Rise of Factor Investing:
Asset Prices, Informational Efficiency, and Security Design*

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Abstract

We model financial innovations such as Exchange-Traded Funds, smart beta products, and many index-based vehicles as composite securities that facilitate trading common factors in assets' liquidation values. Through accessing a larger basket of assets in endogenously-chosen proportions, they can benefit both informed and liquidity traders. Optimal designs feature selecting liquid and representative assets, and attracts all factor investors regardless of their informedness or liquidity needs. Consistent with empirical findings, introducing composite securities leads to higher price variability and co-movements, larger trading costs and synchronicity, and lower asset-specific but higher factor information in prices, especially for illiquid assets. Transparency of composite security trading, distinction between composite bundles and derivatives, and endogenous information acquisition also significantly affect prices and security design.

Key Words: *Composite Securities, ETFs, Smart Beta, Index-Linked investment, Financial Innovation, Security Design, Informational Efficiency.*

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1. Introduction

Since their humble inception more than two decades ago, Exchange Traded Funds (ETFs) and other exchange traded products have proliferated beyond expectations.[1] This development represents the latest episode of the rise of indexing and passive investing over the past 40 years, when "composite securities" such as passive mutual funds and index futures have flourished.[2] In fact, many composite securities have also grown beyond their initial function of tracking large liquid indices. New hybrid forms of active and passive investing, generally referred to as smart betas or alternative indexing or enhanced indexing, represent one hotbed of recent development.[3] As a result, anyone with a brokerage account can now choose from among more than 5,000 different composite securities, covering almost every conceivable asset class, geographic region, market sector, and fashionable trading strategy.

Despite their rapid rise and growing importance, there lacks a clear understanding of why composite securities such as ETFs have become so popular and how they affect the trading and pricing of the underlying assets. Proponents have surely articulated the manifold benefits, but voices of concerns also emerge over their growth.[4] Empirical evidence is far from conclusive and it has been difficult to contemplate the design of composite securities and gauge their impact on asset prices and trading behaviors. This paper aims to fill this gap by developing a parsimonious model that illustrates important economic forces at play when a composite security gets introduced, and complements prior literature by demonstrating that underlying the emergence of many innovative products is the rise of factor investing. It thus provides a unifying framework to analyze various composite securities from an informational perspective, adds theoretical insights on composite security design, and rationalizes recent

[1] By December 2015, they have surpassed the hedgefund industry in AUM with about $3 trillion USD, had 23 consecutive month of positive net flows, and set a record of annual asset gathering for $372 billion USD. As much as 30% of U.S. equity trading volume is attributable to ETFs and 43% of the 183 US-based institutional investors invest at least 10 per cent of their total assets in ETFs, according to Boroujerdi and Fogertey (2015), and ETFGI, a research consultancy's recent Greewich Associates' survey. Madhavan et al. (2016) provides a comprehensive discussion on ETFs.

[2] Stambaugh (2014) describes the trends in passive investing and relates that to declining ownership by individual investors.

[3] See Shores (2015) for more details.

[4] On one extreme, Zingales (2009) argues for the necessity of regulatory protection that would prohibit individual investors from investing in individual stocks and encourage them to invest exclusively in exchange-traded funds (ETFs) or in mutual funds. On the opposite side, John Bogle, founder of Vanguard and creator of the first index fund, has long advocated tighter regulations on ETFs. Luis Aguilar, commissioner of the SEC, also publicly argues for major reform of the ETF industry, after trading in a fifth of all US-listed ETFs and in 257 securities was halted 1,278 times on August 24, 2015, the worst trading day in the US for four years. Details can be found at https://www.sec.gov/news/statement/aguilar-emsac-10-2015.html. Academic articles such as Israeli et al. (2016), Bhattacharya and O'Hara (2015), and \cite{ramaswamy2011market} also point to the reality that ETFs may reduce market efficiency and increase systemic fragility.

empirical findings while making additional predictions.

Specifically, we extend Kyle (1985) and Admati and Pfleiderer (1988) to multiple asset markets and potentially a composite security whose price is simply the weighted sum of underlying asset prices. In a risk-neutral setting, there are two underlying representative assets with liquidation values comprised of two parts: a loading on a systematic factor and an asset-specific component. There are two types of speculators: asset-specific speculator who can costly acquire asset-specific information and a factor speculator who costly acquires factor information. There are also two types of liquidity traders: asset-specific liquidity traders who have exogenous needs for an asset-specific component (and thus the asset), and factor liquidity traders who have exogenous exposure requirements for the common factor.

A group of intermediaries supply composite securities as weighted bundles or derivatives of the underlying assets, to maximize their clients' payoffs and thus the management fees. Composite securities are non-redundant because absent them, investors can only trade a subset of available or knowable assets represented by one underlying asset in the model. This is a reduced-form representation of the high transaction or search costs associated with many assets before the introduction of composite securities. All investors submit orders on each asset based on their information and liquidity needs to a market maker. Orders on composite securities involving trading the underlying assets in the weights specified for the bundle. As in Kyle (1985), market makers are specialized and competitive, and set prices to break even.

We derive and characterize the unique equilibrium with linear trading strategies and pricing rules in which both factor speculator and liquidity traders trade composite securities, and contrast that to the unique linear equilibrium absent composite securities. The factor speculator prefers composite securities because he can exploit his informational advantage without creating too much price impact in a single asset market; factor liquidity traders also prefer composite securities because collectively they are less adversely selected by asset speculators through coordinated trading, even though they still face factor speculators. In this regard, composite securities are quintessentially a factor investing tool because no matter one is informed of systematic news or is simply getting an exposure to the factor, both his own price impact (if he is atomic) and the collective price impact of similar agents are lower compared to the situation in which he only trades a subset of the underlying assets.

We then endogenize composite security design and prove that it is independent of the factor speculator's informedness, or the factor liquidity traders's exposure needs, or the composition of a designer's clients. The optimal weights are proportional to assets' factor exposure but inversely relate to their illiquidity. With transparent composite security trading, a non-trivial set of designs exist that attract factor investors. Therefore, the popularity of passive indexing can be interpreted as a reflection that market-cap weighting approximately picks liquid representative assets. This also helps explain why some smart beta products are gaining momentum: by deviating from market-cap weights, they could better facilitate factor investing or capture factors different from the market. Our findings also suggest that the so-called passive investing is mainly factor investing from an informational perspective, and is not passive for investors because not only is there active decision on which factor or when or how much to invest in, but it is not always predicated on the market being efficient either: trading composite securities relies on the divergence between prices and liquidation values, and the trading itself affects the informational efficiency in prices, which is important in decisions such as managerial compensation, adjustment to capital structure, and real investments.

We find that after introducing composite securities with endogenous designs, asset prices reflect more systematic information and less asset-specific information, because market makers, understanding that the factor speculator now fully exploits his informational advantage, set prices more sensitive to the composite security orders. Moreover, asset-specific speculators may stop acquiring information when factor liquidity traders endogenously switch to trading composite securities (thus providing less camoufladge in the underlying assets market). As long as the asset-specific informational asymmetry is not extremely small relative to the systematic informational asymmetry, introducing composite securities improve the overall pricing efficiency. Consistent with earlier studies, price variability, co-movement, and synchronicity all go up, which is partially due to better reflection of the systematic information. Moreover, we provide conditions under which composite securities are more liquid than the underlying assets in terms of average price impact, or reduces the underlying assets' liquidity.These implications differ from many prior studies but are consistent with recent studies on ETFs.

In our analysis, we also highlight the role of transparent composite security trading, the difference between composite bundles and composite derivatives, and endogenous information acquisition. We say composite security trading is transparent if the market markers

further observe the orders for composite securities, which effectively groups traders into factor investors and non-factor investors. Factor liquidity traders are pitched directly against factor speculators and their equilibrium payoffs are less sensitive to composite security design. Composite derivatives which are just side contracts without directly affecting the demand and supply of underlying assets have asset pricing implications often opposite to that of composite bundles in the baseline model that "lock in" the underlying assets. Finally, endogenous information acquisition further strengthens composite securities' impact on the informational efficiency of asset prices, as asset-specific information acquistion could endogenously decline and systematic information acquisition increase. We underscore that these are important dimensions in which various types of composite securities differ, and should be taken into consideration when designing and regulating composite securities. We also show our main results and intuition remain valid with endogenous noise trading and mixed-strategy equilibrium.

Our paper relates to the fast growing empirical literature on the economic consequences of indexing and trading of composite securities, especially ETFs. Ben-David et al. (2014) and Madhavan and Sobczyk (2014) find evidence that ETFs are more liquid than the underlying basket. Ben-David et al. (2014) and Krause et al. (2014) talk about elevated intraday return volatility. Hamm (2014) finds that ETFs and passive mutual funds deprive underlying securities' liquidity. Bradley and Litan (2011) also voice concerns that ETFs may drain the liquidity of already illiquid stocks and commodities. Da and Shive (2013) show ETF arbitrage causes co-movements. Leippold et al. (2015) documents correlations of underyling assets' returns in the presence of ETFs and index futures. Israeli et al. (2016) provide further informational perspective to the debate by showing that an increase in ETF ownership is associated with higher trading costs, greater return synchronicity, reduced price efficiency, and less information acquisition. Glosten et al. (2015) also find ETF trading increases co-movement and return synchronicity, but argue ETFs actually increase informational efficiency for small stocks. Our model not only produce predictions consistent with these empirical findings but also provides a framework to interpret and reconcile seemingly contradictory results. We also provide novel testable implications beyond ETFs.

Despite the emerging empirical studies on composite securities, few theory papers examine their informational effects or explain their rise in popularity. Subrahmanyam (1991) and Gorton and Pennacchi (1993) are two notable exceptions. Subrahmanyam (1991) highlights how liquidity traders could be better off trading composite securities with mitigated adverse selection. Gorton and Pennacchi (1993) belabor a similar point, but focus on risk-averse

liquidity traders and do not distinguish systematic versus asset-specific information or endogenize information acquisition. Also related are Stambaugh (2014) on the relationship between growth in passive investing and the decline in noise trading, Pan and Zeng (2016) on liquidity mismatch between ETFs and underlying assets, Malamud (2015) on limits to arbitrage of risk-averse APs under symmetric information, and Bhattacharya and O'Hara (2015) on information linkages in ETF markets and fragility. Distinct from these papers and as a complement to them, this paper does not rely on risk-aversion or mispricing due to failure of arbitrage. Furthermore, we endogenize traders in composite securities without exogenously assuming additional noise trading (Subrahmanyam (1991), Bhattacharya and O'Hara (2015), and Malamud (2015)) or informed trading (Gorton and Pennacchi (1993)) in composite securities.[5] While earlier studies typically emphasize the welfare of liquidity traders, we highlight that composite securities attract both informed and liquidity traders, leading to drastically different implications on different types of informational efficiency and investor welfare. Finally, these studies do not examine composite security design or derive the optimal design in closed-form.

More broadly, this paper relates to the literature of financial innovation and security design.[6] In contrast to the general approach in these papers, we focus on the informational impact of an innovation that drastically reduces trading costs and increases access to many assets. Although composite securities in our model reduce asset-specific information sensitivity, they could increase factor information sensitivity, and thus our paper complements studies such as Gorton and Pennacchi (1990) regarding how financial innovations impact liquidity.

The rest of the paper is organized as follows: Section 2 sets up the model and characterizes the sub-equilibrium given the security design. Section 3 endogenizes composite security design. Section 4 presents the model implications on asset prices and informational efficiency. Section 5 extends the model along several dimensions and discusses endogenous information

[5]We basically postulate that the powerful forces of arbitrage render many composite securities simply as bundles of the underlying assets, in order to abstract from short-term freezes of arbitrage liquidity and focus on the long-term informational and pricing effects. In addition to endogenizing the trading choice of liquidity traders, endogenizing the total amount of noise trading a la Hassan and Mertens (2011) may also prove fruitful, but is beyond the scope of this paper.

[6]See Duffie and Rahi (1995), Allen and Gale (1994), and Tufano (2003) for comprehensive surveys of the earlier literature. Recent studies include Kubler and Schmedders (2012), Simsek (2013), and Banerjee and Graveline (2014).

acquisition and liquidity trading, mixed-strategy equilibria, trading transparency, and composite derivatives. Section 6 concludes. All proofs are in the appendix.

2. A Model of Speculative and Liquidity Trading

In this section, we develop a variant of the classic Kyle (1985) model and characterize a set of linear equilibria with and without composite securities (henceforth referred to as "CS"). Given that one important application of the theory is in ETF trading, one can think of CSs in the baseline model as physical ETFs. We differentiate various types of CSs in Section 5.

2.1 Model Setup

Assets and Liquidation Values

For simplicity, we consider an economy with $N = 2$ underlying assets, which could represent two baskets of assets. The liquidation value S_i of asset i , $i = 1, 2$, comprises of a systematic component γ and an asset-specific component ϵ_i:

$$S_i = \bar{S}_i + \beta_i \gamma + \epsilon_i \tag{1}$$

where $\bar{S}_i$ is the expected payoff of the asset, and β_i is security i's exposure to factor γ. Here γ could represent a macroeconomic shock or a industry-wide technology change that affects the payoff of all securities in a systematic way. γ, ϵ_1, and ϵ_2 are mutually independent normally distributed random variables with mean zero and standard deviationsσ_γ, σ_{ϵ_1}, and σ_{ϵ_2} respectively. $\bar{S}_i$ (normalized to zero for the remainder of the paper without loss of generality), β_i and the prior distributions on γ, ϵ_1, and ϵ_2 are public knowledge.

Unless otherwise specified, CSs are bundles of the underlying securities $(w_1 S_1, w_2 S_2)$ with $w_1 + w_2 = 1$, where the weights w_i on each asset are design parameters. The price of a CS is thus the weighted sum of the prices of the underlying assets. We analyse in Section 5 composite derivatives that do not entail trading the underlyings directly.

Market Participants, Information, and Trading

We assume all agents are risk-neutral to focus on the informational aspect of CS trading.[7] There are four types of traders:

[7]Alternatively, we could use a CARA-Normal setup, but the solutions are less analytical and intuitive.

(a) An **asset speculator** for each asset i who can costly acquire imperfect information about ϵ_i, namely a noisy signal $\chi_i = \epsilon_i + \nu_i$, and trade to maximize profit.
(b) A **factor speculator** who can costly acquire imperfect information about γ, namely a noisy signal $\zeta = \gamma + \xi$, and trades to maximize profit.
(c) A unit measure of **asset liquidity traders** for each asset i , who trade to minimize costs to satisfy an exogenous need $\hat{n}_i$ for exposure to ϵ_i, and thus asset i.
(d) A unit measure of **factor liquidity traders**, who trade to minimize costs to satisfy an exogenous need τ for exposure to factor γ. A fraction f_i achieves this exposure using asset i.

where ν_i, ξ, $\hat{n}_i$, and τ are mutually independent and normally distributed with mean zero and standard deviations σ_{ν_i}, σ_ξ, $\sigma_{\hat{n}_i}$, and σ_τ respectively. We refer to (b) and (d) collectively as **factor investors**. We take the information of speculators as given here, and discuss endogenous information acquisition in Section 5.1. As commonly assumed in the literature, liquidity traders trade for reasons outside the model. For instance, a fund manager may need to achieve certain positions in securities required by his clients, or a tech entrepreneur wishes to hedge against the systematic risk in cloud computing.[8] Earlier models tyically assume an aggregate liquidity trading n_i in each market. We basically decompose n_i into asset-specific and factor liquidity trading. Mathematically, $n_i = \hat{n}_i + f_i\tau/\beta_i$, where f_i is the exogenous fraction of τ achieved via trading asset i, and $f_1 + f_2 = 1$. We endogenize f_i in Section 5, which in turn endogenizes the correlation and relative amounts of liquidity trading across asset markets before the introduction of CS.

Technologies to cheaply access and trade large baskets of assets have improved over time, enabling intermediaries to charge low fees to clients to trade portoflios of the underlying assets with specific weights. Passive mutual funds, ETFs, and funds issuing smart beta products are some examples. We model them as a unit measure of competitive **CS sponsors**, who trade the underlying securities to meet clients' demand for CS. To capture the reality that their payoffs are typically proportional to assets under management, we assume they maximize their clients' payoffs, and for the same maximum client payoff, they maximize

[8]Factor liquidity trading here captures in a reduced-form hedging-motivated trading in financial assets, where agents "hedge" against background risks outside the model. For example, risk-averse uninformed traders generate "noisy" demand due to random endowments inSpiegel and Subrahmanyam (1992), and factor liquidity traders with different trading opportunities as in Goldstein et al. (2014) need the exposure to hedge against their positions in other assets.

the percentage of clients' trade through CS, in order to attract maximum assets.[9]

Speculators, liquidity traders, and CS sponsors submit orders to a rational and competitive market maker in each market, who then prices the asset at her expected value based on observed order flow in order to break even. We make the standard assumption that each market maker specializes in one market only and observes the order flow in the market. Orders on composite securities simply translate into orders on the underlying securities with the corresponding weights. After the trading round, assets' liquidation values are realized.

Finally, to model the limited diversification due to constraints on technology or attention, and high transaction or search costs, we assume that investors other than CS sponsors can only trade one single asset. This assumption is realistic–each asset in the model could be thought of as a nonexhaustive basket of assets, and trading the rest would be prohibatively costly due to the transaction cost of trading a large number of additional assets, the limited accessibility of assets to retail investors, or the fact that searching and learning about assets' relevance for a factor is costly under localized information and uncertain market encironment.[10] Note that this assumption would not apply if there are already composite securities such as an index product covering the same factor and with similar designs.

2.2 Equilibrium before Introducting CS

An equilibrium without CS is defined as follows:

- The asset speculator for security i submits an order $x_i = X_i(\chi_i)$ to maximize profit;
- The factor speculator submits order $y_i = Y_i(\zeta)$ (i=1,2) in either asset market 1 and 2, but not both, to maximize profit.
- Knowing the equilibrium trading behavior of each investor, market maker i breaks even in expectation by setting $P_i(\omega_i)$ after receiving a total order ω_i. Specifically,

[9]Alternatively, we could have one representative CS sponsor who maximizes her clients' payoffs. Fixing a client's profit, a higher fraction of trading (or total order flow in shares or dollar amounts) through CS represents a higher fraction of the clients' asset managed at the CS fund, which in turn corresponds to a higher management fee collected. Adding this requirement pins down the unique optimal design, but is not crucial for other results. One may question why CS sponsors do not maximize their clients' trading profts in CS (rather than overall profits), which seems to be a reasonable objective if they charge a carry. We argue that even in this scenario, they must be maximizing the overall profit as well, lest the clients switch to other CS sponsors with better designs. In that regard our specification nests such cases.

[10]Indeed, transaction costs are high in bond markets, REITs, asset-backed securities, etc. For our results to hold, we just need a significant fraction of the investor population to endogenously trade a smaller subset of available assets than what composite securities would allow them.

market makers form a common equilibrium belief on the probability θ_i of the factor speculator trading in asset i, where $\theta_1 + \theta_2 = 1$.

This section characterizes the pure strategy equilibrium (when exists) with either $(\theta_1, \theta_2) = (1, 0)$ or $(\theta_1, \theta_2) = (0, 1)$. Section 5.3 extends the analysis to mixed-strategy equilibrium where an equilibrium always exists.

Following the literature, we focus on equilibria with linear trading and pricing strategies, and conjecture $X_i(\chi_i) = \alpha_i \chi_i$ and $Y_i(\zeta) = \eta_i \zeta$. Without loss of generality, let us examine the equilibrium with $(\theta_1, \theta_2) = (1, 0)$, in which the factor speculator endogenously prefers market 1. Throughout the rest of the paper, unless otherwise noted, we will refer to market 1 as the originally liquid market and market 2 as the originally illiquid market. The following Lemma 1 characterizes this equilibrium.

Lemma 1

When $\lambda_1^N \beta_2^2 < \lambda_2^N \beta_1^2$, *where* $\lambda_1^N = \frac{1}{2\sigma_{n_1}} \sqrt{\left(\sigma_{\epsilon_1}^2 - \sigma_{\nu_1}^2\right) + \beta_1^2 \left(\sigma_\gamma^2 - \sigma_\xi^2\right)}$, $\lambda_2^N = \frac{1}{2\sigma_{n_1}} \sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2}$, *and* $\sigma_{n_i}^2 = \sigma_{\hat{n}_i}^2 + f_i^2 \sigma_\tau^2 / \beta_i^2$, *the asset speculators and factor liquidity traders submit* $X_i(\chi_i) = \frac{\chi_i}{2\lambda_i^N}$ *and* $f_i \frac{\tau}{\beta_i}$ *in market* i, *the factor speculator submits* $Y_1(\zeta) = \frac{\beta_1}{2\lambda_1^N} \zeta$ *in market 1, and the market makers set prices to be* $P_i^N(\omega_i) = \lambda_i^N \omega_i$.

In equilibrium, the asset speculator i*'s and factor speculator's expected profit are* $\Pi_{\epsilon,i}^N \left(\sigma_{\nu_i}^2\right) = \frac{1}{4\lambda_i^N} \left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right)$ *and* $\Pi_\gamma^N \left(\sigma_\xi^2\right) = \frac{\beta_1^2}{4\lambda_1^N} \left(\sigma_\gamma^2 - \sigma_\xi^2\right)$ *respectively; the asset* i *liquidity traders' and the factor liquidity traders' aggregate expected loss are* $C_i = \lambda_i^N \sigma_{\hat{n}_i}^2$ *and* $C_F = \frac{\lambda_i^N f_i \sigma_\tau^2}{\beta_i^2}$ *respectively.*

As in Kyle (1985), speculators' profits are increasing in their informational advantage $\sigma_\gamma^2 - \sigma_\xi^2$ or $\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2$, and decreasing in their price impacts λ_i^N. Liquidity traders face adverse selection from both asset speculators and the factor speculator, as reflected in λ_i^N. For example, the expected loss for factor liquidity traders is

$$E[C_1(\tau)] = \frac{\sigma_\tau}{2\beta_1} \sqrt{\left[\beta_1^2 \left(\sigma_\gamma^2 - \sigma_\xi^2\right) + \left(\sigma_{\epsilon_1}^2 - \sigma_{\nu_1}^2\right)\right] \frac{f_1^2 \sigma_\tau^2 \beta_1^{-2}}{f_1^2 \sigma_\tau^2 \beta_1^{-2} + \sigma_{\hat{n}_1}^2}} \tag{2}$$

$\beta_1^2\left(\sigma_\gamma^2-\sigma_\xi^2\right)+\left(\sigma_{\epsilon_1}^2-\sigma_{\nu_1}^2\right)$ reflects the adverse selection by speculators in the market whereas $\frac{f_1^2\sigma_\tau^2\beta_1^{-2}}{f_1^2\sigma_\tau^2\beta_1^{-2}+\sigma_{\hat{n}_1}^2}$ measures the fraction of adverse selection faced by factor liquidity traders as opposed to asset liquidity traders. $\frac{\sigma_\tau}{\beta_1}$ simply represents how much factor liquidity traders need to trade asset 1. In equilibrium, an infinistimal factor liquidity trader must be indifferent between trading in either market, which is made by the equilibrium allocation of the factions (f_1, f_2). On the one hand, a factor liquidity trader would choose to trade in the market with lower trading cost λ_i^N. On the other hand, as more factor liquidity traders flow into the preferred market i, the higher f_i means that the factor liquidity trader would face more correlated hedging order if he trades in market i, which implies a higher trading cost.

2.3 Introducing CS

Now consider introducing a CS (w_1S_1, w_2S_2) with $w_1 + w_2 = 1$, where the weights w_i on each asset are treated as exogenous for now. Again we focus on symmetric linear equilibra. Previously, the factor speculator cannot fully exploit his informational advantage by trading both assets. But now he can via trading CS, if he so chooses. For the same reason, CS may attract factor liquidity traders. While existing studies typically exogenously add noise traders with the introduction of CS, we contribute to the literature by endogenizing liquidity trading in CS. In this section, we consider the case where factor investors can trade CS and one of the underlying assets. Section 3 discusses the case where factor investors still can only trade one of the three securities, as a reduced representation of incomplete market even after introducing CS.

Asset speculator in market i does not have informational advantage in market j and faces adverse selection from asset speculator j. Similarly, asset liquidity traders in market i do not fulfill their liquidity needs by trading asset j, yet face adverse selection. Therefore neither trades CS because doing so reduces profit or increases trading cost. Suppose through trading CS and potentially one underlying asset, the factor speculator and liquidity traders are effective trading the two underlying assets in proportions $\left(\eta w_1^S, \eta w_2^S\right)$ and $\left(\frac{\tau}{\beta_1}w_1^L, \frac{\tau}{\beta_2}w_2^L\right)$ respectively, where $w_1^S + w_2^S = w_1^L + w_2^L = 1$ for normalization, then market maker i observes the total order

$$\omega_i = \alpha_i\chi_i + w_i^S\eta\xi + \hat{n}_i + w_i^L\frac{\tau}{\beta_i} \tag{3}$$

and sets the price $P_i = \lambda_i^{CS}\omega_i$, where

$$\lambda_i^{CS} = \frac{\alpha_i \sigma_{\epsilon_i}^2 + \beta_i w_i^S \eta \sigma_\gamma^2}{\alpha_i^2 \left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \left(w_i^S\right)^2 \eta^2 \left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \sigma_{\hat{n}_i}^2 + \left(w_i^L\right)^2 \frac{\sigma_\tau^2}{\beta_i^2}} \tag{4}$$

Fixing $\left(w_1^S, w_2^S\right)$, the factor speculator chooses

$$\eta = \arg\max_{\eta} \mathbb{E}\left[\eta\zeta \sum_{i=1,2} w_i^S(\beta_i\gamma + \epsilon_i - P_i\omega_i)|\zeta\right] \tag{5}$$

and gets profit

$$\Pi_\gamma^{CS} = \frac{\left(w_1^S\beta_1 + w_2^S\beta_2\right)^2}{4\left(\lambda_1^{CS}\left(w_1^S\right)^2 + \lambda_2^{CS}\left(w_2^S\right)^2\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{6}$$

The factor liquidity traders' trading cost is

$$C_F^{CS} = \frac{\lambda_1^{CS}\left(w_1^L\right)^2 + \lambda_2^{CS}\left(w_2^L\right)^2}{\left(w_1^L\beta_1 + w_2^L\beta_2\right)^2}\sigma_\tau^2 \tag{7}$$

Lemma 2 (Trading Ratio Duality)

Given equilibrium pricing strategies, the proportion of trading the underlying assets that maximizes the factor speculator's profit also minimizes factor liquidity traders' costs.

The intuition is that both the factor speculator and liquidity traders as a group want to maximize factor exposure per unit of trade, which allows greater exploitation of the informational advantage for the speculator and greater fulfillment of the exposure needs for liquidity traders. Their programs are isomorphic because both traders face the same trading cost, which they minimize through trading in certain proportions of the underlying assets, and therefore both prefer the same trading ratio for all factor investors. Lemma 2 allows us to derive the trading equilibrium with CS.

Proposition 1 (Factor-Investing Equilibrium)

There is a unique equilibrium with CS trading. The asset speculator and liquidity traders

submit $X_i(\chi_i) = \frac{\chi_i}{2\lambda_i^{CS}}$ *and* $\hat{n}_i$ *in market* i*; the factor speculator and liquidity traders effectively trade the underlying assets in proportion* $w_1^S : w_2^S = w_1^L : w_2^L = \frac{\beta_1}{\lambda_1^{CS}} : \frac{\beta_2}{\lambda_2^{CS}}$*, and market makers set the price according to* $P_i(\omega_i) = \lambda_i^{CS}\omega_i$*, where* λ_1^{CS}*,* λ_2^{CS} *and* η *are determined by the following system of equations*

$$\eta = \frac{1}{2}\left(\frac{\beta_1}{\lambda_1^{CS}} + \frac{\beta_2}{\lambda_2^{CS}}\right) \tag{8}$$

$$\frac{1}{4\beta_i^2\sigma_\tau^2}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) + \frac{1}{4\sigma_\tau^2}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) - \frac{1}{\beta_i^2\sigma_\tau^2}\left(\lambda_i^{CS}\right)^2\sigma_{\hat{n}_i}^2 = \left(\frac{\lambda_1^{CS}\lambda_2^{CS}}{\lambda_2^{CS}\beta_1^2 + \lambda_1^{CS}\beta_2^2}\right)^2, \quad i = 1, 2. \tag{9}$$

Holding factor speculator's trading the same as in Lemma 1, factor liquidity traders can achieve at lesat the same payoff as in Lemma 1 by setting $w_i^L = \frac{f_i\beta_j}{f_j\beta_i + f_i\beta_j}$, $i \neq j$. This is reminiscent of the "diversification" benefits of CS trading to discretionary liquidity traders in Subrahmanyam (1991) and Gorton and Pennacchi (1993). The idea is that factor liquidity traders mitigate asset-specific adverse selection by trading the basket rather than the portfolio of individual assets, which implies that no asset speculator holds absolute informational advantage against them in the market. Similarly, when liquidity traders are risk averse, a basket reduces the variance in assets' total value, and thus the cost of adverse selection. The underlying mechanism here is rather different: rather than relying on risk-aversion, or separate market making and mispricing between composite securities and underlying securities, we emphasize the **"coordination" benefit** of CS for factor liquidity traders who are infinitesimal. Without CS, they collectively are trading the underlying assets in proportion f_1:f_2, and a CS can coordinate the traders to trade in a different proportion to maximize the "diversification" benefits. Moreover, once we consider the factor speculator, it is unclear if this "diversification" benefit outweighs the adverse selection from the factor speculator. We thus point out the fact that introducing CS may not always benefit the factor liquidity traders.

Correspondingly, holding factor liquidity traders' trading the same as in Lemma 1, the factor speculator's expected profit is weakly improved because they can achieve the same payoff as in Lemma 1 by setting $(w_1^S, w_2^S) = (1, 0)$. We call this **price impact "diversification" benefit** to the factor speculator, which comes directly from the fact that CS expands accessiblity of underlying assets. The factor specualtor can reduce his price impact by simultaneously trading all assets in optimized proportions. Again, if factor speculator and factor liquidity traders both endogenously decide whether to trade CS, it is not guaranteed that

both are better off. The next section provides the conditions for these benefits to dominate and discusses them in more details when we endogenize CS Design.

The above proposition relies on the fact that adding CS completes the market when $N = 2$. To the extent that CS helps complete the market, we always have the above equilibrium. What is interesting is that when we endogenous CS design in the next section, the factor-investing equilibrium stays the same even with incomplete market. Moreover, we get a unique equilibrium whereas earlier studies typically involve multiple equilibria.

3. Composite Security Design

In this section, we endogenize CS design, prove existence of feasible designs, and characterize the optimal design. In particular, we show that the optimal design is independent factor speculator's informedness, factor liquidity traders' trading needs, and CS sponsors' clients.

Because $w_1^S : w_2^S = w_1^L : w_2^L$, a design with the same weights intuitively attracts all factor investors. The trading profit and costs from CS for factor speculator and liquidity traders can be written as

$$\Pi_\gamma^{CS} = \frac{(w_1\beta_1 + w_2\beta_2)^2}{4\left(\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \quad \text{and} \quad C_F^{CS} = \frac{\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2}{(w_1\beta_1 + w_2\beta_2)^2}\sigma_\tau^2$$

once again we have a duality property because Π_γ^{CS} and C_F^{CS} are exactly inversely related when the design changes.

Proposition 2 (Design Duality)

The design that maximizes the factor speculator's profit from trading CS also minimizes factor liquidity traders' trading cost from CS.

Lemma 2 and Proposition 1 and 2 leads to the following theorem, which shows such a design emerges in equilibrium, and characterizes it.

Theorem 1 (Optimal Design)

There is a unique optimal CS design with weights (w_1, w_2) *satisfying* $w_i = \frac{\beta_i \lambda_j^{CS}}{\beta_i \lambda_j^{CS} + \beta_j \lambda_i^{CS}}, i \neq j,$

where λ_i^{CS}s are the unique solution to equations (8) and (9). All factor investors trade CS in equilibrium.[11]

The optimal design is particularly intuitive: because $w_1 : w_2 = \frac{\beta_1}{\lambda_1^{CS}} : \frac{\beta_2}{\lambda_2^{CS}}$, it puts more weight on liquid (low λ^{CS}) and representative (high β) assets. When the market is complete, this design ensures the infinitesimal traders would not deviate; when the market is incomplete and factor investors can only trade one asset (so infinitesimal traders would not deviate to trade both an underlying and CS), they would also prefer this design because it optimizes their objectives. The following corollary describes the dependence of optimal design on the model primitives.

Corollary 1.1 (Comparative Statics)

Asset i's weight w_i in the optimal design is increasing in $\sigma_{\hat{n}_i}$, β_i, σ_{ϵ_j} and decreasing in σ_{ϵ_i}, β_j, $\sigma_{\hat{n}_j}$, for $i = 1, 2$ and $j = 3 - i$.

Intuitively, Theorem 1 and Corollary 1.1 suggest that a CS designer allocates more weights on assets more exposed to the systematic factor, having less asset-specific information asymmetry, and with more noise trading. The above design is robust to a number of considerations, as summarized in the next theorem.

Theorem 2 (Irrelevance Principles)

The optimal CS design does not depend on the informedness of factor speculator (informedness irrelevance), the exposure requirements of factor liquidity traders (liquidity-needs irrelevance), or the composition of CS sponsors' clients (client irrelevance).

The first two irrelevance principles are based on the intuition discussed in Kyle (1985). If the factor speculator is more informed, he would want to trade more agressively. Knowning this, market makers in equilibrium would set prices more sensitive to order flows, effectively preventing more aggressive trading. Similarly, when factor liquidity traders' exposure needs is large, they have to submit larger orders, but knowning this, market makers set prices less sensitive to order flows. Though these issues affect the overal informational advantage

[11]The optimal design would not be unique if we relax the requirement that CS sponsors want to attract all orders from factor investors, because it is possible to combine CS and one of the underlying assets to achieve the same proportion of trading in the underlying assets.

or liquidity cost, they concern the common factor but not individual assets. Therefore the optimal design that minimizing trading costs is unaffected. Finally, duality implies that if a design maximizes the factor speculator' payoff, it also maximizes factor liquidity traders, and the exact composition of them as CS sponsors' clients does not matter. Duality also ensures that a CS sponsor can find a single design that caters to both factor speculator and liquidity traders, thus attracting all factor investors' trades if he prefers.

We now turn to one central question earlier studies ask: do composite securities such as ETFs reduce the trading cost of discretionary liquidity traders?

Proposition 3 (Factor Investors' Welfare)

The introduction of CS is welfare-improving for both the factor speculator and factor liquidity traders as a group if and only if

$$\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}} > \frac{\beta_1^2}{\lambda_1^N} + \frac{\beta_2^2}{\lambda_2^N}, \tag{10}$$

where the λ's are defined as in Lemma 1 and Proposition 1. In particular, as long as the factor liquidity traders are better off, the factor speculator is better off.

Intuitively, the factor speculator's profit in market i is proportional to β_i^2 and inversly proportional to λ_i , and the factor liquidity traders' cost is inversely proportional to β_i^2 and proportional to λ_i. Factor liquidity traders have less price impact than factor speculator to start with because they are infinitesimal and separate into trading two assets even without CS. Therefore if CS reduces their trading cost, it must reduce the factor speculator's trading cost.

For exogenously given CS design (w_1, w_2), the condition becomes $\frac{\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2}{(w_1\beta_1 + w_2\beta_2)^2} < \frac{\lambda_i^N f_i}{\beta_i^2}$. Many prior studies focus on either the factor liquidity traders' or the factor speculator's payoff, and fix behaviors of the other group the same as before the introduction of CS. In such cases there indeed exists a CS that is benefitial to either group:

Corollary 3.1

When the trading strategies of the factor liquidity traders are exogenously fixed before and after the introduction of CS, there exists a design that leads to higher profit for the factor speculator through trading CSs alone in equilibrium.

Corollary 3.2

When the trading strategy of the factor speculator is exogenously fixed before and after the introduction of CS, there exists a design that results in lower trading costs for factor liquidity traders through trading CSs alone in equilibrium.

However, in a factor-investing equilibrium both the factor speculator and factor liquidity traders switch to CS after its introduction, it is thus no longer guaranteed that both are better off. Therefore the characterization in the Proposition 3 is useful in informing us whether factor investors are better off with the introduction of CS. In fact, when we allow mixed strategies, there always exists designs (including the optimal design) that improve the payoffs of both the factor speculator and liquidity traders, contributing to the popularity of CS.

Finally, to close the loop, we have to prove that feasible designs exist to support factor-investing equilibria with CS. While this is trivial when the market is complete, we show that the optimal design supports the equilibrium even when the market is incomplete.

Proposition 4 (Existence)

There always exists a feasible design that leads to the equilibrium in Proposition 1.

This proposition implies that in general, there are feasible designs that attract factor investors. This potentially helps to explain the drastic increase in CS varieties: not only are there multiple factors to design CSs around (extensive margin), there can be multiple feasible CS designs for the same factor (intensive margin), especially when they serve different clienteles outside the model (e.g. with different tax exemptions). Our results are also consistent with Stambaugh (2014), in that the decline in noise trading in individual assets indeed eclipses with CS trading because factor liquidity traders endogenously switch to CS. Such endogenous security choice by traders, combined with endogenous CS design, leads to a wide array of asset pricing implications the next section discusses.

4. Asset Pricing Implications

We now examine how introducing CS affects informational efficiency in underlying asset markets, price volatility, co-movements, synchronicity, and market liquidity, highlighting the heterogeneous impact on underlying assets with differential levels of adverse selection and liquidity trading.

4.1 Informational Efficiency

It is important to understand how introducing CS affects the informational efficiency of asset prices for several reasons. First, information in asset prices can help real decisions. Managerial compensations are often tied to stock prices, and market price reactions to recent announcements can help the managers revise expansion or merger plans. Second, a more efficient secondary market mitigates lemon's problem in the primary market. Third, informational efficiency also affects incentives for information acquisition.

We define three types of informational efficiency: 1. Overall Efficiency: how prices reflect the intrinsic values of the asset; 2. Systematic Efficiency: how prices incorporate systematic information; 3. Asset-specific Efficiency: how prices reflect firm-specific or asset-specific information. Specifically, we measure informational efficiency as the correlation of price with the overall, the systematic component, and the asset-specific component of the intrinsic value of the asset respectively. We refer to an asset as *relatively liquid* if the factor speculator trades it more than the other asset before the introduction of CS. We refer to the other asset as *relatively illiquid*.

Theorem 3 (Informational Efficiency)

Introducing CS weakly decreases asset-specific efficiency, increases systematic efficiency, and improves overall efficiency. The impact is bigger for relatively illiquid assets.

Before introducing CS, the factor speculator was trading less of the relatively illiquid asset (in our baseline setup with pure strategies, he is not trading it at all), therefore factor information is not fully impounded into the price. On the one hand, CS allows his informational advantage to be better exploited, leading to prices more sensitive to systematic information and an increase in systematic efficiency. On the other hand, asset-specific efficiency goes down for two reasons. First, asset-specific information is overshadowed by the systematic

information incorporated through CS trading. Second, once we allow heterogenous cost and endogenize information acquisition, asset speculators with high information acquisition costs find it no longer profitable to acquire information, leading to a discontinuous drop of asset-specific information content in prices at the introduction of CS. Section 5 discusses this second channel in more details.

Our results differ from Subrahmanyam (1991) which predicts that the introduction of a basket tends to increase the number of security analysts for the most heavily weighted securities in the basket, and prices of such securities will become more informative in the security-specific component. Our findings are consistent with empirical studies on ETFs such as Israeli et al. (2016) which documents that firms experiencing a 1% increase in ETF ownership experience a 21% reduction in the magnitude of their future earnings response coefficients, a measure of the association between current firm-specific returns and future firm-specific earnings. In another study, Glosten et al. (2015) find that ETF trading increases information efficiency for small firms and firms with imperfect competitive capital markets by incorporating aggregate accounting information into stock prices in a timely manner, but find no such effect for big stocks. Consistent with their findings, our model reveals that the relatively illiquid asset experiences a larger increase in systematic informational efficiency, and they tend to be small in size and face imperfect competitive markets in real life.

Regarding overal efficiency, while the introduction of CS trading may decrease asset-specific information in asset prices, it better incorporates systematic informaiton. For the relatively illiquid asset, the latter strictly dominates, and introducing CS improves its price's overall efficiency.

4.2 Volatility, Synchronicity, and Co-movements

How CS trading affects price volatility, synchronicity, and co-movements is also important to investors and regulators. Regulators are especially concerned with potential systemic risk originating from excess volatility and over-correlated price movements. Our framework is useful for analyzing these issues. We define synchronicity as the extent to which the variation in stock returns is attributable to general market and related-industry movements (the systematic component). While our model is static, we can view the liquidation value as the value of assets when the asset specific and systematic information becomes available to all and the assets is fairly priced, then co-movements can be captured in a stylized way

by the correlation between the deviations of current prices from true values for the two assets.

Proposition 5 (Volatility, Synchronicity, and Co-movements)

Introducing CS increases price co-movements of the underlying assets, and increases price variability and synchronicity for the relatively illiquid asset.

While earlier studies in the context of index products arrive at similar conclusions, we emphasize the role of systematic information. Subrahmanyam (1991) predicts that the introduction of a basket will have no effect on the variability of price changes of individual securities, but our model predicts that CSs increase volatility of the underlying securities. This is consistent with Ben-David et al. (2014) who find that stocks owned by CSs exhibit significantly higher intraday and daily volatility, and an increase of one standard deviation in CS ownership is associated with an increase of 19% in intraday stock volatility. However, our model suggests an alternative to the authors' conclusion that CSs attract a new layer of demand shocks to the stock market due to their high liquidity: even wihtout adding non-fundamental shocks, it is possible that stock price variance increases (the "reshuffling hypothesis" in Ben-David et al. (2014)) because when agents with various accuracies of information about a systematic component and factor liquidity traders with various need to hedge against the systematic component get reshuffled to CS trading, price reflect this systematic component better. The fundamental variance in the systematic component shows up fully in the price variance. This is not necessarily an unintended effect if we care about how prices reflect information about the systematic components of asset values.

This result is consistent with findings in Crawford et al. (2012) and **Glosten et al. (2015)** that CS trading increases co-movement and synchronicity which is partly attributable to timely incorporation of aggregate earnings information. We provide a theoretical foundation for their argument that while earlier studies emphasize increases in co-movement and return synchronicity due to non-fundamental factors (see, for example, Vijh (1994), Harris and Gurel (1986), and Barberis et al. (2005)), these increases could be attributed to more systematic information being impounded into prices. Barberis et al. (2005), Da and Shive (2013) also show co-movements go up.

Since we already know that correlation of price and firm-specific component goes down, and correlation of price and systematic firm information goes up for illiquid stocks, synchronicity goes up more for illiquid stocks. Small stocks that are more illiquid tend to have

greater increase in synchronicity relative to large stocks. This could be tested in the data.

4.3 Price Impacts and Liquidity

A large literature on security design has shown that financial innovations can alter market liquidity, and some argue that CSs drain underlying assets' liquidity, but are more liquid themselves. The liquidity of CSs and the impact of their trading on the liquidity of underlying assets are more subtle than it appears.

Proposition 6 (Trading Cost)

The transaction cost in the illiquid market increases after introducing CS, if the systematic information asymmetry is large enough relative to the asset specific information asymmetry. A sufficient condition is

$$\frac{\sqrt{\sigma_{\epsilon_2}^2-\sigma_{\nu_2}^2}}{\sigma_{\hat{n}_2}} < \frac{\sqrt{\frac{1}{4}(\sigma_{\epsilon_2}^2-\sigma_{\nu_2}^2)+\frac{\beta_2^2}{4}(\sigma_\gamma^2-\sigma_\xi^2)}}{\sqrt{\frac{\sigma_\tau^2}{\min\{\beta_1,\beta_2\}^2}+\sigma_{\hat{n}_2}^2}} \tag{11}$$

While CSs attract factor liquidity traders, the latter's demand still manifests in orders in the underlying market because to deliver CS one has to trade the underlying assets. This proposition suggests that when the information aysmmetry in the systematic component is large enough, introducing CS trading would potentially increase the trading cost in the initially illiquid asset market. This is because the presence of CS trading makes the market maker of the illiquid asset set prices more sensitive to orders, out of concern about the adverse selection by the factor speculator who is absent before the CS is introduced. Our findings are consistent with Bradley and Litan (2011) on that CSs may drain the liquidity of already illiquid stocks and commodities. Hamm (2014) also finds that ETFs and passive mutual funds deprive underlying securities' liquidity and interprets the findings in light of demand substitution under adverse selection. We argue this is more significant for assets that are relatively illiquid and has more information asymmetry in the systematic component of liquidation values.

Having analyzed the trading cost in underlying security markets before and after the introduction of CS trading, we also compare the trading cost across underlying security market and CS market. Ben-David et al. (2014) and Madhavan and Sobczyk (2014) find evidence that CS's are more liquid than the underlying basket. We show this is indeed the

case for assets with severe adverse selection and asset liquidity trading. A crucial feature of the CS studied here is that there is no seperate market making for CS. That is, the CS price is determined as a weighted average of the underlying security prices, which implies that we should consider the CS order's price impact in both security markets. To construct the measure of liquidity in the CS market, note that the price of per unit of CS is

$$P^{CS} = w_1 P_1 + w_2 P_2$$

Thus submitting one unit of order in the CS market would induce w_1 unit of order in market 1 and w_2 unit of order in market 2. Hence we define the trading cost of CS as the price impact of this additional order:

$$\begin{aligned} \lambda^{CS} &= w_1 \left(w_1 \lambda_1^{CS} \right) + w_2 \left(w_2 \lambda_2^{CS} \right) \\ &= \lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2 \end{aligned}$$

From the above expression, we note that the CS market has a natural liquidity advantage over the underlying asset markets in that the price impact of an order submitted in the CS market is "diversified".

Proposition 7 (Relative Trading Cost)

CS is more liquid than the underlying asset i ($i = 1, 2$) in a factor-investing equilibrium if and only if

$$\frac{\lambda_i^{CS}}{\lambda_j^{CS}} > \frac{\beta_j - 2\beta_i}{\beta_i} \qquad i \neq j$$

Corollary 7.1 *CS is more liquid than the underlying asset with the larger β. If $\frac{\beta_1}{\beta_2} \in [\frac{1}{2}, 2]$, then CS is more liquid than both underlying assets.*

As long as the two assets have similar amount of exposure to systematic factor, the endogenously designed CS is always more liquid than the underlying assets regardless of information asymmetry in either market.

5. Discussion and Extensions

In the discussion thus far we have made a few assumptions: 1. market makers have no information on CS orders; 2. CS trading involves trading the underlying assets; 3. information is exogenous; 4. the noise trading from factor liquidity traders in each underlying market is exogenous; 5. factor investors have to play pure strategy. We now show how our main

results are robust or even strengthened when we relax these assumptions. In doing so, we also highlight some salient differences between various forms of CS and their implications.

5.1 Endogenous Information Acquisition

We now endogenize information acquisition by allowing multiple asset speculators and heterogeneity in their costs of information acquisition. The case for factor speculators is similar. For simplicity, we assume that there are two asset speculators for each underlying asset: an **insider speculator** endowed with asset-specific information and an **outsider speculator** who can pay an effort cost to observe a noisy signal. Intuitively, the outsider speculator acquires information on ϵ_i only if the signal noise is less than $\sigma^2_{\epsilon_i}$, otherwise he incurs losses trading on the noisy signal. Therefore, there is a lower bound on the information acquisition cost $\underline{C}_{\epsilon,i} = C_{\epsilon,i}\left(\sigma^2_{\epsilon_i}\right)$ that he pays to be informed. An interesting change could accompany the introduction of CS if there are large enough liquidity trading before the introduction and the asset speculator finds it profitable to be informed, yet the fundamental liquidity $\sigma_{\hat{n}_i}$ for asset i is so low that when the CS draws the factor liquidity traders away it is preferable to stay uninformed.

Without composite security, in an equilibrium the factor speculator chooses to trade in market 1, the market order received by market maker of asset 1 could be written as

$$\omega_1 = \alpha_1\epsilon_1 + \hat{\alpha}_1\chi_1 + \eta\gamma + n_1$$

where $X_1(\epsilon_1) = \alpha_1\epsilon_1$ is the order from the insider speculator and $\hat{X}_1(\chi_1) = \hat{\alpha}_1\chi_1$ is the order from the outsider speculator who observes signal $\chi_1 = \epsilon_1 + \nu_1$ and pays $C(\sigma^2_{\nu_1})$, where information cost is increasing in precision $1/\sigma_{\nu_1}$. When both asset speculators are actively trading, the asset specific information is better reveal compared to the case with only one asset speculator. In the appendix, we show that with multiple asset speculators the price variability in the liquid market becomes

$$var(\tilde{P}_1) = \frac{2}{3}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma \tag{12}$$

and the asset specific pricing efficiency is

$$corr\left(\tilde{P}_1, \epsilon_1\right) = \frac{\frac{2}{3}\sigma_{\epsilon_1}}{\sqrt{\frac{2}{3}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma}} \tag{13}$$

Similarly, for the illiquid asset, we have $var(\tilde{P}_2) = \frac{2}{3}\sigma^2_{\epsilon_2}$ and $corr\left(\tilde{P}_1, \epsilon_1\right) = \sqrt{\frac{2}{3}}$.

Now with CS, the outside asset speculator in the illiquid market is likely to quit after the introduction of CS trading. By allowing the factor speculator to exploit his information in both markets, introducing CS trading increases the trading cost for the asset speculator in the relatively illiquid market, where the factor speculator initially does not trade in. This profit reduction is especially severe when the systematic information asymmetry is large compared to the asset specific information aysmmetry.

When the outside speculator rationally opts out of the market, the asset specific information is now only reflected through the insider speculator's trading. This leads to a decrease in asset-specific information acquisition, especially in the relatively illiquid asset. Israeli et al. (2016) find this is indeed the case in terms of declining number of analysts covering specific stocks. Similarly, systematic information acquisition goes up, further strengthening our earlier conclusion that systematic pricing efficiency generally goes up and leading to the prediction that the number of analysts covering systematic news goes up. Mathematically, the asset-specific pricing efficiency $corr\left(P_i, \epsilon_i\right)$ as well as the overall pricing efficiency $corr\left(P_i, \epsilon_i + \beta_i\gamma\right)$ go down with composite securities due to the endogenous drop in information acquisition. The deterioration in asset-specific pricing efficiency is apparant:

$$corr^{CS}\left(P_i, \epsilon_i + \beta_i\gamma\right) = \frac{\sqrt{2}}{2} < \frac{\frac{2}{3}\sigma^2_{\epsilon_i} + \frac{1}{2}\beta_i^2\sigma^2_\gamma}{\sqrt{\frac{2}{3}\sigma^2_{\epsilon_i} + \frac{1}{2}\beta_i^2\sigma^2_\gamma}\sqrt{\frac{1}{2}\left(\sigma^2_{\epsilon_i} + \beta_i^2\sigma^2_\gamma\right)}} = corr^{N}\left(P_i, \epsilon_i + \beta_i\gamma\right)$$

We expect these phenomena to be more salient during times when aggregate demand for factor exposure is highly uncertain and the underlying assets are fundamentally illiquid, which are likely during financial crises.

5.2 Endogenous Noise Trading

So far we have taken the factor liquidity trading f_1 and f_2 as exogenous, though we allow them to endogenously choose to trade CS. This assumption is realistic if one believes that liquidity traders have search costs in finding relevant assets or are not superrational. Nevertheless, it is important to understand the impact of fully endogenous noise trading. Before introducing CS, for factor liquidity traders to not deviate, their trading costs must equalize across the two assets:

$$C_F = \frac{\lambda_1^N f_1 \sigma_{\tau^2}}{\beta_1^2} = \frac{\lambda_2^N f_2 \sigma_{\tau^2}}{\beta_2^2} \tag{14}$$

Intuitively, a factor liquidity trader faces the following tradeoff when making the choice of asset trading: the trading cost λ_i^N and the volume of correlated trading $\frac{f_i \sigma_{\tau^2}}{\beta_i^2}$ in asset market i. Factor liquidity traders prefer the asset with lower trading cost to meet. However, as more and more factor liquidity traders flowing into the market with lower trading cost, the expected cost of trading there would increase as the correlated order submitted in that market increases for any individual factor liquidity trader.[12] In equilibrium, the allocation of the fractions of liquidity traders f_i's would be such that it makes any individual factor liquidity trader indifferent between trading in either market. We also note that the endogenous correlation between noise trading in the two assets is $\frac{f_1 f_2 \sigma_\tau^2}{\sigma_{\hat{n}_1} \sigma_{\hat{n}_2}}$.

To compare the pricing efficiency before and after CS trading in this case, we may think of the factor liquidity traders in the equilibrium without CS trading as all submitting an order pair $\left(\frac{f_1}{\beta_1}, \frac{f_2}{\beta_2}\right)$. Then by the above indifference condition (14), we see that this order pair satisfies

$$\frac{f_1}{\beta_1} / \frac{f_2}{\beta_2} = \frac{\beta_1}{\lambda_1^N} / \frac{\beta_2}{\lambda_2^N} \tag{15}$$

Note that the endogenously designed CS also satisfies $w_1/w_2 = \frac{\beta_1}{\lambda_1^{CS}} / \frac{\beta_2}{\lambda_2^{CS}}$. Hence in an equilibrium without CS trading but factor liquidity traders are allowed to freely choose the market to trade in, the equilibrium allocations of liquidity trade would behave as if the factor liquidity traders are trading CSs. Therefore, if we fix the actions of the factor speculator, an optimally designed CS for factor liquidity traders leads to the same trading costs. From this, we see that introducing CS factor investors without price impact and are fully rational and optimizing would not affect their welfare. That said, in the presence of factor investors with price impacts (the factor speculator in our case), introducing CS could increase their welfare if the condition in equation (10) is met.

[comparative statics and more discussion to be added]

5.3 Mixed Strategies

[12] This effect does not rely on the perfect correlated hedging needs assumption, as long as there is a positive correlation between factor liquidity traders' hedging needs, this effect would exist.

We now allow the factor speculator to adopt a mixed strategy and show our results are robust and are even strengthened.[13] In the appendix we define the equilibrium and show that before introducing CS, the factor speculator randomizes the underlying asset he trades, with probability θ_i to trade asset i. The following figures illustrate how the equilibrium probability of market participation (θ_1, θ_2) are affected by fundamental model parameters.

FIGURE 1. θ_1 to β_1 and β_2

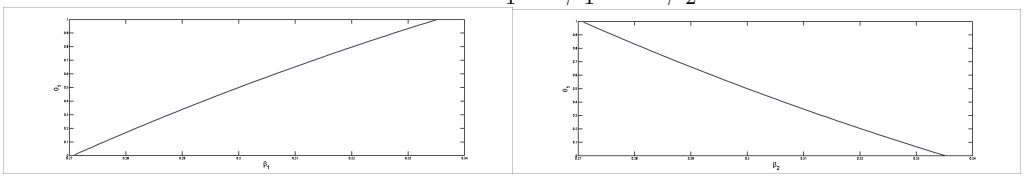


FIGURE 2. θ_1 to σ_{ϵ_1} and σ_{ϵ_2}

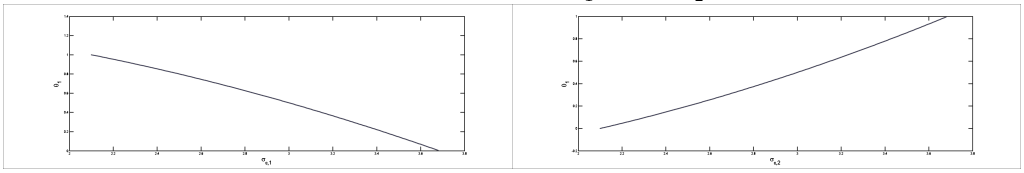
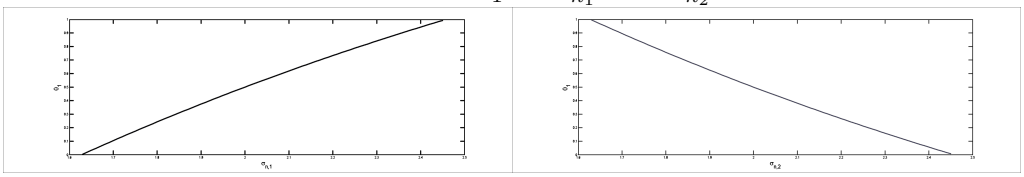


FIGURE 3. θ_1 to $\sigma_{\hat{n}_1}$ and $\sigma_{\hat{n}_2}$



The factor speculator is more likely to trade the asset with low asset-specific information asymmetry (low σ_ϵ^2), high noise trading (high σ_n^2), and high loading on the factor (high β). Allowing the factor speculator to adopt a mixed trading strategy essentially enlarges the feasibility set of the factor speculator's choice space and hence improves his profit from trading. Despite this, introducing CS further "diversifies" his price impact, leading to the asset pricing implications described earlier:

Proposition 8

With mixed trading strategies, equilibria with and without CS always exsit. Introducing CS

[13]We consider the mixed strategy for the equilibrium before CS is introduce. For the equilibrium with CS trading available, it's easy to show that the factor speculator has no incentive to adopt a mixed strategy.

(1) decreases asset-specific efficiency, increases systematic efficiency, and improves overall efficiency if and only if ,

$$\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2\left[2\sigma_{\epsilon_i}^2+\theta_i(3-2\theta)\beta_i^2\sigma_\gamma^2\right]>\theta_i^2\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^3$$

; (2) increases price variability, co-movement, and synchronicity in both underlying assets; (3) has ambiguous impact on the trading cost of underlying assets.

Mixed strategies further strengthen the results on the informational efficiencies in that now the relatively liquid asset's informational efficiencies also change. The condition for the overall efficiency trivially holds for the relatively illiquid asset in pure strategy equilibrium ($\theta_i = 0$), what is interesting is that it also holds for all the relatively liquid assets ($\theta_i > \frac{1}{2}$). Moreover, for the relatively illiquid asset in the mixed-strategy equilibrium, it basically requires $\sigma_{\epsilon_i}^2$ is not too small compared to $\beta_i^2\sigma_\gamma^2$. In fact, $\sigma_{\epsilon_i}^2 \geq \beta_i^2\sigma_\gamma^2$ is a sufficient condition.

The mixed-strategy equilibrium has a particular implication that overall efficiency can actually decrease when, for example, $\sigma_{\epsilon_i}^2 << \beta_i^2\sigma_\gamma^2$. To understanding this result, consider the case where the market maker i believes that the probability θ_i that the factor speculator trades in market i is close to 0, she perceives the order ω_i as coming only from the factor speculator or noise traders. Therefore, the factor speculator would bid very aggressively to fully exploit his information on γ, partially counteracting the fact that information on γ is only incorporated into the market price P_i with small probability. In comparison, if $\sigma_{\epsilon_i}^2 >> 0$, even when the market maker assigns a probability close to 0 to the factor speculator's trading, the factor speculator would not trade as aggressively, because now he worries about the possibility that the market maker may interpret his additional order as reflecting information on ϵ_i. As a consequence, the pricing efficiency could be really low for illiquid market since very little amount of information on γ is revealed in P_i.

5.4 Shades of CS: Transparency

While passive mutual funds only reveal the portoflio, order flow, etc at monthly frequency at best, the shares outstanding and the weights of ETFs are all available at daily frequency, if not higher. Moreover, the ETF arbitrage process also makes authorized participants and fund sponsors visible to the market makers. We model this distinguishing feature of ETFs by allowing market makers to observe order flow from CS sponsors. For simplicity, we focus

on linear equilibria under incomplete market in which investors either trade CS or one underlying asset.

In equilibrium, the market maker of asset i thus conjecture

$$\omega_i = X_i(\chi_i) + w_i m + \hat{n}_i + g_i \frac{\tau}{\beta_i} \tag{16}$$

where $m = Y(\zeta) + n_{H,CS}$ is the observed composite security order. $Y(\zeta)$ denotes the factor speculator's order based on signal ζ, and $n_{H,CS} = \frac{g_3 \tau}{\beta_1 w_1 + \beta_2 w_2}$ denotes CS order from factor liquidity traders. Here we denote by g_i $(i = 1, 2)$ the endogenous fraction of factor liquidity traders that remain in underlying market i and g_3 the fraction of liquidity traders that switch to CS market. As shown later, the perfect observability of CS trading allows market makers to seperate the asset specific adverse selection and the systematic adverse selection. Consequently, CSs do not attract all infinitesimal factor investors. Otherwise, suppose all factor investors trade CSs, then any individual infinitesimal factor liquidity trader finds it profitable to remain trading the underlying asset directly, because he faces no systematic adverse selection in the underlying asset market.

In the appendix we show that market makers utilize information on CS trading and set prices $P_i(\omega_i, m) = \lambda_{1,i}^{CS} \omega_i + \lambda_{2,i}^{CS} m$. We also provide conditions under which feasible CS designs attract the factor speculator and a fraction of liquidity traders. In the resulting equilibrium, the expected trading cost for each factor liquidity trader is

$$E\left[C_{CS}(\tau)\right] = \frac{\sigma_\tau g_3}{2} \sqrt{\sigma_\gamma^2 - \sigma_\xi^2} \tag{17}$$

which is independent of security design (w_1, w_2). Comparing it to the expected loss absent composite securities,

$$E\left[C_1(\tau)\right] = \frac{\sigma_\tau}{2} \sqrt{\left[\left(\sigma_\gamma^2 - \sigma_\xi^2\right) + \left(\sigma_{\epsilon_1}^2 - \sigma_{\nu_1}^2\right) \beta_1^{-2}\right] \frac{f_1^2 \sigma_\tau^2 \beta_1^{-2}}{f_1^2 \sigma_\tau^2 \beta_1^{-2} + \sigma_{\hat{n}_1}^2}}, \tag{18}$$

we see that there is a collective adverse selection substitution: on the one hand, composite securities eliminate the second term under the square root because adverse selection by asset speculators is removed when market makers utilize the order information from CS market; on the other hand, the adverse selection due to the factor speculator is strengthened as $\left(\sigma_\gamma^2 - \sigma_\xi^2\right)$ term is no longer scaled down with composite securities due to the fact that factor liquidity traders can no longer hide in the individual market (we can interpret the scaling factor as a fraction of the total noise). This seems to indicate that whether the factor liquidity traders

trade composite securities really depends on f's, g's and noise ratios, not on the design at all.

Similarly, there is a liquidity substitution effect for the factor speculator whose profit with CS is

$$\Pi_{\gamma}^{CS}\left(\sigma_{\xi}^{2}\right)=\frac{g_{3}\sigma_{\tau}}{2}\sqrt{\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}} \tag{19}$$

, as compared to that without CS

$$\Pi_{\gamma}^{N}\left(\sigma_{\xi}^{2}\right)=\frac{\sqrt{\beta_{1}^{2}\sigma_{\hat{n}_{1}}^{2}+f_{1}^{2}\sigma_{\tau}^{2}}}{2}\frac{\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}}{\sqrt{\beta_{1}^{-2}\left(\sigma_{\epsilon_{1}}^{2}-\sigma_{\nu_{1}}^{2}\right)+\left(\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}\right)}} \tag{20}$$

. On the one hand, liquidity may go down as factor speculator no longer taps into the asset-specific liquidity trading (when $g_{3}\sigma_{\tau}<\sqrt{\beta_{1}^{2}\sigma_{\hat{n}_{1}}^{2}+f_{1}^{2}\sigma_{\tau}^{2}}$), on the other hand, liquidity may improve as he faces less adverse selection by asset speculators ($\sqrt{\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}}>\frac{\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}}{\sqrt{\beta_{1}^{-2}\left(\sigma_{\epsilon_{1}}^{2}-\sigma_{\nu_{1}}^{2}\right)+\left(\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}\right)}}$).

Despite these new channels, the extent introducing CS alters the informationl efficiency is exactly the same as before:

Proposition 9

Introducing CS with transparent trading (1) reduces the asset-specific efficiency, improves the systematic efficiency, and improves the overall efficiency for the relatively illiquid asset; (2) increases asset prices co-movement, and increases price variability and synchronicity of the illiquid asset ; (3) increases the trading cost in the illiquid market if $g_{2}\leq f_{2}$ and may lower the trading cost in the liquid market if $\sigma_{\gamma}^{2}-\sigma_{\xi}^{2}$ is large relative to $\sigma_{\epsilon_{1}}^{2}-\sigma_{\nu_{1}}^{2}$.

Intuitively, introducing CS trading would attract away both the systematic speculator and the factor liquidity traders from underlying market i, hence the adverse selection in market i would deteriorate if the remaining noise trading is low, or if the information on γ is less relevant to the security valuation. Conversely, if information asymmetry in the systematic component γ is large compare to that in the idiosyncratic part, then by attracting away the systematic speculators the introduction of CS may actually alleviate the adverse selection in originally liquid market and hence lower the trading cost in it.

The results on endogenous information acquisition still apply: outsider asset speculators may no longer find it worthwhile to acquire information. This is because with perfect market seperation, market makers of underlying assets are able to identify the orders coming from asset speculators and asset liquidity traders, from those made by factor investors. If the noisiness of factor liquidity traders' hedging need is large relative to that of asset liquidity traders, the outside asset speculator may quit acquiring information if the profit he can gain from the remaining liquidity traders is not sufficient to recoup his incurred information acquisition cost.

Different from our baseline model, equilibrium g_is are independent of (w_1, w_2), implying that as long as the design supports the equilibrium, it does not affect factor investors' payoffs. Therefore, the optimal design could be degenerate. This helps to abcdefg

5.5 Shades of CS: Bundles vs Derivatives

In the case of ETFs, the creation and redemption process dictates that one has to buy the underlying assets to create an ETF. However, these assumptions may not hold for synthetic ETFs and index futures, etc because they trade more like a derivative than a bundle of the underlying assets. To the extent the traders do not always mechanically hedge by trading the underlying assets, orders on composite security may not significantly alter the demand for the underlying assets. To examine the effect of introducing composite derivatives rather than composite bundles, we let the composite security to be in perfectly elastic supply and introduce a separate market maker who absorbs residual demands and sets prices to break even on average. We still do not resort to introducing additional noise trading or speculation, but endogenize them as we do in the baseline model. In the appendix, we show how our main results are modified and how key mechanism and intuition go through.

In the appendix we provide conditions under which properly designed composite derivatives attract both the factor speculator and the factor liquidity traders. In such an equilibrium, with the factor speculator being attracted away from underlying markets, the systematic information is no longer reflected in the underlying security markets after the introduction of the derivative products. Therefore, unlike the ETF-like products as we analyzed above, which tend to help improve the pricing efficiency in underlying security market through better incorporating the systematic information, introducing derivative products is

likely to impair the price discovery in underlying security markets as the systematic information is no longer reflected in the market order received by underlying security market makers. Formally, we summarize the results of our analysis in the following proposition.

Proposition 10

After the introduction of derivative products such as synthetic ETF and index futures, 1) In the originally liquid market, the systematic pricing efficiency and overall pricing efficiency decrease, while the asset specific pricing efficiency weakly increases;2) Comovement in security returns across markets and the synchronicity in underlying security markets weakly goes down; 3) The trading cost in the illiquid market weakly goes up.

These results in turn imply that the physical replication required in composite bundles would have different impact on the underlyings than composite derivatives. To the extent that index futures are less of composite bundles than physical ETFs, this is consistent with the evidence found in Leippold et al. (2015) that demand shocks to ETFs have a stronger impact due to physical replication.

6. Conclusion

In this paper, we develop a theory of composite securities - combinations of underlying assets that facilitate trading larger baskets of assets which are previously too costly to fully access. Similar to discoveries from prior literature, composite securities benefit liquidity traders through coordination. Moreover, they also provide price impact "diversification" and attract traders informed of systematic factor of asset values. The model thus illustrates that financial innovations such as Exchange-Traded Funds (ETFs), smart beta products, and index-based vehicles encourage factor investing. Regardless of their informedness and liquidity needs, factor investors prefer the same composite security design. Characterization of the feasible design space as well as the optimal designs help us understand why such composite securities typically involve liquid assets representative of systematic factors and why ETFs and smart beta products have such popularity. Furthermore, the model generates implications on asset prices and the informational efficiency of the markets that are consistent with recent empirical findings. Finally, transparency in composite security trading distinguishes ETFs from other composite securities, and introducing composite derivatives tend to have opposite implications for asset prices to introducing composite bundles.

Our model is stylized and static, and studying composite security trading in dynamic settings would likely provider richer results. For example, while we focus on how current price reflect currently available information, a dynamic model would allow us to discuss the timing of information incorporate into asset prices. Dynamic rebalancing considerations also favor market-cap weighing. However, the main intuitions should continue holding. The model also produces a number of novel and testable implications. For example, all the predictions on asset prices would be significantly reduced for introduction of ETFs covering an index already covered by passive index mutual funds or another ETF. Another example is that the number of analysts covering specific firms goes down more for illiquid stocks, and the number of analysts covering systematic news goes up when an ETF covering that factor is introduced.

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Appendix

Proof of Lemma 1

For market maker 1, her perceived structure of the net order ω_1 is

$$\omega_1 = \alpha_1\chi_1 + \eta_1\zeta + n_1$$

and she sets the price according to

$$\begin{aligned} P_1^N(\omega_1) &= \bar{S}_1 + E\left[\beta_1\gamma + \epsilon_1 | \alpha_1\chi_1 + \eta_1\zeta + n_1 = \omega_1\right] \\ &= \bar{S}_1 + E\left[\beta_1\gamma + \epsilon_1 | \alpha_1(\epsilon_1 + \nu_1) + \eta_i(\gamma + \xi) + n_i = \omega_i\right] \\ &\equiv \bar{S}_1 + \lambda_1^N\omega_1 \end{aligned} \tag{21}$$

where

$$\lambda_1^N \equiv \frac{\alpha_1\sigma_{\epsilon_1}^2 + \beta_1\eta_1\sigma_\gamma^2}{\alpha_1^2\left(\sigma_{\epsilon_1}^2 + \sigma_{\nu_1}^2\right) + \eta_1^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \sigma_{n_1}^2} \tag{22}$$

is the price impact of order in market 1. At this price, market maker 1 breaks even. Similarly,

$$\omega_2 = \alpha_2\chi_2 + n_2$$

and

$$\begin{aligned} P_2^N(\omega_2) &= \bar{S}_2 + E\left[\beta_2\gamma + \epsilon_2 | \alpha_2\chi_2 + n_2 = \omega_2\right] \\ &\equiv \bar{S}_2 + \lambda_2^N\omega_2 \end{aligned} \tag{23}$$

where

$$\lambda_2^N \equiv \frac{\alpha_2\sigma_{\epsilon_2}^2}{\alpha_2^2\left(\sigma_{\epsilon_2}^2 + \sigma_{\nu_2}^2\right) + \sigma_{n_2}^2} \tag{24}$$

is the price impact of order in market 2. Given the pricing rules, when the asset speculator observes $\chi_1 = \epsilon_1 + \nu_1$ with $\nu_1 \sim \mathcal{N}(0, \sigma_{\nu_1}^2)$, he solves

$$\begin{aligned} & \max_{x_1} \quad E\left[x_1\left(\bar{S}_1 + \beta_1\gamma + \epsilon_1 - P_1^N(\omega_1)\right)|\chi_1\right] \\ = \; & \max_{x_1} \quad E\left[x_1\left(\beta_1\gamma + \chi_1 - \nu_1 - \lambda_1^N\omega_1\right)|\chi_1\right] \end{aligned}$$

taking into account his price impact through his order x_1's impact on the net order ω_1. Since he has no information about γ, thus he conjectures the factor speculator bids $\eta_1\gamma$ which is zero in expectation. This reduces the maximization to

$$\max_{x_1} x_1\left[\chi_1 - \lambda_1^N x_i\right]$$

The optimal choice of x_i is

$$X_i(\chi_i) = \frac{\chi_i}{2\lambda_1^N} \tag{25}$$

thus

$$\alpha_1 = \frac{1}{2\lambda_1^N} \tag{26}$$

Similarly, the factor speculator observeing $\zeta = \gamma + \xi$ with $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$ solves

$$\begin{aligned} & \max_{y_1} \quad E\left[y_1\left(\bar{S}_1 + \beta_1\gamma + \epsilon_1 - P_1(\omega_1)\right)|\zeta\right] \\ = \quad & \max_{y_1} \quad E\left[y_1\left(\beta_1(\zeta - \xi) + \epsilon_1 - \lambda_1^N\omega_1\right)|\zeta\right] \end{aligned}$$

Again, he expects zero order from the asset speculator, yielding

$$\max_{y_1} y_1\left[\beta_1\zeta - \lambda_1^N y_1\right]$$

and the optimal optimal y_1

$$Y_1(\zeta) = \frac{\beta_1}{2\lambda_1^N}\zeta = \eta_1\zeta \tag{27}$$

Plug (46) and (47) into (43) and (45), the price impact in market 1 is

$$\lambda_1^N = \frac{\sqrt{\left(\sigma_{\epsilon_1}^2 - \sigma_{\nu_1}^2\right) + \beta_1^2\left(\sigma_\gamma^2 - \sigma_\xi^2\right)}}{2\sigma_{n_1}} \tag{28}$$

Similarly

$$\lambda_2^N = \frac{\sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2}}{2\sigma_{n_2}} \tag{29}$$

Note that market makers are less sensitive to order flows if she expects speculations on noisier information. The expected profit each informed speculator gains from trading are

$$\Pi_\gamma^N\left(\sigma_\xi^2\right) = \frac{\beta_1^2}{4\lambda_1^N}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{30}$$

for the factor speculator, and for the asset speculator

$$\Pi_{\epsilon,i}^N\left(\sigma_{\nu_i}^2\right) = \frac{1}{4\lambda_i^N}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) \tag{31}$$

For the pure strategy, i.e, $(\theta_1, \theta_2) = (1, 0)$, indeed to be an equilibrium, we need to have

$$\frac{\beta_1^2}{4\lambda_1^N}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) > \frac{\beta_2^2}{4\lambda_2^N}\left(\sigma_\gamma^2 - \sigma_\xi^2\right)$$

The expected loss from trading for each factor liquidity trader in the benchmark model (pure strategy) is

$$C_i(\tau) = \hat{\lambda}_i f_i \left(\frac{\tau}{\beta_i}\right)^2 \tag{32}$$

where

$$\hat{\lambda}_1 = \frac{\sqrt{\left(\sigma_{\epsilon_1}^2 - \sigma_{\nu_1}^2\right) + \beta_1^2\left(\sigma_\gamma^2 - \sigma_\xi^2\right)}}{2\sqrt{\frac{f_i^2}{\beta_i^2}\sigma_\tau^2 + \sigma_{\hat{n_1}}^2}} \tag{33}$$

and f_i's are endogenously determined by

$$\hat{\lambda}_1 f_1 \left(\frac{\tau}{\beta_1}\right)^2 = \hat{\lambda}_2 f_2 \left(\frac{\tau}{\beta_2}\right)^2 \tag{34}$$

■

Proof of Lemma 2

From the expressions for Π_γ^{CS} and C_F^{CS} (equation (6) and (7)), it's easy to see that the optimizers of $\left(w_1^S, w_2^S\right)$ and $\left(w_1^L, w_2^L\right)$ are the same, taking λ as given. Because the market makers set the pricing rules, the equilibrium response by factor investors is to take λ as given, and through trading CS and potentially one underlying asset, adjust the ratios of trading. As a group, factor liquidity traders would prefer the same trading weights as factor speculator prefers because collectively they have price impact. $N = 2$ case involves a complete market, so there is no constraint on how to adjust the ratio. In general with incomplete market, there could be constraints, but the optimizers are the same as long as the constraints are the same for both factor speculator and factor liquidity traders. ■

Proof of Proposition 1 (Uniqueness of Equilibrium)

Since $N = 2$, market is complete, the factor speculator can trade CS and potentially one of the underlying assets, and optimize their proportions of trading the underlying assets to be

$$w_i = \frac{\frac{\beta_i}{\lambda_i^{CS}}}{\frac{\beta_1}{\lambda_1^{CS}} + \frac{\beta_2}{\lambda_2^{CS}}}$$

We can also show that any other weights would result in an infinitesimal factor liquidity trader to deviate. An equilibrium design has to be of this form, and it attracts all factor

investors. For this pair, we have

$$\begin{aligned} \eta &= \frac{w_1\beta_1 + w_2\beta_2}{2\left(w_1^2\lambda_1^{CS} + w_2^2\lambda_2^{CS}\right)} \\ &= \frac{1}{2}\left(\frac{\beta_1}{\lambda_1^{CS}} + \frac{\beta_2}{\lambda_2^{CS}}\right) \end{aligned} \tag{35}$$

Thus

$$\eta w_i = \frac{\beta_i}{2\lambda_i}$$

From equation (4) we get

(36)

$$\frac{1}{4\lambda_i^{CS}}(\sigma_{\epsilon_i}^2+\sigma_{\nu_i}^2)+\frac{w_i^2\left(w_1\beta_1 + w_2\beta_2\right)^2\left(\sigma_\gamma^2+\sigma_\xi^2\right)}{4\left(w_1^2\lambda_1^{CS} + w_2^2\lambda_2^{CS}\right)^2}+\left[\frac{\sigma_\tau^2}{(w_1\beta_1 + w_2\beta_2)^2} + \sigma_{\hat{n}_i}^2\right]\lambda_i^{CS} = \frac{1}{2\lambda_i^{CS}}\sigma_{\epsilon_i}^2+\frac{\beta_i w_i\sigma_\gamma^2\left(w_1\beta_1 + w_2\right.}{2\left(w_1^2\lambda_1^{CS} + w_2^2\lambda_2^{C}\right.}$$

This is a system of equations in $\left(\lambda_1^{CS}, \lambda_2^{CS}\right)$ of power 4. Plugging in the expression for η, we get

$$\frac{1}{4\lambda_i^{CS}}(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2) + \frac{\beta_i^2}{4\lambda_i^{CS}}(\sigma_\gamma^2 - \sigma_\xi^2) = \frac{\frac{\beta_i^2}{\lambda_i^{CS}}\sigma_\tau^2}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}}\right)^2} + \lambda_i^{CS}\sigma_{\hat{n}_i}^2$$

which could be rewritten as

$$\frac{1}{4\beta_i^2\sigma_\tau^2}(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2) + \frac{1}{4\sigma_\tau^2}(\sigma_\gamma^2 - \sigma_\xi^2) - \frac{1}{\beta_i^2\sigma_\tau^2}(\lambda_i^{CS})^2\sigma_{\hat{n}_i}^2 = \frac{1}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}}\right)^2} \tag{37}$$

To simplify notations, denote $\frac{1}{4\beta_i^2\sigma_\tau^2}(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2) \equiv b_i$, $\frac{1}{4\sigma_\tau^2}(\sigma_\gamma^2 - \sigma_\xi^2) \equiv a$ and $\frac{1}{\beta_i^2\sigma_\tau^2}\sigma_{\hat{n}_i}^2 \equiv c_i$. Then the above equations could be written as

$$a + b_i - c_i(\lambda_i^{CS})^2 = \frac{1}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}}\right)^2}$$

Combine the two equations, we have

$$\begin{aligned} \lambda_2^{CS} &= \sqrt{\frac{b_2 - b_1 + c_1\left(\lambda_1^{CS}\right)^2}{c_2}} \\ &\equiv g\left(\lambda_1^{CS}\right) \end{aligned}$$

where function $g(\cdot)$ is an increasing function for $\lambda_1^{CS} \in [0, \infty)$. Plug this back into the original equation, we have

$$a + b_1 - c_1(\lambda_1^{CS})^2 = \frac{1}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{g\left(\lambda_1^{CS}\right)}\right)^2} \tag{38}$$

It's easy to see that the LHS is decreasing in λ_1^{CS} on $[0,\infty)$ and the RHS is increasing in λ_1^{CS} on $[0,\infty)$. Also, when $\lambda_1^{CS} \to 0$, the $LHS \to a + b_1$ and the $RHS \to 0$; when $\lambda_1^{CS} \to \infty$, the $LHS \to -\infty$ and the $RHS \to \infty$. Hence there must exist a unique real root for λ_1^{CS}. The proposition follows. ■

Proof of Corollary 1.1 (Comparative Statics)

λ_1^{CS} and λ_2^{CS} are determined from equations

$$a + b_1 - c_1(\lambda_1^{CS})^2 = \frac{1}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}}\right)^2} \tag{39}$$

and

$$a + b_2 - c_2(\lambda_2^{CS})^2 = \frac{1}{\left(\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}}\right)^2} \tag{40}$$

The two λ's are related through

$$\lambda_2^{CS} = \sqrt{\frac{b_2 - b_1 + c_1\left(\lambda_1^{CS}\right)^2}{c_2}} \tag{41}$$

1. Consider an increase in $\sigma_{\epsilon_1}^2$. The impact on the equation system would be increasing b_1. Then λ_1^{CS} must go up. Otherwise if λ_1^{CS} stays the same or goes down, the by (41) we know that λ_2^{CS} must also stay the same or go down. This would violate equation (39). Given that λ_1^{CS} and equation (40), we know that λ_2^{CS} must go down. Therefore,

$$\frac{\partial \lambda_1^{CS}}{\partial \sigma_{\epsilon_1}^2} > 0; \quad \frac{\partial \lambda_2^{CS}}{\partial \sigma_{\epsilon_1}^2} < 0$$

Similarly, we have

$$\frac{\partial \lambda_1^{CS}}{\partial \sigma_{\epsilon_2}^2} < 0; \quad \frac{\partial \lambda_2^{CS}}{\partial \sigma_{\epsilon_2}^2} > 0$$

Given that

$$w_i = \frac{\frac{\beta_i}{\lambda_i}}{\frac{\beta_i}{\lambda_i} + \frac{\beta_{3-i}}{\lambda_{3-i}}}$$

$i = 1, 2$, we can conclude that

$$\frac{\partial w_i}{\partial \sigma_{\epsilon_i}} < 0; \quad \frac{\partial w_i}{\partial \sigma_{\epsilon_{3-i}}^2} > 0$$

2. Consider an increase in σ_γ^2. The impact on the equation system would be increasing a. Thus equation (41) is not affected. In other words, the changes in λ_1^{CS} and λ_2^{CS} must of the

same directions. Then from equation (39) and (40) it's easy to see that both λ_1^{CS} and λ_2^{CS} must be increasing. That is,

$$\frac{\partial \lambda_1^{CS}}{\partial \sigma_\gamma^2} > 0; \quad \frac{\partial \lambda_2^{CS}}{\partial \sigma_\gamma^2} > 0$$

However, the changes in w_i's are not clear.

3. Consider an increase in $\sigma_{\hat{n}_1}^2$. The impact on the equation system would increasing c_1. Then λ_1^{CS} must go down. Otherwise, from equation (41) we know that λ_2^{CS} must go up, which will make equation (39) not holding. Since λ_1^{CS} goes down, by the same logic as above, from equation (40) we know that λ_2^{CS} must go up. Therefore,

$$\frac{\partial \lambda_1^{CS}}{\partial \sigma_{\hat{n}_1}^2} < 0; \quad \frac{\partial \lambda_2^{CS}}{\partial \sigma_{\hat{n}_1}^2} > 0$$

Similarly, we have

$$\frac{\partial \lambda_1^{CS}}{\partial \sigma_{\hat{n}_2}^2} > 0; \quad \frac{\partial \lambda_2^{CS}}{\partial \sigma_{\hat{n}_2}^2} < 0$$

Therefore, similar as in 1, we have

$$\frac{\partial w_i}{\partial \sigma_{\hat{n}_i}} > 0; \quad \frac{\partial w_i}{\partial \sigma_{\hat{n}_{3-i}}} < 0$$

4. Now consider an increase from β_1 to $\hat{\beta}_1$ with $\hat{\beta}_1 = \theta\beta_1$ where $\theta > 1$. Denote the consequent λ's by $\hat{\lambda}_i^{CS}$. The impact on the equation system would be decreasing b_1 and c_1 by the same fraction. Combine the two equations we get

$$\frac{\left(\hat{\lambda}_1^{CS}\right)^2}{\hat{\beta}_1^2\sigma_\tau^2}\sigma_{\hat{n}_1}^2 - \frac{\left(\hat{\lambda}_2^{CS}\right)^2}{\hat{\beta}_2^2\sigma_\tau^2}\sigma_{\hat{n}_2}^2 = \hat{b_1} - b_2 \tag{42}$$

which is now strictly smaller after the increase in β_1.

First suppose λ_2^{CS} increases. In other words, $\hat{\lambda}_2^{CS} = \gamma\lambda_2^{CS}$ where $\gamma > 1$. Then we must have $\frac{\hat{\lambda}_1^{CS}}{\hat{\beta}_1} < \gamma\frac{\lambda_1^{CS}}{\beta_1}$, otherwise $\frac{\left(\hat{\lambda}_1^{CS}\right)^2}{\hat{\beta}_1^2\sigma_\tau^2}\sigma_{\hat{n}_1}^2 - \frac{\left(\hat{\lambda}_2^{CS}\right)^2}{\hat{\beta}_2^2\sigma_\tau^2}\sigma_{\hat{n}_2}^2$ would be larger than before. Therefore, in this case $\frac{\beta_2}{\lambda_2^{CS}}$ decreases by ratio γ while $\frac{\beta_1}{\lambda_1^{CS}}$ decreases by less than ratio γ. This implies w_1 increases.

Now suppose λ_2^{CS} decreases, in this case from equation (40) we know that $\frac{\beta_1^2}{\lambda_1^{CS}}$ must decrease. Since we know $\hat{\beta}_1 = \theta\beta_1$, this implies that $\hat{\lambda}_1^{CS} > \theta^2\lambda_1^{CS}$. Hence $\frac{\left(\hat{\lambda}_1^{CS}\right)^2}{\hat{\beta}_1^2} > \frac{\left(\lambda_1^{CS}\right)^2}{\beta_1^2}$. Combined with $\hat{\lambda}_2^{CS} < \lambda_2^{CS}$, we know that $\frac{\left(\hat{\lambda}_1^{CS}\right)^2}{\hat{\beta}_1^2\sigma_\tau^2}\sigma_{\hat{n}_1}^2 - \frac{\left(\hat{\lambda}_2^{CS}\right)^2}{\hat{\beta}_2^2\sigma_\tau^2}\sigma_{\hat{n}_2}^2$ must be larger than before, which contradicts the fact $\hat{b_1} - b_2$ is smaller than $b_1 - b_2$. Hence λ_2^{CS} is impossible to decrease.

Summarizing the above analysis, we can now conclude that increases in β_1 increases w_1 while

decrease w_2. ■

Proof of Proposition 3 and Corollaries 3.1 and 3.2

From our characterization of the equilibrium without CS trading, the hedging cost for factor liquidity traders are

$$C_F^N = \frac{\lambda_1^N f_1 \sigma_\tau^2}{\beta_1^2} = \frac{\lambda_2^N f_2 \sigma_\tau^2}{\beta_2^2} \tag{43}$$

In the factor investing equilibrium, the liquidity traders' hedging cost is

$$C_F^{CS} = \frac{\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2}{(w_1\beta_1 + w_2\beta_2)^2} \sigma_\tau^2 \tag{44}$$

Hence CS trading is welfare improving for factor liquidity traders if and only if

$$\frac{\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2}{(w_1\beta_1 + w_2\beta_2)^2} < \frac{\lambda_i^N f_i}{\beta_i^2} \qquad i = 1, 2$$

Substituting in the optimal weights, the condition becomes

$$\frac{\beta_1^2}{\lambda_1^{CS}} + \frac{\beta_2^2}{\lambda_2^{CS}} > \frac{\beta_1^2}{f_1\lambda_1^N} = \frac{\beta_2^2}{f_2\lambda_2^N} = f_1\frac{\beta_1^2}{f_1\lambda_1^N} + f_2\frac{\beta_2^2}{f_2\lambda_2^N}$$

where the last equality uses $f_1 + f_2 = 1$. Now if inequality (10) holds, we want to show that factor speculator's welfare is also improved by the introduction of CS trading. In the original equilibrium, the factor speculator's expected profit is

$$\Pi_\gamma^N = \frac{\beta_1^2}{4\lambda_1^N}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{45}$$

In the FIE after CS is introduced, we have

$$\Pi_\gamma^{CS} = \frac{(w_1\beta_1 + w_2\beta_2)^2}{4\left(\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{46}$$

By inequality (10), we have

$$\begin{aligned} \frac{(w_1\beta_1 + w_2\beta_2)^2}{4\left(\lambda_1^{CS} w_1^2 + \lambda_2^{CS} w_2^2\right)} &> \frac{\beta_1^2}{4\lambda_1^N f_1} \\ &\geq \frac{\beta_1^2}{4\lambda_1^N} \end{aligned}$$

since $f_1 \leq 1$. This implies that $\Pi_\gamma^{CS} > \Pi_\gamma^N$.

If the factor liquidity traders are better off in the factor-investing equilibrium, i.e.

$$\left(\frac{\lambda_1^{CS}w_1^2+\lambda_2^{CS}w_2^2}{(w_1\beta_1+w_2\beta_2)^2}\right)\sigma_\tau^2 \leq \frac{\lambda_1^N f_1\sigma_\tau^2}{\beta_1^2}$$

then it must be true that the factor speculator in the factor-investing equilibrium

$$\frac{(w_1\beta_1+w_2\beta_2)^2}{4\left(\lambda_1^{CS}w_1^2+\lambda_2^{CS}w_2^2\right)}\left(\sigma_\gamma^2-\sigma_\xi^2\right) \geq \frac{\beta_1^2}{4\lambda_1^N}\left(\sigma_\gamma^2-\sigma_\xi^2\right)$$

Therefore, the sufficient and necessary condition for both factor speculator and factor liquidity traders to be better off in the factor-investing equilibrium is simply the one stated in the proposition 3.

Now, note that with the CS weights being (w_1, w_2) the factor liquidity trader's expected cost if he chooses to trade CS is

$$\begin{aligned}
C_{CS}(\tau) &= \left(\frac{\lambda_1 w_1^2+\lambda_2 w_2^2}{(w_1\beta_1+w_2\beta_2)^2}\right)\tau^2 \\
&= \tau^2\left[\left(\frac{w_1}{w_1\beta_1+w_2\beta_2}\right)^2\lambda_1+\left(\frac{w_2}{w_1\beta_1+w_2\beta_2}\right)^2\lambda_2\right] \\
&= \tau^2\left[\frac{\sigma_{\epsilon_1}\left(\frac{w_1}{w_1\beta_1+w_2\beta_2}\right)^2}{2\sqrt{\left(\frac{w_1}{w_1\beta_1+w_2\beta_2}\right)^2\sigma_\tau^2+\sigma_{n_1}^2}}+\frac{\sigma_{\epsilon_2}\left(\frac{w_2}{w_1\beta_1+w_2\beta_2}\right)^2}{2\sqrt{\left(\frac{w_2}{w_1\beta_1+w_2\beta_2}\right)^2\sigma_\tau^2+\sigma_{n_2}^2}}\right] \\
&\equiv \tau^2\left[\frac{\sigma_{\epsilon_1}t_1^2}{2\sqrt{t_1^2\sigma_\tau^2+\sigma_{n_1}^2}}+\frac{\sigma_{\epsilon_2}t_2^2}{2\sqrt{t_2^2\sigma_\tau^2+\sigma_{n_2}^2}}\right] \qquad (47)
\end{aligned}$$

where we denote $t_1 \equiv \frac{w_1}{w_1\beta_1+w_2\beta_2}$ and $t_2 \equiv \frac{w_2}{w_1\beta_1+w_2\beta_2}$. Hence we can write the optimal design problem of the cost-minimizing ETF as

$$\min_{t_1,t_2}\tau^2\left[\frac{\sigma_{\epsilon_1}t_1^2}{2\sqrt{t_1^2\sigma_\tau^2+\sigma_{n_1}^2}}+\frac{\sigma_{\epsilon_2}t_2^2}{2\sqrt{t_2^2\sigma_\tau^2+\sigma_{n_2}^2}}\right]$$

subject to

$$\beta_1 t_1+\beta_2 t_2 = 1$$

For future reference, denote the optimal choice of (t_1, t_2) by (t_1^*, t_2^*). If the factor liquidity trader chooses to trade in underlying security market i, his expected cost is $C_i(\tau)$, and in equilibrium the indifference condition implies $C_1(\tau) = C_2(\tau)$. Based on this fact, we can write the individual factor liquidity trader's expected cost of trading in underlying security

market as

$$C_i(\tau) = f_1 C_1(\tau) + f_2 C_2(\tau)$$

where we haveve also used the fact that $f_1 + f_2 = 1$. Plug in the expression for $C_i(\tau)$ we obtained above, we have

$$C_i(\tau) = \left[\frac{\left(\frac{f_1^2}{\beta_1^2}\right)\sigma_{\epsilon_1}}{2\sqrt{\frac{f_1^2}{\beta_1^2}\sigma_\tau^2 + \sigma_{n_1}^2}} + \frac{\left(\frac{f_2^2}{\beta_2^2}\right)\sigma_{\epsilon_2}}{2\sqrt{\frac{f_2^2}{\beta_2^2}\sigma_\tau^2 + \sigma_{n_2}^2}} \right] \tau^2 \tag{48}$$

Now note that $\left(\frac{f_1}{\beta_1}, \frac{f_2}{\beta_2}\right)$ is a feasible pair satisfying constraint $\beta_1 t_1 + \beta_2 t_2 = 1$, hence by the construction of (t_1^*, t_2^*), we have

$$\left[\frac{\left(\frac{f_1^2}{\beta_1^2}\right)\sigma_{\epsilon_1}}{2\sqrt{\frac{f_1^2}{\beta_1^2}\sigma_\tau^2 + \sigma_{n_1}^2}} + \frac{\left(\frac{f_2^2}{\beta_2^2}\right)\sigma_{\epsilon_2}}{2\sqrt{\frac{f_2^2}{\beta_2^2}\sigma_\tau^2 + \sigma_{n_2}^2}} \right] \tau^2 \geq \tau^2 \left[\frac{\sigma_{\epsilon_1}\left(t_1^*\right)^2}{2\sqrt{\left(t_1^*\right)^2\sigma_\tau^2 + \sigma_{n_1}^2}} + \frac{\sigma_{\epsilon_2}\left(t_2^*\right)^2}{2\sqrt{\left(t_2^*\right)^2\sigma_\tau^2 + \sigma_{n_2}^2}} \right] \tag{49}$$

which implies that

$$C_i(\tau) \geq C_{CS}(\tau)$$

for both $i = 1, 2$. This proves that there always exists an properly designed CS makes the factor liquidity traders better off. ■

Proof of Proposition 4

In an incomplete market, factor investors still trade only one asset (CS included). We know with the optimal design, factor investors cannot be trading only one of the underlying assets in equilibrium, for they can deviate to trading the CS. In an equilibrium where they trade CSs, they would not deviate to trading the underlying assets instead because their payoffs are maximized, neither would asset-specific investors.

More specifically, note that for an infinitesimal factor liquidity trader, if he deviates to trade in underlying asset market i, he would have to submit order $\frac{\tau}{\beta_i}$ units of asset i. His expected cost in trading after making such a deviation is thus

$$\begin{aligned} E\left[C_i(\tau)\right] &= E\left[\frac{\tau}{\beta_i} P_i(\omega_i)\right] \\ &= \left(\frac{\lambda_i^{CS} w_i^2}{\beta_i\left(w_1\beta_1 + w_2\beta_2\right)}\right)\sigma_\tau^2 \end{aligned} \tag{50}$$

When we take $w_i = \frac{\lambda_j^{CS}\beta_i}{\lambda_i^{CS}\beta_j+\lambda_j^{CS}\beta_i}$, $i \neq j$, we have

$$\frac{\lambda_1^{CS}w_1^2 + \lambda_2^{CS}w_2^2}{(w_1\beta_1 + w_2\beta_2)^2} = \frac{\lambda_1^{CS}w_i^2}{\beta_1(w_1\beta_1 + w_2\beta_2)} = \frac{\lambda_2^{CS}w_i^2}{\beta_2(w_1\beta_1 + w_2\beta_2)} \tag{51}$$

Thus under the optimal design, each infinitesimal factor liquidity trader has no incentive to deviate.

For the asset speculator i, after deviating to trading CS, he solves

$$\max_x x\left[w_i\chi_i - \left(\lambda_1^{CS}w_1^2 + \lambda_2^{CS}w_2^2\right)x\right]$$

His optimal expected profit after making such deviation is

$$\tilde{\Pi}_{\epsilon,i}^{CS}\left(\sigma_{\nu_i}^2\right) = \frac{w_i^2}{4\left(\lambda_1^{CS}w_1^2 + \lambda_2^{CS}w_2^2\right)}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) \tag{52}$$

The no deviation condition requires

$$\frac{1}{4\lambda_i^{CS}}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) \geq \frac{w_i^2}{4\left(\lambda_1^{CS}w_1^2 + \lambda_2^{CS}w_2^2\right)}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right)$$

which obviously holds. This is quite intuitive because switching to trading CS means that the asset speculator would suffer adverse selection in trading the other asset, which he does not have private information about.

Similarly, for the factor speculator, if he deviates to trading in underlying market i, his optimal expected profit is

$$E\left[\tilde{\Pi}_{\gamma,i}^{CS}\left(\sigma_\xi^2\right)\right] = \frac{\beta_i^2}{4\lambda_i^{CS}}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{53}$$

The no deviation conditions for the factor speculator thus require

$$\frac{(w_1\beta_1 + w_2\beta_2)^2}{4\left(\lambda_1^{CS}w_1^2 + \lambda_2^{CS}w_2^2\right)} \geq \frac{\beta_i^2}{4\lambda_i^{CS}} \tag{54}$$

for $i = 1, 2$. When we take $w_i = \frac{\lambda_j^{CS}\beta_i}{\lambda_i^{CS}\beta_j+\lambda_j^{CS}\beta_i}$, $i \neq j$, the inequalities (weakly) hold, thus there exists a feasible set of designs.■

Proof of Theorem 3

For the endogenous CS design (w_1, w_2) characterized in section 3, by construction we can easily show

$$\frac{w_1^2\lambda_1^{CS}}{w_1\beta_1} = \frac{w_2^2\lambda_2^{CS}}{w_2\beta_2}$$

which implies the factor speculator's trading aggressiveness η is

$$\begin{aligned}\eta &= \frac{w_1\beta_1 + w_2\beta_2}{2\left(w_1^2\lambda_1^{CS} + w_2^2\lambda_2^{CS}\right)} \\ &= \frac{w_i\beta_1}{2w_i^2\lambda_i^{CS}} \\ &= \frac{\beta_1}{2w_i\lambda_i^{CS}} \qquad (55)\end{aligned}$$

for $i = 1, 2$. This allows us to rewrite the equations that characterize the factor investing equilibrium as

$$\frac{1}{4}(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2) + \frac{\beta_i^2}{4}(\sigma_\gamma^2 + \sigma_\xi^2) + \sigma_{n_i}^2\left(\lambda_i^{CS}\right)^2 = \frac{1}{2}\sigma_{\epsilon_i}^2 + \frac{\beta_i^2}{2}\sigma_\gamma^2 \qquad (56)$$

for $i = 1, 2$, where

$$\sigma_{n_i} = \sqrt{\frac{\sigma_\tau^2}{(w_1\beta_1 + w_2\beta_2)^2} + \sigma_{\hat{n}_i}^2} \qquad (57)$$

Based on equation (56), we can now study how the introduction of CS trading affects the pricing efficiency in underlying asset markets. As in section 3, unless otherwise noted, I will refer to market 1 as the originally liquid market and market 2 as the originally illiquid market. Before the introduction of the CS, the systematic pricing efficiency in the illiquid market is

$$corr^N(P_2, \gamma) = 0$$

After the CS is made available for trading, the systematic information is now revealed in the originally illiquid market through the trading behavior of the factor speculator. Specifically, we have

$$\begin{aligned}cov^{CS}(P_2, \gamma) &= cov\left(\lambda_2^{CS}\eta\zeta w_2, \gamma\right) \\ &= \frac{\lambda_2^{CS} w_2\left(w_1\beta_1 + w_2\beta_2\right)}{2\left(w_1^2\lambda_1^{CS} + w_2^2\lambda_2^{CS}\right)}\sigma_\gamma^2 \\ &> 0\end{aligned}$$

Hence the systematic pricing efficiency is strictly improved in the illiquid market. Similarly, before the CS trading is introduced, the asset specific pricing efficieny in the illiquid market

is

$$corr^N(P_2, \epsilon_2) = \frac{\sqrt{2}}{2}$$

Now in equilibrium, the asset specific pricing efficiency in market 2 is

$$\begin{aligned} corr^{CS}(P_2, \epsilon_2) &= \frac{cov(P_2, \epsilon_2)}{\sqrt{var(P_2)}\sigma_{\epsilon_2}} \\ &= \frac{cov\left(\lambda_2^{CS}\frac{\chi_2}{2\lambda_2^{CS}}, \epsilon_2\right)}{\sqrt{var(P_2)}\sigma_{\epsilon_2}} \\ &= \frac{\sigma_{\epsilon_2}}{2\sqrt{var(P_2)}} \end{aligned}$$

Note that

$$\begin{aligned} var^{CS}(P_2) &= var\left(\lambda_2^{CS}\left(\frac{\chi_2}{2\lambda_2^{CS}} + \eta w_2 \zeta + \frac{\tau w_2}{\beta_1 w_1 + \beta_2 w_2} + \hat{n}_2\right)\right) \\ &= \left(\lambda_2^{CS}\right)^2\left[\left(\frac{1}{2\lambda_2^{CS}}\right)^2(\sigma_{\epsilon_2}^2 + \sigma_{\nu_2}^2) + w_2^2\eta^2(\sigma_\gamma^2 + \sigma_\xi^2) + \frac{w_2^2\sigma_\tau^2}{(w_1\beta_1 + w_2\beta_2)^2} + \sigma_{\hat{n},2}^2\right] \\ &= \frac{1}{2}\sigma_{\epsilon_2}^2 + \lambda_2^{CS}\beta_2 w_2 \eta \sigma_\gamma^2 \\ &> \frac{1}{2}\sigma_{\epsilon_2}^2 \end{aligned}$$

Hence we have

$$corr^{CS}(P_2, \epsilon_2) < \frac{\sigma_{\epsilon_2}}{2\sqrt{\frac{1}{2}\sigma_{\epsilon_2}^2}} = \frac{\sqrt{2}}{2}$$

The overall pricing efficiency of the illiquid asset in the original equilibrium is

$$corr^N(P_2, \epsilon_2 + \beta_2\gamma) = \frac{\frac{1}{2}\sigma_{\epsilon_2}}{\sqrt{\frac{1}{2}\left(\sigma_{\epsilon_2}^2 + \beta_2^2\sigma_\gamma^2\right)}}$$

After the introduction of the optimally designed CS, we can show that the overall pricing efficiency of the illiquid asset is

$$corr^{CS}(P_2, \epsilon_2 + \beta_2\gamma) = \frac{\sqrt{2}}{2}$$

which is obviously larger than that in the original equilibrium.

For the relatively liquid asset, before the introduction of CS, we have

$$corr^N(P_1, \gamma) = \frac{\frac{1}{2}\beta_1\sigma_\gamma}{\sqrt{\frac{1}{2}\sigma_{\epsilon_1}^2 + \frac{1}{2}\beta_1^2\sigma_\gamma^2}}$$

$$corr^N(P_1, \epsilon_1) = \frac{\frac{1}{2}\sigma_{\epsilon_1}}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma}}$$

$$corr^N(P_1, \epsilon_1 + \beta_1\gamma) = \frac{\sqrt{2}}{2}$$

After the CS introduction, the price volatility in the liquid market becomes

$$\begin{aligned} var^{CS}(P_1) &= var\left(\lambda_1^{CS}\left(\frac{\chi_1}{2\lambda_1^{CS}} + \eta w_1\zeta + \frac{\tau w_1}{\beta_1 w_1 + \beta_2 w_2} + \hat{n}_1\right)\right) \\ &= \frac{1}{4}(\sigma^2_{\epsilon_1} + \sigma^2_{\nu_1}) + \frac{\beta_1^2}{4}(\sigma^2_\gamma + \sigma^2_\xi) + \sigma^2_{n_1}\left(\lambda_1^{CS}\right)^2 \\ &= \frac{1}{2}\sigma^2_{\epsilon_1} + \frac{\beta_1^2}{2}\sigma^2_\gamma \end{aligned}$$

Therefore,

$$\begin{aligned} corr^{CS}(P_1, \epsilon_1) &= \frac{cov\left(\lambda_1^{CS}\frac{\chi_1}{2\lambda_1^{CS}}, \epsilon_1\right)}{\sqrt{var(P_1)}\sigma_{\epsilon_1}} \\ &= \frac{\frac{1}{2}\sigma_{\epsilon_1}}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma}} \end{aligned}$$

$$\begin{aligned} corr^{CS}(P_1, \gamma) &= cov\left(\lambda_1^{CS}\eta\zeta w_1, \gamma\right) \\ &= \frac{\frac{1}{2}\beta_1\sigma_\gamma}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma}} \end{aligned}$$

$$\begin{aligned} corr^{CS}(P_1, \epsilon_1 + \beta_1\gamma) &= \frac{cov\left(\lambda_1^{CS}\frac{\chi_1}{2\lambda_1^{CS}} + \lambda_1^{CS}\eta\zeta w_1, \epsilon_1\right)}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_1} + \frac{1}{2}\beta_1^2\sigma^2_\gamma}\sqrt{\sigma^2_{\epsilon_1} + \beta_1^2\sigma^2_\gamma}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

which are exactly the same as those before the introduction of CS. ■

Proof of Proposition 5 (Volatility, Synchronicity, and Co-movements)

For price variability, the liquid asset is not affected by the introduction of CS:

$$var^N(P_1) = var^{CS}(P_1) = \frac{1}{2}\left(\sigma^2_{\epsilon,1} + \beta_1^2\sigma^2_\gamma\right) \tag{58}$$

The illiquid asset's price volatility is larger after the introduction of CS:

$$\begin{aligned} var^{CS}(P_2) &= \frac{1}{2}\sigma_{\epsilon_2}^2 + \frac{\beta_2^2}{2}\sigma_\gamma^2 \\ &> \frac{1}{2}\sigma_{\epsilon_2}^2 \\ &= var^N(P_2) \end{aligned}$$

The synchronicity in the originally illiquid market is obviously enhanced since before the CS is introduced, no price variation in market 2 is attributable to systematic factor variation. In the original equilibrium, the asset price comovement is

$$\begin{aligned} cov\left(P_1, P_2\right) &= cov\left(\lambda_1^N \frac{f_1\tau}{\beta_1}, \lambda_2^N \frac{f_2\tau}{\beta_2}\right) \\ &= \frac{\lambda_1^N \lambda_2^N f_1 f_2}{\beta_1\beta_2}\sigma_\tau^2 \end{aligned} \tag{59}$$

where the fractions f_1 and f_2 satisfies

$$\frac{\lambda_1^N f_1}{\beta_1^2} = \frac{\lambda_2^N f_2}{\beta_2^2} \tag{60}$$

In this equilibrium, since the systematic information is only reflected in the price of liquid asset, the price comovement only comes from the common hedging needs for factor liquidity traders. In the factor investing equilibrium with composite security, the price comovement is

$$\begin{aligned} cov^{CS}\left(P_1, P_2\right) &= cov\left(\lambda_1^{CS}\left(\eta w_1\zeta + \frac{\tau w_1}{\beta_1 w_1 + \beta_2 w_2}\right),\ \lambda_2^{CS}\left(\eta w_2\zeta + \frac{\tau w_2}{\beta_1 w_1 + \beta_2 w_2}\right)\right) \\ &= \lambda_1^{CS}\lambda_2^{CS} w_1 w_2 \eta^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \frac{\lambda_1^{CS}\lambda_2^{CS} w_1 w_2}{\left(\beta_1 w_1 + \beta_2 w_2\right)^2}\sigma_\tau^2 \end{aligned} \tag{61}$$

For the endogenously designed CS weights (w_1, w_2) such that

$$w_1/w_2 = \frac{\beta_1}{\lambda_1^{CS}} / \frac{\beta_2}{\lambda_2^{CS}}$$

Then we have

$$\begin{aligned} \frac{\lambda_1^{CS}\lambda_2^{CS} w_1 w_2}{\left(\beta_1 w_1 + \beta_2 w_2\right)^2} &= \frac{\lambda_1^{CS}\lambda_2^{CS}}{\beta_1\beta_2} \cdot \left(\frac{\beta_1 w_1}{\beta_1 w_1 + \beta_2 w_2}\right) \cdot \left(\frac{\beta_2 w_2}{\beta_1 w_1 + \beta_2 w_2}\right) \\ &\equiv \frac{\lambda_1^{CS}\lambda_2^{CS}\hat{f}_1\hat{f}_2}{\beta_1\beta_2} \end{aligned}$$

where $\hat{f}_i \equiv \frac{\beta_i w_i}{\beta_1 w_1 + \beta_2 w_2}$, $i = 1, 2$. In addition, we have

$$\begin{aligned} \frac{\lambda_1^{CS} \hat{f}_1}{\beta_1^2} &= \frac{\lambda_1^{CS}}{\beta_1 (\beta_1 w_1 + \beta_2 w_2)} \\ &= \frac{\lambda_2^{CS}}{\beta_2 (\beta_1 w_1 + \beta_2 w_2)} \\ &= \frac{\lambda_2^{CS} \hat{f}_2}{\beta_2^2} \end{aligned}$$

From equation (56) we know that for this particular pair (w_1, w_2),

$$\lambda_i^{CS} = \frac{1}{2\sigma_{n_i}} \sqrt{\frac{1}{4}(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2) + \frac{\beta_i^2}{4}(\sigma_\gamma^2 - \sigma_\xi^2)} \tag{62}$$

where

$$\begin{aligned} \sigma_{n_i} &= \sqrt{\frac{w_i^2 \sigma_\tau^2}{(\hat{w}_1 \beta_1 + \hat{w}_2 \beta_2)^2} + \sigma_{\hat{n}_i}^2} \\ &= \sqrt{\frac{\beta_i^2 \sigma_\tau^2}{\hat{f}_i^2} + \sigma_{\hat{n}_i}^2} \end{aligned}$$

for $i = 1, 2$. Therefore, we can conclude for this pair of $(\hat{w}_1, \hat{w}_2)$ we have

$$\frac{\lambda_1^{CS} \lambda_2^{CS} \hat{f}_1 \hat{f}_2}{\beta_1 \beta_2} > \frac{\lambda_1^N \lambda_2^N f_1 f_2}{\beta_1 \beta_2} \tag{63}$$

■

Proof of Proposition 6 (Trading Cost)

In the original equilibrium, the trading cost in the illiquid market is

$$\lambda_2^N = \frac{1}{2\sigma_{n_2}} \sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2} \tag{64}$$

where

$$\sigma_{n_2}^2 = \sigma_{\hat{n}_2}^2 + f_2^2 \sigma_\tau^2 / \beta_2^2$$

In the factor investing equilibrium with endogenously designed CS weights (w_1, w_2) satisfying $\frac{w_1}{w_2} = \frac{\beta_1}{\lambda_1^{CS}} / \frac{\beta_2}{\lambda_2^{CS}}$, the trading cost in market 2 is

$$\lambda_2^{CS} = \frac{1}{2\sigma_{n_2}} \sqrt{\frac{1}{4}(\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2) + \frac{\beta_2^2}{4}(\sigma_\gamma^2 - \sigma_\xi^2)} \tag{65}$$

where

$$\sigma_{n_2} = \sqrt{\frac{\sigma_\tau^2}{(\hat{w}_1\beta_1 + \hat{w}_2\beta_2)^2} + \sigma_{\hat{n},2}^2}$$

Thus if the information asymmetry in systematic component $\sigma_\gamma^2 - \sigma_\xi^2$ is large enough compare to that in asset specific component $\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2$, we can get

$$\lambda_2^{CS} > \lambda_2^N$$

A sufficient condition is

$$\frac{\sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2}}{\sigma_{\hat{n},2}} < \frac{\sqrt{\frac{1}{4}(\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2) + \frac{\beta_2^2}{4}(\sigma_\gamma^2 - \sigma_\xi^2)}}{\sqrt{\frac{\sigma_\tau^2}{\min\{\beta_1,\beta_2\}^2} + \sigma_{\hat{n},2}^2}} \tag{66}$$

■

Proof of Proposition 7 and Corollary 7.1

Plug in $w_i = \frac{\frac{\beta_i}{\lambda_i}}{\frac{\beta_i}{\lambda_i} + \frac{\beta_j}{\lambda_j}}$, then $\lambda^{CS} < \lambda_1^{CS}$ is equivalent to

$$\left(\frac{\beta_1}{\lambda_1^{CS}} + \frac{\beta_2}{\lambda_2^{CS}}\right)^2 > \frac{\beta_i^2}{\left(\lambda_i^{CS}\right)^2} + \frac{\beta_{3-i}^2}{\lambda_1^{CS}\lambda_2^{CS}} \tag{67}$$

which could be further reduced to

$$\frac{\lambda_i^{CS}}{\lambda_j^{CS}} > \frac{\beta_j - 2\beta_i}{\beta_i}$$

First, we notice that this definitely holds if $\beta_j < \beta_i$. Even if $\beta_j > \beta_i$, as long as $\beta_j \leq 2\beta_i$, CS is more liquid.■

Section 5.1 Discussion on Pricing Efficiency with Multiple Asset Speculators

With multiple asset speculators, market maker 1 would then set the asset price according to

$$P_1(\omega_1) = \bar{S}_1 + \lambda_1^N \omega_1$$

where

$$\lambda_1^N \equiv \frac{(\alpha_1 + \hat{\alpha}_1)\sigma_{\epsilon_1}^2 + \eta\beta_1\sigma_\gamma^2}{(\alpha_1 + \hat{\alpha}_1)^2\sigma_{\epsilon_1}^2 + \hat{\alpha}_1^2\sigma_{\nu_1}^2 + \eta^2\sigma_\gamma^2 + \sigma_{n_1}^2} \tag{68}$$

Given this pricing rule, the proficient asset speculator solves

$$\max_{x_1} x_1\left(\epsilon_1 - \lambda_1^N\left(x_1 + \hat{\alpha}_1\epsilon_1\right)\right)$$

which implies

$$x_1^* = \frac{\left(1 - \lambda_1^N \hat{\alpha}_1\right) \epsilon_1}{2\lambda_1^N}$$

Hence we have

$$\alpha_1 = \frac{1 - \lambda_1^N \hat{\alpha}_1}{2\lambda_1^N} \tag{69}$$

Similarly, the less proficient asset speculator solves

$$\max_{\hat{x}_1} \hat{x}_1 \left(\chi_1 - \lambda_1^N \left(\hat{x}_1 + \alpha_1 \chi_1\right)\right)$$

which implies

$$\hat{x}_1^* = \frac{\left(1 - \lambda_1^N \alpha_1\right) \chi_1}{2\lambda_1^N}$$

Hence we have

$$\hat{\alpha}_1 = \frac{1 - \lambda_1^N \alpha_1}{2\lambda_1^N} \tag{70}$$

The factor speculator solves

$$\max_{y} y \left(\beta_1 \gamma - \lambda_1^N y\right)$$

which implies

$$y^* = \frac{\beta_1 \gamma}{2\lambda_1^N}$$

Hence

$$\eta = \frac{\beta_1}{2\lambda_1^N} \tag{71}$$

From the above results we have

$$\lambda_1^N(\alpha_1 + \hat{\alpha}_1) = \frac{2}{3}$$

and

$$\begin{aligned} \left(\lambda_1^N\right)^2 \left[(\alpha_1 + \hat{\alpha}_1)^2 \sigma_{\epsilon_1}^2 + \hat{\alpha}_1^2 \sigma_{\nu_1}^2 + \eta^2 \sigma_\gamma^2 + \sigma_{n_1}^2\right] &= \lambda_1^N \left[(\alpha_1 + \hat{\alpha}_1)\sigma_{\epsilon_1}^2 + \eta\beta_1\sigma_\gamma^2\right] \\ &= \frac{2}{3}\sigma_{\epsilon_1}^2 + \frac{1}{2}\beta_1^2\sigma_\gamma^2 \end{aligned}$$

Hence

$$\begin{aligned} cov\left(P_1, \epsilon_1\right) &= cov\left(\lambda_1^N \left(\alpha_1 \epsilon_1 + \hat{\alpha}_1 \chi_1 + \eta\gamma + n_1\right), \epsilon_1\right) \\ &= \frac{2}{3}\sigma_{\epsilon_1}^2 \end{aligned}$$

and

$$\begin{aligned} var\left(P_1\right) &= \left(\lambda_1^N\right)^2\left[(\alpha_1+\hat{\alpha}_1)^2\sigma_{\epsilon_1}^2+\hat{\alpha}_1^2\sigma_{\nu_1}^2+\eta^2\sigma_\gamma^2+\sigma_{n_1}^2\right] \\ &= \frac{2}{3}\sigma_{\epsilon_1}^2+\frac{1}{2}\beta_1^2\sigma_\gamma^2 \end{aligned}$$

Thus the asset specific pricing efficiency in market 1 is

$$\begin{aligned} corr^N\left(P_1,\epsilon_1\right) &= \frac{cov\left(P_1,\epsilon_1\right)}{\sqrt{var\left(P_1\right)var\left(\epsilon_1\right)}} \\ &= \frac{\frac{2}{3}\sigma_{\epsilon_1}}{\sqrt{\frac{2}{3}\sigma_{\epsilon_1}^2+\frac{1}{2}\beta_1^2\sigma_\gamma^2}} \end{aligned}$$

Similarly, we can calculate the asset specific pricing efficiency in market 2 as follows

$$corr^N\left(P_2,\epsilon_2\right)=\sqrt{\frac{2}{3}}$$

■

Section 5.3 Discussion on Mixed Strategy Equilibrium

We consider a linear equilibrium without CS trading comprised of the following:

- The asset speculator submits an order $x_i = X_i(\chi_i)$ in security i only to maximize his profit;
- The factor speculator submits order $y_i = Y_i(\zeta)$ (i=1,2) in either security market 1 and 2 with probability $\theta_1$and θ_2, where $\theta_1+\theta_2=1$, to maximize his profit.
- Market maker i forms consistent equilibrium belief about θ_i and commits to linear pricing $P_i(\omega_i)$, in order to make zero expected profit.

We need to require the market makers to pre-commit to linear pricing schemes and break even ex ante, which is different from the typical setup requiring market makers break even ex post. The latter does not admit any linear equilibrium. Again, we will restrict our attention to a linear equilibrium in which

$$X_i(\chi_i)=\alpha_i\chi_i$$
$$Y_i(\zeta)=\eta_i\zeta$$

The equilibrium in which the factor investors are allowed to adopt a mixed strategy is then characterized as follows:

Mixed Strategy Equilibrium

In a mixed strategy equilibrium, the factor speculator would adopt a randomizing strategy in which he trades in market i with probability θ_i. The equilibrium consists of the asset speculator i's order submission $X_i(\chi_i) = \alpha_i \chi_i$, the factor speculator's order submitted in market i, $Y_i(\zeta) = \eta_i \zeta$ and the market maker i set the price according to

$$\bar{P}_i(\omega_i) = \bar{S}_i + \left(\theta_i \tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)\omega_i \tag{72}$$

where α_i, η_i, $\tilde{\lambda}_i$, $\hat{\lambda}_i$ and θ_i are determined by the following equations system:

$$\tilde{\lambda}_i = \frac{\alpha_i \sigma_{\epsilon_i}^2 + \beta_i \eta_i \sigma_\gamma^2}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \eta_i^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \sigma_{n_i}^2} \tag{73}$$

$$\hat{\lambda}_i = \frac{\alpha_i \sigma_{\epsilon_i}^2}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \sigma_{n_i}^2} \tag{74}$$

$$\frac{1}{2\left(\theta_i \tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)} = \alpha_i \tag{75}$$

$$\frac{\beta_i}{2\left(\theta_i \tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)} = \eta_i \tag{76}$$

$$\frac{\beta_1^2}{4\left(\theta_1 \tilde{\lambda}_1 + \theta_2 \hat{\lambda}_1\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) = \frac{\beta_2^2}{4\left(\theta_2 \tilde{\lambda}_2 + \theta_1 \hat{\lambda}_2\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{77}$$

$$\theta_1 + \theta_2 = 1 \tag{78}$$

where $i = 1, 2$.

Proof of Proposition 8

We first note each trader's payoff as the following: The expected profit from trading for the asset speculator and the factor speculator are

$$\Pi_{\epsilon,i}^M\left(\sigma_{\nu_i}^2\right) = \frac{1}{4\left(\theta_i \tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) \tag{79}$$

and

$$\Pi_{\gamma,i}^M\left(\sigma_\xi^2\right) = \frac{\beta_i^2}{4\left(\theta_i \tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{80}$$

respectively. And the trading cost for the liquidity traders are

$$C_H^M = \frac{\left(\theta_i\tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right) f_i\sigma_\tau^2}{\beta_i^2} \tag{81}$$

and

$$C_i^M = \left(\theta_i\tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i\right)\sigma_{\hat{n}_i}^2 \tag{82}$$

It's easy to see that in mixed strategy equilibrium, the average Kyle's lambda $\bar{\lambda}_i \equiv \theta_i\tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i$ in market i plays an important role in determining market i's liquidity and pricing efficiency. While it's hard to analytically solve for $\bar{\lambda}_i$, the following lemma provides bounds for the trading cost $\bar{\lambda}_i$ in market i.

Lemma

We can bound the equilibrium trading cost $\bar{\lambda}_i \equiv \theta_i\tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i$ *in market* i *by*

$$\frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) < \bar{\lambda}_i^2\sigma_{n,i}^2 < \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 - \sigma_\xi^2\right) \tag{83}$$

Proof

Note that

$$\begin{aligned}\bar{\lambda}_i &= \theta_i\tilde{\lambda}_i + (1-\theta_i)\hat{\lambda}_i \\ &= \frac{\theta_i\left(\alpha_i\sigma_{\epsilon_i}^2 + \beta_i\eta_i\sigma_\gamma^2\right)}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \eta_i^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \sigma_{n_i}^2} + \frac{(1-\theta_i)\alpha_i\sigma_{\epsilon_i}^2}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \sigma_{n_i}^2}\end{aligned}$$

Plug in the expression for bidding agressiveness α_i and η_i as given by equation (6) and (7), the above equation could be rewritten as

$$1 = \frac{\frac{\theta_i}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right)}{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \bar{\lambda}_i^2\sigma_{n_i}^2} + \frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \bar{\lambda}_i^2\sigma_{n_i}^2} \tag{84}$$

Hence if we plug in $\bar{\lambda}_i^2\sigma_{n,i}^2 = \frac{1}{4}\left(\sigma_{\epsilon,i}^2 - \sigma_{\nu,i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 - \sigma_\xi^2\right)$, the RHS of equation (84)becomes

$$\begin{aligned} RHS &= \theta_i + \frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 - \sigma_\xi^2\right)} \\ &< \theta_i + \frac{\frac{1-\theta_i}{2}\sigma_{\epsilon,i}^2}{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right)} \\ &= \theta_i + (1-\theta_i) \\ &= 1 \end{aligned}$$

Since the RHS of equation (74) is decreasing in $\bar{\lambda}_i^2\sigma_{n_i}^2$, this proves that we must have

$$\bar{\lambda}_i^2\sigma_{n_i}^2 < \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 - \sigma_\xi^2\right)$$

By the same method, we can also show that

$$\bar{\lambda}_i^2\sigma_{n_i}^2 > \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right)$$

which finishes the proof. ■

Coming back to the proposition, the first claim is trivial. After CS trading is introduced, the price volatility increases and thus the correlation between P_i and ϵ_i decreases, since the covariance $cov\left(P_i, \epsilon_i\right)$ remains the same.

The movement in the correlation between P_i and γ is less obvious, since although the price volatility increases, the covariance between security price and systematic information also increases. To show that CS trading increases $corr\left(P_i, \gamma\right)$, we just need to show

$$\frac{\frac{\theta_i\beta_i}{2}\sigma_\gamma}{\sqrt{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \bar{\lambda}_i^2\sigma_{n,i}^2}} < \frac{\frac{\beta_i}{2}\sigma_\gamma}{\sqrt{\frac{1}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right)}}$$

or

$$\bar{\lambda}_i^2\sigma_{n_i}^2 > \frac{\theta_i^2}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right) - \frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) - \frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2 + \sigma_\xi^2\right)$$

To show this inequality, simply replace $\bar{\lambda}_i^2\sigma_{n_i}^2$ with the $\frac{\theta_i^2}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)-\frac{1}{4}\left(\sigma_{\epsilon_i}^2+\sigma_{\nu_i}^2\right)-\frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)$ into the equation (84). Then the RHS of equation (84) becomes

$$\begin{aligned}\frac{\frac{\theta_i}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}{\frac{\theta_i^2}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}+\frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{\theta_i^2}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)-\frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)} &> \frac{\frac{\theta_i}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}{\frac{\theta_i^2}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}\\ &= \frac{1}{\theta_i}\\ &> 1\end{aligned}$$

Hence it proves the desired inequality since the RHS of equation (84) is decreasing in $\bar{\lambda}_i^2\sigma_{n_i}^2$.

Next we show that the overall price efficiency would be improved, i.e.

$$corr_N\left(P_i,\beta_i\gamma+\epsilon_i\right)<corr_E^L\left(P_i,\beta_i\gamma+\epsilon_i\right)$$

when the information aysmmtry in ϵ_i is not too small compare to the information asymmetry in γ. Specifically, we will show that if $\sigma_{\epsilon,i}^2>\beta_i^2\sigma_\gamma^2$, the pricing efficiency in market i is strictly improved after the introduction of CS trading. Mathematically, we just need to show

$$\frac{\frac{1}{2}\sigma_{\epsilon_i}^2+\frac{1}{2}\theta_i\beta_i^2\sigma_\gamma^2}{\sqrt{\frac{1}{4}\left(\sigma_{\epsilon_i}^2+\sigma_{\nu_i}^2\right)+\frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)+\bar{\lambda}_i^2\sigma_{n_i}^2}}<\frac{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}{\sqrt{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}}$$

or

$$\bar{\lambda}_i^2\sigma_{n_i}^2>\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^{-1}\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2-\frac{1}{4}\left(\sigma_{\epsilon_i}^2+\sigma_{\nu_i}^2\right)-\frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)$$

Again, to show the above inequality, simply replace $\bar{\lambda}_i^2\sigma_{n_i}^2$ with the RHS into equation (84). Then the RHS of equation (84) becomes

$$\begin{aligned}&\frac{\frac{\theta_i}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)}{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^{-1}\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2}+\frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^{-1}\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2-\frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)}\\ >\ &\theta_i\left(\frac{\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2}{\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2}\right)^2+\frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^{-1}\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2-\frac{\theta_i\beta_i^2}{4}\sigma_\gamma^2}\end{aligned}$$

By the monotonicity of the RHS of (84) as a function of $\bar{\lambda}_i^2\sigma_{n_i}^2$, we just need to show

$$\theta_i\left(\frac{\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2}{\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2}\right)^2+\frac{\frac{1-\theta_i}{2}\sigma_{\epsilon_i}^2}{\frac{1}{2}\left(\sigma_{\epsilon_i}^2+\beta_i^2\sigma_\gamma^2\right)^{-1}\left(\sigma_{\epsilon_i}^2+\theta_i\beta_i^2\sigma_\gamma^2\right)^2-\frac{\theta_i\beta_i^2}{4}\sigma_\gamma^2}>1 \tag{85}$$

To simplify notation, denote $a\equiv\sigma_{\epsilon_i}^2$ and $b\equiv\beta_i^2\sigma_\gamma^2$. Then the above inequality could be rewritten as

$$\frac{\theta\left(a+b\right)^2}{\left(a+\theta b\right)^2}+\frac{\left(1-\theta\right)a\left(a+b\right)}{\left(a+\theta b\right)^2-\frac{\theta}{2}b\left(a+b\right)}>1$$

which is equivalent to

$$\frac{\theta(a+b)^2\left[(a+\theta b)^2-\frac{\theta}{2}b(a+b)\right]+(a+\theta b)^2(1-\theta)a(a+b)}{\theta(a+b)^2\left[(a+\theta b)^2-\frac{\theta}{2}b(a+b)\right]}>1$$

Doing some algebra, we can show that the numerator could be reduced to $(a+b)\left[(a+\theta b)^3-\frac{\theta^2}{2}b(a+b)^2\right]$. Thus we just need to show

$$(1-\theta)(a+\theta b)^3>\frac{\theta^2}{2}(a+b)^3-\frac{\theta}{2}(a+b)(a+\theta b)^2$$

or

$$(a+\theta b)^2[2a+(3\theta-2)\theta b]>\theta^2(a+b)^3$$

which could be further rewritten as

$$\left(\frac{a}{\theta}+b\right)(a+\theta b)\left[\frac{2}{\theta}a+(3-2\theta)b\right]>(a+b)^3 \tag{86}$$

We want to show the inequality holds for all $\theta\in(0,1)$, under the assumption that $a\geq b$.
If $\theta>\frac{1}{3}$, we have

$$a+\theta b>\frac{2}{3}(a+b)$$

and

$$\begin{aligned}\frac{2}{\theta}a+(3-2\theta)b &> 2a\\ &> \frac{3}{2}(a+b)\end{aligned}$$

Obviously, we always have $\frac{a}{\theta}+b>a+b$, hence we have

$$\begin{aligned}\left(\frac{a}{\theta}+b\right)(a+\theta b)\left[\frac{2}{\theta}a+(3-2\theta)b\right] &> 1\cdot\frac{2}{3}\cdot\frac{3}{2}(a+b)^3\\ &= (a+b)^3\end{aligned}$$

If $\theta\leq\frac{1}{3}$, we have

$$\begin{aligned}\frac{a}{\theta}+b &> 3a+b\\ &> 2(a+b)\end{aligned}$$

$$\begin{aligned}a+\theta b &> a\\ &> \frac{1}{2}(a+b)\end{aligned}$$

and

$$\frac{2}{\theta}a+(3-2\theta)b>2(a+b)$$

hence we also have the inequality holds. Now if an asset is fundamentally illiquid, the asset speculator would quit information acquisition and trading. This potentially leads to lower pricing efficiency. For pricing efficiency to fall after the introduction of CS trading, we need

$$corr_E^H\left(P_i, \beta_i\gamma + \epsilon_i\right) < corr_N\left(P_i, \beta_i\gamma + \epsilon_i\right)$$

or

$$\sqrt{\frac{\beta_i^2}{2}\sigma_\gamma^2} < \frac{\frac{1}{2}\sigma_{\epsilon_i}^2 + \frac{\theta_i\beta_i^2}{2}\sigma_\gamma^2}{\sqrt{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \bar{\lambda}_i^2\sigma_{n_i}^2}}$$

By Lemma 2, it is sufficient to prove

$$\sqrt{\frac{\beta_i^2}{2}\sigma_\gamma^2} \quad < \quad \frac{\frac{1}{2}\sigma_{\epsilon_i}^2 + \frac{\theta_i\beta_i^2}{2}\sigma_\gamma^2}{\sqrt{\frac{1}{4}\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \frac{\theta_i\beta_i^2}{4}\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \frac{1}{4}\left(\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2\right) + \frac{\beta_i^2}{4}\left(\sigma_\gamma^2 - \sigma_\xi^2\right)}}$$

which would be true if we have

$$\sqrt{\frac{\beta_i^2}{2}\sigma_\gamma^2} < \frac{\frac{1}{2}\sigma_{\epsilon_i}^2 + \frac{\theta_i\beta_i^2}{2}\sigma_\gamma^2}{\sqrt{\frac{1}{2}\sigma_{\epsilon_i}^2 + \frac{\beta_i^2}{2}\sigma_\gamma^2}} \tag{87}$$

$\sigma_{\epsilon_i}^2 > \frac{1-2\theta_i+\sqrt{5-4\theta_i}}{2}\beta_i^2\sigma_\gamma^2$ is sufficient to ensure the above inequality.■

Discussion on Transparent Trading (Section 5.4)

We again focus on the linear equilibiurm where $X_i(\chi_i) = \alpha_i\chi_i$ and $Y(\zeta) = \eta\zeta$, and characterize the equilibrium as follows:

Under transparent CS trading, if the variance in hedging demand is large enough, a unique FIE exists. Specifically, the asset speculator would submit order $X_i(\chi_i) = \frac{\chi_i}{2\lambda_{1,i}^{CS}}$, *the factor speculator and factor liquidity traders would submit order* $Y(\zeta) = \frac{1}{2}\left[\sum_i w_i\left(w_i\lambda_{1,i}^{CS} + \lambda_{2,i}^{CS}\right)\right]^{-1}\left(\sum_i w_i\beta_i\right)\zeta$ *and* $n_{H,CS} = \frac{\tau}{\beta_1 w_1 + \beta_2 w_2}$ *for CS respectively, where*

$$\lambda_{1,i}^{CS} = \frac{\sqrt{\sigma_{\epsilon_i}^2 - \sigma_{\nu_i}^2}}{2\sqrt{\sigma_{\hat{n}_i}^2 + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}} \quad \textit{and} \quad w_i\lambda_{1,i}^{CS} + \lambda_{2,i}^{CS} = \frac{\beta_i\sqrt{\sigma_\gamma^2 - \sigma_\xi^2}}{2\frac{g_3\sigma_\tau}{w_1\beta_1 + w_2\beta_2}}$$

Market maker i *sets price according to* $P_i(\omega_i, m) = \lambda_{1,i}^{CS}\omega_i + \lambda_{2,i}^{CS}m$, *where* ω_i *is the market order in market* i *and* m *is the trading volume in CS market. The measure of factor liquidity traders that trade in market* i *is* g_i *for* $i = 1, 2$, *and the remaining* g_3 *of factor liquidity traders are trading in CS market.*

The expected profit from trading for asset speculator and factor speculator are

$$\hat{\Pi}^{CS}_{\epsilon_i}\left(\sigma^2_{\nu_i}\right)=\frac{\left(\sigma^2_{\hat{n}_i}+\frac{g_i^2\sigma_\tau^2}{\beta_i^2}\right)^{\frac{1}{2}}}{2}\sqrt{\sigma^2_{\epsilon_i}-\sigma^2_{\nu_i}}\quad and\quad \hat{\Pi}^{CS}_{\gamma}\left(\sigma^2_{\xi}\right)=\frac{\left(\sigma^2_{\gamma}-\sigma^2_{\xi}\right)^{\frac{1}{2}}}{2}g_3\sigma_\tau$$

The expected loss for factor liquidity traders who trade composite securities is

$$\hat{C}_{F,CS}=\frac{\sqrt{\sigma^2_{\gamma}-\sigma^2_{\xi}}}{2}g_3\sigma_\tau$$

and that for those who remain trading in market i is

$$\hat{C}_{F,i}=\frac{\lambda^{CS}_{1,i}g_i\sigma^2_\tau}{\beta_i^2}$$

In equilibrium, g_is are endogenously determined by each infinitesimal factor liquidity trader's indifference conditions

$$C_{F,CS}=C_{F,1}=C_{F,2}$$

and

$$g_1+g_2+g_3=1$$

Finally, as long as

$$\max_{i=1,2}\frac{\sqrt{\sigma^2_{\hat{n}_i}+g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{\sqrt{\sigma^2_{\epsilon_i}-\sigma^2_{\nu_i}}}\beta_i^2<\frac{g_3\sigma_\tau}{\sqrt{\sigma^2_{\gamma}-\sigma^2_{\xi}}}<\sum_i\frac{\sqrt{\sigma^2_{\hat{n}_i}+g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{\sqrt{\sigma^2_{\epsilon_i}-\sigma^2_{\nu_i}}}\beta_i^2 \tag{88}$$

, there always exists a feasible design (w_1,w_2) that attracts both factor speculator and factor liquidity traders and supports the equilibrium.

Proofs:

Under the linear equilibrium the order received by market maker i can be further written as

$$\omega_i=\alpha_i\chi_i+w_i\eta\zeta+\hat{n}_i+w_in_{H,CS}$$

She sets

$$\begin{aligned}P_i(\omega_i,m)&=E\left[\bar{S}_i+\beta_i\gamma+\epsilon_i|\omega_i=\alpha_i\chi_i+w_i\eta\zeta+\hat{n}_i+w_in_{H,ETF},\ m=\eta\zeta+n_{H,CS}\right]\\&=\bar{S}_i+E\left[\beta_i\gamma+\epsilon_i|\omega_i-w_im=\alpha_i\chi_i+\hat{n}_i,\ m=\eta\zeta+n_{H,CS}\right]\\&=\bar{S}_i+E\left[\epsilon_i|\omega_i-w_im=\alpha_i\chi_i+\hat{n}_i\right]+E\left[\beta_i\gamma|m=\eta\zeta+n_{H,CS}\right]\end{aligned}$$

$$= \bar{S}_i + \lambda_{1,i}^{CS}\omega_i + \lambda_{2,i}^{CS} m \tag{89}$$

where

$$\lambda_{1,i}^{CS} \equiv \frac{\alpha_i \sigma_{\epsilon_i}^2}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \left(\sigma_{\hat{n}_i}^2 + g_i^2 \frac{\sigma_\tau^2}{\beta_i^2}\right)} \tag{90}$$

and

$$\lambda_{2,i}^{CS} \equiv \frac{\beta_i \eta \sigma_\gamma^2}{\eta^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + \left(\frac{g_3\sigma_\tau}{w_1\beta_1 + w_2\beta_2}\right)^2} - \frac{w_i\alpha_i\sigma_{\epsilon_i}^2}{\alpha_i^2\left(\sigma_{\epsilon_i}^2 + \sigma_{\nu_i}^2\right) + \left(\sigma_{\hat{n}_i}^2 + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}\right)} \tag{91}$$

and $m = \eta\zeta + n_{H,CS}$ is the observed CS order. The last equality is based on the orthoganality of $\alpha_i\chi_i + \hat{n}_i$ to γ and the orthoganality of $\eta\zeta + n_{CS}$ to ϵ_i. Given this pricing rule, we can now study the optimal trading strategy for each type of investors in equilibrium. For the asset speculator who observes $\chi_i = \epsilon_i + \nu_i$ with $\nu_i \sim \mathcal{N}(0, \sigma_{\nu_i}^2)$, he solves

$$\begin{aligned} & \max_{x_i} \quad E\left[x_i\left(\beta_i\gamma + \epsilon_i - \lambda_{1,i}\left(x_i + w_i\eta\zeta + \hat{n}_i + w_i n_{H,CS}\right) - \lambda_{2,i}\left(\eta\zeta + n_{H,CS}\right)\right)|\chi_i\right] \\ = \; & \max_{x_i} \quad x_i\left(\chi_i - \lambda_{1,i}x_i\right) \end{aligned}$$

where the simplification is based on the fact that the asset speculator has no information regarding ζ. The optimal order he should submit is then

$$X_i(\chi_i) = \frac{\chi_i}{2\lambda_{1,i}} \tag{92}$$

which indicates

$$\alpha_i = \frac{1}{2\lambda_{1,i}} \tag{93}$$

Plug this into the expression of $\lambda_{1,i}$, we can solve for $\lambda_{1,i}$ as follows

$$\lambda_{1,i}^{CS} = \frac{\sqrt{\sigma_{\epsilon,i}^2 - \sigma_{\nu,i}^2}}{2\sqrt{\sigma_{\hat{n},i}^2 + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}} \tag{94}$$

While the profit maximization for the asset speculator stays mostly the same, it becomes structurally different for the factor speculator who observes $\zeta = \gamma + \xi$ with $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$. After observing signal ζ, the γ-informed investor needs to decide the number of ETF shares

y to trade so as to

$$\begin{aligned} & \max_y \quad E\left[y\sum_{i=1,2} w_i\left(\bar{S}_i+\beta_i\gamma+\epsilon_i-P_i\left(\alpha_i\chi_i+w_iy+\hat{n}_i+w_in_{H,CS},\ y+n_{H,CS}\right)\right)|\zeta\right] \\ = & \max_{\hat{y}} \quad E\left[\hat{y}\sum_{i=1,2} w_i\left(\beta_i\gamma+\epsilon_i-\lambda_{1,i}\left(\alpha_i\chi_i+w_iy+\hat{n}_i+w_in_{H,CS}\right)-\lambda_{2,i}\left(y+n_{H,CS}\right)\right)|\zeta\right] \\ = & \max_{\hat{y}} \quad y\sum_{i=1,2} w_i\left(\beta_i\zeta-y\left(w_i\lambda_{1,i}^{CS}+\lambda_{2,i}^{CS}\right)\right) \end{aligned}$$

The optimal order he should submit is then

$$Y(\zeta)=\frac{1}{2}\left[\sum_i w_i\left(w_i\lambda_{1,i}^{CS}+\lambda_{2,i}^{CS}\right)\right]^{-1}\left(\sum_i w_i\beta_i\right)\zeta \tag{95}$$

which implies

$$\eta=\frac{1}{2}\left[\sum_i w_i\left(w_i\lambda_{1,i}^{CS}+\lambda_{2,i}^{CS}\right)\right]^{-1}\left(\sum_i w_i\beta_i\right) \tag{96}$$

From the expression of $\lambda_{1,i}^{CS}$ and $\lambda_{2,i}^{CS}$, it's easy to see that

$$w_i\lambda_{1,i}^{CS}+\lambda_{2,i}^{CS}=\frac{\beta_i\eta\sigma_\gamma^2}{\eta^2\left(\sigma_\gamma^2+\sigma_\xi^2\right)+\left(\frac{g_3\sigma_\tau}{w_1\beta_1+w_2\beta_2}\right)^2} \tag{97}$$

Plug this result into equation (96), we get

$$\eta=\frac{\eta^2\left(\sigma_\gamma^2+\sigma_\xi^2\right)+\left(\frac{g_3\sigma_\tau}{w_1\beta_1+w_2\beta_2}\right)^2}{2\eta\sigma_\gamma^2}$$

which gives us

$$\eta=\frac{\frac{g_3\sigma_\tau}{w_1\beta_1+w_2\beta_2}}{\sqrt{\sigma_\gamma^2-\sigma_\xi^2}} \tag{98}$$

and

$$w_i\lambda_{1,i}^{CS}+\lambda_{2,i}^{CS}=\frac{\beta_i\sqrt{\sigma_\gamma^2-\sigma_\xi^2}}{2\frac{g_3\sigma_\tau}{w_1\beta_1+w_2\beta_2}} \tag{99}$$

With the equilibrium pricing rule and optimal trading strategy for each investor we derived above, we can now calculate the expected profit each informed investor can make from following their equilibrium trading strategy. For the asset speculator, by submitting order

$X_i(\chi_i) = \alpha_i \chi_i$, the expected profit he can gain from trading in market i is

$$\begin{aligned}
\Pi^{CS}_{\epsilon,i}\left(\sigma^2_{\nu,i}\right) &= E\left[\frac{\chi_i}{2\lambda_{1,i}}\left(\beta_i\gamma + \epsilon_i - \lambda_{1,i}\left(\frac{\chi_i}{2\lambda_{1,i}} + w_i\eta\zeta + \hat{n}_i + w_i n_{H,CS}\right) - \lambda_{2,i}\left(\eta\zeta + n_{H,CS}\right)\right)\right] \\
&= \frac{1}{4\lambda_{1,i}}\left(\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}\right) \\
&= \frac{\left(\sigma^2_{\hat{n}_i} + \frac{g_i^2\sigma_\tau^2}{\beta_i^2}\right)^{\frac{1}{2}}}{2}\sqrt{\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}} \qquad (100)
\end{aligned}$$

For the equilibrium described above to be valid, we need to verify that each trader has no incentive to deviate. Taking the equilibrium trading costs $\lambda^{CS}_{1,i}$ and $\lambda^{CS}_{2,i}$ as given, if the asset speculator i switches to trading the CS shares, he solves

$$\max_x x\left[w_i\chi_i - \frac{\sqrt{\sigma^2_\gamma - \sigma^2_\xi}}{2g_3\sigma_\tau}\left(w_1\beta_1 + w_2\beta_2\right)^2 x\right]$$

The maximum expected profit he can gain is

$$\tilde{\Pi}^{CS}_{\epsilon,i}\left(\sigma^2_{\nu_i}\right) = \frac{w_i^2}{2\left(w_1\beta_1 + w_2\beta_2\right)^2}\frac{g_3\sigma_\tau}{\sqrt{\sigma^2_\gamma - \sigma^2_\xi}}\left(\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}\right) \qquad (101)$$

Thus for the asset speculator i to have no incentive to deviate, we need

$$\tilde{\Pi}^{CS}_{\epsilon,i}\left(\sigma^2_{\nu_i}\right) < \Pi^{CS}_{\epsilon,i}\left(\sigma^2_{\nu_i}\right)$$

Thus the weights (w_1, w_2) need to satisfy

$$\frac{w_i^2}{\left(w_1\beta_1 + w_2\beta_2\right)^2}\frac{g_3\sigma_\tau}{\sqrt{\sigma^2_\gamma - \sigma^2_\xi}} < \frac{\sqrt{\sigma^2_{\hat{n}_i} + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{\sqrt{\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}}}$$

We know that the factor speculator has no incentive to deviate, when

$$\frac{\sqrt{\sigma^2_\gamma - \sigma^2_\xi}}{2}g_3\sigma_\tau > \frac{\sqrt{\sigma^2_{\hat{n}_i} + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{2\sqrt{\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}}}\beta_i^2\left(\sigma^2_\gamma - \sigma^2_\xi\right)$$

for $i = 1, 2$. Hence, as long as

$$\max_i \frac{\sqrt{\sigma^2_{\hat{n}_i} + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{\sqrt{\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}}}\beta_i^2 < \frac{g_3\sigma_\tau}{\sqrt{\sigma^2_\gamma - \sigma^2_\xi}} < \sum_i \frac{\sqrt{\sigma^2_{\hat{n}_i} + g_i^2\frac{\sigma_\tau^2}{\beta_i^2}}}{\sqrt{\sigma^2_{\epsilon_i} - \sigma^2_{\nu_i}}}\beta_i^2 \qquad (102)$$

, there always exists a non-trivial set of feasible weights (w_1, w_2) such that a factor investing equilibrium exists. ■

Notice that the g_is are independent of (w_1, w_2). This property allows us to extend Theorem 1 to a more general setting where before introducing a CS, with probability p market makers will be able to observe CS orders. To attract the factor liquidity traders, a CS sponsor designs to minimize

$$\begin{aligned} V(w_1, w_2) &= E\left[C_{CS}(w_1, w_2)\right] \\ &= pE\left[C_{CS}(w_1, w_2)|\text{observable CS order}\right] + (1-p)E\left[C(w_1, w_2)|\text{unobservable CS order}\right] \\ &= pC_F^{CS}\left(w_1, w_2\right) + (1-p)\hat{C}_{F,CS}\left(w_1, w_2\right) \end{aligned}$$

where $C_F^{CS}\left(w_1, w_2\right)$ is the factor liquidity traders' trading cost in the opaque setting we analyzed in section 2 and $\hat{C}_{F,CS}\left(w_1, w_2\right)$ is the trading cost under the transparent setting we characterized above. The independence of function $\hat{C}_{F,CS}$ to CS design (w_1, w_2) immediately implies

$$\arg\min_{(w_1,w_2)} V(w_1, w_2) = \arg\min_{(w_1,w_2)} C_F^{CS}\left(w_1, w_2\right)$$

Thus most of the CS design results carry through.

Proof of Proposition 9

With transparent CS trading, the pricing efficiency of asset i is captured by

$$var_T^{CS}(P_i) = \frac{1}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right)$$

$$corr_T^{CS}(P_i, \epsilon_i) = \frac{\frac{1}{2}\sigma_{\epsilon_i}}{\sqrt{\frac{1}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right)}}$$

$$corr_T^{CS}(P_i, \gamma) = \frac{\frac{1}{2}\beta_i\sigma_\gamma}{\sqrt{\frac{1}{2}\left(\sigma_{\epsilon_i}^2 + \beta_i^2\sigma_\gamma^2\right)}}$$

$$corr_T^{CS}(P_i, \epsilon_i + \beta_i\gamma) = \frac{\sqrt{2}}{2}$$

Compared with those in the equilibrium before CS is introduced, we immediately get the pricing efficiency implications in (1).

Following the same reasoning in proposition 5, we can show that the asset price co-movement and synchronicity also increase after the CS been introduced. For trading cost of the illiquid asset, note that before CS is introduced,

$$\lambda_2^N = \frac{\sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2}}{\sqrt{\sigma_{\hat{n}_2}^2 + \frac{f_2^2\sigma_\tau^2}{\beta_2^2}}} \tag{103}$$

In the FIE with transparent CS trading, the trading cost in the illiquid market is

$$\lambda_{1,2}^{CS} = \frac{\sqrt{\sigma_{\epsilon_2}^2 - \sigma_{\nu_2}^2}}{2\sqrt{\sigma_{\hat{n}_2}^2 + g_2^2 \frac{\sigma_\tau^2}{\beta_2^2}}} \tag{104}$$

Hence trading cost increases if and only if $f_2 > g_2$. ■

Section 5.5 Discussion on Bundles vs Derivatives and Proof for Proposition 10

Consider derivatives such as synthatic ETFs and index futures, the trading of which does not involve a direct purchase and sale of the underlying assets. Suppose that in equilibrium the factor speculator and factor liquidity traders trade in the derivative market while the asset speculators are trading in the underlying asset markets. Unlike the previous model we have analyzed, there is a market maker in the derivative market that seperately determines the price of the derivative security. Knowing that only the factor speculator and factor liquidity traders are trading in the derivative market, the derivative market maker would expect the market order he receives to be only reflecting systematic information.

Again, we restrict our attention to linear equilibrium in which asset speculator i submits order $X_i(\chi_i) = \alpha_i \chi_i$ in market i and factor speculator submits order $Y(\zeta) = \eta\zeta$ in the derivative market. The derivative security tracks the payoff of $(w_1 S_1, w_2 S_2)$. factor liquidity traders with hedging demand τ would need to trade $n_H(\tau) = \frac{\tau}{w_1\beta_1 + w_2\beta_2}$ units of derivative security. Given the equilibrium trading strategies of each group of traders, the derivative market maker would set the price of derivative according to

$$P(m) = (w_1 \bar{S}_1 + w_2 \bar{S}_2) + \lambda_D m$$

where m is the market order of derivative security and λ_D is given by

$$\lambda_D = \frac{(w_1\beta_1 + w_2\beta_2)\,\eta\sigma_\gamma^2}{\eta^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + var\left(n_H(\tau)\right)}$$

This could be derived from

$$\begin{aligned} P(m) &= (w_1 \bar{S}_1 + w_2 \bar{S}_2) + E\left[w_1(\beta_1\gamma + \epsilon_1) + w_2(\beta_2\gamma + \epsilon_2) | \eta\zeta + n_H(\tau) = m\right] \\ &= (w_1 \bar{S}_1 + w_2 \bar{S}_2) + E\left[w_1(\beta_1\gamma + \epsilon_1) + w_2(\beta_2\gamma + \epsilon_2) | \eta(\gamma + \xi) + n_H(\tau) = m\right] \\ &= (w_1 \bar{S}_1 + w_2 \bar{S}_2) + \frac{(w_1\beta_1 + w_2\beta_2)\,\eta\sigma_\gamma^2}{\eta^2\left(\sigma_\gamma^2 + \sigma_\xi^2\right) + var\left(n_H(\tau)\right)} m \end{aligned} \tag{105}$$

Given the derivative market maker's pricing rule, the objective for the factor speculator is thus to

$$\begin{aligned} & \max_y \quad E\left[y\sum_{i=1,2} w_i\left(\bar{S}_i+\beta_i\gamma+\epsilon_i-P_i\left(y+n_H(\tau)\right)\right)|\zeta\right] \\ = & \max_y \quad E\left[y\sum_{i=1,2} w_i\left(\beta_i(\zeta-\xi)+\epsilon_i-\lambda_D\left(y+n_H(\tau)\right)\right)|\zeta\right] \\ = & \max_y \quad y\sum_{i=1,2} w_i\left(\beta_i\zeta-\lambda_D y\right) \end{aligned}$$

hence the optimal order the factor speculator should submit is

$$y^*=\frac{w_1\beta_1+w_2\beta_2}{2\lambda_D}\zeta$$

which implies that

$$\eta=\frac{w_1\beta_1+w_2\beta_2}{2\lambda_D} \tag{106}$$

Solve for λ_D we get

$$\begin{aligned} \lambda_D &= \left(w_1\beta_1+w_2\beta_2\right)\frac{\sqrt{\sigma_\gamma^2-\sigma_\xi^2}}{2\sqrt{var\left(n_H(\tau)\right)}} \\ &= \left(w_1\beta_1+w_2\beta_2\right)^2\frac{\sqrt{\sigma_\gamma^2-\sigma_\xi^2}}{2\sigma_\tau} \end{aligned} \tag{107}$$

and therefore

$$\eta=\frac{\sigma_\tau}{\left(w_1\beta_1+w_2\beta_2\right)\sqrt{\sigma_\gamma^2-\sigma_\xi^2}} \tag{108}$$

Given the factor speculator's trading strategy and market maker's pricing rule, the price volatility in the derivative market is

$$\begin{aligned} var(P_D) &= var\left(\lambda_D(\eta\zeta+n_H(\tau))\right) \\ &= \frac{\left(w_1\beta_1+w_2\beta_2\right)^2}{4}var(\zeta)+\lambda_D^2 var\left(n_H(\tau)\right) \\ &= \frac{\left(w_1\beta_1+w_2\beta_2\right)^2}{4}\left(\sigma_\gamma^2+\sigma_\xi^2\right)+\frac{\left(w_1\beta_1+w_2\beta_2\right)^2}{4}\left(\sigma_\gamma^2-\sigma_\xi^2\right) \\ &= \frac{\left(w_1\beta_1+w_2\beta_2\right)^2}{2}\sigma_\gamma^2 \end{aligned} \tag{109}$$

and the correlation between derivative price and the NAV of the underlying is

$$\begin{aligned} cov\left(P_D,\ w_1S_1+w_2S_2\right) &= cov\left(\lambda_D(\eta\zeta+n_H(\tau)),\ \left(w_1\beta_1+w_2\beta_2\right)\gamma\right) \\ &= \frac{\left(w_1\beta_1+w_2\beta_2\right)^2}{2}\sigma_\gamma^2 \end{aligned} \tag{110}$$

Hence the pricing efficiency is

$$\begin{aligned} corr\left(P_D,\ w_1S_1+w_2S_2\right) &= \frac{cov\left(P_D,\ w_1S_1+w_2S_2\right)}{\sqrt{var(P_D)}\sqrt{var(w_1S_1+w_2S_2)}} \\ &= \frac{\left(w_1\beta_1+w_2\beta_2\right)\sigma_\gamma}{\sqrt{2\left(w_1\beta_1+w_2\beta_2\right)^2\sigma_\gamma^2+2\left(w_1^2\sigma_{\epsilon_1}^2+w_2^2\sigma_{\epsilon_2}^2\right)}} \end{aligned} \tag{111}$$

Underlying security markets

In this equilibrium, only the asset speculator and the underlying liquidity traders are left in the underlying security markets. This is exactly the same as the illiquid market without ETF trading. The pricing rule in each underlying security market is

$$P_i(\omega_i)=\bar{S}_i+\lambda_i\omega_i$$

where

$$\lambda_i=\frac{\sigma_{\epsilon_i}}{2\sigma_{\hat{n}_i}}$$

The price variation in market i is

$$\begin{aligned} var_N(P_i) &= var\left(\lambda_i\omega_i\right) \\ &= var\left(\lambda_i\left(\alpha_i\epsilon_i+n_i\right)\right) \\ &= \frac{1}{2}\sigma_{\epsilon_i}^2 \end{aligned}$$

The covariance between P_i and ϵ_i is

$$\begin{aligned} cov_N(P_i,\epsilon_i) &= cov\left(\lambda_iw_i,\epsilon_i\right) \\ &= \frac{1}{2}\sigma_{\epsilon_i}^2 \end{aligned}$$

and the covariance between P_i and γ is

$$corr_N(P_2,\gamma)=0$$

Hence

$$\begin{aligned} corr_N(P_2, \epsilon_2) &= \frac{\frac{1}{2}\sigma^2_{\epsilon_2}}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_2}}\sigma_{\epsilon_2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

and

$$\begin{aligned} corr_N(P_i, \beta_i\gamma + \epsilon_i) &= \frac{\frac{1}{2}\sigma^2_{\epsilon_i}}{\sqrt{\frac{1}{2}\sigma^2_{\epsilon_i}}\sqrt{\sigma^2_{\epsilon_i} + \beta_i^2\sigma^2_{\gamma}}} \\ &= \frac{\frac{1}{2}\sigma_{\epsilon_i}}{\sqrt{\frac{1}{2}\left(\sigma^2_{\epsilon_i} + \beta_i^2\sigma^2_{\gamma}\right)}} \end{aligned}$$

Additionally, we can show that sufficient conditions that ensures the above equilibrium to be a legitimate one involves: (a) The volatility in hedging demand is high enough; and (b) The asset specific information asymmetry $\sigma^2_{\epsilon_i}$ is large enough compare to the systematic information asymmetry σ^2_{γ}. Condition (a) ensures that the factor speculator can make enough profit after switching to derivative market and condition (b) ensures that the factor liquidity traders are faced with less amount of adverse selection after switching to derivative market. In fact, these two conditions also hold for the main part of the paper, in which we focus on the bundle securities.