Research Statement

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My primary fields of interest are industrial organization and economics of innovation along with more broad interests in applied microeconomic theory. Much of my research focuses on how firms strategically interact with one another given an underlying economic environment and describing the impacts of such strategic interaction between firms on market outcomes.

In particular, two current research projects fall into the category of economics of innovation. In my job market paper, *Optimal Use of Patents and Trade Secrets for Complex Innovations*, I examine the optimal protection mode of an innovator who has a "complex" innovation. In this case, the innovator may protect the innovation through a mix of patents and trade secrets. Even though a substantial amount of evidence shows that such combined use of patents and trade secrets are prevalent, the existing intellectual property right literature has given scant attention to this type of protection. My key contribution is to show under what conditions such combined use is optimal and then draw welfare implications. This is an important finding because a carefully designed patents and trade secrets policy without considering a potential for a patent-secret mix may overpay the innovator.

Recently, non-practicing entities (NPEs), which acquire but do not produce patents from various sources, received much criticism for making deceptive patent infringement claims. However, in my paper, *A Role of NPEs when Patent Ownership is Fragmented*, I suggest a mechanism through which non-practicing entities can help individual innovators appropriate from the innovators' intellectual properties in a non-deceptive way.

My third research paper uncovers an issue in asymmetric information models. Asymmetric information is now well known to cause perfectly competitive market outcomes to no longer be efficient; i.e., market failure. Credible certification, which substantiates unobservable characteristics of products, is generally considered a standard solution to such market failure. In the paper, *Welfare Effects of Certification under Latent Adverse Selection*, my coauthors and I show that such conventional wisdom may be wrong. In particular, we show that the certification may decrease overall welfare if the certification technology is too costly or too inaccurate.

In aggregate, my contributions to the economics of innovation, industrial organization and information economics literatures include offering new insights in designing optimal patent and certification policies and incorporating the analysis and welfare implications of firms' strategic

behavior, which is currently under-explored in the literature. While I would like to pursue this direction of research further in the future, I am also open to any other joint research projects with both theoretical and empirical researchers with similar interests.

Optimal Use of Patents and Trade Secrets for Complex Innovations

In this paper, the optimal protection of "complex" innovations that involve multiple complementary components is considered. The innovator can protect each component through either patent or trade secret protection, so that the complete innovation may be protected through a mix of patents and trade secrets. The innovator faces potential entrants who might acquire the components either by costly imitation or through licensing. It is shown that: (i) secrecy is optimal when the patent length is relatively short; otherwise, a combined use of patents and trade secrets is optimal. (ii) The innovator is (weakly) over-rewarded compared to an inventor with a simple innovation (with identical profitability and imitation cost). (iii) A "Strict Utility Requirement" policy (that precludes the use of a patents-secrets mix) enhances allocative efficiency ex-post but stifles R&D incentives ex-ante.

Welfare Effects of Certification under Latent Adverse Selection (with Anthony Creane and Thomas D. Jeitschko)

Asymmetric information is a classic example of market failure that undermines perfectly competitive market outcomes: the "lemons" market. Credible certification, that substantiates unobservable characteristics of products that consumers value, is often considered as a potential solution to such market failure. This paper examines welfare effects of certification in markets in which there is asymmetric information, but without an adverse selection problem. We analyze the market equilibrium when the certification technology becomes available and contrast this with the equilibrium without certification. We find that despite an improvement in allocative efficiency, overall welfare may decrease due to the possibility of certification when such certification is either too costly or too inaccurate.

A Role of NPEs when Patent Ownership is Fragmented

This paper suggests a mechanism by which a non-practicing entity (NPE) enhances returns to individual inventors. The basic premises of the paper are that: (i) a product, manufactured by a practicing entity (PE) must adopt all complementary innovations patented and owned by individual inventors; and (ii) an individual innovator accurately anticipates the equilibrium

licensing fee but does not know which other innovations are required to produce the products. When litigation costs are small, the individual inventors can collect license fees without any help from the NPE. Whereas when litigation costs are large, I consider a setting in which the individual inventors sell their patents sequentially because litigation is not feasible. I show that (i) if two PEs compete to buy the patents, the two PEs buy the patents at negligible prices; and (ii) if a PE and the NPE compete to buy the patents, the NPE helps some individual inventors sell their patents at the price equal to the licensing fee.

Optimal Use of Patents and Trade Secrets for Complex Innovations

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Abstract

In this paper, the optimal protection of "complex" innovations that involve multiple complementary components is considered. The innovator can protect each component through either patent or trade secret protection, so that the complete innovation may be protected through a mix of patents and trade secrets. The innovator faces potential entrants who might acquire the components either by costly imitation or through licensing. It is shown that: (i) secrecy is optimal when the patent length is relatively short; otherwise, a combined use of patents and trade secrets is optimal. (ii) The innovator is (weakly) over-rewarded compared to an inventor with a simple innovation (with identical profitability and imitation cost). (iii) A "Strict Utility Requirement" policy (that precludes the use of a patents-secrets mix) enhances allocative efficiency ex-post but stifles R&D incentives ex-ante.

Keywords: patents, trade secrets, intellectual property rights, patent-secret mix.

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1 Introduction

Firms protect their innovations not only by patents but also by secrecy.[1] These two protection modes help the firms appropriate returns from their innovations through

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[1]More generally, Hall et al. (2014) point out that there are two different types of intellectual property rights (IPRs): formal IPRs including patents, trademarks, designs and copyright, and informal IPRs including secrecy, confidentiality agreements and lead time advantage. In this paper, I focus only on patents and secrecy.

different mechanisms. Patents grant patents holders a legal right to exclude others from using protected innovations for a fixed period of time but require disclosure of all information. In contrast, trade secrets do not have a fixed time period and do not require any disclosure of information but they are vulnerable to independent reverse engineering.[2]

In the intellectual property rights literature, initially, patents received far more attention than trade secrets. Subsequently, several authors have highlighted trade secrets as an important appropriability method (e.g. Levin et al. (1987) and Cohen et al. (2000)), and analyzed the secrecy mechanism as a viable alternative to patents (e.g. Friedman et al. (1991)). However, most of this work does not consider a potential for a combined use of patents and trade secrets, the focus of this paper.[3]

To fill this gap in the literature, this paper considers a situation in which an innovator has a "complex" innovation that involves multiple complementary components. The innovator can protect each component either by a patent or by secrecy, i.e., the innovation may be protected through a mixed use of patents and trade secrets. In such a setting we ask the following questions. Under which condition, does the innovator opt for the mix use? Would the innovator with the complex innovation earn more than an innovator with a simple innovation when the mix use of patents and trade secrets is feasible? Is restricting her choice set to the binary one, either patenting or relying on secrecy, socially desirable?

Substantial anecdotal evidence suggests that patents and trade secrets are combined in many settings. Premarin, which has been sold by Wyeth (merged with Pfizer in 2009), has been the most popular hormone-therapy drug for menopausal women since 1942. Wyeth filed two patents to protect the innovation in the early 1940s. However, the company effectively extended the duration for its exclusive position in the industry beyond the length of the patents. The reason for this success was that no rivals could uncover closely secured trade secrets, more specifically the so-called Brandon Process: a secret chemical process extracting conjugated estrogen from pregnant mare urine (Jorda (2007) and Orly (2013)).

Jorda (2007) also attributes a success story of GE in the business of industrial (or synthetic) diamonds to the use of patents in conjunction with trade secrets. GE patented a part of its technology for making diamonds, and therefore the disclosed

[2]Coca-Cola's secret formula, which has been locked away in a vault for more than 90 years, is a good example of well guarded trade secrets.

[3]Hall et al. (2014) offer a comprehensive survey on firms' choice between formal and informal IPRs including both theoretical and empirical work.

information was publicly available to its rival companies after the expiration of patents. However, as GE also closely kept the remaining part secret, it could maintain a dominant position in that market. Pan (2010) reports another example; an Illinois based energy company, Coskata, applied for patents to protect technology on a bio-reactor while keeping the identity of the micro-organism fed into the bio-reactor secret. In addition, there are many other examples showing a similar strategy pattern in the food industry (see the introduction in Belleflamme and Bloch (2013)) and in the organic chemical industry in 19th century (Arora (1997)). In fact, the web pages of many patent attorneys and law firms advise their customers to use this type of strategy. [4]

This paper first examines the best protection mode for an innovator facing potential imitators after the discovery of a complex innovation. To this end, it is important to figure out the inventor's ability to appropriate returns from innovation under each protection modes: the all-by-patent mode, the all-by-secrecy mode and the optimal patent-secret mix. Instead of imposing ad hoc assumptions in that regard, my framework extends Henry and Ponce (2011)'s work in the context of complex innovations. In this model, entry times of the imitators under each protection modes are endogenously determined in equilibrium. More specifically, the length of patents (T) and the rates of entry by the imitators (λ), which are driven by imitation costs (κ), are key variables governing the entry times of the imitators under patent and trade secret protection respectively.

The first key finding of the paper is that the all-by-secrecy is optimal when the patent length is relatively short, otherwise, the optimal combined use of patents and trade secrets is optimal. To understand this result, it should be first noted that the all-by-patent is dominated by any patent-secret mix because the inventor can effectively extend the duration of monopoly beyond T by mixing the two protection mechanisms; due to the perfect complementarity of the components, patented components make entry by the imitators infeasible during the patent period, and the remaining components kept by secrecy delay entry after T. Moreover, the optimal patent-secret mix requires patenting just the component with the minimal imitation cost because the prolonged monopoly duration is determined by the aggregate imitation costs of all components kept under secrecy. Therefore, the optimal protection mode has a binary feature: depending on T, choosing either the all-by-secrecy or the

[4]For example, visit http://www.peloquinlaw.com/combining_protection.html. (retrieved on 8/4/2016)

optimal patent-secret mix; the optimal patent-secret mix is more advantageous than the all-by-secrecy in the sense that the mix takes advantage of the patent system, but it is also less advantageous in the sense that it requires the inventor to disclose a part of the innovation, thereby increasing the rates of entry. (Perfect Complementarity and conclude when T is)

Next, this paper compares the optimal decision and the payoff to the innovator with the complex innovation (the "complex innovator" hereafter) with that to another inventor with a "simple" innovation comprised of a single component (the "simple innovator" hereafter). *Ceteris paribus* (i.e., with identical profitability and imitation costs), the complex innovator is more inclined to patent because she can take advantage of the patent system with only a partial disclosure of the whole innovation. Moreover, the complex innovator is (weakly) over-rewarded compared to the simple innovator. When T is relatively short, so that both inventors rely only on secrecy, they earn the same expected payoff regardless of whether the innovation can be fragmented or not, but if T falls into the region in which the complex innovator opts for the optimal patent-secret mix, the complex innovator earns strictly more than the simple innovator does.

In addition to the positive analysis of ex-post innovation problem, this paper further derives ex-ante and ex-post policy implications of imposing a "Strict Utility Requirement" (SUR). For ex-ante welfare analysis, we also take into account ex-ante investment incentives of the innovator in R&D stage. Under the SUR, the patent office turns down all patent applications that discloses only a part of innovation in compliance, and thus the inventor cannot use a patent-secret mix. The complex inventor's payoff schedule converts into one that is exactly same as that of the simple inventor's. Whereas the SUR always enhances ex-post welfare, the ex-ante welfare implication is ambiguous. This is because the availability of the combined use of patents and trade secrets might boost the inventor's ex-ante investment incentive.

Related Literature: There is a large body of literature viewing patents and trade secrets as mutually exclusive alternatives in a variety of contexts. Anton and Yao (2004) study the innovator's choice between patenting and relaying on secrecy, focusing on signaling (or strategic disclosure) aspects. Kultti et al. (2007) and Kwon (2012) consider a similar problem in a patent race model in which two or more innovators compete to develop the same innovation simultaneously, and they make some normative arguments regarding socially desirable patent policies. Denicolo and Franzoni (2004) focus on whether granting prior user right to the first inven-

tor (i.e., the subsequent innovator is able to patent and exclude the first inventor when the first inventor resorted to trade secrets) increases social welfare or not. Erkal (2005) and Bhattacharya and Guriev (2006) consider a cumulative innovation framework. More specifically, Erkal (2005) considers two consecutive R&D races between two firms in which the winner of the first race has an option to protect it either by a patent or by secrecy, and compares several patent policy regimes. Bhattacharya and Guriev (2006) build a model in which the upstream inventor has two potential licensees in the downstream market. They show which of the two protection modes is more efficient from the inventor's and the social planner's point of view. All of these papers offer great insights in understanding distinctions between patents and trade secrets as substitutes for each other, but neglect the possibility of the combined use of those two mechanisms.

Although there is a nascent strand of literature exploring the patent-secrecy mix mechanisms, research papers on this topic are still sparse. Lee (2015) envisions secrecy as a mechanism for entry deterrence, but patents as a more nuanced instrument of transferring entrants' surplus to the inventor or deterring entry. He shows that the mix use of protection can arise in equilibrium when the two mechanisms perform different functions. Unlike his model, I show similar results that the two mechanisms could be complements with the opposite assumption that knowledge covered by trade secrets can be transferred or licensed to the potential imitators while patents only deter entry. The two papers most closely related to my own approach are Cugno and Ottoz (2006) and Belleflamme and Bloch (2013). Both models start with a situation in which the inventor has come up with a complex innovation, and show that the mix of patents and trade secrets may provide optimal protection. They, however, consider an innovation which can be fragmented continuously. Cugno and Ottoz (2006) consider a framework in which it is a prerequisite for an imitator's entry, given a patent-secrecy portfolio, to not only uncover a part protected by secrecy but also find a way to invent around the other part protected by patents. Belleflamme and Bloch (2013) study a broader set of competitive configurations (e.g. an imitator may be able to get some surplus even with access to only a small portion of the innovation) as well as protection modes including dynamic patenting. Albeit my paper considers complex innovations with finite number of perfectly complementary components, it is based on a micro-foundation which determines the strength of each protection mode endogenously, and thus provides a detailed mechanism by which the evolution of the imitation game and inventor's profit is fully described.

Moreover, my framework also explicitly considers the ex-ante investment incentive of the inventor, which is absent in their works.

The remainder of the paper is organized as follows. In section 2, the base model and assumptions are introduced. Section 3 provides the main analysis including the calculation of the inventor's expected payoffs under each protection mode, to derive the inventor's optimal decision and to compare her payoff to the one of the simple innovator. In Section 4, policy implications regarding SUR are drawn. Section 5 contains two possible extensions and checks the robustness of the main results, and the final section presents the conclusion.

2 The Model

This section introduces the basic setup of the model, which builds upon the framework in Henry and Ponce (2011). Consider an infinite horizon model in which Δ represents the length of each time period. More specifically, we consider the continuous time framework which could be understood as the limit of a discrete time framework as Δ approaches to 0.

There are three players: an innovator, denoted by i, and two potential imitators, indexed by $b \in \{j,k\}$. All parties are assumed to be risk neutral agents maximizing their (expected) discounted payoffs, and have the same per-period discount factor, $\delta := e^{-r\Delta}$, where $r > 0$ represents a discount rate.

The innovator already has a "complex" innovation at time period $t = 0$ allowing her to introduce a new product to the market. Here, the complex innovation is comprised of two perfectly complementary components of knowledge: x and y, which differ by the extent of difficulties in reverse engineering.[5] In the beginning, the innovator can protect each component by a patent or by secrecy in the beginning; there are four possible choices: protecting all components by a patent, all by trade secrets, x by secrecy and y by a patent, and y by secrecy and x by a patent. Note that the choice set of innovation protection incorporates some implicit assumptions about patent examination process. I assume that the novelty requirement is strict in the sense that the patent office treats an invention already on sale as a prior art and thus rejects a patent application for that.[6] Consequently, the inventor cannot

[5]We relax this assumption and consider N component model in section 5.

[6]After the America Invents Act was enacted in 2011 (AIA), some have argued that invention claims on a "secret" sale is no longer considered as a prior art, based upon a new interpretation

rely on a dynamic protection method combining patents and trade secrets, protecting x by a patent and y by secrecy initially, for example, and then at some point later trying to get another patent for y. However, I assume the weak utility requirement, which allows the inventor to get a patent even when hiding part of the invention and disclosing only the other part.

An imitator can "enter" the market and start to earn profits in the market only with access to both knowledge components. There are two different ways for the imitators to "adopt" (or learn) the knowledge components; they can either buy a component from the innovator through a contract or imitate the component through a reverse engineering process. The imitation costs of x and y are given by κ_x and κ_y respectively. Without loss of generality, I assume that $\kappa_y \geq \kappa_x > 0$. Put differently, the component y is more difficult or costly to imitate.

Regardless of whether a firm is the innovator or an imitator, the per-period profit is the same for all selling in the market and is determined only by the number of firms in the market; π_n represents a single firm's per-period profit with n firms in the market. Then, the present value of stream of all profit flows per firm with n firms in the market is equal to $\Pi_n \equiv \pi_n / r$. All parameter values, including profits given the number of firms (π_n), imitation costs (κ_x, κ_y and κ) and the discount rate (r), are common knowledge. Throughout the paper, I maintain the following assumption stating that joint industry profits are dissipated by more entrants.

Assumption 1: $\Pi_1 > 2\Pi_2 > 3\Pi_3$.

Furthermore, to make the analysis non-trivial, I assume that the sum of imitation costs is not so high that both imitators have an incentive to enter the market.

Assumption 2: $\Pi_3 > \kappa := \kappa_x + \kappa_y$, *where* $\kappa_y \geq \kappa_x > 0$.

The game unfolds as follows under each protection mode chosen by the innovator. First, when the inventor protects both components by the patent, it confers a temporal monopoly to the inventor for a fixed term T in exchange for disclosing information about the innovation. As a result, the imitators become aware of how the innovation works without incurring any costs, but have to wait to enter the market until the patent expires. In summary, the imitators choose their entry times after T without incurring any costs in all-by-patent scenario.

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of an amended clause about the prior art. There have been considerable number of debates supporting or opposing the view. Here, I accept the old meaning. For more details, see for example, http://patentlyo.com/patent/2012/10/did-the-aia-eliminate-secret-prior-art.html.

In the second scenario of all-by-secrecy protection, as soon as an imitator uncovers how to "adopt" the two components through licensing or imitating, he is able to enter the market. Henry and Ponce (2011) suggest two possible types of contracts: transferable and nontransferable. Under a transferable contract, unlike a non transferable one, an imitator can resell the knowledge to the remaining imitator. Since Henry and Ponce (2011) show that offering transferable contracts is optimal and the only way for the inventor to earn a stream of profit similar to the one obtained through patent protection, I will focus only on transferable contracts.[7] Transferable contracts specify the price levels offered by the seller (the inventor) to the buyers (the imitators) before the first purchase takes place and by the sellers (the inventor and the first buyer) to the remaining buyer after that moment for each knowledge component.

A more detailed description of the game under all-by-secrecy mode is given below. In any time period t from the beginning until the first knowledge adoption occurs for one of the two components, the game unfolds as follows: (i) The innovator i announces prices for x and y, p_{ib}^{xt} and p_{ib}^{yt}, to buyer $b \in \{j,k\}$. (ii) Given the offered prices, the imitators simultaneously choose to learn each component by accepting the contract (c_{ib}^t), to learn each component by imitating (m_b^t) or to wait (w_b^t). Thus, the action set of an imitator b at time t is given by $A_b^t = \{c_{ib}^t, m_b^t, w_b^t\} \times \{c_{ib}^t, m_b^t, w_b^t\}$ in which the first and second elements represent his action for component x and y respectively. Suppose now that the first knowledge adoption arises by one of the two imitators. Without loss of generality, say imitator j learned the component x. In that case, there would then be two sellers, i and j, for component x while there is still only one seller, i, for component y. From this period on, until somebody acquires another component, the game proceeds as follows in each period: (i) Two sellers of x offer prices, p_{ik}^{xt} and p_{jk}^{xt}, to k, and the sole seller i for component y announces prices, p_{ij}^{yt} and p_{ik}^{yt}. (ii) Given the price offers, the imitator j chooses $a_j^t \in A_j^t = \{c_{ij}^t, m_j^t, w_j^t\}$ for the remaining component y, while, at the same time, the imitator k chooses $a_k^t \in A_k^t = \{c_{ik}^t, c_{jk}^t, m_k^t, w_k^t\} \times \{c_{ik}^t, m_k^t, w_k^t\}$. Note that now k has three different options for getting component x: by buying from i (c_{ik}^t), by buying from j (c_{jk}^t) and by imitating (m_k^t).

[7] It is not difficult to find a technology transfer agreement in which a licensor grant a licensee a right to sublicense or resell the licensed technology (a transferable contract). Such technology transfer agreements can be found in the archive of the U.S. Securities and Exchange Commission webpage. (For example, see https://www.sec.gov/Archives/edgar/data/1223112/000119312510190208/dex1074.htm. Retrieved: Oct/12/2016.)

The game goes on in a similar manner until everybody possesses both knowledge components and enters the market.

At time period t, every agent observes and recalls the history up to $t - \Delta$. The equilibrium concept, which will be used for the analysis, is symmetric Markov perfect equilibria (MPE) in which each agent's strategy is optimal given the state variable and other agents' strategy. The state variable is denoted by $\theta^t := \left(\theta^{xt}, \theta^{yt}\right)$ at time t in which θ^{xt} (θ^{yt}) indicates which imitator has obtained the component x (y) in addition to the inventor, i.e., $\theta^{xt}, \theta^{yt} \in \{\{i\}, \{i,j\}, \{i,k\}, \{i,j,k\}\}$.

Lastly, the last case to be considered is that of the patent-secret mix. For expositional simplicity, assume that x is protected by a patent and y by secrecy. Then, the imitators can use x freely after T, while they still need to adopt y for entry. Then, the imitators will adopt y in a similar way as was described in the all-by-secrecy mode. But now the state variable θ^t should also specify whether the current time period t is before or after the patent expires. If $t \geq T$, the imitators start to sell as soon as they adopt y. If $t < T$, however, the remaining duration until patent expiration is different at each t, which results in different payoff structures given the same entry time after T. Thus, under this scenario I derive a symmetric MPE of the game in which the state variable is $\theta^t := (\tau, \theta^{yt})$. Here, the first element indicates whether it is before or after the expiration of the patent. If it is before T, then, furthermore, the first element specifies at which period the players are currently. (i.e., $\tau = t$ if $t < T$ and $\tau = T_+$ otherwise).

3 Optimal Use of Patents and Trade Secrets

This section explores how the innovator would optimally protect her complex innovation by using patents and trade secrets. To this end, I solve the innovator's profit maximizing problem depending on modes of protection: patenting both components, relying only on secrecy, and mixing patents and trade secrets. A comparison of profits is provided next, followed by an illustration of how the payoff of the complex innovator relates to that of the simple innovator.

3.1 Expected Payoffs in Each Protection Mode

3.1.1 Patenting Both Components

I study the case in which all components are protected by the patent first. Here, notice that we only consider ironclad patents which grant the inventor an exclusive right to use the innovation until T. In addition, the imitators do not need to pay to use the innovation (i.e., both components) once the patent expires. Consequently, at time T it is optimal for the imitators to enter the market immediately and simultaneously. Hence, the discounted expected payoff of the inventor under protecting all by patent mode is given by

$$V_i^P = (1 - e^{-rT})\Pi_1 + e^{-rT}\Pi_3 - f$$

in which f represents a patent application fee.

3.1.2 Relying only on Secrecy

How much can the innovator appropriate from her innovation when she relies only on trade secrets? Before investigating each class of subgames, for expositional simplicity, let us define a subgame, in which there is one seller for a component, as a monopoly game with respect to that component. Similarly, define a subgame, in which there are two sellers for a component, as a competitive game with respect to that component.[8] Thus, for example, if there are two sellers for one component and there is only the original innovator for the other component, then it is her considered a monopoly game with respect to the former and a competitive game with respect to the latter. Working backwards, I consider a subgame which is competitive with respect to one component and monopolistic with respect to the other.

Lemma 1 *(i) In the unique MPE of a competitive game with respect to both components or with respect to only one last component, the component(s) with two sellers is (are) sold to the remaining buyer(s) at a zero price(s) immediately; (ii) In an MPE of a game which is competitive with respect to one component and monopolistic with respect to the other, the component with two sellers is sold to the remaining buyer at a zero price, and MPE is unique up to equilibrium payoff regardless of the timing of its sale.*

Whenever there is competition in selling a knowledge component, it is sold to

[8]Recall that as only transferable contracts are considered, the first adopter becomes an additional seller for the corresponding component.

the remaining buyer at 0 price, and it can be assumed that the timing of the sale, in general, is immediate. The intuition behind result (i) is straightforward. For expositional ease, suppose $\theta = (\{i,j\},\{i,j\})$. Since price offers are stationary in MPE and entry is profitable for k, entry by him is immediate if he buys from one of the sellers or imitates both components. Anticipating that, i and j compete in prices to sell the two components, and thus equilibrium prices are driven to 0. Now, take a step back and consider a subgame in result (ii). For expositional ease, say j has just adopted x, i.e., $\theta = (\{i,j\},\{i\})$. Note that since x and y are perfect complements, the stream of market profits for each firm depends entirely on the timing of the first adoption of y after which the entry by both imitators follows promptly. Then, it is never optimal for k to buy x at some positive prices or to imitate x now taking price offers and imitators' adoption decision for y as given. This is because k can wait instead until the first adoption of y arises and buy it at 0 price. Therefore, k buys x from one of the sellers at 0 prices in equilibrium, which allows one to assume that a subgame with $\theta = (\{i,j\},\{i\})$ proceeds to a subgame with $\theta = (\{i,j,k\},\{i\})$ instantly. The remaining analysis for the subgame directly follows from the main results of Henry and Ponce (2011) in which they consider a single component innovation. Even if, for simplicity, the following lemma characterizes a unique MPE for only the case in which the first adoption has occurred for x, it is applicable in a similar way to the other case as well.[9]

Lemma 2 *(Henry and Ponce (2011)) In a unique symmetric MPE in a monopolistic game with respect to y only (i.e., $\theta = (\{i,j,k\},\{i\})$), (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for the remaining component y by setting $p^y_{ib} = \kappa_y$ for $b \in \{j,k\}$; (ii) entry times of each imitator follow the exponential distribution with hazard rate, $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$; (iii) the expected equilibrium payoffs of the inventor and of the imitators are $V_i^{y|x} = \mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3 + \kappa_y)$, where $\mu^{y|x} := r/(r+2\lambda^{y|x}) \in (0,1)$, and $V_b^{y|x} = \Pi_3 - \kappa_y$ respectively.*

Given the occurrence of the first adoption of x, the discounted expected payoff of the inventor consists of three parts; she will earn monopoly profits for a random duration until the first adoption for y arises ($\mu^{y|x}\Pi_1$), revenues from selling y when the first adoption for y takes place ($(1-\mu^{y|x})\kappa_y$), and triopoly profits thereafter

[9] For $\theta = (\{i\},\{i,j\})$ or $(\{i\},\{i,k\})$, instead the inventor sets $p^x_{ib} = \kappa_x$ for $b \in \{j,k\}$; the hazard rate is $\lambda^{x|y} = r(\Pi_3 - \kappa_x)/\kappa_x$; the expected payoffs of the inventor and the imitators are $V_i^{x|y} = \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3 + \kappa_x)$, where $\mu^{y|x} := r/(r+2\lambda^{y|x})$ and $V_b^{x|y} = \Pi_3 - \kappa_x$.

$((1-\mu^{y|x})\Pi_3)$ in which $y|x$ denotes a variable associated with a monopolistic game with respect to y given x was adopted by both imitators. A delay in the imitators' entry can be attributed to the equilibrium outcome of balancing the benefits and costs of immediate entry as next explained. From Lemma 1 and result (i) in Lemma 2, we know that the first and second adopter pay κ_y and 0 to learn component y respectively while they end up obtaining the same market profits because of almost simultaneous entry. Thus, on the one hand, an imitator has an incentive to wait to save the cost of knowledge acquisition. On the other hand, delaying entry entails costs as well: they need to sacrifice market profits which would have been earned by immediate entry. Hence, in symmetric equilibrium, the imitators mix waiting and buying y from the inventor in a way that such benefits and costs are offset exactly. A speed of entry is described by the hazard rate of entry, $\lambda^{y|x}$, and $\mu^{y|x}$ represents the natural duration of monopoly under secrecy protection.[10] Simple comparative statics shows that the higher imitation costs of y (κ_y), the later the entry by the imitators.

Now consider the game starting at $t=0$, in which the inventor is the only seller of both knowledge components, x and y. Unlike the single innovation component case, the inventor can induce several different sequences of adoption by manipulating price offers, which will results in different speeds of entry and timings of knowledge sale.

Lemma 3 *In the monopoly game with respect to both components, the inventor never charges $p_{ib}^{xt} > \kappa_x$ and $p_{ib}^{yt} > \kappa_y$ for $b \in \{j,k\}$.*

Lemma 3 states that it is never optimal for the inventor to charge such high prices for both components, letting the imitators incur imitation costs rather than buying from the inventor. Once one rules out the price structure in Lemma 3, there remain three candidates for the symmetric equilibrium price structures: (1) $p_{ib}^{xt} \leq \kappa_x$ and $p_{ib}^{yt} > \kappa_y$, (2) $p_{ib}^{xt} > \kappa_x$ and $p_{ib}^{yt} \leq \kappa_y$ and (3) $p_{ib}^{xt} \leq \kappa_x$ and $p_{ib}^{yt} \leq \kappa_y$ for $b \in \{j,k\}$. The price structure (1) induces a step-by-step adoption sequence (denoted by $x \to y$) in which the imitators adopt x first and then y next. (i.e., mixing cw and ww in the current stage). Similarly, (2) induces the imitators to adopt y first and then x next (denoted by $y \to x$). Lastly, (3) induces an all-or-nothing adoption sequence (denoted by xy) in which the imitators adopt x and y together (i.e., mixing cc and ww). To see why, suppose we have the price structure (1). The imitators are aware that if a

[10] In some other papers, e.g., Denicolo and Franzoni (2004) and Belleflamme and Bloch (2013), $\mu^{y|x}/r$ is instead defined as the discounting adjusted duration of monopoly under secrecy protection.

first adoption occurs for x, they promptly enter a subgame with $\theta = (\{i,j,k\},\{i\})$ in which $p^y_{ib} = \kappa_y$ and they end up getting $\Pi_3 - \kappa_y$. Therefore, the imitators are willing to adopt only x for now. [11]

It turns out that inducing the imitators to follow the sequence xy is optimal for the inventor. The proposition below summarizes the results.[12]

Proposition 1 *In a unique symmetric MPE when the inventor only relies on secrecy, (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for both components by setting $p^x_{ib} = \kappa_x$ and $p^y_{ib} = \kappa_y$ for $b \in \{j,k\}$; (ii) entry times of each imitator follow the exponential distribution with hazard rate, $\lambda^{xy} = r(\Pi_3 - \kappa)/\kappa$; (iii) the expected equilibrium payoffs of the inventor and of the imitators are $V_i = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa)$, where $\mu^{xy} := r/(r + 2\lambda^{xy}) \in (0,1)$, and $V_b = \Pi_3 - \kappa$ respectively.*

In the appendix, I show that the optimal payoffs of the inventor under sequence xy and $y \to x$ are given by, respectively,

$$V_i^{xy} = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa)$$

where μ^{xy} and λ^{xy} are as in result (ii), and

$$V_i^{y\to x} = \mu^y\Pi_1 + (1-\mu^y)\left\{\kappa_y + \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3 + \kappa_x)\right\}$$

where $\mu^y = r/(r+2\lambda^y)$, $\lambda^y = r(\Pi_3 - \kappa)/\kappa_y$, $\mu^{x|y} = r/(r+2\lambda^{x|y})$ and $\lambda^{x|y} = r(\Pi_3 - \kappa_x)/\kappa_x$. Here, y stands for variables associated with an adoption game for y given optimally chosen price structure (2), conditional on not having any adoption for both, and $x|y$ for variables associated with an adoption game for x, given that the adoption of y occurs. Subtracting $V_i^{y\to x}$ from V_i^{xy} shows the trade-offs between the two

[11] One might ask why they don't mix *cm* and *ww* instead. In fact, the imitators are indifferent to the choice between mixing *cm* and *ww* and mixing *cw* and *ww* because we are assuming the continuous time framework, in which case I assume the tie-break rule here is to choose knowledge sale rather than imitation. Note that the inventor can include a clause in the contract promising she will pay back an arbitrarily small ϵ amount in the next stage if *cw* is chosen.

[12] There exist trivial asymmetric MPE as well. However, in the literature, papers touching on a variant of wars of attrition game (or timing game more generally) with the continuous time framework like mine mostly focus only on symmetric equilibrium. For example, see Choi (1998), Fudenberg and Tirole (1991) p.117~130, the footnote 7 in Henry and Ponce (2011) p.966 and the cited papers therein.

mechanisms.

$$\begin{aligned} V_i^{xy} - V_i^{y\to x} &= \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}(\Pi_1 - \Pi_3) \\ &\quad -(\mu^{xy} - \mu^y)\kappa_y - \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}\kappa_x \end{aligned}$$

One can readily see that the expected time until entry is longer under xy than under $y \to x$, i.e., $\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y} > 0$. This is because, roughly speaking, the imitators are more aggressive in entering in an $x \to y$ sequence, in which they may have a chance to split the imitation costs, than in xy. Thus, the advantage of xy relative to $y \to x$ results from a longer duration of monopoly (the first term on RHS). However, there is a disadvantage associated with xy: the inventor collects revenues from knowledge sales later (the second and third terms on RHS). Given assumptions 1 and 2, though, the advantages outweigh the disadvantage, and hence the sequence xy is optimal. Let V_i^S denote the maximized payoff of the inventor under protecting all-by-secrecy mode. Then,

$$V_i^S = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa).$$

It is worth noting that the profit flow of the innovator under secrecy-only protection resembles the one under patent-only protection in that both mechanisms yield a monopoly profit for some time and a triopoly profit after the simultaneous entry by the imitators.

3.1.3 Mixing Patents and Secrecy

The final category to analyze is the case where the innovator patents one component and keeps the other secret. The analysis is analogous in either of the two possible variations in this category, i.e., x patented and y kept secret, or y patented and x kept secret. For expositional simplicity, let us examine the former case of the two, in which case the imitators cannot use x until time T but can do so freely afterwards. Thus, due to the perfect complementarity of x and y, the imitators' entry during the patent term is not feasible. However, the timing of entry after T is closely related with the timing of adoption of component y.

Let us consider a subgame starting at T. There are two scenarios to investigate: (i) a competitive game with respect to y, (i.e., the first adoption of y has occurred before T), $\theta = (T_+, \{i,j\})$ or $(T_+, \{i,k\})$, and (ii) a monopoly game with

respect to y, $\theta = (T_+, \{i\})$. Notice that these subgames have a stationary property as true of the games analyzed using the secrecy mechanism, and thus the analysis therein is directly applicable. From lemma 1 and 2, we can infer that under scenario (i), at time T the discounted expected payoffs of the inventor and the imitators are $V_i|_{\theta=(T_+,\{i,j\})} = \Pi_3$ and $V_b|_{\theta=(T_+,\{i,j\})} = \Pi_3$ respectively since the entry by the second imitator is immediate. In contrast, under scenario (ii), the adopting game for y begins from this period on, and thus, $V_i|_{\theta=(T_+,\{i\})} = (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3 + \kappa_y)$ and $V_b|_{\theta=(T_+,\{i\})} = \Pi_3 - \kappa_y$ as in lemma 2.

With the results above in mind, we will derive the maximum payoff of the inventor. Observe first that eliminating the chance of the first adoption of y during the patent term strictly increases the inventor's payoff as it extends the duration of monopoly even beyond T. Second, the inventor is able to make the imitators to preclude the imitators from adopting y before T by setting p_{ib}^{yt} relatively high for $b \in \{j,k\}$ and $t < T$. To understand this point, conditional on not having the first adoption, at time $t < T$ an imitator j would get $e^{-r(T-t)}(\Pi_3 - \kappa_y)$ in a worst-case scenario in which no adoption takes place until time T either by himself or by the rival imitator, k. Instead, if j decides to obtain y now, his payoff would be $e^{-r(T-t)}\Pi_3 - \min\{p_{ij}^{yt}, \kappa_y\}$ which is strictly less than his minimum possible payoff unless the inventor offers quite low price for y. Put differently, if $p_{ib}^{yt} > e^{-r(T-t)}\kappa_y$ for all $t < T$, adopting y earlier than T is strictly dominated by waiting for the imitators.

The discussion above is summarized in the following proposition.

Proposition 2 *Suppose the inventor protects x by patenting and y by relying on secrecy. In any symmetric MPE, (i) it is optimal for the inventor to induce the imitators not to adopt y until the patent expires ($t < T$) by setting high enough prices and to set $p_{ib}^{y} = \kappa_y$ for $b \in \{j,k\}$ thereafter ($t \geq T$); (ii) entry times of each imitator follow the exponential distribution with hazard rate, $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$ starting from T; (iii) the expected equilibrium payoffs of the inventor and of the imitators are $V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\{(1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3 + \kappa_y)\} - f$, where $\mu^{y|x} := r/(r + 2\lambda^{y|x}) \in (0,1)$, and $V_b = e^{-rT}(\Pi_3 - \kappa_y)$ respectively.*

Proposition 2 states that under the patent-secret mix the monopoly period is extended beyond the term of the patent in the expected sense, depending on the imitation cost of the component kept by secrecy. It is also certain that the component protected by secrecy is not adopted by the imitators until T for sure as the adopting game for the component begins after T.

It remains to examine which mix use is more profitable: either patenting x and keeping y secret as in the previous discussion or patenting y and keeping x secret. As shown in corollary 1, the inventor (weakly) prefers the former over the latter because the more costly the component kept secret, the longer duration of monopoly beyond T while the term of patent is invariant regardless of which component to patent.

Corollary 1 *When the inventor uses a patent-secret mix, (i) it is optimal for her to keep the component y, which is the more costly to imitate, secret if $\kappa_y > \kappa_x$; (ii) she is indifferent between protecting x by a patent and y by secrecy and protecting y by a patent and x by secrecy if $\kappa_y = \kappa_x$.*

If we denote the maximum payoff of the inventor under the mixed protection mode by V_i^M,

$$V_i^M = (1 - e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{y|x}\Pi_1 + (1 - \mu^{y|x})(\Pi_3 + \kappa_y)\right\} - f.$$

This result is comparable to that found by Anton and Yao (2004). They show that an innovator may patent small innovations but keep secret big innovations with the view that patents and trade secrets are mutually exclusive substitutes. Though my model also suggests that the inventor may optimally protect the more important y (more costly to imitate) by secrecy and x with a patent, such behavior results from the combined use of those two protection mechanisms here. In addition, the reasons for doing that in the two models are different. Here, keeping y secret is to generate a longer duration of monopoly, whereas, in their framework, where the strategic disclosure works as a signaling devise, the inventor faced with limited legal protection is reluctant to disclose big innovations as they are valuable enough to make the imitator want to imitate.

3.2 Optimal Protection Decision and Complex vs. Simple Innovations

In this section, I derive one of the key findings of this study by comparing the inventor's payoffs. If the patent filing fee is too high, the inventor would always avoid patenting. In order to exclude those trivial cases, we impose the following assumption.

Assumption 3: $f < \Pi_1 - V_i^S$.

First, note that

$$V_i^M - V_i^P = e^{-rT}\left\{\mu^{y|x}(\Pi_1 - \Pi_3) + (1-\mu^{y|x})\kappa_y\right\} > 0$$

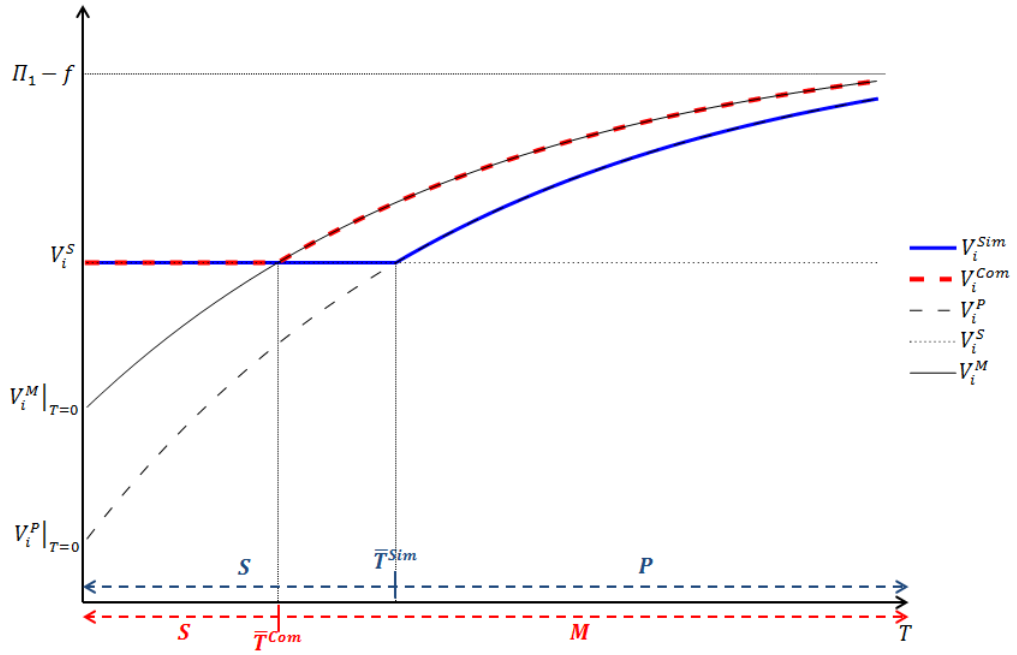
for all $T > 0$. There are two different elements which make using a patent and a trade secret together more appealing than using a patent alone. After the patent expires, not only can the inventor be a monopolist for some more random periods (the first term in the brace), but she also can collect revenues from the sale of y (the second term in the brace). Consequently, the complex innovator never protects both components by a patent. Then, by comparing V_i^S and V_i^M, we obtain the following proposition.

Proposition 3 *(i) There exists a unique patent length $\overline{T}^{Com}$ such that the inventor protects the complex innovation through the optimal patent-secrecy mix (as described in corollary 1) if the patent length is relatively long ($T \geq \overline{T}^{Com}$) and otherwise only by secrecy ($T < \overline{T}^{Com}$). (ii) Holding all other variables constant, $\overline{T}^{Com}$ decreases as the composition of the imitation costs become more asymmetric, i.e., κ_y increases and κ_x decreases by the same amount.*

There are two things facets of optimal protection characteristics worth noting. First, the inventor's optimal protection has a binary feature: either all-by-secrecy or patenting x and keeping y secret. Second, *ceteris paribus* the inventor becomes more aggressive in patenting (i.e., $\overline{T}^{Com}$ gets smaller) under a more asymmetric structure of the imitation costs. The intuition behind this result is as follows. V_i^S depends not on the composition of the imitation costs but on the aggregate imitation costs (see proposition 1), and thus it is invariant to such cost changes. However, V_i^M increases as κ_y increases and κ_x decreases by the same amount because term T is given the same as before while the expected duration for monopoly beyond T will be lengthened. As a result, the threshold patent length at which V_i^S and V_i^M are equal gets shorter.

What if the discovery rate of secrecy (λ) is exogenously given regardless of what portion of an innovation is kept secret? When λ is exogenous and constant, as long as at least one component has been protected by secrecy, keeping either x or y secret generates the same λ. Likewise, patenting either x or y makes the inventor a monopolist until T. Given that x and y are perfect complements, to take advantage of both mechanisms, the inventor would always choose to combine both of them in use. In addition, determining which component to patent and which to keep secret is

Figure 1: Payoff Comparison between the Complex Innovator and the Simple Innovator



Note: Vertical and horizontal axes represent the innovator's payoffs and length of patent, respectively. The two dashed lines near the horizontal axis shows the optimal protection mode for the simple innovator (upper blue) and for the complex innovator (lower red).

never a concern for the inventor because she is indifferent to the choice between the two possible mixing methods. Lastly, an imitator can uncover the inventor's trade secrets even during the patent term, which is in sharp contrast to the prediction of this paper that under a patent-secret mix entry never takes place until the patent expires.

Now, let us compare the payoffs of inventors having the complex innovation (the "complex innovator" hereafter) and a simple innovation (the "simple innovator" hereafter)respectively. The simple innovation consists of a single and not separable component of which imitation cost is κ. We also assume that the identical profit structure for the simple innovation. Note that, the available protection for the innovator who has a simple innovation is binary: either protecting all-by-patenting or all-by-secrecy. Then, it is easy to see that the simple innovator's equilibrium payoff is $V_i^{Sim}(T) = \max\{V_i^S, V_i^P(T)\}$ while the complex innovator's is $V_i^{Com}(T) = \max\{V_i^S, V_i^M(T)\}$ in which V_i^P, V_i^S and V_i^M are defined as in section 3.1.

Proposition 4 *There exists a unique patent length $\overline{T}^{Sim}$ such that the inventor protects the simple innovation only by a patent if the patent length is relatively long ($T \geq \overline{T}^{Sim}$) and*

only by secrecy otherwise ($T < \overline{T}^{Sim}$). Moreover, $\overline{T}^{Com} < \overline{T}^{Sim}$.

As shown in figure 1, both the complex innovator (red dashed line) and the simple innovator (blue line) have the same payoff schedules until $\overline{T}^{Com}$. If T is intermediate i.e., $T \in [\overline{T}^{Com}, \overline{T}^{Sim})$, the payoff of the complex innovator starts to grow, while the simple innovator's is still fixed. When T exceeds $\overline{T}^{Sim}$, the payoff of the simple innovator also begins to increase but never catches up to the complex innovator's. From the optimal protection choice depicted in the bottom, we also find that the complex innovator is more inclined to apply for a patent.

One important observation is that without the patent system the inventor would get the same payoffs as the simple inventor would. To understand this, recall that the equilibrium payoff of the inventor under secrecy protection does not depend on the composition of imitation costs but only on the aggregate costs (proposition 1). This implies that it is not *per se* the existence of a number of perfectly complementary components which may make the complex innovator more profitable. Coexistence of the patent system and the secrecy mechanism is crucial to have that result.

4 Strict Utility Requirement

Up to this point, only the positive analysis has been conducted. In particular, it was shown that the inventor with a complex innovation is (weakly) over-rewarded. However, if the patent examination office imposes a strict utility requirement (SUR), under which the innovator needs to disclose both components to get a patent, it can transform the inventor's payoff schedule into that of another investor having a simple innovation. This section addresses policy implications of SURs whether they are socially desirable or not.

There are numerous research studies investigating optimal patent policy regarding various policy variables (e.g., the patent scope or the term of patent). However, there is no clear consensus on this topic, and results heavily depend on underlying environments and assumptions.[13] For the sake of a thought experiment, we only consider the length of patent as the policy instrument of interest. We then consider a simple model in which the social planner decides whether to impose an SUR or not after the discovery (ex-post welfare implication) and before the discovery (ex-ante welfare implication) of the complex innovation in that order. For simplicity,

[13]For example, Gilbert and Shapiro (1990) and Gallini (1992) show sharply contrasting results.

throughout this section we assume the two components are symmetric in terms of imitation costs. (i.e., $\kappa_x = \kappa_y = \kappa/2$)

4.1 Ex-post Welfare Implication of SUR

In order to first calculate ex-post welfare after the discovery of the innovation with and without SUR, given the patent length, T, we denote the ex-post welfare functions under each protection mode, P, M and S, by $\widetilde{W}^P(T)$, $\widetilde{W}^M(T)$ and $\widetilde{W}^S$ as follows

$$\begin{aligned}
\widetilde{W}^P(T) &= (1-e^{-rT})W_1 + e^{-rT}W_3 \\
\widetilde{W}^M(T) &= (1-e^{-rT})W_1 + e^{-rT}\left\{\mu^1 W_1 + (1-\mu^1)W_3\right\} \\
\widetilde{W}^S &= \mu^2 W_1 + (1-\mu^2)W_3
\end{aligned}$$

where μ^1 and μ^2 are the natural duration of monopoly generated by keeping 1 component and 2 components by secrecy, respectively.[14] As is usually assumed in the literature, W_1 and W_3 represent social welfare, defined as the sum of industry profits and consumer surplus under monopoly and triopoly market structure respectively, and social welfare is higher under a triopoly situation than under a monopoly one, i.e., $W_1 < W_3$.

As the inventor's optimal decision is opting for S if $T \leq \overline{T}^{Sim}$ and P otherwise under SUR, ex-post welfare with SUR is

$$\widetilde{W}^{SUR}(T) = \begin{cases} \widetilde{W}^S & \text{if } T \leq \overline{T}^{Sim} \\ \widetilde{W}^P(T) & \text{otherwise} \end{cases}.$$

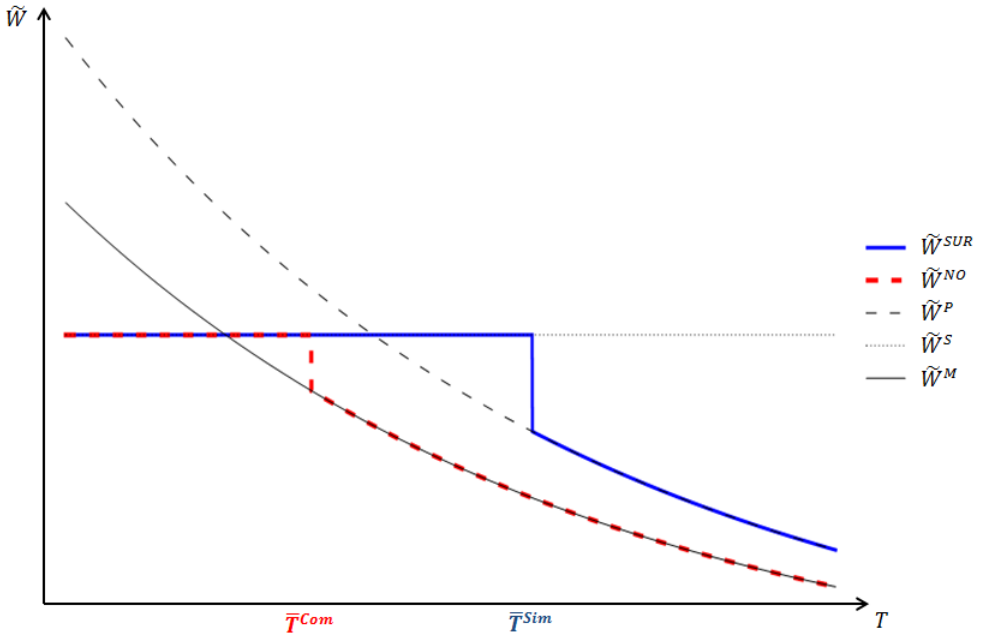
Similarly, ex-post welfare without SUR is given by

$$\widetilde{W}^{NO}(T) = \begin{cases} \widetilde{W}^S & \text{if } T \leq \overline{T}^{Com} \\ \widetilde{W}^M(T) & \text{otherwise} \end{cases}.$$

The schedules of the two welfare functions can be compared by first identifying their characteristics. Observe first that $\widetilde{W}^M$ is always less than $\widetilde{W}^P$ because the patent-secret mix always entails a longer duration for a monopoly. Next, while $\widetilde{W}^S$

[14] $\mu^1 = r/(r+2\lambda^1)$, $\lambda^1 = r(\Pi_3 - \kappa/2)/(\kappa/2)$, $\mu^2 = r/(r+2\lambda^2)$, $\lambda^2 = r(\Pi_3 - \kappa)/\kappa$.

Figure 2: Ex-post Welfare with and without SUR



Note: Vertical and horizontal axes represent respectively social welfare and length of patent

does not vary by the patent length T, $\widetilde{W}^M$ and $\widetilde{W}^P$ decrease as T increases. Last, and most interestingly, there is a sudden drop in welfare for $\widetilde{W}^{SUR}$ (or $\widetilde{W}^{NO}$) at the threshold, $\overline{T}^{Sim}$ (or $\overline{T}^{Com}$). The reason for this discontinuity of the function, $\widetilde{W}^{SUR}$, is that while the optimal payoff of the inventor remains the same in the two protection modes, the expected monopoly duration is shorter in S. To see this, it is useful to compare the natural duration of the secret, μ^2 and the natural patent length, $(1-e^{-r\overline{T}^{Sim}})$ where $\overline{T}^{Sim} = (1/r)\ln[(\Pi_1-\Pi_3)/\{(1-\mu^2)(\Pi_1-\Pi_3-\kappa)-f\}]$ by the symmetric imitation cost structure and the definition of $\overline{T}^{Sim}$.[15] Then, it is easy to see that

$$(1-e^{-r\overline{T}^{Sim}})-\mu^2 = \frac{(1-\mu^2)\kappa+f}{\Pi_1-\Pi_3} > 0.$$

The intuition is simple: under the secrecy protection mode as the inventor has another channel of appropriating her innovation, i.e., selling the knowledge components, a shorter monopoly is required for her to achieve the expected payoff she would have earned under the patent only protection mode, with the threshold $\overline{T}^{Sim}$ (i.e., $V_i^P(\overline{T}^{Sim})$). This implies when $T=\overline{T}^{Sim}$ so that the inventor is indifferent between the secrecy only protection mode (S) and the patent only protection mode (P), ex-post welfare is higher under S, i.e., $\widetilde{W}^S > \widetilde{W}^P(\overline{T}^{Sim})$. A similar logic can be

[15] See the proof of proposition 4.

applied to show $\widetilde{W}^S > \widetilde{W}^M(\overline{T}^{Com})$.

From the discussion above, it is clear that $\widetilde{W}^{SUR}$ and $\widetilde{W}^{NO}$ must be drawn as depicted in figure 2. The figure shows that $\widetilde{W}^{NO}$ is equal or less than $\widetilde{W}^{SUR}$ for all T, implying that availability of a patent-secret mix decreases ex-post welfare, and thus, imposing SUR is advisable.

Proposition 5 *Imposing the strict utility requirement is socially desirable ex-post (strictly desirable for $T > \overline{T}^{Com}$).*

4.2 Ex-ante Welfare Implication of SUR

This section turns from the above examination of ex-post welfare implications of SURs to the ex-ante ones, taking into account the inventor's investment incentives. To this end, the first question is to determine which T will be chosen by the social planner facing an innovator when only the patent protection mode is available. Then, such T is a solution of the following problem:

$$\max_T W^P(T) = \hat{I}^P \cdot \widetilde{W}^P - D(\hat{I}^P)$$

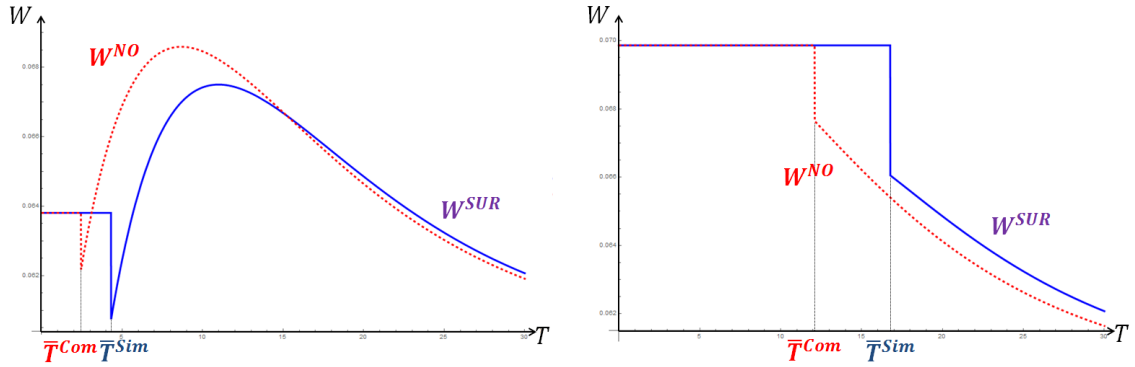
subject to

$$\hat{I}^P = \arg\max_I I \cdot V_i^P - D(I)$$

where I is the probability that the innovation process turns out to be successful; $D(I)$ is a cost function of such R&D process given I; $\hat{I}^P$ is the optimal investment decision of the inventor given T. Furthermore, let us assume that the objective function subject to the optimal investment decision is strictly quasi-concave in T, and so the solution exists uniquely. There is a trade-off of giving more incentive for the inventor to invest by increasing T, which increases the probability that the innovation project is successful but also decreases ex-post social welfare by extending the duration for which the inventor is a monopolist. Thus, if we denote the solution of the above problem by $T^{P*} \in (0, \infty)$, it is set by the social planner in such a way that those two marginal effects are balanced. We can consider the same problem with mixed protection. Let $T^{M*} \in (0, \infty)$ be the solution of such social planner's problem.

Then, the ex-ante social welfare functions with and without SUR are defined

Figure 3: An Example of Ex-ante Welfare with and without SUR



Case (I): $\kappa = .04$ Case (II): $\kappa = .085$

Note: Vertical and horizontal axes represent social welfare and length of patent, respectively. W^{SUR} and W^{NO} are the ex-ante welfare functions given T with and without SUR respectively.

again by the optimal decision rules of the innovator:

$$W^{SUR}(T) = \begin{cases} W^S & \text{if } T \leq \overline{T}^{Sim} \\ W^P(T) & \text{otherwise} \end{cases}$$

and

$$W^{NO}(T) = \begin{cases} W^S & \text{if } T \leq \overline{T}^{Com} \\ W^M(T) & \text{otherwise} \end{cases}.$$

Even if the socially optimal length of patent is well defined for P and M, a complete analysis for all underlying primitives is somewhat challenging. First, given T, optimally chosen investment levels under P, M and S are all different. Moreover, due to the discontinuity nature of the ex-post welfare functions introduced in the previous subsection, the ex-ante welfare functions are also not continuous and even, at times, not monotone. Therefore, in what follows, I will demonstrate, by offering an example, that policy implications are sensitive to parameter values of the model and ambiguous in general by offering an example.

Example: Consider the following two cases which only differ by the imitation costs: for case (I), $\kappa = .04$ and for case (II), $\kappa = .085$. Except for that difference, the two cases share the same modeling assumptions: $D(I) = I^2$, $\Pi_1 = .4$, $\Pi_3 = .1$, $W_1 = .5$, $W_3 = .8$, $r = .1$ and $f = 0$. Figure 3 shows welfare functions for the two cases. Note that the policy implication to be drawn is that imposing SUR is socially desirable whenever $W^{SUR} > W^{NO}$. First, consider case (I) in which the strength of secrecy

mechanism (relatively low κ), by which the lower bound of ex-ante investment incentives is determined, is relatively low. In this situation, $\overline{T}^{Com}$ and $\overline{T}^{Sim}$ are less than T^{M*} and T^{P*}, respectively, and this implies the patent system should contribute to boost the inventor's investment incentive. Hence, in this case, for some relatively intermediate T, allowing the inventor to choose M may increase the social welfare. However, the policy implication is not monotone in T. Roughly speaking, imposing SUR increases social welfare over either the lower or the upper tail range of T but decreases it over the intermediate range of T. In contrast, in case (II), the strength of secrecy is so high that even without the patent system, the inventor is willing to invest aggressively. Therefore, adding M option to the innovator's choice set always lowers social welfare. This example illustrates the policy implication of SUR is, in general, ambiguous.

5 Extensions and Robustness

This section considers two extensions to check the robustness of the main findings. The complex innovation is comprised of $N(>2)$ components rather than 2 components is first discussed, followed by an exploration of the case in which patent rights are probabilistic.

5.1 N Component Innovation

We have assumed so far, for simplicity, that the complex innovation has just two complementary components. Here, suppose we have a complex innovation which is comprised of N different complementary components. The imitation costs of each component are given by $\kappa_1 \leq \ldots \leq \kappa_n \leq \ldots \leq \kappa_N$ for $n \in \{1,2,\ldots,N\}$, and let $\kappa \equiv \sum_{n=1}^{N} \kappa_n < \Pi_3$ denote the aggregate imitation cost. I show that the main results we had for the model with 2 components carry over to this situation.

When the inventor applies for a patent containing all N components, her payoff remains the same as before:

$$V_i^P = (1 - e^{-rT})\Pi_1 + e^{-rT}\Pi_3 - f.$$

Regarding the secrecy only mode and the patent-secret mix, equilibria in each scenario also shows a pattern that is similar to the 2 component model. We formally

characterize the equilibria in the next two lemmas.

Lemma 4 *In the N component model, when the inventor relies only on secrecy, the unique symmetric MPE is characterized as follows: (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for all N components by setting* $p_{ib}^n = \kappa_n$ *for all* $n \in \{1,2,\ldots,N\}$ *for* $b \in \{j,k\}$*; (ii) entry times of each imitator follow the exponential distribution with the hazard rate,* $\lambda = r(\Pi_3 - \kappa)/\kappa$*; (iii) the expected equilibrium payoff of the inventor is* $V_i^S = \mu\Pi_1 + (1-\mu)(\Pi_3 + \kappa)$ *where* $\mu = 1/(r+2\lambda)$*.*

Lemma 5 *In the N component model, (i) the optimal patent-secret mix is patenting only component 1, which is the least costly to imitate, and keeping all the other components by secrecy; (ii) the inventor induces the imitators not to adopt all components except component 1 until the patent expires* $(t < T)$ *by setting high enough prices and sets* $p_{ib}^n = \kappa_n$ *for all* $n \in \{2,3,\ldots,N\}$ *for* $b \in \{j,k\}$ *thereafter* $(t \geq T)$*; (ii) entry times of each imitator follow the exponential distribution with the hazard rate,* $\lambda^{-1} = r(\Pi_3 - \kappa_{-1})/\kappa_{-1}$ *starting from T; (iii) the expected equilibrium payoff of the inventor is* $V_i^M = (1-e^{-rT})\Pi_1 + e^{-rT}\{\mu^{-1}\Pi_1 + (1-\mu^{-1})(\Pi_3 + \kappa_{-1})\} - f$ *where* $\mu^{-1} = 1/(r+2\lambda^{-1})$ *and* $\kappa_{-1} = \sum_{n=2}^{N}\kappa_n$*.*

V_i^S is the same expression as in the 2 component model, and V_i^M is also similar to the counterpart of the 2 component model. Hence, we reach to the same conclusion that the optimal protection mode is either S or the optimal M because P is always dominated by the optimal M, and thus the complex innovator weakly earns more than the simple innovator.

The N component model also offers some additional insights through a simple comparative static analysis. Assuming symmetric imitation costs, as the innovation becomes more complex (larger N), both the expected duration of prolonged monopoly (higher λ^{-1}) and the inventor's expected payoff (higher V_i^M) increase in the optimal patent-secrecy mix, and, as a result, the threshold equating V_i^S and V_i^M ($\overline{T}^{Com}$) gets shorter.

Proposition 6 *In the N component model, (i) there exists a unique patent length* $\overline{T}^{Com}$ *such that the inventor protects the complex innovation through the optimal patent-secret mix (as described in Lemma 4) if the patent length is relatively long* $(T \geq \overline{T}^{Com})$ *and only by secrecy otherwise* $(T < \overline{T}^{Com})$*; (ii) when all components are symmetric in terms of imitation cost, i.e.,* $\kappa_1 = \ldots = \kappa_n = \ldots = \kappa_N$*,* $\overline{T}^{Com}$ *shortens as the innovation becomes more complex (i.e., as N rises).*

5.2 Probabilistic Patents

How much of the previous results would be changed when we assume that patents are probabilistic rather than ironclad? Put differently, what if filing for a patent no longer absolutely assures the inventor of T-period monopoly position, but only provides some probability? Here, assuming patents are probabilistic, we determine under which conditions the combined use of patents and secrecy turns out to be optimal, at least for some values of patent length, and thus the complex innovator is still (weakly) over-rewarded.

To formalize the idea, consider the following simplified version of a post-grant review in the U.S. patent system or an opposition proceeding in the European patent system.[16] Right after a patent is granted to the inventor, the imitators have a chance to challenge it at the patent office. For simplicity, assume that the challenge process takes place instantly without any delay at 0 cost, which implies it is optimal for the imitators to challenge every patent granted. Let us further assume that $f = 0$ and that the two components are symmetric in terms of imitation cost ($\kappa_x = \kappa_y = \kappa/2$) and the probability that each component will be upheld as valid through the trial proceeding when the components are challenged. Denote by α the probability that at least one component survives the challenge and is upheld as valid when the inventor has chosen P. Similarly, define $\rho\alpha$ as the probability that a component is upheld as valid when the inventor has chosen M where ρ is a measure of the inverse degree of complementarity between the two components in obtaining a valid patent and takes a value in $[1/2, 1]$. [17]

Then, the equilibrium payoff of the inventor under S remains as in the basic model, i.e., $V_i^S = \mu\Pi_1 + (1-\mu)(\Pi_3 + \kappa)$, whereas the equilibrium payoffs under

[16] United States Patent and Trademark Office states "Post grant review is a trial proceeding conducted at the Board to review the patentability of one or more claims in a patent on any ground that could be raised under § 282(b)(2) or (3). Post grant review process begins with a third party filing a petition on or prior to the date that is 9 months after the grant of the patent or issuance of a reissue patent." (For more details, visit https://www.uspto.gov/patents-application-process/appealing-patent-decisions/trials/post-grant-review. Last retrieved on Aug/19/2016.)

[17] To see this, note that the joint distribution of trial results for the two components is as follows since x and y are symmetric in terms of validity:

Component $x \setminus y$	Valid	Not valid
Valid	A	B
Not valid	B	C

Then, $\alpha = A + 2B$ and $\rho\alpha = A + B$, and thus, $\rho = (A+B)/(A+2B)$ which takes a value between $1/2$ and 1. For example, depending on whether the trial results of x and y are perfectly positively correlated ($B = 0$), independent ($B = \rho\alpha(1-\rho\alpha)$) or perfectively negatively correlated ($B = 1/2$), ρ equals to 1, $(1-\sqrt{1-\alpha})/\alpha$, or $1/2$ respectively.

the other two protection modes change. When the inventor opts for P, she earns monopoly profit until T and triopoly profit afterwards only with probability α, and with the complementary probability, $1-\alpha$, the imitators can enter the market from the beginning. Hence,

$$V_i^P = \alpha\left\{(1-e^{-rT})\Pi_1 + e^{-rT}\Pi_3\right\} + (1-\alpha)\Pi_3.$$

When the inventor chooses M, she becomes a monopolist until T and then the adopting game for the remaining component by the imitators follows right after T if the patent is upheld as valid (with probability $\rho\alpha$). Otherwise (with the complementary probability $1-\rho\alpha$), the same adopting game starts from the beginning. Thus,

$$V_i^M = \rho\alpha\left\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\right\} + (1-\rho\alpha)V^1,$$

where $V^1 = \{\mu^1\Pi_1 + (1-\mu^1)(\Pi_3+\kappa/2)\}$, $\mu^1 = r/(r+2\lambda^1)$ and $\lambda^1 = r(\Pi_3-\kappa/2)/(\kappa/2)$.[18]

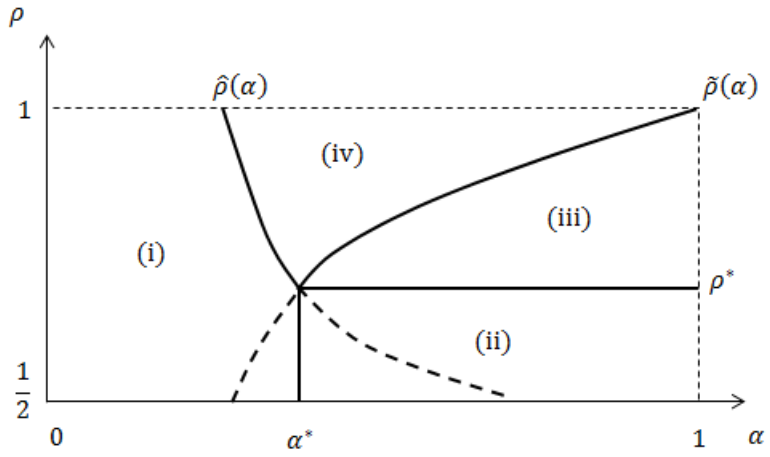
In order to find the optimal protection mode depending on the patent length, first observe that if $T=0$, for all α and ρ,

$$V_i^S > V_i^M|_{T=0} > V_i^P|_{T=0}.$$

This tells us that when T is 0, it is better to protect as many components as possible by secrecy so that the inventor can have a longer random monopoly period regardless of α and ρ. This is somewhat obvious: recall that we had the same ranking between the payoffs when $T=0$ even in the basic model, and patenting, moreover, now became less attractive due to the probabilistic nature of patent rights. By the continuity of payoff functions, when patent duration is relatively short, S would be still optimal compared to M or P. However, as T increases, the ranking between the payoffs may be reversed since, for all α and ρ,

[18] Alternatively, we can assume that patent rights are probabilistic in the sense that patent examiners at the patent office reject a claim (component), which is included in a patent application and does not meet one of the patentability requirements, with a certain probability. (In the U.S. patent law, the patentability requirements are patentable subject matter, novelty, non-obviousness and utility). If rejected claims are disclosed to the public, the equilibrium payoffs will be the same as those described in the post-grant review scenario. If rejected claims are not disclosed to the public, and thus if the imitators still needs to incur imitation costs or to buy from the inventor in order to adopt those components, the equilibrium payoffs with P and M are modified as follows respectively: $V_i^P = (2\rho-1)\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}\Pi_3\} + 2(1-\rho)\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\} + (1-\alpha)V_i^S$ and $V_i^M = \rho\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\} + (1-\rho\alpha)V_i^S$. The qualitative results do not, however, change under this alternative scenario.

Figure 4: Equilibrium Protection Constellations



$$\frac{\partial V_i^P}{\partial T} = \alpha r e^{-rT}(\Pi_1 - \Pi_3) > \frac{\partial V_i^M}{\partial T} = \rho\alpha r e^{-rT}(\Pi_1 - V^1) > \frac{\partial V_i^S}{\partial T} = 0$$

where the first inequality follows from the fact that $\rho \leq 1$ and $V^1 > \Pi_3$. In fact, the full optimal schedule of protection mode over $T \in [0,\infty)$ crucially depends upon the ranking of V_i^P, V_i^M and V_i^S in the limit (i.e., $T \to \infty$). Thus, to fully characterize equilibrium constellation, let us define the following thresholds: α^*, $\hat{\rho}(\alpha)$ and $\tilde{\rho}(\alpha)$ such that respectively

$$\begin{aligned}
\lim_{T\to\infty} V_i^P(\alpha) \gtreqless V_i^S &\iff \alpha \gtreqless \alpha^*, \\
\lim_{T\to\infty} V_i^M(\alpha,\rho) \gtreqless V_i^S &\iff \rho \gtreqless \hat{\rho}(\alpha), \\
\lim_{T\to\infty} V_i^M(\alpha,\rho) \gtreqless \lim_{T\to\infty} V_i^P(\alpha) &\iff \rho \gtreqless \tilde{\rho}(\alpha).
\end{aligned}$$

Then, the equilibrium protection constellations are as shown in Proposition 7 and illustrated in figure 4.

Proposition 7 *In the probabilistic patents model, given* $(\alpha,\rho) \in (0,1] \times [1/2,1]$*, the optimal protection modes chosen by the inventor are as follows:*
(i) if $\alpha \leq \alpha^*$ *and* $\rho \leq \hat{\rho}(\alpha)$*, S always;*
(ii) if $\alpha > \alpha^*$ *and* $\rho \leq \rho^*$*, S if T is small and P otherwise;*
(iii) if $\alpha > \alpha^*$ *and* $\rho^* < \rho < \tilde{\rho}(\alpha)$*, S if T is small, M if T is intermediate and P otherwise;*
(iv) if $\tilde{\rho}(\alpha) \leq \rho$ *and* $\hat{\rho}(\alpha) < \rho$*, S if T is small and M otherwise*

where $\alpha^* = \frac{V_i^S - \Pi_3}{\Pi_1 - \Pi_3} \in (0,1)$, $\hat{\rho}(\alpha) = \frac{V_i^S - V^1}{\alpha(\Pi_1 - V^1)}$, $\tilde{\rho}(\alpha) = \frac{\alpha(\Pi_1 - \Pi_3) - (V^1 - \Pi_3)}{\alpha(\Pi_1 - V^1)}$ *and* $\rho^* = \tilde{\rho}(\alpha^*) = \hat{\rho}(\alpha^*) \in (1/2, 1)$.

Proposition 7 confirms that in the probabilistic patents model the combined use of patents and trade secrets (for some T) arises in equilibrium as before insofar as the probability of getting a valid patent with the two components is relatively high (i.e., relatively high α), and the gap between such probabilities with the two components and with a component is not that high (i.e., relatively high ρ). That corresponds to the area (iii) and (iv) in figure 4, and in this region it is also robust that the complex innovator is weakly over-rewarded.

It is noteworthy that now there is a pair (α, ρ) under which P dominates M for all T (area (ii)). Recall that in the main analysis, either M or P ensured obtaining a patent, and, as a result, P was always inferior to M as it only made the inventor give up a chance to prolong the monopoly period beyond T by disclosing all information. That is no longer the case since P has an advantage over M in the sense that it has a higher probability of acquiring a patent, and the degree of that advantage depends on ρ. More specifically, for each $\alpha > \alpha^*$, the optimal pattern of the inventor's strategy changes, as ρ increases, from the one in (ii) of proposition 7 to the one in (iii) and finally to the one in (iv). Another interesting point is that some dubious innovations, which would not have been patented at all if they were a simple innovation, might be patented if ρ is high (corresponding to the subarea of (iv) such that $\alpha < \alpha^*$).

6 Conclusion

This paper studies the optimal use of patents and trade secrets by an inventor who has a complex innovation. I introduce the mechanism of how the patent-secret mix can extend the duration of monopoly beyond the length of patent, and thus dominate the patent protection. For that reason, the inventor's optimal choice has a binary feature: either opting for the combined use of a patent and secrecy or secrecy only. The optimality of patent-secret mix is carried over to the case of probabilistic patent rights as well on a broad set of parameters.

As one may easily expect, with a richer set of protection modes, including the mixed one, the inventor who has the complex innovation is over-rewarded compared to another inventor who has a simple innovation. However, the policy implication of imposing SUR considering the ex-ante investment incentives is ambiguous

and sensitive to underlying parameters. Therefore, unfortunately, this paper informs us that designing optimal intellectual property rights policies calls for a case-by-case scrutiny with the patent-secret mix as the inventor's protection strategy. In this regard, it would be also beneficial to conduct a similar analysis to illuminate how and when firms optimally use a patent-secret mix in different innovation frameworks, such as, a cumulative innovation setting, a patent portfolio setting and a complementary innovation setting associated with patent pooling.

Appendix: Omitted Proofs of Propositions and Lemmas

Proof of Lemma 1

Result (i): I explore all possible competitive games with respect to a knowledge component in turn. First, consider a competitive game with respect to both components. There are two possible scenarios: (1) an imitator is a seller for both components besides i, i.e., $\theta = (\{i,j\},\{i,j\})$ or $(\{i,k\},\{i,k\})$; (2) each imitator is a seller for different components in addition to i, i.e., $\theta \in (\{i,j\},\{i,k\})$ or $(\{i,k\},\{i,j\})$. Assume scenario (1) first and say j the seller without loss of generality, i.e., $\theta = (\{i,j\},\{i,j\})$. I will show that both of components are sold immediately at 0 prices in equilibrium. The proof for this claim consists of the following three steps, and these can be applied to the rest of three cases in a similar manner.

Step 1: Given price offers, k's best respond is

$$\begin{cases} c_{\underline{s}k}c_{\underline{s}k} & \text{if } p^x_{\underline{s}} \leq \kappa_x \text{ and } p^y_{\underline{s}} \leq \kappa_y \\ c_{\underline{s}k}m_k & \text{if } p^x_{\underline{s}} \leq \kappa_x \text{ and } p^y_{\underline{s}} > \kappa_y \\ m_k c_{\underline{s}k} & \text{if } p^x_{\underline{s}} > \kappa_x \text{ and } p^y_{\underline{s}} \leq \kappa_y \\ m_k m_k & \text{otherwise.} \end{cases}$$

where $\underline{s}$ denote the seller who offers the lower price. This is straightforward from the fact (1) that including w_k for any components is never a best response because the immediate entry, which is possible only when k adopts x and y, is profitable for k and (2) that it is optimal for k to pick the cheapest way of adopting a component.

Step 2: In any MPE, all components are sold to k through contracts offered by one of the two sellers. Suppose not, which implies either $p^x_{\underline{s}} > \kappa_x$ or $p^y_{\underline{s}} > \kappa_y$. For simplicity, say $p^x_{\underline{s}} > \kappa_x$. Then, it is profitable for either i or j to set $p^x = \kappa_x$ at which k buys x rather than imitating because it increases her or his payoff by κ_x without

changing the timing of entry by k at all since it takes place anyway in that period.

Step 3: In a unique MPE, $p^x_{ik} = p^x_{jk} = 0$ and $p^y_{ik} = p^y_{jk} = 0$. Again, suppose it were not the case. If both sellers offer a pair of positive prices for a component, at least one of the two has an incentive to set a price just below the rival seller's and collect more revenue without affecting the present value of each firm's profit stream form the market. Similarly, if one of them offers a strictly positive price, but the other offers 0, then the latter has an incentive to increase her price. Thus, in any competitive game with respect to both components, there is a unique MPE where all remaining buyers obtains knowledge components still not in their possession at 0 price immediately.

Next, consider another class of competitive games in which there are two sellers for a component, and the other was obtained by all firms. For simplicity, assume that all firms learned x. Then, there are two possible cases to consider, $\theta = (\{i,j.k\},\{i,j\})$ or $(\{i,j.k\},\{i,k\})$. The analysis is analogous to the previous one but simpler than that because it is now sufficient to consider only one remaining component to be sold.

Result (ii): Consider a game that is competitive with respect to a component but monopolistic with respect to the other. I show that the remaining buyer of the former component purchases it at a 0 price in an MPE. For simplicity, say x was adopted by j at time t, i.e., $\theta = (\{i,j\},\{i\})$. First, note that until the first adoption takes place for y as well, i is the monopolist in the market because of perfect complementarity of x and y. If k chooses to wait for x, he starts to earn triopoly profit once the first adoption for y takes place since he can get x for free almost immediately (from result (i)). Conversely, if k chooses either buy or imitate at time t, he enters a subgame of $\theta = (\{i,j,k\},\{i\})$ right away in which he ends up earning the same profit stream with imitators' actions for y remaining the same as in the previous case. As so far as the minimum price of x offered to k is positive, imitating or buying x at time t is dominated by waiting, in any MPE of this subgame, x should be sold to k at 0 price and the equilibrium payoff is independent of the timing of sale of x. ■

Proof of Lemma 2

Together with lemma 1 in this paper, see lemma 2 and proposition 2 in Henry and Ponce (2011) except calculating the buyers' payoffs in equilibrium. Note that

$$V_b^{y|x} = \frac{\lambda^{y|x}(\Pi_3 - \kappa_y) + \lambda^{y|x}\Pi_3}{r + 2\lambda^{y|x}}.$$

Once $\lambda^{y|x}$ in lemma 2 is plugged into the above equation, it follows that $V_b^{y|x} = \Pi_3 - \kappa_y$. ■

Proof of Lemma 3

Suppose there is an MPE in which the innovator offers $p_{ib}^x > \kappa_x$ and $p_{ib}^y > \kappa_y$ for $b \in \{j,k\}$. Then, in any equilibrium it must be the case that the first adoption of a component by an imitator[19] is associated with imitation by himself rather than buying knowledge through a contract. Given that if the seller offers a price equal to its imitation cost for that component, the only thing changes would be the mode of adoption of it from imitating to buying with all other aspects of the equilibrium unchanged. Then, clearly by doing this the seller will be strictly better off because now she additionally obtains revenue from selling knowledge. This contradicts the supposition that $p_{ib}^x > \kappa_x$ and $p_{ib}^y > \kappa_y$ for $b \in \{j,k\}$ are MPE price offers. ■

Proof of Proposition 1

Step 1: First, let us calculate the expected payoffs to the innovators and imitators given different price combinations. If $p_{ib}^x \leq \kappa_x$ and $p_{ib}^y > \kappa_y$, m_b^x is never optimal because at least b can resort to c_{ib}^x to adopt x allowing b to save imitation cost, keeping all other aspects of the game unchanged. By the same token, c_{ib}^y is never optimal as b can instead choose m_b^y. Then, candidates for an equilibrium is among $c_{ib}^x w_b^y$, $w_b^x w_b^y$, $c_{ib}^x m_b^y$ and $w_b^x m_b^y$. However, if the equilibrium should be associated either with $c_{ib}^x m_b^y$ or $w_b^x m_b^y$, i always has a profitable deviation of adjusting $p_{ib}^y = \kappa_y$, which increases i's revenue from selling y without changing other aspects of the game. Since that is a clear contradiction, it must be the case that b chooses either $c_{ib}^x w_b^y$ or $w_b^x w_b^y$ if this is an equilibrium. This implies that knowledge adoption takes place for x first and for y later in turn in equilibrium. ($x \to y$)

Let $F_b : [0.\infty) \times [0,\kappa_x] \to [0,1]$ refer to the distribution function of entry times of b. Taking F_{-b} as given and conditional on no adoption for x and y until time t, the expected benefit of waiting infinitesimal dt measure of time more (comparing to purchasing x now) is given by $f_{-b} p_{ib}^x dt/(1 - F_{-b})$ since b can get x freely instead of paying p_{ib}^x if $-b$ enters during this time window. However, there are also costs associated with waiting. b needs to sacrifice some surplus which he would otherwise

[19]Depending on an equilibrium strategy profile, a component and an imitator need to be modified as components and the imitators respectively.

have been able to earn, $r(\Pi_3 - \kappa_y - p^x_{ib})dt$. [20] In equilibrium the above two terms should be equal, which yields

$$\frac{f_{-b}}{1 - F_{-b}} = \frac{r(\Pi_3 - \kappa_y - p^x_{ib})}{p^x_{ib}}.$$

Note that $\lambda_{-b} := f_{-b}/(1 - F_{-b}) = r(\Pi_3 - \kappa_y - p^x_{ib})/p^x_{ib}$ is stationary, and thus adoption strategies of the imitators follow an exponential distributions with the hazard rates.

The timing of the first adoption for x is a random variable which follows exponential distribution with the hazard rate of $\lambda_j + \lambda_k$ since it is the minimum of adoption times of the two imitators which are independently and exponentially distributed with hazard rates of λ_j and λ_k respectively given $p^x = (p^x_{ij}, p^x_{ik}) \in [0, \kappa_x]^2$. By applying the technique used by Henry and Ponce (2011) along with lemma 2 [21], if we write down the expected payoff of the inventor,

$$V_i^{x \to y}(p^x) = \frac{r\Pi_1}{r + \lambda_j + \lambda_k} + \frac{(\lambda_j + \lambda_k)V_i^{y|x}}{r + \lambda_j + \lambda_k} + \frac{\lambda_j p^x_{ij} + \lambda_k p^x_{ik}}{r + \lambda_j + \lambda_k}.$$

One can easily check that $V_i^{x \to y}(p^x)$ is strictly increasing in both price levels. In order to see this, observe

$$\frac{\partial V_i^{x \to y}}{\partial p^x_{ij}} = \frac{\lambda_j}{\phi} - \frac{1}{\phi^2}\frac{\partial \lambda_j}{\partial p^x_{ij}}\left\{r\Pi_1 - rV_i^{y|x} - (r + \lambda_j)p^x_{ik} + \lambda_j p^x_{ij}\right\}$$

where $\phi := r + \lambda_j + \lambda_k$. Since $\partial\lambda_j/\partial p^x_{ij} < 0$, it suffices to check the term inside of the squared bracket is positive. Plugging in the expression for $V_i^{y|x}$ from lemma 2 and $r(\Pi_3 - \kappa_y - p^x_{ik})$ for $\lambda_j p^x_{ik}$ and collecting terms,

$$\begin{array}{rl} & r\Pi_1 - rV_i^{y|x} - (r + \lambda_j)p^x_{ik} + \lambda_j p^x_{ij} \\ = & \frac{r(2\Pi_1 - 4\Pi_3 - \kappa_y)(\Pi_3 - \kappa_y)}{(2\Pi_3 - \kappa_y)} + \lambda_j p^x_{ij} \end{array},$$

and the above term is positive as $2\Pi_1 - 4\Pi_3 - \kappa_y > 0$ by assumption 1 and 2. Hence $V_i^{x \to y}$ is maximized when $p^x_{ib} = \kappa_x$ and $p^y_{ib} > \kappa_y$ for $b \in \{j, k\}$.

It is easy to apply the same logic for the rest of price structures to find out

[20] Recall from lemma 1 and lemma 3 that once the first adoption takes place for x, they enter the subsequent subgame immediately in which imitators' expected payoffs in equilibrium is $\Pi_3 - \kappa_y$.

[21] See the proof of Proposition 2 in Henry and Ponce (2011) for more details

optimal pairs of prices. If the innovator wants a component to be adopted by the imitators in the current state, she charges a price equal to its imitation cost. Otherwise, a price for that component is higher than its imitation cost.

Step 2: Candidates for symmetric MPE and corresponding maximized expected payoffs of the innovator are as follows. If $p_{ib}^x = \kappa_x$ and $p_{ib}^y > \kappa_y$ in the beginning (inducing an adoption sequence of x first and y later, $x \to y$),

$$V_i^{x\to y} = \mu^x \Pi_1 + (1-\mu^x)\left\{\kappa_x + \mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3 + \kappa_y)\right\}$$

where $\mu^x = r/(r+2\lambda^x)$, $\lambda^x = r(\Pi_3 - \kappa)/\kappa_x$, $\mu^{y|x} = r/(r+2\lambda^{y|x})$ and $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$. If $p_{ib}^x > \kappa_x$ and $p_{ib}^y = \kappa_y$, (inducing an adoption sequence of y first and x later, $y \to x$),

$$V_i^{y\to x} = \mu^y \Pi_1 + (1-\mu^y)\left\{\kappa_y + \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3 + \kappa_x)\right\}$$

where $\mu^y = r/(r+2\lambda^y)$, $\lambda^y = r(\Pi_3 - \kappa)/\kappa_y$, $\mu^{x|y} = r/(r+2\lambda^{x|y})$ and $\lambda^{x|y} = r(\Pi_3 - \kappa_x)/\kappa_x$. If $p_{ib}^x = \kappa_x$ and $p_{ib}^y = \kappa_y$, (inducing simultaneous adoption for both components, xy),

$$V_i^{xy} = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa)$$

where $\mu^{xy} = r/(r+2\lambda^{xy})$ and $\lambda^{xy} = r(\Pi_3 - \kappa)/\kappa$. One can verify that an imitator's payoff is the same in all cases as $V_b^{x\to y} = V_b^{y\to x} = V_b^{xy} = \Pi_3 - \kappa$.[22]

Step 3: Now we show that in the monopoly game with respect to both components, it is optimal for the inventor to induce a simultaneous adoption by setting $p_{ib}^{x*} = \kappa_x$ and $p_{ib}^{y*} = \kappa_y$ since $V_i^{xy} > V_i^{y\to x} \geq V_i^{x\to y}$.[23] (The last inequality is strict if and only if $\kappa_x < \kappa_y$.)

[22] In order to derive this, plug in corresponding hazard rates into the following three: $V_b^{x\to y} = \lambda^x\{(\Pi_3 - \kappa) + (\Pi_3 - \kappa_y)\}/(r+2\lambda^x)$, $V_b^{y\to x} = \lambda^y\{(\Pi_3 - \kappa) + (\Pi_3 - \kappa_x)\}/(r+2\lambda^y)$ and $V_b^{xy} = \lambda^{xy}\{(\Pi_3 - \kappa) + \Pi_3\}/(r+2\lambda^{xy})$.

[23] In fact, when $p_{ib}^x = \kappa_x$ and $p_{ib}^y = \kappa_y$, not only the sequence xy but any other sequences can take place because buyers are indifferent among the following actions: cc, cw, wc and ww. If cw or wc is included in an optimal behavioral strategy of b given $p_{ib}^x = \kappa_x$ and $p_{ib}^y = \kappa_y$ with strictly positive probability, i would lower prices for each component by ϵ in order for b to mix only adopting or waiting for both and thereby to increase her expected payoff. (p_{ib}^x and p_{ib}^y approach to κ_x and κ_y from below, but cannot hit that point.) With that said, if we do not specify optimal behavior of the imitators in this case, a price equilibrium may not exist. To guarantee the existence of equilibrium, we assume that the imitators behave in such a way that the inventor's expected payoff is maximized when they are indifferent among several actions. Another simple justification, which yields a similar consequence, would be that the inventor can sell knowledge components through a bundling.

First, let us show $V_i^{xy} > V_i^{y\to x}$. If we take $V_i^{y\to x}$ from V_i^{xy}, we get

$$V_i^{xy} - V_i^{y\to x} = \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}(\Pi_1 - \Pi_3 - \kappa_x) - (\mu^{xy} - \mu^y)\kappa_y.$$

By replacing μ^{xy}, μ^y and $\mu^{x|y}$ with the expressions in step 2, we have

$$\begin{aligned} V_i^{xy} - V_i^{y\to x} &= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)}(\Pi_1-\Pi_3-\kappa_x) \\ &\quad -\frac{(2\Pi_3-\kappa-\kappa_x)\kappa-(2\Pi_3-\kappa)\kappa_y}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa)}\kappa_y \\ &= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)(\Pi_1-\Pi_3-\kappa_x)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} - \frac{2(\Pi_3-\kappa)\kappa_x\kappa_y}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa)} \\ &= \frac{2\kappa_x\kappa_y\left[(\Pi_3-\kappa)(\Pi_1-\Pi_3-\kappa_x)-(\Pi_3-\kappa)(2\Pi_3-\kappa_x)\right]}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} \\ &= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)(\Pi_1-3\Pi_3)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} > 0 \end{aligned}$$

since all terms in the numerator and the denominator are positive by assumption 1 and 2.

Next, to show $V_i^{y\to x} \geq V_i^{x\to y}$, write the difference between the two as follows:

$$\begin{aligned} V_i^{y\to x} - V_i^{x\to y} &= \left\{(1-\mu^x)(1-\mu^{y|x}) - (1-\mu^y)(1-\mu^{x|y})\right\}(\Pi_1-\Pi_3) \\ &\quad +(1-\mu^y)\left\{\kappa_y + (1-\mu^{x|y})\kappa_x\right\} - (1-\mu^x)\left\{\kappa_x + (1-\mu^{y|x})\kappa_y\right\}. \end{aligned}$$

Substitution of μ^x, $\mu^{y|x}$, μ^y and $\mu^{x|y}$ given in step 2 into the above equation gives, after a similar tedious algebra as shown above,

$$V_i^{y\to x} - V_i^{x\to y} = \frac{4\kappa_x\kappa_y(\Pi_1-3\Pi_3)(\Pi_3-\kappa)(\kappa_y-\kappa_x)}{\{2(\Pi_3-\kappa)+\kappa_x\}\{2(\Pi_3-\kappa)+\kappa_y\}(2\Pi_3-\kappa_x)(2\Pi_3-\kappa_y)},$$

which is 0 if $\kappa_x = \kappa_y$ and strictly positive otherwise.

Step 1 through 3 completes the proof since we only consider symmetric MPE. However, one might wonder whether there exists an asymmetric sequence generating higher payoff than V_i^{xy}. Step 4 demonstrates that when the inventor induces asymmetric sequences, the inventor's payoff is a weighted average of some two of the three expected payoffs: V_i^{xy}, $V_i^{y\to x}$ and $V_i^{x\to y}$ implying it is never optimal for her to do so.

Step 4: Here, I only offer a proof for the case in which the inventor induces j to choose $x \to y$ ($p_{ij}^x = \kappa_x$, $p_{ij}^y > \kappa_y$) and k to choose $y \to x$ ($p_{ik}^x > \kappa_x$, $p_{ik}^y = \kappa_y$), which gives enough description on how to prove the remaining cases. Let $V_i^{x\to y/y\to x}$ denote the expected payoff of the investor gets from this pricing strategy. Then,

$$\begin{aligned} V_i^{x\to y/y\to x} &= \frac{r}{r+\lambda^x+\lambda^y}\Pi_1 + \frac{\lambda^x}{r+\lambda^x+\lambda^y}\left\{\kappa_x + (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3+\kappa_y)\right\} \\ &\quad + \frac{\lambda^y}{r+\lambda^x+\lambda^y}\left\{\kappa_y + (1-\mu^{x|y})\Pi_1 + \mu^{x|y}(\Pi_3+\kappa_x)\right\}. \end{aligned}$$

Rearranging terms and writing it again in terms of $V_i^{x\to y}$ and $V_i^{y\to x}$ completes the proof.

$$\begin{aligned} V_i^{x\to y/y\to x} &= \frac{1}{2(r+\lambda^x+\lambda^y)} \times \Big[r\Pi_1 + 2\lambda^x\left\{\kappa_x + (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3+\kappa_y)\right\} \\ &\quad + r\Pi_1 + 2\lambda^y\left\{\kappa_y + (1-\mu^{x|y})\Pi_1 + \mu^{x|y}(\Pi_3+\kappa_x)\right\}\Big] \\ &= \frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)}V_i^{x\to y} + \frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)}V_i^{y\to x} \end{aligned}$$

where $\frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)}$ and $\frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)} \in (0,1)$, and $\frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)} + \frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)} = 1$. ■

Proof of Corollary 1

If the inventor patents x and keeps y secret, from the proposition 2,

$$V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3+\kappa_y)\right\} - f.$$

On the contrary to this, if the inventor chooses the opposite mix use,

$$V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3+\kappa_x)\right\} - f.$$

If we subtract the second expression from the first, we have

$$\begin{aligned} & e^{-rT}\left\{(\mu^{y|x}-\mu^{x|y})(\Pi_1-\Pi_3) + (1-\mu^{y|x})\kappa_y - (1-\mu^{x|y})\kappa_x\right\} \\ = & \frac{2\left\{(\Pi_1+\Pi_3-2\kappa)\Pi_3 + \kappa_x\kappa_y\right\}(\kappa_y-\kappa_x)}{e^{rT}(2\Pi_3-\kappa_x)(2\Pi_3-\kappa_y)} > 0 \end{aligned}$$

by assumption 1 and 2. ■

Proof of Proposition 3

Result (i): In the text, we have already shown that protecting all by patent is dominated by the optimal mix use of a patent and secrecy. Thus, we only need to show the existence of $\overline{T}^{Com}$. Note that $V_i^M|_{T=0} < V_i^S$, $\lim_{T\to\infty} V_i^M > V_i^S$ by assumption 3 and V_i^M is strictly increasing in T. Therefore, there exists a unique $\overline{T}^{Com} \in (0,\infty)$ which solves $V_i^M = V_i^S$. More specifically,

$$\overline{T}^{Com} = \frac{1}{r}\ln\frac{(1-\mu^{y|x})(\Pi_1 - \Pi_3 - \kappa_y)}{(1-\mu^{xy})(\Pi_1 - \Pi_3 - \kappa) - f}.$$

Result (ii): Given $\overline{T}^{Com}$, note that as κ_y increases and κ_x decreases by the same amount, $\mu^{y|x}$ increases. Thus, the numerator of the log on RHS decreases while all other things remain constant, which causes $\overline{T}^{Com}$ to decrease. ■

Proof of Proposition 4

For the simple innovation, the innovator's optimal payoff is $\max\{V_i^P, V_i^S\}$. Since $V_i^P|_{T=0} < V_i^S$, $\lim_{T\to\infty} V_i^P > V_i^S$ by assumption 3 and V_i^P is strictly increasing in T, there exists a unique $\overline{T}^{Sim} \in (0,\infty)$ which solves $V_i^P = V_i^S$, and

$$\overline{T}^{Sim} = \frac{1}{r}\ln\frac{\Pi_1 - \Pi_3}{(1-\mu^{xy})(\Pi_1 - \Pi_3 - \kappa) - f}.$$

Hence, it follows that the optimal protection mode is patenting if $T < \overline{T}^{Sim}$ and relying on trade secrets otherwise. Taking away $\overline{T}^{Com}$, which was obtained in proposition 4, from $\overline{T}^{Sim}$, we have

$$\overline{T}^{Sim} - \overline{T}^{Com} = \frac{1}{r}\ln\frac{\Pi_1 - \Pi_3}{(1-\mu^{y|x})(\Pi_1 - \Pi_3 - \kappa_y)}.$$

Then, since $\Pi_1 - \Pi_3 > (1-\mu^{y|x})(\Pi_1 - \Pi_3 - \kappa_y)$, it follows $\overline{T}^{Sim} > \overline{T}^{Com}$. ■

Proof of Lemma 4

Let us start from the case $N = 3$. Once we rule out the price structure in which the inventor charges higher than imitations costs for all components by the same logic in lemma 3, equilibrium candidate price structures fall into one of the following three

categories: trying to sell (1) one component (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} > \kappa_2$ and $p_{ib}^{3t} > \kappa_3$), (2) two components (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} \leq \kappa_2$ and $p_{ib}^{3t} > \kappa_3$) and (3) all three components (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} \leq \kappa_2$ and $p_{ib}^{3t} \leq \kappa_3$) for $b \in \{j,k\}$ given that no adoption has occurred. Note that (1) is analogous to the 2 component model because once an imitator adopts a component, they enter a subgame in which the remaining two components are sold in the all-or-nothing adoption fashion as in proposition 2. Since we showed that in proposition 2 that the step-by-step adoption sequence in the two component model is less profitable than the all-or-nothing adoption sequence, the inventor prefers price structure (3) to (1). Also, with a very similar reason, (3) will be chosen over (2).

Applying this logic recursively, it is straightforward to see that for all $N > 3$, the inventor will sell knowledge in all-or-nothing fashion by setting $p_{ib}^{nt} = \kappa_n$ for all $n \in \{1,\ldots,N\}$ and $b \in \{j,k\}$. Hence, slight modification of proposition 2 allows us to have lemma 4 as stated in the body. ■

Proof of Lemma 5

Define components kept by secrecy in a patent-secret mix as N_S. Then, N_S should be an element in $2^{\{1,\ldots,N\}}$, where $2^{\{1,\ldots,N\}}$ is a power set of $\{1,\ldots,N\}$, excluding two extreme protection modes: patenting all ($N_S = \emptyset$) and keeping all-by-secrecy ($N_S = \{1,\ldots,N\}$). We know that if all components in N_S are not adopted by the imitators until T, the inventor's discounted expected payoff starting from T is given by

$$\mu^{N_S}\Pi_1 + (1-\mu^{N_S})(\Pi_3 + \kappa_{N_S})$$

where $\kappa_{N_S} = \sum_{n\in N_S}\kappa_n$, $\mu^{N_S} = 1/(r+2\lambda^{N_S})$ and where and $\lambda^{N_S} = r(\Pi_3 - \kappa_{N_S})/\kappa_{N_S}$ from the lemma 4. Also, note that it is optimal for the inventor to choose $p_{ib}^{nt} > e^{-r(T-t)}\kappa_n$ for all $n \in N_S$ by the same logic in proposition 2. Then, the optimal patent-secret mix is a solution to the following problem:

$$\max_{N_S \in 2^{\{1,\ldots,N\}}\setminus\{\{1,\ldots,N\},\emptyset\}} (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{N_S}\Pi_1 + (1-\mu^{N_S})(\Pi_3+\kappa_{N_S})\right\} - f.$$

It is easy to see that the solution N_S^* maximizes κ_{N_S}, and so $N_S^* = \{2,\ldots,N\}$, i.e., it is optimal patenting only component 1 least costly to imitate and keeping all the other components as trade secrets. ■

Proof of Proposition 6

As the proof for the point (i) is exactly the same as the one in proposition 3, here only the proof for the point (ii) is provided. When $\kappa_1 = \kappa_2 = \ldots = \kappa_N = \kappa/N$,

$$\overline{T}^{Com} = \frac{1}{r} \ln \frac{(1-\mu^{-1})(\Pi_1 - \Pi_3 - (1-\frac{1}{N})\kappa)}{(1-\mu)(\Pi_1 - \Pi_3 - \kappa) - f}.$$

Only the numerator in the log is a function of N. Since μ^{-1} increases in N, $\overline{T}^{Com}$ gets shorter as N increases. ■

Proof of Proposition 7

We will first show (α^*, ρ^*) is in the interior of $(0,1] \times [0,1/2]$, which implies each area is not an empty set, and then investigate equilibrium protection mode in each area in turn.

Step 1: We show that (α^*, ρ^*) is in the interior of $[0,1] \times [1/2,1]$. Since $\alpha^* = (V_i^S - \Pi_3)/(\Pi_1 - \Pi_3) \in (0,1)$ is obvious, it is sufficient to show that $\rho^* = \tilde{\rho}(\alpha^*) = \hat{\rho}(\alpha^*) = \frac{(V_i^S - V^1)(\Pi_1 - \Pi_3)}{(V_i^S - \Pi_3)(\Pi_1 - V^1)} \in (1/2,1)$. First, $\rho^* < 1$ since $(V_i^S - \Pi_3)(\Pi_1 - V^1) - (V_i^S - V^1)(\Pi_1 - \Pi_3) = (\Pi_1 - V_i^S)(V^1 - \Pi_3) > 0$. Next, observe $\rho^* > 1/2$ is equivalent to

$$2(V_i^S - V^1)(\Pi_1 - \Pi_3) - (V_i^S - \Pi_3)(\Pi_1 - V^1) \quad > \quad 0.$$

After some algebra, one can easily verify that the above inequality is equivalent to the following condition:

$$(\Pi_1 - \Pi_3)(V_i^S - V^1) - (\Pi_1 - V_i^S)(V^1 - \Pi_3) > 0.$$

Since $\Pi_1 - \Pi_3 > \Pi_1 - V_i^S$, it remains to show $V_i^S - V^1 > V^1 - \Pi_3$. Now to conclude, note that

$$\begin{aligned} V_i^S - V^1 - (V^1 - \Pi_3) &= \\ (\mu^2 - 2\mu^1)(\Pi_1 - \Pi_3) - (\mu^2 - \mu^1)\kappa &= \\ \frac{(\Pi_1 - 3\Pi_3)\kappa^2}{(4\Pi_3 - \kappa)(2\Pi_3 - \kappa)} &> 0. \end{aligned}$$

where the second equality follows from replacing μ^2 and μ^1 with the expressions

given in the body and rearranging terms.

Step 2: Patterns of optimal protection in each area are as follows.

Area (i): For all T, $V_i^S > V_i^P$ and $V_i^S > V_i^M$ given $\alpha \leq \alpha^*$ and $\rho < \hat{\rho}(\alpha)$. Hence, S is trivially optimal for the inventor for all T.

Area (iv): Given $\tilde{\rho}(\alpha) \leq \rho \leq 1$, $V_i^M > V_i^P$ for all T given α, which implies P is never optimal. Also, since we consider the area $\hat{\rho}(\alpha) < \rho \leq 1$ where $V_i^S > V_i^M|_{T=0}$ but $V_i^S < \lim_{T\to\infty} V_i^M$ given α, there exists a threshold $\overline{T}^M$ such that $V_i^S = V_i^M|_{T=\bar{T}^M}$ by the Intermediate Value Theorem. Moreover, since V_i^M is monotonically increasing in T the threshold is unique. Thus, the optimal protection mode is S for $T \leq \overline{T}^M$ and M for $T > \overline{T}^M$.

Area (ii) and (iii): First, consider the following subarea where $\rho < \hat{\rho}(\alpha)$ and $\alpha > \alpha^*$. Here, applying a similar logic used in a proof for area (iv), we can show that the optimal protection mode is S for $T \leq \overline{T}^P$ and M for $T > \overline{T}^P$ where $\overline{T}^P$ is such that $.V_i^S = V_i^P|_{T=\bar{T}^P}$. Next, consider the rest of the area. Here, for any (α,ρ) $\lim_{T\to\infty} V_i^P > \lim_{T\to\infty} V_i^M > V_i^S$, and recall that $V_i^S > V_i^M|_{T=0} > V_i^P|_{T=0}$. Therefore, by continuity of payoff functions we know that S is optimal when T is small and P is optimal when T is large. One remaining question is whether there are some intermediate T values under which M is optimal. The previous statement is true if and only if $\overline{T}^P > \overline{T}^M$ where $\overline{T}^P$ and $\overline{T}^M$ are defined as above. That is because in that case as T increases V_i^M surpasses V_i^S first while V_i^P is still less than V_i^S even if it is overtaken by V_i^P eventually. To see $\overline{T}^P > \overline{T}^M$ is equivalent to $\rho > \rho^*$, observe the following:

$$\begin{aligned} & \overline{T}^P > \overline{T}^M \\ \iff & \tfrac{1}{r}\ln\tfrac{\alpha(\Pi_1-\Pi_3)}{\alpha(\Pi_1-\Pi_3)-(V_i^S-\Pi_3)} > \tfrac{1}{r}\ln\tfrac{\rho\alpha(\Pi_1-V^1)}{\rho\alpha(\Pi_1-V^1)-(V_i^S-V^1)} \\ \iff & \tfrac{\alpha(\Pi_1-\Pi_3)-(V_i^S-\Pi_3)}{\alpha(\Pi_1-\Pi_3)} < \tfrac{\rho\alpha(\Pi_1-V^1)-(V_i^S-V^1)}{\rho\alpha(\Pi_1-V^1)} \\ \iff & \rho > \tfrac{(V_i^S-V^1)(\Pi_1-\Pi_3)}{(V_i^S-\Pi_3)(\Pi_1-V^1)} = \rho^* \end{aligned} .$$

In sum, in area (iii) the optimal protection mode is S if T is small, M if T is intermediate and P otherwise whereas S if T is small and P otherwise in area (ii). ■

References

Anton, J. J., Yao, D. A. 2004. Little Patents and Big Secrets: Managing Intellectual Property. The RAND Journal of Economics, 35, 1–22.

Arora, A. 1997. Patents, Licensing, and Market Structure in the Chemical Industry. Research Policy, 26, 391–403.

Belleflamme, P., Bloch, F. 2013. Dynamic Protection of Innovations through Patents and Trade Secrets. CESifo Working Paper, No. 4486.

Bhattacharya, S., Guriev, S. 2006. Patents vs. Trade Secrets: Knowledge Licensing and Spillover. Journal of the European Economic Association, 4, 1112–1147.

Choi, J. P. 1998. Patent Litigation as an Information Transmission Mechanism. The American Economic Review, 88, 1249–1263.

Cohen, W., Nelson, R., Walsh, J. 2000. Protecting Their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not).Technical report, National Bureau of Economic Research, Cambridge, MA.

Cugno, F., Ottoz, E. 2006. Trade Secret vs. Broad Patent: The Role of Licensing. Review of Law and Economics, 2, 209–221.

Denicolo, V., Franzoni, A. L. 2004. Patents, Secrets, and the First-Inventor Defense. Journal of Economics & Management Strategy, 13, 517–538.

Erkal, N. 2005. The Decision to Patent, Cumulative Innovation, and Optimal Policy. International Journal of Industrial Organization, 23, 535–562.

Friedman, D. D., Landes, W. M., Posner, R. A. 1991. Some Economics of Trade Secret Law. Journal of Economic Perspectives, 5, 61–72.

Fudenberg, D., Tirole, J. 1991. Game Theory. Cambridge, MA, The MIT Press, 1–603.

Gallini, N. T. 1992. Patent Policy and Costly Imitation. The RAND Journal of Economics, 23, 52–63.

Gilbert, R., Shapiro, C. 1990. Optimal Patent Length and Breadth. The RAND Journal of Economics, 21, 106–112.

Hall, B., Helmers, C., Rogers, M., Sena, V. 2014. The Choice between Formal and Informal Intellectual Property: A Review. Journal of Economic Literature, 52, 375–423.

Henry, E., Ponce, C. J. 2011. Waiting to Imitate: On the Dynamic Pricing of Knowledge. Journal of Political Economy, 119, 959–981.

Jorda, K. F. 2007. Trade Secrets and Trade-Secret Licensing.. In A. Krattiger, R. T. Mahoney, L. Nelsen eds. Intellectual Property Management in Health and Agricultural Innovation: A Handbook of Best Practices, MIHR: Oxford, U.K., and PIPRA: Davis, 1043–1057.

Kultti, K., Takalo, T., Toikka, J. 2007. Secrecy versus Patenting. RAND Journal of Economics, 38, 22–42.

Kwon, I. 2012. Patent Races with Secrecy. Journal of Industrial Economics, 60, 499–516.

Lee, J. 2015. Patents, Secrets, and Competitive Pressure. Working Paper, 1–32.

Lemley, M., Shapiro, C. 2005. Probabilistic patents. Journal of Economic Perspectives, 19, 75–98.

Levin, R. C., Klevorick, A. K., Nelson, R. R., Winter, S. G., Gilbert, R., Griliches, Z. 1987. Appropriating the Returns from Industrial Research and Development. Brookings Papers on Economic Activity, 1987, 783–831.

Orly, L. 2013. Filing for a Patent Versus Keeping Your Invention a Trade Secret. Harvard Business Review.

Pan, S. P. 2010. Hybrid Use of Trade Secret and Patent Protection in Green Technology. Bloomberg Law Reports, 3.