Heterogeneous Beliefs and School Choice Mechanisms

Adam Kapor Christopher A. Neilson Seth D. Zimmerman

January 9, 2017

Abstract

This paper studies how welfare and academic outcomes in centralized school choice depend on the assignment mechanism when participants are not fully informed. We use a survey of school choice participants to estimate a model of school choice that incorporates preference heterogeneity, strategic behavior, and subjective beliefs about admissions chances. We evaluate the equilibrium effects of switching to a strategy-proof deferred acceptance algorithm, and of improving the information available to households. Subjective beliefs differ from rational expectations values and predict choice behavior. Switching to a deferred acceptance algorithm raises welfare, but has limited effects on the distribution of test scores.

*Kapor: Columbia University, ak4003@columbia.edu; Neilson: Princeton University, cneilson@princeton.edu; Zimmerman: University of Chicago Booth School of Business, seth.zimmerman@chicagobooth.edu. We thank Sherri Davis-Googe and Carolyn Ross-Lee for their many contributions to survey design and implementation. We thank Fred Benton for his assistance with data access. We thank Parag Pathak, Maria Marta Ferreyra, Nikhil Agarwal, Eric Budish, and seminar participants at the University of Chicago, Princeton, Columbia, SITE, APPAM, the 2016 Northwestern Interactions Workshop, and the Cowles Foundation for helpful comments. We thank Fabiola Alba, Alejandra Aponte, Felipe Arteaga Ossa, Kevin DeLuca, and Julia Wagner for their excellent research assistance. We gratefully acknowledge financial support from the Cowles Foundation, the Yale Program in Applied Economics and Policy, and Princeton University Industrial Relations Section. All errors are our own.

1 Introduction

Many cities in the US and abroad use centralized school choice mechanisms to assign students to schools. Most centralized assignment mechanisms work by eliciting rank-order lists of schools from applicants and then making school assignments based on a combination of coarse priorities and random lotteries. However, districts differ in the extent to which their chosen assignment algorithms reward informed strategic play by choice participants. Charlotte, Barcelona, and Beijing use mechanisms that reward strategic play, while Boston, New York, and Denver use mechanisms which aim to make truthfully reporting one's preferences a dominant strategy.[1] Which type of mechanism is preferable is a central debate in the literature on school choice mechanism design. Mechanisms that reward informed strategic play can raise welfare by allowing participants to express the intensity of their preferences as opposed to just the ordering (Abdulkadiroğlu et al., 2011), but they can also lead to costly application mistakes and inequitable outcomes if some participants lack the information or sophistication to strategize effectively (Pathak and Sönmez, 2008).

Despite the critical role of beliefs and strategic play in the welfare comparison between the two mechanism types, there is little empirical evidence on what families know about school choice and how this affects the allocation of students to schools. This paper studies how welfare and academic outcomes depend on the assignment mechanism when school choice participants are not fully informed. We combine a new household survey measuring the preferences, sophistication, and beliefs of potential school choice participants with administrative records of choice and academic outcomes to estimate a model of school choice in which families make decisions on the basis of subjective beliefs about admissions chances. We use our model estimates to evaluate the equilibrium effects of improving the information available to households in a mechanism that rewards strategic play, and of switching from such a mechanism to a strategy-proof deferred acceptance algorithm. Coupling survey data with an empirical model of school choice allows us to document the errors families make when participating in strategic school choice, and to quantify the tradeoff between welfare-reducing mistakes and families' ability to express cardinal preferences in terms of both aggregate welfare and equity.

We conduct our study in the context of the public school district in New Haven, Connecticut (henceforth NHPS). At the time of our study, NHPS had used the same centralized mechanism to assign students to schools for at least 18 years. The assignment mechanism rewards strategic

[1] Boston, New York, Denver: Abdulkadiroğlu et al. (2005a,b, 2015b). Barcelona: Calsamiglia and Guell (2014); Charlotte: Hastings et al. (2009); Beijing: He (2012). See Pathak and Sönmez (2013) for a discussion of incentives to report truthfully in these mechanisms.

play by giving applicants higher admissions priority at schools they rank higher on their application forms. Admissions priorities also depend on whether students live in the neighborhood zone for a school, and whether they have a sibling at the school.

We begin by using our survey to describe participants' subjective beliefs and strategic sophistication, and the relationship between beliefs and choice behavior. We have two main descriptive findings. The first is that many families misunderstand the assignment mechanism and make errors in their estimates of the admissions probabilities associated with different application portfolios. Less than twenty percent of participants respond correctly to questions about the ordering of priority groups by sibling priority, neighborhood priority, and submitted preference ranking. The mean absolute difference between subjective and rational expectations admissions probabilities is about 30 percentage points, with larger values for students from low-SES backgrounds. Consistent with the hypothesis that families do not understand the assignment mechanism, respondents underestimate how much ranking a school lower on their application reduces admissions chances, and how much having sibling or neighborhood priority increases them.

Our second descriptive finding is that subjective beliefs about admissions probabilities predict choice behavior and placement outcomes. Conditional on rational expectations admissions chances, students with subjective beliefs in the upper tercile are 28 percentage points more likely to rank their most-preferred school first on their application than students with subjective beliefs in the bottom tercile. Though 54% of students are 'revealed strategic' in the sense that they report a school other than the one identified as their most-preferred option as their first choice on their submitted rank list, a majority of revealed strategic students submitted a first choice with a lower admissions probability than their most-preferred option. And students with average absolute belief errors of greater than the median value are 20 percentage points less likely to be placed in their most-preferred school, a 40% decline from a base rate of 50%.

We next use an empirical model of school choice to study the equilibrium effects of alternative school choice policies. Motivated by our survey results, households in our model maximize expected utility given their subjective beliefs about admissions probabilities, not rational expectations beliefs. Because we cannot ask families about the admissions probabilities associated with each possible application portfolio, we develop a parsimonious model of belief formation that captures key features of our survey results. In the model, students' beliefs about their own admissions rankings relative to cutoff rankings for admission to each school are equal to the true values plus a shift term. The shift term depends on a) the student's priority at a target school, b) the school's rank on a student's submitted application, c) a student level shock that is common across all schools, and d) idiosyncratic person-school components. Intuitively, the first two terms allow us to capture

systematic misunderstanding of the assignment mechanism, while the latter two allow, respectively, for levels of optimism to vary across students and for errors in belief about school-specific demand.

We incorporate subjective beliefs into a model of choice in which households choose whether to participate in choice and, if they participate, what application to submit. The model allows for correlated heterogeneous preferences across schools. We estimate the model using an MCMC procedure (McCulloch and Rossi, 1994; Agarwal and Somaini, 2014) that incorporates both survey and administrative data. The survey data help us overcome the challenges associated with separately identifying beliefs and preferences described by Agarwal and Somaini (2014) without imposing strong assumptions on applicants' equilibrium play. For surveyed students, the model fits both administrative records of submitted applications and survey reports of beliefs and preferences. The model also uses belief errors to rationalize choices for unsurveyed households.

With parameter estimates in hand, we study two sets of counterfactual simulations. To evaluate welfare in these counterfactuals, we consider each student's expected utility, according to the utility he or she gets from placement at each school and the rational-expectations chances associated with their lottery application. The first counterfactual exercise simulates a switch to a student-proposing deferred acceptance algorithm. The second considers a best-case informational intervention allowing households to play the Bayes Nash equilibrium in the game induced by the New Haven mechanism.

Results from these exercises show that the potential welfare gains from strategic play are outweighed by the costs of application mistakes. If households had rational expectations beliefs, switching from a deferred acceptance assignment mechanism to the New Haven mechanism would raise mean welfare by the equivalent of 0.05 fewer miles traveled in the kindergarten choice market and 0.20 fewer miles traveled in the grade 9 market. However, given households' actual beliefs, the switch would reduce mean welfare by 0.09 miles for kindergarteners and 0.13 miles for ninth graders. From an equity perspective, higher average welfare under deferred acceptance is driven by shifts upward in roughly the lower three quarters of the individual expected welfare distribution. Median welfare under deferred acceptance is the equivalent of 0.357 (0.276) fewer miles traveled higher for kindergarten (grade nine) households.

We also use our counterfactual simulations to study the distribution of test score value added across students under different assignment mechanisms. Though changes in expected value added under counterfactual policies are correlated with changes in expected utility, we find little evidence that a change to the deferred acceptance mechanism or a best-case informational intervention would yield aggregate gains in school value added or redistribute value added towards low-SES students. We interpret these findings with caution because, in contrast to our utility model, our model of school value added does not allow for student-school specific match effects. This is consistent with

most existing studies of school value added but rules out positive sum trades in school assignments across students.

Our findings have policy implications for both school districts and market designers. From the district perspective, counterfactual simulations show that the strategic New Haven mechanism is preferable to deferred acceptance only if belief errors can be scaled down by roughly 40% relative to what we observe in the data. Given the extensive outreach efforts New Haven already conducts, it is unclear what form such an intervention would take. From the perspective of a market designer, our findings indicate that accounting for belief errors is important when making welfare comparisons across mechanisms. Alternate estimates obtained by imposing rational expectations beliefs in both estimation and simulation would reverse our finding that the deferred acceptance mechanism yields higher aggregate welfare than the benchmark mechanism in our setting.

The paper proceeds as follows. Section 2 describes our contribution to existing literature. Section 3 and Section 4 describe the New Haven school district and our survey instrument, respectively. Section 5 describes our model of student behavior, section 6 describes estimation, and 7 describes results and counterfactuals. Section 8 concludes.

2 Literature on School Choice and Mechanism Design

This paper's primary contribution is to bring observations of beliefs and preferences to the analysis of the welfare properties of school choice mechanisms. Our approach brings new evidence to a major debate in the school choice mechanism design literature: whether districts with centralized choice should employ student-optimal stable matching mechanisms, which do not give incentives to misreport preferences, or alternative mechanisms that reward informed strategic play. The most frequently studied mechanism of the latter type is the immediate acceptance mechanism, also known as the 'Boston' mechanism (Abdulkadiroğlu et al., 2006). A theoretical literature provides conditions under which all students prefer the Boston mechanism to the student-optimal stable matching mechanism, and others under which it is (weakly) worse for all students (Ergin and Sonmez, 2006; Abdulkadiroğlu et al., 2011).[2] Which mechanism will perform best in a particular district is therefore an empirical question. Intuitively, the answer depends on whether applicants' ability to express cardinal preferences through strategic play in the Boston mechanism outweighs the welfare costs of strategic mistakes due to misunderstandings about the mechanism or lack of information about demand conditions. Observations of beliefs help us quantify this tradeoff.

[2]See also Pathak and Sönmez (2008), who provide a model in which sophisticated students benefit, and naive students suffer, from the Boston mechanism, and Pathak (2011) for a review.

Allowing for subjective beliefs proves to be important for comparisons across assignment mechanisms. In the absence of data on beliefs, a growing empirical literature has generally found that mechanisms that reward informed strategic play outperform deferred acceptance in revealed-preference welfare measures under the assumption that participants are informed and sophisticated, or deviate from optimal behavior in specific ways. For example, Agarwal and Somaini (2014) assume, as a baseline specification, that participants are fully rational and correctly anticipate their chances in the lottery when choosing applications. Alternatively, Calsamiglia and Guell (2014) consider school choice under a Boston mechanism in Barcelona. They allow two types of participants: one type is sophisticated and informed while the other type uses a rule of thumb to determine choices. Calsamiglia et al. (2014), He (2012), and Abdulkadiroğlu et al. (2015b) take similar approaches. We show that accounting for application mistakes in an empirically guided way reverses the welfare comparison between deferred acceptance and a mechanism that rewards strategic play. To the best of our knowledge this is the first paper to collect belief and preference data from actual and potential school choice participants.[3]

In addition to collecting survey data, we contribute to the school choice mechanism design literature by developing methods for analyzing belief data and preference data in the context of a model of school choice. Our estimation strategy builds on Agarwal and Somaini (2014), who present a framework for tractably estimating school choice models using MCMC. We innovate by specifying a parsimonious model of belief formation and integrating it into the choice framework.

This paper contributes to a second strand of literature by evaluating the effect of school choice policy on the distribution of achievement test scores. We combine our counterfactual simulations with estimates of school test score value added that resemble Deming (2014). We show that, as in Deming (2014), value added estimates are strong predictors of test score gains for students assigned to schools through random lotteries. Previous work estimating preferences in strategic school choice mechanisms has focused on parent satisfaction (measured by, e.g., distance-metric utility) while ignoring achievement, even as an extensive parallel literature uses data from school lotteries to estimate achievement effects while ignoring satisfaction.[4] We show that changes across mechanisms

[3]Two recent papers incorporate some survey elements to unpack school choice participation decisions and reports. Dur et al. (2015) make use of data on the frequency with which students access a school choice website to proxy for strategic and sincere participants in a school choice mechanism. Students who visit the site multiple times are assumed to be sophisticated, while those visiting only once are assumed sincere. de Haan et al. (2015) measure cardinal utility in Amsterdam using a survey that asks students to assign points to each school, with the top choice receiving 100 points, but do not ask about beliefs. Neither paper incorporates survey data on beliefs into a model of household behavior.

[4]See Cullen et al. (2006) for seminal early work, and Abdulkadiroğlu et al. (2015a) for recent advances. Several other recent papers estimate preferences for school characteristics such as school quality and distance and use

in utility and our measure of school quality are positively correlated, but that changing assignment mechanisms does not have large effects on equity in school quality by socioeconomic background.

Our final contribution is to describe the procedure New Haven uses to assign students to schools. To the best of our knowledge this process, which we call the 'New Haven mechanism' and describe in detail in Section 3, has not been documented elsewhere. In the New Haven mechanism, schools' preferences over students are determined first by priority group (e.g., sibling or neighborhood), then by submitted rank, and finally by lottery number. The New Haven mechanism rewards informed strategic play, but to a lesser extent than the Boston mechanism. We therefore expect our findings to understate the welfare losses due to less-than-fully informed play that would be observed in a Boston mechanism setting.

3 Empirical Setting

3.1 The school choice process in New Haven

Three features of the school choice process in New Haven make it a useful context in which to study beliefs and preferences for school choice participants. First, New Haven was an early adopter of centralized school choice, and the assignment process the district uses has remained fairly stable over time. The first choice-based magnet school opened in New Haven in 1970, and the number of school choice options expanded rapidly in the mid-1990s following a state-wide push to reduce school segregation (Huelin, 1996). New Haven has assigned students to schools using a centralized mechanism since at least 1997.[5] New Haven adopted centralized school choice several years before New York, which introduced a centralized application in 2003, and other cities such as Denver, New Orleans, Newark, and Washington DC, which built on the New York example (Abdulkadiroğlu et al., 2015a). The school choice system includes both district-run magnet schools and charter schools run by outside operators, such as 'no excuses' charter brand Achievement First.

Second, the school district conducts extensive outreach to publicize the process, including events for parents and children outside of school hours, in-school open-houses, and published documentation on procedures and past outcomes. The school choice process in New Haven follows a consistent pattern from year to year. The process begins in January of the academic year preceding the year of school assignment. Students and families can learn more about schools and the choice process by

these estimates to conduct welfare analysis and counterfactual simulations in decentralized or non-strategic settings (Hastings et al., 2009; Neilson, 2013; Walters, 2014; He, 2012; Dinerstein and Smith, 2014).

[5] We have verified the use of the centralized mechanism as far back as 1997 by inspection of the code used to run the process.

visiting open houses at different schools or by attending one of several 'magnet fairs' where schools set up information booths. The school district provides students with a magnet school guide that includes a description of the rules of choice and data on available seats and applicant counts by priority group from the previous year. This guide is available in English and Spanish, both in print and on an NHPS website. Students typically submit their applications in February, and receive notice of their placements in March or early April. These first two points suggest that parents and students have had ample time and opportunity to learn about the mechanism from the district and from each other, so the distribution of beliefs and preferences we observe is likely to reflect a long-run steady state.

Third, and finally, the vendor the district uses to implement school choice is employed by many other districts around the country. Between 1997 and 2013, the school assignment mechanism was implemented by an independent contractor working for NHPS. For the 2013-2014 school year, NHPS switched to the school choice vendor Smart Choice Technologies, which also administers school choice programs in Bridgeport CT, Hartford CT, Syracuse NY, New Orleans LA, and Tulsa OK, among others (Smart Choice, 2016). The third point suggests that the practices we observe in New Haven may have external validity in the sense that they are used in other districts as well.

The primary entry points in most district schools are kindergarten and ninth grade. In our analysis, we restrict attention to families living in New Haven with children enrolled in eighth grade or pre-K. In the 2014-2015 school year, when we conducted our survey, there were 1484 such potential ninth graders and 1746 potential kindergarteners. 40% of kindergarteners and 66% of eighth graders participate in choice. Just under half of all choice participants came from these two grades. See Appendix Figure A1 for the grade distribution of choice participants.

From this population, students who do not leave the city or enroll in private school may enter a lottery to enroll in one of 12 high schools or 34 elementary/middle schools that offer kindergarten. Most of these schools are administered by the district, but the total includes two charter high schools and three charter elementary schools. Many of the schools reserve some seats for suburban applicants. The remaining seats are available only to within-city applicants. Consistent with our sample frame, we focus on the seats reserved for within-city applicants.

3.2 The New Haven mechanism

Most school choice mechanisms use some form of coarse priorities to favor certain applicants. In New Haven, each student is assigned a priority at each school, which is a number between one and four:

$$priority_{ij} = \begin{cases} 1 & \text{if i lives in the neighborhood and has a sibling at j, and j gives neighborhood priority} \\ 2 & \text{if i lives in the neighborhood of j, and j gives neighborhood priority} \\ 3 & \text{if i has a sibling at j} \\ 4 & \text{otherwise} \end{cases}$$

Not all schools give neighborhood priority. Two high schools, Hillhouse and Wilbur Cross, give neighborhood priority, but the remaining high schools are classified as magnet schools, which give priority for siblings only. Similar priority structures are in place in Boston, Cambridge, New York, Barcelona, Beijing, and other cities.

The mechanism assigns students to schools using the following algorithm:

1. Consider each student's first choice submission. Each school ranks applicants up to its capacity, in order of priority group, using random lottery numbers as a tiebreaker. Each school provisionally accepts students up to its capacity and rejects the rest of its applicants.

2. Consider the next listed choice of students who were rejected in the previous step, together with the applications provisionally assigned in the previous step. Make provisional assignments at each school in order of a) priority group and b) submitted rank, again using lottery numbers as a tiebreaker.

3. Repeat Step 2 until all students are assigned to schools or have been considered and rejected at each listed school.

The mechanism assigns each student to at most one school. Students may choose to accept or decline this placement. If they decline, they have the option to enroll in a neighborhood school with unfilled seats[6] or leave NHPS.

Table 1 describes placement outcomes and priority groups in 2015. Two thirds of participants placed in their first-listed school, and 13% of applicants were unplaced. Most students submitted applications to schools where they had neither sibling nor neighborhood priority. High school students typically do not apply to schools where they have neighborhood priority because neighborhood high schools are not fully subscribed and are available outside of the choice process. In contrast, many neighborhood Kindergartens are oversubscribed.

[6]There is no guarantee that this will be the student's zoned school. If the student's own zoned neighborhood school has remaining seats, the student has the option to enroll there. If not the student may enroll at some nearby school that has excess capacity.

Table 1: Placement outcomes and priority groups by grade

	All	K	9
A. Participation and placement			
Participates	0.52	0.401	0.660
Places First	0.666	0.701	0.640
Places Second	0.113	0.127	0.102
Places Third	0.052	0.066	0.042
Places Fourth	0.041	0.057	0.029
Unplaced	0.129	0.049	0.187
B. Priorities			
Sib and Nbd	0.029	0.069	0.000
Neighborhood	0.077	0.184	0.001
Sibbling	0.104	0.183	0.047
None	0.789	0.563	0.950
N	3230	1746	1484

Notes: Placement outcomes and priority group in 2015 by grade. Placement outcomes are conditional on participation. Priorities average across all submitted applications.

This mechanism, which we label the 'New Haven' mechanism, differs from standard deferred acceptance and immediate acceptance algorithms. To compare the New Haven mechanism to Boston and deferred acceptance mechanisms, we employ a cutoff representation of matching algorithms introduced by Azevedo and Leshno (2016) for stable matchings and extended to a class of 'report-specific priority plus cutoff' mechanisms by Agarwal and Somaini (2014). The cutoff representation also provides a starting point for our model of belief errors.

The cutoff representation of the New Haven mechanism is as follows. The mechanism assigns student i a 'report-specific priority' at school j:

$$rsp_{ij} = 4 * priority_{ij} + rank_{ij}.$$

Ties are broken with uniform random draws that assign each student a score at each school:

$$score_{ij} = rsp_{ij} + z_{ij}, \;\; z_{ij} \sim U[0,1].$$

The resulting assignment is characterized by cutoffs π_j that fill schools' capacities when each student is matched to his earliest-listed school at which $score_{ij} < \pi_j$. If a school is undersubscribed,

its cutoff is set above all applicants' scores. The New Haven mechanism is a mapping from profiles of applications to distributions over cutoffs $\pi \in \mathbb{R}^J$.

The New Haven mechanism differs from Boston and student-optimal stable matching ("SOSM") mechanisms in the construction of rsp_{ij}. In New Haven, report-specific priority depends lexicographically on the exogenous priority $priority_{ij}$ and the rank that the student assigns to the school. In the Boston mechanism, this lexicographic order is reversed. In the student-optimal stable matching mechanism, report-specific priorities depend on the exogenous priority group only.

$$\begin{aligned} rsp_{ij}^{SOSM} &= priority_{ij} \\ rsp_{ij}^{\text{Boston}} &= (rank_{ij}, priority_{ij}) \\ rsp_{ij}^{\text{New Haven}} &= (priority_{ij}, rank_{ij}) \end{aligned}$$

Neighborhood and sibling priority play a relatively more important role and submitted rank lists a relatively less important role in determining report-specific priority in the New Haven mechanism than the Boston Mechanism. The New Haven Mechanism differs from the SOSM mechanism in that the tiebreaking rule within priority groups depends on submitted ranks.

3.3 Belief errors in the New Haven mechanism

We model belief errors using the cutoff representation of the New Haven mechanism. The probability that applicant i will be assigned to school j given that he submits report a to the mechanism is

$$P_{ija} = Pr(z_{ij} \leq \pi_j - rsp_{ij}(a),\ z_{ij'} > \pi'_j - rsp_{ij'}(a) \text{ for all } j' \prec_a j)$$

Inaccurate beliefs about P_{ija} may arise because students mis-estimate $rsp_{ij}(a)$ or the distribution of cutoff values π_j. Mistaken beliefs about these two quantities can arise from similar thought processes. For example, households who do not understand how priority groups and submitted rankings jointly determine rsp_{ij} will have inaccurate beliefs about their own values of $rsp_{ij}(a)$ and also about π_j even given full knowledge of other households' submitted applications.

Errors in beliefs about π_j and rsp_{ij} sum to alter beliefs about admissions probabilities. Let $\tilde{P}_{ija}$ denote student i's belief about the probability of admission to j given report a, and $\tilde{rsp}_{ij}(a)$ and $\tilde{\pi}_j = \pi_j + \Delta\pi_j$ be his beliefs about his response-specific priority and the cutoff score for admission, respectively, with $\Delta\pi_j \in \mathbb{R}$. Then

$$\tilde{P}_{ija} = Pr(z_{ij} \leq \pi_j - rsp_{ij}(a) - shift_{ij}(a),\ z_{ij'} > \pi'_j - rsp_{ij'}(a) - shift_{ij'}(a) \text{ for all } j' \prec_a j)$$

where $shift_{ij}(a) = \pi_j - \tilde{\pi}_j - (rsp_{ij}(a) - r\tilde{s}p_{ij}(a))$. The $shift_{ij}(a)$ term incorporates errors in beliefs about both rsp_{ij} and π_j. Rather than trying to distinguish between these two closely related sources of error, our empirical model takes a parsimonious approach and focuses on the $shift_{ij}$ term itself. This choice does not restrict the distribution of deviations of subjective beliefs from rational expectations values. In Section 4 we present descriptive evidence on the distributions of belief errors.

3.4 The New Haven School District

3.4.1 Measuring student SES

Before presenting the results of our household survey, we describe our core measures of student background and academic achievement. NHPS serves a low-income, majority-minority student population. The district is roughly 90% black or Latino, and students score an average of two thirds of a student-level standard deviation (henceforth SD) below statewide means on standardized tests. See Table A1 for more detail. The district has community eligibility for free lunch, meaning that all students may receive free meals in school each day regardless of own eligibility status. Roughly 80% of students are individually eligible, but this is based on survey measures that focus on ensuring the district maintains community eligibility.

One goal of this paper is to describe how school choice mechanisms affect the distribution of welfare by student background, but standard measures of socioeconomic status (SES) such as race/ethnicity or free lunch status are coarse in our context. We address this issue by creating an SES measure based on home sale prices. We first regress real (2015 dollars) per-square-foot sale prices of homes in New Haven on time dummies, using all home sales from 2005-2015 and obtained residuals. We then compute the implied price per square foot at each location using a normal kernel with bandwidth 0.05 miles. This measure closely tracks median census tract income but is a better predictor of test score value added and placed schools and of belief errors, as defined in Section 4. See Appendix Figure A2 for details. The fine-grained nature of the SES measure lets us differentiate between students from different kinds of backgrounds even within the same neighborhood catchment zones. See Figure A3 for a map of SES rankings across the district.

3.4.2 Measuring school quality

We evaluate the effect of changes in school choice mechanisms on the distribution of academic achievement using a measure of test score value added. Our approach here follows Deming (2014). We model test scores for student i in school j in year t, Y_{ijt}, as arising from the process

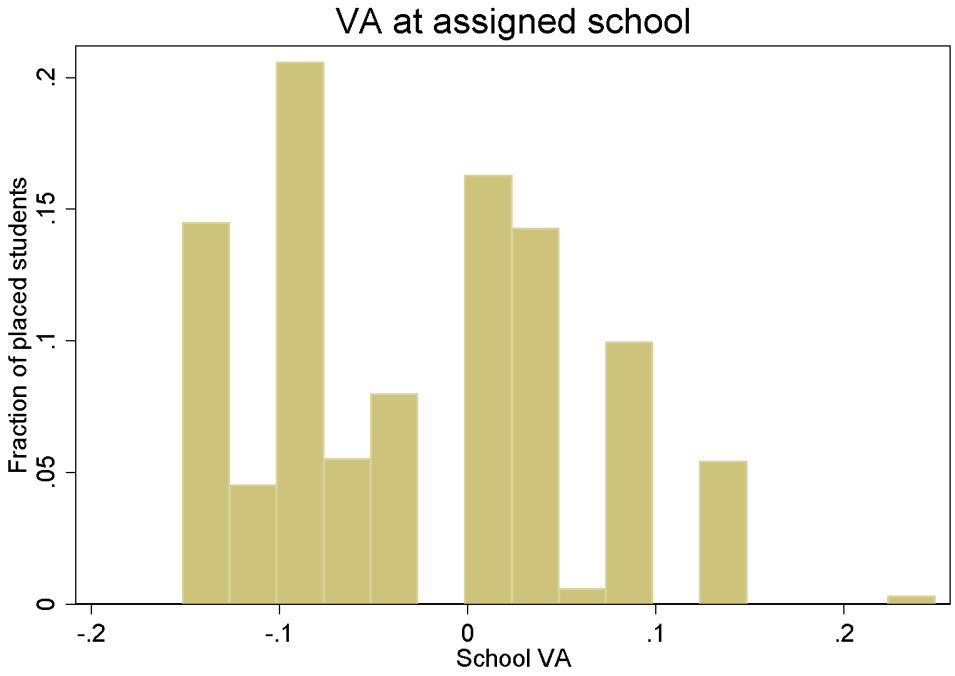
$$\begin{aligned} Y_{ijt} &= X'_{ijt}\beta^{test} + v_{ijt}^{test}. \\ v_{ijt}^{test} &= \mu_j^{test} + \theta_{jt}^{test} + \epsilon_{ijt}^{test} \end{aligned} \tag{1}$$

where Y_{ijt} is the average of standardized math and reading scores on state accountability tests, X_{ijt} is a set of observable characteristics that includes SES tercile, race/ethnicity, gender, grade, year, and a cubic function of lagged test scores. Scores are standardized using grade-specific means and standard deviations for district students in each year t. The residual term v_{ijt}^{test} is the sum of a 'school effect' μ_j^{test}, a time-varying school-specific shock θ_{jt}^{test}, and an idiosyncratic student-specific error ϵ_{ijt}^{test}. We recover best linear predictors of the μ_j^{test} term by computing school-specific mean residuals and shrinking them back towards zero. Here we follow the literature on teacher effects (Kane and Staiger, 2008; Chetty et al., 2014). In the main text, we focus on value added estimates that do not allow for drift in school effects over time.

We estimate test score specifications using data on school enrollment and test scores for the years 2007 through 2013. We exclude 2011 and 2012 outcome years due to lack of data availability. During this period, students took state exams in grades three through eight and again in grade ten. To expand the set of data that can be used to estimate scores, we use eighth grade scores as lag scores for tenth graders and dummy out missing values of lag scores.

Figure 1 displays the distribution of value added measures by school. The distribution is weighted by the count of assigned students in the 2015 school lottery, so the mean need not be (and in fact is not) zero. The standard deviation of the distribution is 0.08, and the gap between the best schools and the worst schools is about 0.36. The schools with the highest value added estimates are neighborhood schools in high-SES neighborhoods and high-peforming 'No Excuses' charters. See Appendix B.1 for more detail on value added estimation.

Figure 1: Distribution of school VA estimates



Notes: Figure displays the distribution of school value added, weighted by the count of students assigned to the school in the 2015 school lottery. Students who are unplaced or do not participate are assigned to their neighborhood school, or to the nearest undersubscribed school to their home if their neighborhood school is full.

Our measures of test score value added are helpful in evaluating the effects of mechanism choice only if they predict test score gains for students assigned to different schools. We test this using approaches similar to Deming (2014) and Angrist et al. (2015). Results are described in Appendix B.2. We find that our value added estimates accurately predict average test score gains for students randomly assigned to schools through lotteries. An important caveat is that our measures of test score effects do not allow for student-specific heterogeneous effects. This is typical in the literature on school valued added, but rules out positive-sum trades in school assignments across students. Our comparisons across assignment mechanisms on this measure will focus on equity across demographic groups as opposed to gains in aggregate test score performance.

4 Household Survey

4.1 Survey overview

During the summer of 2015, we conducted in-person interviews at 212 households with parents or guardians of children who had been enrolled in pre-kindergarten and/or eighth-grade in NHPS during the 2014-15 school year. To construct the sample, we drew 600 candidate households, stratifying by zoned elementary school.

Representativeness is important here because belief errors may be correlated with survey non-response. Panels A, B, and C of Table 2 describe demographic characteristics for the population, the 600 target households, and the 212 respondents. The first column describes the population, the second column the sample of households we intended to survey, and the third the households who we successfully surveyed. Panels A and B show that surveyed households are statistically indistinguishable from the population on race/ethnicity and SES background. Panel C shows that we slightly oversampled English-language learners and bilingual students relative to the population. See Figure A4 for evidence that our sample matched the geographic distribution of students across the district.

Panel D of Table 2 describes balance on school choice participation. Households who participate may list up to four schools on their application. Survey participants are statistically indistinguishable from the population in terms of the probability of participating in the centralized mechanism and the number of schools listed on the application.

Table 2: Balance in Socieconomic Characteristics

Category	Population Mean	Sample Mean	Surveys Mean	Mean test Pop. vs Surveys	P-value
A. SES quartile					
1st quartile	0.250	0.244	0.269	0.020	0.508
2nd quartile	0.250	0.276	0.231	-0.020	0.508
3rd quartile	0.250	0.246	0.217	-0.035	0.253
4th quartile	0.250	0.234	0.283	0.035	0.253
B. Race					
Asian	0.021	0.025	0.028	0.007	0.470
Black	0.482	0.483	0.495	0.014	0.690
Latino	0.396	0.401	0.382	-0.015	0.662
Other	0.008	0.005	0.000	-0.008	0.184
White	0.093	0.085	0.094	0.002	0.927
C. Educational program					
Biling/Dual	0.013	0.018	0.038	0.026**	0.001
No ELL	0.948	0.928	0.892	-0.060**	0.000
Regular/ESL	0.039	0.054	0.071	0.034*	0.014
SPED	0.134	0.134	0.184	0.053*	0.028
D. Number of applications by grade					
Grade K					
0	0.599	0.591	0.554	-0.048	0.360
1	0.056	0.050	0.076	0.013	0.588
2	0.060	0.070	0.076	0.022	0.388
3	0.068	0.077	0.043	-0.028	0.308
4	0.217	0.211	0.250	0.040	0.361
Grade 9					
0	0.340	0.340	0.275	-0.058	0.198
1	0.095	0.103	0.117	0.019	0.509
2	0.146	0.160	0.125	-0.022	0.508
3	0.164	0.160	0.175	0.011	0.747
4	0.255	0.237	0.308	0.050	0.229

Notes: $N = 3230$ (population), 598 (intended to survey), 212 (survey participants). $^{**}p < 0.01$, $^{*}p < 0.05$, $^{+}p < 0.1$. P-value for joint test (F) is 0.002. 'Population' is universe of NHPS students in K and 9th grade lotteries. 'Sample' is the set of candidate households selected for survey inclusion. 'Surveys' is the set of successfully surved households.

We conducted the survey as a tablet app, with randomly-generated questions tailored to each household. The survey procedures and question text are presented in Appendix C.1. We do not incentivize 'correct' beliefs, e.g. by paying people to state beliefs that are close to rational expectations chances.

4.2 Information acquisition, preferences, and revealed strategic play

We first describe the informational environment facing potential school choice participants. Students appear to take advantage of the informational resources the district provides. Panel A of Table 3 displays the fraction of students who reported using different resources to inform their school choice decision. Nearly two thirds of potential participants reported reading the choice catalog provided by the district, which contains descriptions of schools and information on demand from the previous year. Other common sources of information are the school choice website, which includes information similar to the catalog, school visits, counselors, and teachers. Over 90% of households report using some administrative information source, defined here to include a visit to a school or choice fair, reading the choice catalog or choice website, or talking to a counselor. Students consider a broad range of schools. At least 40% of surveyed students considered each school in the district at both the Kindergarten and grade nine levels. More than three-fourths of students in each grade level reported considering a 'no excuses' Achievement First charter school. See Tables A2 and A3 for evidence statistics for each school.

Though respondents consult a variety of information sources and consider broad sets of schools, they are unlikely to answer questions about how the assignment mechanism works correctly. Panel B of Table 3 presents the fraction of students who correctly answer questions about the ordering of priority groups and the role of rank in the choice mechanism. Only 17% of respondents correctly identified the neighborhood priority group as being preferred to the sibling priority group, and only 18% correctly stated that a student rejected from her first choice school has a (weakly) lower chance of admission at her second choice school than if she had ranked the second choice school first. There were four possible responses to the first question and three to the second, so correct answer rates are worse than under random guessing.

Table 3: Sources of information, understanding of choice rules, and revealed strategic play

	All	K	9
A. Sources of information			
Visit fair	0.358	0.294	0.405
Visit school	0.483	0.448	0.508
Visit website	0.592	0.614	0.576
Talk to teacher	0.538	0.286	0.556
Talk to counselor	0.481	0.286	0.495
Talk to friend	0.396	0.143	0.414
Read catalog	0.658	0.706	0.624
Read newspaper	0.243	0.224	0.256
Any admin. source	0.913	0.900	0.924
B. Understanding choice rules			
Get priority ordering	0.167	0.209	0.137
Get mechanism	0.179	0.226	0.143
C. Strategic play			
Revealed strategic	0.539	0.439	0.586
Mistaken strategic	0.306	0.128	0.388
N	212	92	120

Notes: Panel A describes means of dummy variables equal to one if students used the listed information source. 'Any admin. source' is a dummy equal to one if a respondent reports visiting a fair or a school, reading the website or school choice catalog, or talking to a counselor. Panel B describes means of dummy variables equal to one if students responded correctly to questions about priority ordering (neighborhood vs. sibling) and the importance of the submitted rank to admissions outcomes, respectively. 'Revealed strategic' is a dummy equal to one for students who had a first choice different than their stated most-preferred school. 'Mistaken strategic' is a dummy equal to one if a student was revealed strategic and their admissions chances would have been higher at their most-preferred school than their first-listed school.

Survey data on preferences suggest that participants play strategically, but often make errors in play. To elicit household preferences, we asked parents which school they would have chosen for their child if they were guaranteed admission to every school in the district. We then asked what they would have chosen if this school were unavailable but all other schools were available. We define a respondent to be revealed strategic if they applied to school j but stated that $j' \neq j$ was their most preferred school if they could enroll anywhere. As reported in Panel C of Table 3, 54% of survey

respondents who submitted applications were revealed strategic in this sense.[7] However, a majority (57%, or 31% of respondents who participated in choice) of revealed strategic respondents submitted first choice to applications to schools where the rational expectations admissions probability was below that for their most-preferred school. We describe how we calculate rational expectations admissions probabilities in detail in the next section.

4.3 Beliefs about admissions chances

We next document respondents' beliefs about admissions chances and compare them to objective measures of admissions probabilities. Findings from this descriptive analysis suggest an important role for belief errors in determining the allocation of students to schools, and motivate modeling choices in Section 5.

To provide a benchmark with which to compare subjective beliefs, we estimate rational-expectations admissions chances for the kindergarten and ninth-grade lotteries. Our measure of rational expectations admissions chances represent the beliefs about admissions chances that an agent would have if he knew his own report-specific priority, the rules of the mechanism, schools' capacities, the number of other applicants, and the underlying distribution of preference lists and report-specific priorities for other applicants, but did not know which preference lists and priorities had been drawn from this distribution. We calculate these probabilities by resampling $n = 200$ markets, drawing individuals, together with their applications and priority types, iid with replacement from the population. In each resampled market, we calculate the market-clearing cutoffs. Given a realization of the cutoff vector, we calculate admissions chances for each student. For example, if an individual has $rsp_{ij} = 41$ and lists j first, if the cutoff is $\underline{\pi}_j = 41.4$ then the individual has a .4 chance of placing in j. For each individual i, we compute the propensity to place in each school j under the individual's observed application and the given cutoff vector, and then average these chances over the resampled market-clearing cutoffs.

With rational expectations chances in hand, we define the following measures: let $optimism_{ija}$ denote the difference between i's subjective belief about his admissions chance at j under application a and the rational-expectation chance:

$$optimism_{ija} = \hat{p}_{ija} - p_{ija}^{\text{true}}$$

We also consider absolute error $|optimism_{ija}|$.

[7] We report rates at which each school is most-preferred, first listed, and listed at all in Tables A2 and A3.

The survey asked each of the 212 respondents about their beliefs for schools ranked first and second on two hypothetical applications, for a total of four elicited beliefs per respondent. Since some participants declined to answer some questions, we obtained a total of 786 elicited beliefs about admission to some school j under an application that listed j. We chose hypothetical applications that contained a mix of nearby schools, high-performing schools, and popular schools at the district level. The distribution of rational expectations admissions probabilities at the hypothetical applications is similar to the distribution of rational expectations probabilities for the actual applications that students in our sample submitted. See Figures A5 and A6 for the distribution of rational expectations probabilities in hypothetical and submitted applications.

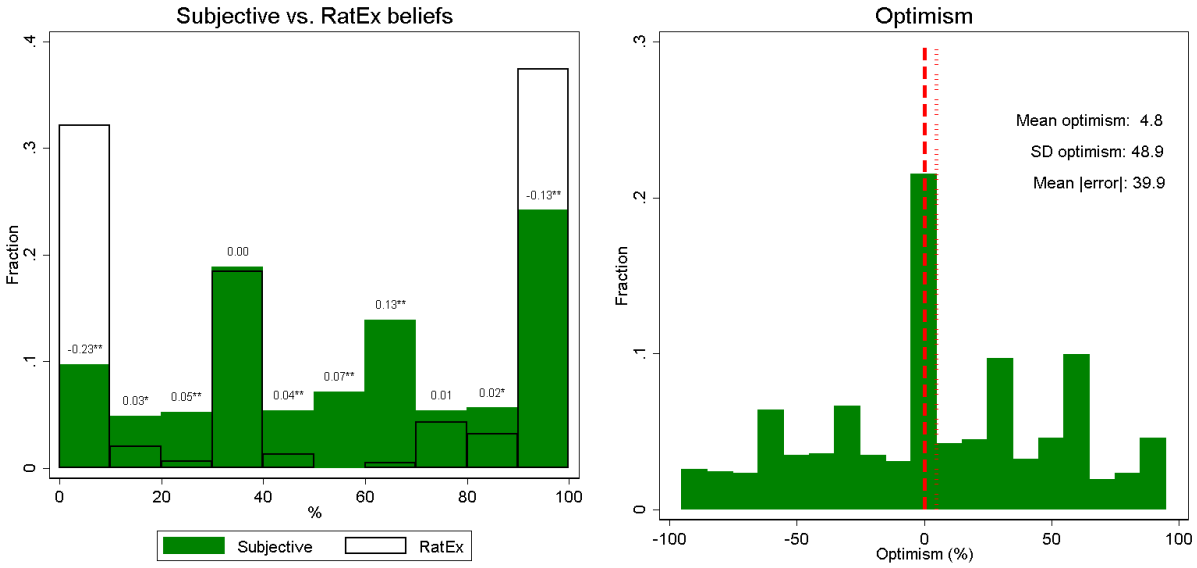
The survey elicited subjective probabilities in bins with widths of 10 percentage points (1 to 10%, 11 to 20%,...,91-100%). For second-ranked options, the survey elicited beliefs conditional on non-admission to the first ranked option. To facilitate graphical comparison between rational expecations and subjective probabilities, we place the (conditional) rational expectations probabilities in the same set of bins as the subjective probabilities. When computing averages of subjective expecations and differences between rational expectations and subjective expectations, we set subjective expectations to the midpoint of the reported bin.

The left panel of Figure Figure 2 plots the distribution of rational expectations and subjective beliefs for the sample of hypothetical applications. Above each bar is printed the difference between the share of subjective beliefs and the share of rational expectations beliefs in the bin, along with the standard error of the difference. Differences betwen subjective and rational expecations shares are large and statistically significant in many bins. The most notable difference between the two distributions is that households are much less likely to think admissions chances are close to zero or one than if they held rational expecations beliefs.

A consequence of this pattern is that many households think they have relatively large chances of admission when rational expectations are close to zero, or that they are likely to be rejected when admissions rates are close to one.[8] Cases where the rational expectations chance of admission was 0.1% or less but respondents believed their chances of admission were 25% or more accounted for 11 percent responses. We label these 'false positive' beliefs. Cases where applicants thought their chances of admission were below 50% but their true probability was at least 99.9% account for a further 8 percent of responses. We label these 'false negative' beliefs.

[8]The finding that households overestimate the likelihood of low-probability events (i.e., being rejected from a school with a rational expectations probability near one, or accepted at a school with a likelihood near zero) is common in behavioral economics; see e.g. Barberis (2013).

Figure 2: Distributions of beliefs and belief errors



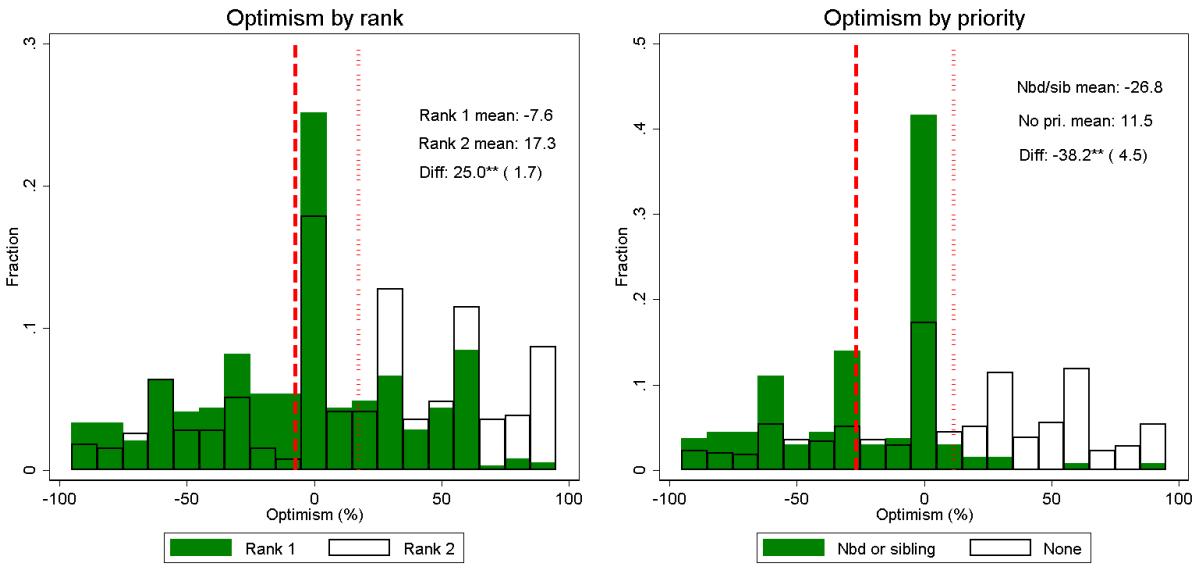
Notes: N=786. Left panel: distribution of subjective and rational expectations beliefs in bins of width 10 pp. Listed values are differences between the fraction of subjective and rational expectations beliefs in the bin. Significance from tests of equality of shares in subj. and RatEx distributions: +0.10 * 0.05 ** 0.01. Right panel: distribution of optimism (subjective minus RatEx belief) in bins of width 10. In both panels, subjective and RatEx beliefs for second-ranked options are conditional on non-admission to the first-ranked choice.

The right panel of Figure 2 plots the distribution of (conditional) optimism in the sample of hypothetical applications. Beliefs are on average correctly centered around zero, but the spread around this value is wide. The mean absolute error in conditional beliefs is 38 percentage points, while the mean absolute error in unconditional beliefs is 29 percentage points. The optimism distribution implies a distribution of shifts in beliefs about own application score relative to admissions cutoff scores $shift_{ij}(a)$ that is also centered near zero. See Figure A7.

Figure 3 shows that optimism is systematically related to rank and priority group. The left panel shows the distribution of optimism by submitted rank. Households are an average of 22 percentage points more optimistic about second-ranked options than first ranked options. This reflects a large decline in rational expectations probabilities between the first and second ranked choices coupled with a smaller decline in subjective beliefs. See Figure A8 for more detail. Similarly,

as reported in the right panel of Figure 3, households who have neighborhood or sibling priority are 34 percentage points less optimistic than households that do not. See Figure A9 for subjective and rational expectations belief distributions by priority. The observed distribution of optimism suggests that a realistic model of belief errors should allow for systematic variation by priority and rank as well as for scatter within these groups. We return to this point in Section 5.

Figure 3: Optimism by rank and priority



Notes: N=786. Left panel: distribution of optimism by submitted rank. Right panel: distribution of optimism by priority group (have neighbor or sibling priority vs. do not have priority). Significance for reported tests of cross-group differences in means:+0.10 * 0.05 ** 0.01. Bars show shares of population within bins of width 10. In both panels, optimism for second-ranked options is conditional on non-admission to the first-ranked choice.

We now consider the correlates of belief errors. Table 4 presents results from OLS regressions of optimism, absolute error, and false positive and false negative beliefs on student characteristics and descriptors of a household's interaction with the choice process. These descriptors include an indicator for participation in school choice, an indicator equal to one if a respondent reports looking up application counts from previous years, an indicator equal to one if the application we are asking

about is the respondent's most preferred school, and the count of information sources an applicant drew from when making a choice. We focus on unconditional probability measures from this point forward because these will be the inputs into our model of school choice.

Students from high-SES backgrounds have similar levels of optimism to other students but absolute errors that are lower by eight percentage points, or 28% of the sample mean of 29 percentage points. They are six percentage points (54%) less likely to have false positive beliefs and five percentage points (64%) less likely to have false negative beliefs. There is little variation in belief errors by race.

Table 4: Correlates of belief errors

	Optimism	Absolute error	False Pos.	False Neg.
High SES	-0.25	-8.21**	-0.06**	-0.06**
	(2.41)	(2.01)	(0.02)	(0.02)
Black	-3.33	1.52	0.01	0.02
	(2.47)	(2.17)	(0.02)	(0.02)
White/Asian	-2.81	1.92	0.01	0.02
	(3.98)	(3.25)	(0.03)	(0.03)
Grade 9	5.76*	-0.19	-0.02	-0.04*
	(2.57)	(2.04)	(0.02)	(0.02)
1st ranked	-17.90**	9.58**	-0.16**	0.12**
	(2.91)	(1.42)	(0.02)	(0.02)
Have priority	-16.56**	4.11+	-0.04	0.10**
	(4.41)	(2.36)	(0.03)	(0.03)
Filed an application	-1.96	0.47	0.02	0.03
	(2.31)	(2.07)	(0.02)	(0.02)
looked up past demand	5.39*	-3.63+	-0.02	-0.03+
	(2.22)	(2.06)	(0.02)	(0.02)
Most-preferred school	3.84	-1.49	0.03	-0.02
	(3.58)	(1.79)	(0.03)	(0.02)
Count info sources used	-0.35	-0.19	0.00	-0.01*
	(0.73)	(0.69)	(0.01)	(0.01)
Constant	12.14**	26.02**	0.22**	0.06*
	(3.33)	(3.02)	(0.03)	(0.03)
Observations	786	786	786	786

Notes: Significance: +0.10 * 0.05 ** 0.01. Linear regressions of error type listed in columns on control variables listed in rows. '1st ranked' and 'have priority' are dummies equal to one if a school was ranked first on the hypothetical application and if a student had some priority at that school, respectively. 'Filed an application' is and indicator equal to one if a student participated in choice. 'Looked up application past demand' is dummy equal to one if respondent reported looking up application counts and capacities from prior years. 'Most-preferred school' is a dummy equal to one if an application is to the school listed as most-preferred. 'Count of info sources used' counts sources of information respondents reported consuming.

There is also little evidence that search strategy, interest in choice, or preference for a particular school are related to belief errors. While students who looked up past demand have slightly lower absolute errors on average, the relationship between belief errors and the count of sources students drew upon when making their choice is weak. Similarly, there is no relationship between belief errors and students' choice to file an application, or their stated preference for a school. A possible explanation is that search for accurate information about one's own admissions chances is costly

and unrelated to other elements of the search process. These results motivate a model of belief errors in which belief accuracy does not depend on students' preference for a school or participation in choice.

4.4 Belief errors, application strategies, and placement outcomes

We next examine the relationship between belief errors, submitted applications, and placement outcomes. Panel A of Table 5 describes how subjective beliefs affect submitted applications conditional on rational expecations beliefs. We take as an outcome a dummy variable equal to one if a household that participates in choice lists its stated most-preferred school first on the application. We focus on the sample of choice participants for whom we elicit a subjective belief about their most-preferred school in the first-ranked spot on an application.

The first column of Panel A shows that subjective and rational expectations beliefs are postively but not perfectly correlated in this sample. A ten percentage point increase in rational expecations beliefs is associated with a 2.8 percentage point increase in subjective beliefs. The second through fourth columns regress the dummy for ranking the most-preferred option first on dummies for terciles of the subjective expectations distribution, with, respectively, no controls, controls for a second-degree polynomial in rational expectations admissions chances, and controls for the rational expecations polynomial and individual characteristics. Students in the upper tercile of the subjective expectations distribution are between 22 and 31 percentage points more likely to rank their most-preferred option first than those in the bottom tercile. This difference is statistically significant at the five percent level in the specifications with controls, and has a p-value of 0.101 in the specification without controls. In contrast, regression results indicate that rational expectations are weakly correlated with choice.

Figure 4 provides graphical evidence that subjective beliefs predict choice while rational expectations beliefs do not. After residualizing each variable using controls for SES, race/ethnicity, and grade, we regress the indicator for listing the most-preferred school first on subjective and RatEx beliefs. We plot the linear fit and binned means by quintile. We observe an upward slope in subjective beliefs and a flat slopein RatEx beliefs.

Table 5: Correlates of Placement Outcomes

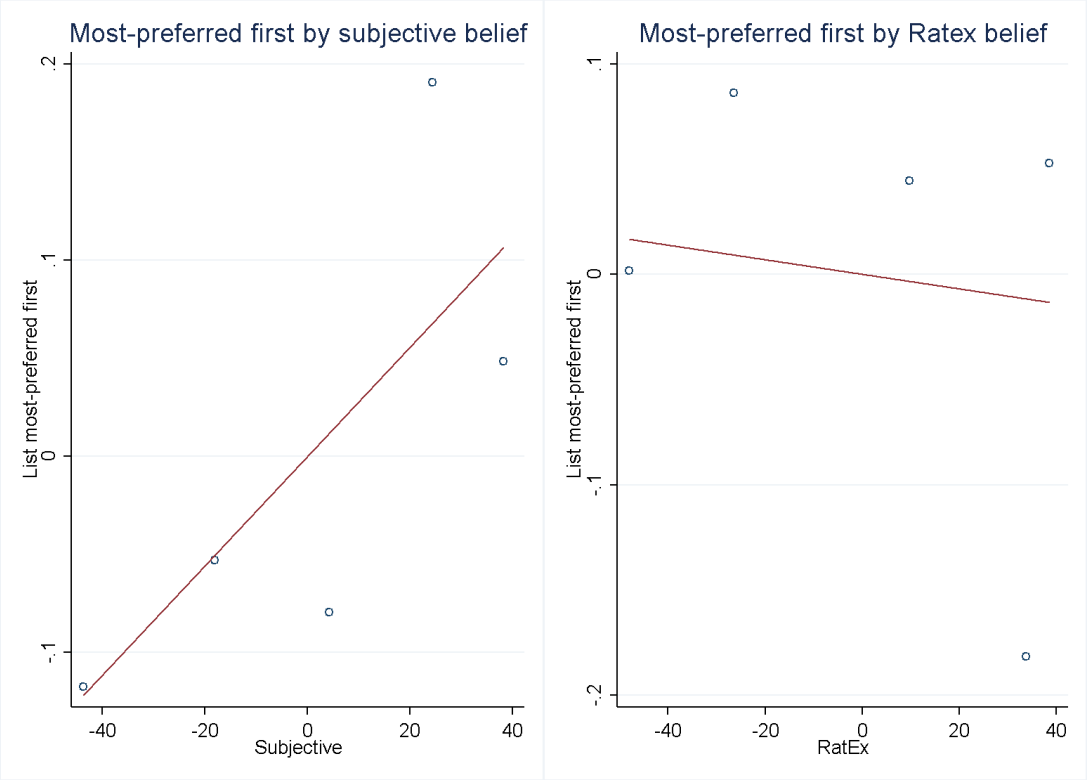
A. Choices and subjective beliefs	Subjective belief	List MP first	List MP first	List MP first
Middle tercile Subjective		0.109	0.089	0.094
		(0.142)	(0.141)	(0.143)
Upper tercile subjective		0.22	0.283*	0.306*
		(0.133)	(0.133)	(0.132)
RatEx	0.277**		0.011	0.017+
	(0.104)		(0.008)	(0.009)
Ratex^2/100			-0.011	-0.017*
			(0.007)	(0.008)
Observations	76	76	76	76
B. Placement by error	Place	First listed	Most preferred	VA placed
|error|>median	0.03	0.019	-0.199*	-0.003
	(0.056)	(0.086)	(0.087)	(0.013)
High SES	0.105+	0.174+	0.029	0.035*
	(0.059)	(0.090)	(0.090)	(0.013)
Black	0.007	-0.008	0.022	0.02
	(0.063)	(0.094)	(0.093)	(0.014)
White	0.024	0.005	-0.023	0.024
	(0.086)	(0.130)	(0.143)	(0.022)
Grade 9	-0.157**	-0.002	-0.127	-0.075**
	(0.042)	(0.093)	(0.095)	(0.015)
Constant	0.937**	0.622**	0.514**	-0.007
	(0.051)	(0.109)	(0.115)	(0.018)
Observations	120	120	120	110

Notes: Significance: +0.10 * 0.05 ** 0.01. Linear regressions of column variable on variables listed in rows. Observations at applicant level. Panel A: Sample is students who participated in choice for whom we recovered beliefs about admissions probabilities at their most-preferred school in the first-ranked spot. Column 1 regresses subjective beliefs on RatEx beliefs. Columns 2-4 regress indicators for ranking the most-preferred school first on dummies for terciles of the subjective belief distribution. Column 2 has no controls. Column 3 adds controls for a polynomial in RatEx beliefs. Column 4 adds demographic controls. Panel B: sample is surveyed choice participants. 'Place' is dummy for placing any listed school. 'First listed' is dummy for placing in first-listed school. 'Most-preferred' is dummy for placing in reported most-preferred school. 'VA placed' is test score VA estimate for placed school. This variable is missing for 10 observations. $|error|$ >median is a dummy equal to one if households' average absolute belief error over first choice schools across hypothetical applications is above the median value.

We also find that belief errors predict students' placement in the schools they most want to attend. We estimate linear probability regressions of indicator variables for any placement, placed in first choice school, and placed in most-preferred school on an indicator variable equal to one if a respondent's mean absolute belief error was above the median value, and student demographics.

We also present results from an OLS regression of value-added at the placed school on those same covariates. Observations are the respondent level, and the sample consists of choice participants.

Figure 4: Choices by subjective and RatEx beliefs



Notes: N=76. Linear fits and binned means within quintiles from regressions of the dummy for listing the most-preferred school first on subjective and rational expectations beliefs. All variables are residualized using controls for SES, race/ethnicity, and grade.

Panel B of Table 5 presents results. Belief errors are not strongly related to receiving any placement or to placing in the first-listed school. However, large belief errors reduce the probability of placement the most-preferrred program by 20 percentage points, on a base of 50 percent. Students from high-SES backgrounds are more likely to receive a placement and to place in their first choice school, and to place in schools with higher average value-added. However, conditional on the controls for belief error, they are not more likely to place into their most-preferred school.

To summarize, we use two main points from the descriptive analysis to inform our model of school choice. First, households behave strategically, but do so based on subjective beliefs that are often in error. Second, belief errors are larger for students from low-SES backgrounds, and vary systematically with ranking and priority group, but not with participation or reported preference for a school. A realistic model of belief formation should incorporate variation across SES, priority group, and rank, but can potentially abstract from a guided search framework in which households seek out information about admissions probabilities on the basis of preferences.

5 Model

Our model consists of three stages. First, applicants learn their preferences over schools and costs of applying to schools. Second, they choose whether to participate in the school choice process and, if they participate, what report to submit. Third, the lottery runs and participants receive placements. If there is space, nonparticipants are assigned to their neighborhood school, otherwise they are given a place where a spot is open.

Students $i \in I$ have underlying preferences over schools $j \in J$ according to:

$$u_{ij} = X_{ij}\beta + \epsilon_{ij}$$

where X_{ij} includes distance to the school from home $distance_{ij}$ and school characteristics, as well as interactions between student characteristics and school attributes such as academic quality. The errors ϵ_i are distributed according to

$$\epsilon_i \sim MVN(0, \Sigma),$$

iid across households.

In practice, each student has a fallback school that he will be placed in, or a distribution of fallback schools, if he does not receive a placement through the lottery. Each student therefore has an outside option u_{i0} which consists of the choice between attending this fallback school and leaving the district. We normalize the value of this outside option: $u_{i0} = 0$.

Once a student is placed in school j, he has the option to decline his placement. At the time of this decision, students receive a shock to preferences for j and for the outside option, giving a utility

$$U_{ij} = u_{ij} + \epsilon_{ij}^{e}$$

where the enrollment-time shock ϵ^e_{ij} has an extreme value distribution with scale parameter $\frac{1}{\lambda}$. The probability of accepting an offer is therefore

$$P(u_{ij} + \epsilon^e_{ij} > \epsilon^e_{i0}) = \frac{\exp(\lambda u_{ij})}{1 + \exp(\lambda u_{ij})}.$$

An alternative modeling choice would be to treat the choice to accept a placement as exogenous. We prefer our approach because descriptive evidence shows that applicants are more likely to accept placements at more-preferred schools. See Table A5 for details.

The expected value of school j at the time of matriculation is given by

$$v_{ij} = E(\max\{U_{ij}, U_{i0}|u_{ij}\}) = \frac{1}{\lambda}\log(1 + \exp(\lambda u_{ij})).$$

Let p_{ija} denote i's subjective estimate of the probability that he will be placed in school j if he submits report a to the mechanism. Students for whom $|a| = 0$ are those who do not participate in school choice.

To allow for nonparticipation and short application lists, we allow for a cost of receiving a placement. If i receives a placement in any inside school j, he receives a (possibly negative) payment

$$b_i \sim N(\mu_b, \sigma^2_b).$$

We interpret b_i as the cost of the actions i must take to accept or decline a placement. These include finding and getting in touch with the school placement office or the assigned school.

In this case, i's decision solves

$$\max_a \left(\sum_j p_{ija}(v_{ij} + b_i) \right).$$

We allow people to have mistaken beliefs about their priority or, equivalently, about schools' cutoffs, and about the role of priority and the rank of applications. We let

$$shift_{ijr} = \eta^0_i + \eta^r_i(r - \overline{r}_j) + \eta^{priority}_i \left(priority_{ij} - \overline{priority}_j \right) + \eta_{ij} + \eta_{ijr} \tag{2}$$

denote i's error about his own admissions ranking. Here, r is the rank of j on application a for student i, and $\overline{r}_j$ is the average rank of applications. Similarly, $priority_{ij}$ is $i's$ priority at j and $\overline{priority}_j$ is the average in the data. This functional form nests several relevant cases. For example, $\eta^r_i = 0$ means students understand how priority groups affect choices, while $\eta^r_i = -1$ if students

do not believe assignment probability depends on rank, as in a DA mechanism. $\eta_i^{priority} = -4$ corresponds to the case where students' beliefs about admissions probabilities do not change with changes in their priority group, while η_i^0 captures individual-specific optimism or pessimism and η_{ij}^0 captures idiosyncratic person-school error.

We assume $\eta_{ij} \sim N(0, \sigma^2_{\eta_{school}})$ iid across j, and $\eta_{ijr} \sim N(0, \sigma^2_{\eta_{school \times round}})$ i.i.d. The remaining terms are distributed according to

$$(\eta_i^0, \eta_i^r, \eta_i^{\text{priority}}) \sim N(\overline{\eta}, \Sigma^\eta).$$

This specification allows us to capture many types of errors. For example, people who misunderstand priorities may also misunderstand the importance of rank. Following our descriptive analysis, we allow for separate parameters for students from high- and low-SES backgrounds.

We now describe the preference specifications in detail. We consider the 9th grade and Kindergarten markets separately. In the 9th grade, X_{ij} includes a full vector of school dummies δ_j. In addition it includes an indicator for the zoned school, the distance to the zoned school, and an interaction term $1(\text{high value added}_j) \times 1(\text{low SES}_i)$.[9] In high school, the fallback school is the student's zoned school, which in practice is never oversubscribed. Students who wish to attend their zoned school are encouraged not to submit a lottery application. Moreover, we have confirmed that it is not possible to select one's own zoned school in the online version of the application. Therefore the zoned high school is properly part of the outside option. Because the relative value of placing in an inside school depends on the zoned school and the distance to it, we control for these characteristics.[10] The covariance matrix Σ is unrestricted. It therefore subsumes random coefficients on school indicators.

Because the Kindergarten market includes 34 schools, it would be impractical to estimate an unrestricted covariance matrix for preferences across schools, or mean effects for each school. We instead let X_{ij} consist of the distance between household i and school j and indicators for whether school j is the zoned school for i, whether j is an Achievement First charter, whether j is a charter at all, whether j is in the top tercile of the test score value added distribution, and whether j is a high- or medium demand school.[11] We also include interactions between the charter indicator and

[9] A school has high value added if it is in the top third of the value-added distribution in our population. A household has low SES if it is in the botton two thirds of the SES distribution in our population.

[10] Including zoned-school dummies and distance-to-zoned-school in each inside option is equivalent to parameterizing the outside option with those terms.

[11] To classify schools as high- or medium- demand, we rank them by the number of first-choice applications they received, and include dummies for being in the top and second terciles. In addition to the dummy for the high value added category, we include an indicator for schools for which we do not have a value added estimate. These are new

student SES category.

Rather than an unrestricted covariance matrix Σ, we specify a random effects model:

$$\varepsilon_{ij} = X_{ij}^{rc\prime}\beta_i^{rc} + \varepsilon_{ij}^*,$$

The terms X_{ij}^{rc} consist of indicators for charter status, for high value added, and for high demand. These classifications ((charter vs. public, high vs. low value added, and popular vs. niche schools) are important dimensions of differentiation among schools. The random coefficients are distributed as $\beta_i^{rc} \sim N(0, \Sigma_{rc})$. We place no restriction on Σ_{rc}. The remaining error component ε^* is independent but heteroskedastic across $j = 1, \ldots, J$ with distribution $\varepsilon_{ij}^* \sim N(0, \sigma_j^2)$.

In the Kindergarten lottery, in contrast to 9th grade, many zoned schools are oversubscribed in practice, and Kindergarten households are encouraged to apply to their local zoned school if they wish to attend it. If not placed, Kindergarten applicants will be assigned to a school that has space remaining, not necessarily their zoned school. As a result the zoned school is included among the inside options.

6 Estimation

We use a Bayesian Markov-Chain Monte Carlo (MCMC) procedure to estimate the model and sample from the posterior distribution of counterfactual outcomes. Similar methods have been used successfully in the marketing and industrial organization literatures to model consumers' demand for goods (McCulloch and Rossi, 1994) and have been applied successfully to centralized school choice (Agarwal and Somaini, 2014). Our strategy extends these methods to make use of surveyed beliefs and preferences as well as data on the decision to accept or decline a placement.

We use a three-step procedure. In the first step, we estimate the distribution of market-clearing cutoffs at each school, which determine the rational-expectations chances of admission at each school conditional on a priority vector and a report. Second, we use the survey together with the estimated rational-expectations chances to estimate the parameters governing the belief distribution. Third, we estimate the remaining parameters. To do so, we use data augmentation to pick utility vectors and beliefs for each individual consistent with their choices, introduce prior distributions for the model parameters, and use MCMC in order to sample from the posterior distribution of parameters conditional on the data. In order to obtain distributions of outcomes under counterfactuals, we simulate alternative policies at many points drawn from this posterior distribution. This approach

schools.

allows us to model belief errors even for non-surveyed individuals. Intuitively, the survey plays the critical role in pinning down the distribution of belief errors, but belief errors help rationalize observed choices for both surveyed and non-surveyed students.

6.1 Recovering admissions chances

Our approach is similar to Agarwal and Somaini (2014). Within each market (defined here by grades) we draw a large number (e.g. $N = 100$) of resampled markets by sampling from the population i.i.d. with replacement. Each resampled market is therefore a list of individuals with a participation decision, a report if they participated in the lottery, and a priority at each school. In each resampled market, we solve for market-clearing cutoffs by running the New Haven algorithm.

The distribution of cutoffs feeds into our results in two places. First, the cutoffs $\left\{\pi_j^{(k)}\right\}_{k=1,\ldots,N}$ allow us to calculate rational-expectations admissions chances, which serve as a benchmark in our descriptive analysis. Student i's chance of being placed in school j under report a_i is given by

$$\begin{aligned} P_{ij}(a_i) &\equiv Pr(placement_i(a_i) = j) = Pr\left(score_{ij} < \pi_j, score_{ij'} > \pi_{j'}\ \forall j' \text{ s.t. } j' \succ_a j\right) \\ &\approx \frac{1}{N}\sum_k \int 1\left(score_{ij} < \pi_j^{(k)}\right) 1\left(score_{ij'} > \pi_{j'}^{(k)}\ \forall j' \text{ s.t. } j' \prec_a j\right) dF(score_i | rsp_i, a_i). \end{aligned}$$

Second, the true cutoffs are inputs into our model of subjective beliefs about admissions chances.

6.2 Recovering belief parameters

Having obtained the distribution of cutoffs in each grade, we combine the ninth-grade and kindergarten surveys to estimate the parameters $\sigma_{\eta_j}, \sigma_{\eta_{jr}}$, Σ^η, and $\overline{\eta}$. These parameters govern the distribution of individual belief shocks and random terms η_i. To recover them, we first compute initial values of $shift_{ijr}(a)$ for each individual, school, round and hypothetical application for which we elicited a subjective belief.[12] We then estimate equation 2. In practice we use a Gibbs sampler, iteratively updating $\sigma_{\eta_{school}}, \sigma_{\eta_{school*round}}$, Σ^η, and $\overline{\eta}$ each conditional on the other parameters. This procedure can be interpreted as approximating the maximum-likelihood estimates of these parameters.

[12] We first find values for first-round values $shift_{ij1}$, then compute later-round shifts. We compute $shift_{ijr}$ conservatively. If a household reports an interior subjective belief of, say, 50 to 60 %, we pick the value of $shift$ that would make the subjective belief be the midpoint of the given interval (55%). If a household reports a subjective belief in the 90-100% or 0-10% intervals, and the rational expectations chance lies in that interval, we set $shift_{ijr} = 0$.

6.3 Recovering preference parameters

Before we describe the MCMC procedure in detail, we discuss the normalizations that we make, and the restrictions implied by households' optimal application decisions, accept/decline decisions, and reported first and second choices.

6.3.1 Normalization

We have already imposed the location normalization $u_{i0} = 0$. The following scale normalization is also useful. Define $\tilde{u} = \lambda u$, $\tilde{v} = \lambda v$, and $\tilde{\mu} = \lambda \mu$. Let

$$\tilde{u} = X\tilde{\beta} + \tilde{\epsilon}^a,$$

and fix

$$\tilde{\beta}_{dist} = -1.$$

By construction $\tilde{\epsilon}$ has a standard Gumbel distribution. The probability of accepting an offer, conditional on $\tilde{u}$, is then

$$s_{ij} = \frac{\exp(\lambda u_{ij})}{1 + \exp(\lambda u_{ij})} = \frac{\exp(\tilde{u}_{ij})}{1 + \exp(\tilde{u}_{ij})}.$$

The expected value of an offer gives

$$\tilde{v}_{ij} = \log(1 + \exp(\tilde{u}_{ij})).$$

Because $\tilde{\beta}_{dist} = -1$, welfare in units of miles traveled is given by

$$\tilde{v}_{ij} = \tilde{v}_{ij}/|\tilde{\beta}_{dist}| = v_{ij}/\beta_{dist}.$$

6.3.2 Optimality of applications

Let $\tilde{v}_i = (\tilde{v}_{i1}, \ldots, \tilde{v}_{iJ}, \tilde{b}_i)$ denote the vector of inclusive values of admission to each of the J schools, and let $p_i(a)$ denote the vector of i's subjective beliefs about admissions chances under report a. Agarwal and Somaini (2014) observe that a report a is optimal for agent i if and only if $v_i \cdot p_i(a) \geq v_i \cdot p_i(a')$ for all reports a'. Hence, given the matrix $\Gamma_i = (p_i(a) - p_i(a_1), \ldots, p_i(a) - p_i(a_N))$, a report is optimal if and only if $\Gamma_i' * (v_i + b_i) \geq 0$. Equivalently, a report is optimal if and only if

$$\Gamma_i' * \left(\tilde{v}_i + \tilde{b}_i\right) \geq 0.$$

6.3.3 Accept/decline decision and reported preferences

In the survey we elicit households' first and second choices if parents could choose any school, unconstrained by admissions chances. We allow for measurement error in elicited preferences: If i says that j_1 is the household's first choice, then

$$u_{ij_1} + \epsilon_{ij_1}^{survey} > u_{ij} + \epsilon_{ij}^{survey} \ \forall j$$

Similarly, if j_2 is the household's second choice, then

$$u_{ij_2} + \epsilon_{ij_2}^{survey} > u_{ij} + \epsilon_{ij}^{survey} \ \forall j \neq j_1.$$

Scaling by λ without loss, we assume the measurement error is drawn iid from a normal distribution:

$$\tilde{\epsilon}_{ij} = \lambda \epsilon_{ij}^{survey} \sim N(0, \tilde{\sigma}^2_{survey}), \ iid.$$

We also make use of the decision to accept or decline a placement. If i accepts a placement in j, then we require $u_{ij} + \epsilon^e_{ij} > \epsilon^e_{i0}$. If i receives and declines a placement in j, we require $u_{ij} + \epsilon^e_{i0} < \epsilon^e_{ij}$. Define

$$\tilde{\epsilon}^e_i = \lambda * \left(\epsilon^e_{ij} - \epsilon^e_{i0}\right).$$

By construction $\tilde{\epsilon}^e_i$ has a standard logistic distribution.

We can write these constraints in matrix form as

$$\Gamma'_{i,(shock)} * \begin{pmatrix} \tilde{u}_i \\ \tilde{\epsilon}_i^{survey} \\ \tilde{\epsilon}^e_i \end{pmatrix} \geq 0.$$

If i reported first and second choices, then the first column of $\Gamma_{i,(shock)}$ contains 1's in the j_1th and $(J + j_1)$th places, and -1 in the j_2th and $(j + j_2)$th places.[13] The next $J - 1$ columns similarly require

$$u_{i,j_2} + \epsilon_{i,j_2}^{survey} > u_{i,j} + \epsilon_{i,j}^{survey} for j \neq j_1, j_2.$$

If i was placed in school j, then the final column of $\Gamma_{i,(shock)}$ contains 1 in the jth place and -1 in the final place.

[13] If i reported a first but not a second choice, we similarly construct $\Gamma_{i,(shock)}$ using the resulting inequalities.

6.3.4 Starting values

We first construct feasible belief shifts $shift_{ij}$ for all i and j. Where the survey provides no constraints, we start at $shift_{ij} = 0$, i.e. at the rational-expectations value. We pick points interior to the relevant intervals when households report beliefs.

Next, given the feasible beliefs, we use linear programming techniques to construct strictly feasible utilities $\tilde{u}_i \in \mathbb{R}^J$ and placement payoff terms $b_i \in \mathbb{R}$. A utility vector $\tilde{u}_i$ and benefit $\tilde{b}_i$ are (strictly) feasible if the observed report a_i is optimal conditional on the beliefs p_i, that is if $\Gamma_i(p) * \left(\tilde{v}_i + \tilde{b}_i\right)' > 0$. We allow the set of possible reports to include an empty list, which we interpret as nonparticipation.

Finally, we use linear programming again to pick strictly feasible enrollment-time shocks $\tilde{\epsilon}_i^e$ and measurement errors $\tilde{\epsilon}_i^{survey}$.

We now describe the prior distributions and MCMC procedure that we use to estimate the remaining parameters. We first present the procedure for the simpler ninth-grade specification, then discuss the modifications we make when we estimate the Kindergarten model.

6.3.5 Prior distributions

We begin with prior distributions over the preference parameters and belief parameters. We place priors directly on $\tilde{\beta}$, $\tilde{\Sigma}$, $\tilde{\mu}_b = \lambda\mu_b$, $\tilde{\sigma}_b = \lambda\sigma_b$, and $\tilde{\sigma}_{survey}$ as well as on the belief parameters separately by SES category. In order to minimize the priors' influence on our estimates, we choose the following diffuse (flat) priors:

$$\begin{aligned}
\tilde{\beta} &\sim N(0, 100 * I) \\
\tilde{\Sigma} &\sim IW(J, I) \\
\tilde{\sigma}_{survey}, \tilde{\sigma}_b &\sim InverseGamma(1,1)\ iid \\
\overline{\eta} &\sim N(0, 100 * I) \\
\Sigma^{\eta} &\sim IW(4, I) \\
\sigma^2_{\eta_{school}}, \sigma^2_{\eta_{school*round}} &\sim InverseGamma(1,1)\ iid
\end{aligned}$$

We assume that the priors are independent.

6.3.6 MCMC iteration

Next, we iterate through the following steps, which consist of sampling from the conditional posterior distributions of utilities, utility shocks, beliefs, application costs, and model parameters:

1. Draw mean-utility parameters $\beta^{(s+1)}$ and mean benefit $\mu_b^{(s+1)}$ from the distribution of $\tilde{\beta}|\tilde{u}^{(s)}, \tilde{\Sigma}^{(s)}$ and $\tilde{\mu}_b|\tilde{b}^{(s)}, \tilde{\sigma}_b^{(s)}$
2. Draw variance of benefit term $(\tilde{\sigma}_b^2)^{(s+1)}$ from the distribution of $\sigma_c^2|\mu_b^{(s+1)}, b^{(s)}$.
3. Draw variance of shocks to reported preferences σ^2_{survey} from the distribution of $\sigma^2_{survey}|\tilde{\epsilon}^{survey}$.
4. Draw covariance matrix $\Sigma^{(s+1)}$ from the distribution of $\Sigma|\beta^{(s+1)}, u^{(s)}$.
5. For each individual in the dataset:
   (a) Draw utility $u_i^{(s+1)}$ from the posterior distribution of $\tilde{u}_i$ given β, Σ, i's decision to accept or decline his placement (if offered one), and constraints implied by the optimality of i's report.
   (b) Draw $\tilde{b}_i^{(s+1)}$ from the posterior distribution of $\tilde{b}_i$ given $\tilde{v}_i(\tilde{u}_i^{(s+1)})$ and constraints implied by the optimality of i's report.
   (c) Draw shock realizations $\tilde{\epsilon}_i^{survey}$ and $\tilde{\epsilon}_i^e$ from their posterior distributions given $\tilde{u}_i$ and the constraints $\Gamma'_{i,(shock)} * \begin{pmatrix} u_i \\ \epsilon_i^{survey} \\ \epsilon_i^e \end{pmatrix} \geq 0$.
   (d) Draw belief random effects η_i^0, $\eta_i^{priority}$, η_i^{round}, and $\{\eta_{ij}\}_{j\in J}$ from their posterior distribution given $shift_i$, $\overline{\eta}$, Σ_η, $\sigma^2_{\eta_{school\times round}}$, and $\sigma^2_{\eta_{school}}$.
   (e) Draw $shift_i$ from its posterior distribution conditional on η_i^0, $\eta_i^{priority}$, η_i^{round}, $\{\eta_{ij}\}_{j\in J}$, $\tilde{v}_i$, $\tilde{b}_i$, and the constraints imposed by the survey.

Utilities: In order to update utilities, for each individual we iterate through the various schools, updating the terms $\tilde{u}_{ij}$ sequentially. Because $\tilde{u}_i$ is jointly normal, the distribution of $u_{ij}|u_{i,-j}, \beta, \Sigma$ is normal with known mean and variance.

The restriction $\Gamma_i' * (v_i + b_i) \geq 0$ implies that $\tilde{v}_{ij}$ must belong to a (known) interval whose endpoints depend on $\tilde{v}_{i,-j}$ and $\tilde{b}_i$.[14] Recall that $\tilde{v}_{ij} = \log(1+\exp(\tilde{u}_{ij}))$ is a monotone transformation

[14] Similarly, b_i must belong to an interval with known endpoints that depend on $\tilde{v}_i$.

of $\tilde{u}_{ij}$. Therefore, conditional on the optimality of the report and the current values of other variables and parameters, updating u_{ij} consists of drawing from a truncated normal distribution.

Beliefs: In order to update beliefs subject to the constraints provided by the survey and by optimality, we take a standard Metropolis-Hastings step using a symmetric normal proposal density. For each individual, we draw a vector $\Delta(shift_i) \sim N(0, \sigma_{proposal} * I)$, and construct a new proposal $shift_i' = shift_i + \Delta(shift_i)$. We update to the proposed draw with the appropriate Metropolis-Hastings acceptance probability. We reject the proposal with probability 1 if it violates the constraints imposed by the survey or causes the observed report to become non-optimal.[15] We tune the variance of the proposal density so that roughly one third of draws are accepted.

Kindergarten: In order to estimate the random-effects model, we modify the procedure in the following ways:

1. Consistent with the restrictions we place on Σ, we specify i.i.d. $InverseGamma(1,1)$ priors on σ_j^2 and an $InverseWishart(4, I)$ prior on the random coefficients' covariance matrix, Σ^{rc}.
2. We augment the data with draws of β_i^{rc} for each i.
3. We introduce a step to update these draws: For each individual, draw $\beta_i^{rc\ (s+1)}$ from the posterior distribution of $\beta_i^{rc}|v_i^{(s+1)}, \beta^{(s+1)}, \{\sigma_j^2\}_{j\in J}^{(s+1)}$.
4. The update of the covariance matrix is modified. We draw σ_j^2 for each school j from its posterior, conditioning on u, β, and the matrix of random coeffcients β^{rc}
5. When updating u_i and b_i, we condition on β_i^{rc}.
6. When updating β, we condition on β^{rc}.

Implementation: To obtain belief parameters we use a chain of 80000 draws, discarding the first 20000 to allow for burn-in. In estimating preferences, we use a chain of 40000 iterations for the ninth-grade market and 20000 iterations for Kindergarten. We discard the first half of the draws in order to allow for burn-in.

[15] Individuals' belief errors $\eta_{ijr} = shift_{ijr} - \eta_i^0 - \eta_i^{priority} * priority_{ij} - \eta_i^{round} * r - \eta_{ij}$ are distributed according to truncated normal distributions. If the report is optimal and consistent with the survey, the density of η_{ijr} is proportional to a normal density.

7 Results

7.1 Estimation results

Table 6 reports estimates and credible intervals for model parameters. For each parameter we show .025, .5, and .975 quantiles of the posterior distribution. While we recover a full distribution, the median may be taken as a point estimate. Panel A displays estimates of belief model parameters for high and low-SES students. To interpret the magnitudes, note that that there is an interval of length 1 for each report-specific priority type such that if the cutoff lies in this interval, the type is rationed. Trace plots are reported in Figures A10 and A11 and off-diagonal elements of covariance matrices are reported in Table A6.

Focusing first on idiosyncratic school and school-rank specific errors, we find that σ_{η_j} and $\sigma_{\eta_{jr}}$ converge to values far from zero. For low-SES students, the standard deviation of these error terms are roughly 2.1 and 1.2, respectively, while for high-SES students the values are roughly 1.7 and 1.2. These are sufficiently large to lead to mistaken beliefs about the round in which the capacity constraint binds. Households also make errors that are systematically correlated with their priority at a school and the round in which they apply to a school. We estimate $\mu_{priority}$ for low-SES households at -2.6. The true effect of admissions priority group on admission score is 4, so households underestimate the effect of sibling or neighborhood priority on admissions score by more than half. Similarly, the true effect of ranking a school one spot higher on one's application is to reduce the application score by 1. A μ_{round} estimate of roughly -0.5 indicates that students underestimate this effect by half. Errors of both types are smaller for high-SES households. Estimates of $\sigma_{\eta_{pri}}$ and $\sigma_{\eta_{round}}$ indicate that there is substantial heterogeneity across households in the effects of round and priority group on belief errors.

Table 6: Parameter Estimates

Variable	.025	.5	.975	Variable	.025	.5	.975
A. Belief parameters		*Low SES*				*High SES*	
σ_{η_0} (low SES)	0.72	1.12	1.566	σ_{η_0} (high SES)	0.479	0.847	1.347
$\sigma_{\eta_{pri}}$ (low SES)	1.122	1.714	2.375	$\sigma_{\eta_{pri}}$ (high SES)	1.071	1.711	2.631
$\sigma_{\eta_{round}}$ (low SES)	1.612	1.869	2.206	$\sigma_{\eta_{round}}$ (high SES)	1.523	1.857	2.285
σ_{η_j} (low SES)	1.829	2.071	2.393	σ_{η_j} (high SES)	1.412	1.73	2.075
$\sigma_{\eta_{jr}}$ (low SES)	1.112	1.2	1.296	$\sigma_{\eta_{jr}}$ (high SES)	1.07	1.178	1.307
μ_0 (low SES)	-0.213	0.158	0.544	μ_0 (high SES)	-0.601	-0.108	0.342
$\mu_{priority}$ (low SES)	-3.251	-2.58	-1.916	$\mu_{priority}$ (high SES)	-3.114	-2.222	-1.228
μ_{round} (low SES)	-0.838	-0.472	-0.084	μ_{round} (high SES)	-0.76	-0.264	0.26
B. Preference parameters		*Grade K*				*Grade 9*	
1(low SES)	-0.833	-0.531	-0.23	1(low SES)	-1.926	-0.388	0.888
High value added	-0.119	0.01	0.113	Distance to zoned	-0.358	0.105	0.318
Missing value added	0.866	1.216	1.585	1(default$_i$ = Cross)	0.299	1.373	2.601
Own zone	1.52	1.753	2.022	High VA × low SES	-1.329	1.598	5.309
AF charter	-0.983	-0.478	0.041	δ AF charter	-2.328	-0.168	2.966
Any charter	-0.466	-0.036	0.345	δ CG charter	-6.307	-3.44	-0.959
High demand	2.104	2.626	3.188	δ CoOp Arts	0.93	3.107	5.403
medium demand	1.099	1.552	2.045	δ Engi/Sci Mag.	-0.611	1.454	4.471
Charter X low SES	0.095	0.513	0.851	δ HSC	-3.263	0.058	1.931
Constant	-1.649	-0.968	-0.384	δ Career Acad.	-0.248	1.414	4.426
μ_b	-5.069	-4.749	-4.455	δ Hillhouse	-9.73	-6.238	-0.945
σ_b	0.383	0.484	0.622	δ Hyde School	-0.753	2.255	5.327
σ_{survey}	1.382	1.791	2.222	δ Metro Business	-1.059	1.784	4.224
				δ New Haven Acad.	-1.398	1.072	3.639
				δ Riverside Acad.	-11.252	-5.031	0.06
				δ Cross HS	-6.856	-3.121	1.146
				μ_b	-3.259	-2.339	-1.252
				σ_b	0.528	0.809	1.729
				σ_{survey}	4.551	7.023	9.799

Notes: Quantiles of distribution of posterior mean for parameters listed in the rows. Panel A: belief model by student SES. 'High SES' is top third of SES distribution. Off-diagonal elements of covariance matrices reported in Table A6. Panel B: preference parameter estimates by grade. Coefficient on miles traveled is normalized to -1. Appendix Tables A7, A8, and A9 provide 90% credible intervals for elements of the utility shock covariance matrices Σ. Grade 9: The coefficients on Wilbur Cross and Hillhouse apply only to students who are not zoned into these schools. The coefficient on the own zoned school is set equal to zero.

Panel B of Table 6 presents estimates of the parameters governing household preferences. Estimates from the Grade K model are in the left three columns, while estimates from the Grade 9 model are in the right three columns. To interpret the coefficients, recall that the coefficient on miles traveled is equal to -1 and that the mean utility of the 'no placement' outcome, which includes placement the choice to leave the district, is normalized to zero. See Tables A7, A8, and A9 for the utility shock covariance matrix Σ and Figures A12 through A16 for trace plots.

We focus our discussion on estimates of the Grade K model. A coefficient of roughly -0.5 on the $1(lowSES)$ variable indicates that low-SES households have a lower taste for inside-option schools. The credible interval for households' taste for schools with high value added spans zero, as do the credible intervals for charter school taste main effects. We do observe a stronger preference for charters among low-SES students. On average, receiving a placement is costly, with $\mu_b \in (-5.1, -4.5)$, and a standard deviation of $\sigma_b \in (0.4, 0.6)$. Measurement error in reported preferences has a standard deviation of roughly 1.8 miles traveled, suggesting that elicited first- and second-choice data is informative but not perfectly so.

Results from our HS model are similar in that we observe differences in preferences across schools relative to the outside option. The school with the highest mean utility term is a magnet school specializing in arts. The school with the lowest mean utility term is the neighborhood high school with the lower-SES catchment zone.

7.2 Welfare analysis and counterfactual simulations

We now turn to an analysis of households' welfare and test scores under observed and counterfactual policies. Our procedure estimates the joint distribution of parameters and utilities. Using this distribution, we are able to compute each household's expected welfare according to its utility and the true rational-expectations admissions chances under the application it submitted. We compute average utility at every 10th iteration along the Markov chain after the burn-in period, and divide by $|\tilde{\beta}_{dist}|$ to measure welfare in miles traveled.

In the first counterfactual, we simulate outcomes under deferred acceptance. We evaluate outcomes under deferred acceptance in two ways. In the first, we maintain the limit of at most four schools per application. Under the resulting 'truncated deferred acceptance' procedure it need not be optimal to report truthfully (Fack et al., 2015). We simulate equilibrium play under this mechanism, making the assumption that households have rational expectations beliefs. In the second approach, we simulate play under the same truncated deferred acceptance procedure with 'naive' play, in which households list their most-preferred schools in order regardless of beliefs, but stop if

$v_{ij} + b_i < 0$ for the best remaining j, or if they run out of spaces on the application form. This addresses the concern that imposing rational expectations beliefs may overstate the gains from deferred acceptance. We consider the naive deferred acceptance counterfactual for a variety of application lengths.

Our second counterfactual considers the effects of informational interventions by shrinking the $shift_{ijr}$ error terms by factors ranging from zero to one and then solving for the equilibrium of the New Haven mechanism. A factor of one corresponds to a best-case informational intervention, with $shift_{ijr} = 0$ for all ijr. A factor of zero corresponds to baseline case. An alternate interpretation of the best-case intervention counterfactual is as the result of providing a strategic and informed 'proxy' player with each applicant's cardinal utilities and allowing the proxy player to submit the application list (Budish and Cantillon, 2012).

There may be multiple equilibria under rational expectations and under "sophisticated" truncated deferred acceptance. We select an equilibrium as follows. We start with the distribution of cutoffs π^0 that we recovered from the data in step 1. We then compute optimal applications for each household. Given the new applications and our resampled draws, we compute a new distribution of cutoffs π'. We obtain new cutoffs $\pi^1 = (1-\alpha)\pi^0 + \alpha\pi'$ for $\alpha \in (0,1)$ pointwise in each resampled market, and compute optimal applications given π^1. We iterate this procedure until convergence. We take $\alpha = 0.9$ as a starting value and decrease this value as we iterate.

7.2.1 Aggregate welfare in policy counterfactuals

Table 7 describes the posterior distribution of mean welfare in the market for the benchmark case, the rational expectations counterfactual and the sophisticated DA counterfactual, as measured in miles traveled. In the first column, labeled 'NH', we display quantiles of this distribution under the New Haven mechanism. The second column, 'RatEx,' shows quantiles of the posterior distribution under optimal reports with rational-expectations beliefs in the New Haven mechanism. The third column, 'DA,' describes the posterior distribution under sophisticated deferred acceptance, while columns four and five present the differences between the RatEx and DA mechanisms and baseline NH mechanism.

Table 7: Distance-Metric Welfare: Benchmark and Counterfactuals

quantile	NH	RatEx	DA	RatEx - NH	DA - NH	No survey DA-NH
A. Grade K						
0.05	1.844	1.975	1.925	0.126	0.072	-0.116
0.25	1.877	2.019	1.974	0.134	0.084	-0.113
0.5	1.937	2.076	2.024	0.142	0.092	-0.108
0.75	1.989	2.133	2.085	0.148	0.098	-0.103
0.95	2.027	2.183	2.136	0.162	0.111	-0.099
B. Grade 9						
0.05	1.067	1.299	1.127	0.216	0.042	-0.294
0.25	1.148	1.397	1.198	0.254	0.066	-0.264
0.5	1.243	1.587	1.383	0.337	0.13	-0.255
0.75	1.388	1.741	1.54	0.376	0.16	-0.246
0.95	1.486	1.878	1.676	0.412	0.19	-0.234

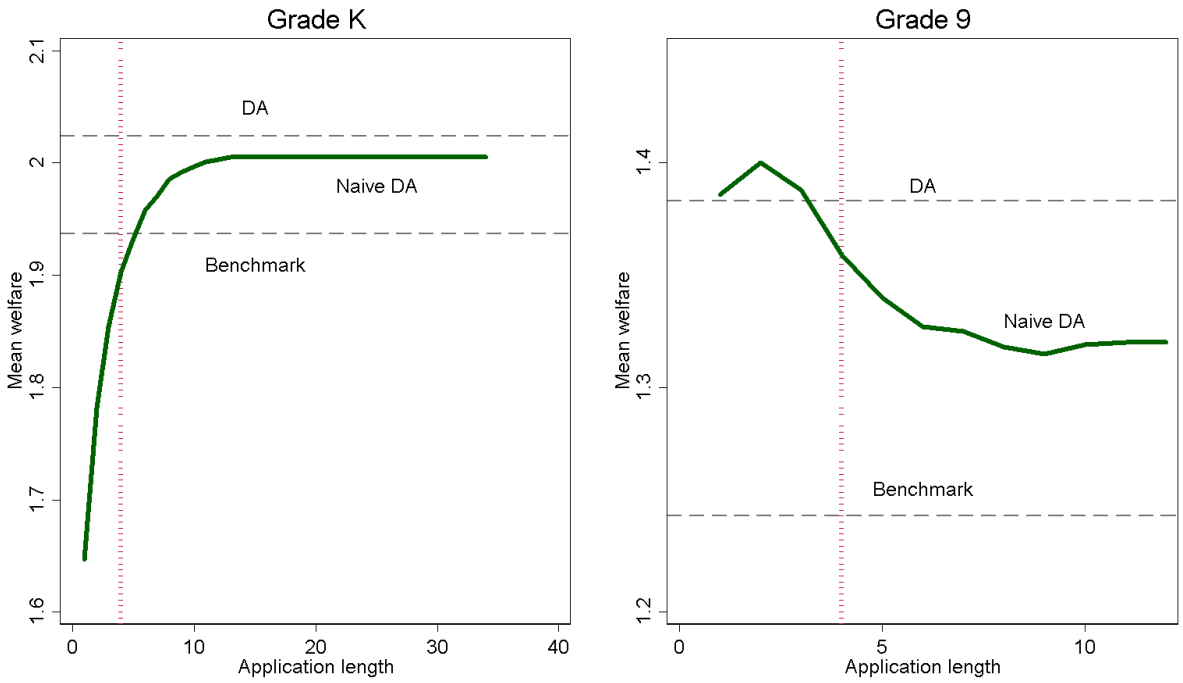
Notes: This table displays quantiles of the posterior distribution of mean welfare by grade in baseline case and under policy counterfactuals. Welfare is measured using miles traveled as the numeraire good. 'NH' is baseline New Haven mechanism given observed beliefs. 'RatEx' is the New Haven mechanism under rational expectations beliefs. 'DA' is the sophisticated deferred acceptance mechanism. 'RatEx-NH' and 'DA-NH' columns compares welfare differences under the listed mechanisms. 'Alt. prefs DA-NH' column compares welfare under the sophisticated DA and NH mechanisms using model estimates based on rational expectations beliefs.

Aggregate welfare improves in both counterfactuals in both grades K and 9. Taking the median as a point estimate, the average grade K household would be made better off by the equivalent of 0.14 fewer miles traveled under rational expecations and 0.09 fewer miles traveled under sophisticated deferred acceptance. The equivalent values for grade 9 are 0.34 and 0.13. 95% posterior probability intervals for these differences do not cover zero.

Figure 5 presents results from the naive DA counterfactual. The vertical axis is the median value of the posterior mean welfare distribution, and the horizontal axis is the number of schools households are allowed to rank on their application. Mean welfare estimates from the benchmark New Haven mechanism case and the sophisticated DA case with an application length of four are marked by horizontal lines. In the kindergarten market, welfare under naive DA is well below both the benchmark and sophisticated DA values for small numbers of schools. This is because many of the 34 schools in the kindergarten market have few spots, so students often go unplaced when their applications consist only of their most-preferred schools. Welfare for naive DA exceeds

benchmark welfare when a fifth school is added to the application, and then plateaus at a level just below sophisticated DA at application lengths of 10 or more. In the grade 9 market, welfare under naive DA is above benchmark welfare at all application lengths, and above sophisticated DA for very short applications. Our conclusion is that a switch from the New Haven mechanism to deferred acceptance is welfare improving even under imperfect play, although robustness to naive play depends on application length in the kindergarten market.

Figure 5: Welfare under naive DA by list length

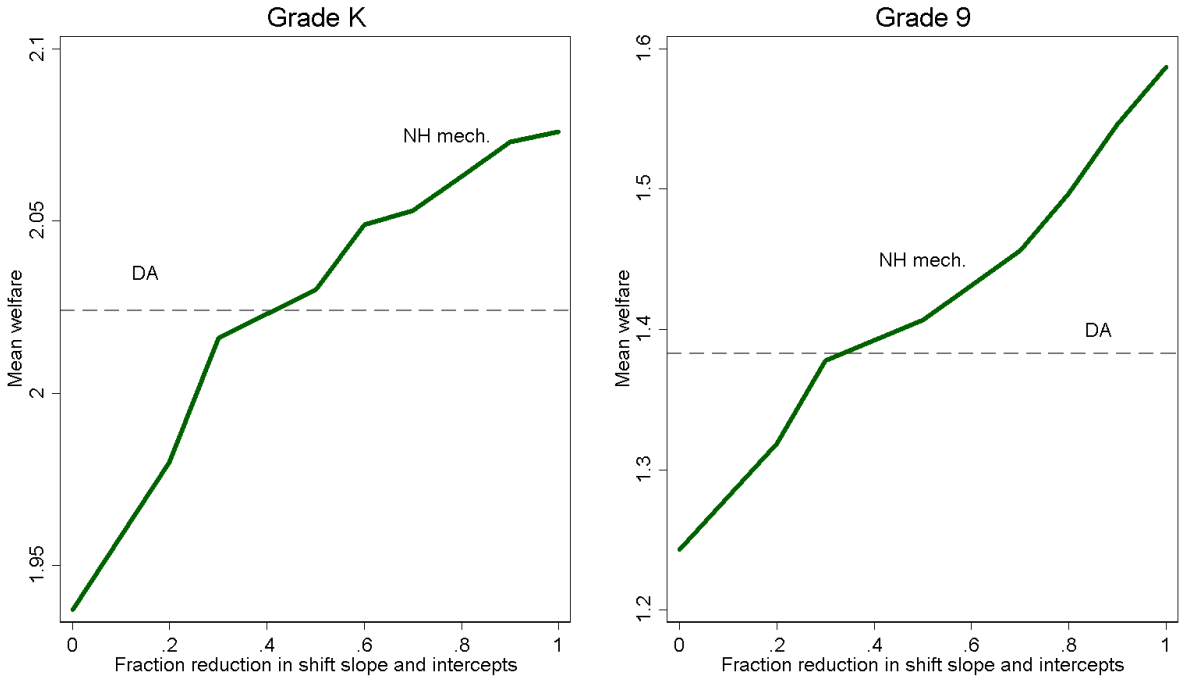


Notes: median of posterior mean welfare distribution (vertical axis) under naive DA policy counterfactual by application length (horizontal axis) and grade. 'Benchmark' line is median of posterior mean welfare under the NH mechanism and observed beliefs with an application length of four. 'DA' is welfare under the sophisticated DA counterfactual at an application length of four.

Data on subjective beliefs are important for market designers trying to choose the welfare-maximizing assignment mechanism. The sixth column of Table 7 compares average welfare under

the deferred acceptance and New Haven mechanisms using model estimates obtained without using survey data. We impose rational expectations beliefs in estimation and in counterfactual simulations. These estimates reverse the welfare comparison between DA and NH mechanisms, with the NH mechanism outperforming DA by 0.11 miles traveled in the kindergarten market and 0.26 miles traveled in the grade 9 market. This finding is consistent with results from Agarwal and Somaini (2014) and Calsamiglia et al. (2014). For example Agarwal and Somaini (2014) estimate a welfare loss of 0.07 additional miles traveled when switching from the Cambridge mechanism under rational expectations to deferred acceptance.

Figure 6: Mean welfare by reduction in variance of shift term



Notes: median of posterior distribution of mean welfare under the New Haven mechanism (vertical axis) by fraction reduction in $shift_{ijr}$ terms (horizontal axis) and grade. Horizontal lines are median of posterior mean welfare dist. under sophisticated DA counterfactual.

We next ask how effective an informational intervention would have to be to cause the New

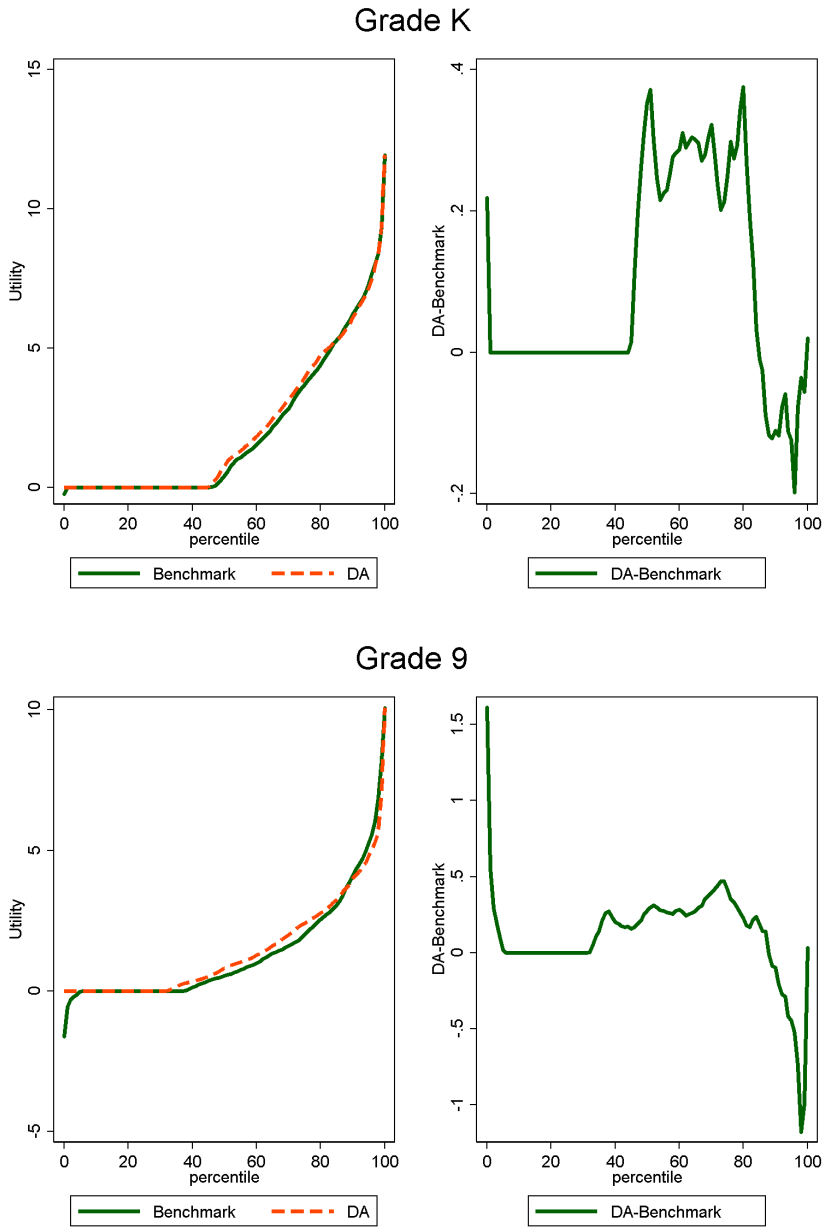
Haven mechanism to raise aggregate welfare relative to deferred acceptance. We scale all shift terms by values ranging from zero to one and simulate counterfactual welfare distribution in each case. Figure 6 presents results from this exercise. The horizontal axis is the fraction reduction in the shift term, and the dashed line is the mean of the welfare distribution from the deferred acceptance procedure. For mean welfare under the New Haven mechanism to break even with the deferred acceptance level requires roughly a 40% scale-down of shift terms in both markets. Given the school district's extensive outreach efforts at baseline, it is unclear what an intervention with this effect would look like.

7.2.2 Distributional impacts of policy counterfactuals

One of the arguments in favor of deferred acceptance mechanisms is that even if they do not raise welfare on average, they help produce a more equitable distribution of welfare across participants. We explore this idea by examining the distribution of welfare across households under the baseline and deferred acceptance mechanisms. For each household, we compute mean welfare by averaging the household's welfare across MCMC iterations. Figure 7 reports mean welfare for households in each centile of the welfare distribution under New Haven and deferred acceptance. Recall that welfare is normalized to zero for unplaced households. In both Kindergarten and grade 9, students are weakly better off through roughly the bottom three quarters of the welfare distribution, while welfare is lower at top quantiles. Gains in the median are larger than average gains. Welfare at the 50th percentile of the distribution rises by 0.36 (0.28) for the kindergarten (grade 9) market under deferred acceptance. See Tables A10 and A11 for more details.

A related point is that mechanisms that low-income households may be disadvantaged by mechanisms that reward strategic play. We consider this point in Table 8. Qualititatively, changes in mean welfare from switching to rational expectations play or to a deferred acceptance mechanism are similar across SES groups and grades. We observe increases in welfare in each case. However, in grade 9, these increases are larger for low-SES students, while for kindergarten, the reverse is true.

Figure 7: Percentiles of the welfare distribution



Notes: Left panel: posterior mean welfare by centile of welfare distribution under benchmark and DA policies. Right panel: centile-by-centile differences in welfare between DA and benchmark policies.

Table 8: Mean Distance-Metric Welfare by SES

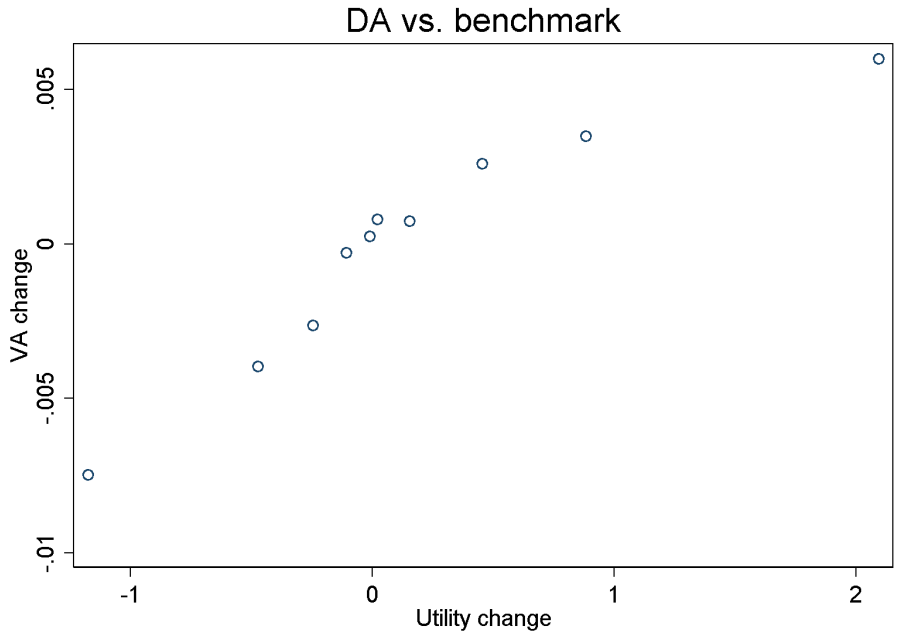
quantile	benchmark	RatEx	DA	RatEx - NH	DA - NH
A. Grade K					
Low SES					
0.05	1.899	2.027	1.975	0.114	0.055
0.25	1.937	2.068	2.013	0.127	0.07
0.5	2.027	2.146	2.091	0.133	0.078
0.75	2.054	2.188	2.136	0.144	0.089
0.95	2.119	2.259	2.198	0.157	0.101
High SES					
0.05	1.683	1.831	1.794	0.131	0.092
0.25	1.739	1.881	1.843	0.143	0.103
0.5	1.782	1.934	1.899	0.154	0.116
0.75	1.85	2.012	1.969	0.169	0.13
0.95	1.94	2.108	2.072	0.191	0.149
B. Grade 9					
Low SES					
0.05	1.178	1.426	1.252	0.241	0.057
0.25	1.259	1.532	1.347	0.286	0.099
0.5	1.348	1.724	1.521	0.365	0.157
0.75	1.488	1.87	1.667	0.401	0.187
0.95	1.645	2.046	1.844	0.45	0.223
High SES					
0.05	0.838	1.022	0.864	0.155	-0.02
0.25	0.938	1.14	0.959	0.19	0.016
0.5	1.016	1.295	1.109	0.261	0.069
0.75	1.163	1.466	1.255	0.308	0.107
0.95	1.276	1.598	1.4	0.354	0.147

Notes: Quantiles of the posterior distribution of mean welfare for low-SES and high-SES households. Low-SES households are the those in the the bottom 2/3 of the distribution of SES, high-SES are the top 1/3. Welfare is measured using miles traveled as the numeraire good. 'benchmark' is baseline New Haven mechanism given observed beliefs. 'RatEx' is the New Haven mechanism under rational expectations beliefs. 'DA' is the sophisticated deferred acceptance mechanism. 'RatEx-NH' and 'DA-NH' columns compares welfare differences under the listed mechanisms.

7.2.3 Counterfactual test score effects

We now turn to the effects of policy changes on the test score value added of the schools to which students are assigned. As with welfare, we compute the average value added of the assigned school for each household by averaging across MCMC iterations. Unlike our utility model, our measures of test score value added do not allow positive-sum trades of school assignments between students. The only way a change in assignment mechanism can generate increases in aggregate test score production is by reducing congestion. We therefore focus our analysis on a) the relationship between test score value added and utility outcomes, and b) the effects of changes in mechanism on test score inequality.

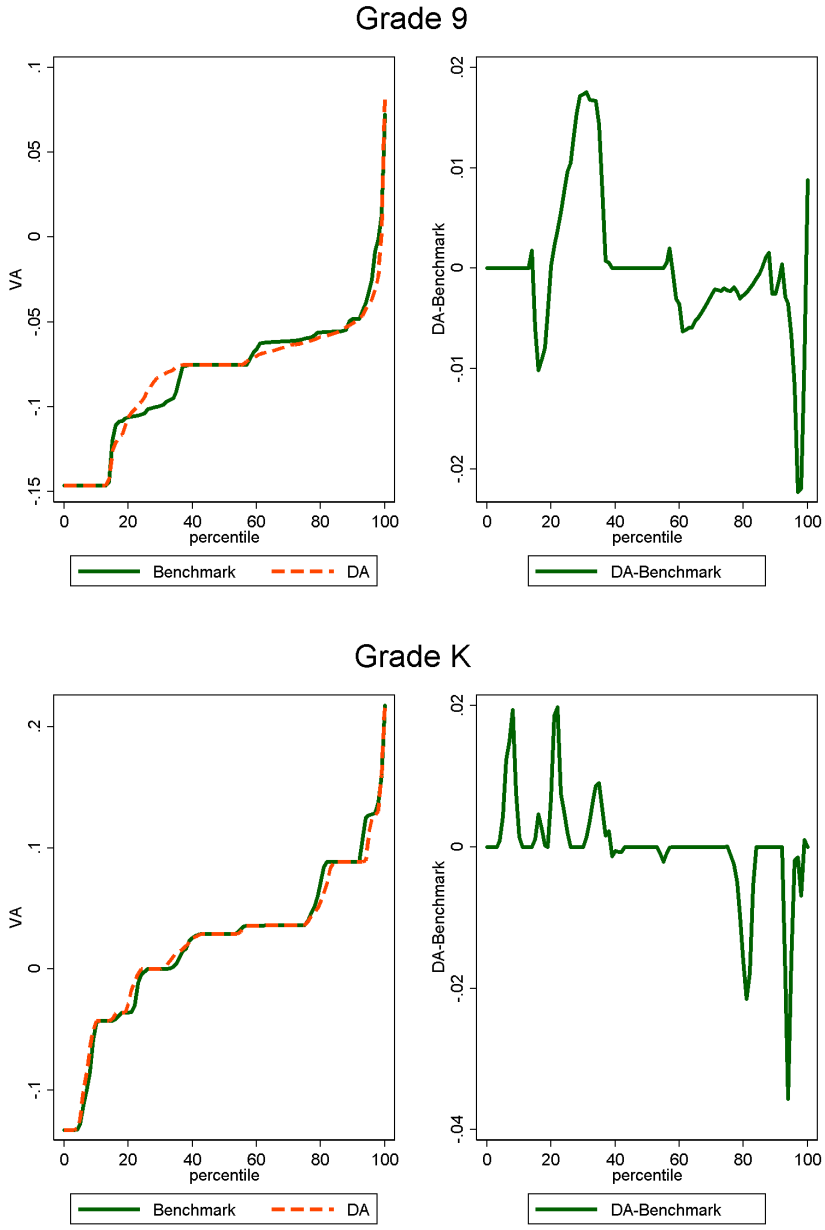
Figure 8: Percentiles of the value added distribution



Notes: Change in test score value added measured in student-level SDs (vertical axis) by change in mean welfare from switch to DA from benchmark (horizontal axis). Points are deciles of the welfare change distribution. Sample: Kindergarten and Grade 9. Figure residualizes on grade and neighborhood fixed effects before plotting.

We have three main findings. First, gains in welfare from a switch to the deferred acceptance mechanism are associated with gains in test score value added. Figure 8 shows the mean change in test score value added at each decile of the change in utility associated with the switch to deferred acceptance. The graph pools across grades. Value added range rises steadily with change in welfare through the utility change distribution. The magnitude of this increase is small: a move from the bottom to the top decile of the utility change distribution is associated with about a 0.01 SD increase in test score value added. Second, the switch to deferred acceptance compresses the distribution of expected test score value added at the time of application. Figure 9 shows mean test score value added at each quantile of the value added distribution in the benchmark case and under deferred acceptance. Quantiles below the median are higher under DA, while quantiles above the median are generally lower. Third, changing the choice mechanism produces little if any redistribution of test score value added from high SES to low SES households. Table 9 displays means and 95% credible intervals for value added gap between high SES and low-SES students under different counterfactual assignment mechanisms. Credible intervals for changes from the benchmark are narrow and span zero in every case.

Figure 9: Percentiles of the value added distribution



Notes: Left panel: centiles of distribution of test score value added (measured in student-level SDs) at placed school under benchmark NH mechanism and DA mechanism. Right panel: centile by centile differences between DA and benchmark mechanisms. Placed school VA computed using school-level VA measures weighted by student-level placement propensities.

Table 9: Value-Added: High-SES - Low-SES

quantile	benchmark	RatEx	DA	RatEx - NH	DA - NH
A. Grade K					
0.025	0.075	0.072	0.071	-0.003	-0.004
0.5	0.075	0.074	0.074	-0.001	-0.001
0.975	0.075	0.077	0.075	0.002	0.001
B. Grade 9					
0.025	0.021	0.018	0.018	-0.003	-0.003
0.5	0.021	0.02	0.021	-0.001	0.0
0.975	0.021	0.023	0.023	0.002	0.002

Notes: Quantiles of distribution of posterior mean of the gap between test-score VA at placed schools for high- and low-SES households. Low-SES households are the those in the the bottom 2/3 of the distribution of SES. Test score VA measured using student-level SDs.

8 Conclusions

This paper studies the performance of a centralized school choice mechanism that rewards strategic behavior when households have heterogeneous beliefs about placement probabilities. We conduct a household survey asking actual and potential choice choice participants about their preferences and beliefs, and link our survey data to administrative records of the school choice process. We use our linked data to describe heterogeneity in beliefs and to estimate a model of school choice that allows for belief and preference heterogeneity. Our survey data allow us to analyze the effects of counterfactual policies without making strong assumptions on applicants' equilibrium play. The counterfactual policies we consider highlight the tradeoff between applicants' ability to express preference intensity in mechanisms that reward strategic play and the increased likelihood of welfare-reducing application mistakes.

Our descriptive findings show that subjective beliefs drive application behavior, that school choice participants make large errors about the probabilities of admission associated with actual and hypothetical application portfolios, and that participants who make large errors are less likely to be placed in their most-preferred schools. Counterfactual policy simulations based on model

estimates that incorporate survey data indicate that the ordering of deferred and immediate acceptance mechanisms by welfare outcomes depends on the accuracy of students' beliefs about admissions chances. Though the immediate acceptance mechanism is preferable when students have rational expectations about choice probabilities, the deferred acceptance mechanism raises welfare given the distribution of belief errors we observe in our data. The costs of application mistakes in the strategic mechanism outweigh the benefits of increased expressiveness.

We find that gains in test score value added are correlated with gains in welfare from switching between the benchmark and deferred acceptance mechanisms, and that this change in mechanism reduces the share of students who submit applications with very low expected value added at the placed school. However, we find little evidence that changes in the centralized choice mechanism will reduce the gap in school quality between low- and high-SES students.

The main conclusion we draw from our findings is that policymakers should consider the informational environment in their district when selecting a centralized assignment mechanism. We show that, in New Haven, a market designer estimating preferences and simulating policy counterfactuals without allowing for application mistakes would have reversed the welfare comparison of the New Haven and deferred acceptance mechanisms.

Given the SES gradient we observe in belief errors, our specific findings are most relevant for lower-income districts. Within this set of districts, however, we view New Haven as close to a best case scenario with respect to the information available to participants. The centralized choice procedure had been in place for at least 18 years at the time we conducted our survey, and the school district conducts extensive outreach aimed at helping students and parents learn about the process. We would expect choice participants in districts where choice has been more recently adopted or where the district conducts more limited outreach to have, if anything, less accurate beliefs. Whether further informational interventions can push students closer to fully informed strategic decision making and, if so, what such interventions might look like, is a topic for future research. Our finding that only a large reduction in the magnitude of belief errors would yield welfare gains relative to deferred acceptance suggests that designing an informational intervention that outperforms a switch to deferred acceptance in terms of aggregate welfare may prove challenging.

References

Abdulkadiroğlu, Atila, Joshua D. Angrist, Yusuke Narita, and Parag A. Pathak, "Research Design Meets Market Design: Using Centralized Assignment for Impact Evaluation," Working Paper 21705, National Bureau of Economic Research November 2015.

_ , **Nikhil Agarwal, and Parag A. Pathak**, "The Welfare Effects of Coordinated Assignment: Evidence from the NYC HS Match," Working Paper 21046, National Bureau of Economic Research March 2015.

_ , **Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez**, "The Boston Public School Match," *American Economic Review*, May 2005, *95* (2), 368–371.

_ , _ , **and** _ , "The New York City High School Match," *American Economic Review*, May 2005, *95* (2), 364–367.

_ , **Parag Pathak, Alvin E. Roth, and Tayfun Sonmez**, "Changing the Boston School Choice Mechanism," Working Paper 11965, National Bureau of Economic Research January 2006.

_ , **Yeon-Koo Che, and Yosuke Yasuda**, "Resolving Conflicting Preferences in School Choice: The "Boston Mechanism" Reconsidered," *American Economic Review*, February 2011, *101* (1), 399–410.

Agarwal, Nikhil and Paulo Somaini, "Demand Analysis using Strategic Reports: An application to a school choice mechanism," Working Paper 20775, National Bureau of Economic Research December 2014.

Angrist, Joshua D., Peter Hull, Parag A. Pathak, and Christopher Walters, "Leveraging Lotteries for School Value-Added: Testing and Estimation," 2015, (20775).

Azevedo, Eduardo and Jacob Leshno, "A Supply and Demand Framework for Two-Sided Matching Markets," *Journal of Political Economy*, 2016. Forthcoming.

Barberis, Nicholas, "The psychology of tail events: Progress and challenges," *The American Economic Review*, 2013, *103* (3), 611–616.

Budish, Eric and Estelle Cantillon, "The multi-unit assignment problem: Theory and evidence from course allocation at harvard," *The American economic review*, 2012, *102* (5), 2237–2271.

Calsamiglia, Caterina and Maia Guell, "The Illusion of School Choice: Empirical Evidence from Barcelona," Working Papers 712, Federal Reserve Bank of Minneapolis May 2014.

_ , **Chao Fu, and Maia Güell**, "Structural Estimation of a Model of School Choices: the Boston Mechanism vs. Its Alternatives," Working Papers 2014-21, FEDEA December 2014.

Chetty, Raj, John N Friedman, and Jonah E Rockoff, "Measuring the impacts of teachers I: Evaluating bias in teacher value-added estimates," *The American Economic Review*, 2014, *104*

(9), 2593–2632.

Cullen, Julie Berry, Brian A Jacob, and Steven Levitt, "The Effect of School Choice on Participants: Evidence from Randomized Lotteries," *Econometrica*, 2006, *74* (5), 1191–1230.

de Haan, Monique, Pieter A. Gautier, Hessel Oosterbeek, and Bas van der Klaauw, "The Performance of School Assignment Mechanisms in Practice," IZA Discussion Papers 9118, Institute for the Study of Labor (IZA) June 2015.

Deming, David J, "Using school choice lotteries to test measures of school effectiveness," *The American Economic Review*, 2014, *104* (5), 406–411.

Dinerstein, Michael and Troy Smith, "Quantifying the Supply Response of Private Schools to Public Policies," *SIEPR Working Paper*, 2014.

Dur, UM, Robert G Hammond, and Thayer Morrill, "Identifying the Harm of Manipulable School-Choice Mechanisms," Technical Report, mimeo 2015.

Ergin, Haluk and Tayfun Sonmez, "Games of School Choice under the Boston Mechanism," *Journal of Public Economics*, January 2006, *90* (1-2), 215–237.

Fack, Gabrielle, Julien Grenet, and Yinghua He, "Beyond Truth-Telling: Preference Estimation with Centralized School Choice," CEPR Discussion Papers 10907, C.E.P.R. Discussion Papers October 2015.

Hastings, Justine, Thomas Kane, and Douglar Staiger, "Heterogeneous Preferences and the Efficacy of Public School Choice," 2009. Unplublished Working Paper.

He, Yinghua, "Gaming the Boston School Choice Mechanism in Beijing," TSE Working Papers 12-345, Toulouse School of Economics (TSE) May 2012.

Huelin, Rob, "New Haven School Join Drive to Diversify," 1996. The Yale Herald, published February 23.

Kane, Thomas J and Douglas O Staiger, "Estimating teacher impacts on student achievement: An experimental evaluation," Working Paper 14607, National Bureau of Economic Research 2008.

McCulloch, Robert and Peter Rossi, "An exact likelihood analysis of the multinomial probit model," *Journal of Econometrics*, 1994, *64* (1-2), 207–240.

Neilson, Christopher A., "Targeted Vouchers, Competition Among Schools, and the Academic Achievement of Poor Students," 2013. Princeton University Working Paper.

Pathak, Parag, "The Mechanism Design Approach to Student Assignment," *Annual Review of Economics*, 2011, *3* (1), 513–536.

Pathak, Parag A and Tayfun Sönmez, "School admissions reform in Chicago and England: Comparing mechanisms by their vulnerability to manipulation," *The American Economic Review*, 2013, *103* (1), 80–106.

Pathak, Parag A. and Tayfun Sönmez, "Leveling the Playing Field: Sincere and Sophisticated Players in the Boston Mechanism," *American Economic Review*, September 2008, *98* (4), 1636–52.

Smart Choice, "Our Clients," https://smartchoicetech.com/clients/ 2016. Accessed 2016-08-05.

Walters, Christopher, "The Demand for Effective Charter Schools," Working Paper 20640, National Bureau of Economic Research October 2014.

A Appendix A

A.1 Additional Tables and Figures

Table A1: NHPS school district characteristics

	Population Mean
Asian	0.021
Black	0.482
Latino	0.396
Other	0.008
White	0.093
No Spec. Ed.	0.866
Spec. Ed.	0.134
F/R lunch	0.77
Reading	-0.69
Math	-0.66

Notes: School district characteristics in 2014-2015. Test score data from Neilson and Zimmerman (2014). Test score units are student-level standard deviations relative to statewide mean.

Table A2: Summary Statistics: Ninth-Grade Survey and Lottery Applications

School	Considered	In App.	1st in App.	1st Unconstr.
Achievement First Amistad HS	75.7	9.5	0.0	6.8
Common Ground Charter	64.9	17.6	8.1	2.7
Coop. Arts and Humanities	74.3	52.7	23.0	17.6
Engineering and Science Univ. HS	51.4	18.9	8.1	6.8
High School in the Community	63.5	20.3	4.1	5.4
Hill Regional Career	86.5	56.8	29.7	16.2
Hillhouse	83.8	5.4	0.0	2.7
Hyde School	62.2	25.7	5.4	8.1
Metropolitan Business Academy	70.3	43.2	9.5	9.5
New Haven Academy	75.7	25.7	6.8	6.8
Riverside Education Academy	46.0	2.7	1.4	0.0
Wilbur L. Cross High School	86.5	9.5	4.1	8.1

Notes: N=74 students in ninth-grade survey who participated in the lottery and matched to lottery data. All figures are percentages out of N. "Considered" equals 1 when the parent stated that he or she considered this school as a possible choice for his/her child. We asked parents for their top choice if they could choose any school and be guaranteed admission. We then asked what school they could choose if this school were full but all other schools guaranteed admission. We refer to these two schools as "unconstrained" choices.

Table A3: Summary Statistics: Kindergarten Survey and Lottery Applications

School	Considered	In App.	1st in App.	1st Unconstr.
Amistad Academy Elementary	87.5	25.0	12.5	25.0
Augusta Lewis Troup School	55.0	2.5	0.0	0.0
Barnard Environmental Studies School	62.5	15.0	2.5	7.5
Beecher Museum School	60.0	7.5	0.0	0.0
Bishop Woods Executive Academy	62.5	7.5	2.5	0.0
Booker T. Washington Academy Charter	52.5	10.0	2.5	0.0
Brennan-Rogers	42.5	17.5	7.5	7.5
Celentano Biotech Health and Medical	55.0	12.5	5.0	5.0
Christopher Columbus Family Academy	70.0	10.0	7.5	2.5
Clinton Avenue School	65.0	7.5	2.5	0.0
Conte-West Hills	57.5	12.5	5.0	2.5
Davis Street Arts and Academics	52.5	2.5	0.0	0.0
East Rock Community	72.5	10.0	2.5	2.5
Edgewood	65.0	5.0	0.0	0.0
Elm City College Preparatory Charter	62.5	20.0	2.5	2.5
Elm City Montessori	50.0	5.0	2.5	2.5
Fair Haven School	75.0	7.5	2.5	0.0
Hill Central School	65.0	7.5	2.5	2.5
Jepson Multi-Age	50.0	7.5	0.0	0.0
John C. Daniels	62.5	12.5	2.5	2.5
John S. Martinez	65.0	17.5	7.5	10.0
King-Robinson School: an IB World School	45.0	12.5	2.5	2.5
Lincoln-Bassett Community School	62.5	5.0	0.0	0.0
Mauro-Sheridan Sci/Tech/Communications	45.0	2.5	2.5	0.0
Nathan Hale School	57.5	5.0	2.5	0.0
Quinnipiac Real World Math STEM	47.5	10.0	5.0	2.5
Roberto Clemente Leadership Academy	70.0	10.0	5.0	2.5
Ross Woodward Classical Studies	62.5	0.0	0.0	0.0
Strong 21st Century Communications	50.0	5.0	0.0	0.0
Truman School	67.5	5.0	2.5	0.0
West Rock Authors Academy	45.0	0.0	0.0	0.0
Wexler-Grant Community School	55.0	2.5	0.0	0.0
Wintergreen	45.0	7.5	0.0	0.0
Worthington Hooker School	50.0	12.5	10.0	15.0

Notes: N=40 students in kindergarten survey who participated in the lottery and matched to lottery data. All figures are percentages out of N. "Considered" equals 1 when the parent stated that he or she considered this school as a possible choice for his/her child. We asked parents for their top choice if they could choose any school and be guaranteed admission. We then asked what school they could choose if this school were full but all other schools guaranteed admission. We refer to these two schools as "unconstrained" choices.

Table A4: Beliefs and errors by demographic group

	All	High SES	Low SES	Non-black	Black
A. Errors by type					
Belief>25% | ratex< 0.1%	0.112	0.073	0.134	0.107	0.116
Belief<50% | ratex>99.9%	0.078	0.045	0.096	0.064	0.091
B. RatEx beliefs					
Mean	38.3	39	37.8	38.7	37.8
SD	39.8	39.2	40.2	40	39.7
p25	0.1	0.1	0.1	0.1	0.1
p75	78.2	78.2	80.7	80.7	78.2
C. Subjective beliefs					
Mean	40	40	39	41	38
SD	32	32	32	32	31
p25	11	12	11	13	9
p75	65	65	65	65	65
D. Optimism					
Mean	1	1	2	3	0
SD	38	32	41	37	39
p25	-15	-14	-21	-15	-18
p75	23	19	26	25	23
|Mean|	29	24	32	28	29

Notes: N=786. Table displays beliefs and belief errors of the type listed in the rows for demographic groups listed in the columns. Panel A: mean rates of 'false positive' beliefs (first row) and 'false negative' beliefs (second row). See section 4.2 for definitions. Panels B through D: distribution of RatEx beliefs, subjective beliefs, and optimism, respectively. Units are percentage points. 'p25' and 'p75' are percentiles of distribution. 'High SES' is top third of SES distribution.

Table A5: Probability of Accepting a Placement: Probits

	Accept	Accept	Accept
rank = 2	-0.470	-0.368	-0.580
	(5.29)**	(3.61)**	(5.73)**
rank = 3	-0.123	-0.017	-0.383
	(0.92)	(0.12)	(2.46)*
rank = 4	-0.156	-0.042	-0.574
	(1.10)	(0.28)	(3.51)**
neighbor	0.310	0.252	0.195
	(3.46)**	(2.69)**	(1.27)
sibling	1.004	0.949	0.873
	(6.73)**	(6.26)**	(5.48)**
both	0.768	0.695	0.477
	(3.49)**	(3.12)**	(1.78)+
admissions chance		0.003	
		(2.05)*	
Constant	0.828	0.616	
	(18.58)**	(5.49)**	
N	2,055	2,055	2,055
School FE	No	No	Yes

Notes: + $p < 0.1$; * $p < 0.05$; ** $p < 0.01$. t-statistics in parentheses. Probit regressions of the decision to accept a placement, conditional on being placed in a school, on the rank of the school within the individual's application vector and controls. Omitted category is rank 1. Students are more likely to accept placements at first-listed schools. All specifications control for the presence of a sibling at the school ('sibling'), an indicator for neighborhood priority ('neighbor'), and their interaction ('both'). The second controls for the rational-expectations admissions chance. The third specification adds school fixed effects.

Table A6: Median and 90% credible intervals for belief model covariances Σ^{η}.

Variable	η_0	η_{pri}	η_{round}	η_j	η_{jr}
η_0	(0.63,1.25, 2.19)				
η_{pri}	(-2.24,-1.22, -0.48)	(1.48,2.94, 5.1)			
η_{round}	(0.09,0.76, 1.51)	(-0.41,0.38, 1.24)	(2.71,3.49, 4.58)		
η_j	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(3.48,4.29, 5.41)	
η_{jr}	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(1.27,1.44, 1.63)

Variable	η_0	η_{pri}	η_{round}	η_j	η_{jr}
η_0	(0.63,1.25, 2.19)				
η_{pri}	(-2.24,-1.22, -0.48)	(1.48,2.94, 5.1)			
η_{round}	(0.09,0.76, 1.51)	(-0.41,0.38, 1.24)	(2.71,3.49, 4.58)		
η_j	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(3.48,4.29, 5.41)	
η_{jr}	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(0.0,0.0, 0.0)	(1.27,1.44, 1.63)

Notes: Median and 90% credible intervals for belief error covariance terms. Upper panel: Low SES. Lower panel: High SES.

Table A7: 90% Credible Intervals, σ_j^2 (Grade K)

Variable	σ^2)
Amistad (1)	(8.86, 11.04)
A.L. Troup (2)	(4.91, 8.03)
Barnard (3)	(6.03, 7.59)
Beecher Museum School (4)	(8.37, 10.7)
Bishop Woods (5)	(8.74, 10.95)
Booker T Washington (6)	(8.85, 10.97)
Brennan-Rogers (7)	(16.36, 22.88)
Celentano (8)	(8.04, 10.18)
Columbus (9)	(6.08, 7.72)
Clinton Avenue (10)	(7.61, 9.7)
Conte-West Hills (11)	(5.76, 7.35)
Davis Street (12)	(8.4, 10.48)
East Rock Community (13)	(6.16, 7.78)
Edgewood (14)	(6.35, 8.06)
Elm City College Prep (15)	(8.43, 10.46)
Elm City Montessori (16)	(8.34, 10.78)
Fair Haven School (17)	(7.5, 9.09)
Hill Central School (18)	(7.69, 9.6)
Jepson Multi-Age (19)	(8.61, 10.61)
John C. Daniels (20)	(6.41, 8.14)
John S Martinez (21)	(6.36, 8.14)
King-Robinson (22)	(15.37, 24.31)
Lincoln-Bassett (23)	(1.7, 3.87)
Mauro-Sheridan (24)	(8.32, 10.25)
Nathan Hale (25)	(8.76, 10.85)
Quinnipiac (26)	(6.53, 10.46)
Roberto Clemente (27)	(9.93, 14.72)
Ross Woodward (28)	(8.8, 10.96)
Strong (29)	(4.54, 7.46)
Truman (30)	(7.87, 11.93)
West Rock (31)	(7.62, 12.48)
Wexler-Grant (32)	(3.8, 6.82)
Wintergreen (33)	(8.89, 11.31)
Worthington Hooker (34)	(8.08, 10.08)

Notes: 90% credible intervals for variance of $\{\epsilon_j^*\}_{j \in J}$.

Table A8: 90% Credible Intervals for random coefficients, Σ (Grade K)

Variable	High VA	Charter	High demand	Medium demand
High VA	(1.93, 2.41)			
Charter	(-0.82, -0.54)	(2.63, 3.27)		
High demand	(-0.85, -0.63)	(-0.22, -0.05)	(0.86, 1.13)	
Medium demand	(-0.27, -0.11)	(-0.3, -0.14)	(0.09, 0.24)	(0.92, 1.18)

Notes: 90% credible intervals for preference covariance matrix in grade K estimation.

Table A9: 90% Credible Intervals, Σ (Grade 9)

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Achievement First Amistad HS (1)	(4.65, 12.61)					
Common Ground Charter (2)	(13.06, 20.02)	(22.2, 66.38)				
Coop. Arts and Humanities (3)	(1.19, 7.25)	(5.37, 12.24)	(10.81, 20.7)			
Engineering & Science Univ. HS (4)	(1.42, 4.9)	(3.46, 11.0)	(-9.02, -1.21)	(2.97, 12.74)		
High School in the Community (5)	(4.24, 14.71)	(15.21, 27.51)	(5.56, 14.9)	(-1.48, 4.5)	(11.57, 34.88)	
Hill Regional Career (6)	(7.33, 13.55)	(12.57, 22.44)	(10.01, 18.33)	(-0.1, 6.78)	(10.45, 20.64)	(26.53, 68.13)
Hillhouse (7)	(2.06, 9.18)	(1.47, 19.5)	(-6.49, 2.44)	(2.71, 8.57)	(-14.22, -0.25)	(5.17, 17.53)
Hyde School (8)	(0.49, 8.81)	(-11.67, 7.77)	(-13.66, 0.78)	(5.46, 23.59)	(4.69, 23.75)	(24.29, 60.33)
Metropolitan Business Academy (9)	(0.71, 8.38)	(3.39, 16.02)	(6.52, 13.64)	(-3.32, 3.2)	(9.25, 20.45)	(13.19, 25.7)
New Haven Academy (10)	(-1.29, 3.29)	(-2.62, 4.42)	(3.66, 8.53)	(-3.77, 0.99)	(5.36, 18.7)	(7.85, 14.29)
Riverside Education Academy (11)	(-5.13, 7.94)	(-0.29, 18.36)	(7.26, 17.58)	(-15.43, -1.35)	(-3.43, 9.6)	(-35.59, 9.86)
Wilbur L. Cross High School (12)	(-11.14, -3.49)	(-44.1, -10.88)	(3.06, 10.53)	(-14.41, -3.38)	(-12.78, 0.88)	(6.35, 19.72)

Variable	(7)	(8)	(9)	(10)	(11)	(12)
Achievement First Amistad HS (1)						
Common Ground Charter (2)						
Coop. Arts and Humanities (3)						
Engineering & Science Univ. HS (4)						
High School in the Community (5)						
Hill Regional Career (6)						
Hillhouse (7)	(5.64, 35.26)					
Hyde School (8)	(-0.92, 18.29)	(51.07, 137.28)				
Metropolitan Business Academy (9)	(-12.11, 2.85)	(12.38, 28.32)	(12.66, 30.7)			
New Haven Academy (10)	(-12.78, -3.02)	(10.5, 22.11)	(7.02, 15.18)	(6.61, 18.42)		
Riverside Education Academy (11)	(-17.65, 7.83)	(-77.67, -3.42)	(-11.22, 11.87)	(-6.97, 3.48)	(7.43, 47.46)	
Wilbur L. Cross High School (12)	(-10.68, -0.63)	(4.36, 23.04)	(2.79, 13.32)	(2.91, 12.07)	(-8.82, 9.54)	(11.1, 60.84)

Notes: 90% credible intervals for preference covariance matrix in grade 9 estimation.

Table A10: Heterogeneity in Distance-Metric Welfare, Grade K

quantile	(benchmark)	(RatEx)	(DA)	(Naive DA)	RatEx - NH	DA - NH	Naive DA - NH
0.025	0.0	0.0	0.0	0.0	-0.537	-0.754	-1.358
0.05	0.0	0.0	0.0	0.0	-0.27	-0.458	-0.968
0.1	0.0	0.0	0.0	0.0	-0.005	-0.142	-0.458
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.355	0.824	0.712	0.649	0.0	0.0	0.0
0.75	3.617	3.93	3.836	3.44	0.042	0.014	0.002
0.9	6.116	6.126	5.982	5.69	0.549	0.421	0.164
0.95	7.129	7.105	7.006	6.955	1.275	1.113	0.648
0.975	8.044	7.985	7.928	7.96	1.917	1.784	1.315

Notes: quantiles of posterior welfare distribution by policy counterfactual. For each person in the population, we calculate the posterior mean welfare. This table displays quantiles of this distribution over individuals, under the benchmark, under Bayes Nash equilibrium of the New Haven mechanism (RatEx), under sophisticated student-proposing deferred acceptance (DA), and under naive DA.

Table A11: Heterogeneity in Distance-Metric Welfare, Grade 9

quantile	(benchmark)	(RatEx)	(DA)	(Naive DA)	RatEx - NH	DA - NH	Naive DA - NH
0.025	-0.282	0.0	0.0	0.0	-0.839	-1.312	-1.375
0.05	-0.06	0.0	0.0	0.0	-0.469	-0.853	-0.88
0.1	0.0	0.0	0.0	0.0	-0.16	-0.503	-0.509
0.25	0.0	0.0	0.0	0.0	0.0	-0.162	-0.163
0.5	0.516	1.125	0.792	0.755	0.0	0.0	0.0
0.75	1.952	2.67	2.394	2.333	0.473	0.232	0.156
0.9	3.982	4.154	3.868	3.833	1.212	0.957	0.817
0.95	5.107	4.947	4.655	4.647	2.004	1.594	1.464
0.975	6.021	5.564	5.299	5.253	2.749	2.347	2.188

Notes: quantiles of posterior welfare distribution by policy counterfactual. For each person in the population, we calculate the posterior mean welfare. This table displays quantiles of this distribution over individuals, under the benchmark, under Bayes Nash equilibrium of the New Haven mechanism (RatEx), under sophisticated student-proposing deferred acceptance (DA), and under naive DA.

Table A12: Value-Added: Grade K

quantile	(NH)	(RatEx)	(DA)	RatEx - NH	DA - NH
A. Low SES					
0.025	0.01	0.008	0.009	-0.002%	-0.001%
0.5	0.01	0.009	0.01	-0.001%	-0.0%
0.975	0.01	0.01	0.011	0.0%	0.001%
B. High SES					
0.025	0.085	0.082	0.082	-0.003%	-0.003%
0.5	0.085	0.084	0.084	-0.001%	-0.001%
0.975	0.085	0.085	0.085	0.001%	0.0%

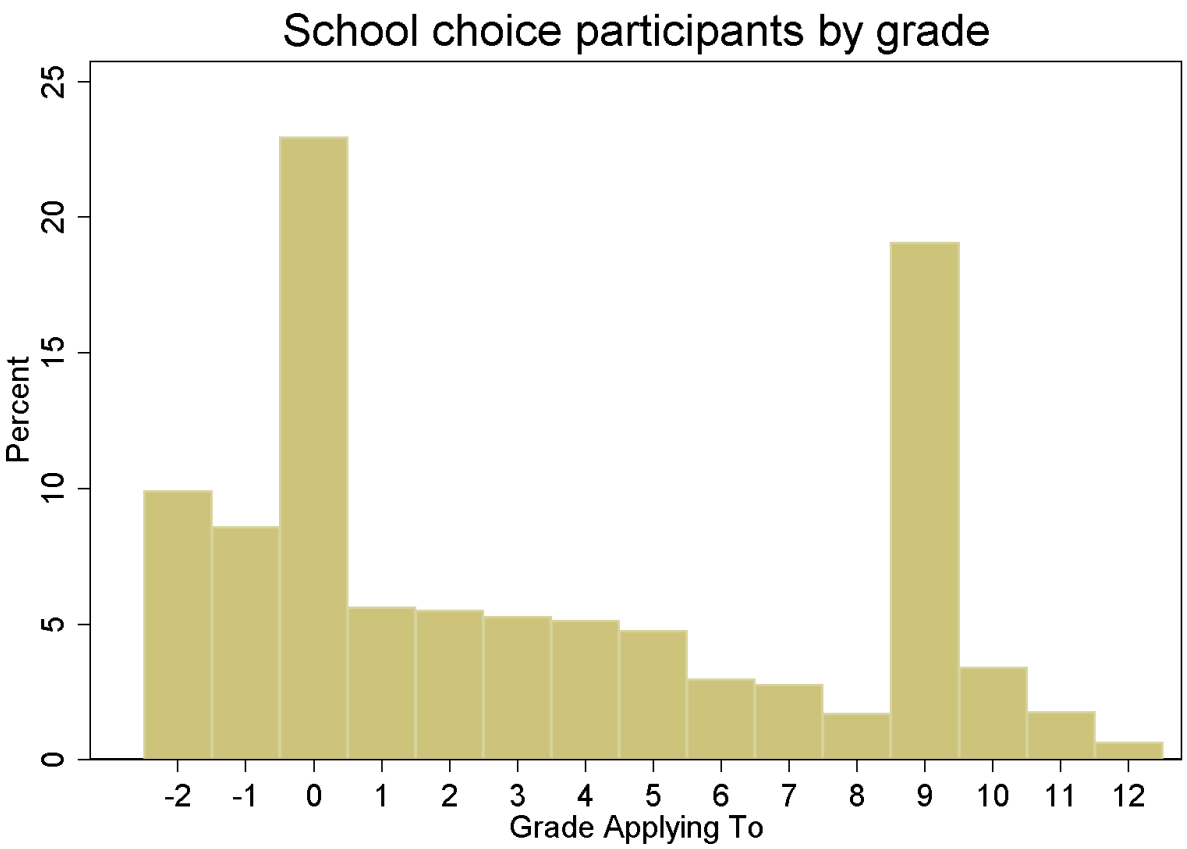
Notes: This table displays quantiles of value-added of the "placed" school for low-SES households only under different policy counterfactuals. Low-SES households are the those in the the bottom 2/3 of the distribution of SES. NH mechanism is benchmark case. RatEx is NH mechanism with rational expectations beliefs. DA is sophisticated student-proposing deferred acceptance.

Table A13: Value-Added: Grade 9

quantile	(benchmark)	(RatEx)	(DA)	RatEx - NH	DA - NH	
A. Low SES						
0.025	-0.14	-0.143	-0.142	-0.003%	-0.001%	-0.002%
0.5	-0.14	-0.142	-0.141	-0.001%	-0.0%	-0.001%
0.975	-0.14	-0.14	-0.139	0.0%	0.001%	0.001%
B. High SES						
0.025	0.085	0.082	0.082	-0.003%	-0.003%	
0.5	0.085	0.084	0.084	-0.001%	-0.001%	
0.975	0.085	0.085	0.085	0.001%	0.0%	

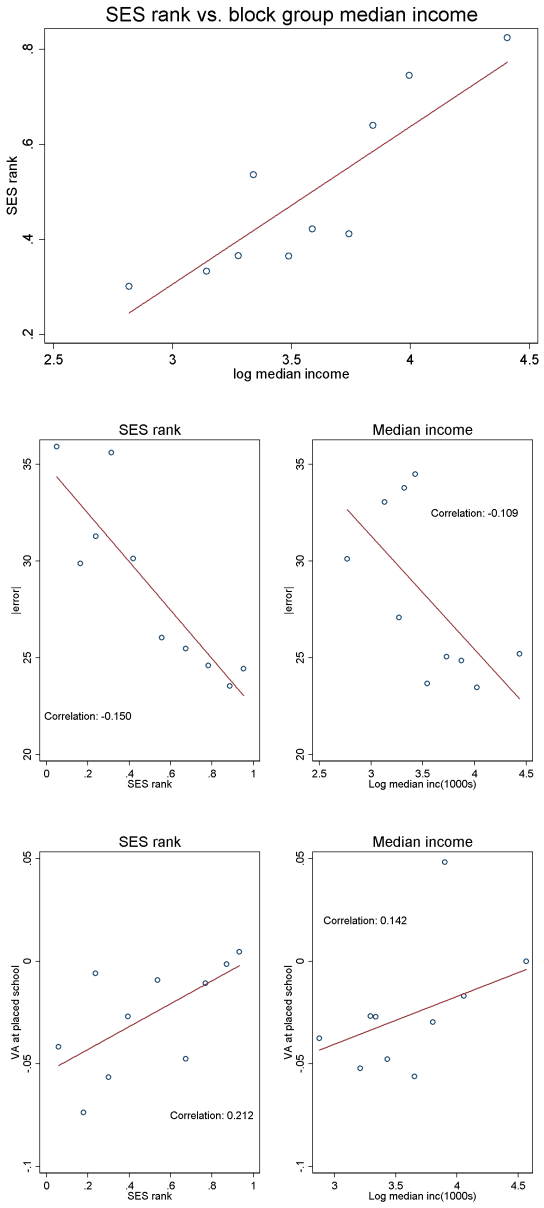
Notes: This table displays quantiles of value-added of the "placed" school for high-SES households only under different policy counterfactuals. High-SES households are those in the top 1/3 of the distribution of SES. NH mechanism is benchmark case. RatEx is NH mechanism with rational expectations beliefs. DA is sophisticated student-proposing deferred acceptance.

Figure A1: School choice participation by grade



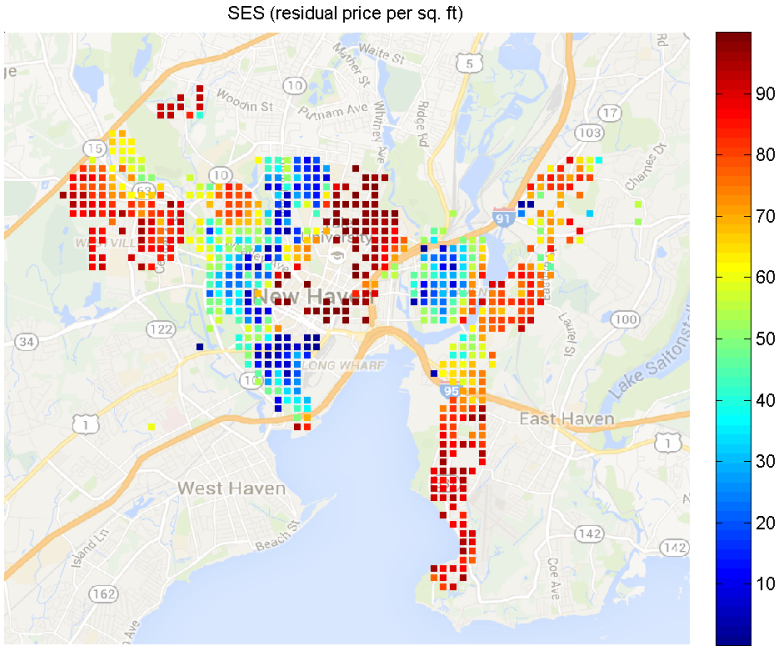
Notes: Percent of school choice participants entering listed grade in the 2015 school lottery.

Figure A2: Benchmarking SES measures



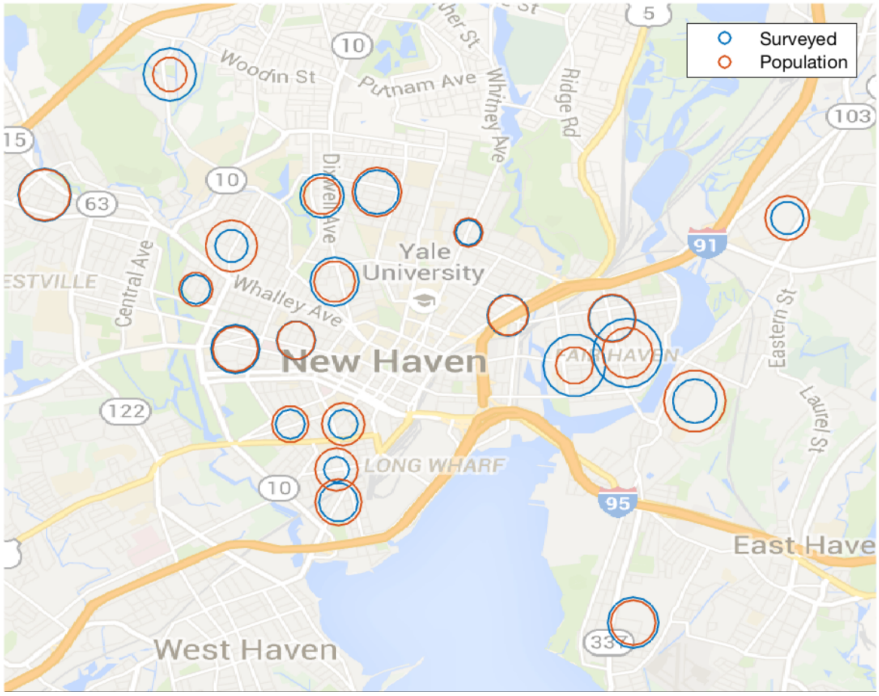
Notes: Upper panel: SES rank by decile of block group median income. Block group median income measured in logs of 1000s of dollars. Middle panel: mean absolute belief error by decile of SES ranking (left) and decile of census tract median income (right). Bivariate student-weighted correlations reported in graphs. Lower panel: VA of placed school by decile of SES ranking (left) and decile of census tract median income (right). Bivariate correlations reported in graphs.

Figure A3: SES rank in population



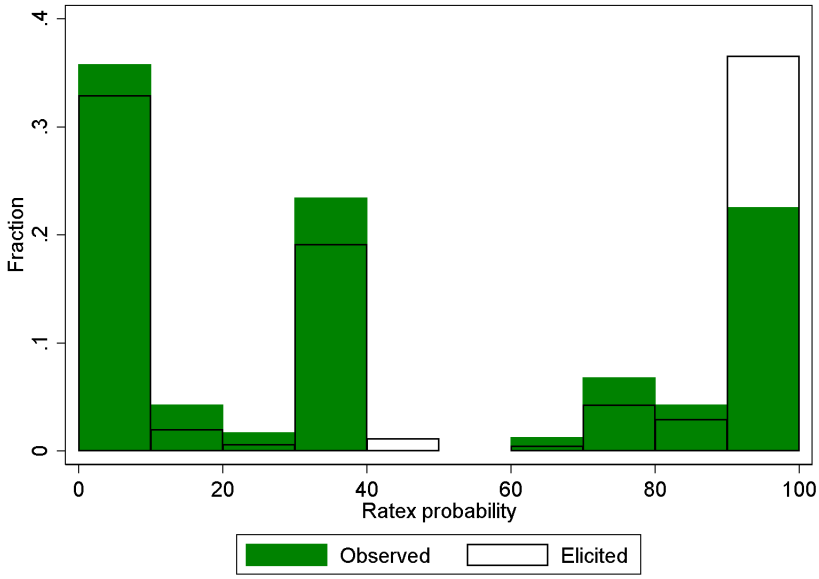
Notes: This figure displays SES rankings based on home price per square foot. 100 is the highest ranking and 0 the lowest. We compute rankings within each (centered) block using a normal kernel with bandwidth 0.05 miles. Estimates based on detrended home sale price data for the years 2005-2015.

Figure A4: Share of Students within each School Zone



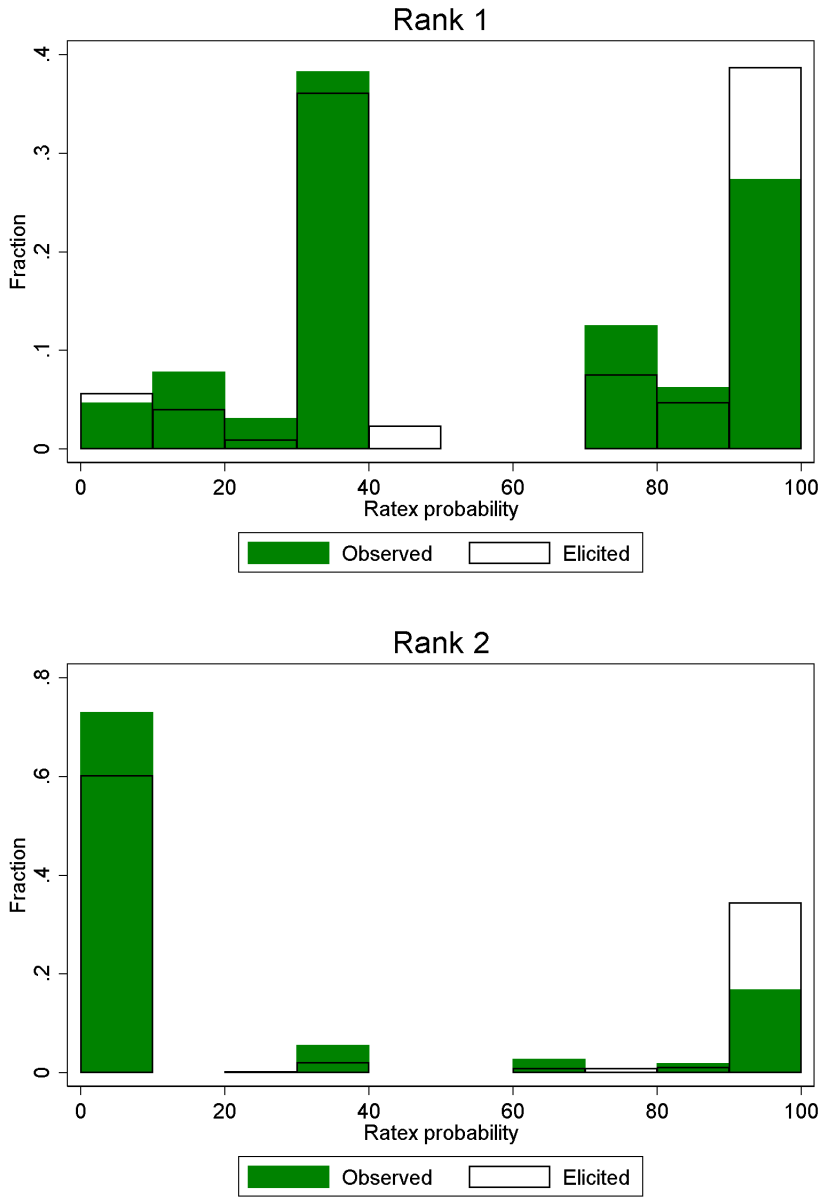
Notes: This figure displays the geographic distribution of sample universe and surveyed population. Size of circles reflect shares of population and surveyed individuals, respectively. Each point is centered at the location of an neighborhood zoned elementary/middle school, and reports data for students residing in that zone.

Figure A5: Ratex admissions probabilities of actual and hypothetical applications



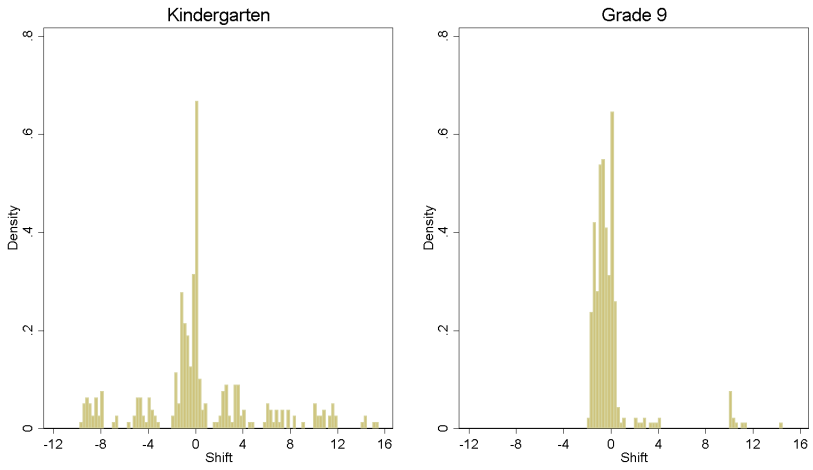
Notes: N=235 for observed applications and N=848 for elicited (hypothetical) applications. Sample for observed applications is submitted application-ranks for choice participants in surveyed sample. Sample for elicited applications is hypothetical application-ranks in surveyed sample. Bins have width 10pp.

Figure A6: Ratex admissions probabilities of actual and hypothetical applications by rank



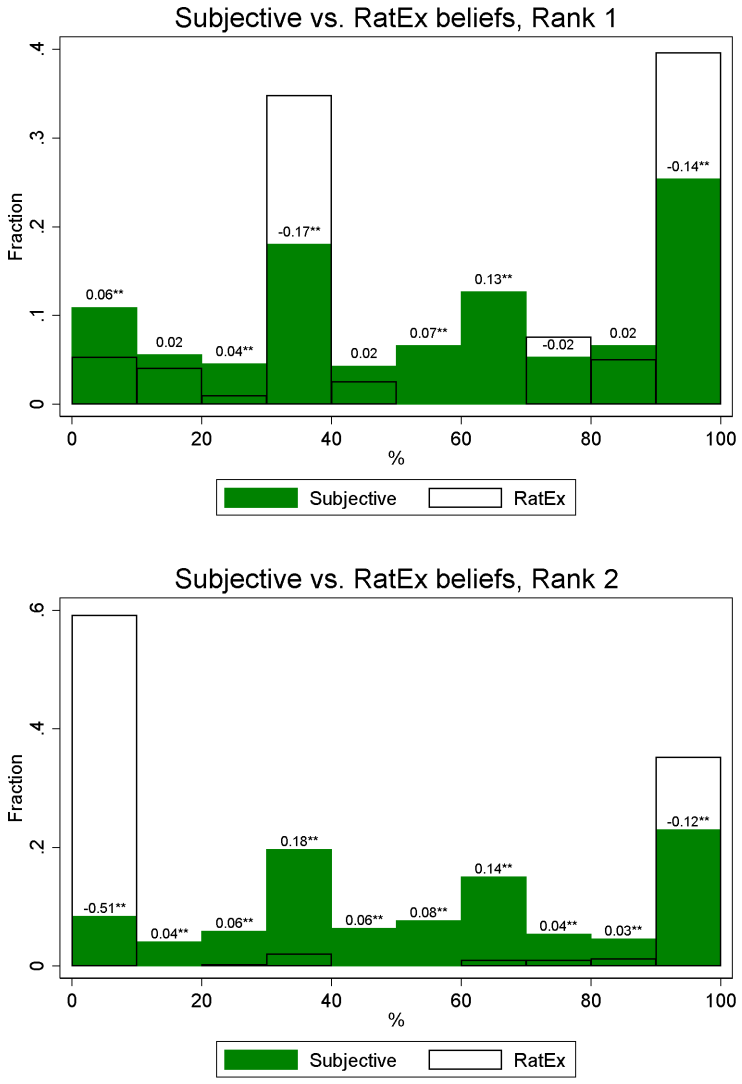
Notes: N=235 for observed applications and N=848 for elicited (hypothetical) applications. Sample for observed applications is submitted application-ranks for choice participants in surveyed sample. Sample for elicited applications is hypothetical application-ranks in surveyed sample. Bins have width 10pp.

Figure A7: Distribution of application score shift terms by grade



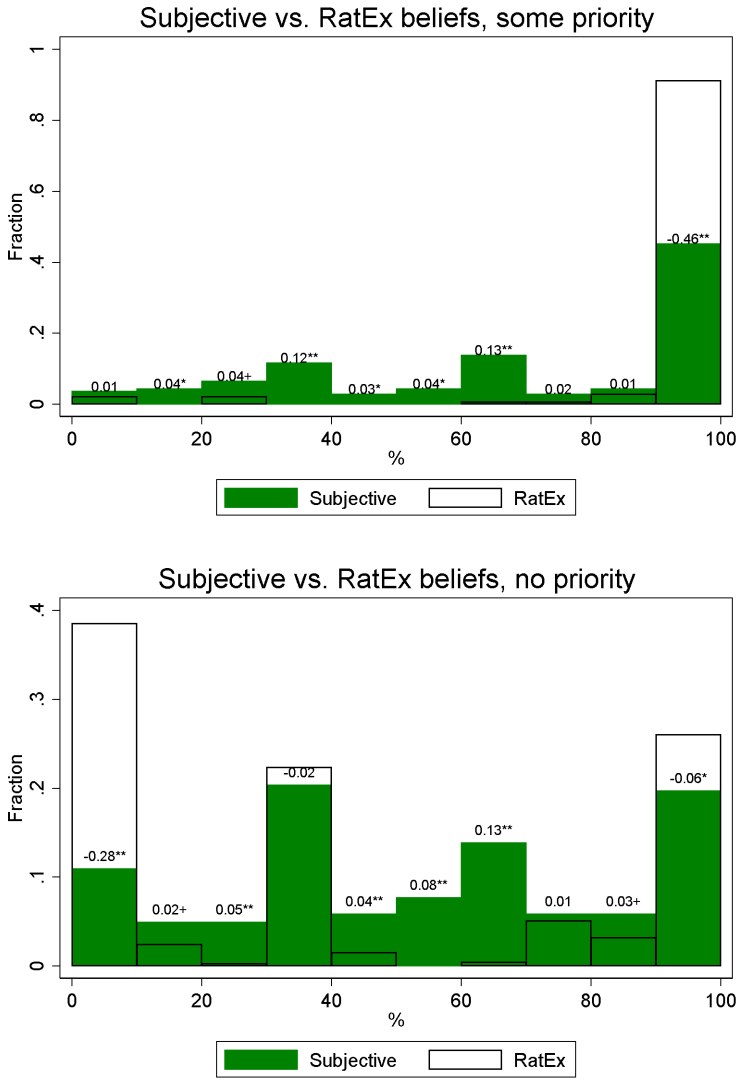
Notes: N=690. Distribution of $shift_{ij}(a)$ terms for hypothetical applications by grade. See section 3.3 for a definition of the $shift_{ij}(a)$. We compute $shift_{ij}(a)$ conservatively. If a household reports an interior subjective belief of, say, 50 to 60%, we pick the value of shift that would make the subjective belief be the midpoint of the given interval (55%). If a household reports a subjective belief in the 90-100% or 0-10% intervals, and the rational expectations chance lies in that interval, we set $shift_{ij}(a) = 0$.

Figure A8: Distribution of RatEx and subjective beliefs by rank



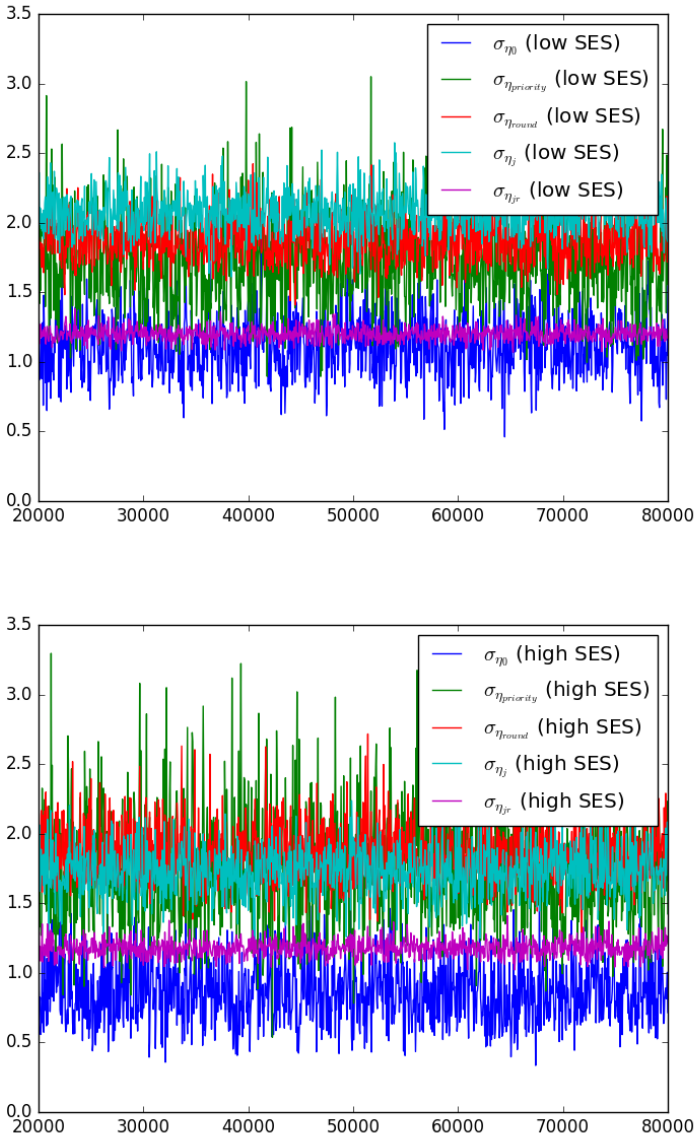
Notes: N=786. Distribution of subjective and rational expectations beliefs in bins of width 10pp split by submitted rank. Listed values are differences between the fraction of subjective and rational expectations beliefs in the bin. Significance from tests of equality of shares in subj. and RatEx distributions: +0.10 * 0.05 ** 0.01.

Figure A9: Distribution of RatEx and subjective beliefs by priority



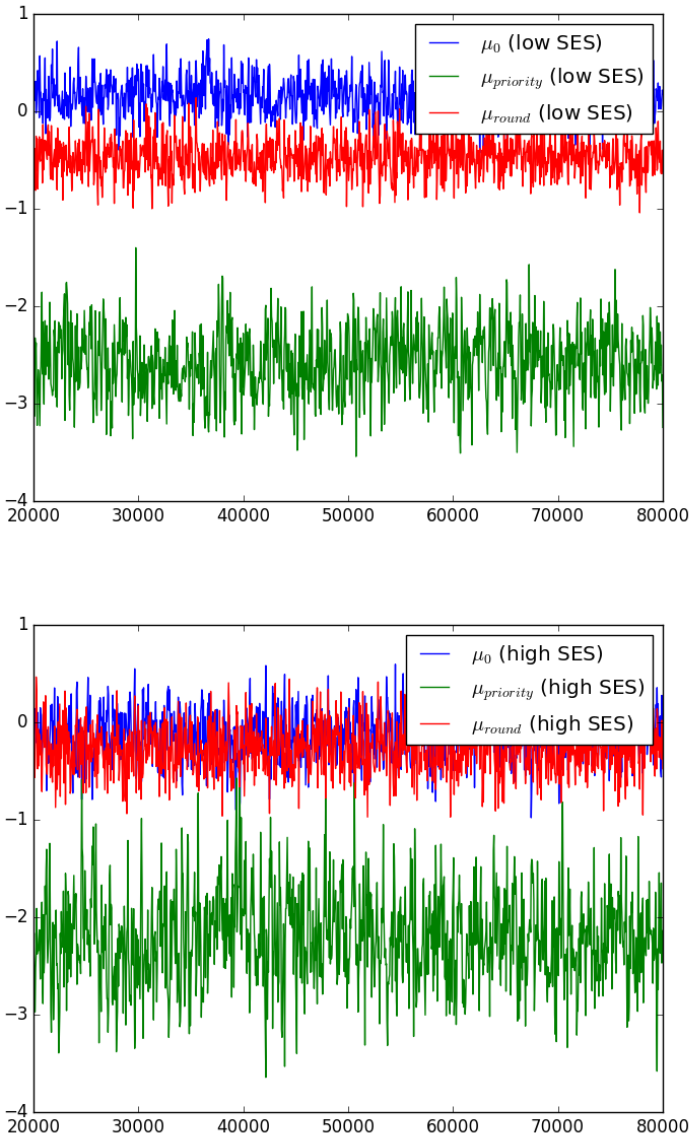
Notes: N=786. Distribution of subjective and rational expectations beliefs in bins of width 10pp split by applicant priority at hypothetical school. 'Some priority' sample is applicants with neighborhood priority, sibling priority, or both. Listed values are differences between the fraction of subjective and rational expectations beliefs in the bin. Significance from tests of equality of shares in subj. and RatEx distributions: +0.10 * 0.05 ** 0.01.

Figure A10: Trace Plots: Belief variances



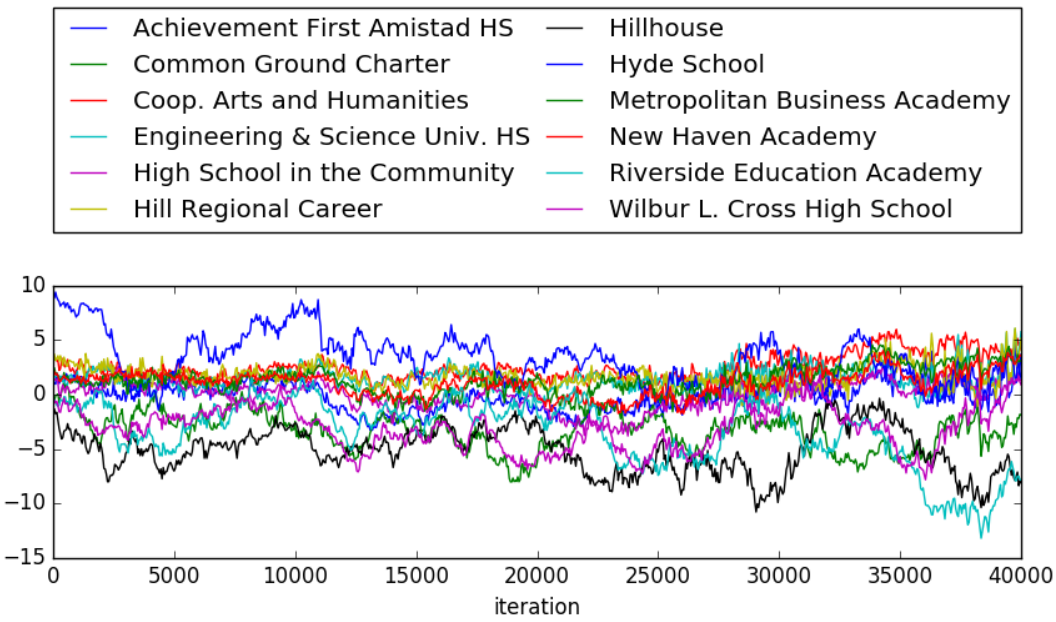
Trace plots for estimates of belief variance parameters for low-SES households (upper panel) and high-SES households (lower panel). See section 6 for estimation details.

Figure A11: Trace Plots: Belief means



Trace plots for estimates of belief mean shift parameters for low-SES households (upper panel) and high-SES households (lower panel). See section 6 for estimation details.

Figure A12: Trace Plots: δ_j (grade 9)



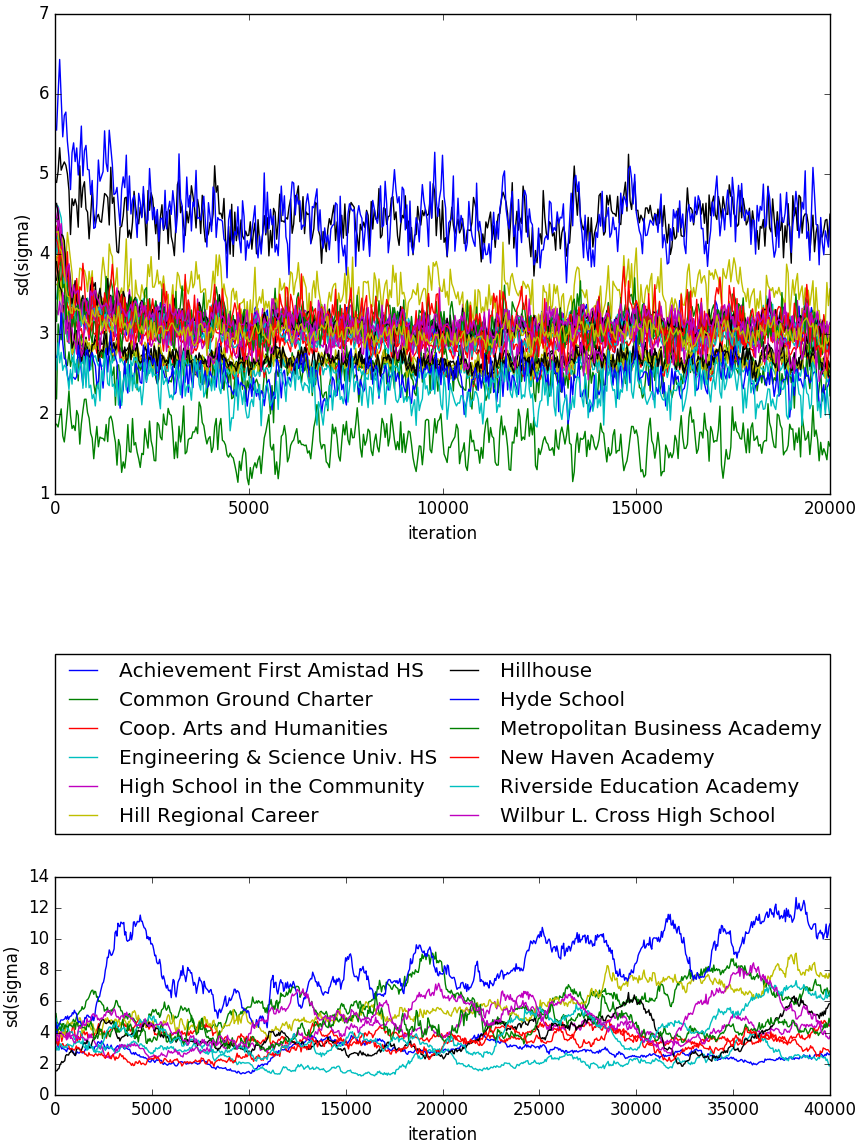
Trace plots for estimates of school-specific preference shifters δ_j in grade 9 estimation. See section 6 for estimation details.

Figure A13: Trace Plots: Other Utility Parameters



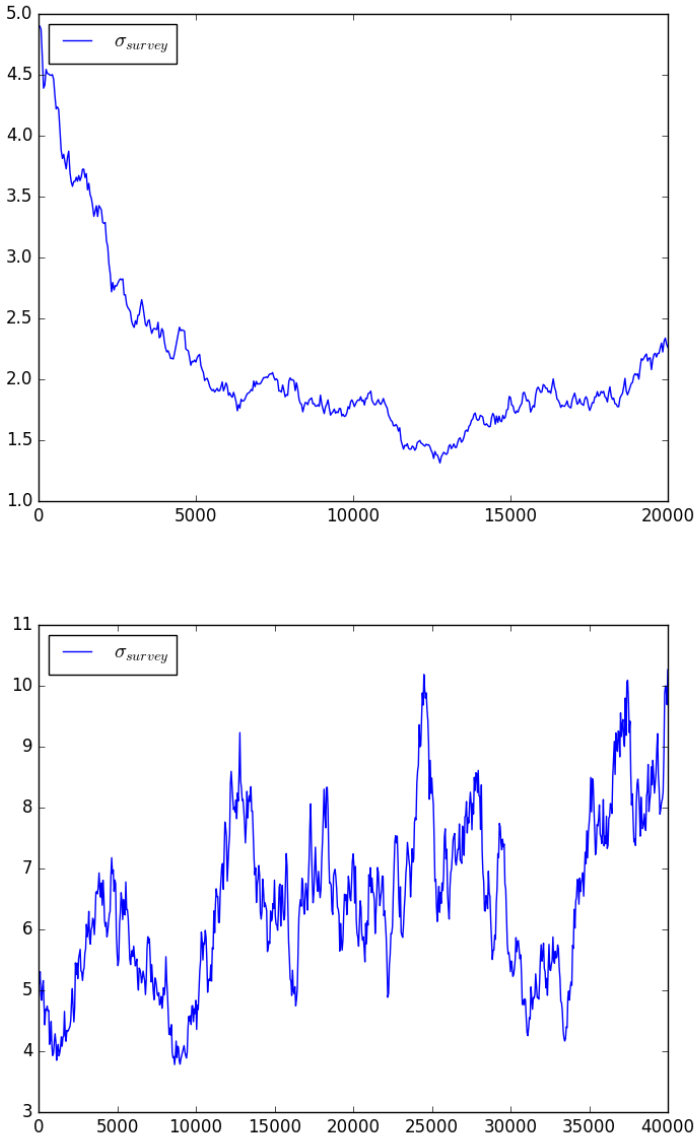
Trace plots for miscellaneous utility parameters from grade K model (upper panel) and grade 9 model (lower panel) . See section 6 for estimation details. See 5 for parameter definitions.

Figure A14: Trace Plots: Preference Shocks $\sqrt{\Sigma_{(j,j)}}$



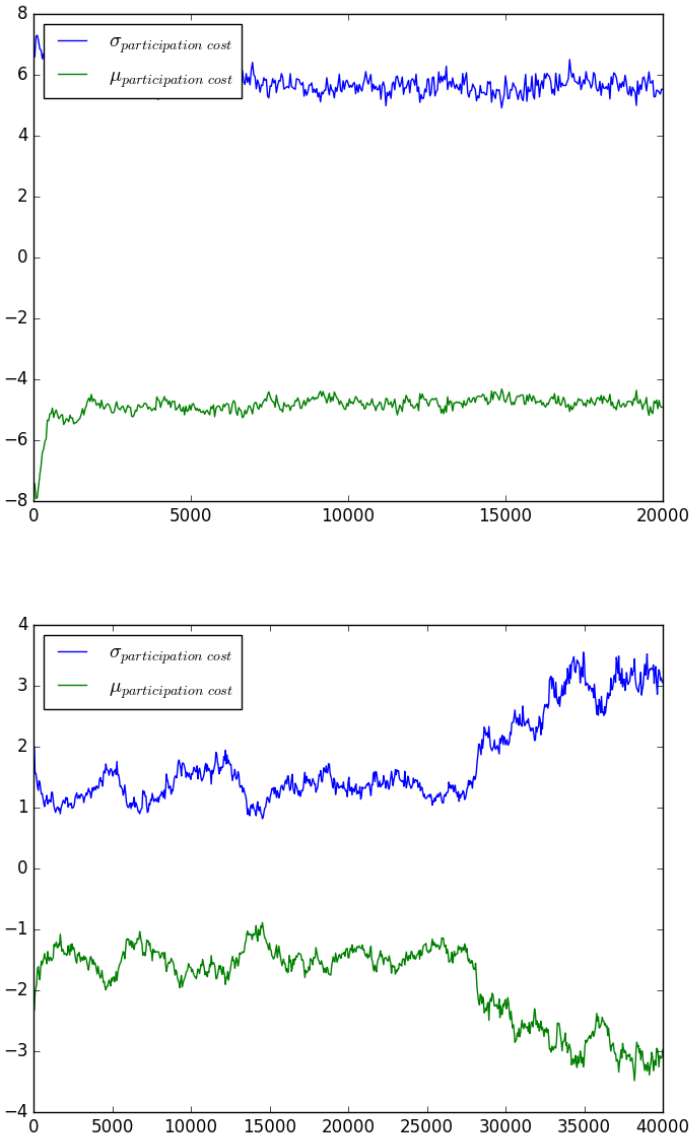
Trace plots for preference variance matrix terms in grade K (upper panel) and grade 9 (lower panel). See section 6 for estimation details. See 5 for parameter definitions.

Figure A15: Trace Plots: Measurement Error σ_{survey}



Trace plots for survey measurement error variance parameter in grade K (upper panel) and grade 9 (lower panel). See section 6 for estimation details. See 5 for parameter definitions.

Figure A16: Trace Plots: Placement Cost



Trace plots for placement cost parameters in grade K (upper panel) and grade 9 (lower panel). See section 6 for estimation details. See 5 for parameter definitions.

B Appendix B: Value added estimation

B.1 VA

We measure value added using a procedure that most closely follows Deming (2014). We model test scores for student i in school j in year t, Y_{ijt}, as arising from the process

$$\begin{aligned} Y_{ijt} &= X'_{ijt}\beta^{test} + v_{ijt}^{test}. \\ v_{ijt}^{test} &= \mu_j^{test} + \theta_{jt}^{test} + \epsilon_{ijt}^{test} \end{aligned} \tag{3}$$

where Y_{ijt} is the average of standardized math and reading scores on state accountability tests, X_{ijt} is a set of observable characteristics that includes SES tercile, race/ethnicity, gender, grade, year, and baseline test scores. Scores are standardized using observed grade-specific means and standard deviations for district students in each year t. The residual term v_{ijt} is the sum of a school-specific component μ_j^{test} that is constant over time, a time-varying school-specific shock θ_{jt}^{test}, and an idiosyncratic student-specific error ϵ_{ijt}^{test}.

We recover best predictors of the μ_j^{test} term by estimating school-specific mean residuals and shrinking them back towards zero. Here we follow Kane and Staiger (2008) and Chetty, Friedman, and Rockoff (2014a,b). We consider both models in which school effects are constant over time and models in which there is 'drift' in school effects. Models with drift identify school effects in particular year based on autocorrelations in mean residuals across years.

We estimate test score specifications using data on school enrollment and test scores for the years 2007 through 2013. During this period, students took state exams in grades three through eight and again in grade ten. To expand the set of data that can be used to estimate scores, we use eighth grade scores as lag scores for tenth graders and dummy out observations for which lag scores are missing.

In the main text, we characterize schools using the no-drift value added estimates. Figure 1 in the main text displays the distribution of value added measures by school. The distribution is weighted by the count of assigned students in the 2015 school lottery, so the mean need not be (and in fact is not) zero. The standard deviation of the distribution is 0.08, and the gap between the best schools and the worst schools is about 0.36. Table B1 presents additional descriptive statistics on the distribution of value added for placed students in the 2015 school lottery.

Table B1: Distribution of value added at placed schools

Grade	min	p25	p50	p75	max	N
9	-0.127	-0.120	-0.091	-0.052	0.127	1423
K	-0.139	-0.038	0.028	0.046	0.224	1540
Total	-0.139	-0.091	-0.041	0.034	0.224	2963

Notes:N grade 9: 1423. N grade K: 1589. Estimates of no-drift VA estimates by grade. Sample: students placed in schools in 2015 school lottery for which we can compute VA.

B.2 Benchmarking VA estimates using school lotteries

We benchmark our value added estimates by comparing them to observed outcomes for students randomly assigned to schools by placement lotteries. We use two benchmarking procedures. The first is as follows:

1. Identify all lotteries l with an admissions probabilty between zero and one in each application year. A lottery here is defined by target school/grade, priority group, and submitted preference ranking. These are the groups within which admission is determined only by a random number. Lotteries include only students not admitted to a higher-ranked choice.

2. For each lottery l, compute $\overline{VA}_l^{out}$, the mean value added at the schools where rejected students in lottery l are placed. Let VA_l be the value added at the target school for lottery l, and $\overline{GAP}_l = VA_l - \overline{VA}_l^{out}$ be the difference between value added at the target school and average value added at the placed school for rejected applicants.

3. Estimate instrumental variables specifications of the form

$$VA_{ijtl} = \pi_0 W_{il} \overline{GAP}_l + \pi_1 X_i + \kappa_{jt} + e_{ijtl} \tag{4}$$

$$Y_{ijtl} = \delta_0 VA_{ijtl} + \delta_1 X_{ijt} + \tau_{jt} + u_{ijtl} \tag{5}$$

Here, VA_{ijtl} is measured value added for student i enrolled in school j in outcome year t who participated in lottery l, W_{il} is an indicator variable equal to one if i was a winner in lottery l and zero otherwise, X_i a set of covariates that includes race, gender, and student SES, and

κ_{jt} is a set of fixed effects for the interactions between lottery l and outcome year t. Y_{ijtl} is the observed test score for student i in lottery l enrolled in school j in year t.

This specification instruments for value added at the school where a student is enrolled using variation in school assignment generated by lottery outcomes. We exploit variation in the effects of a lottery win on value added generated by differences across lotteries in value added at the assigned school and the schools where lottery losers are assigned. The prediction of our value added model is that the coefficient δ_0 should be equal to one– i.e., that test scores should rise one for one with value added at the school where students enroll.

We implement the benchmarking procedure using value added measures with and without drift. In each case, we compute separate value added measures for each school in each outcome year t using data from all other outcome years. This is to avoid a mechanical relationship between the value added measure and observed test outcomes in that year. We two-way cluster our standard errors by lottery and by individual.

Our benchmarking procedure closely follows Deming (2014). Two differences are worth noting. First, Deming uses in the gap between value added at the target school and students' neighborhood schools rather than the average of assigned schools for rejected students. We choose the latter because neighborhood schools are a (relatively) weak predictor of school assignment in our context of centralized and comprehensive school choice. Second, Deming focuses on students' first-ranked school while we consider all marginal applications.

Panels A and B of Table B2 show results from this exercise. Panel A shows estimates using VA measures with drift. The first two columns show the estimated effects of the excluded instrument $W_{il}\overline{GAP}_l$ on value added at the enrolled school. This value is roughly 0.5. If compliance with school assignment were perfect and students could not switch schools in subsequent years, it would be one. The second column shows the effect of the instrument on own test scores. This value is also approximately 0.5. The third column reports the IV estimate of 0.99. This is consistent with the prediction of the value added model of test score production, though we note that the estimate is imprecise.

Panel B shows parallel estimates for no-drift value added estimates results are similar, though the IV estimates of 1.46 is somewhat larger. This pattern echoes findings from Deming (2014), who also finds that VA estimates that do not adjust for drift have somewhat lower predictive accuracy that those that do. We note that differences in predictive power across shrinkage procedures are not statistically significant either here or in Deming (2014).

Table B2: VA benchmarking by lottery ID

	Enrolled VA	Test score	Test Score
A. Drift			
Admit X Gap	0.523**	0.518	
	(0.063)	(0.316)	
Placed VA			0.992+
			(0.551)
N	4254	4254	4254
N lotteries	307	307	307
B. No drift			
Admit X Gap	0.476**	0.693+	
	(0.055)	(0.373)	
Placed VA			1.455+
			(0.756)
N	4984	4984	4984
N lotteries	351	351	351
C. AANP, no drift			
Admit top quartile	0.039**	0.102	
	(0.005)	(0.082)	
Placed VA			2.639
			(2.072)
N	1929	1929	1929

We also consider an alternate benchmarking procedure based on Angrist et al. (2015; henceforth AANP). AANP show that school assignment is exogenous conditional on assignment propensities, and that this insight can be used to pool students in different priority groups or who submitted different applications into larger lottery blocks. We implement this approach here and use it to study the effects of admission to a school in the top quartile of the value added distribution on test scores.

Results are reported in panel C of Table B2. Admission to a school in the top quartile of the value added distribution raises value added at the enrolled schoool by 0.039. This value is relatively small because many students admitted to high value-added schools have next-best options that also perform well on this metric. Test score rise by a (statistically insignificant) 0.102, yielding an

imprecise IV estimate of the effect of VA at the enrolled school on own score of 2.6. Power is reduced here because the procedure does not exploit variation in counterfactual options across lotteries as in panels A and B.

Our conclusion from this section is that as in Deming (2014), estimates of school value added are strongly predictive of test score outcomes for randomly assigned students.

C Appendix C: Fieldwork and Survey Questionnaire

C.1 Data and Fieldwork

C.1.1 Administrative Data

New Haven Public Schools is a large school district similar to many urban school districts across the United States. Administrative student-level data was given to us with permission from New Haven Public Schools (NHPS). The data contain information for approximately 20,000 students, and includes student race, gender, school lunch status, test scores, and other information. NHPS is a majority-minority school district - nearly 90% of students are either black or Hispanic - and more than 80% are eligible for free lunch.

C.1.2 New Haven Home Sales Data

Data on home prices used to create measures of SES come from the City of New Haven's official tax assessor's website. This dataset includes the universe of residential homes. A sales history is included for each property listed on the websites, along with various property characteristics such as street address, home square footage, acerage, number of rooms, and more. Table C1 shows summary statistics for all home sales in New Haven between 2005 and 2015. In order to construct our measure of student socioeconomic status (SES), we first regress real (2015 dollar) per-square-foot sale prices of New Haven homes on time dummies and obtained residuals. We then compute the implied price per square foot at each location using a normal kernel with bandwidth .05 miles. See Section 3.4 and Figure A3 in the text for more details.

C.1.3 Data Collection

In addition to administrative records provided to us by NHPS, we designed and conducted a household survey named *"New Haven School Choice Survey: Empowering Choice through Information*

Table C1: New Haven Home Sales Data

	2005-2015	2005-2009	2010-2015
Mean Price ($1,000s)	216.9	240.1	176.5
Median Price ($1,000s)	193.6	222.7	136.0
Square Feet	2,032	1,983	2,118
Acreage	0.119	0.115	0.126
Bedrooms	4.37	4.35	4.41
Bathrooms	2.16	2.15	2.19
Rooms	8.35	8.22	8.57
High Quality	0.323	0.306	0.353
N	11,221	7,112	4,109

Notes: This table shows summary statistics (means) for all residential property sales in New Haven between 2005 and 2015. Prices are in 2015 dollars. Data were collected from the City of New Haven's offical tax assessor website database (see http://gis.vgsi.com/newhavenct/).

and Understanding". Funded by the Industral Relations Section of Princeton University and the Cowles Foundation at Yale University, we worked together with the district to collect data directly from potential participants in the school choice mechanism. NHPS provided student-level administrative data that contained the universe of the sample population and also information relevant for contacting the families. We conducted in-person interviews with over 200 families. We interviewed parents and guardians of students at their homes in an anonymous questionnaire that last approximately 20 minutes. This survey was conducted in collaboration with NHPS and experienced field personnel with prior experience conducting similar surveys in projects done with JPAL LAC. The field work was conducted during August to October of 2015. The survey was implemented on Samsung Galaxy Tab 7 and programmed using SurveyCTO. The household survey was designed and tested in small focus groups three occasions during the two months prior to the field work.

The team of surveyors was composed by ten active members who were recruited using online advertisement and Yale University's public spaces for flyers. All of them received a two-day training that prepared them for the use of the tablet and regulation regarding interacting with human subjects. Almost half of the surveyors were bilingual English and Spanish speakers which was useful given a significant proportion of the population in New Haven is Hispanic. The two-day training covered the following topics:

- **Day 1:** Introduction regarding data confidentiality and safety. Logistics procedures were discussed.
- **Day 2:** Practical training of the instrument in a random neighborhood where we tested their skills to approach the families and their accuracy while using the instrument.
- **CITI Certificate:** All surveyors had to complete an on-line course for IRB purposes where they learned about dealing with private and confidential data.

Each of these items were mandatory before going to the field. Also, all personnel were supervised by a field coordinator who monitor and verified the data collection process. Each surveyor was assigned with a route sheet that included the closest selected households in a neighborhood in order to facilitate their work. The field coordinator selected the routes of households to reach certain geographic and demographic coverage goals.

C.1.4 Recruiting Participants

Parents' participation was voluntary and there was no compensation (neither monetary nor non-monetary) for their participation.

- In partnership with NHPS, the district contacted the households via phone-calls to announce their participation in the project.
- When the surveyors visited each house, they announced the project and handed in a business card (see Figure C1) with study contact information. Parents or guarding who agreed to participate signed an informed consent.
- In case of finding no one at home, we left a door hanger with contact information (see Figure C2)
- Also, surveyors had the chance to re-schedule the interview if the respondent had time issues at the moment.

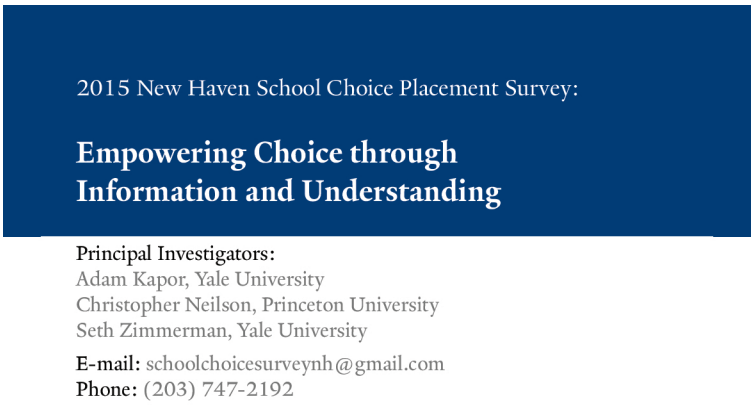


Figure C1: Business Card used during Fieldwork

2015 New Haven School Choice Placement Survey

Empowering Choice through Information and Understanding

We're sorry we missed you!

We are a team of researchers from Yale and Princeton Universities who are collaborating with New Haven Public Schools to conduct a survey about your experiences with school choice.

We want to hear from you! **Please call or text us at 203-747-2192** and we can find a time to talk that is convenient for you.

Thanks!



Figure C2: Door Hanger used during Fieldwork