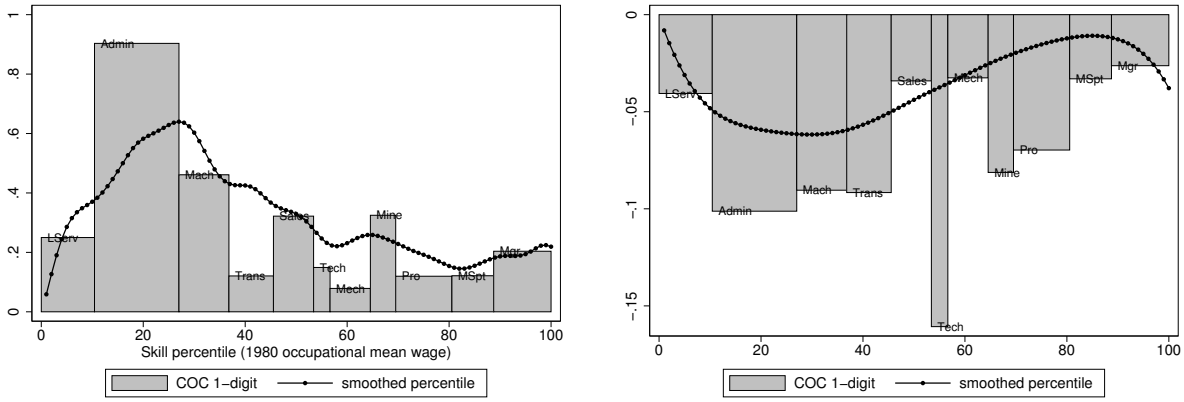
Fig. 1: Job and Wage Polarization, 30 years.

2 Facts

When obtaining our empirical moments, we subsume mining and construction into manufacturing, and government into services, and the agricultural sector is dropped. That is, aggregate output and capital correspond to the moments computed without agriculture, as is the case for employment shares.[4]

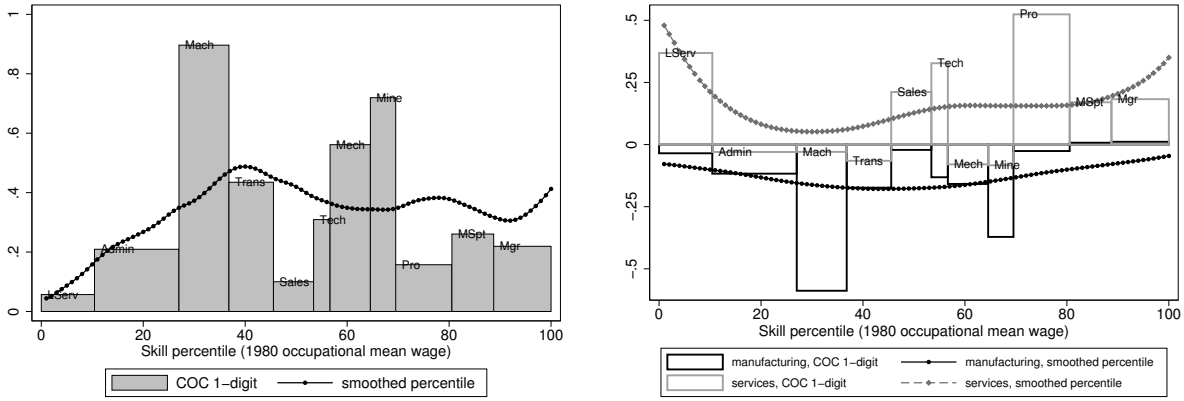
1. Jobs and wages have polarized, figure 1 Source: U.S. Census and ACS 2010, replicates and extends Autor and Dorn (2013). Occupations are ranked by their 1980 mean wage.
2. Routinizable jobs correlate with structural change, figure 2. Source: U.S. Census and ACS 2010. Left panel replicated from Autor and Dorn (2013), right panel shows change in manufacturing employment by occupation cell.
3. Manufacturing has relatively higher share of intermediate occupations, of which low-to-middle skill jobs have been relatively shrinking, figure 3. The right panel compares employment polarization by manufacturing and services, separately. Source: U.S. Census and ACS 2010.
4. Structural change continues in the U.S., figure 4. Source: BEA (value-added), NIPA Table 6 (persons involved in production). Manufacturing includes mining and construction.
5. Both employment share and relative wages for managers are increasing: figure 5. See appendix for definition of managers in the census.

[4]Agriculture shares are only about 2% of both total output and employment, and has been more or less flat since the 90s.



(a) Routinization (b) Manufacturing Employment

Fig. 2: Routinization and Structural Change, 30 years.



(a) Manufacturing (b) Employment Polarization by Sector

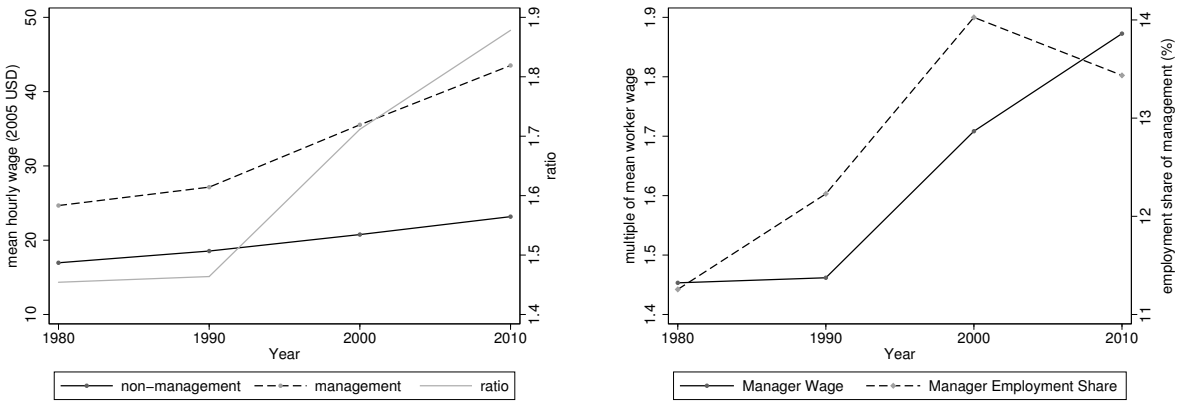
Fig. 3: Manufacturing employment shares across skill percentiles.



(a) GDP, Value-Added

(b) Aggregate Employment Shares

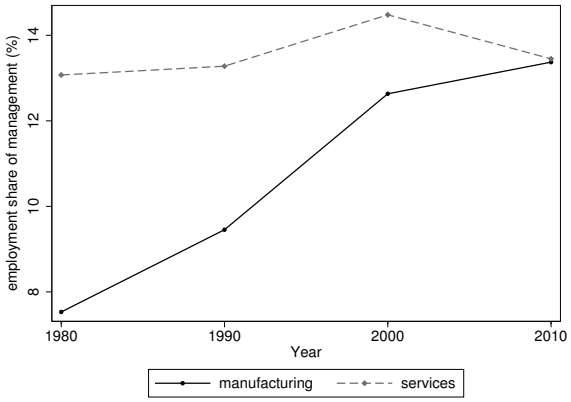
Fig. 4: Structural Change, 1970-2013.



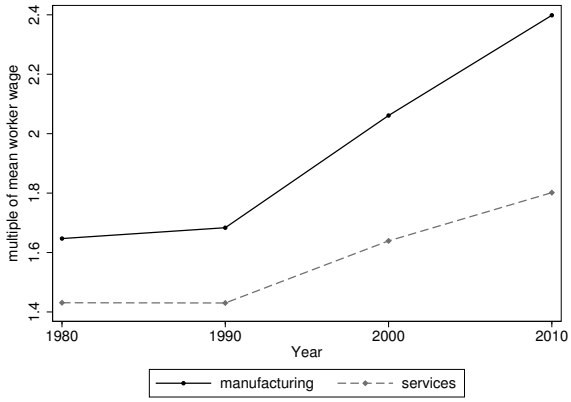
(a) Wage Levels

(b) Relative Employment and Wages

Fig. 5: Managers vs Workers

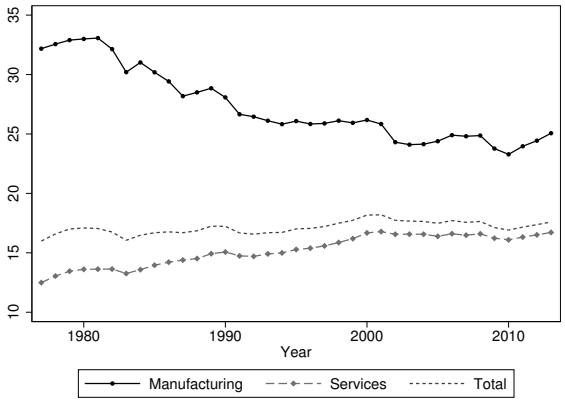


(a) Manager Employment Share

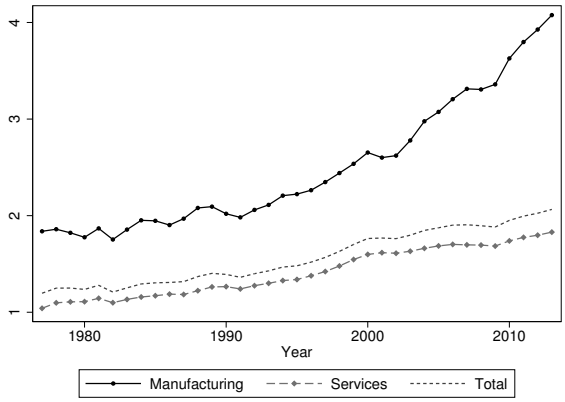


(b) Relative Manager Wage

Fig. 6: Managers by Sector



(a) Average No. of Workers



(b) Average Value-added Output

Fig. 7: Establishment Size by Sector

6. Moreover, this occurs faster in the manufacturing sector: figure 6.
7. Average size of establishments have been rising, but shrinking in the manufacturing sector: figure 7. Source: BDS 1980-2013.
8. Manager definition. Middle-management has been growing (relative to top management): figure 8

3 Model

There are a continuum of individuals each endowed with two types of skill, $(h, z) \in \mathcal{H} \times \mathcal{Z}$. Worker human capital, h, is used to produced worker tasks. Managerial talent, z, is a skill for organizing tasks. Without loss of generality, we assume that the mass

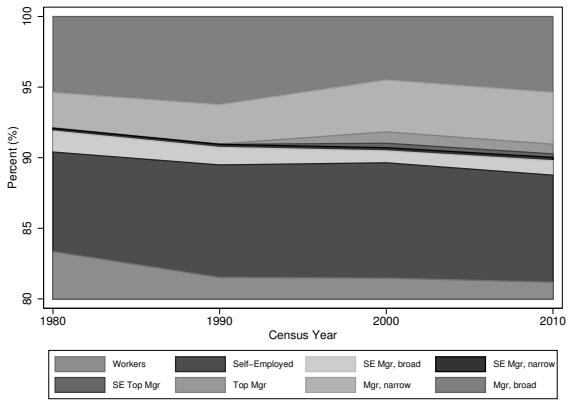


Fig. 8: Managers in the Data

of individuals is 1, with associated distribution function $\mu(h, z)$.

There are 2 sectors $i \in \{m, s\}$.[5] In each sector, goods are produced by teams. A manager uses his own managerial skill and physical capital to organize a continuum of worker tasks $j \in \mathcal{J} = [0, J]$. (From now on, a "task" should be understood as a "worker task".) Each task requires both physical and worker human capital, and how much of each is allocated to a task is decided by the manager. Aggregating over the goods produced by all managers in a sector yields total sectoral output.

Within a sector, a manager with a higher z produces more goods with the same amount of tasks, but task intensities may differ across sectors. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M(z) k_{iz}(z)^{\alpha} z^{1-\alpha}, \qquad x_h(z) = \left[\int_{j=0}^{J} \nu_i(j)^{\frac{1}{\sigma}} \tau_i(j;z)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{1b}$$

$$\tau_i(j;z) = M(j) k_{ih}(j;z)^{\alpha} \left[\int_{\mathrm{h}_i(j;z)} b(h,j) d\mu\right]^{1-\alpha}, \tag{1c}$$

with $\{\nu_i(j), \eta_i\} \in (0,1)$ for all sector i and task j, and $\int_j \nu_i(j) dj = 1$. The $\tau_i(j;z)$ is the amount of task j output produced for a manager with skill z in sector i. This manager uses physical capital $k_{iz}(z)$ for himself, and allocates $k_{ih}(j;z)$ and hires a set of workers $\mathrm{h}_i(j;z)$ for task j. The function $b(h,j)$ is the contribution of a worker with human capital h assigned to task j.

[5] In our application, the two sectors stand in for "manufacturing" and "services," respectively. However, our model can be extended to incorporate any countably finite number N of sectors. We use the subscripts m and s to avoid confusing them with the subscripts for tasks.

Assumption 1 (Log-supermodularity) *The function $b : \mathcal{H} \times \mathcal{J} \mapsto \mathbb{R}^+$ is strictly positive and twice-differentiable, and is log-supermodular. That is, for all $h' > h$ and $j' > j$:*

$$\log b(h', j') + \log b(h, j) > \log b(h', j) + \log b(h, j'). \tag{2}$$

Assumption 1 will ensure that high h-workers sort into high j-tasks in any equilibrium. Integrating $b(h, j)$ over h of the workers in the set $\mathrm{h}_i(j; z)$ yields the total amount of human capital input used in task j by a manager with z in sector i.

We will refer to the managerial task as "task z," which is vertically (in a hierarchical sense) differentiated from tasks $j \in \mathcal{J}$, which are horizontally differentiated among themselves. The substitutability between tasks is captured by the elasticity parameter σ, and ω captures the elasticity between all workers and managers. The $M(j)$'s, $j \in \mathcal{J}^z \equiv \mathcal{J} \cup \{z\}$, capture task-specific TFP's, which are sector-neutral.

Now let $\mathcal{Z}_i$ denote the set of individuals working as managers in sector $i \in \{m, s\}$. Output in each sector is then

$$Y_i = A_i \int_{\mathcal{Z}_i} y_i(z) d\mu, \tag{3}$$

where A_i is an exogenous, sector-level productivity parameter. Final goods are produced by combining output from both sectors according to a CES aggregator:

$$Y = G(Y_m, Y_s) = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\gamma_m + \gamma_s = 1$. We will assume $\epsilon < 1$.[6]

Several points are in order. In contrast to Acemoglu and Autor (2011), who assume a continuum of tasks and a discrete number of skills, we assume both as a continuum. While the implications are comparable, our formulation is more suitable for exploring employment shares across tasks. The model is also comparable to Goos et al. (2014), who show (empirically) that relative price changes in task-specific capital, representing routinization, can drive employment polarization. However, they do not model skill and hence cannot explain selection on skills nor changes in the wage distribution. The worker side of our model is similar to Costinot and Vogel (2010), but we have multiple sectors and also include capital as a production factor. Lastly, in contrast to all these models, we treat managers as a special occupation, which generates additional insights both theoretically and quantitatively.

[6]The estimated ϵ between the manufacturing and service sector (broadly defined) is close to zero, as we show in section 6.2.

3.1 Planner's Problem

We assume complete markets for solve a static planner's problem. A planner allocates aggregate capital K and all individuals into sectors $i \in \{m, s\}$ and tasks $j \in [0, J]$. Formally, define $h_i(j; z)$ as the amount of human capital the planner allocates to task j in sector i under a manager with skill z. Also define $l_{ih}(s, j)$ as the number of individuals with skill $s = (h, z)$ that the planner assigns to task j, and $l_{iz}(s)$ the number of individuals with skill s the planner assigns as managers, in sector i. Then the planner's problem is to choose factor allocation rules $[k_{iz}(z), k_{ih}(j; z), h_i(j; z)]$ and assignment rules $[l_{ih}(s, j), l_{iz}(s)]$ to maximize current output (4) subject to (1) and for all $i \in \{m, s\}$ and $j \in \mathcal{J}$,

$$
\begin{aligned}
Y_i &= A_i \int y_i(z) l_{iz}(s) ds \\
K &= K_m + K_s \equiv \sum_{i \in \{m,s\}} \int \left\{ \left[k_{iz}(z) + \int_j k_{ih}(j; z) dj \right] \cdot l_{iz}(s) \right\} ds \\
H_i(j) &\equiv \int b(h, j) l_{ih}(s, j) ds = \int \left[\int_{\mathrm{h}_i(j;z)} b(h, j) d\mu \right] \cdot l_{iz}(s) ds, \\
d\mu(s) &= \sum_{i \in \{m,s\}} \left[\int l_{ih}(s, j) dj + l_{iz}(s) \right] ds
\end{aligned} \tag{5}
$$

where K_i and is the amount of capital allocated to sector i, and $H_i(j)$ the total productivity of workers allocated to task j in sector i. For existence of a solution, we assume that

Assumption 2 *There exists a strictly positive mass of jobs such that $b(0, j) > 0$ and individuals such that $b(h, J) > 0$.*

The following assumption is needed for uniqueness:

Assumption 3 *The domain of skills $\mathcal{H} \times \mathcal{Z} = [0, h_M] \times [0, \infty)$, where $h_M < \infty$ is the upperbound of skill h. The measure $\mu(h, z)$ is differentiable and $d\mu(h, z) > 0$ is continuous on $\mathcal{H} \times \mathcal{Z}$.*

Assumption 3 implies that we can write

$$
\mu(\tilde{h}, \tilde{z}) = \int^{\tilde{h}} \int^{\tilde{z}} dF(z|h) dG(h) = \int^{\tilde{h}} \left[\int^{\tilde{z}} df(z|h) dz \right] g(h) dh,
$$

where (G, g), the marginal cumulative and probability density functions of h, and (F, f), the c.d.f and p.d.f. of z conditional on h, are continuous.

The optimal factor allocation rules across managers, $[k_{iz}(z), k_{ih}(j; z), h_i(j; z)]$, are straightforward: they are chosen to equalize marginal products across managers with

different z. Since we assume a constant returns to scale technology, we can write sectoral technologies as

$$Y_i = A_i \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{6a}$$

$$X_{iz} = M(z) K_{iz}^{\alpha} Z_i^{1-\alpha}, \qquad X_{ih} = \left[\int_j \nu_i(j)^{\frac{1}{\sigma}} T_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{6b}$$

where K_{iz} is the total amount of capital allocated to managers, $Z_i \equiv \int z l_{iz}(s) ds$, X_{ih} is a sectoral task aggregate, and

$$T_i(j) = M(j) K_{ih}(j)^{\alpha} H_i(j)^{1-\alpha}, \tag{7}$$

where $K_{ih}(j)$ is the total amount of capital allocated to task j in sector i. We characterize the solution in the following order:

1. Optimal physical capital allocations across tasks within a sector.
2. Optimal worker allocations across tasks within a sector.
3. Optimal manager-worker allocations within a sector.
4. Optimal capital and labor allocations across sectors

which will also allow us to show existence and uniqueness of the solution.

Capital allocation within sectors Given sectoral capital K_i, the planner equalizes marginal product across tasks:

$$\begin{aligned} & MPK_{ih}(0) = MPK_{ih}(j) \\ \Rightarrow \quad & \frac{MPT_i(0) \cdot \alpha T_i(0)}{K_{ih}(0)} = \frac{MPT_i(j) \cdot \alpha T_i(j)}{K_{ih}(j)} \\ \Rightarrow \quad & \frac{MPT_i(j) \cdot T_i(j)}{MPT_i(0) \cdot T_i(0)} = \frac{K_{ih}(j)}{K_{ih}(0)} \quad \equiv \quad \pi_{ih}(j) = \underbrace{\left[\frac{\nu_i(j)}{\nu_i(0)} \right]}_{\equiv V_i(j)}^{\frac{1}{\sigma}} \cdot \left[\frac{T_i(j)}{T_i(0)} \right]^{\frac{\sigma-1}{\sigma}}, \end{aligned} \tag{8}$$

where $MPT_i(j)$ is the marginal product of $T_i(j)$ w.r.t. X_i, and $\pi_{ih}(j)$ is the capital input ratio between task j and 0. Due to the Cobb-Douglas assumption, $\pi_{ih}(0)$ divided by task output ratios is the marginal rate of technological substitution ($MRTS$) between tasks j and 0; furthermore, $\pi_{ih}(j)$ divided by either factor input ratios in tasks j and 0 measures the $MRTS$ of that factor between tasks j and 0. (For capital, this is equal to 1.) Given (8) we can write

$$X_{ih} = \nu_i(0)^{\frac{1}{\sigma-1}} \underbrace{\left[\int_j \pi_{ih}(j) dj \right]}_{\equiv \Pi_{ih}}^{\frac{\sigma}{\sigma-1}} T_{i0}. \tag{9}$$

Of course, MPK must also be equalized across the managerial task and the rest:

$$\begin{aligned} & MPK_{iz} = MPK_{ih}(j) = MPK_{ih}(0) \\ \Rightarrow \quad & \frac{MPX_{iz} \cdot \alpha X_{iz}}{K_{iz}} = \frac{MPX_{ih} \cdot MPT_i(0) \cdot \alpha T_i(0)}{K_{ih}(0)} \\ \Rightarrow \quad & \frac{MPX_{iz} \cdot X_{iz}}{MPX_{ih} \cdot MPT_i(0) \cdot T_i(0)} = \frac{K_{iz}}{K_{ih}(0)} \quad \equiv \quad \pi_{iz} = \left(\frac{\eta_i}{1-\eta_i}\right)^{\frac{1}{\omega}} \cdot \left(\frac{X_{iz}}{X_{ih}}\right)^{\frac{\omega-1}{\omega}} \cdot \Pi_{ih}, \end{aligned} \tag{10}$$

that is, π_{iz} does not vary with j. This allows us to write, using (9),

$$\begin{aligned} Y_i &= A_i(1-\eta_i)^{\frac{1}{\omega-1}} \left[1 + \pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} X_{ih} \\ &= A_i(1-\eta_i)^{\frac{1}{\omega-1}} \nu_i(0)^{\frac{1}{\sigma-1}} \cdot \Pi_{K_i}^{\frac{\omega}{\omega-1}} \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} T_i(0) \end{aligned} \tag{11}$$

where $\Pi_{K_i} \equiv \Pi_{ih} + \pi_{iz}$.

Sorting workers across tasks within sectors Since we assume $b(h,j)$ is strictly log-supermodular, Assumptions 1-3 imply that there exists a continuous assignment function $\hat{j} : [0, h_M] \mapsto \mathcal{J}$ s.t. $\hat{j}'(h) > 0$, and $\hat{j}(0) = 0$, $\hat{j}(h_M) = J$.[7] That is, there is positive sorting of workers into tasks, and workers of skill j are assigned to job $\hat{j}(h)$. And since $\hat{j}'(h) > 0$ we can also define its inverse $\hat{h} : \mathcal{J} \mapsto [0, h_M]$.

It should also be clear that $[\hat{j}(h), \hat{h}(j)]$ are identical across sectors, and hence are not subscripted by i: Otherwise, the planner would be able to reallocate h across sectors and increase output. This implies that the planner's problem of choosing $l_{ih}(s,j)$ has two parts: One of choosing $l_h(s,j)$, the number of individuals with skill s the planner assigns to task j *regardless of sector*, and the other of choosing $q_{ih}(j)$, the fraction of task j workers that are allocated to sector i, which satisfies $\sum_{i \in \{m,s\}} q_{ih}(j) = 1$. Hence we can write

$$l_{ih}(s,j) = q_{ih}(j) \cdot l_h(s,j).$$

Sorting managers and workers within a sector Since individuals are heterogeneous in 2 dimensions, the key is to get a cutoff rule $\tilde{z}(h)$ s.t. for every h, individuals with z above $\tilde{z}(h)$ become managers and below become workers. Since we know that workers sort positively into j, we can also define $\hat{z}(j) = \tilde{z}(\hat{h}(j))$. Then the planner's problem is to get this implied rule $\hat{z}(j)$ for all $j \in \mathcal{J}$; again, the rule must be identical across sectors.

The rule $\hat{z}(j)$ is chosen so that the marginal product of the threshold manager is equalized in either task. So for each j, that rule must satisfy:

$$MPX_{iz} \cdot \frac{(1-\alpha)X_{iz}}{Z_i} \cdot \hat{z}(j) = MPX_{ih} \cdot MPT_i(j) \cdot \frac{(1-\alpha)T_i(j)}{H_i(j)} \cdot b(\hat{h}(j), j)$$

[7] For a more formal proof, refer to Lemma 1 in Costinot and Vogel (2010).

$$\Rightarrow \quad \hat{z}(j)/b(\hat{h}(j),j) = \pi_{ih}(j)Z_i/\pi_{iz}H_i(j). \tag{12}$$

Without loss of generality, we normalize $b(0,0) = 1$ to obtain

$$\hat{z} \equiv \hat{z}(0) = \tilde{z}(0) = Z_i/\pi_{iz}H_i(0) \tag{13}$$

and using (13) in (10) we obtain

$$\pi_{iz} = \underbrace{\frac{\eta_i \nu_i(0)^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i}}_{\equiv v_{iz}} \cdot \left(\tilde{M}(z)\cdot \hat{z}^{1-\alpha}\right)^{\omega-1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}} \tag{14}$$

where $\tilde{M}(z) \equiv M(z)/M(0)$, while (12) can be rewritten

$$\hat{z}(j) = \hat{z}\cdot b(\hat{h}(j),j)\cdot \pi_{ih}(j)H_i(0)/H_i(j). \tag{15}$$

Characterizing assignment rules First consider the rule $l_{iz}(s)$. Since the rule $\tilde{z}(h) = \hat{z}(\hat{j}(h))$ does not depend on sector, this implies that

$$\int l_{iz}(s)ds = q_{iz}\int \left[1 - F(\tilde{z}(h)|h)\right] dG(h) \tag{16}$$

where q_{iz} are sectoral weights that satisfy $\sum_{i\in\{m,s\}} q_{iz} = 1$. So unlike the case with $l_{ih}(s,j)$, any solution that satisfies $l_{iz}(s) = 0$ whenever $z < \hat{z}(j)$ and (16) is optimal. Hence, only the planner's choices of $\hat{z}(j)$ and q_{iz} are unique, not the rule $l_{iz}(s)$ for all $s \in \mathcal{S}$. The planner does not care how managers are allocated across sectors but only about how the total Z is divided across sectors, where

$$Z \equiv \int\int_{\tilde{z}(h)} z dF(z|h)dG(h) = \sum_{i\in\{m,s\}} q_{iz}Z_i.$$

Now consider the planner's choice of $l_h(s,j)$. The feasibility constraint (5) and existence of $[\hat{h}(j),\hat{z}(j)]$ implies that the number of people with skill s assigned to task j is

$$l_h(s,j)ds = \delta\left[j - \hat{j}(h)\right]\cdot \mathcal{I}\left[z \le \tilde{z}(h)\right] d\mu$$

where $\delta(\cdot)$ is the Dirac delta function and $\mathcal{I}$ an indicator function. Hence the allocation rule is completely determined by the assignment functions $[\hat{h}(j),\hat{z}(j)]$, and the productivity of all workers assigned to task $j = \hat{j}(h)$ is

$$H(j) = \int b(h,\hat{j}(h'))\cdot \delta\left[j - \hat{j}(h')\right]\cdot F(\tilde{z}(h')|h')dG(h')$$

and we can use the change of variables $j' = \hat{j}(h')$ to instead integrate over j':

$$H(j) = \int b(\hat{h}(j'),j')\cdot \delta\left[j - j'\right]\cdot F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j'))\cdot \hat{h}'(j')dj'$$

$$= b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j)) \cdot \hat{h}'(j) \tag{17}$$

At the optimal allocation, there is no marginal gain from switching any worker to another job. Therefore for any $j' = j + dj$,

$$\begin{aligned}
\frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j), j) \geq & \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j), j'), \\
\frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j'), j') \geq & \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j'), j),
\end{aligned}$$

with equality if $|dj| = 0$. Substituting for $H_i(j)$ using (17), we obtain

$$\frac{b(\hat{h}(j'), j')}{b(\hat{h}(j), j')} \geq \frac{\pi_{ih}(j')}{\pi_{ih}(j)} \cdot \frac{q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j)}{q_{ih}(j')F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j'))\hat{h}'(j')} \geq \frac{b(\hat{h}(j'), j)}{b(\hat{h}(j), j)},$$

so we obtain, as $|dj| \to 0$,

$$\left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) = d \log \left\{ \pi_{ih}(j) \Big/ \left[q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j) \right] \right\} \Big/ dj.$$

Now using the total derivative of $b(\hat{h}(j), j)$:

$$d \log b(\hat{h}(j), j)/dj = \left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) + \partial \log b(\hat{h}(j), j)/\partial j, \tag{18}$$

and applying $\pi_{ih}(0) = 1$, we obtain

$$H_i(j)/\pi_{ih}(j)H_i(0) = \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \equiv B_j(j; \hat{h}) \tag{19}$$

where B_j is a functional of j and the function $\hat{h}$. Applying this to (8) we obtain

$$\pi_{ih}(j) = v_{ih}(j) \Big/ \left[\tilde{M}(j) B_j(j; \hat{h})^{1-\alpha} \right]^{1-\sigma}. \tag{20}$$

where $\tilde{M}(j) \equiv M(j)/M(0)$, while applying it to (15) yields

$$\hat{z}(j) = \hat{z} \cdot b(\hat{h}(j), j)/B_j(j; \hat{h}).$$

Note that (18) also implies that

$$b(h, \hat{j}(h))/B_j(\hat{j}(h); \hat{h}) = B_h(h; \hat{j}) \equiv \exp\left[\int_0^h \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh' \right] \tag{21}$$

so we can also write

$$\hat{z}(j) = \hat{z} \cdot B_h(\hat{h}(j); \hat{j}), \quad \tilde{z}(h) = \hat{z} \cdot B_h(h; \hat{j}). \tag{22}$$

In what follows, we suppress the dependence of (B_j, B_h) on $(\hat{h}, \hat{j})$ unless necessary.

Within sector solution and sectoral production function Given $[q_{ih}(j), q_{iz}]$, the optimal allocation within a sector is thus described by $[\hat{h}(j), \hat{z}]$ that solves

$$\hat{h}'(j) = \frac{H_i(0) \cdot v_{ih}(j)}{q_{ih}(j)} \Big/ \left\{ [\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j)) \right\} \tag{23a}$$

$$\hat{z}^{\alpha+\omega(1-\alpha)} = \frac{q_{iz}}{H_i(0) \cdot v_{iz}} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{1-\sigma}} \cdot \tilde{M}(z)^{1-\omega} \cdot Z \tag{23b}$$

along with the boundary condition $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$, which implies

$$H_i(0) = h_M \Big/ \int v_{ih}(j) \Big/ \left\{ q_{ih}(j) \cdot [\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j)) \right\} dj \tag{23c}$$

The ODE (23a) is separable as $C(\hat{h})d\hat{h} = D(j)dj$, which is easily solvable and where the functions (C, D) are represented by the integrands in

$$\begin{aligned} \Pi_{ih} &= \int \left\{ v_{ih}(j) \Big/ \left[\tilde{M}(j) B_j(j)^{1-\alpha} \right]^{1-\sigma} \right\} dj \\ &= \int q_{ih}(\hat{j}(h)) \cdot B_h(h) F(\tilde{z}(h)|h) g(h) dh \Big/ H_i(0). \end{aligned} \tag{23d}$$

The functions $(B_j(j), B_h(h), \hat{z}(j), \tilde{z}(h))$, which represent relative wages in equilibrium, are defined in (19) and (21)-(22). Note that all these objects are determined independently of the amount of physical capital, and also the total amount of labor in either sector—all that matters are relative masses *across tasks*. To see this more clearly, define $\psi_i \equiv (1-\eta_i)^{\frac{1}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}}$ and rewrite (11) to obtain

$$Y_i = \psi_i \cdot \Pi_{K_i}^{\frac{\omega}{\omega-1}} \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} M(0) K_i(0)^{\alpha} H_i(0)^{1-\alpha} \tag{24}$$

Sectoral capital can be written as

$$K_i = K_i(0) \cdot (\Pi_{ih} + \pi_{iz}) = K_i(0) \cdot \Pi_{K_i}. \tag{25}$$

From (13) we can write

$$Z_i = q_{iz} L_z \bar{z} = L_{iz} \bar{z} = H_i(0) \cdot \hat{z} \pi_{iz}, \quad \text{where} \quad L_z \equiv \int_{z > \tilde{z}(h)} d\mu, \quad \bar{z} = Z / L_z,$$

and using (17) and (19)-(20) we obtain

$$\begin{aligned} \int \left[H_i(j) \Big/ b(\hat{h}(j), j) \right] dj &= \int q_{ih}(\hat{j}(h)) F(\tilde{z}(h)|h) g(h) dh \\ &= H_i(0) \cdot \int \left[\pi_{ih}(j) / B_h(\hat{h}(j)) \right] dj \quad = L_i - L_{iz} \end{aligned}$$

and rearranging we obtain

$$L_i = H_i(0) \cdot \underbrace{[\Pi_{il} + (\hat{z}/\bar{z})\pi_{iz}]}_{\Pi_{L_i}} \quad \text{where} \quad \Pi_{il} \equiv \int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj, \tag{26}$$

and (24) becomes

$$Y_i = \underbrace{M(0) \cdot \psi_i \cdot \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1}}_{\Phi_i\text{: TFP}} K_i^{\alpha} L_i^{1-\alpha}. \tag{27}$$

Hence, we recover a sectoral production function in which sector TFP is completely separated from sectoral capital and labor. Sectoral TFP, Φ_i, can be decomposed into 3 parts: $M(0)$, that is common across both sectors; ψ_i, which is sector-specific but exogenous; and the part determined by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by the allocation rules $[\hat{h}(j), \hat{z}]$. Since these rules depend only on the relative masses of individuals across tasks *within* a sector, they do not depend on the employment shares *across* sectors (nor capital)—that is, sectoral TFP's are independent of sectoral capital and labor shares.

In fact, it is the TFP's that determine sectoral input shares. Sectors only differ in how intensely they use each task, i.e., the mass of individuals allocated to each task. Across sectors, employment shares are determined so that the MPK and MPL are equalized:

$$\kappa \equiv \frac{K_s}{K_m} = \frac{L_s}{L_m} = \left(\frac{\gamma_s}{\gamma_m}\right)^{\frac{1}{\epsilon}} \left(\frac{Y_s}{Y_m}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\Phi_s}{\Phi_m}\right)^{\epsilon-1} \tag{28}$$

where κ is the capital input ratio between sector s and m. Hence relative employment between sectors is completely determined by the relative endogenous TFP ratio between the two sectors. Since the Φ_i's are just functions of $[\hat{h}(j), \hat{z}]$ as seen in (27), so is κ and hence sectoral employment L_i since

$$L_m = 1/(1+\kappa), \; L_s = \kappa/(1+\kappa). \tag{29}$$

Hence, the aggregate levels of K or L have no impact whatsoever on the assignment rules nor employment shares either.

3.2 Equilibrium Existence and Uniqueness

Since there are no frictions in our economy, a solution to the planner's problem coincides with an equilibrium, so existence and uniqueness of an equilibrium is equivalent to a unique solution to the planner's problem. As a final step, the planner needs to ensure that the within-sector allocations are consistent with the between sector allocations.

That is, the weights used to split μ—$[q_{ih}(j), q_{iz}]$—must be consistent with (28). In equilibrium, these are equivalent to the the labor market clearing conditions.

First, we can use (23) to find the $[q_{ih}(j), q_{iz}]$ that ensures that the within-sector allocation rules $[\hat{h}(j), \hat{z}]$ are equal across sectors. Let $q_h(j)$ and q_z denote the shares of employment in tasks j and z in the services sector, respectively; then $q_{mh}(j) = 1 - q_h(j)$ and $q_{mz} = 1 - q_z$. The rule for splitting workers across tasks is given by (23a):

$$\begin{aligned} q_h(0) \cdot v_{sh}(j) / q_h(j) &= [1 - q_h(0)] \cdot v_{mh}(j) / [1 - q_h(j)] \\ \Leftrightarrow \qquad q_h(j) &= 1 \Big/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\nu_m(j)}{\nu_s(j)}\right] \end{aligned} \tag{30a}$$

and $q_h(0)$ must solve (23c), so the dependence of the between-sector allocation rule on the within-sector rule comes only through $q_h(0)$. Second, (23b) implies

$$\begin{aligned} q_z \Pi_{sh}^{\frac{\sigma-\omega}{1-\sigma}} \Big/ q_h(0) v_{sz} &= (1 - q_z) \Pi_{mh}^{\frac{\sigma-\omega}{1-\sigma}} \Big/ [1 - q_h(0)] v_{mz} \\ \Leftrightarrow \qquad q_z &= 1 \Big/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1 - \eta_s)}{(1 - \eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}\right] \end{aligned} \tag{30b}$$

where

$$V_{ih} \equiv \nu_i(0) M(0)^{\sigma-1} \Pi_{ih} = \int \left\{ \nu_i(j) \cdot \left[\tilde{M}(j) \cdot B_j(j)^{1-\alpha}\right]^{\sigma-1} \right\} dj, \tag{31}$$

which also depends on the within-sector allocation rule. Note that given a within-sector rule, the between-sector allocation rules $[q_h(j), q_z]$ only depend on the task intensity parameters $[\nu_i(j), \eta_i]$, which are uniquely determined given $q_h(0)$. Lastly, $q_h(0)$ must solve the implied sectoral shares in (28) given (26):

$$\begin{aligned} &\frac{q_h(0)}{1 - q_h(0)} \equiv Q(q_h(0)) \\ =&\frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\psi_s}{\psi_m}\right)^{\epsilon-1} \cdot \left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{(\sigma-\omega)(1-\epsilon)}{(1-\sigma)(1-\omega)}} \cdot \left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\left(\alpha+\frac{\omega}{1-\omega}\right)(1-\epsilon)} \cdot \left(\frac{\Pi_{L_s}}{\Pi_{L_m}}\right)^{-[\alpha+\epsilon(1-\alpha)]}. \end{aligned} \tag{32}$$

So to establish uniqueness, we need to show that within-sector allocations are unique, and that the $[q_h(0), q_z] \in (0, 1)$ that solve (30b) and (32) are unique. We need more assumptions on the distribution and the production function $b(h, j)$ that will serve as sufficient conditions for a unique equilibrium: Assumption 4.1 will guarantee that the within-sector equilibrium is unique, and Assumptions 4.2-3 and 5.2 that the between-sector equilibrium is unique.

Assumption 4 *For all* $(h, z) \in \mathcal{H} \times \mathcal{Z}$,

1. $g'(h) \leq 0$ *and* $f'(z|h) \leq 0$,

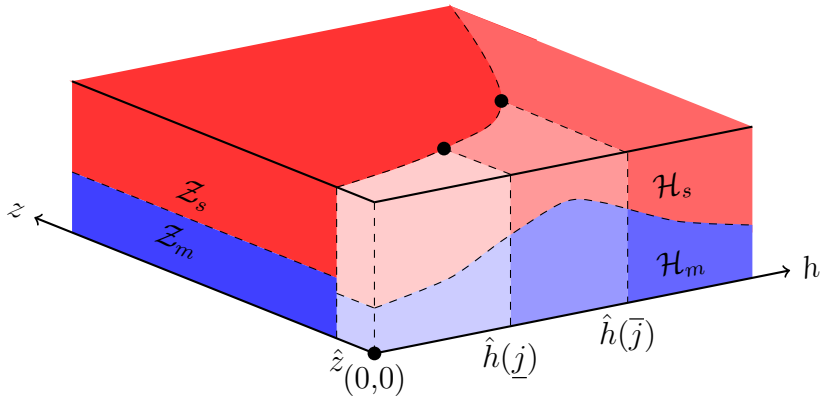


Fig. 9: Equilibrium

2. $F(z|h)/\left[1-F(z|h)\right] \leq z/\left[\int_z z' f(z'|h)dz'\right]$, *and*
3. $zf(z|h)/F(z|h) \geq (1-\alpha)(1-\omega)$.

Assumption 4.1 means that there are less people at higher levels of skill (the p.d.f.'s of h and z are declining everywhere), which is a common assumption and also supported by most empirical evidence. Conversely, Assumptions 4.2-3 mean that the conditional distribution of z is declining but not too much, in the sense that it still has fat tails for all values of h.

Assumption 5 *For all* $(h,j) \in \mathcal{H} \times \mathcal{J}$,

1. $\partial b(h,j)/\partial h > 0$
2. *for all* $\varepsilon > 0$, $0 < \partial^2 \log b(h,j)/\partial h \partial j < \varepsilon$.

Assumption 5.1 captures the notion that h-workers perform better in any task; in particular under this assumption, $\tilde{z}(h)$ in (22) is a strictly increasing function. That the cross partial is larger than 0 is already implied by Assumption 1. Assumption 5.2 means that there is just enough log-supermodularity so that workers positively sort into tasks.

Theorem 1 *Under Assumptions 1-5, the solution to the planner's problem,* $[\hat{h}(j), q_h(j)]_{j=0}^{J}$ *and* $(\hat{z}, q_z)$*, exists and is unique.*

Proof: See Appendix A.1. □

The resulting equilibrium skill allocation is depicted in Figure 10.[8] The thresholds determine the tasks, and employment is split across sectors while preserving the means

[8]For illustrative purposes, the figure assumes that $\mu(z,h)$ is uniform.

for each task. The different masses of sectoral employment across tasks are due to the task intensity parameters $[\nu_i(j), \eta_i]$.

3.3 Equilibrium wages and prices

The solution $[\hat{h}(j), \hat{z}]$ gives all the information needed to derive equilibrium prices (which are unique). The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = \left[Y_i/\gamma_i Y\right]^{-\frac{1}{\epsilon}}. \tag{33}$$

Sectoral output prices can be obtained by applying the sectoral output in (25)-(27) in (33) at the solution to the planner's problem.

Let R denote the rental rate of capital and $w_h(h)$ the wage of a worker with skill h. The rental rate R is given either by the dynamic law of motion for aggregate capital, or fixed in a small open economy. Since capital and labor input ratios are equalized across sectors, $w_h(0)$ can be found from either sector from

$$w_h(0) = \frac{1-\alpha}{\alpha} \cdot \frac{RK_i(0)}{H_i(0) \cdot b(0,0)} = \frac{1-\alpha}{\alpha} \cdot \frac{\Pi_{L_i}}{\Pi_{K_i}} \cdot RK,$$

where the second equality follows from (25)-(26), (28), and since we have normalized $b(0,0)$ and the population to 1.

Similarly, all workers earn their marginal product, so for any two workers with skills $h \neq h'$ we can write

$$\begin{aligned}
\log w_h(h') - \log w_h(h) &= \log\left[\frac{MPT_i(\hat{j}(h')) \cdot T_i(\hat{j}(h'))}{MPT_i(\hat{j}(h)) \cdot T_i(\hat{j}(h))} \cdot \frac{H_i(\hat{j}(h))}{H_i(\hat{j}(h'))} \cdot \frac{b(h', \hat{j}(h'))}{b(h, \hat{j}(h))}\right] \\
&= \log\left[\frac{\pi_{ih}(\hat{j}(h'))}{\pi_{ih}(\hat{j}(h))} \cdot \frac{H_i(\hat{j}(h))}{H_i(\hat{j}(h'))} \cdot \frac{b(h', \hat{j}(h'))}{b(h, \hat{j}(h))}\right] \\
&= \log\left[B_h(\hat{j}(h'))/B_h(\hat{j}(h))\right],
\end{aligned}$$

where the second equality follows from (8) and the third from (19) and (21). So letting $h' = h + dh$ and sending $|dh| \to 0$, we obtain

$$w_h(h) = w_h(0) \cdot B_h(h), \tag{34}$$

so Assumption 5.1 implies that $w(h)$ is strictly increasing in h.

For all $h \in \mathcal{H}$, threshold managers with skill $z = \tilde{z}(h)$ are indifferent between becoming a worker or manager, so we can determine a "managerial efficiency wage" w_z that satisfies

$$w_z \tilde{z}(h) = w_h(h) \quad \Rightarrow \quad w_z = w_h(0)/\hat{z}, \tag{35}$$

and (22) and (34) guarantee that the first equation holds for all h.

4 Comparative Statics

The sectoral technology representation (27) implies that this model has similar implications as Ngai and Pissarides (2007): the sector with the larger TFP shrinks. The major difference is that these TFP's are endogenous.

What is more interesting is the implications of growth in task-specific TFP's.[9]. In particular, we are interested in the effect of routinization, which we model as an increase in $M(j)$ for all $j \in \mathcal{J}_1 \equiv [\underline{j}, \overline{j}]$, where $0 < \underline{j} < \overline{j} < J$. We will refer to this group of tasks as "routine jobs." This is illustrated in a series of comparative statics, which is possible since the equilibrium is unique and skill distribution continuous (under assumption 3). We proceed as follows:

4.1 Wage and Job Polarization

First, we will focus on the comparative statics of $[\hat{h}(j), \hat{z}]$ within a sector i. Our exercise assumes that there is an increase in the exogenous productivity, $M(j)$, of intermediate j-tasks. Since $M(j)$'s are separate from the worker's human capital h, this can be interpreted as a rise in capital-augmented TFP for middle-skill jobs, or "routinization." This is a similar exercise to Lemma 6 in (Costinot and Vogel, 2010), except that we have modeled capital and more importantly, the vertically differentiated manager.[10]

Proposition 1 (Routinization and Polarization) *Let $\mathcal{J}^1 \equiv (\underline{j}, \overline{j}) \subset \mathcal{J}$, where $0 < \underline{j} < \overline{j} < J$. Suppose $[q_h(j), q_z]$ is held constant, and that $M(j)$ uniformly grows to $M^1(j) = M(j)e^m$ for all $j \in \mathcal{J}^1$, where $m > 0$. Then under Assumptions 1-4,*

1. *if $\sigma < 1$, there exists $j^* \in \mathcal{J}^1$ such that $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (0, j^*)$ and $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j^*, J)$, and*
2. *if $\omega < \sigma < 1$ and Assumption 5.2 holds, there exists some $\varepsilon > 0$ s.t. $\tilde{z}^1(h) < \tilde{z}(h)$ for all $0 < m < \varepsilon$ and $h \in \mathcal{H}$.*

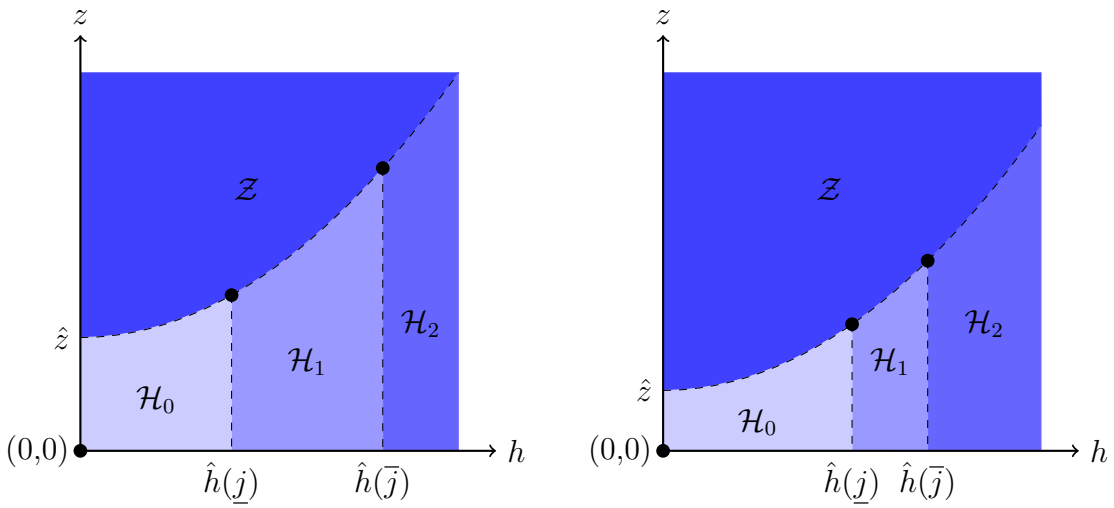
Proof: See Appendix A.2 □

Part 1 implies that among worker tasks, capital and labor flow out from intermediate tasks into the extremes (job polarization), and also that their relative wages decline (wage polarization), since from (34),

$$\log\left[\frac{w^1(h)}{w^1(h^*)}\right] - \log\left[\frac{w(h)}{w(h^*)}\right] = \int_{h^*}^{h}\left[\frac{\partial \log b(h, \hat{j}^1(h))}{\partial h} - \frac{\partial \log b(h, \hat{j}(h))}{\partial h}\right] dh$$

[9] This is akin to the price of task-specific capital falling in Goos et al. (2014), or extreme-biased technological change in Costinot and Vogel (2010)

[10] It can also be viewed as extending (Goos et al., 2014)—who do model capital—to a continuum of tasks. However, theirs is an empirical model in which wages are exogenously varied according to the data, while labor is task-specific.



(a) Equilibrium wt low M_1

(b) Equilibrium wt high M_1

Fig. 10: Comparative Static, Within-Sector

is positive for all $j \neq j^* \in \mathcal{J}^1$. Part 2 implies that capital and labor flow into management from *all* worker tasks, and (35) means that each manager earns a higher efficiency wage per skill (vertical polarization). The within-sector comparative static is depicted in figure 10.

The mechanism for part 1 is the same as in Goos et al. (2014): when $\sigma < 1$, the exogenous rise in productivity causes factors to flow out to other tasks since tasks are complementary, and we get employment polarization. As in Costinot and Vogel (2010), this also leads to wage polarization in the presence of positive sorting. What is new in our model is that this happens even in the presence of the vertically differentiated management task, and that with stronger complementarity between workers and managers than among workers, similar forces drive vertical polarization.[11]

4.2 Structural change

Previous models of structural change either rely on a special non-homogeneous form of demand (rise in income shifting demand for service products) or relative technology differences across sectors (rise in manufacturing productivity relative to services, combined with complementarity between the two types of goods, shifting production to services). Our model is also technology driven, but transformation arises from a

[11] A change in factor neutral TFP, $M(j)$, is different from an increase in the amount of skill working in any given task. Since we are modeling worker skill as human capital, the qualitative effect of a change in TFP is the same as if it were only capital-augmenting. Changes in the skill distribution can have different effects as shown in Costinot and Vogel (2010), which we do not pursue here.

skill neutral increase in task productivities, or routinization. Most importantly, in contrast to recent papers arguing that sectoral productivity differences can explain the skill premia or polarization (Buera et al., 2015; Bárány and Siegel, 2016), we argue exactly the opposite—that routinization can explain sectoral productivity differences and structural change.

Decomposing Polarization Define the "unnormalized" total labor productivity

$$V_{L_i} = (1-\eta_i)\nu_i(0)M(0)^{\sigma-1}\Pi_{L_i} = \int V_i(j)dj + V_i(z),$$

where the weights $V_i(j)$ for each task $j \in \mathcal{J}^z$ are

$$V_i(j) = (1-\eta_i)\nu_i(j)\left[M_h(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}/B_h(\hat{h}(j)) \tag{36a}$$

$$V_i(z) = \eta_i M_z^{\omega-1}\cdot V_{ih}^{\frac{\sigma-\omega}{\sigma-1}}\cdot \hat{z}^{\alpha+\omega(1-\alpha)}/\bar{z}. \tag{36b}$$

and V_{ih} was defined in (31). Then (26) implies that the amount of labor in each task across both sectors can be expressed as

$$L(j) = \sum_{i\in\{m,s\}} \frac{L_i(j)}{L_i}\cdot L_i = \sum_{i\in\{m,s\}} \frac{V_i(j)}{V_{L_i}}\cdot L_i.$$

Using this expression, the change in the total amount of labor in each task can be decomposed similarly as in Goos et al. (2014).[12] Now, we consider the same exercise as in Proposition 1, that $M(j)$ grows to $M_h^1(j) = M(j)e^m$ for all $j \in \mathcal{J}^1 \equiv (\underline{j}, \bar{j})$. Let Δ_x denote the log-derivative of x w.r.t. m, then

$$\begin{aligned}\Delta_{L(j)} &= \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)}\cdot\left[\Delta_{V_i(j)} - \Delta_{V_{L_i}} + \Delta_{L_i}\right] \\ &= \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)}\cdot\left\{\underbrace{\Delta_{V_i(j)} - \int_{\mathcal{J}^z}\left[\frac{V_i(j')}{V_{L_i}}\cdot\Delta_{V_i(j')}\right]dj'}_{W_{ij}} + \Delta_{L_i}\right\}.\end{aligned} \tag{37}$$

A change in the $V_i(j)$'s occurs even holding L_i's constant, shifting the term W_{ij}. This leads to "within-sector polarization," as we saw in Proposition 1 that intermediate tasks are shrinking and management is growing. To compare the sectoral differences in its impact, we compare the $(\Delta_{V_i(j)}, \Delta_{V_{L_i}})$'s.

First note from (36) that the $\Delta_{V_i(j)}$'s are sector-neutral and common across sectors, except for $\Delta_{V_i(z)}$. Under Assumption 5.2, (45)-(46) in Appendix A.2 imply

$$\Delta_{V_i(j)} = \sigma - 1 < 0 \quad \forall j \in \mathcal{J}^1 \quad \text{and 0 otherwise.} \tag{38a}$$

[12] Our decomposition differs from theirs because our decomposition is driven by selection on skills, while in that paper labor is task-specific. Their thought experiment is to separate employment effects from keeping industry *output* fixed versus when it is allowed to vary, while ours is to separate the within-sector selection effect from between-sector allocation rules.

So for workers, any difference in how the share of task j employment evolves differentially across sectors depends only on $\Delta_{V_{L_i}}$, the sum of within-sector employment shifts, weighted by the employment shares of all tasks within a sector $V_i(j)/V_{L_i} = L_i(j)/L_i$. Since we know that intermediate jobs are the ones that are declining, from (26) a measure of the speed of polarization among workers is the total change in their employment:

$$(1-\eta_i)\cdot\Delta_{V_{il}}V_{il} = \int_{\mathcal{J}} V_i(j)\cdot\Delta_{V_i(j)}dj = (\sigma-1)\cdot\int_{\underline{j}}^{\bar{j}} V_i(j)dj$$
$$\text{where}\quad V_{il}\equiv\nu_i(0)M(0)^{\sigma-1}\cdot\Pi_{il} = \int\nu_i(j)\left[M(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}/B_h(\hat{h}(j)),$$

and we have used (45)-(46) in Appendix A.1, which were implied by Assumption 5.2. So we can compare the speeds of polarization across the two sectors from

$$\Delta_{V_{ml}} - \Delta_{V_{sl}} = (\sigma-1)\cdot\int_{\underline{j}}^{\bar{j}}\left\{\left[\frac{\nu_m(j)}{V_{ml}} - \frac{\nu_s(j)}{V_{sl}}\right]\cdot\left[M(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}\Big/B_h(\hat{h}(j))\right\}dj. \tag{38b}$$

Manager employment has sector-differential effects through V_{ih}: Under Assumption 5.2 and using (52) in Appendix A.2, we obtain

$$\Delta_{V_{m(z)}} - \Delta_{V_{s(z)}} = (\sigma-\omega)\cdot\int_{\underline{j}}^{\bar{j}}\left\{\left[\frac{\nu_m(j)}{V_{mh}} - \frac{\nu_s(j)}{V_{sh}}\right]\cdot\left[M(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}\right\}dj. \tag{38c}$$

Equation (38b)-(38c) implies there is a sense in which a sufficient condition for both horizontal and vertical polarization to be faster in manufacturing, as in the data, is $\omega<\sigma<1$ and $\nu_{mh}(j) >> \nu_{sh}(j)$ for all $j\in\mathcal{J}^1$.

Lemma 1 *Suppose $\omega<\sigma<1$ in Proposition 1, so that we get both horizontal and vertical polarization within sectors. Then*

1. *both horizontal and vertical polarization is faster in manufacturing in the sense that $\Delta_{V_{ml}} < \Delta_{V_{sl}} < 0$ and $\Delta_{V_{m(z)}} > \Delta_{V_{s(z)}} > 0$ if $L_m(j)/L_m > L_s(j)/L_s$, or*
2. *equivalently, there exists $\bar{\nu}\in(0,1)$ such that for all $\nu_m(j)-\nu_s(j)\geq\bar{\nu}$, $j\in\mathcal{J}^1$.* □

The proof is straightforward by inspection of (36)-(38) and is omitted. The assumptions in the lemma mean that the manufacturing sector is more reliant on the intermediate "routine" jobs ($j\in\mathcal{J}^1$), and that services is more reliant on managers. Part 1 makes assumptions on endogenous variables because we do not know the value of task productivities $[M(j),\hat{h}(j),\hat{z}]$, so are not guaranteed to hold in general; part 2 gives the general parametric restrictions.

Note that (38) holds regardless of the value of $q_h(0)$, so W_{ij} in (37) gives the equilibrium change in within-sector employment shares coming from a change in the selection rules $[\hat{h}(j), \hat{z}]$. Clearly, a change in the rules will also alter the last term, Δ_{L_i}, which captures between-sector reallocation. Specifically, from (29) and (32) we obtain

$$\Delta_{L_s} = L_m \cdot \Delta_\kappa = L_m \cdot \left\{ \Delta_{V_{L_s}} - \Delta_{V_{L_m}} + \Delta_Q \right\}. \tag{39a}$$

The term $\Delta_{L_s} - \Delta_{L_m}$ is the first-order force of structural change which comes only from the change in selection rules. But since this takes us off the between-sector equilibrium, $q_h(0)$ must shift to satisfy the equilibrium condition (32). The net amount of structural change will depend on whether the selection effect is overturned or reinforced by a change in $q_h(0)$.

Since $Q(q_h(0))$ in (32) changes monotonically from 0 to ∞ in $q_h(0)$, we only need to consider the direction of change of the RHS off-equilibrium. Using (38), the log-derivative of the RHS of (32) can be written

$$\begin{aligned}(1-\epsilon) &\left\{ \frac{\sigma - \omega}{(1-\sigma)(1-\omega)} \cdot \left(\Delta_{V_{sh}} - \Delta_{V_{mh}}\right) + \left(\alpha + \frac{\omega}{1-\omega}\right) \left(\Delta_{\Pi_{K_s}} - \Delta_{\Pi_{K_m}}\right) \right\} \\ &- \left[\alpha + \epsilon(1-\alpha)\right] \left(\Delta_{V_{L_s}} - \Delta_{V_{L_m}}\right). \end{aligned} \tag{39b}$$

Under Lemma 1, the term with $\Delta_{V_{ih}}$'s is positive from (38b). Terms with $\Delta_{\Pi_{K_i}}$ are determined by

$$\Delta_{\Pi_{K_i}} \Pi_{K_i} = \Pi_{ih} \Delta_{V_{ih}} + \pi_{iz} \Delta_{\pi_{iz}}, \tag{39c}$$

$$\Delta_{\pi_{sz}} - \Delta_{\pi_{sz}} = \frac{\sigma - \omega}{1-\sigma} \left(\Delta_{V_{mh}} - \Delta_{V_{sh}}\right). \tag{39d}$$

Clearly, capital polarizes along with labor, both horizontally and vertically; and the speed is faster in manufacturing if a similar condition as in Lemma 1 holds. Lemma 2 summarizes when structural change is in the direction of shifting capital and labor from manufacturing to services.

Lemma 2 (Structural Change) *Suppose $\omega < \sigma < 1$ in Proposition 1, so that we get both horizontal and vertical polarization within sectors. Then there exists $(\bar{\nu}, \bar{\eta}) \in (0,1)$ such that for all $\nu_m(j) - \nu_s(j) \geq \bar{\nu}$, $j \in \mathcal{J}^1$, and $\eta_s - \eta_m \geq \bar{\eta}$,*

$$\Delta_{V_{L_s}} > \Delta_{V_{L_m}}, \ \Delta_{\Pi_{K_s}} > \Delta_{\Pi_{K_m}}, \text{ and } \Delta_{L_s} > 0. \qquad \square$$

The proof is straightforward from inspection of (36)-(39) and is omitted. In Lemma 2, the additional assumption on within-sector manager shares η_i are needed to ensure that the degree of vertical polarization does not overturn the decline coming from horizontal polarization among workers.

The astute reader would have noticed that what the task-specific productivities effectively do is shift relative employment shares over time as if the weights $[\nu_i(j), \eta_i]$ were changing, so the two are not separately identified in a static equilibrium. Since the model is constructed so that the weights capture an initial distribution of employment shares, while task-specific TFP changes drive employment share changes over time, these assumptions are valid insofar as they hold throughout our observation period in the data.[13]

Note in Lemma 2 that even if there is a decline in $q_h(0)$ due to the negative effect coming from last term in (39b) dominating the positive effect from the first two terms, it can never overturn the direction of structural change in (39a) as long as $\epsilon < 1$. Hence, structural change can be thought of as having two parts, analogously to Goos et al. (2014): as some tasks become more productive than others, selection on skills ensures that less resources are allocated to it when we have complementarity across tasks (Proposition 1). And if one sector uses the tasks that have become more productive more intensively, resources reallocate across sectors even in the absence of a change in the sectoral allocation rule.

But as the manufacturing sector becomes more productive, in equilibrium the price of its output falls relative to services. So if manufacturing and service outputs are complementary, i.e. $\epsilon < 1$, more resources are allocated toward services; this is the insight from Ngai and Pissarides (2007).

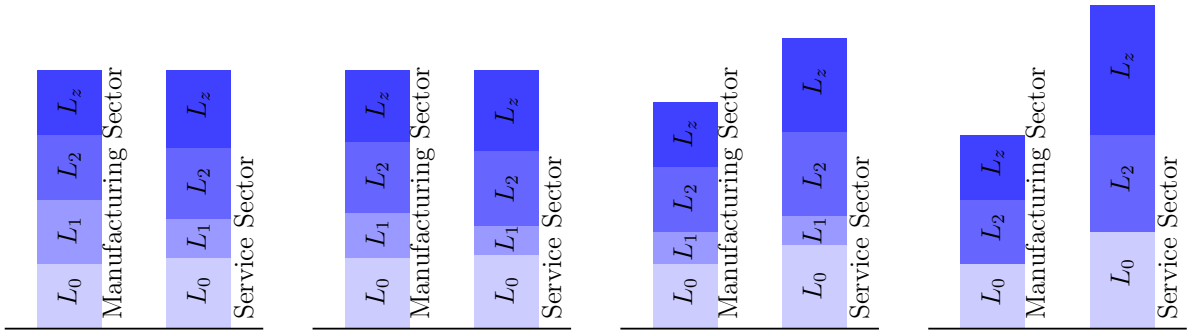
It is worth noting from (38)-(39) that the dependence of structural change depends differently on the productivity of capital and labor, in contrast to both Ngai and Pissarides (2007) and Goos et al. (2014). This is because capital is homogenous while labor is not, which is measured in two different types of skill units. Since labor productivity is determined by the sorting of individuals on skill, how task-specific TFP changes sectoral capital and labor input ratios differ not only according to differences in the reliance of sectors on certain tasks, but also according to changes in the selection rules.

Of course from (37), it is clear that structural change (change in Δ_{L_i}) also contributes to polarization. To see this more precisely, rewrite (37) using (29) as

$$\Delta_{L(j)} = \Delta_{V_i(j)} - \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)} \cdot \Delta_{V_{L_i}} + \left[\frac{L_s(j)}{L(j)} \cdot L_m - \frac{L_m(j)}{L(j)} \cdot L_s\right] \Delta_\kappa \quad (40)$$

$$\Rightarrow \quad L(j)\left(\Delta_{L(j)} - \Delta_{V_i(j)}\right) = -\sum_{i\in\{m,s\}} L_i(j)\Delta_{V_{L_i}} + \left[\frac{V_s(j)}{V_s} - \frac{V_m(j)}{V_m}\right] L_m L_s \Delta_\kappa.$$

[13] When we calibrate the model to 1980 data—for which we need assume that $M(j) = M$—the calibration naturally admits that $\eta_m < \eta_s$ and $\nu_m(j) > \nu_s(j)$ for a wide range of intermediate jobs. Furthermore, since occupational employment ratios across sectors are more or less preserved for most occupations up to 2010, so the model is insensitive to the year of normalization.



(a) Low M_1 (b) High M_1, κ fixed (c) High M_1 (d) BGP m_1

Fig. 11: Comparative Statics, Across-Sectors

Lemma 3 *Suppose Lemma 1 holds. Then structural change also contributes to polarization.*

Proof: Under Lemma 2, the term in the square brackets in (40) is negative for $j \in \mathcal{J}^1$, and positive for $j = z$. □

This is intuitive. If manufacturing is more reliant on intermediate tasks (that is, if it is more routine-intense), and shrinks as a result of polarization, this leads to even more polarization.

Lemmas 1-3 are depicted in the first 3 subplots in Figure 11. In Figure (a), manufacturing is depicted as having a higher share in intermediate tasks, and services in managers. As we move from (a) to (b), sectoral employment shares are held fixed, and intermediate tasks shrink in both sectors. The change in employment shares is larger in manufacturing due to lemma 1. This leads to structural change in (c), according to lemma 2. Because manufacturing has a higher share in intermediate tasks, shrinking its size contributes to polarization (not depicted).

4.3 Polarization or Structural Change?

One may argue that it is not task productivities that lead to structural change, but advances in sector-specific productivities that lead to polarization. While it is most likely in reality that both forces are in play, in the context of our model, as long as technologies are either task- or sector-specific (that is, there are no task- *and* sector-specific technologies), sector-specific productivity shifts does not lead to polarization within sectors.

To see this, consider an exogenous change in the manufacturing sector's exogenous productivity, A_m. As in Ngai and Pissarides (2007), a rise in A_m changes κ at a rate of $1-\epsilon$, that is, manufacturing shrinks. But it is easily seen that none of the thresholds change, and hence neither do the Φ_i's (the endogenous sectoral TFP's). So polarization can only arise by the reallocation of labor across sectors that use different mixes of tasks. To be precise, from (37),

$$\frac{d\log L(j)}{d\log A_m} = (1-\epsilon)\cdot\frac{d\log L(j)}{d\log \kappa} = (1-\epsilon)\left[\frac{L_s(j)}{L(j)}L_m - \frac{L_m(j)}{L(j)}L_s\right] < 0. \tag{41}$$

Note that $d\log L(j)/d\log\kappa$ is equal to the term in square brackets in (40), and negative for $j\in\mathcal{J}^1$. Hence, polarization only occurs because manufacturing shrinks. The reason is that in our micro-founded model, tasks are aggregated up into sectoral output, not the other way around.

Equation (41) also puts a bound on how much sectoral shifts alone can account for job polarization. For example, in the data, manufacturing employment fell from approximately 33% to 19% from 1980 to 2010. If this were solely due to a change in A_m, this means that (denoting empirical values with hats):

$$\frac{d\log\hat{\kappa}}{d\log A_m} \approx 14/67 + 14/33 \approx 0.63.$$

Now denote all routine jobs as $j=1$, then we can approximate

$$\frac{d\hat{L}_1}{d\log A_m} \approx 0.63\cdot\left[\hat{L}_{s1}\hat{L}_m - \hat{L}_{m1}\hat{L}_s\right] = 0.63\cdot\left[\frac{\hat{L}_{s1}}{\hat{L}_1}\cdot 0.33 - \frac{\hat{L}_{m1}}{\hat{L}_1}\cdot 0.67\right].$$

In Section 6, we measure the employment share of routine, manufacturing jobs and routine, service jobs (as a share of total employment; that is, L_{m1} and L_{s1}) in 1980 were 26% and 33%, respectively (refer to Table 1). So

$$\frac{d\hat{L}_1}{d\log A_m} = 0.63\left[0.33\cdot 0.33 - 0.26\cdot 0.67\right] = -0.04,$$

that is, a change in A_m alone would imply an approximately 4 percentage point drop in routine jobs from 1980 to 2010. As shown in Table 1, the actual drop was 13 percentage points.

5 Dynamics

The above result implies that on a dynamic path in which M_1 grows at a constant rate, polarization happens faster than structural change. This implies that in the limit, task 1 vanishes, structural change ceases, but both sectors still employ non-trivial amounts of labor, unlike previous models of structural change.

Such a dynamic version of the model is a straightforward extension of the neoclassical growth model. Assume that aggregate labor L grows at rate n, and a representative household with CRRA preferences

$$\int_0^\infty \exp(-\rho t) \cdot \frac{c(t)^{1-\theta} - 1}{1-\theta} dt$$

where $c_t = C_t/L_t$, and a law of motion for aggregate capital

$$\dot{K}_t = Y_t - \delta K_t - C_t,$$

and for simplicity let us assume that $\dot{M}_1/M_1 = m_1$ and $M_0 = M_2 = M_z$, $\dot{M}_0/M_0 = m$. Then from (28), we can also write the aggregate production function as

$$\begin{aligned} Y_t &= Y_{st} \cdot \left[\gamma_m^{\frac{1}{\epsilon}} \left(\frac{Y_{mt}}{Y_{st}} \right)^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} = \Phi_{st} \cdot L_{st} K_t^\alpha L_t^{1-\alpha} \cdot \left(\gamma_s^{\frac{1}{\epsilon}} L_{st}^{-1} \right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}} \cdot K_t^\alpha L_t^{1-\alpha} \end{aligned}$$

where Φ_{st}, L_{st}, the endogenous sectoral TFP and employment share of services at time t, are functions of $(\hat{z}_t, \hat{h}_{1t}, \hat{h}_{2t})$.

Now define

$$\Phi_t^{1-\alpha} \equiv \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}}$$

the endogenous aggregate (Harrod-neutral) TFP and its growth rate $g_t \equiv \dot{\Phi}_t/\Phi_t$. As in the RCK model, define the normalized consumption and capital per efficiency unit of labor

$$\hat{c}_t \equiv C_t/\Phi_t L_t \qquad \hat{k}_t \equiv K_t/\Phi_t L_t,$$

so output per efficiency unit of labor is

$$\hat{y}_t \equiv Y_t/\Phi_t L_t = f(\hat{k}_t) \equiv \hat{k}_t^\alpha.$$

The dynamic equilibrium is characterized by

$$\begin{aligned} \dot{\hat{c}}_t &= \frac{1}{\theta} \cdot \left[f'(\hat{k}_t) - (n + \delta + \rho + g_t \theta) \right] \cdot \hat{c}_t \\ \dot{\hat{k}}_t &= f(\hat{k}_t) - (n + \delta + g_t)\hat{k}_t - \hat{c}_t \\ g_t &\equiv \frac{\dot{\Phi}_t}{\Phi_t} = g\left(\hat{h}_{1t}, \hat{h}_{2t}, \hat{z}_t \right). \end{aligned}$$

So instead of having sectoral shares as in Acemoglu and Guerrieri (2008), we have endogenously evolving TFP which pins down the sectoral shares at every instant. Using (27) and (**??**), the endogenous growth rate g_t becomes

$$(1-\alpha) g_t = m + \sum_i L_{it} \cdot \left[\frac{\omega - \sigma}{(\omega - 1)(\sigma - 1)} \cdot \frac{\dot{\Pi}_{ih}}{\Pi_{ih}} + \left(\frac{\omega}{\omega - 1} - \alpha \right) \cdot \frac{\dot{\Pi}_{K_i}}{\Pi_{K_i}} + (\alpha - 1) \cdot \frac{\dot{\Pi}_{L_i}}{\Pi_{L_i}} \right].$$

On a BGP, g_t must be constant. Hence it must be that $(\hat{h}_1, \hat{h}_2, \hat{z})$ no longer evolve: Clearly this happens when $\hat{h}_1 = \hat{h}_2$, or from (**??**)-(**??**),

$$\hat{h}_2 - \chi = \frac{(\bar{h}_2 - \chi)L_{i2}}{L_{i0}} \cdot \left[\frac{\nu_{i0}}{\nu_{i2}} \cdot \left(\frac{\hat{h}_2 - \chi}{\bar{h}} \right)^{(1-\alpha)(1-\sigma)} \right]$$

$$\Rightarrow \qquad \frac{\nu_{i0} L_{i2}}{\nu_{i2} L_{i0}} = \frac{(\hat{h}_2 - \chi)^{\sigma + \alpha(1-\sigma)}}{\bar{h}_2 - \chi},$$

assuming $\bar{h} = 1$. Then $\hat{z}$ is determined by (13), sectoral-task employment masses are determined by (Π_{K_i}, Π_{L_i}) according to (**??**), and on a BGP

$$g^* = \frac{m}{1-\alpha}.$$

The long-run dynamics is depicted in figure 11(d), where both polarization and structural change continue until task 1 vanishes.

6 Quantitative Analysis

The goal of our quantitative analysis is to quantify how much of the observed changes in employment and wage shares from 1980 to 2010 can be explained by task-level productivity growth, and relate such productivity growth to empirically measurable sources. Whenever possible, we fix parameters to their empirical counterparts, and separately estimate the aggregate technology (4) from time series data on sectoral price and output ratios. Then we choose most model parameters to fit the 1980 data exactly, including a parametric skill distribution of (h, z). The rest of the model parameters, which includes the elasticity parameters (σ, ω), are calibrated to empirical time trends from 1980 to 2010.

6.1 Occupations and Skills

In the quantitative analysis, we assume that there are 10, rather than 3, (horizontally differentiated) worker task/occupations. (There is still only one management task.) Recall that in task 0, workers can only utilize $\bar{h}$ regardless of their own level of human capital, and in task 1 they simply use all of their human capital. Each of the additional worker tasks is characterized by a skill-loss parameter $\{\chi_j\}_{j=2}^{9}$.[14]

We assume each of these 11 occupations (10 worker + 1 manager occupation) in the model broadly correspond to the 11 one-digit occupation categories in the census, discussed in Section 2 and summarized in Table 1. The 10 worker occupation

[14] The characterization of the equilibrium is exactly the same as before.

Ranked by mean wage (except management)	SOC Code	Employment Shares 1980	2010	Manufacturing		Total Wage Shares 1980	2010	Manufacturing	
Low Skill Services	400	**10.44**	**13.92**	**0.59**	**0.23**	**6.75**	**7.60**	**0.52**	**0.16**
Middle Skill		**59.09**	**46.48**	**25.86**	**12.93**	**53.43**	**35.90**	**24.76**	**10.02**
Administrative Support	300	16.57	14.13	3.47	1.53	12.90	9.60	2.90	1.15
Machine Operators	700	9.81	3.75	8.79	3.02	8.21	2.39	7.37	1.91
Transportation	800	8.73	6.64	3.80	2.28	7.73	4.15	3.37	1.46
Sales	240	7.87	9.37	0.79	0.62	7.40	8.45	1.06	0.85
Technicians	200	3.23	3.86	1.00	0.57	3.35	4.33	1.13	0.66
Mechanics & Construction	500	7.91	6.02	4.44	3.19	8.40	4.88	4.91	2.61
Miners & Precision Workers	600	4.97	2.71	3.58	1.73	5.43	2.10	4.03	1.38
High Skill		**19.22**	**26.16**	**3.87**	**3.64**	**24.20**	**33.98**	**6.07**	**5.51**
Professionals	40	11.02	16.51	1.73	1.45	13.36	20.78	2.59	2.12
Management Support	20	8.20	9.65	2.14	2.20	10.84	13.20	3.48	3.39
Management	1	**11.26**	**13.44**	**2.47**	**2.59**	**15.62**	**22.52**	**4.22**	**5.81**

Table 1: Occupation Groups used for Calibration
Source: US Decennial Census (1%), 1980-2010.

groups can further be broadly grouped into low/medium/high skill tasks, or manual/routine/abstract jobs, according to the mean wages of each occupation group and routinization indices.

In Table 1, the left panel shows the SOC of each occupation with a short job description, and the middle and right panels their employment and total wage shares in 1980 and 2010, respectively. For the employment and wage share panels, the first two columns shows the size of each occupation in 1980 and 2010 as a fraction of total employment/wages. The next two columns show the size of each occupation within manufacturing as a fraction of total employment/wages.

These were already depicted graphically in Section 2, and will be the bulk of our target moments in the calibration. The only other target moment is the growth rate of aggregate output.

Parametric Skill Distribution For the quantitative analysis, we assume a parametric skill distribution that is type IV bivariate Pareto (Arnold, 2014). Specifically, the c.d.f. we assume is

$$\mu(h,z) = 1 - \left[1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right]^{-a}.$$

We normalize $\gamma_z = 1$, since we cannot separately identify both skills from the skill-specific TFP's. This is consistent with an establishment size distribution that is Pareto, and a wage distribution that is hump-shaped with a thin tail, as depicted in Figure 12.

6.2 Aggregate Production Function

The aggregate production function is estimated outside of the model. For the estimation, we only look at the manufacturing and service sectors, where manufacturing

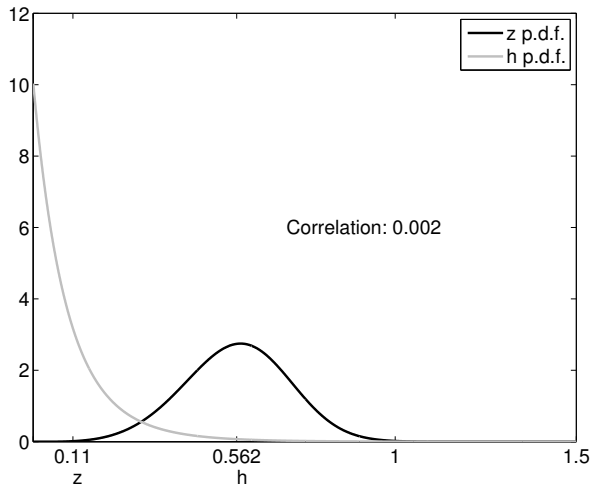


Fig. 12: Calibrated Skill Distribution

We use a type IV bivariate Pareto distribution to model the distribution over worker and manager skills (h, z). The figure depicts the marginal distributions each skill, and also their mean values below the x-axis. The implied Pearson correlation coefficient between the two skills is low, at 0.002.

includes mining and construction, and government is included in services. We estimate the parameters (γ_m, ϵ) from the system of equations

$$\begin{aligned}\log\left(\frac{p_m Y_m}{PY}\right) &= \log\gamma_m + (1-\epsilon)\log p_m - \log\left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right] + u_1 \\ \log(Y) &= c + \frac{\epsilon}{\epsilon-1}\log\left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2\end{aligned}$$

where $\gamma_s \equiv 1-\gamma_m$, using non-linear SUR (seemingly unrelated regression), on all years of real and nominal sectoral output observed in the BEA accounts.

Real production by sector is computed by a cyclical expansion procedure as in (Herrendorf et al., 2014) using production value-added to merge lower level industries (as opposed to consumption value-added, as in their analysis).[15] The price indices are implied from nominal versus real sectoral quantities. We try different base years as well: 1947, 1980, and 2005, corresponding to columns (1)-(3) in Table 2. This is to check the robustness of the choice of base years: 1947 is the first year the required data is available, 1980 is the first year in our model, and 2005 is chosen as a year close to present but before the crisis.

As shown there, the values are in a similar range as in Herrendorf et al. (2014), although ϵ is not significant with 2005 as a base year. For the calibration, we will use take the values of (γ_m, ϵ) in column (1) values as a benchmark.

The capital income share α is computed as 1-(labor income/total income), and fixed at 0.360. Since we do not model investment, for the calibration we also need the

[15]The constant c is included since it is not levels, but relative changes that identify ϵ.

	(1)	(2)	(3)
γ_m	0.371**	0.346**	0.258**
	(0.003)	(0.005)	(0.004)
ϵ	0.003**	0.002**	0.003
	(0.000)	(0.000)	(0.004)
AIC	-550.175	-551.264	-550.866
RMSE_1	0.106	0.106	0.106
RMSE_2	0.039	0.039	0.039

Standard errors in parentheses
† $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table 2: Aggregate Production Function

The manufacturing share parameter γ_m and elasticity parameter between manufacturing and services, ϵ, are estimated off the time series of output and price ratios from 1947 to 2013, which are available from the BEA accounts. The service share parameter γ_s is assumed to equal $1 - \gamma_m$. For details of the estimation, we closely follow Herrendorf et al. (2014).

level of total capital stock (for manufacturing and services) for each decade, which we take from the Fixed Assets Table and directly plugged into the model. Since we do not model population growth, in practice we we normalize output per worker in 1980, y_{1980} to unity, and plug in $K_t = k_t/y_{1980}$ for $t \in \{1980, 1990, 2000, 2010\}$, where k_t is capital per worker in year t.

6.3 Calibration

Given the aggregate production function, we simulate two equilibria: one each for 1980 and 2010. Except for capital per worker, which we take exogenously from the data, any change between 1980 to 2010 is only due to exogenous task-TFP growth.

Most of the parameters are calibrated so that our our model equilibrium fits the 1980 data moments in Table 1 exactly. We then calibrate the elasticity parameters (σ, ω) and the 11 constant TFP growth rates $\{m_j\}_{j=z,0}^{9}$ to fit the time trends in aggregate output and employment shares from 1980 to 2010.

All model parameters are summarized in Table 3, except for the skill loss parameters χ_j, task intensity parameters (η_i, ν_{ij}), and task-TFP growth rates m_j, which are tabulated in Table 4.

Below, we explain in detail how these parameters are recovered.

Calibrating the distribution For any guess of $(\gamma_h, a, \{\chi_j\}_{j=2}^{9})$, we can find $(\hat{z}, \{\hat{h}_j\}_{j=1}^{9})$ that exactly match observed employment shares by occupation in 1980, by integrating over the guessed skill distribution. Given the thresholds, we then compute

	Parameter	Value	Target
Fixed from data	K_{1980}	2.895	Computed from BEA/NIPA data
	K_{2010}	4.235	
	α	0.361	
	γ	0.371	Estimated in section 6.2
	ϵ	0.003	
Fit to 1980	$M_j \equiv M$	0.985	Output per worker, normalization
	A_m	1.112	Manufacturing employment share
	A_s	1.000	Normalization
	ν_{ij} (18)	Table 4	Witin-sector employment shares by occupation
	$\eta_i(2)$		Within-sector manager share
	χ_j (8)	Table 4	Relative wages by occupation
	a	10.087	
	γ_h	0.216	
	γ_z	1.000	Normalizations;
	$\bar{h}$	1.000	Not separately identified from M_j
Fit to 2010	σ	0.704	Witin-sector employment shares by occupation
	ω	0.341	
	m_j	Table 4	Output per worker growth and employment shares by occupation

Table 3: Parameters

All parameters valued 1 are normalizations.

the model-implied relative wages using (**??**),

$$\frac{w_1\bar{h}_1}{w_0\bar{h}} = \frac{\bar{h}_1}{\hat{h}_1}, \quad \frac{w_2(\bar{h}_2 - \chi_2)}{w_1\bar{h}_1} = \frac{\bar{h}_2 - \chi_2}{\bar{h}_1(1 - \chi_2/\hat{h}_2)}, \quad \frac{w_z\bar{z}}{w_0\bar{h}} = \frac{\bar{z}}{\hat{z}},$$

and similarly for $j \in \{3, \dots 9\}$. The LHS is the ratio of mean wages by occupation, which we observe from the data. The RHS is a function only of the thresholds, which themselves are functions of (γ_h, a, χ_j). Hence, we iterate over $(\gamma_h, a, , \{\chi_j\}_{j=2}^9)$ so that the model-implied ratios match observed mean wage ratios exactly.

For the rest of the calibration, we fix these three parameters that govern the skill distribution. By construction, we already know the thresholds $(\hat{z}, \{\hat{h}_j\}_{j=1}^9)$ that fit 1980 employment shares by occupation exactly. And since the skill distribution is fixed by the data, we can similarly compute the thresholds that fit 2010 employment shares by occupation. Denote these two sets of thresholds as $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$, respectively. Note that these thresholds are determined solely by the exogenously assumed skill distribution and the data, independently of our model equilibria.

Ranked by mean wage (except management)	χ_j	Emp Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j	RTI
Low Skill Services	**-**	**0.016**	**0.136**	**-0.731**	**-0.211**
Middle Skill		**0.816**	**0.524**		
Administrative Support	-	0.088	0.173	2.930	2.445
Machine Operators	0.001	0.256	0.015	9.122	0.602
Transportation	0.002	0.119	0.081	4.348	-0.576
Sales	0.003	0.026	0.123	0.012	0.483
Technicians	0.005	0.034	0.040	-1.144	-0.269
Mechanics & Construction	0.006	0.159	0.065	2.315	-0.630
Miners & Precision Workers	0.007	0.134	0.027	6.328	0.639
High Skill		**0.168**	**0.340**		
Professionals	0.009	0.070	0.195	-2.248	-0.725
Management Support	0.010	0.098	0.146	-0.489	-0.655
Management	**-**	**0.076**	**0.130**	**-0.017**	**-1.103**

Table 4: Calibrated Employment Weights and Growth Rates

Normalizations Before we simulate the model equilibrium and calibrated the remaining parameters, some normalizations are in order. We have already normalized $\gamma_z = 1$. For notational convenience, we will denote the 1980 levels of the TFP's by (M, A_i) and denote their 2010 levels by multiplying them by their respective growth rates. For example, the manager-task TFP in 2010 is $M(1 + m_z)^{30}$.

1. We normalize $\bar{h} = 1$ since it is not separately identified from M_0. This can be seen in (**??**).
2. We also normalize $M_j \equiv M$ for all $j \in \{0, 1, \ldots, z\}$ for 1980, since it is not separately identified from (η_i, ν_{ij}) in a static equilibrium. This is implied by the production technology we assume in (6)-(7).

The rest of the parameters are calibrated so that simulated moments from the model's 1980 and 2010 equilibria match the data.

Calibrated within the model There are 35 parameters that remain to be calibrated: the elasticity parameters (σ, ω), TFP parameters (M, A_m), task intensities $(\eta_i, \{\nu_{ij}\}_{j=1}^9)$ for $i \in \{m, s\}$, and the task-TFP growth rates $\{m_j\}_{j=z,0}^9$.[16]

Given the aggregate production function, skill distribution, normalizations and thresholds ($\mathbf{x}_{1980}$, we can recover the remaining parameters as follows.

(A) Guess (σ, ω).

[16]There are only 9 intensity parameters to calibrate per sector, since we assume $\sum_{j=0}^9 \nu_{ij} = 1$.

(B) Given the guess, first fit 1980 moments:

   (a) Guess (M, A_m).

   (b) Plug in the threshold values $\mathbf{x}_{1980}$, and the empirical values of $(L_{iz}, L_{i0}, \ldots, L_{i9})$—the employment shares of each occupation in sector $i \in \{m, s\}$—from Table 1, into (**??**) and (13). Then we recover all the ν_{ij}'s from (**??**)-(**??**), and the η_i's from (13)-(14) in closed form (since all M_j's are assumed to be equal). This ensures that the 1980 equilibrium exactly fits within-sector employment shares by occupation (20 parameters, 20 moments).

   (c) Repeat from (a) until we exactly fit the manufacturing employment share in 1980, and output per worker of 1.[17] Since (27)-(28) are monotone in (M, A_m), the solution is unique (2 parameters, 2 moments).

(C) Given the parameters recovered from the 1980 equilibrium, plug in threshold values $\mathbf{x}_{2010}$ into (**??**). Then find $\{m_j\}_{j=z,0}^{9}$ so that the 2010 equilibrium exactly fits employment shares by occupation (but not necessarily by sector), and also output per worker, in 2010 (11 parameters, 11 moments).

(D) Repeat from (1) to minimize the distance between the within-sector employment shares by occupation implied by the 2010 model equilibrium and the data (2 parameters, 11 moments).

Note that since all other moments are fit exactly, in essence we are only calibrating the two parameters (σ, ω) in (A) to match the 11 moments in (D).[18]

The resulting parameters are tabulated in Tables 3-4. The last column of Table 4 shows the empirical RTI indices constructed from Autor and Dorn (2013), which are also visualized in Figure 19.

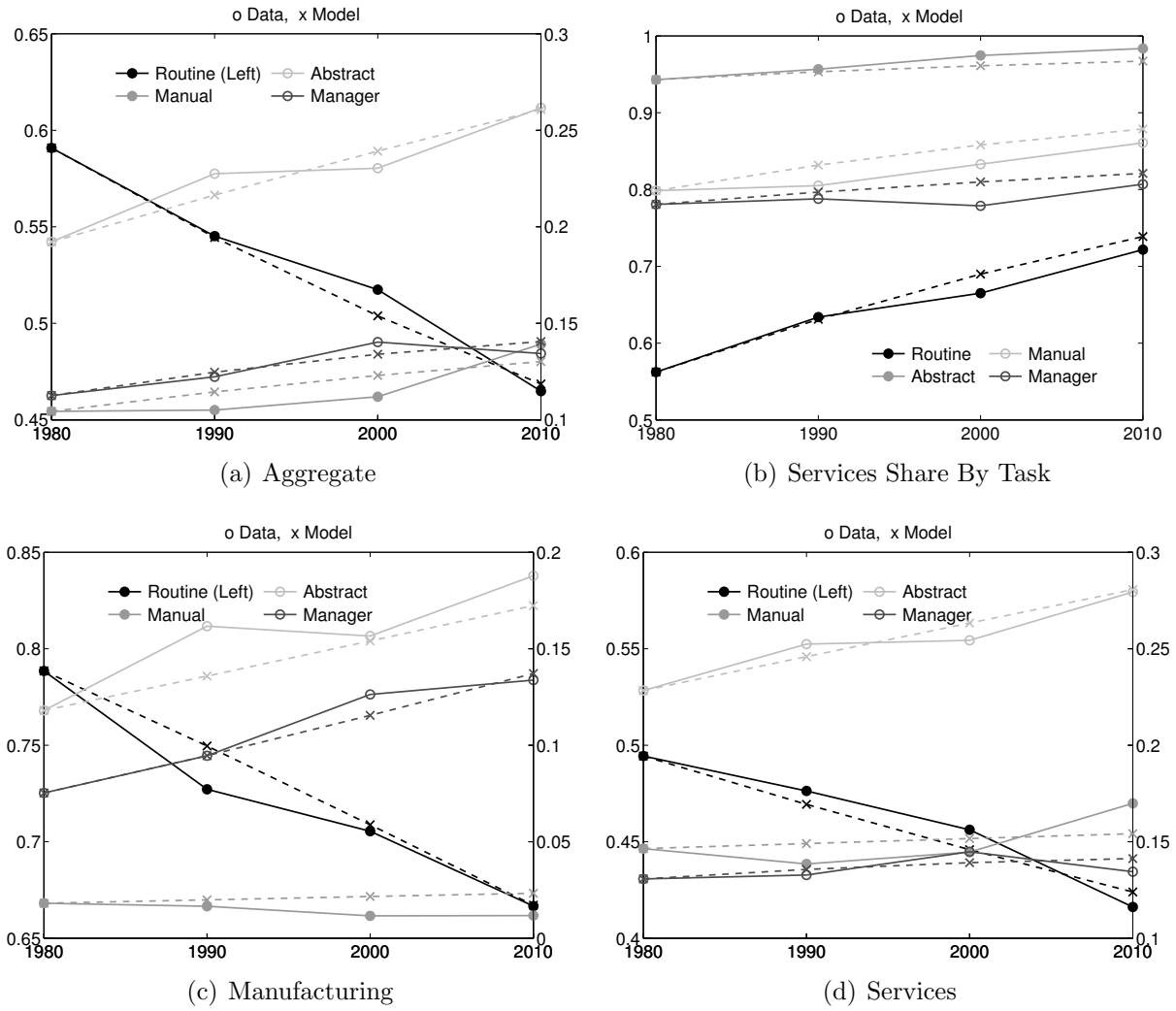
6.4 Model Fit

Figure 13 plots the model implied trends in employment shares across tasks, in aggregate and by sector, against the data. When computing the simulated paths for 1990 and 2000, we plug in the empirical values of $K_t = k_t/y_{1980}$ and the level of task-TFP's implied by the calibrated growth rates, and compute the respective equilbria allocations.

As we explained above, the aggregate trend can be solved in closed form using the same number of parameter and moments, so it is not surprising that we obtain a more or

[17] The latter must be matched since the value of K_{1980} we plug in from the data was normalized by 1980's output.

[18] When targeting 2010 employment shares and output per capita, we in fact target the linear trend from 1980 to 2010 rather than their exact values. However, since most trends are in fact linear, using the exact values barely change any of our results.



(a) Aggregate

(b) Services Share By Task

(c) Manufacturing

(d) Services

Fig. 13: Data vs. Model, Employment Shares by Task

less exactly fit as seen in Figure 13(a). On the other hand, while we target the starting points for all the shares (services employment share, and within-sector employment shares by task), these 11 trends were calibrated using only the 2 elasticity parameters (σ, ω). Nonetheless, the calibrated model more or less exactly replicates structural change by occupation, as seen in Figure 13(b) and also within-sector polarization, as seen in Figures 13(c)-(d).

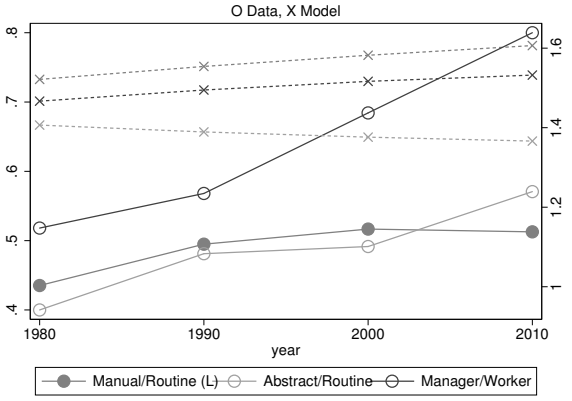
The fit is not as satisfactory for relative wages, plotted in figure 14. Here we plot the mean relative wages in aggregate and by sector. Here, the only targeted moments were the 1980 average wage ratios in Figure 14(a); all other moments were not. Manual and abstract wages are relative to routine jobs, and manager wages are relative to all workers.

Recall that in the model, efficiency wages $(w_z, w_1, \ldots, w_9)$ are equal since we assume



(a) Aggregate

(b) Manufacturing



(c) Services

Fig. 14: Data vs. Model, Relative Wages by Task

that individuals are indifferent across sectors, and we constrained our attention to equilibria in which mean skill levels within occupations were equal across sectors. Hence, within relative wages are equal across sectors for any given occupation; the only reason they differ in Figures 14(b)-(c) is because we aggregate sub-occupation groups 3 broader categories; and for managers since they are compared against all workers. Compared to the data, all model-implied relative wages are too low in manufacturing, and too high in services. Hence to some degree, our model is missing something that causes relative wages to be lower in services.

Some explanation is in order. Due to the multiple layers of complementary between occupations and sectors ($[\omega, \sigma, \epsilon] < 1$), the calibrated growth rates m_j are smaller for those occupations that are growing, as shown in Table 4. Then for all occupations in the middle ($j = 1, \ldots, 9$), whether their relative wages grow or shrink depends on the magnitude of negative selection (that comes from having more or less low-skill workers

from the left-side of the h-skill distribution) and positive selection (that comes from having more or less high-skill workers from the right-side of the h-skill distribution), since whether the employment share grows or shrinks, it will either absorb or lose employment from both sides of the distribution. This was proven in Proposition 1 and in Figure 10, can be seen from $\hat{h}_1$ increasing and $\hat{h}_2$ decreasing following a rise in the TFP among routine jobs.

For manual jobs, average skill is assumed to be constant at $\bar{h}$, and it is only those workers with the lowest h skill that work in this job. It turns out that routine jobs as a whole display enough negative selection so that the wages of manual workers relative to routine workers rise, although only slightly. This is in fact consistent with the data in aggregate and in the services sector, although in the manufacturing sector, manual wages slightly dropped.

We also proved in Proposition 1 that as long as $\omega < \sigma$, which it is (Table 3), managers' relative wages would rise relative to workers as long as their task-TFP grew slower than workers'. However, the quantitative magnitude of this rise is small. This is because as the employment share of managers grow, there is a negative selection along z-skills. In Figure 10, this can be seen from $\hat{z}$ decreasing to increase the mass of managers.

According to our model assumptions, all workers with the highest h-skill work in the highest-skill worker occupation ("professionals" in the data). Since their task-specific TFP grew relatively less, the average skill of workers in this occupation becomes lower, since employment growth leads to negative selection. Consequently, both because of lower TFP growth and negative selection, relative wages decline for abstract jobs, in contrast to the data.

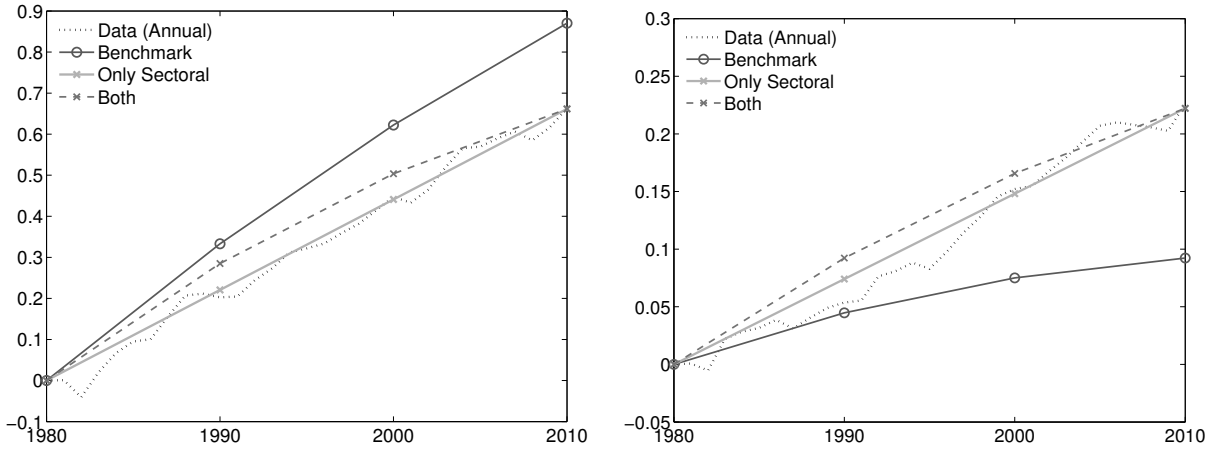
Overall, the model targeted only to aggregate moments delivers a good fit by task even within and across sectors in terms of employment shares, but not in terms of relative wages. In what follows, we focus only on employment shares and investigate how much each of these trends is explained by task-specific TFP's, and its implications for other outcomes such as sectoral TFP's.

7 Results

7.1 Counterfactuals

In this subsection, we analyze the role of task-specific TFP's on structural change, , we conduct two counterfactuals:

(1) First, we set all task-specific TFP growth to be equal, that is, we set $m_j = m$, and instead let both (A_m, A_s) change at rates (a_m, a_s). We jointly recalibrate



(a) Log TFP per Worker, Manufacturing

(b) Log TFP per Worker, Services

Fig. 15: Benchmark vs. Counterfactuals, TFP

1980 levels are normalized to 0, so the slope of the lines are the growth rates.

(m, a_m, a_s) to match the empirical growth rate of TFP (i.e., the Solow residuals) in aggregate, and also in manufacturing and services, from 1980 to 2010. This yields the model's predictions in the absence of any exogenous, task-specific TFP growth.

(2) Second, we allow both exogenous task- and sector-specific TFP growth, and re-calibrate $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ to match the change in employment shares *and* the empirical growth rates of TFP in aggregate, manufacturing and services, from 1980 to 2010. This gives the model the best chance to explain the data.

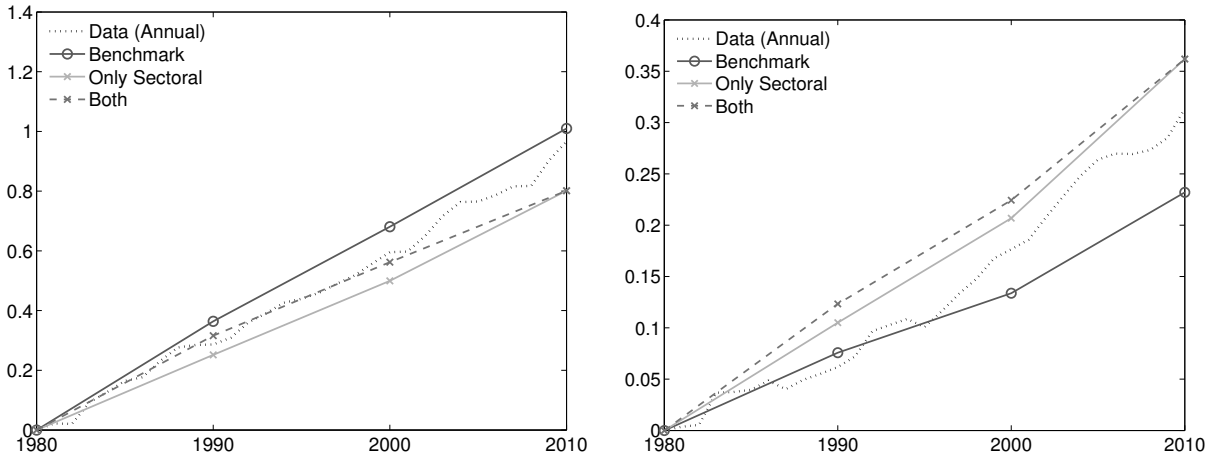
For both counterfactuals, we keep all other parameters at their benchmark values in Table 3-4, and only recalibrate the growth rates.

We focus on sectoral TFP's since in our model, structural change only results from the differential TFP growth across sectors—expressed in closed form in (27)—whether it is exogenous (caused by a_m and/or a_s) or endogenous (as in Section 4.2). The recalibrated parameters for the counterfactual scenarios are summarized in Appendix Table 5.

TFP and output growth In Figure 15, we compare the path of log sectoral TFP in the data, in our benchmark calibration, and two counterfactual scenarios. All scenarios match aggregate TFP and GDP growth from 1980-2010 in the calibration as shown in Appendix Figure 22, so we do not discuss them here.[19]

In our benchmark, we over shoot the growth rate of manufacturing TFP by about

[19]Denote aggregate TFP as Z_t. Since $Y_t = Z_t K_t^{\alpha}$ (labor is normalized to one), and we plug in the empirical values of K_t for all calibrations, matching aggregate TFP and GDP are the same things.



(a) Log GDP per Worker, Manufacturing

(b) Log GDP per Worker, Services

Fig. 16: Benchmark vs. Counterfactuals, GDP per Worker

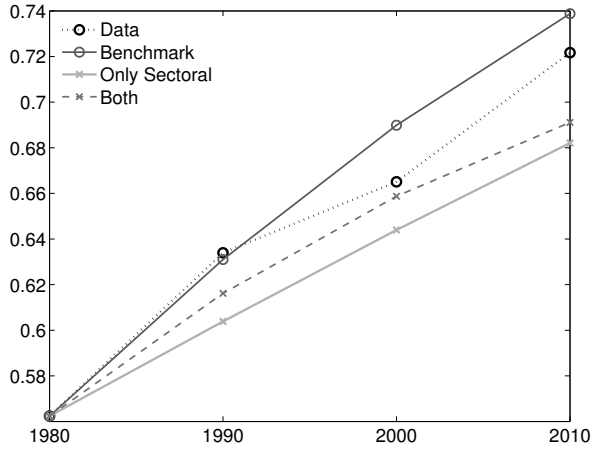
1980 levels are normalized to 0, so the slope of the lines are the growth rates.

half a percentage point, while undershooting the services TFP growth rate by about half a percentage point. However, note that when we look at the growth rates of sectoral output, in Figure 16, this gap almost disappears. This is because while the model assumes that the sectoral capital input shares are equal to labor input shares, as shown in (28) earlier, in the data they are not. In fact, counterfactuals (1) and (2) in Figure 16 show that when sectoral TFP growth is matched exactly, manufacturing output grows more slowly, and services output more quickly, compared to the data. This implies that capital input ratio between manufacturing and services grew slower than the labor input ratio, although the differences are small.[20]

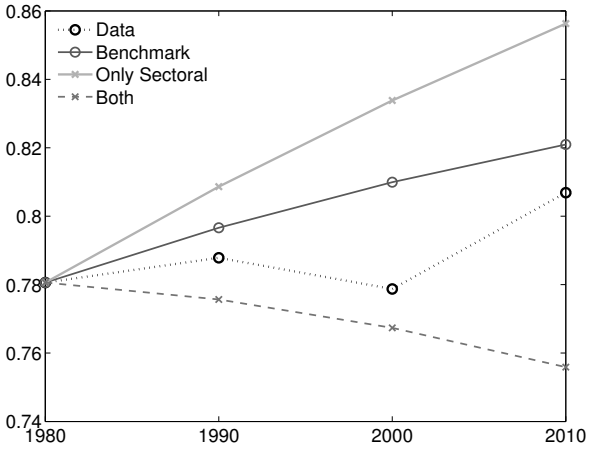
Structural Change and Polarization Since structural change is solely determined by output ratios (Section 4.2, equation (28)), the fact that sectoral output growth nearly tracks the data implies that the the benchmark model more or less fully explains structural change (in terms of employment), as shown in Figure 17(c). Both counterfactuals (1) and (2) undershoot the full extent of structural change, since sectoral output growth is too low and high in manufacturing and services, respectively. Moreover, when we look at structural change within occupation categories, the benchmark model outperforms both counterfactuals (1) and (2), especially among managers.

Lastly, we investigate whether exogenous growth in sectoral TFP's can explain polarization, as we also discussed in Section 4.3. In Figure 18, we see that sectoral forces alone can account for about 15-20 percent of horizontal and vertical polarization.

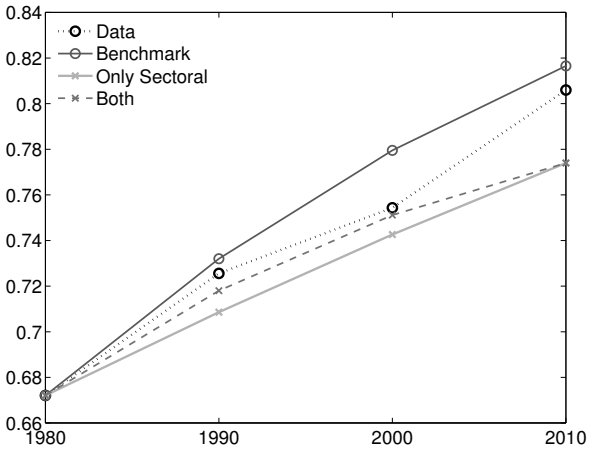
[20] If sectoral production functions remain Cobb-Douglas, this means that capital intensity is higher in manufacturing, as analyzed in Acemoglu and Guerrieri (2008).



(a) Services Employment Share, Routine Jobs



(b) Services Employment Share, Managers

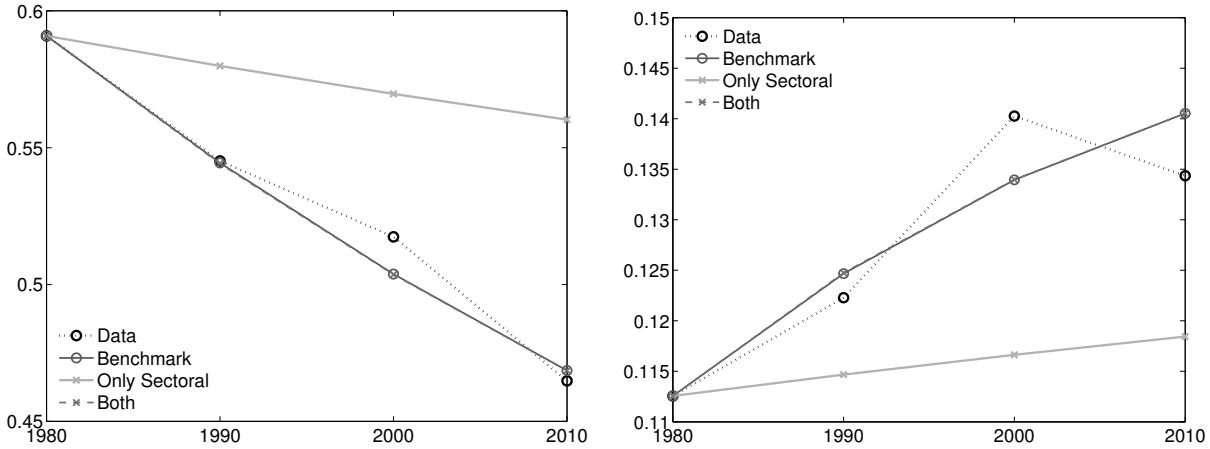


(c) Services Employment Share, All Jobs

Fig. 17: Benchmark vs. Counterfactuals, Structural Change

However, remember that this would not cause any changes in *within* sector employment shares by occupation.

In sum, task-specific TFP growth can more or less fully account for sectoral output growth, and consequently for the observed levle of structural change from 1980 to 2010. Due to the vertical and horizontal polarization induced by changes in task-specific TFP's, employment shifts to the sector that uses the routine task less and management more intensively. Conversely, sector-specific productivities can only account for 15-20 percent of polarization, and furthermore we have shown, both analytically and quantitatively, that it does not cause any polarization within sectors, that contrary to the data.



(a) Routine Employment Share

(b) Manager Employment Share

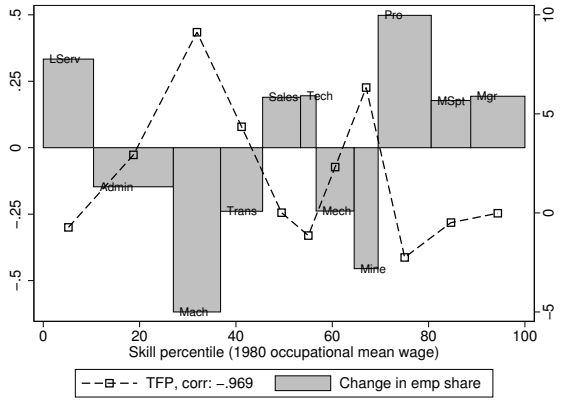
Fig. 18: Benchmark vs. Counterfactuals, Polarization

7.2 What are Task-Specific Productivities?

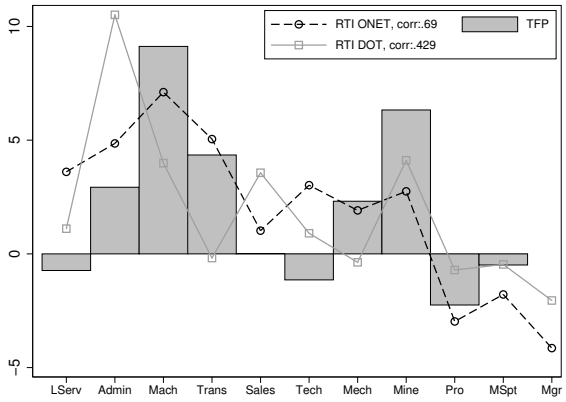
Despite having skill selection, horizontally and vertically differentiated jobs, and multiple sectors, Figure 19(a) shows that the bulk of the changes in occupational employment shares are still directly accounted for by task-specific TFP's, with a (negative) correlation of 0.97. This is also confirmed from a simple regression analysis shown in Appendix Table 6. This leads us to conclude that in order to understand changes in the employment structure, it is important to identify what these task-specific TFP's are.

How much of the variation in the TFP growth rates can be explained by routinization? As a first pass, we correlate the TFP growth rates with the RTI measure used in Autor and Dorn (2013), which itself is constructed from Autor et al. (2003) using the DOT, and the RTI measure from Acemoglu and Autor (2011), which was constructed from O*NET. While the TFP growth rates are positively correlated with both indices, and more strongly with the latter, it is visually clear that there is much left to be explained. More precisely, both the correlation and R^2's are still quite low, as can be seen in Appendix Table 6.

What about college? Skill-biased technological change (SBTC) has been a usual suspect for changes in the employment structure since since Katz and Murphy (1992), e.g. Krusell et al. (2000); Buera et al. (2015). In the SBTC literature, "skill" is usually a stand-in for whether or not an individual went to college, or obtained a college degree. However, as is evident from Figure 20(a), neither the fraction of college graduates within each occupation in 1980, nor the change in the fraction of graduates from 1980 to 2010, have much of a relationship with the task-specific TFP's calibrated from our

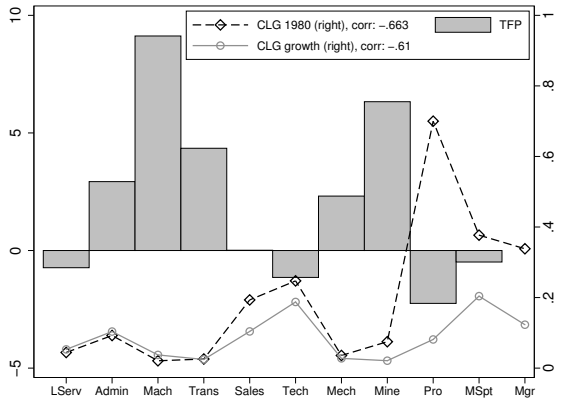


(a) Task-specific TFP growth and Employment Shares

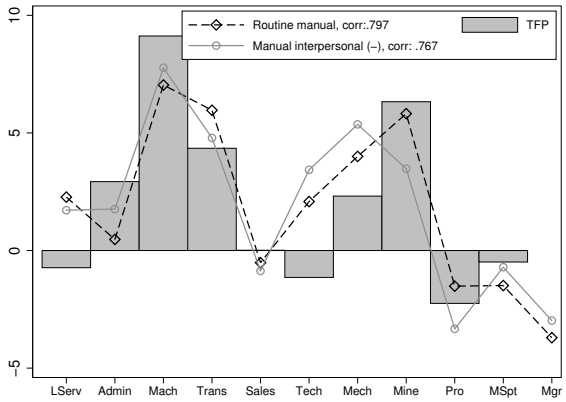


(b) Task-specific TFP growth and RTI

Fig. 19: Employment Shares, Task TFP growth and the Routinization Index



(a) Task-specific TFP Growth and College



(b) Task-specific TFP Growth and O*NET

Fig. 20: Task TFP growth, College Shares and O*NET-based indices

model. Since the TFP's more or less entirely explain the employment shifts observed in the data, this means college can not explain occupational employment shifts. Moreover, as is clearer in Appendix Table 6, the correlation between employment shifts and college measures are *negative*, that is, those occupations with more college graduates, or in which the college graduate share grew the fastest, in fact *shrank*. This is the opposite of most of propositions made in the SBTC literature.

What we do find, however, is that the TFP growth rates correlate strongly with disaggregated components of the RTI index in O*NET, in particular the routine-manual and interpersonal skills indices. Appendix Table 6 shows that the R^2 for both are also high. This means that those occupations with a higher share of routine-manual tasks have shrunk, while those with higher share of interpersonal tasks have grown.

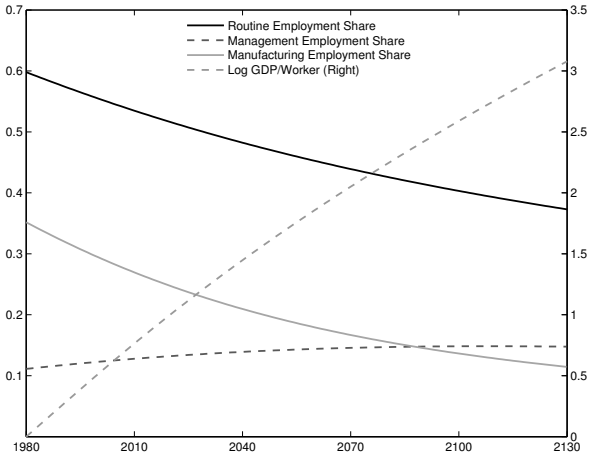


Fig. 21: Long-run Dynamics

While we conclude that productivity growth has been high in routine-manual tasks and low among interpersonal tasks, and that this can explain a significant part of shifts in occupational employment, polarization, and consequently structural change, it is evident from the regressions that this is not the end of the story. The unexplained part of task-specific TFP growth may also come from endogenous changes in the distribution of skill, and in an open economy setting from off-shoring, both of which we have abstracted from.[21]

7.3 Long-Run Growth Path

Lastly, we show the long-run dynamics of the model from Section 5, extended to the 10 horizontally-differentiated tasks we have analyzed thus far. Assuming a CRRA coefficient of $\theta = 2$, and that the economy starts in 1980, we target an asymptotic interest rate of 2%, implying an approximately equal discount rate ρ. The depreciation rate is set to $\delta = 0.065$, as computed from the NIPA accounts. As can be seen, both routine and manufacturing continue to decline, but the speed of the decline in manufacturing slows down as routine jobs continue to disappear. Likewise, managerial employment continues to rise, albeit at a slower pace. Finally, note that the first 150 years displays structural change and near balanced growth, consistently with the Kuznets and Kaldor facts.

[21] In an open economy setting, cheaper foreign labor would be observationally equivalent to higher productivity.

8 Conclusion

We presented a new model which encompasses job polarization, structural change, and a modified span of control technology. We showed analytically and the quantitatively that the model can be a useful tool for analyzing macroeconomic dynamics.

A Proofs

A.1 Proof of Theorem 1

We first show that for fixed $[q_h(j), q_z] \in (0,1)$, the within-sector equilibrium is unique. For an arbitrary guess of $\hat{z}(j)$, Assumptions 1-3 imply existence of a solution to the differential equation (23a) by Picard-Lindelöf's existence theorem. Similarly, a solution to (23b) exists by Brouwer's fixed point theorem once we apply a minimum value for $\hat{z} \geq \underline{z} > 0$ such that the denominator does not converge to zero.

To show that the solution is unique, we first prove the following Lemma:

Lemma 4 *Suppose $[q_{ih}(j), q_{iz}]$ are fixed and that $[\hat{h}(j), \hat{z}]$ and $[\hat{h}^1(j), \hat{z}^1]$ are both an equilibrium for one sector. For any connected subset $\mathcal{J}^1 \subseteq \mathcal{J}$, $\hat{h}$ and $\hat{h}^1$ can never coincide more than once on $\mathcal{J}^1$.*

Proof: We proceed by contradiction as in Lemmas 3-6 in Costinot and Vogel (2010). Suppose **(i)** $\hat{h}(j_a) = \hat{h}^1(j_a)$ and $\hat{h}(j_b) = \hat{h}^1(j_b)$ such that both $(j_a, j_b) \in \mathcal{J}^1$. Without loss of generality, we assume that $j_a < j_b$ are two adjacent crossing points. Then, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that

1. **(ii)** $\hat{h}^{1\prime}(j_a) \geq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \leq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j_a, j_b)$;
   or
2. **(ii)** $\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \geq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j_a, j_b)$.

Consider case **1**. Condition **(ii)** implies

$$\hat{h}^{1\prime}(j_b)/\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_b)/\hat{h}'(j_a)$$

and using (18)-(19) and (23a), and applying $\hat{h}^1(j) = \hat{h}(j)$ for $j \in \{j_a, j_b\}$ we obtain

$$\begin{aligned} 0 < [\alpha + \sigma(1-\alpha)] \cdot \left[\int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}^1(j'), j')}{\partial j'} dj' - \int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \\ \leq \log \left[F(\hat{z}^1(j_b)|\hat{h}(j_b))/F(\hat{z}(j_b)|\hat{h}(j_b)) \right] - \log \left[F(\hat{z}^1(j_a)|\hat{h}(j_a))/F(\hat{z}(j_a)|\hat{h}(j_a)) \right] \end{aligned} \tag{42}$$

where the first inequality follows since (2), the log-supermodularity of b, implies

$$\partial \log b(h^1, j)/\partial j > \partial \log b(h, j)/\partial j \qquad \forall h^1 > h, \tag{43}$$

and applying **(iii)**. Next, since (22) and Assumption 5.1 implies that $\hat{z}'(j) = \tilde{z}'(h)\hat{h}'(j) > 0$, Assumption 4.1 implies that the strict inequality in (42) holds only if

$$\hat{z}^1(j_b)/\hat{z}(j_b) > \hat{z}^1(j_a)/\hat{z}(j_a) \quad \Leftrightarrow \quad \log \left[\tilde{z}^1(h_b)/\tilde{z}^1(h_a) \right] > \log \left[\tilde{z}(h_b)/\tilde{z}(h_a) \right]$$

where we have written $h_x \equiv \hat{h}(j_x)$ for $x \in \{a, b\}$. Plugging in for $\tilde{z}(\cdot)$ using (22) we obtain

$$\Leftrightarrow \quad \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' > \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

and since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(iii)** implies that $\hat{j}^1(h) < \hat{j}(h)$ for all $h \in (h_a, h_b)$. But (2), the log-supermodularity of b, implies

$$\partial \log b(h, j^1)/\partial h < \partial \log b(h, j)/\partial h, \qquad \forall j^1 < j. \tag{44}$$

a contradiction. Case **2** is symmetric. □

Lemma 4 implies, in particular, that any within-sector equilibria must have identical $\hat{h}(j)$, since $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$ in all equilibria. Moreover, the lemma also implies that $\hat{h}(j)$ is determined independently of $\hat{z}$, which is uniquely determined by $\hat{h}(j)$ given (23). Hence, the within-sector equilibrium is unique, and furthermore, the solution to q_z in (30b) is also unique given $q_h(0)$.

Existence of the between sector equilibrium is straightforward, since the LHS of (32) increases smoothly from 0 to ∞ as $q_h(0)$ varies from 0 to 1, while the RHS is always positive and strictly bounded regardless of the value of $q_h(0)$. To show uniqueness then, it suffices to show that the RHS cannot cross LHS more than once. We will consider the log derivatives of the RHS of (32) term by term.

Let Δ_x denote the log-derivative of x w.r.t. $q_h(0)$. Since Assumption 5.2 implies that

$$\Delta_{B_j(j)} = \int_0^j \frac{\partial^2 \log b(\hat{h}(j'), j')}{\partial h \partial j'} \cdot \frac{d\hat{h}(j')}{dj'} \cdot dj' < \varepsilon \tag{45}$$

for all $\varepsilon > 0$, we obtain from (20) that

$$\Delta_{\pi_{ih}} = (1 - \alpha)(\sigma - 1) \cdot \Delta_{B_j(j)} \approx 0$$

so $\Delta_{\Pi_{ih}} \approx 0$. Likewise, Assumption 5.2 also implies that

$$\Delta_{B_h(h)} = \int_0^h \frac{\partial^2 \log b(h', \hat{j}(h'))}{\partial h' \partial j} \cdot \frac{d\hat{j}(h')}{dh'} \cdot dh' < \varepsilon \tag{46}$$

for all $\varepsilon > 0$. This implies that $\hat{h}(j)$ is not affected by the choice of $q_h(0)$, and it is independent of the determination of $\hat{z}$ by Lemma 4. Intuitively, Assumption 5.2 makes the model behave as if there were no log-supermodularity. Then since we assume a constant returns technology, all worker allocations approach constant multiples of H_0 and does not depend on its particular value. Since $\Delta_{\Pi_{ih}} \approx 0$, $\Delta_{\Pi_{K_i}}$ only depends on $\Delta_{\hat{z}}$ since from (14) and (25),

$$\Delta_{\pi_{iz}} = (1 - \alpha)(\omega - 1)\Delta_{\hat{z}} \quad \Rightarrow \quad \Delta_{\Pi_{K_i}} \Pi_{K_i} = \pi_{iz} \cdot (1 - \alpha)(\omega - 1)\Delta_{\hat{z}},$$

Similarly, $\Delta_{\Pi_{L_i}}$ only depends on $\Delta_{\hat{z}}$ as well, since from (22) and (46) we obtain

$$\Delta_{\tilde{z}(h)} = \Delta_{\hat{z}} + \Delta_{B_h(h)} \approx \Delta_{\hat{z}}. \tag{47}$$

so using Leibniz' rule,

$$\Delta_Z \cdot Z = -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \tag{48}$$

$$\Delta_{L_z} \cdot L_z = -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h) \cdot f(\tilde{z}(h)|h)\right] g(h)dh,$$

$$\Rightarrow \quad \Delta_{\bar{z}} = \Delta_Z - \Delta_{L_z} = \Delta_{\hat{z}} \cdot \underbrace{\int \left\{\tilde{z}(h)\left[1/L_z - \tilde{z}(h)/Z\right] \cdot f(\tilde{z}(h)|h)\right\} g(h)dh}_{\equiv \Lambda \in (0,1)}$$

where the inequality follows from selection and Assumption 4.2, so using this and (46), from (26) we obtain

$$\Delta_{\Pi_{L_i}} \Pi_{L_i} = (\hat{z}/\bar{z})\pi_{iz} \cdot \left[\alpha + \omega(1-\alpha) - \Lambda\right] \Delta_{\hat{z}}.$$

Now rearranging (23b), plugging in (48), and using (23a) at $j = 0$ we obtain

$$\left\{\alpha + \omega(1-\alpha) + \hat{z}f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh\right\} \Delta_{\hat{z}}$$
$$= \Delta_{q_z} - 1 \equiv \Gamma(X),$$

since $H_s(0) = q_h(0)H(0)$, $\Delta_{\hat{h}'(0)} = 0$ as it does not vary with $q_h(0)$, and $\Gamma(X)$ is defined from (30b):

$$\Gamma(X) = q_h(0)(X-1)/\left[q_h(0) + (1-q_h(0))X\right],$$
$$\text{where} \quad X \equiv \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1-\eta_s)}{(1-\eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}.$$

So it follows that the log-slope of the RHS in (32) is

$$-\Bigg\{(1-\epsilon)(1-\alpha)[\alpha + \omega(1-\alpha)] \cdot \left[\frac{\pi_{sz}}{\Pi_{K_s}} - \frac{\pi_{mz}}{\Pi_{K_m}}\right]$$
$$+ (\hat{z}/\bar{z})[\alpha + \epsilon(1-\alpha)][\alpha + \omega(1-\alpha) - \Lambda] \cdot \left[\frac{\pi_{sz}}{\Pi_{L_s}} - \frac{\pi_{mz}}{\Pi_{L_m}}\right]\Bigg\}$$
$$\times \frac{\Gamma(X)}{\alpha + \omega(1-\alpha) + \hat{z}f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh}.$$

The log-slope of the LHS in (32) is $1/\left[1 - q_h(0)\right]$, which increases from 1 to ∞ as $q_h(0)$ increases from 0 to 1, and is larger than $\Gamma(X)$ for all $X > 0$. Hence it suffices to show that the absolute value of all terms multiplying $\Gamma(X)$ are less than 1, which is true in particular due to Assumption 4.3.

Intuitively, what the planner cares about is the marginal products of Z and H in total. So when the distribution of z has a fat tail, the response of $\hat{z}$ to the choice of $q_h(0)$ is minimal as it changes Z smoothly along its entire support.

A.2 Proof of Proposition 1

Part 1. By Lemma 4, we know that no crossing can occur on $(0, \underline{j})$ or $(\bar{j}, J)$, since $\hat{h}$ and $\hat{h}^1$ already coincide at the boundaries 0 and J. Similarly, we also know from Theorem 1 that it can never be the case that there is no crossing ($\hat{h}^1(j) > \hat{h}(j)$ or $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in \mathcal{J} \setminus \{0, J\}$). Hence, there must be a single crossing in $\mathcal{J}^1$ since Lemma 4 also rules out multiple crossings.

At this point, the only possibility for j^* not to exist is if instead, there exists a single crossing j^{**} such that **(i)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (0, j^{**})$ and **(ii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j^{**}, J)$. If so, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that $\hat{h}^{1\prime}(0) < \hat{h}'(0)$, $\hat{h}^{1\prime}(j^{**}) > \hat{h}'(j^{**})$ and $\hat{h}^{1\prime}(J) < \hat{h}'(J)$. This implies

$$\hat{h}^{1\prime}(j^{**})/\hat{h}^{1\prime}(0) \geq \hat{h}'(j^{**})/\hat{h}'(0), \qquad \hat{h}^{1\prime}(J)/\hat{h}^{1\prime}(j^{**}) \leq \hat{h}'(J)/\hat{h}'(j^{**}). \tag{49}$$

Let us focus on the first inequality. Using (19) and (23a) we obtain

$$0 > [\alpha + \sigma(1-\alpha)] \cdot \left[\int_0^{j^{**}} \frac{\partial \log b(\hat{h}^1(j), j)}{\partial j} dj - \int_0^{j^{**}} \frac{\partial \log b(\hat{h}(j), j)}{\partial j} dj \right] \tag{50}$$

$$\geq (1-\sigma)m + \log \left[F(\hat{z}^1(j^{**})|\hat{h}(j^{**}))/F(\hat{z}(j^{**})|\hat{h}(j^{**})) \right] - \log \left[F(\hat{z}^1(0)|\hat{h}(0))/F(\hat{z}(0)|\hat{h}(0)) \right]. \tag{51}$$

where the first inequality follows from (43), and applying **(i)**. Since $m > 0$, if $\sigma \in (0, 1)$, Assumptions 4.1 and 5.1 imply that the strict inequality in (50) holds only if

$$\int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' < \int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

where we have written $h^{**} \equiv \hat{h}(j^{**})$. And since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(i)** implies that $\hat{j}^1(h) > \hat{j}(h)$ for all $h \in (0, h^{**})$. But this violates (44), the log-supermodularity of b. The case for the second inequality in (49) is symmetric.

Part 2. Let Δ_x denote the log-derivative of x w.r.t. m. From (23d),

$$\begin{aligned} \Delta_{\Pi_{ih}} \cdot \Pi_{ih} &= (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj + \int \left\{ \pi_{ih}(j) \cdot (1-\alpha)(\sigma - 1) \cdot \Delta_{B_j(j)} \right\} dj \\ &\approx (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj \end{aligned} \tag{52}$$

where the approximation follows from Assumption 5.2 and (45). Hence $\Delta_{\Pi_{ih}} < 0$ if $\sigma < 1$. Rearranging (23b) and using (23a) at $j = 0$ we obtain

$$0 > \frac{\sigma - \omega}{1 - \sigma} \cdot \Delta_{\Pi_{ih}} - \Delta_{\hat{h}'(0)} = \left[\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) \right] \Delta_{\hat{z}} - \Delta_Z \tag{53}$$

where the inequality holds if $\omega < \sigma < 1$, and since we know from part 1 that $\Delta_{\hat{h}'(0)} \geq 0$. Now suppose $\Delta_{\hat{z}} \geq 0$. Then for (53) to hold it must be the case that $\Delta_Z > 0$, but from (48), $\Delta_Z \leq 0$ if $\Delta_{\hat{z}} \geq 0$, a contradiction. Hence, $\hat{z}^1 < \hat{z}$, and $\tilde{z}^1(h) < \tilde{z}(h)$ for all h by (47).

B Additional Tables and Figures

Ranked by mean wage (except management)	(1) -	(2) m_j	BM m_j	Data
Low Skill Services	**1.973**	**-2.726**	**-0.731**	
Middle Skill				
Administrative Support	1.973	1.252	2.930	
Machine Operators	1.973	10.018	9.122	
Transportation	1.973	3.326	4.348	
Sales	1.973	-1.895	0.012	
Technicians	1.973	-2.484	-1.144	
Mechanics & Construction	1.973	1.742	2.315	
Miners & Precision Workers	1.973	6.367	6.328	
High Skill				
Professionals	1.973	-3.973	-2.248	
Management Support	1.973	-1.973	-0.489	
Management	**1.973**	**-1.438**	**-0.017**	
Aggregate TFP growth	**1.030**	**1.030**	**1.030**	**1.030**
a_m (Manu TFP growth)	0.252	0.252	2.943	2.229
a_s (Serv TFP growth)	-1.205	2.021	0.308	0.743

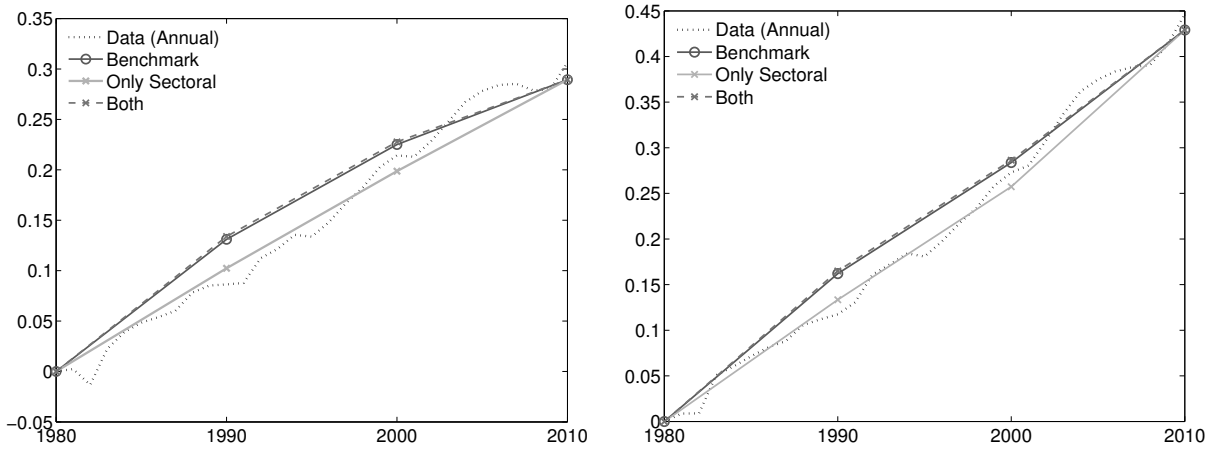
Table 5: Recalibrated TFP Growth Rates for Counterfactuals

Column (1) stands for the counterfactual in which we set $m_j = m$ and calibrate (a_m, a_s) to match sectoral TFP's, and (2) for when we let $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ all vary simultaneously. "BM" stands for the benchmark calibration. For all scenarios, aggregate GDP growth (and consequently TFP growth) is matched exactly, shown in the first row of the bottom panel. For the "BM" and "Data" columns, the a_m and a_s rows show the empirical growth rates of the manufacturing and services sectors' TFP's, respectively.

		ΔL_j					
		TFP	- 9.584 ***				
		R^2	0.939				
RTI (DOT)	0.429						
	(0.268)						
Routine manual		0.797**		0.618			
		(0.206)		(0.527)			
Manual interpersonal			-0.767**	-0.192			
			(0.192)	(0.549)			
College share 1980					-11.142*		-7.994**
					(3.599)		(2.269)
ΔCollege share 1980-2010						-33.673*	-20.295*
						(17.410)	(13.547)
Constant	1.061	0.377	3.281**	1.065	4.035*	4.818*	5.204*
	(0.941)	(0.738)	(0.970)	(2.339)	(1.401)	(1.674)	(1.759)
R^2	0.184	0.635	0.588	0.640	0.439	0.372	0.539

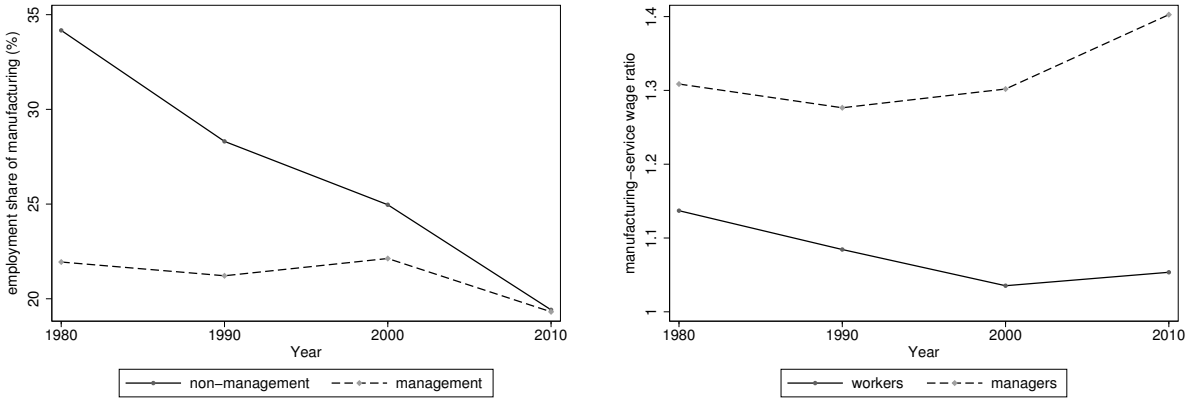
Standard errors in parentheses, $^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$

Table 6: Task-Specific TFP Growth, Employment, and Empirical Measures
The first panel shows the results from regressing employment share changes on the calibrated task-specific TFP growth rates, m_j. The second panel shows the results from regression the growth rates on various empirical measures.



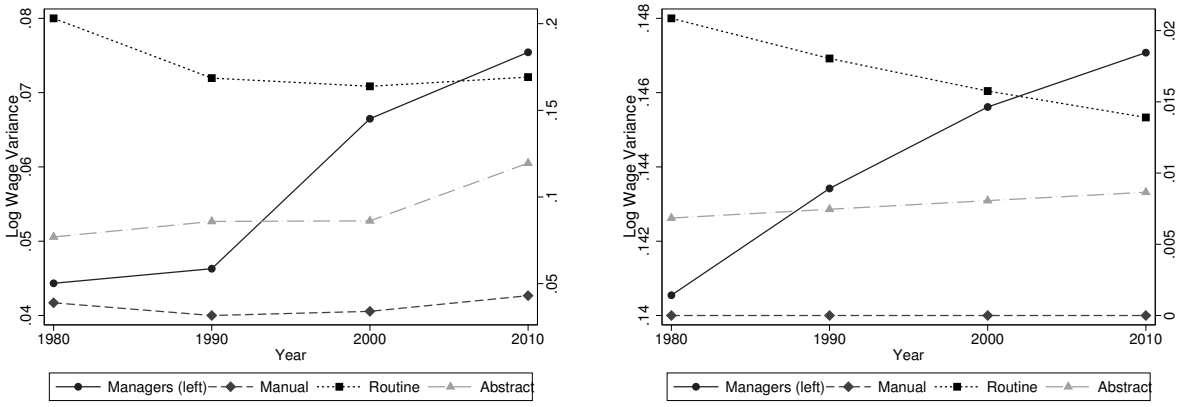
(a) Log TFP Growth (b) Log GDP per Worker Growth

Fig. 22: Aggregate Output and TFP Growth



(a) Manufacturing Employment Share (b) Manufacturing-Services Average Wage Ratio

Fig. 23: Manufacturing vs. Services by Occuaption



(a) Data (b) Model

Fig. 24: Within Task Wage Inequality

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