Heterogeneous Beliefs and School Choice

Adam Kapor Christopher A. Neilson Seth D. Zimmerman

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Abstract

Many school districts in the United States and abroad offer students a choice of schools, with seats at highly demanded schools apportioned using a centralized mechanism with random rationing. This paper studies how welfare and academic outcomes depend on the assignment mechanism when school choice participants are not fully informed of their admissions chances. We survey school choice participants about their preferences and beliefs, and use our results to estimate an empirical model of school choice that incorporates heterogeneity in preferences, strategic behavior, and subjective beliefs about admissions chances. We then use the estimated model to evaluate the equilibrium effects of a) switching to the strategy-proof student-proposing deferred acceptance algorithm, and b) improving the information available to households about the existing lottery mechanism. Survey data show that beliefs about admissions probabilities are correctly centered but have large mean absolute errors. Participants with above-median absolute errors are 57% less likely to place in their most-preferred school. Model estimates suggest that switching to a deferred acceptance algorithm would raise total welfare by reducing the chances of very bad welfare outcomes, but would have limited effects on the distribution of student test scores.

*Kapor: Columbia University, ak4003@columbia.edu; Neilson: Princeton University, cneilson@princeton.edu; Zimmerman: University of Chicago Booth School of Business, seth.zimmerman@chicagobooth.edu. We thank Sherri Davis-Googe and Carolyn Ross-Lee for their many contributions to survey design and implementation. We thank Fred Benton for his assistance with data access. We thank Fabiola Alba, Alejandra Aponte, Felipe Arteaga Ossa, Kevin DeLuca, and Julia Wagner for their excellent research assistance. We gratefully acknowledge financial support from the Cowles Foundation, the Yale Program in Applied Economics and Policy, and Princeton University Industrial Relations Section. All errors are our own.

1 Introduction

Many cities in the United States and abroad have replaced neighborhood-based school assignment with policies offering students a choice of schools within broader geographic areas. However, space in desirable schools is limited, and school choice policies must ration seats somehow. A common approach is to conduct a centralized assignment process that elicits rank-order lists of schools from applicants and uses a combination of coarse priorities and random lotteries to assign seats at schools with excess demand.[1] Centralized assignment mechanisms have the potential to raise welfare and improve academic outcomes by reducing congestion in the assignment process and by helping students match to schools they like. Theoretical work emphasizes that, depending on the assignment mechanism used, families who do not know their own priority group, the number of slots available, or the distribution of other students' preferences and priorities may benefit less from school choice than those who do (Pathak and Sönmez, 2008). However, there is limited empirical evidence on what students do and do not know about the school choice process and how this affects the allocation of students to schools under different mechanisms.

This paper studies how welfare and academic outcomes depend on the assignment mechanism when school choice participants are not fully informed of their admissions chances. We combine a new household survey measuring the preferences, sophistication, and beliefs of potential school choice participants with administrative records of school choice and academic outcomes to estimate a model of school choice participation. We conduct this study in the context of the public school district in New Haven, Connecticut (henceforth NHPS), which for at least 18 years has used a centralized mechanism that rewards fully-informed strategic behavior. Similar procedures advantaging informed and strategic participants are employed by Cambridge MA, Charlotte NC, and Beijing, among other cities.

We focus on three sets of questions. First, we use our survey data to describe families' preferences over schools and their beliefs about the school choice process. We consider both beliefs about admissions probabilities and understanding of the assignment mechanism. Second, we consider the effects of replacing the current mechanism with a deferred acceptance procedure, in which it is always optimal to report one's true preferences, on student welfare and academic achievement. Third, we ask whether there is scope for reducing inequality and/or improving educational outcomes by providing information about school choice. We consider a set of policies that scale households'

[1] Centralized school assignment mechanisms are used in Boston, New York, Chicago, New Orleans, Cambridge MA, and Charlotte NC school districts, among others. For studies in these cities, see Abdulkadiroğlu et al. (2005a,b), Cullen et al. (2006), Agarwal and Somaini (2014), and Deming et al. (2014).

deviations from informed strategic play along a spectrum with observed behavior at one end and optimal strategic play (i.e., a 'best-case' intervention) at the other.

A descriptive analysis of our survey and administrative data shows that many families misunderstand the assignment mechanism and make errors in their estimates of the admissions probabilities associated with different application portfolios. Less than twenty percent of participants respond correctly to questions about the ordering of priority groups by sibling, neighborhood, and submitted preference ranking. Beliefs about admissions probabilities are correctly centered, with subjective beliefs exceeding rational expectations by only about 3 percentage points on average. However, the mean absolute difference between elicited and observed admissions probabilities is 30 percentage points. For students in the upper tercile of SES, the mean absolute error is 25 percentage points, compared to 32 percentage points for students from the lower two terciles. We further find that respondents underestimate both the decrease in admissions chances associated with ranking a school second as opposed to first and the increase in admissions chances that accrues to students in a sibling or neighborhood priority group. These findings are consistent with the hypothesis that families do not understand the assignment mechanism.

Beliefs about admissions affect the applications students submit and their school placement outcomes. Though 54% of students are 'revealed strategic' in the sense that they report a school other than the one identified as their most-preferred option as their first choice on their submitted rank list, a majority of revealed strategic students submitted a first choice with a lower admissions probability than their most-preferred option. Students with average absolute belief errors of greater than the median value (roughly 25 percentage points) are 27 percentage points less likely to be placed in their most-preferred school, a 57% decline from a base rate of 47%.

We explore the effects of subjective expectations on the distribution of students across schools by combining our survey data with administrative records of the choice process to estimate an empirical model of school choice. Households in the model maximize expected utility given their subjective beliefs about admissions probabilities. However, their beliefs may be mistaken due to uncertainty about the admissions mechanism, their own priority groups, or demand conditions in the district. We focus on a parsimonious model of belief formation in which students' beliefs about their own admissions rankings relative to a cutoff ranking for admission to each school are equal to the true value plus a shift term. We model the shift term as depending on a) the student's priority at a target school, b) the school's rank on a student's submitted application, c) a student level shock that is common across all schools, and d) an idiosyncratic person-school match-specific component. Intuitively, the first two terms allow us to capture systematic misunderstanding of the assignment mechanism, while the latter two allow, respectively, for levels of optimism to vary across students

and for errors in belief about school-specific demand.

Consistent with our descriptive analysis, model estimates suggest that individuals routinely misestimate the marginal admissions round in which a school will fill its places: the standard deviation of the belief shift term is equal to roughly one admissions round. A student whose admissions are shifted by one round might think a school will fill up with students from the zoned neighborhood who listed a school second on their application, when in fact it fills up with students from the neighborhood who listed it first. Students from low-SES backgrounds have have belief shift terms with a standard deviation 25% larger than those from high-SES backgrounds.

With parameter estimates in hand, we study two sets of counterfactual simulations. To evaluate these counterfactuals, we consider each student's expected utility, according to the utility he or she gets from placement at each school and the rational-expectations chances associated with their lottery application. Our first counterfactual exercise shows that, given households' errors about their admissions chances, switching to the student-proposing deferred acceptance algorithm raises total welfare. Gains in overall welfare are driven by a shift upward in the lower quantiles of the counterfactual welfare distribution. Our second set of simulations shows that a best-case informational intervention, one that would allow households to play Bayes Nash equilibrium in the game induced by the New Haven mechanism, would further raise total welfare relative to the deferred acceptance case. However, to obtain gains relative to deferred acceptance requires a reduction in the variance of the belief error term by more than 80% relative to what we observe in the data.

We also use our counterfactual simulations to study the distribution of test score value added across students under different assignment mechanisms. Though changes in expected value added under counterfactual policies are correlated with changes in expected utility, we find little evidence that a change to the deferred acceptance mechanism or a best-case informational intervention would yield aggregate gains in school value added or redistribute value added towards low-SES students.

The paper proceeds as follows. Section 2 describes our contribution to existing literature. Section 3 and Section 4 describe the New Haven school district and our survey instrument, respectively. Section 5 describes our model of student behavior, section 6 describes estimation, and 7 describes results and counterfactuals. Section 8 concludes.

2 Literature on School Choice and Mechanism Design

This paper's primary contribution is to bring direct observations of beliefs to the analysis of the welfare properties of school choice mechanisms, and to develop a method for analyzing belief data in the context of a model of school choice. A central debate in the literature on school choice

mechanism design is whether districts adopting centralized choice should employ student-optimal stable matching mechanisms, which do not give incentives to misreport preferences, or immediate acceptance mechanisms (also known as 'Boston' mechanisms), which reward informed strategic play (Abdulkadiroğlu et al., 2006). Cities including Boston, New York, and Denver have adopted student-optimal stable matching mechanisms (Abdulkadiroğlu et al., 2005a,b, 2015b), while cities such as Charlotte NC, Barcelona, and Beijing use immediate acceptance mechanisms.[2] A theoretical literature provides conditions under which all students prefer the Boston mechanism to the student-optimal stable matching mechanism, and others under which it is (weakly) worse for all students (Ergin and Sonmez, 2006; Abdulkadiroğlu et al., 2011).[3] Which mechanism will perform best in a particular district or set of districts is therefore an empirical question, the answer to which depends on the distribution of preferences, beliefs, and strategic sophistication in the population of participants.

In the absence of data on beliefs, a growing empirical literature has generally found that the Boston mechanism outperforms deferred acceptance in revealed-preference welfare measures under the assumption that participants are informed and sophisticated, or deviate from optimal behavior in specific ways. For example, Agarwal and Somaini (2014) assume, as a baseline specification, that participants are fully rational and correctly anticipate their chances in the lottery when choosing applications. Alternatively, Calsamiglia and Guell (2014) consider school choice under a Boston mechanism in Barcelona. They allow two types of participants: one type is sophisticated and informed while the other type uses a rule of thumb to determine choices. Calsamiglia et al. (2014), He (2012), and Abdulkadiroğlu et al. (2015b) take similar approaches.

By directly eliciting both preferences and beliefs about admissions probabilities under different hypothetical application portfolios, we are able to analyze the effects of counterfactual policy changes without making strong assumptions on applicants' equilibrium play. To the best of our knowledge this is the first paper to collect such data from actual and potential school choice participants.[4] Our findings suggest that the ordering of deferred and immediate acceptance mechanisms by welfare

[2] Barcelona: Calsamiglia and Guell (2014); Charlotte: Hastings et al. (2009); Denver: Abdulkadiroğlu et al. (2015a); Beijing: He (2012).

[3] See also Pathak and Sönmez (2008), who provide a model in which sophisticated students benefit, and naive students suffer, from the Boston mechanism, and Pathak (2011) for a review.

[4] Two recent papers use incorporate survey elements to unpack school choice participation decisions and reports. Dur et al. (2015) make use of data on the frequency with which students access a school choice website to proxy for strategic and sincere participants in a school choice mechanism. Students who visit the site multiple times are assumed to be sophisticated, while those visiting only once are assumed sincere. de Haan et al. (2015) measure cardinal utility in Amsterdam using a survey that asks students to assign points to each school, with the top choice receiving 100 points, but do not ask about beliefs.

outcomes depends on the accuracy of students' beliefs about admissions chances. Though the immediate acceptance mechanism is preferable when students have rational expectations about choice probabilities, the deferred acceptance mechanism raises welfare given the distribution of belief errors we observe in our data. The gains we observe come in large part from increases in quantiles below the median of the welfare distribution, consistent with the idea that one benefit of student-optimal stable mechanisms is to reduce the prevalance of major choice errors.

Our estimation strategy incorporates both survey and administrative data. The survey data help us overcome the challenges associated with separately identifying beliefs and preferences described by Agarwal and Somaini (2014) and estimate parameters governing the distribution of belief errors. A two-step procedure coupling MCMC and data augmentation then uses belief errors to rationalize observed choice data for both surveyed and unsurveyed students.

We make an additional contribution to the mechanism design literature by describing the procedure New Haven uses to assign students to schools. To the best of our knowledge this process, which we call the 'New Haven mechanism' and describe in detail in Section 3, has not been documented elsewhere. The New Haven mechanism rewards informed strategic play, but to a lesser extent than the Boston mechanism. We therefore expect our findings to understate the welfare losses due to less-than-fully informed play that would be observed in a Boston mechanism setting.

This paper also contributes to existing work by evaluating the effect of school choice policy on the distribution of achievement test scores. To do this, we combine our counterfactual simulations with OLS estimates of school test score value added that most closely resemble Deming (2014). Previous work estimating preferences in strategic school choice mechanisms has focused on parent satisfaction (measured in terms of, e.g., distance-metric utility) while ignoring achievement, even as an extensive parallel literature uses data from school lotteries to estimate school-specific test score effects while ignoring satisfaction. See Cullen et al. (2006) for seminal early work, and Abdulkadiroğlu et al. (2015a) for recent advances. Several other recent papers estimate preferences for school characteristics such as school quality and distance and use these estimates to conduct welfare analysis and counterfactual simulations in decentralized or non-strategic settings (Hastings et al., 2009; Neilson, 2013; Walters, 2014; He, 2012; Dinerstein and Smith, 2014). Though changes in expected value added under counterfactual policies are correlated with changes in expected utility, we find little evidence that a change to the deferred acceptance mechanism or a best-case informational intervention would yield aggregate gains in school value added or redistribute value added towards low-SES students. We interpret these findings with caution because, in contrast to our utility model, our model of school value added does not allow for student-school specific match effects. This is consistent with most existing studies of school value added but rules out positive sum trades in

school assignments across students.

3 Empirical Setting

3.1 The school choice process in New Haven

Three features of the school choice process in New Haven make it a useful context in which to study beliefs and preferences for school choice participants. First, New Haven was an early adopter of centralized school choice, and the assignment process the district uses has remained fairly stable over time. The first choice-based magnet school opened in New Haven in 1970, and the number of school choice options expanded rapidly in the mid-1990s following a state-wide push to reduce school segregation (Huelin, 1996). New Haven has assigned students to schools using a centralized New Haven mechanism since at least 1997.[5] New Haven adopted centralized school choice several years before New York, which introduced a centralized application in 2003, and other cities such as Denver, New Orleans, Newark, and Washington DC, which built on the New York example (Abdulkadiroğlu et al., 2015a). The school choice system includes both district-run magnet schools and charter schools run by outside operators, such as 'no excuses' charter brand Achievement First.

Second, the school district conducts extensive outreach to publicize the process, including events for parents and children outside of school hours, in-school open-houses, and published documentation on procedures and past outcomes. The school choice process in New Haven follows a consistent pattern from year to year. The process begins in January of the academic year preceding the year of school assignment. Students and families can learn more about schools and the choice process by visiting open houses at different schools or by attending one of several 'magnet fairs' where schools set up information booths. The school district provides students with a magnet school guide that includes a description of the rules of choice and data on available seats and applicant counts by priority group from the previous year. This guide is available in English and Spanish, both in print and on an NHPS website. Students typically submit their applications in February, and receive notice of their placements in March or early April. These first two points suggest that parents and students have had ample time and opportunity to learn about the mechanism from the district and from each other, so the distribution of beliefs and preferences we observe is likely to reflect a long-run steady state.

Third, and finally, the vendor the district uses to implement school choice is employed by many

[5] We have verified the use of the centralized mechanism as far back as 1997 by inspection of the code used to run the process.

other districts around the country. Between 1997 and 2013, the school assignment mechanism was implemented by an independent contractor working for NHPS. For the 2013-2014 school year, NHPS switched to the school choice vendor Smart Choice Technologies, which also administers school choice programs in Bridgeport CT, Hartford CT, Syracuse NY, New Orleans LA, and Tulsa OK, among others (Smart Choice, 2016). The third point suggests that the practices we observe in New Haven may have external validity in the sense that they are used in other districts as well.

The primary entry points in most district schools are kindergarten and ninth grade. In our analysis, we restrict attention to families living in New Haven with children enrolled in eighth grade or pre-K. In the 2014-2015 school year, when we conducted our survey, there were 1480 such potential ninth graders and 1743 potential kindergarteners. 40% of kindergarteners and 66% of eighth graders participate in choice. As reported in Appendix Figure A1, just under half of all choice participants come from these two grades.

From this population, students who do not leave the city or enroll in private school may enter a lottery to enroll in one of 12 high schools or 34 elementary/middle schools that offer kindergarten. Most of these schools are administered by the district, but the total includes two charter high schools and three charter elementary schools. Many of the schools reserve some seats for suburban applicants. The remaining seats are available only to within-city applicants. Consistent with our sample frame, we focus on the seats reserved for within-city applicants.

3.2 The New Haven mechanism

Most school choice mechanisms use some form of coarse priorities to favor certain applicants. In New Haven, each student is assigned a priority at each school, which is a number between one and four:

$$priority_{ij} = \begin{cases} 1 & \text{if i lives in the neighborhood and has a sibling at j, and j gives neighborhood priority} \\ 2 & \text{if i lives in the neighborhood of j, and j gives neighborhood priority} \\ 3 & \text{if i has a sibling at j} \\ 4 & \text{otherwise} \end{cases}$$

Not all schools give neighborhood priority. Two high schools, Hillhouse and Wilbur Cross, give neighborhood priority, but the remaining high schools are classified as magnet schools, which give priority for siblings only. Similar priority structures are in place in Boston, Cambridge, New York,

Barcelona, Beijing, and other cities.

The mechanism assigns students to schools using the following algorithm:

1. Take each applicant's first choice submission and make provisional assignments in order of priority group, using random lottery numbers as a tiebreaker.
2. For unassigned students, move to the next listed choice, and make provisional assignments in order of a) priority group and b) submitted rank, again using lottery numbers as a tiebreaker.
3. Repeat Step 2 until all students are assigned to schools or have been considered and rejected at each listed school.

The mechanism assigns each student to at most one school. Students may choose to accept or decline this placement. If they decline, they may enroll in a neighborhood school with unfilled seats or leave NHPS.

Table 1 describes placement outcomes and priority groups in 2015. Two thirds of participants placed in their first-listed school, and 13% of applicants are unplaced. Most students submit applications to schools where they have neither sibling nor neighborhood priority. High school students typically do not apply to schools where they have neighborhood priority because these schools are available as default options.

Table 1: Placement outcomes and priority groups by grade

	All	K	9
A. Participation and placement			
Participates	0.52	0.401	0.660
Places First	0.666	0.701	0.640
Places Second	0.113	0.127	0.102
Places Third	0.052	0.066	0.042
Places Fourth	0.041	0.057	0.029
Unplaced	0.129	0.049	0.187
B. Priorities			
Sib and Nbd	0.029	0.069	0.000
Nbd	0.077	0.184	0.001
Sib	0.104	0.183	0.047
None	0.789	0.563	0.950
N	3230	1746	1484

Notes: Placement outcomes and priority group in 2015 by grade. Placement outcomes are conditional on participation. Priorities average across all submitted applications.

This mechanism, which we label the 'New Haven' mechanism, differs from standard deferred acceptance and immediate acceptance algorithms. In what follows, we employ a cutoff representation of this matching algorithm introduced by Azevedo and Leshno (2016) for stable matchings and extended to a class of "report-specific priority plus cutoff" mechanisms by Agarwal and Somaini (2014). The cutoff representation simplifies comparisons between New Haven, Boston, and deferred acceptance mechanisms, and also provides a starting point for our model of belief errors.

The cutoff representation of the New Haven mechanism is as follows. The mechanism assigns student i a "report-specific priority" at school j:

$$rsp_{ij} = 4 * priority_{ij} + rank_{ij}.$$

Ties are broken with uniform random draws that assign each student a score at each school:

$$score_{ij} = rsp_{ij} + z_{ij},\ z_{ij} \sim U[0,1].$$

The mechanism finds cutoffs π_j that fill schools' capacities when each student is matched to his earliest-listed school at which $score_{ij} < \pi_j$. If a school is undersubscribed, its cutoff is set above all applicants' scores. Each student is offered a place in at most one school. Each student may accept his/her placement, or decline and enroll in a default school which is assigned based on the student's residence, or leave NHPS.

The New Haven mechanism differs from Boston and student-optimal stable matching ("SOSM") mechanisms in the construction of rsp_{ij}. In New Haven, report-specific priority depends lexicographically on the exogenous priority $priority_{ij}$ and the rank that the student assigns to the school. In the Boston mechanism, this lexicographic order is reversed. In the student-optimal stable matching mechanism, report-specific priorities depend on the exogenous priority group only.

$$\begin{aligned} rsp_{ij}^{SOSM} &= priority_{ij} \\ rsp_{ij}^{\text{Boston}} &= (rank_{ij}, priority_{ij}) \\ rsp_{ij}^{\text{New Haven}} &= (priority_{ij}, rank_{ij}) \end{aligned}$$

Neighborhood and sibling priority play a relatively more important role and submitted rank lists a relatively less important role in determining response-specific priority in the New Haven mechanism than the Boston Mechanism.

3.3 Belief errors in the New Haven mechanism

We model belief errors using the cutoff representation of the New Haven mechanism. The probability that applicant i will be assigned to school j given that he submits report a to the mechanism and is not admitted to a higher-ranked choice is $P_{ija} = Pr(z_{ij} \leq \pi_j - rsp_{ij}(a))$. Inaccurate beliefs about P_{ija} may arise because students mis-estimate $rsp_{ij}(a)$ or cutoff values π_j. Mistaken beliefs about these two quantities can arise from similar thought processes. For example, students who do not understand how priority groups and submitted rankings jointly determine rsp_{ij} will have inaccurate beliefs about their own values of $rsp_{ij}(a)$ and also about π_j even given full knowledge of other students' submitted applications.

Errors in beliefs about π_j and rsp_{ij} sum to alter beliefs about admissions probabilities. Let $\tilde{P}_{ija}$ denote student i's belief about the probability of admission to j given report a and non-admission to a higher-listed choice, and $\tilde{rsp}_{ij}(a)$ and $\tilde{\pi}_j$ be his beliefs about his response-specific priority and the cutoff score for admission, respectively. Then

$$\tilde{P}_{ija} = Pr(z_{ij} \leq \pi_j - rsp_{ij}(a) - shift_{ij}(a))$$

where $shift_{ij}(a) = \pi_j - \tilde{\pi}_j - (rsp_{ij}(a) - \tilde{rsp}_{ij}(a))$. The $shift_{ij}(a)$ term incorporates errors in beliefs about both rsp_{ij} and π_j. Rather than trying to distinguish between these two closely related sources of error, our empirical model takes a more parsimonious approach and focuses on the $shift_{ij}$ term itself. This choice does not restrict the distribution of deviations of subjective beliefs from rational expectations values. In Section 4 we present descriptive evidence on the distributions of errors in beliefs about probabilities and the $shift_{ij}$.

3.4 The New Haven School District

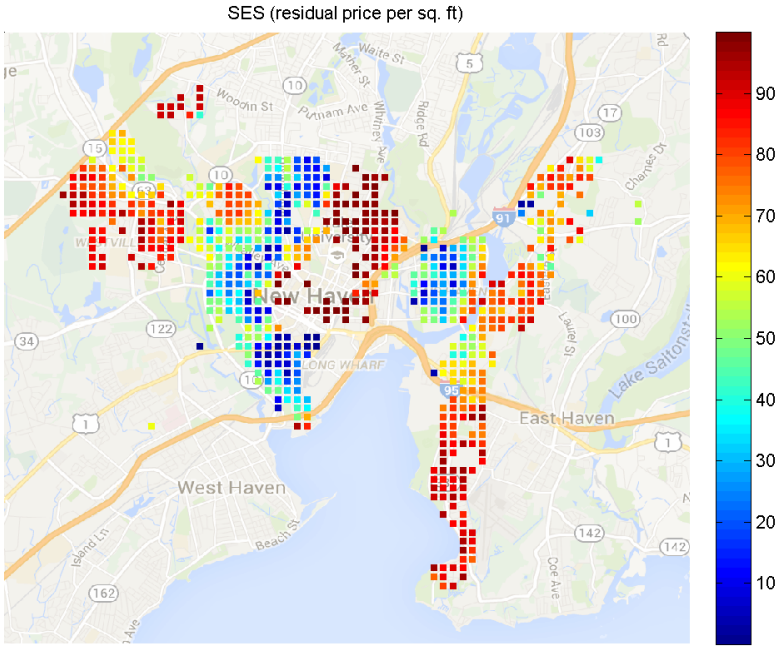
3.4.1 Measuring student SES

NHPS serves a low-income, majority-minority student population. The district is roughly 90% black or Latino, and students score an average of two thirds of a student-level standard deviation (henceforth SD) below statewide means on standardized tests. See Table A1 for more detail. The district has community eligibility for free lunch, meaning that all students may receive two free meals in school each day regardless of own eligibility status. Roughly 80% of students are individually eligibile, but this is based on survey measures that focus on ensuring the district maintains community eligibility.

One goal of this paper is to describe how school choice mechanisms affect the distribution of

welfare by student background. That standard measures of socioeconomic status (SES) are very coarse in our context makes this challenging. Our approach is to create a measure of student socioeconomic status (SES) based on home sale prices. We first regress real (2015 dollars) per-square-foot sale prices of homes in New Haven on time dummies, using all home sales from 2005-2015 and obtained residuals. We then compute the implied price per square foot at each location using a normal kernel with bandwidth .05 miles. Figure 1 plots SES rankings, which range from 0 (the lowest-priced housing in the district) to 100 (the highest-priced). High-SES neighborhoods surrounding Yale University and on the coast are visible in dark red, while the lower-SES neighborhoods are visible in blue. Appendix Figure A2 shows that the our SES measure closely tracks median census tract income but is a better predictor of belief errors, as defined in Section 4. The fine-grained nature of our SES measure allows us to differentiate between students from different kinds of backgrounds even within the same neighborhood catchment zones.

Figure 1: SES rank in population



Notes: This figure displays SES rankings based on home price per square foot. 100 is the highest ranking and 0 the lowest. We compute rankings within each (centered) block using a normal kernel with bandwidth 0.05 miles. Estimates based on detrended home sale price data for the years 2005-2015.

3.4.2 Measuring school quality

We complement our analysis of student welfare using a simple measure of school quality: test score value added. We take a mean-residual measure of school quality that most closely follows Deming (2014). Specifically, we model test scores for student i in school j in year t, Y_{ijt}, as arising from the process

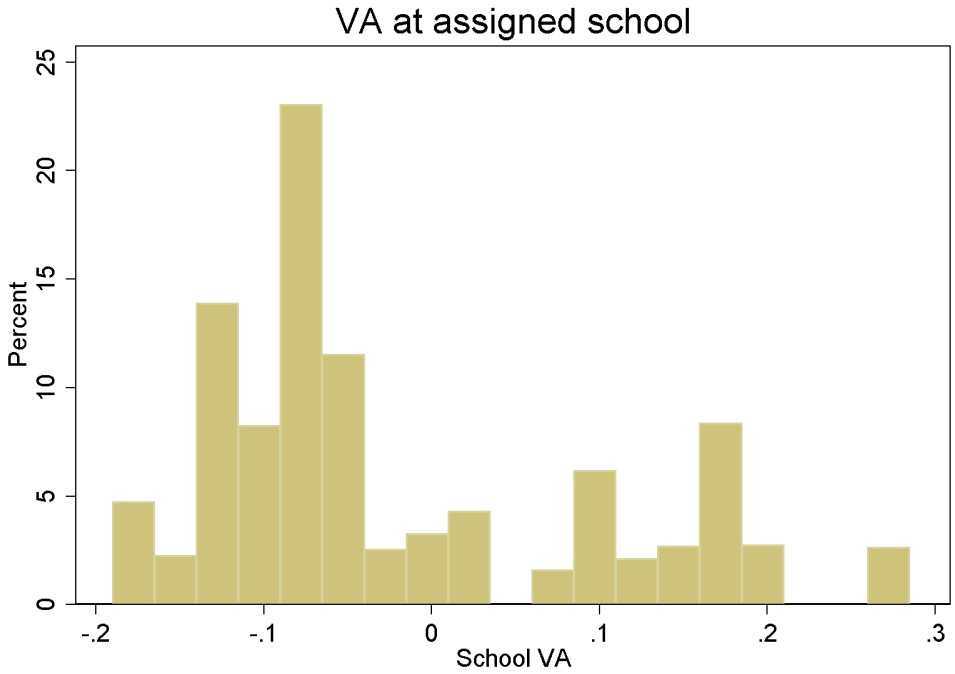
$$Y_{ijt} = X'_{ijt}\beta^{test} + v_{ijt}^{test}.$$
$$v_{ijt}^{test} = \mu_j^{test} + \theta_{jt}^{test} + \epsilon_{ijt}^{test} \quad (1)$$

where Y_{ijt} is the average of standardized math and reading scores on state accountability tests, X_{ijt} is a set of observable characteristics that includes SES tercile, race/ethnicity, gender, grade, year, baseline characteristics, and school-level means of these variables. Scores are standardized using observed means and standard deviations for district students in each year t. The residual term v_{ijt} is the sum of a school-specific component μ_j^{test} that is constant over time, a time-varying school-specific shock θ_{jt}^{test}, and an idiosyncratic student-specific error ϵ_{ijt}^{test}. We recover best predictors of the μ_j^{test} term by estimating school-specific mean residuals and shrinking them back towards zero. Here we follow Kane and Staiger (2008) and Chetty, Friedman, and Rockoff (2014). Our model of school effects does not allow for drift in school effects over time.

We estimate test score specifications using data on school enrollment and test scores for the years 2007 through 2013. During this period, students took state exams in grades three through eight and again in grade ten. To expand the set of data that can be used to estimate scores, we use eighth grade scores as lag scores for tenth graders.

Figure 2 displays the distribution of value added measures by school. The distribution is weighted by the count of assigned students in the 2015 school lottery, so the mean need not be (and in fact is not) zero. The standard deviation of the distribution is 0.12, and the gap between the best schools and the worst schools is just under 0.5. The schools with the highest value added estimates are neighborhood schools in high-SES neighborhoods and high-peforming 'No Excuses' charters.

Figure 2: Distribution of school VA estimates



Notes: This figure displays the distribution of school value added, weighted by the count of students assigned to the school in the 2015 school lottery.

We interpret our value-added results with caution. Students may select into schools in a way that is correlated with test score levels or heterogeneous treatment effects. Our OLS estimates would then not reflect causal effects of schools, or would only do so for the subset of students who attend these schools. That said, evidence from Deming (2014) indicates that value added measures of this type are strong predictors of test score outcomes for students randomized between schools by lotteries. Future drafts will incorporate both lottery and OLS estimates of school effects into a single analysis, following Angrist et al. (2015).

4 Household Survey

4.1 Survey overview

During the summer of 2015, we conducted in-person interviews at 212 households with parents or guardians of children who had been enrolled in pre-kindergarten and/or eighth-grade in NHPS during the 2014-15 school year. In order to construct the sample, we drew 600 households, stratifying by zoned elementary school.

Representativeness is important here because belief errors may be correlated with survey non-response. In Table 2 we show sample means and balance across the population, the 600 target households, and the 212 respondents. The first column shows that within the relevant grade levels the district is approximately 50% black, 10% white, and 40% Latino. The second column shows means in the sample of households we intended to survey, while the third shows means among the households who we successfully surveyed. These households are statistically indistinguishable from the population on race, although the second panel shows that we oversampled English-language learners and bilingual students relative to the population. See Figure A3 for additional evidence that our sample matched the geographic distribution of students across the district.

Table 2: Balance in Socieconomic Characteristics

Category	Population Mean	Sample Mean	Surveys Mean	Mean test Pop. vs Surveys	P-value
A. SES quartile					
1st quartile	0.250	0.244	0.269	0.020	0.508
2nd quartile	0.250	0.276	0.231	-0.020	0.508
3rd quartile	0.250	0.246	0.217	-0.035	0.253
4th quartile	0.250	0.234	0.283	0.035	0.253
B. Race					
Asian	0.021	0.025	0.028	0.007	0.470
Black	0.482	0.483	0.495	0.014	0.690
Latino	0.396	0.401	0.382	-0.015	0.662
Other	0.008	0.005	0.000	-0.008	0.184
White	0.093	0.085	0.094	0.002	0.927
C. Educational program					
Biling/Dual	0.013	0.018	0.038	0.026***	0.001
No ELL	0.948	0.928	0.892	-0.060***	0.000
Regular/ESL	0.039	0.054	0.071	0.034**	0.014
SPED	0.134	0.134	0.184	0.053**	0.028

Notes: $N = 3230$ (population), 598 (intended to survey), 212 (survey participants). $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$. P-value for joint test (F) is 0.002.

In New Haven, students have the option not to enter the lottery. Each student has a default high school, determined by his residential location, and a default elementary school or the opportunity to be placed in a school with excess capacity, if his neighborhood school is full. Households who participate may list up to four schools on their application. Survey participants are statistically indistinguishable from the population on the probability of participating in the centralized mechanism. Table 3 shows balance on participation decisions. Sixty percent of potential kindergarteners and thirty four percent of potential ninth-graders do not apply to any school.

Table 3: Participation and application length by grade

Number of Applications	Population Mean	Sample Mean	Surveys Mean	Mean test Pop. vs Surveys	P-value
A. Grade K					
0	0.599	0.591	0.554	-0.048	0.360
1	0.056	0.050	0.076	0.013	0.588
2	0.060	0.070	0.076	0.022	0.388
3	0.068	0.077	0.043	-0.028	0.308
4	0.217	0.211	0.250	0.040	0.361
B. Grade 9					
0	0.340	0.340	0.275	-0.058	0.198
1	0.095	0.103	0.117	0.019	0.509
2	0.146	0.160	0.125	-0.022	0.508
3	0.164	0.160	0.175	0.011	0.747
4	0.255	0.237	0.308	0.050	0.229

Grade K: $N = 1746$ (population), 298 (intended to survey), 92 (survey participants). $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$. P-value for joint test (F) is 0.079. Grade 9: $N = 1484$ (population), 300 (intended to survey), 120 (survey participants). $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$. P-value for joint test (F) is 0.377

We conducted the survey as a tablet app, with randomly-generated questions tailored to each household. The survey procedures and question text are presented in Appendix A.2. We chose not to incentivize "correct" beliefs, e.g. by paying people to state beliefs that are close to rational-expectations chances. From parents' perspective there may be considerable ambiguity in the school choice process, which may affect the interpretation of bets that parents place.

4.2 Information acquisition, preferences, and revealed strategic play

We first describe the informational environment facing potential school choice participants. Panel A of Table 4 describes the fraction of students who reported using different resources to inform their school choice decision. Nearly two thirds of potential participants reported reading the choice catalog provided by the district, which contains descriptions of schools and information on demand from the previous year. Other sources of information used by many students in choice grades are the

school choice website, which includes information similar to the catalog, school visits, counselors, and teachers. Students also consider a broad range of schools. See Appendix Tables A2 and A3 for evidence here. The district's efforts to inform students about school choice are successful in that students take advantage of the resources provided.

Table 4: Sources of information and understanding of choice rules

	All	K	9
A. Sources of information			
Visit fair	0.358	0.294	0.405
Visit school	0.483	0.448	0.508
Visit website	0.592	0.614	0.576
Talk to teacher	0.556		0.556
Talk to counselor	0.495		0.495
Talk to friend	0.414		0.414
Read catalog	0.658	0.706	0.624
Read newspaper	0.243	0.224	0.256
B. Understanding choice rules			
Get priority ordering	0.167	0.209	0.173
Get mechanism	0.179	0.226	0.162
C. Strategic play			
Revealed strategic	0.539	0.439	0.586
Mistaken strategic	0.306	0.128	0.388
N	212	92	109

Notes: Panel A describes means of dummy variables equal to one if students used the listed information source. Panel B describes means of dummy variables equal to one if students responded correctly to questions about priority ordering (neighborhood vs. sibling) and the importance of the submitted rank to admissions outcomes, respectively. 'Revealed strategic' is a dummy equal to one for students who had a first choice different than their stated most-preferred school. 'Mistaken strategic' is a dummy equal to one if a student was revealed strategic and their admissions chances would have been higher at their most-preferred school than their first-listed school.

Though respondents consult a wide variety of information sources and consider broad sets of schools, they are unlikely to answer questions about how the assignment mechanism works correctly. Panel B of Table 4 presents the fraction of students who correctly answer questions about the ordering of priorities groups and the role of rank in the choice mechanism. Only 17% of respondents correctly identified the neighborhood priority group as being preferred to the sibling priority group,

and only 18% correctly noted that a student rejected from her first choice school has a (weakly) lower chance of admission at her second choice school than if she had ranked the second choice school first. There were four possible responses to the first question and three to the second, so correct answer rates are worse than under random guessing.

Our data on student preferences suggests that participants play strategically, but often make errors in play. To elicit "unconstrained" preferences, we first asked parents which school they would have chosen for their child if they were guaranteed admission to every school in the district. We then asked what they would have chosen if this school were unavailable but all other schools were available. We define a respondent to be revealed strategic if they applied to school j but stated that $j' \neq j$ was their most preferred school if they could enroll anywhere. As reported in Panel C of Table 4, 54% of survey respondents who submitted applications were revealed strategic in this sense.[6] However, a majority (57%, or 31% respondents who participated in choice) of revealed strategic respondents submitted first choice to applications to schools where the rational expectations admissions probability was below that for their most-preferred school. We describe how we calculate rational expectations admissions probabilities in detail in the next section. This finding is consistent with the hypothesize that inaccurate beliefs about admissions chances can limit participants' ability to strategize effectively.

4.3 Beliefs about admissions chances

We next document respondents' beliefs about admissions chances and compare them to objective measures of admissions probabilities. Findings from this descriptive analysis suggest an important role for belief errors in determining the allocation of students to schools, and motivate modeling choices in Section 5.

To provide a benchmark with which to compare subjective beliefs, we estimate rational-expectations admissions chances for the kindergarten and ninth-grade lotteries. These rational-expectations admissions chances represent the beliefs about admissions chances that an agent would have if he knew his own report-specific priority, the rules of the mechanism, schools' capacities, and the number of other applicants but did not know their preference lists or report-specific priorities. We calculate them by resampling $n = 200$ markets, drawing individuals, together with their applications and priority types, iid with replacement from the population. In each resampled market, we calculate the market-clearing cutoffs. Given a vector of cutoffs, we calculate admissions chances for each student. For example, if an individual has $rsp_{ij} = 41$ and lists j first, if the cutoff is $\underline{\pi}_j = 41.4$ then

[6] We report rates at which each school is most-preferred, first listed, and listed at all in Tables A2 and A3.

the individual has a .4 chance of placing in j. For each individual i, we compute the propensity to place in each school j under the individual's observed application and the given cutoff vector, and then average these chances over the resampled market-clearing cutoffs.

This procedure differs slightly from the procedure used by Agarwal and Somaini (2014). In particular, we resample cutoffs and obtain smooth chances for applicants if their report-specific priority type is rationed at school j. In contrast, Agarwal and Somaini (2014) average over simulated placement outcomes from resampled markets, rather than cutoffs.

With rational expectations chances in hand, we define the following measures: let "optimism" denote the difference between i's subjective belief about his admissions chance at j under application a and the rational-expectation chance:

$$optimism_{ij} = \hat{p}_{ij} - p_{ij}^{\text{true}}$$

We consider also the absolute error $|optimism_{ij}|$.

The survey elicited up to four beliefs from each of our 212 participants, but some participants declined to answer some questions, giving a total of 786 elicited beliefs about admission to some school j under an application that listed j. We chose hypothetical applications that contained a mix of nearby schools, high-performing schools, and popular schools at the district level. The distribution of rational expectations admissions probabilities at the hypothetical applications closely matched the distribution of rational expectations probabilities for the actual applications that students in our sample submitted. See Figure A4 for details.

Table 5 describes the relationship between subjective and rational expectations beliefs. Respondents may err by mis-estimating their own response specific priority relative to the marginal admissions round, or mis-estimating the number of other students in the marginal admissions round. The upper two rows show that a fairly large share of applicants make the first type of error. Cases where the rational expectations chance of admission was 0.1% or less but respondents believed their chances of admission were 25% or more accounted for 11 percent of applications about which we elicited beliefs. Cases where applicants thought their chances of admission were below 50% but their true probability was at least 99.9% account for a further 5 percent of applications. Rates of both types of 'round errors' are substantially higher for low-SES students than for high-SES students. Students from low-SES backgrounds make 'false positive' errors in 13% of elicited beliefs and 'false negative' errors on 7%.

Table 5: Errors by Demographic Group

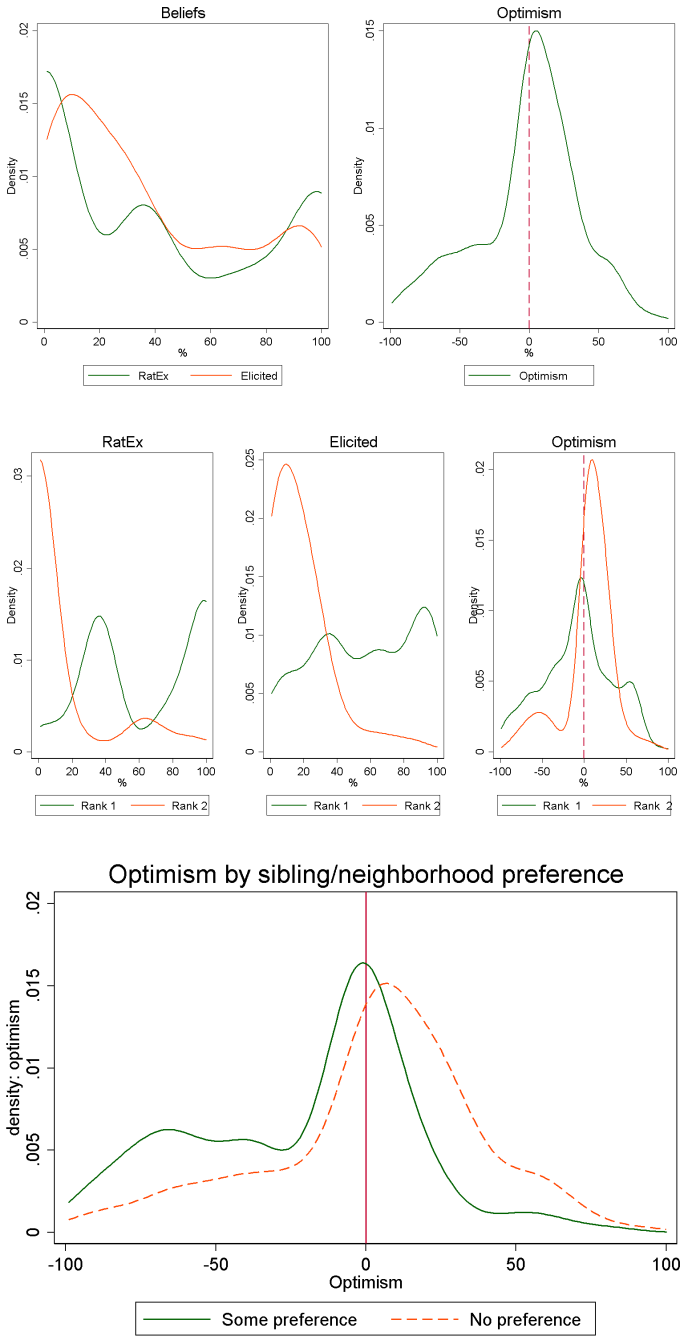
	All	High-SES	Low-SES	Non-black	Black
A. Errors by type					
Belief>25% | ratex≤ 0.1	0.11	0.09	0.13	0.12	0.11
Belief<50% | ratex≥ 99.9	0.05	0.03	0.07	0.04	0.07
B. Subjective beliefs					
Mean	40.9	41.6	40.6	42.9	39
SD	31.5	31.7	31.5	32	31
p25	13.5	13.5	13.5	15	10.5
p50	35	35	35	35	33
p75	65	65	65	66.5	65
C. RatEx Beliefs					
Mean	38.2	39.1	37.8	38.6	37.8
SD	39.7	39.1	40.1	39.8	39.7
p25	0.1	0.1	0.1	0.1	0.1
p50	35.6	35.6	35.6	35.6	35.6
p75	76	76	80.8	81.8	76
D. Optimism					
Mean	2.7	2.5	2.8	4.3	1.1
SD	38.7	32.8	41.4	37.3	39.9
p25	-14.9	-12.5	-20.6	-14.5	-17.1
p50	5.7	4.9	7	8.4	4.9
p75	25.9	23.1	27.9	27	25.9
E. Absolute error					
Mean	29.6	24.6	32.3	28.9	30.4
SD	25	21.8	26.1	24	25.9
p25	7.5	5.5	9.4	8.9	7
p50	24.5	19.4	25.9	24.5	24.5
p75	45.9	35.9	52.3	44.9	48.9

Notes: N=786. The rows labeled 'Understands sibling and neighborhood priorities' and 'Understands ranking priorities' report the fraction of survey-takers responding correctly to questions about the ordering of priority groups. The rows labeled 'Understands sibling and neighborhood priorities' and 'Understands ranking priorities' report the fraction of survey-takers responding correctly to questions about the ordering of priority groups. The first asked respondents whether a student with neighborhood preference would have a better chance of getting into an over-subscribed school than a student with sibling preference, or vice versa (correct answer: neighborhood preference). The second question asked whether an applicant who submitted an application which ranked school A first and school B second but did not get into A would have higher or lower chances of admission than a student who submitted an application which ranked school B first (correct answer: lower chance). Subjective beliefs, ratex beliefs, optimism, and absolute error are measured in percentage point units. High-SES is upper tercile of SES ranking.

Panels two through five of Table 5 present the distributions of subjective beliefs, rational expectations beliefs, and belief errors. Units are percentage points. The distributions of subjective and rational expectations beliefs have similar means. The cross-application average of subjective ad-

missions chances is 41%, compared to 38% for rational expectations. This implies that on average, optimism is close to zero (specifically, 3 percentage points). However, as we report in the fourth panel of Table 5, beliefs are widely distributed around this average. The standard deviation of optimism in the population is 39 percentage points. The standard deviation is larger from students from low-SES backgrounds (41 percentage points) than for students from high-SES backgrounds (33 percentage points), while the standard deviations for black and non-black students are similar (39 and 37 percentage points, respectively). The mean absolute error is 30% in the population, 32% for low-SES students, and 25% for high-SES students.

Figure 3: Distributions of Beliefs, Belief Errors, and Shift Terms



Notes: N=786. Kernel density estimates using Gaussian kernel with bandwidth 10.

Figure 3 explores the distribution of beliefs and belief errors in more detail. The upper panel pools across all submitted ranks and priority groups. In this sample, the distribution of rational expectations probabilities has larger peaks at 0 and 1 than the distribution of subjective expectations, and the distribution of errors is unimodal and centered around zero. The lower two panels disaggregate by submitted rank and priority group, respectively. The middle panel shows that respondents systematically underestimate admissions probabilities at schools listed first on the application, while over-estimating those probabilities at schools listed second. The mean of the optimism distribution is 12 percentage points higher for second-listed schools. Similarly, respondents underestimate the benefit associated with having either neighborhood or sibling priority. The mean of the optimism distribution is shifted 24 percentage points downward for students with sibling or neighborhood priority. The optimism distributions we observe are consistent with the findings that students misunderstand the role of ranking and neighborhood priority in the choice mechanism. They suggest that a realistic model of belief errors should allow for systematic variation by priority and rank as well as for scatter within these groups. We return to this point in Section 5.

Table 6 considers the determinant of optimism and absolute error in more detail. It presents results from OLS regressions of the indicated error type on indicators for SES background, race, and grade. Alternate specifications also include descriptors for an individual's interaction with the choice process, including an indicator for participation, an indicator equal to one if a respondent reports looking up application counts from previous years, an indicator equal to one if the application we are asking about is the respondent's most preferred school, and the count of information sources an applicant drew from when making a choice. Students from high-SES backgrounds have lower absolute errors but similar levels of optimism. There is little relationship between race and optimism or absolute error after conditioning on SES. Students who participate in choice are less optimistic but have similar absolute errors to students who do not, while students who looked up applicant counts are more optimistic but do not have lower absolute errors.

Notably, the relationship between preference for a school and belief optimism or error is weak, and using information from more sources is not correlated with changes in optimism or reductions in absolute error. A possible explanation for this finding is that search for accurate information about one's own admissions chances is costly and unrelated to most other elements of the search process. Students may find it relatively easy to learn about characteristics of schools, but find it harder to learn about characteristics of the assignment process. We use this fact to motivate a model of belief errors in which belief accuracy does not depend on students' preference for a school.

Table 6: Correlates of Errors

	Optimism		Absolute error	
High-SES	-0.65	-0.66	-7.66**	-7.53**
	(2.27)	(2.25)	(1.92)	(1.96)
Black	-2.05	-2.18	0.48	0.50
	(2.30)	(2.28)	(1.87)	(1.94)
Grade 9	11.35**	10.96**	-3.42+	-3.29+
	(2.32)	(2.26)	(1.87)	(1.91)
Partipates in choice		-7.11*		2.06
		(2.74)		(2.24)
Looked up application counts		4.78*		-2.88
		(2.21)		(1.97)
Preferred school		4.03		-2.15
		(3.85)		(1.77)
Sources of information		-0.26		-0.24
		(0.73)		(0.68)
Constant	-1.76	-1.43	33.75**	34.53**
	(2.12)	(2.88)	(1.89)	(2.43)
N	786	782	786	782

Notes: Significance: +0.10 * 0.05 ** 0.01. Linear regressions of error type listed in columns on control variables listed in rows. 'Looked up application counts' is dummy equal to one if respondent reported looking up application counts from prior years. 'Preferred school' is a dummy equal to one if an application is to the school listed as most-preferred. 'Sources of information' counts sources of information respondents reported consuming.

4.4 Belief errors, application strategies, and placement outcomes

We next consider the relationship between belief errors and placement outcomes. We estimate linear probability regressions of indicator variables for any placement, placed in first choice school, and placed in most-preferred school on an indicator variable equal to one if a respondent's mean absolute belief error was above the median value, and student demographics. We also present results from an OLS regression of value-added at the placed school on those same covariates. Observations are the respondent level.

Table 7 presents results. We find that belief errors are not strongly related to receiving any placement or to placing in the first-listed school. However, large belief errors reduce the probability of placement the most-preferrred program by 27 percentage points, on a base of 48%. Students from high-SES backgrounds are more likely to receive a placement and to place in their first choice school, and to place in schools with higher average value-added. However, conditional on the controls for

belief error, they are not more likely to place into their most-preferred school.

Table 7: Correlates of Placement Outcomes

	Any	First	Most preferred	Value added
$\|error\|$ >median	-0.069	-0.036	-0.273**	-0.024
	(0.052)	(0.083)	(0.082)	(0.026)
High SES	0.100*	0.132	-0.002	0.062*
	(0.049)	(0.085)	(0.084)	(0.027)
Black	0.010	-0.051	0.009	0.011
	(0.053)	(0.084)	(0.081)	(0.026)
Grade 9	-0.161**	-0.036	-0.061	-0.124**
	(0.041)	(0.090)	(0.089)	(0.030)
Constant	0.995**	0.707**	0.513**	0.019
	(0.037)	(0.100)	(0.099)	(0.035)
N	128	128	128	109

Notes: Significance: +0.10 * 0.05 ** 0.01. Linear regressions of outcome listed in columns on control variables listed in rows. See text for variable definitions. Observations at applicant level.

As a final piece of evidence to motivate our structural model, we observe that there is a relationship between applications and decisions to accept placements, which is consistent with a model in which people list schools they prefer more highly on average. Table 8 shows probit regressions of the decision to accept a placement, conditional on being placed in a school, on the rank of the school within the individual's application vector and controls. We find that people are more likely to accept placements to their first-listed school, suggesting that the first-listed school is more strongly preferred on average. In all specifications, we control for the presence of a sibling at the school, and an indicator for neighborhood priority ("neighbor"), as well as their interaction. These variables shift a household's priority at a school. In the second specification we control for the rational-expectations admissions chance. One may be concerned that first-round applications are to a different set of schools than lower-ranked applications. In the third specification, we include school fixed effects. The results are robust to including fixed effects.

Table 8: Probability of Accepting a Placement: Probits

	Accept	Accept	Accept
rank = 2	-0.470	-0.368	-0.580
	(5.29)**	(3.61)**	(5.73)**
rank = 3	-0.123	-0.017	-0.383
	(0.92)	(0.12)	(2.46)*
rank = 4	-0.156	-0.042	-0.574
	(1.10)	(0.28)	(3.51)**
neighbor	0.310	0.252	0.195
	(3.46)**	(2.69)**	(1.27)
sibling	1.004	0.949	0.873
	(6.73)**	(6.26)**	(5.48)**
both	0.768	0.695	0.477
	(3.49)**	(3.12)**	(1.78)+
admissions chance		0.003	
		(2.05)*	
Constant	0.828	0.616	
	(18.58)**	(5.49)**	
N	2,055	2,055	2,055
School FE	No	No	Yes

Notes: + $p < 0.1$; * $p < 0.05$; ** $p < 0.01$

To summarize, we have two core descriptive findings. First, we observe that belief errors are large: the average absolute error is 30 percentage points, with even larger values for students from low-SES backgrounds. We also observe that belief errors shift with submitted ranking and with priority group, but not with students' stated preference for a school. These observations inform our choice of parametric form for the $shift_{ij}(a)$ term in the next section. Second, we find suggestive evidence that belief errors matter both for the applications that students submit and for the schools in which they are placed. For example, school choice participants with belief errors above the median value are 17 percentage points less likely to place into their most-preferred school. These findings suggest that changes in the informational environment and/or choice mechanism may have important implications for welfare outcomes.

5 Model

Our model consists of three stages. First, applicants learn their preferences over schools and costs of applying to schools. Second, they then choose whether to participate in the school choice process and, if they participate, what report to submit. Third, the lottery runs and participants receive placements. If there is space, nonparticipants are assigned to their neighborhood school, otherwise they are given a place where a spot is open.

Students $i \in I$ have underlying preferences over schools $j \in J$ according to:

$$u_{ij} = \delta_j + X_{ij}\beta + \epsilon_{ij}$$

where X_{ij} includes distance to the school from home as well as interactions between student demographic characteristics and school attributes such as academic quality. The errors ϵ_i are distributed according to

$$\epsilon_i \sim MVN(0, \Sigma),$$

iid across households.

In practice, each student has a default school that he will be placed in if he does not receive a placement through the lottery. Each student therefore has an outside option u_{i0} which consists of the choice between attending the default school and leaving the district. We normalize the value of this outside option: $u_{i0} = 0$.

Once a student is placed in school j, he has the option to decline his placement. At the time of this decision, students receive a shock to preferences for j and for the outside option, giving a utility

$$U_{ij} = u_{ij} + \epsilon^e_{ij}$$

where the enrollment-time shock ϵ^e_{ij} has an extreme value distribution with scale parameter $\frac{1}{\lambda}$. The probability of accepting an offer is therefore

$$P(u_{ij} + \epsilon^e_{ij} > \epsilon^e_{i0}) = \frac{\exp(\lambda u_{ij})}{1 + \exp(\lambda u_{ij})}.$$

The expected value of school j at the time of matriculation is given by

$$v_{ij} = E(\max\{U_{ij}, U_{i0} | u_{ij}\}) = \frac{1}{\lambda} \log\left(1 + \exp(\lambda u_{ij})\right).$$

Let p_{ija} denote i's subjective estimate of the probability that he will be placed in school j if he submits report a to the mechanism. Students for whom $|a| = 0$ are those who do not participate in school choice.

To allow for nonparticipation and short application lists, we allow for a cost of receiving a placement. If i receives a placement in any inside school j, he receives a (possibly negative) payment

$$b_i \sim N(\mu_b, \sigma_b^2).$$

We interpret b_i as the cost of the actions i must take to accept or decline a placement. These include finding and getting in touch with the school placement office or the assigned school.

In this case, i's decision solves

$$\max_a \left(\sum_j p_{ija}(v_{ij} + 1(j \neq default_i) \cdot b_i) \right).$$

We allow people to have mistaken beliefs about their priority or, equivalently, about schools' cutoffs, while maintaining the structure of the mechanism. In the current draft, we use a simplified model of belief errors. Motivated by the finding that the distribution of households' beliefs about admissions chances is centered very near their rational-expectation values, we let

$$shift_{ij} = \nu_i + \eta_{ij}$$

denote $i's$ error in beliefs about his own admissions ranking. We assume $\nu_i \sim N(0, \sigma_\nu^2)$, iid across individuals, and $\eta_{ij} \sim N(0, \sigma_\eta^2)$, iid across matches. The term ν_i allows for correlation in errors within i. For example, i may be pessimistic in general. Applicant i believes that he will qualify for a place in j under report a if

$$rsp_{ij}(a) + shift_{ij} + z_{ij} \leq \pi_j,$$

where z_ij has a standard uniform distribution. Note that i's beliefs coincide with rational expectations exactly when $shift_{ij} = 0$.

Future drafts will use a richer belief error model in which

$$shift_{ijr} = \eta_i^0 + \eta_i^r(r - \overline{r}_j) + \eta_i^{priority}\left(priority_{ij} - \overline{priority}_j\right) + \eta_{ij}$$

Here, r is the rank of j on application a for student i, and $\overline{r}_j$ is the rank of the marginal round

for j. Similarly, $priority_{ij}$ is $i's$ priority at j and $\overline{priority}_j$ is the marginal priority group at j. This expanded functional form nests several relevant cases. For example, $\eta_i^r = 0$ means students understand how priority groups affect choices, while $\eta_i^r = -1$ if students do not believe assignment probability depends on rank, as in a DA mechanism. $\eta_i^{priority} = -4$ corresponds to the case where students do not beliefs about admissions probabilities with changes in their priority group, while η_i^0 captures individual-specific optimism or pessimism and η_{ij}^0: idiosyncratic person-school error.

Following our descriptive analysis, we divide the sample into two groups on the basis of our SES measure: "high-SES" (top tercile), and "low-SES" (the remaining two thirds). We allow for separate parameters σ_η and σ_ν for the two groups.

6 Estimation

We use a Bayesian Markov-Chain Monte Carlo (MCMC) procedure to estimate the model and sample from the posterior distribution of counterfactual outcomes. Similar methods have been used successfully in the marketing and industrial organization literatures to consumers' demand for goods (McCulloch and Rossi, 1994) and have been applied successfully to centralized school choice (Agarwal and Somaini, 2014). Our strategy extends these methods to make use of surveyed beliefs as well as data on the decision to accept or decline a placement.

We use a two-step procedure. In the first step, we estimate the distribution of market-clearing cutoffs at each school, which determine the rational-expectations chances of admission at each school conditional on a priority vector and a report. We then use data augmentation to pick utility vectors and beliefs for each individual consistent with their choices, introduce prior distributions for the model parameters, and use MCMC in order to sample from the posterior distribution of parameters conditional on the data. In order to obtain distributions of outcomes under counterfactuals, we simulate alternative policies at many points drawn from this posterior distribution. This approach allows us to model belief errors even for non-surveyed individuals. Intuitively, the survey plays the critical role in pinning down the distribution of the belief error parameters, but belief errors help rationalize observed choices for both surveyed and non-surveyed students.

6.1 Recovering admissions chances

Our approach is similar to Agarwal and Somaini (2014). Within each market (e.g. eighth-graders) we draw a large number (e.g. $N = 100$) of resampled markets by sampling from the population i.i.d. with replacement. Each resampled market is therefore a list of individuals with a participation

decision, a report if they participated in the lottery, and a priority at each school. In each resampled market, we solve for market-clearing cutoffs by running the New Haven algorithm.

The distribution of cutoffs feeds into our results in three places. First, the cutoffs $\left\{\pi_j^{(k)}\right\}_{k=1,\ldots,N}$ allow us to calculate rational-expectations admissions chances, which serve as a benchmark in our descriptive analysis. In particular, student i's chance of being placed in school j under report a is given by

$$\begin{aligned} P_{ij}(a) \equiv Pr(placement_i(a) = j) &= Pr\left(score_{ij} < \pi_j, score_{ij'} > \pi_{j'}\ \forall j' \text{ s.t. } j' \succ_a j\right) \\ &\approx \frac{1}{N}\sum_k \int 1\left(score_{ij} < \pi_j^{(k)}\right) 1\left(score_{ij'} > \pi_{j'}^{(k)} \forall j' \text{ s.t. } j' \succ_a j\right) dF(score_i | rsp_i). \end{aligned}$$

Second, the probability that i is placed in j under i's observed report a_i is his *propensity* to be placed in j, which is needed for estimating the effects on test scores of attending j. Finally, the true cutoffs are inputs into our model of subjective beliefs about admissions chances.

6.2 Recovering preference and belief parameters

Before we describe the MCMC procedure in detail, we discuss the normalizations that we make, and the restrictions implied by households' optimal application decisions, accept/decline decisions, and reported first and second choices.

6.2.1 Normalization

We have already imposed the location normalization $u_{i0} = 0$. Importantly, we include in X_{ij} an indicator for i's neighborhood school, and the distance d_{i0} between i's home and zoned school.

The following scale normalization is useful. Define $\tilde{u} = \lambda u$, $\tilde{v} = \lambda v$, and $\tilde{\mu} = \lambda \mu$.

Let

$$\tilde{u} = X\tilde{\beta} + \tilde{\epsilon}^a,$$

and fix

$$\tilde{\beta}_{dist} = -1.$$

By construction $\tilde{\epsilon}$ has a standard Gumbel distribution. The probability of accepting an offer, conditional on $\tilde{u}$, is then

$$s_{ij} = \frac{\exp(\lambda u_{ij})}{1 + \exp(\lambda u_{ij})} = \frac{\exp(\tilde{u}_{ij})}{1 + \exp(\tilde{u}_{ij})}.$$

The expected value of an offer gives

$$\tilde{v}_{ij} = \log(1 + \exp(\tilde{u}_{ij})).$$

Because $\tilde{\beta}_{dist} = -1$, welfare in units of miles traveled is given by

$$\tilde{v}_{ij} = \tilde{v}_{ij}/|\tilde{\beta}_{dist}| = v_{ij}/\beta_{dist}.$$

6.2.2 Optimality of applications

Let $\tilde{v}_i = (\tilde{v}_{i1}, \ldots, \tilde{v}_{iJ}, \tilde{b}_i)$ denote the vector of inclusive values of admission to each of the J schools, and let $p_i(a)$ denote the vector of i's subjective beliefs about admissions chances under report a. Agarwal and Somaini (2014) observe that a report a is optimal for agent i if and only if $v_i \cdot p_i(a) \geq v_i \cdot p_i(a')$ for all reports a'. Hence, given the matrix $\Gamma_i = (p_i(a) - p_i(a_1), \ldots, p_i(a) - p_i(a_N))$, a report is optimal if and only if $\Gamma_i' * (v_i + b_i) \geq 0$. Equivalently, a report is optimal if and only if

$$\Gamma_i' * \left(\tilde{v}_i + \tilde{b}_i\right) \geq 0.$$

6.2.3 Accept/decline decision and reported preferences

In the survey we elicit households' first and second choices if parents could choose any school, unconstrained by admissions chances. We allow for measurement error in elicited preferences: If i says that j_1 is the household's first choice, then

$$u_{ij_1} + \epsilon_{ij_1}^{survey} > u_{ij} + \epsilon_{ij}^{survey} \ \forall j$$

Similarly, if j_2 is the household's second choice, then

$$u_{ij_2} + \epsilon_{ij_2}^{survey} > u_{ij} + \epsilon_{ij}^{survey} \ \forall j \neq j_1.$$

Scaling by λ without loss, we assume the measurement error is drawn iid from a normal distribution:

$$\tilde{\epsilon}_{ij} = \lambda \epsilon_{ij}^{survey} \sim N(0, \tilde{\sigma}^2_{survey}), \ iid.$$

We also make use of the decision to accept or decline a placement. If i accepts a placement in j, then we require $u_{ij} + \epsilon_{ij}^e > \epsilon_{i0}^e$. If i receives and declines a placement in j, we require $u_{ij} + \epsilon_{i0}^e < \epsilon_{ij}^e$.

Define

$$\tilde{\epsilon}_i^e = \lambda * \left(\epsilon_{ij}^e - \epsilon_{i0}^e\right).$$

By construction $\tilde{\epsilon}_i^e$ has a standard logistic distribution.

We can write these constraints in matrix form as

$$\Gamma'_{i,(shock)} * \begin{pmatrix} \tilde{u}_i \\ \tilde{\epsilon}_i^{survey} \\ \tilde{\epsilon}_i^e \end{pmatrix} \geq 0.$$

If i reported first and second choices, then the first column of $\Gamma_{i,(shock)}$ contains 1's in the j_1th and $(J + j_1)$th places, and -1 in the j_2th and $(j + j_2)$th places.[7] The next $J - 1$ columns similarly require

$$u_{i,j_2} + \epsilon_{i,j_2}^{survey} > u_{i,j} + \epsilon_{i,j}^{survey} for j \neq j_1, j_2.$$

If i was placed in school j, then the final column of $\Gamma_{i,(shock)}$ contains 1 in the jth place and -1 in the final place.

6.2.4 Starting values

We first construct feasible belief shifts $shift_{ij}$ for all i and j. Where the survey provides no constraints, we start at $shift_{ij} = 0$, i.e. at the rational-expectations value. We pick points interior to the relevant intervals when households report beliefs.

Next, given the feasible beliefs, we use linear programming techniques to construct strictly feasible utilities $\tilde{u}_i \in \mathbb{R}^J$ and placement payoff terms $b_i \in \mathbb{R}$. A utility vector $\tilde{u}_i$ and benefit $\tilde{b}_i$ are (strictly) feasible if the observed report a_i is optimal conditional on the beliefs p_i, that is if $\Gamma_i(p) * \left(\tilde{v}_i + \tilde{b}_i\right)' > 0$. We allow the set of possible reports to include an empty list, which we interpret as nonparticipation.

Finally, we use linear programming again to pick strictly feasible enrollment-time shocks $\tilde{\epsilon}_i^e$ and measurement errors $\tilde{\epsilon}_i^{survey}$.

6.2.5 Prior distributions

We begin with prior distributions over the preference parameters[8] and belief parameters. We place priors directly on $\tilde{\beta}$, $\tilde{\Sigma}$, $\tilde{\mu}_b = \lambda\mu_b$, $\tilde{\sigma}_b = \lambda\sigma_b$, and $\tilde{\sigma}_{survey}$ as well as on the belief parameters

[7] If i reported a first but not a second choice, we similarly construct $\Gamma_{i,(shock)}$ using the resulting inequalities.

[8] In order to capture the mean utility terms δ we include dummy variables for each school in X_i.

$\sigma_\eta^{low}, \sigma_\nu^{low}, \sigma_\eta^{high}, \sigma_\nu^{high}$. In order to minimize the priors' influence on our estimates, we choose the following diffuse (flat) priors:

$$\tilde{\beta} \sim N(0, 100 * I)$$
$$\tilde{\Sigma} \sim IW(J, I)$$
$$\sigma_\nu, \sigma_\eta, \tilde{\sigma}_{survey}, \tilde{\sigma}_b \sim InverseGamma(1,1)\ iid$$

We assume that the priors are independent.

6.2.6 MCMC iteration

Next, we iterate through the following steps, which consist of sampling from the conditional posterior distributions of utilities, beliefs, application costs, achievement types, and model parameters:

1. Draw mean-utility parameters $\beta^{(s+1)}$ and mean benefit $\mu_b^{(s+1)}$ from the distribution of $\tilde{\beta}|\tilde{u}^{(s)}, \tilde{\Sigma}^{(s)}$ and $\tilde{\mu}_b|\tilde{b}^{(s)}, \tilde{\sigma}_b^{(s)}$
2. Draw variance of benefit term $(\tilde{\sigma}_b^2)^{(s+1)}$ from the distribution of $\sigma_c^2|\mu_b^{(s+1)}, b^{(s)}$.
3. Draw covariance matrix $\Sigma^{(s+1)}$ from the distribution of $\Sigma|\beta^{(s+1)}, u^{(s)}, \alpha^{(s)}$.
4. Draw belief variance parameters $\sigma_\nu^{(s+1)}$, $\sigma_\eta^{(s+1)}$ from their posterior distribution given $shift_{ij}$ for $i \in I$, $j \in J$.
5. For each individual in the dataset:
   (a) Draw utility $u_i^{(s+1)}$ from the posterior distribution of $\tilde{u}_i$ given β, Σ, i's decision to accept or decline his placement (if offered one), and constraints implied by the optimality of i's report.
   (b) Draw $\tilde{b}_i^{(s+1)}$ from the posterior distribution of $\tilde{b}_i$ given $\tilde{v}_i(\tilde{u}_i^{(s+1)})$ and constraints implied by the optimality of i's report.
   (c) Draw shock realizations $\tilde{\epsilon}_i^{survey}$ and $\tilde{\epsilon}_i^e$ from their posterior distributions given $\tilde{u}_i$ and the constraints $\Gamma'_{i,(shock)} * \begin{pmatrix} u_i \\ \epsilon_i^{survey} \\ \epsilon_i^e \end{pmatrix} \geq 0$.

(d) Draw errors in beliefs from the posterior distribution of beliefs ν_i, η_i conditional on γ, $\tilde{v}_i$, and $\tilde{b}_i$.

Utilities: In order to update utilities, for each individual we iterate through the various schools, updating the terms $\tilde{u}_{ij}$ sequentially. Because $\tilde{u}_i$ is jointly normal, the distribution of $u_{ij}|u_{i,-j}, \beta, \Sigma$ is normal with known mean and variance.

The restriction $\Gamma_i' * (v_i + b_i) \geq 0$ implies that $\tilde{v}_{ij}$ must belong to a (known) interval whose endpoints depend on $\tilde{v}_{i,-j}$ and $\tilde{b}_i$.[9] Recall that $\tilde{v}_{ij} = \log(1+\exp(\tilde{u}_{ij}))$ is a monotone transformation of $\tilde{u}_{ij}$. Therefore, if we use only the information from the optimality of the report and the current values of other variables and parameters, updating u_{ij} consists of drawing from a truncated normal distribution.

Beliefs: In order to update beliefs subject to the constraints provided by the survey, we take a standard Metropolis-Hastings step using a symmetric normal proposal density. For each individual, we draw a vector $\Delta(shift_i) \sim N(0, \sigma_{proposal} * I)$, and construct a new proposal $shift_i' = shift_i + \Delta(shift_i)$. We update to the proposed draw with the appropriate Metropolis-Hastings acceptance probability. We reject the proposal with probability 1 if it violates the constraints imposed by the survey or causes the observed report to become non-optimal.[10] We tune the variance of the proposal density so that roughly one third of draws are accepted. We similarly draw $\nu_i' = \nu_i + \Delta(\nu_i)$ where $\Delta\nu_i \sim N(0, \sigma_{propose,\nu})$ and accept or reject according to the the appropriate Metropolis-Hastings acceptance probability.

Implementation: We use a chain of 24000 iterations. We discard the first 12000 draws in order to allow for burn-in.

7 Results

7.1 Estimation results

We report parameter estimates and credible intervals for grade 8 students in Table 9. Parameter estimates for kindergarten students are available upon request. We show .025, .5, and .975 quantiles

[9] Similarly, b_i must belong to an interval with known endpoints that depend on $\tilde{v}_i$.

[10] Individuals' belief components ν_i and η_{ij} are distributed according to truncated normal distributions. If the report is optimal and consistent with the survey, the densities of η and ν are proportional to normal densities.

of the posterior distribution.[11] While we recover a full distribution, the median may be taken as a point estimate. To interpret the coefficients recall that the coefficient on miles traveled is equal to -1.

Table 9: Parameter Estimates

Quantile	.025	.5	.975
1(default$_i$ = *cross*)	-0.609	0.17	1.257
1(low SES)	-0.807	-0.327	0.351
1(low SES) x 1($grade_j \in A, B, C$)	-0.731	-0.096	0.508
distToZoned	0.267	0.4	0.519
δ Achievement First Amistad HS (1)	-0.697	0.701	1.998
δ Common Ground Charter (2)	-5.218	-1.73	1.38
δ Coop. Arts and Humanities (3)	1.125	2.334	3.424
δ Engineering & Science Univ. HS (4)	-3.47	1.088	2.37
δ High School in the Community (5)	-1.145	0.402	1.565
δ Hill Regional Career (6)	1.054	2.265	3.43
δ Hillhouse (7)	-5.092	-3.342	-0.231
δ Hyde School (8)	4.22	6.703	8.968
δ Metropolitan Business Academy (9)	-0.35	1.139	2.361
δ New Haven Academy (10)	-2.054	-0.156	1.01
δ Riverside Education Academy (11)	-5.213	-2.388	-0.867
δ Wilbur L. Cross High School (12)	-8.892	-2.893	2.165
μ_b	0.677	1.076	1.357
σ_b	0.633	1.01	1.291
σ_{survey}	2.48	3.179	4.279
σ_ν(low SES)	0.003	0.003	0.004
σ_ν(high SES)	0.002	0.003	0.004
σ_η(low SES)	1.375	1.582	1.739
σ_η(high SES)	1.175	1.264	1.345

Notes: The coefficients on Wilbur Cross and Hillhouse apply only to students who are not zoned into these schools. The coefficient on the own zoned school is set equal to zero.

Table A4 in the appendix shows 90% credible intervals for each element of the utility shock covariance matrix Σ.

[11] In the appendix we provide trace plots for each variable.

We first consider the parameters governing the deviation of subjective beliefs from rational expectations values. Figure A5 shows a trace plot of belief parameters along the chain. Our estimates of σ_η converge to values far from zero. To interpret the magnitude, note that that there is an interval of length 1 for each report-specific priority type such that if the cutoff lies in this interval, the type is rationed. We find that the student-level components of errors in beliefs are small, but the idiosyncratic components have standard deviations of roughly 1.5 and 1.2 for low- and high-SES households respectively, indicating that households are likely to be mistaken about the round in which the capacity constraint binds, with low-SES households making larger errors. The model estimates are qualitatively and quantitatively similar to the descriptive estimates of the distribution of the shift term reported in Section 4.

Next, we examine the parameters that relate to all non-default schools. Recall that the mean utility of students' own zoned school is normalized to zero. To allow for differences across zones, we let the taste for all inside schools differ: the coefficient on $1(default_i = cross)$ allows students in the Wilbur Cross zone to have a higher taste for all non-neighborhood schools than students in the Hillhouse zone. We find a point estimate of .17 miles traveled, but the credible interval covers zero. We find also that distance from the zoned school increases households' taste for all other schools relative to the outside option, which includes the zoned school.

We allow for differences in application patterns across SES groups to reflect differences in tastes as well as beliefs. In particular, we allow low-SES households to have lower taste for non-default schools relative to the outside option. We find that the point estimate is negative but the interval covers zero. In addition, we allow low-SES households to have a different taste for low-performing schools. To construct this measure, we use the "school report cards" provided by the Connecticut Coalition for Achievement Now.[12] Five of the twelve high schools receive grades of "B" or "C", while the remainder receive "D" or "F" grades. Our estimates do not show a difference by SES in taste for lower-graded schools.

We find that utilities differ systematically across schools. Riverside, which is an alternative school, has mean utility of 2.4 additional miles traveled at the point estimate, while Coop Arts and Hill Regional Career have high mean utilities equivalent to 2.3 *fewer* miles traveled. Hyde is a geographic outlier in that it is located in North Haven, 10 to 15 miles from most households; the high mean utility of 6.7 fewer miles traveled does not imply that it is the most preferred. We find that, on average, receiving a placement is costly, with $\mu_b \in (.7, 1.4)$, but the standard deviation is equivalent to approximately one mile as well. Measurement error in reported preferences has a

[12] http://http://reportcards.conncan.org/

standard deviation of roughly 3 miles traveled, suggesting that elicited first- and second-choice data is informative but not perfectly so.

7.2 Welfare analysis and counterfactual simulations

We now turn to an analysis of households' welfare and test scores under observed and counterfactual policies. Our procedure estimates the joint distribution of parameters and utilities. Using this distribution, we are able to compute each household's expected welfare according to its utility and the true rational-expectations admissions chances under the application it submitted. We compute average utility at every 10th iteration along the Markov chain after the burn-in period, and divide by $|\tilde{\beta}_{dist}|$ to measure welfare in miles traveled.

In the first counterfactual, we simulate outcomes under deferred acceptance. To evaluate deferred acceptance, we maintain the limit of at most four schools per application. Under the resulting "truncated deferred acceptance" procedure it need not be optimal to report truthfully (see Fack et al. (2015)). Accordingly, in the second counterfactual we simulate equilibrium play under this mechanism, together with rational expectations. Finally, as imposing rational expectations may overstate the gains from deferred acceptance, we simulate play under the same truncated deferred acceptance procedure with truthful play, in which households list their most-preferred schools in order, up to a maximum of 4, but stopping if $v_{ij} + b_i < 0$ for the best remaining j, regardless of beliefs.

In the second counterfactual, we scale the variance of the shift error terms by values ranging from zero to one and solve for an equilibrium of the New Haven mechanism. A scaling value of zero corresponds to a best-case informational intervention, with $shift_{ij} = 0$ for all i and j. A scaling value of one corresponds to baseline case. An alternate interpretation of the best-case intervention counterfactual is as the result of providing a strategic and informed 'proxy' player with each applicant's cardinal utilities and allowing the proxy player to submit the application list (Budish and Cantillon, 2012).

There may potentially be multiple equilibria under rational expectations and under "sophisticated" truncated deferred acceptance. We select an equilibrium as follows. We start with the distribution of cutoffs π^0 that we recovered from the data in step 1. We then compute optimal applications for each household. Given the new applications and our resampled draws, we compute a new distribution of cutoffs π'. We obtain new cutoffs $\pi^1 = (1-\alpha)\pi^0 + \alpha\pi'$ for $\alpha \in (0,1)$ pointwise in each resampled market, and compute optimal applications given π^1. We iterate this procedure until convergence.

7.2.1 Counterfactual welfare distributions

Table 10 displays the posterior distribution of mean welfare in the market, as measured in miles traveled. In the first column, labeled "benchmark", we display quantiles of this distribution under the New Haven mechanism. The second column, "RatEx", shows quantiles of the posterior distribution under optimal reports with rational-expectations beliefs in the New Haven mechanism. The third and fourth columns show the distribution of mean welfare under deferred acceptance. The final three columns show the distribution of welfare differences between rational expectations and the benchmark, and between deferred acceptance ("rational" and "naive") and the benchmark, respectively.

Table 10: Distance-Metric Welfare: Benchmark and Counterfactuals

quantile	benchmark	RatEx	DA	Naive DA	RatEx - NH	DA - NH	NDA - NH
A. Grade 9							
0.05	1.777	1.965	1.881	1.851	0.166	0.065	0.047
0.25	2.032	2.213	2.117	2.09	0.174	0.081	0.053
0.5	2.129	2.318	2.234	2.208	0.182	0.093	0.068
0.75	2.306	2.487	2.412	2.385	0.191	0.106	0.079
0.95	2.429	2.654	2.541	2.503	0.211	0.117	0.083
B. Grade K							
0.05	10.654	10.803	10.71	10.219	0.101	0.018	-0.522
0.25	12.226	12.437	12.365	11.825	0.118	0.036	-0.402
0.5	15.073	15.182	15.125	14.771	0.137	0.056	-0.327
0.75	20.249	20.411	20.316	19.929	0.155	0.072	-0.286
0.95	40.875	41.012	40.929	40.678	0.208	0.122	-0.186

Notes: This table displays quantiles of the posterior distribution of mean welfare. Welfare is measured using miles traveled as the numeraire good.

For high school students, aggregate welfare improves under both counterfactual policies. Taking the median as a point estimate, the average household would be made better off by the equivalent of .18 fewer miles traveled under rational expectations, and by .07 to .09 fewer miles traveled under deferred acceptance. 95% posterior probability intervals for these differences do not cover zero. For Kindergarten students, the story is somewhat more nuanced. Aggregate welfare improves in with the best-case informational intervention and with sophisticated truncated deferred acceptance.

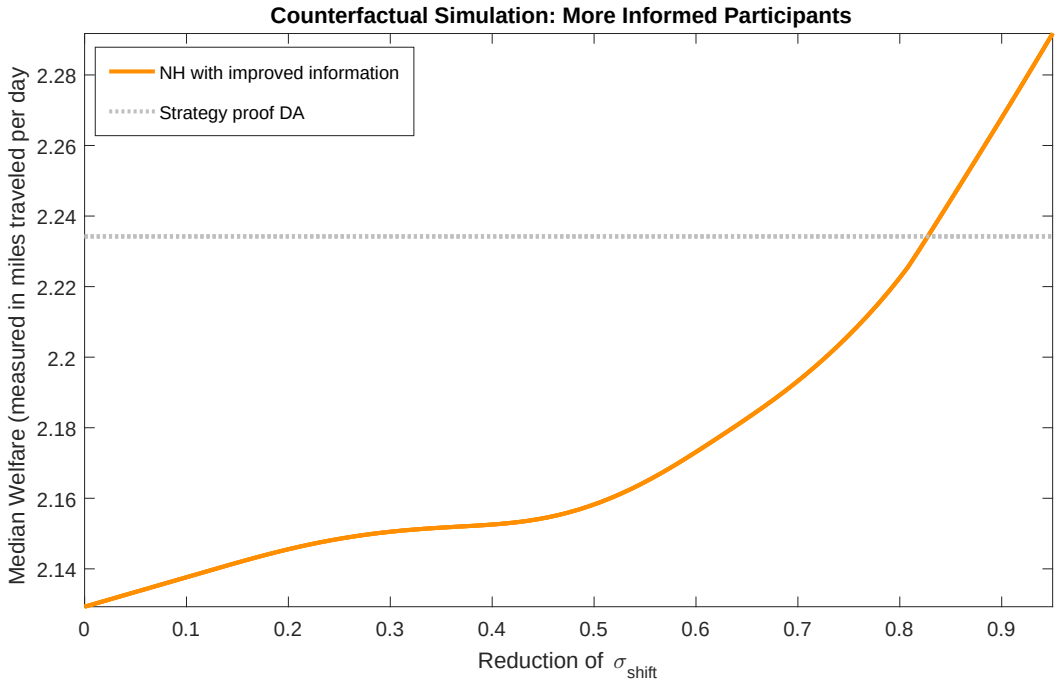
The median gain is 0.137 miles in the former case and 0.056 miles in the latter, with neither 95% posterior probability interval spanning zero. However, the naive DA assignment process produces large welfare losses. Intuitively, this is because there are more schools overall and more schools with low out-of-neighborhood admissions probabilities in the Kindergarten market. Students who list only their four most-preferred schools are likely to receive their default placement. We interpret this finding as evidence that the length of the lists applicants are permitted to submit is potentially important. In what follows, we focus on the truncated deferred acceptance counterfactual, with the idea can reduce the gap between truncated and naive deferred acceptance by expanding the maximum number of preferences students can list on the application.[13]

Our finding that the strategic New Haven mechanism produces higher aggregate welfare under rational expectations than would deferred acceptance is consistent with results from (Agarwal and Somaini, 2014; Calsamiglia et al., 2014). For example Agarwal and Somaini (2014) estimate a welfare loss of 0.07 additional miles traveled when switching from the Cambridge mechanism under rational expectations to deferred acceptance. For us, the equivalent estimates are 0.09 miles (high school) and 0.08 miles (Kindergarten). However, given the understanding and beliefs students actually have, the welfare comparison is reversed, with deferred acceptance outperforming the strategic New Haven mechanism in terms of aggregate welfare.

In practice, a best-case informational intervention that induces rational expectations play is likely not achievable. To better understand how informational interventions that fall short of this standard affect welfare, we scale the variance of the shift parameters by values ranging from zero to one and simulate counterfactual welfare distribution in each case. Figure 4 presents results from this exercise for ninth graders. (Future drafts will include results for Kindergarten as well.) The horizontal axis represents the fraction reduction in the variance term, and the dashed line represents the mean of the welfare distribution from the deferred acceptance procedure. We find that for mean welfare under the New Haven mechanism to break even with the deferred acceptance level requires a more than 80% reduction in the variance of belief errors. Given the school district's extensive outreach efforts at baseline, it is unclear what kind of intervention with such an effect would look like.

[13] NHPS has in fact expanded the number of schools applicants are allowed to rank to 5 for the 2016-2017 cycle. Future drafts will conduct counterfactual analyses of changes in the number of schools applicants are allowed to list.

Figure 4: Mean welfare by reduction in variance of shift term

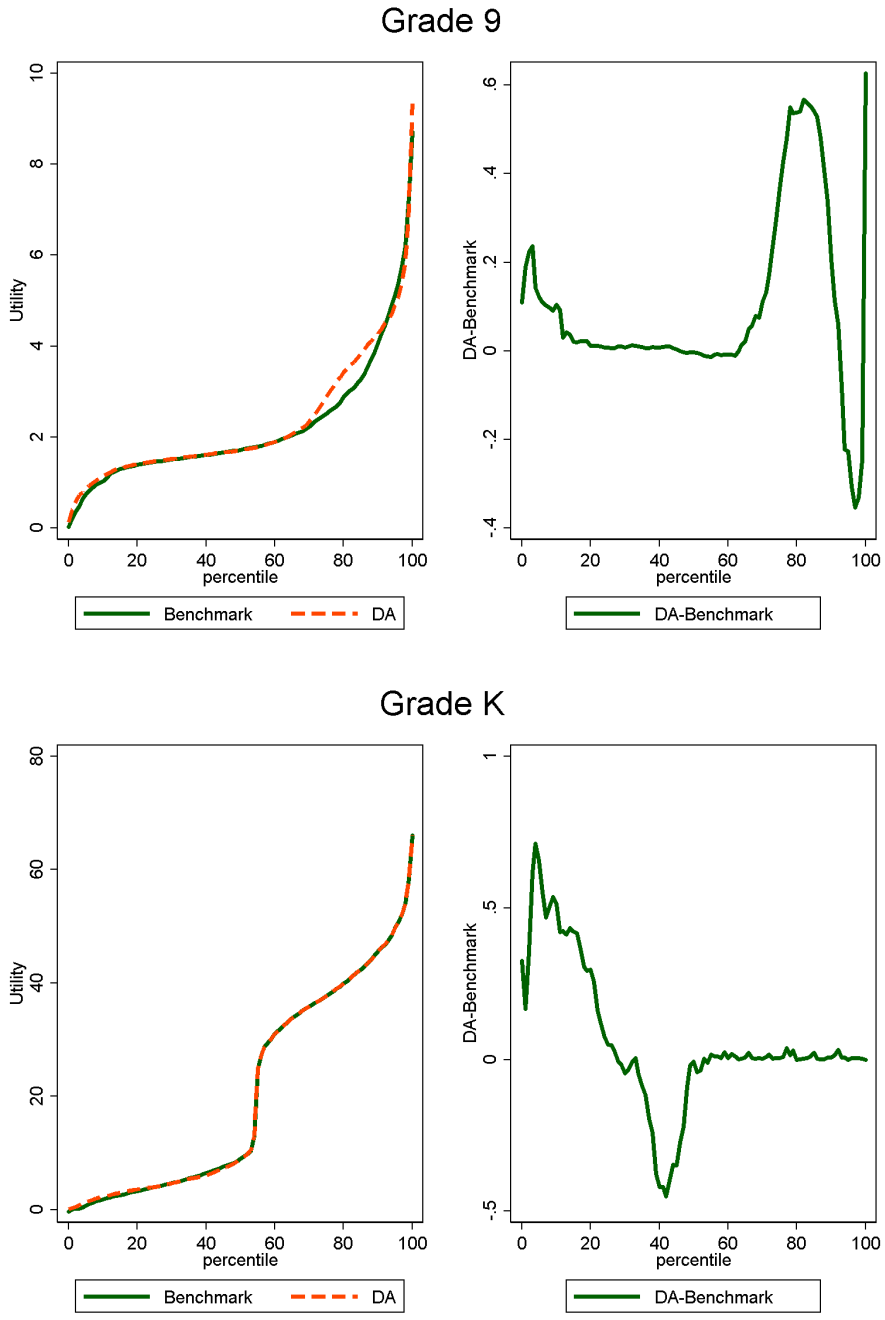


Horizontal axis: fraction reduction in variance of shift term σ_{shift}.

We next consider the effects of the switch to a deferred acceptance mechanism on the distribution of welfare. For each household, we compute household-level mean welfare by averaging the household's welfare across MCMC iterations. The increases in mean welfare we observe when switching from the benchmark New Haven mechanism to deferred acceptance are driven in large part increases in the lower quantiles of the welfare distribution. Figure 5 reports mean welfare for households in each quantile of the welfare distribution. For both Kindergarten and grade 9, we observe a shift upward in roughly the bottom quarter of the welfare distribution under a switch to deferred acceptance. In the grade 9 market there are also welfare gains in the upper part of the distribution, while in the Kindergarten market percentiles above the median are nearly unchanged. One of the arguments in favor of deferred acceptance mechanisms in the context of school choice is that even if they do not raise welfare on average, they help students avoid very bad outcomes. Our findings are consistent with this idea.

Finally, we discuss the distribution of welfare by SES background. We split the population into two bins based on our SES measure: high-SES households (those in the top third of the distribution), and the rest. Qualititatively, changes in mean welfare from switching to rational expectations play or to a deferred acceptance mechanism are similar across SES groups and grades. We observe increases in welfare in each case. However, in grade 9, these increases are larger for low-SES students, while for kindergarten, increases are larger for high-SES students.

Figure 5: Percentiles of the welfare distribution



Left panel: Welfare by percentile of welfare distribution under benchmark and DA policies. Right panel: difference between

Table 11: Mean Distance-Metric Welfare: Low-SES

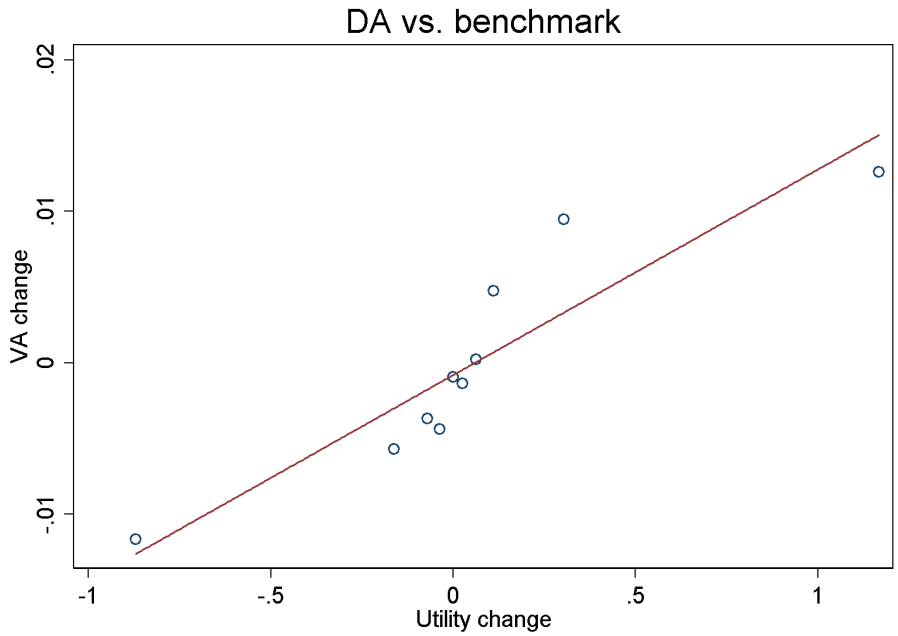
quantile	benchmark	RatEx	DA	Naive DA	RatEx - NH	DA - NH	NDA - NH
A. Low SES							
Grade 9							
0.05	1.817	2.018	1.933	1.897	0.169	0.07	0.048
0.25	2.03	2.231	2.122	2.1	0.187	0.087	0.064
0.5	2.142	2.352	2.259	2.216	0.196	0.108	0.076
0.75	2.297	2.5	2.417	2.388	0.212	0.122	0.086
0.95	2.416	2.646	2.542	2.505	0.231	0.136	0.108
Grade K							
0.05	11.055	11.196	11.094	10.598	0.085	-0.006	-0.604
0.25	12.523	12.794	12.715	12.099	0.111	0.016	-0.458
0.5	15.299	15.407	15.342	14.945	0.128	0.043	-0.391
0.75	21.256	21.412	21.306	20.904	0.155	0.061	-0.334
0.95	42.4	42.53	42.439	42.177	0.181	0.082	-0.215
B. High SES							
Grade 9							
0.05	1.692	1.857	1.779	1.76	0.123	0.038	0.016
0.25	2.002	2.152	2.064	2.044	0.14	0.054	0.037
0.5	2.103	2.261	2.174	2.157	0.158	0.07	0.047
0.75	2.323	2.46	2.385	2.353	0.168	0.081	0.06
0.95	2.453	2.638	2.544	2.509	0.197	0.101	0.089
Grade K							
0.05	9.776	9.942	9.868	9.42	0.112	0.05	-0.41
0.25	11.485	11.723	11.665	11.075	0.136	0.068	-0.291
0.5	14.536	14.687	14.596	14.344	0.152	0.092	-0.248
0.75	19.117	19.266	19.21	18.826	0.166	0.099	-0.192
0.95	37.824	37.976	37.909	37.682	0.238	0.179	-0.128

Notes: This table displays quantiles of the posterior distribution of mean welfare for low-SES and high-SES households. Low-SES households are the those in the the bottom 2/3 of the distribution of SES, high-SES are the top 1/3. Welfare is measured using miles traveled as the numeraire good.

7.2.2 Counterfactual test score effects

We now turn to the effects of policy changes on the test score value added of the schools to which students are assigned. As with welfare, we compute the average value added of the assigned school for each household by averaging across MCMC iterations. Unlike our utility model, our measures of test score value added do not allow for heterogeneous benefits based on student-school match. In this context, there are no positive-sum trades of school assignments between students, and the only way a change in assignment mechanism can generate increases in aggregate test score production is by reducing congestion. We therefore focus our analysis on a) the relationship between test score value added and utility outcomes, and b) the effects of changes in mechanism on test score inequality.

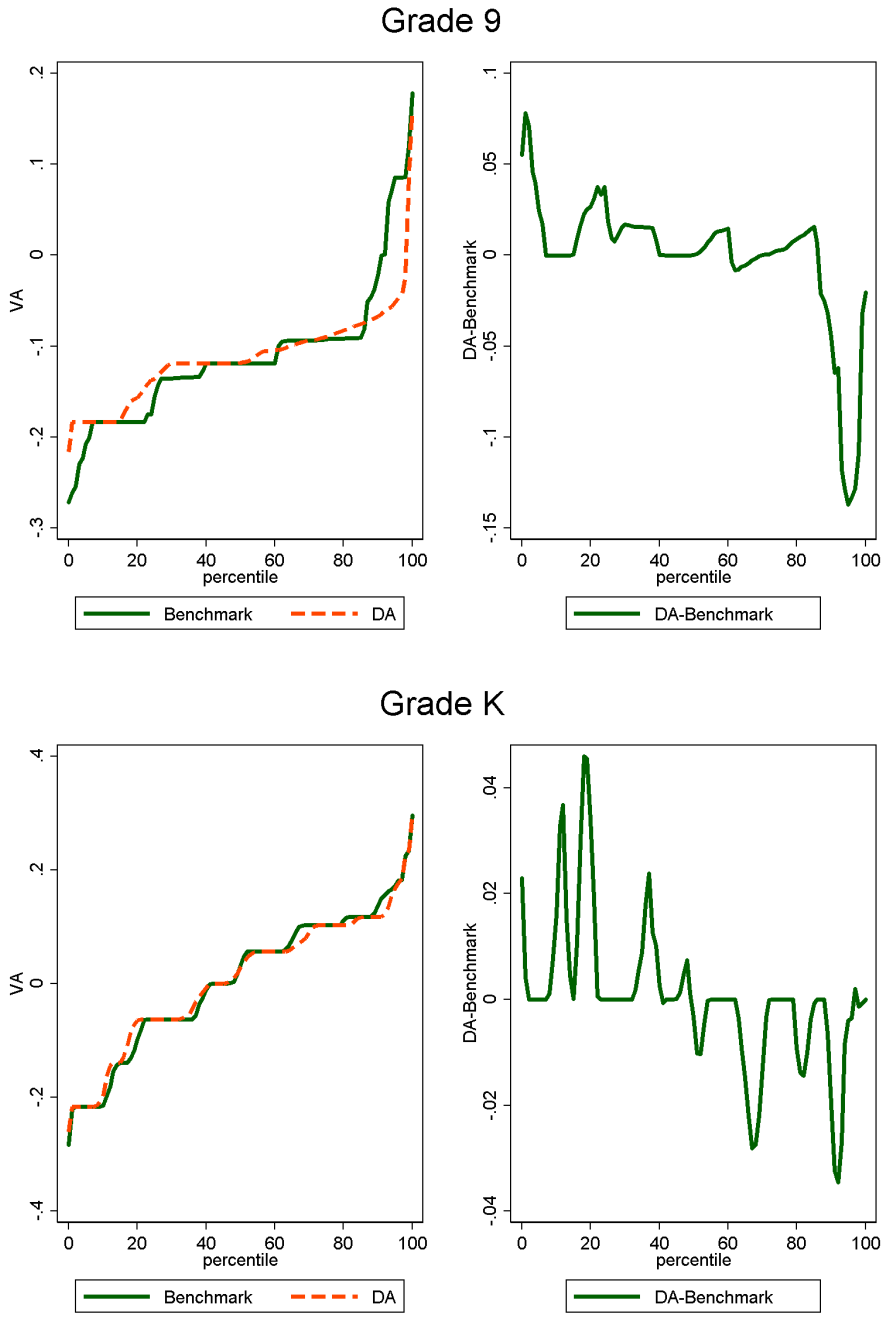
Figure 6: Percentiles of the value added distribution



Change in test score value added by change in welfare from switch to DA from benchmark. Points are deciles of the welfare change distribution. Sample: Kindergarten and Grade 9. Figure residualizes on grade and neighborhood fixed effects before plotting

We have three main findings. First, gains in welfare from a switch to the deferred acceptance mechanism are closely associated with gains in test score value added. Figure 6 shows the mean change in test score value added at each decile of the change in utility associated with the switch to deferred acceptance. The graph pools across grades. Value added range rises steadily with change in welfare. The magnitude of this increase is fairly small: a move from the bottom to the top decile of the utility change distribution is associated with about a 0.03 SD increase in test score value added. Second, the switch to deferred acceptance compresses the distribution of expected test score value added at the time of application. Figure 7 shows mean test score value added at each quantile of the value added distribution in the benchmark case and under deferred acceptance. Quantiles below the median are higher under DA, while quantiles above the median are generally lower. This is consistent with the idea that under deferred acceptance fewer households submit applications where the probability of admission to a high quality school is close to zero. Third, changing the choice mechanism produces little if any redistribution of test score value added from high SES to low SES households. Table 12 displays means and 95% credible intervals for value added gap between high SES and low-SES students under different counterfactual assignment mechanisms. Credible intervals for changes from the benchmark are very narrow and span zero in every case.

Figure 7: Percentiles of the value added distribution



Left panel: Value added by percentile under benchmark and DA policies. Right panel: difference between DA and value added.

Table 12: Value-Added: High-SES - Low-SES

quantile	benchmark	RatEx	DA	Naive DA	RatEx - NH	DA - NH	NDA - NH
A. Grade 9							
0.025	0.022	0.021	0.022	0.021	-0.002	-0.0	-0.002
0.5	0.022	0.022	0.024	0.024	0.0	0.002	0.001
0.975	0.022	0.025	0.026	0.025	0.003	0.004	0.003
B. Kindergarten							
0.025	0.087	0.082	0.082	0.082	-0.005	-0.005	-0.005
0.5	0.087	0.085	0.085	0.084	-0.002	-0.002	-0.003
0.975	0.087	0.088	0.088	0.088	0.001	0.0	0.001

Notes: This table displays quantiles of value-added "gap" between high- and low-SES households. Low-SES households are the those in the the bottom 2/3 of the distribution of SES.

8 Conclusions

This paper studies the performance of a centralized school choice mechanism that rewards strategic behavior when participants have heterogeneous beliefs about how their reports to the mechanism map to placement probabilities. To do so, we conduct a household survey asking actual and potential choice choice participants about their preferences and beliefs, and link our survey data to administrative records of the school choice process. We use our linked data to describe heterogeneity in beliefs and to estimate a model of school choice that allows for belief heterogeneity. Our survey data help us overcome challenges associated with separately identifying beliefs and preferences, and allow us to analyze the effects of counterfactual policy changes without making strong assumptions on applicants' equilibrium play.

Our descriptive findings show that school choice participants make large errors about the probabilities of admission associated with actual and hypothetical application portfolios, and that participants who make large errors are less likely to be placed in their most-preferred schools. Though beliefs are correctly centered, the average absolute difference between subjective and rational expectations beliefs about admissions probabilities is roughly 30 percentage points, with larger errors for low-SES students. Students with absolute errors greater than the median are 17 percentage

points less likely to be placed in their most-preferred school, compared to a base rate of 33%. Counterfactual policy simulations based on model estimates that incorporate our survey data indicate that the ordering of deferred and immediate acceptance mechanisms by welfare outcomes depends on the accuracy of students' beliefs about admissions chances. Though the immediate acceptance mechanism is preferable when students have rational expectations about choice probabilities, the deferred acceptance mechanism raises welfare given the distribution of belief errors we observe in our data. These gains are driven by reduced probabilities of very low welfare outcomes.

We find that gains in test score value added are correlated with gains in welfare from switching between the benchmark and deferred acceptance mechanisms, and that this change in mechanism reduces the share of students who submit applications with very low expected value added at the placed school. However, we find little evidence that changes in the centralized choice mechanism will reduce the gap in school quality between low- and high-SES students.

We conclude with a discussion of external validity and policy relevance. The main conclusion we draw from our findings is that policymakers designing school choice processes should consider the informational environment in their district when selecting an assignment mechanism. Given the SES gradient we observe in belief errors, our specific findings are likely most relevant for lower-income districts. Within this set of districts, however, we view New Haven as close to a best case scenario with respect to the information available to participants. A centralized immediate acceptance choice procedure had been in place for at least 18 years at the time we conducted our survey, and the school district conducts extensive outreach aimed at helping students and parents learn about the process. We would expect potential choice participants in districts where choice has been more recently adopted or where the district conducts more limited outreach to have, if anything, noisier expectations. Whether further informational interventions can push students closer to fully informed strategic decision making and, if so, what such interventions might look like, is a topic for future research. Our finding that only a large reduction in the variance of belief errors relative to the baseline level would yield welfare gains relative to a deferred acceptance suggests that designing an informational intervention that outperforms a switch to deferred acceptance in terms of aggregate welfare may prove challenging.

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A Appendix A

A.1 Additional Tables and Figures

Table A1: NHPS school district characteristics

	Population Mean
Asian	0.021
Black	0.482
Latino	0.396
Other	0.008
White	0.093
No Spec. Ed.	0.866
Spec. Ed.	0.134
F/R lunch	0.77
Reading	-0.69
Math	-0.66

Notes: School district characteristics in 2014-2015. Test score data from Neilson and Zimmerman (2014). Units are student-level standard deviations relative to statewide mean.

Table A2: Summary Statistics: Ninth-Grade Survey and Lottery Applications

School	Considered	In App.	1st in App.	1st Unconstr.
Achievement First Amistad HS	75.7	9.5	0.0	6.8
Common Ground Charter	64.9	17.6	8.1	2.7
Coop. Arts and Humanities	74.3	52.7	23.0	17.6
Engineering and Science Univ. HS	51.4	18.9	8.1	6.8
High School in the Community	63.5	20.3	4.1	5.4
Hill Regional Career	86.5	56.8	29.7	16.2
Hillhouse	83.8	5.4	0.0	2.7
Hyde School	62.2	25.7	5.4	8.1
Metropolitan Business Academy	70.3	43.2	9.5	9.5
New Haven Academy	75.7	25.7	6.8	6.8
Riverside Education Academy	46.0	2.7	1.4	0.0
Wilbur L. Cross High School	86.5	9.5	4.1	8.1

Notes: N=74 students in ninth-grade survey who participated in the lottery and matched to lottery data. All figures are percentages out of N. "Considered" equals 1 when the parent stated that he or she considered this school as a possible choice for his/her child. We asked parents for their top choice if they could choose any school and be guaranteed admission. We then asked what school they could choose if this school were full but all other schools guaranteed admission. We refer to these two schools as "unconstrained" choices.

Table A3: Summary Statistics: Kindergarten Survey and Lottery Applications

School	Considered	In App.	1st in App.	1st Unconstr.
Amistad Academy Elementary	87.5	25.0	12.5	25.0
Augusta Lewis Troup School	55.0	2.5	0.0	0.0
Barnard Environmental Studies School	62.5	15.0	2.5	7.5
Beecher Museum School	60.0	7.5	0.0	0.0
Bishop Woods Executive Academy	62.5	7.5	2.5	0.0
Booker T. Washington Academy Charter	52.5	10.0	2.5	0.0
Brennan-Rogers	42.5	17.5	7.5	7.5
Celentano Biotech Health and Medical	55.0	12.5	5.0	5.0
Christopher Columbus Family Academy	70.0	10.0	7.5	2.5
Clinton Avenue School	65.0	7.5	2.5	0.0
Conte-West Hills	57.5	12.5	5.0	2.5
Davis Street Arts and Academics	52.5	2.5	0.0	0.0
East Rock Community	72.5	10.0	2.5	2.5
Edgewood	65.0	5.0	0.0	0.0
Elm City College Preparatory Charter	62.5	20.0	2.5	2.5
Elm City Montessori	50.0	5.0	2.5	2.5
Fair Haven School	75.0	7.5	2.5	0.0
Hill Central School	65.0	7.5	2.5	2.5
Jepson Multi-Age	50.0	7.5	0.0	0.0
John C. Daniels	62.5	12.5	2.5	2.5
John S. Martinez	65.0	17.5	7.5	10.0
King-Robinson School: an IB World School	45.0	12.5	2.5	2.5
Lincoln-Bassett Community School	62.5	5.0	0.0	0.0
Mauro-Sheridan Sci/Tech/Communications	45.0	2.5	2.5	0.0
Nathan Hale School	57.5	5.0	2.5	0.0
Quinnipiac Real World Math STEM	47.5	10.0	5.0	2.5
Roberto Clemente Leadership Academy	70.0	10.0	5.0	2.5
Ross Woodward Classical Studies	62.5	0.0	0.0	0.0
Strong 21st Century Communications	50.0	5.0	0.0	0.0
Truman School	67.5	5.0	2.5	0.0
West Rock Authors Academy	45.0	0.0	0.0	0.0
Wexler-Grant Community School	55.0	2.5	0.0	0.0
Wintergreen	45.0	7.5	0.0	0.0
Worthington Hooker School	50.0	12.5	10.0	15.0

Notes: N=40 students in kindergarten survey who participated in the lottery and matched to lottery data. All figures are percentages out of N. "Considered" equals 1 when the parent stated that he or she considered this school as a possible choice for his/her child. We asked parents for their top choice if they could choose any school and be guaranteed admission. We then asked what school they could choose if this school were full but all other schools guaranteed admission. We refer to these two schools as "unconstrained" choices.

Table A4: 90% Credible Intervals, Σ

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Achievement First Amistad HS (1)	(9.46, 18.27)					
Common Ground Charter (2)	(9.07, 17.08)	(10.95, 37.1)				
Coop. Arts and Humanities (3)	(8.96, 15.61)	(3.79, 11.87)	(14.89, 22.31)			
Engineering & Science Univ. HS (4)	(13.16, 27.86)	(11.31, 31.87)	(10.58, 23.68)	(16.96, 63.4)		
High School in the Community (5)	(9.91, 16.88)	(7.58, 18.03)	(13.09, 20.05)	(12.93, 26.47)	(14.66, 24.5)	
Hill Regional Career (6)	(12.36, 22.38)	(7.77, 17.78)	(15.56, 24.04)	(15.48, 30.72)	(16.34, 24.94)	(24.18, 40.76)
Hillhouse (7)	(-6.47, 0.32)	(-7.03, -0.7)	(-6.49, 0.36)	(-8.72, 2.72)	(-6.52, 0.44)	(-5.49, 2.58)
Hyde School (8)	(11.6, 18.79)	(2.58, 11.09)	(10.33, 19.12)	(18.52, 41.88)	(11.8, 20.72)	(18.62, 28.29)
Metropolitan Business Academy (9)	(10.67, 19.14)	(6.25, 16.88)	(14.61, 22.74)	(14.22, 29.81)	(16.44, 26.14)	(19.5, 30.64)
New Haven Academy (10)	(10.06, 18.63)	(3.76, 13.24)	(13.58, 23.15)	(13.51, 31.34)	(15.52, 26.64)	(17.49, 29.07)
Riverside Education Academy (11)	(3.38, 8.17)	(3.63, 12.36)	(5.2, 9.98)	(3.42, 13.82)	(6.02, 12.65)	(-1.5, 8.35)
Wilbur L. Cross High School (12)	(-17.12, -1.66)	(-24.12, -1.86)	(-7.84, 1.25)	(-47.09, -3.22)	(-10.85, 0.68)	(-9.47, 3.88)

Variable	(7)	(8)	(9)	(10)	(11)	(12)
Achievement First Amistad HS (1)						
Common Ground Charter (2)						
Coop. Arts and Humanities (3)						
Engineering & Science Univ. HS (4)						
High School in the Community (5)						
Hill Regional Career (6)						
Hillhouse (7)	(0.8, 4.5)					
Hyde School (8)	(-4.89, 6.23)	(24.88, 49.78)				
Metropolitan Business Academy (9)	(-6.55, 1.29)	(16.56, 27.09)	(19.12, 30.65)			
New Haven Academy (10)	(-6.94, 1.85)	(18.54, 32.59)	(18.12, 31.33)	(17.46, 36.42)		
Riverside Education Academy (11)	(-5.64, -0.68)	(-4.14, 6.1)	(5.01, 12.05)	(4.05, 12.25)	(5.63, 12.38)	
Wilbur L. Cross High School (12)	(-2.5, 5.65)	(-24.59, -0.57)	(-10.42, 1.55)	(-11.91, 0.86)	(-7.08, -1.87)	(1.83, 44.77)

Table A5: Heterogeneity in Distance-Metric Welfare

quantile	(benchmark)	(RatEx)	(DA)	(Naive DA)	RatEx - NH	DA - NH	Naive DA - NH
0.05	0.381	0.702	0.617	0.576	-0.769	-1.005	-0.981
0.1	0.709	0.934	0.838	0.813	-0.476	-0.66	-0.679
0.25	1.026	1.25	1.119	1.101	-0.069	-0.283	-0.292
0.4	1.458	1.496	1.465	1.46	0.0	-0.106	-0.12
0.5	1.714	1.797	1.713	1.709	0.013	0.0	0.0
0.6	2.487	2.97	2.824	2.762	0.296	0.123	0.065
0.75	3.988	4.284	4.228	4.105	0.757	0.748	0.69
0.9	4.992	4.829	4.799	4.74	1.064	1.076	1.021
0.95	5.766	5.458	5.389	5.308	1.608	1.638	1.474

Notes: For each person in the population, we calculate the posterior mean welfare. This table displays quantiles of this distribution over individuals, under the benchmark, under Bayes Nash equilibrium of the New Haven mechanism, and under student-proposing deferred acceptance.

Table A6: Value-Added: Low-SES

quantile	(benchmark)	(RatEx)	(DA)	(Naive DA)	RatEx - NH	DA - NH	Naive DA - NH
0.025	-0.14	-0.141	-0.142	-0.143	-0.001	-0.003	-0.003
0.5	-0.14	-0.14	-0.14	-0.142	0.0	-0.001	-0.002
0.975	-0.14	-0.138	-0.139	-0.14	0.002	0.0	-0.0

Notes: This table displays quantiles of value-added of the "placed" school for low-SES households only. Low-SES households are the those in the the bottom 2/3 of the distribution of SES.

Table A7: Value-Added: High-SES

quantile	(benchmark)	(RatEx)	(DA)	(Naive DA)	RatEx - NH	DA - NH	Naive DA - NH
0.025	-0.118	-0.119	-0.118	-0.12	-0.002	-0.0	-0.003
0.5	-0.118	-0.117	-0.116	-0.118	0.001	0.001	-0.001
0.975	-0.118	-0.115	-0.115	-0.117	0.003	0.003	0.001

Notes: This table displays quantiles of value-added of the "placed" school for high-SES households only. High-SES households are those in the top 1/3 of the distribution of SES.

Figure A1: School choice participation by grade

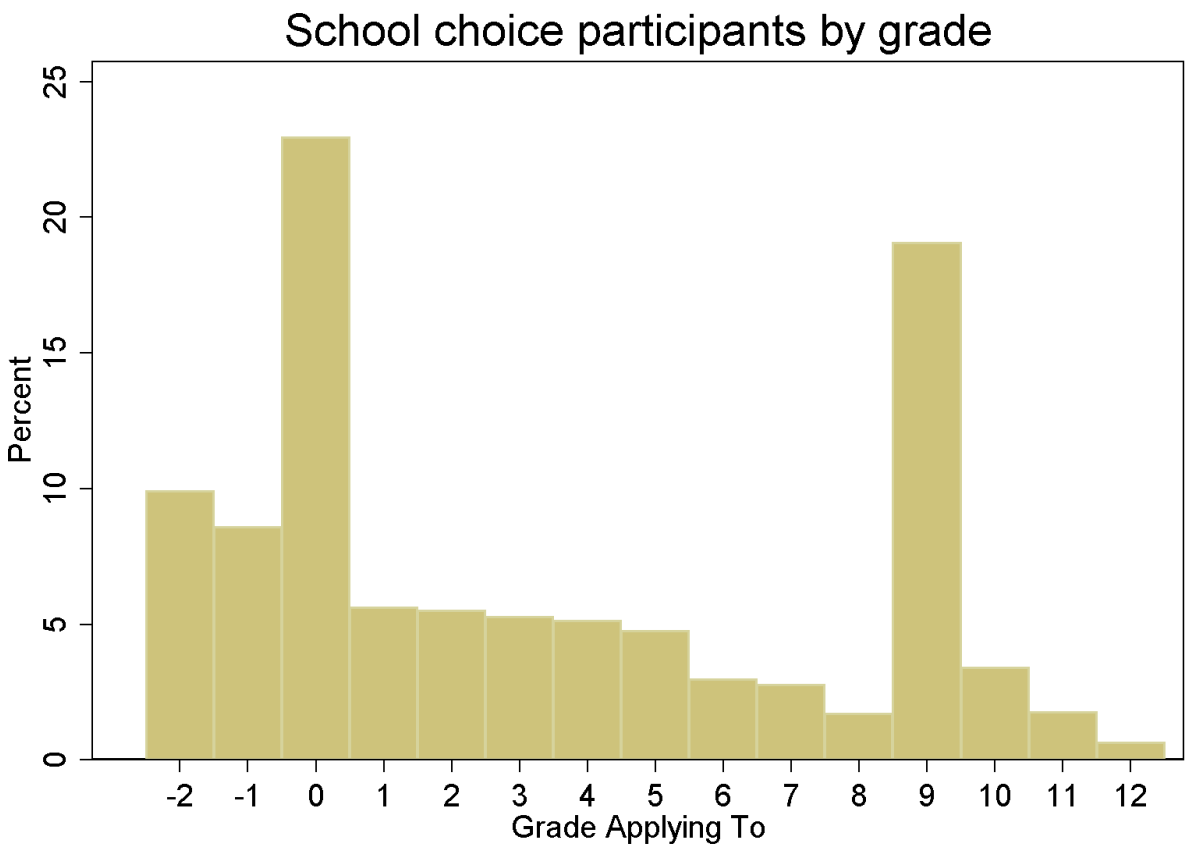
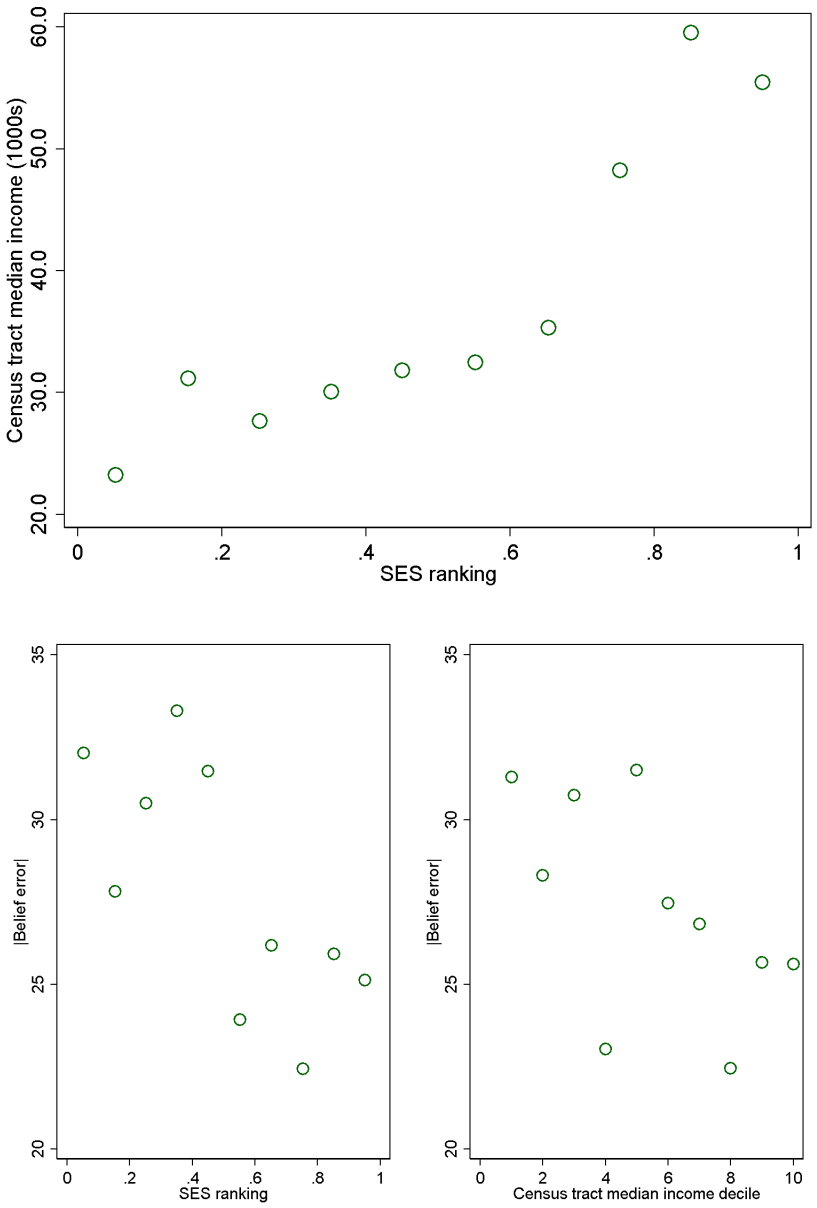
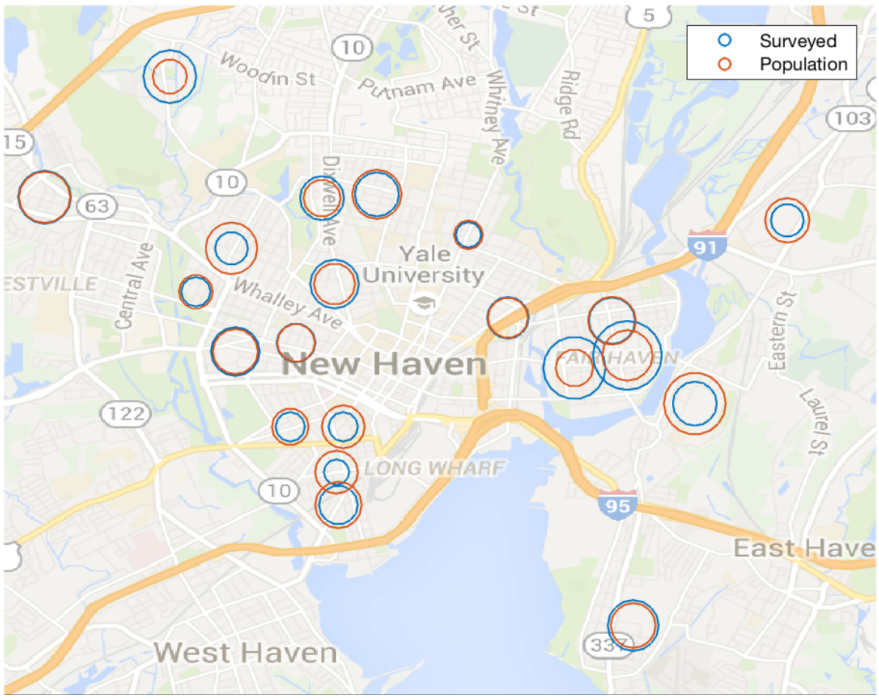


Figure A2: Benchmarking SES measures



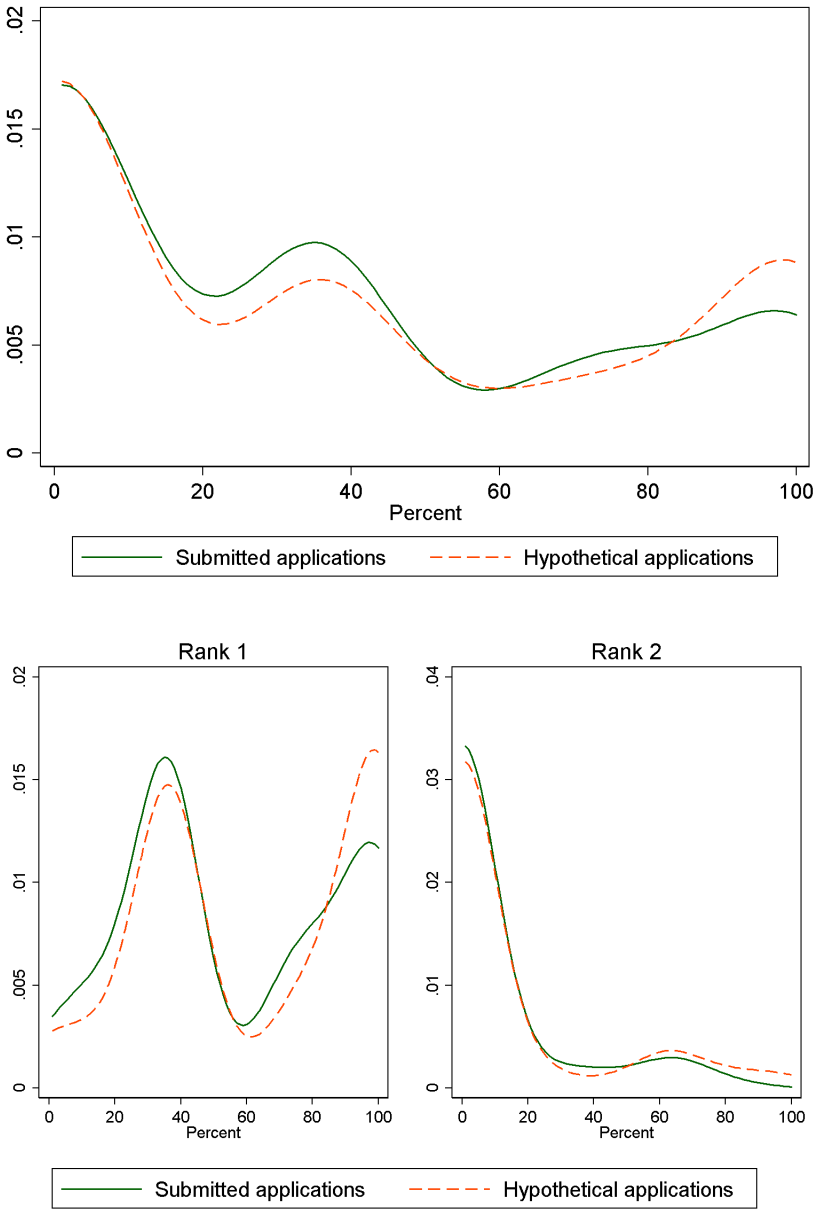
Notes: Upper panel: Median census tract income by decile of SES ranking. Lower panel: mean absolute belief error by decile of SES ranking (left) and decile of census tract median income (right).

Figure A3: Share of Students within each School Zone



Notes: This figure displays geographic distribution of sample universe and surveyed population. Size of circles reflect shares of population and surveyed individuals, respectively. Each point is centered at the location of an neighborhood zoned elementary/middle school, and reports data for students residing in that zone.

Figure A4: Ratex admissions probabilities of actual and hypothetical applications



Notes: N=786. Upper panel: ratex admission probabilities pooling by submitted rank. Lower panel: ratex admission probabilities for first-listed and second-listed choices.

Figure A5: Trace Plots: Beliefs

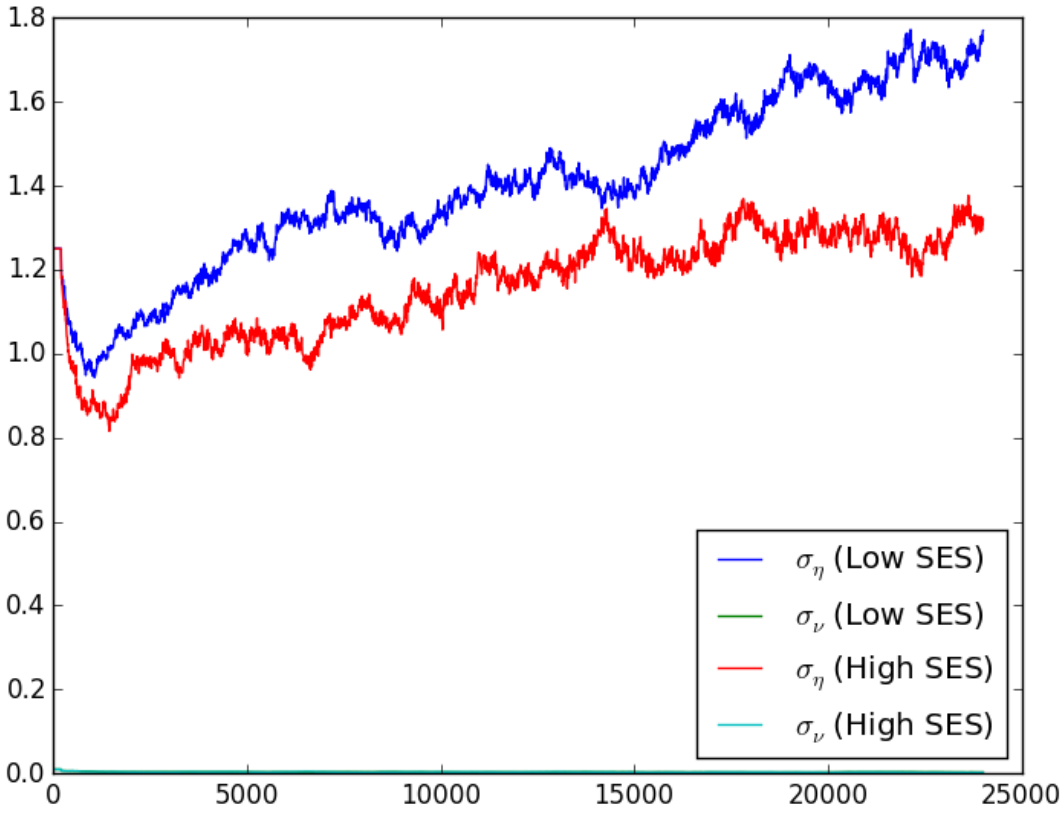


Figure A6: Trace Plots: δ_j

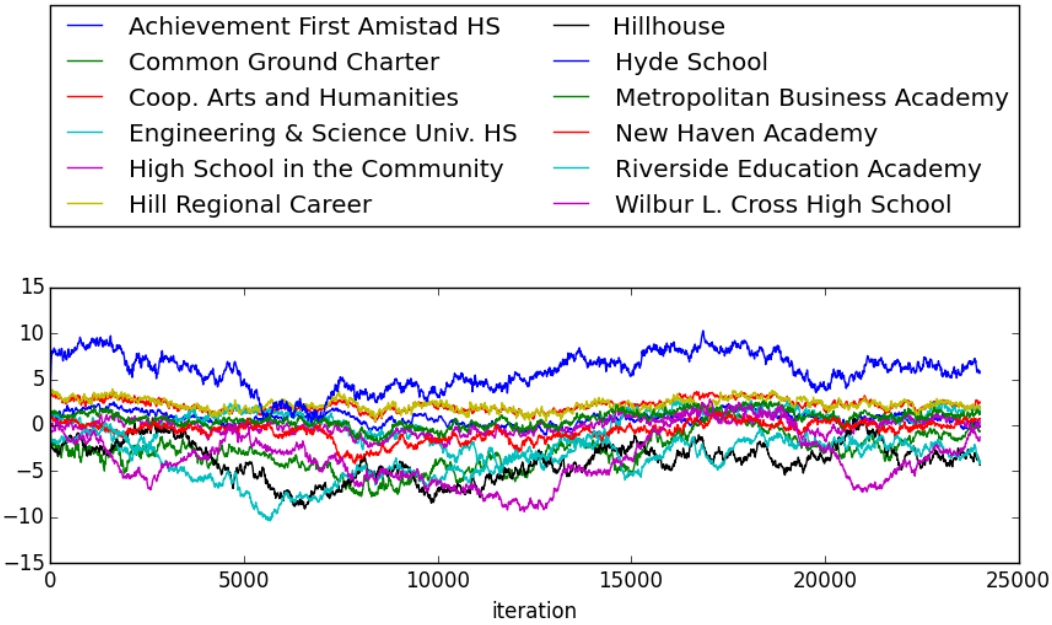


Figure A7: Trace Plots: Other Utility Parameters

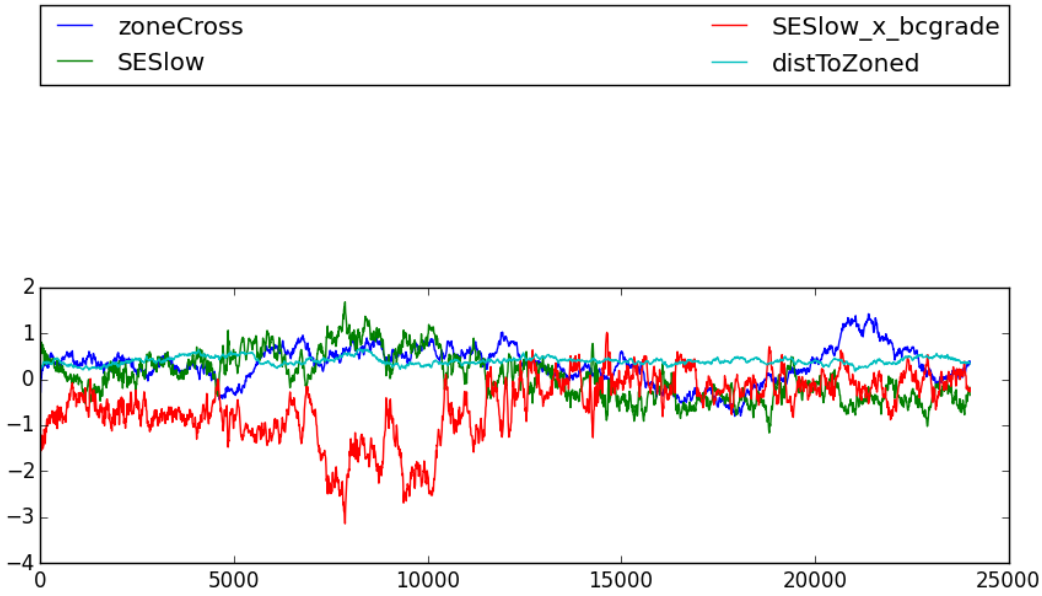


Figure A8: Trace Plots: Preference Shocks $\sqrt{\Sigma_{(j,j)}}$

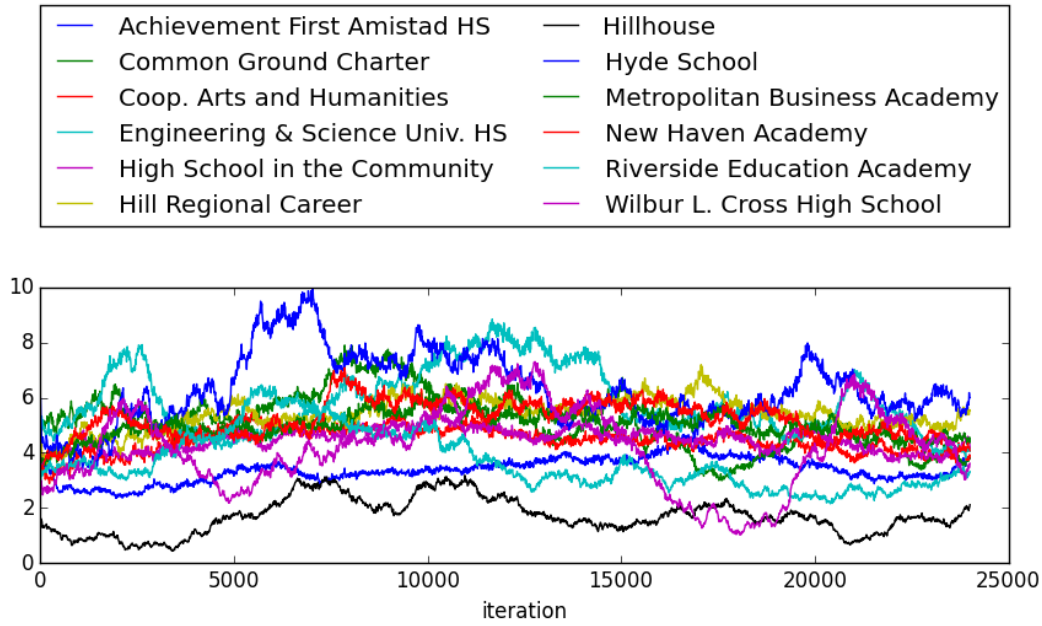


Figure A9: Trace Plots: Measurement Error σ_{survey}

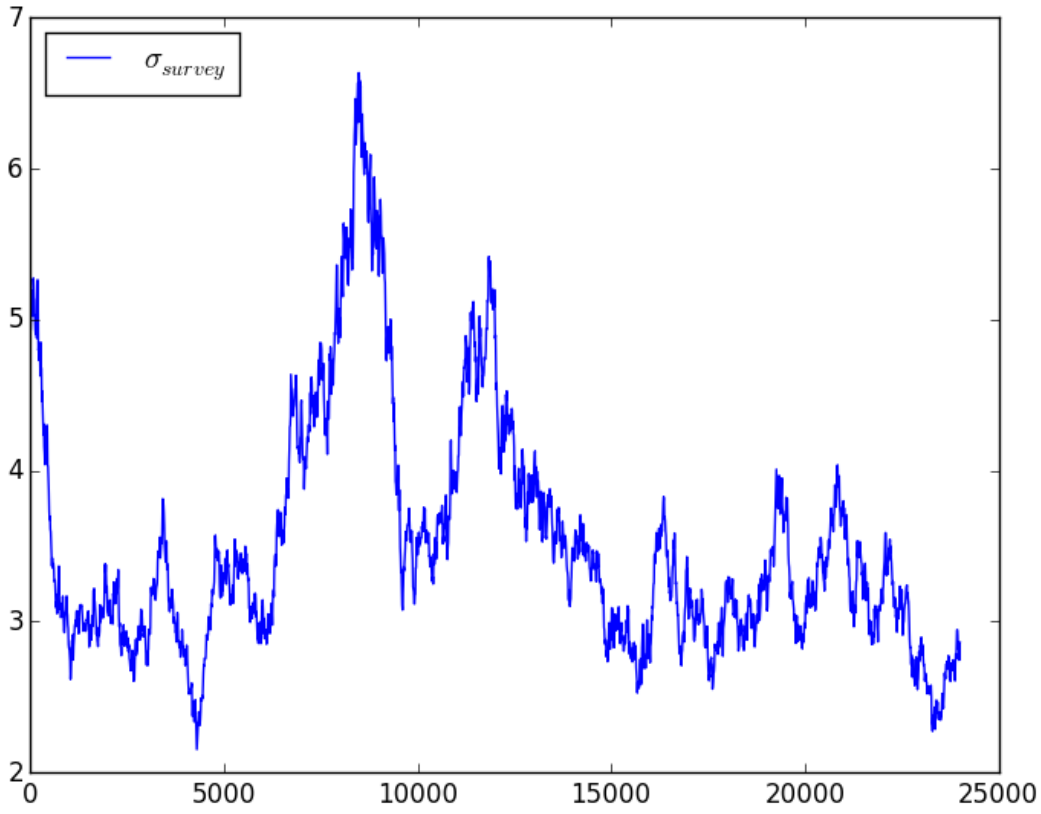
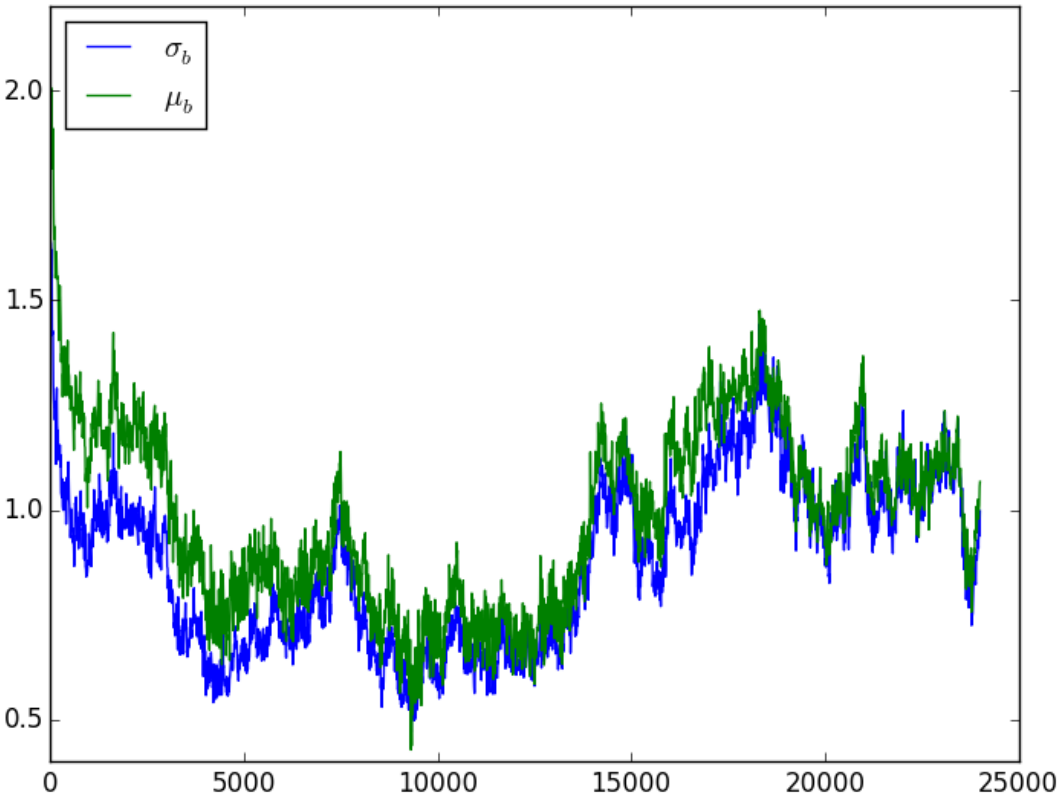


Figure A10: Trace Plots: Placement Cost



A.2 Data and Fieldwork

A.2.1 Administrative Data

New Haven Public Schools is a large school district similar to many urban school districts across the United States. Administrative student-level data was given to us with permission from New Haven Public Schools (NHPS). The data contain information for approximately 20,000 students, and includes student race, gender, school lunch status, test scores, and other information. NHPS is a majority-minority school district - nearly 90% of students are either black or Hispanic - and more than 80% are eligible for free lunch.

A.2.2 New Haven Home Sales Data

Data on home prices used to create measures of SES come from the City of New Haven's official tax assessor's website. This dataset includes the universe of residential homes. A sales history is included for each property listed on the websites, along with various property characteristics such as street address, home square footage, acerage, number of rooms, and more. Table A8 shows summary statistics for all home sales in New Haven between 2005 and 2015. In order to construct our measure of student socioeconomic status (SES), we first regress real (2015 dollar) per-square-foot sale prices of New Haven homes on time dummies and obtained residuals. We then compute the implied price per square foot at each location using a normal kernel with bandwidth .05 miles. See Section 3.4 and Figure 1 in the text for more details.

A.2.3 Data Collection

In addition to administrative records provided to us by NHPS, we designed and conducted a household survey named *"New Haven School Choice Survey: Empowering Choice through Information and Understanding"*. Funded by the Industral Relations Section of Princeton University and the Cowles Foundation at Yale University, we worked together with the district to collect data directly from potential participants in the school choice mechanism. NHPS provided student-level administrative data that contained the universe of the sample population and also information relevant for contacting the families. We conducted in-person interviews with over 200 families. We interviewed parents and guardians of students at their homes in an anonymous questionnaire that last approximately 20 minutes. This survey was conducted in collaboration with NHPS and experienced field personnel with prior experience conducting similar surveys in projects done with JPAL LAC.

Table A8: New Haven Home Sales Data

	2005-2015	2005-2009	2010-2015
Mean Price ($1,000s)	216.9	240.1	176.5
Median Price ($1,000s)	193.6	222.7	136.0
Square Feet	2,032	1,983	2,118
Acreage	0.119	0.115	0.126
Bedrooms	4.37	4.35	4.41
Bathrooms	2.16	2.15	2.19
Rooms	8.35	8.22	8.57
High Quality	0.323	0.306	0.353
N	11,221	7,112	4,109

Notes: This table shows summary statistics (means) for all residential property sales in New Haven between 2005 and 2015. Prices are in 2015 dollars. Data were collected from the City of New Haven's offical tax assessor website database (see http://gis.vgsi.com/newhavenct/).

The field work was conducted during August to October of 2015. The survey was implemented on Samsung Galaxy Tab 7 and programmed using SurveyCTO. The household survey was designed and tested in small focus groups three occasions during the two months prior to the field work.

The team of surveyors was composed by ten active members who were recruited using on-line advertisement and Yale University's public spaces for flyers. All of them received a two-day training that prepared them for the use of the tablet and regulation regarding interacting with human subjects. Almost half of the surveyors were bilingual English and Spanish speakers which was useful given a significant proportion of the population in New Haven is Hispanic. The two-day training covered the following topics:

- **Day 1:** Introduction regarding data confidentiality and safety. Logistics procedures were discussed.
- **Day 2:** Practical training of the instrument in a random neighborhood where we tested their skills to approach the families and their accuracy while using the instrument.
- **CITI Certificate:** All surveyors had to complete an on-line course for IRB purposes where they learned about dealing with private and confidential data.

Each of these items were mandatory before going to the field. Also, all personnel were supervised by a field coordinator who monitor and verified the data collection process. Each surveyor was assigned with a route sheet that included the closest selected households in a neighborhood in order to facilitate their work. The field coordinator selected the routes of households to reach certain geographic and demographic coverage goals.

A.2.4 Recruiting Participants

Parents' participation was voluntary and there was no compensation (neither monetary nor non-monetary) for their participation.

- In partnership with NHPS, the district contacted the households via phone-calls to announce their participation in the project.
- When the surveyors visited each house, they announced the project and handed in a business card (see Figure A11) with study contact information. Parents or guarding who agreed to participate signed an informed consent.

- In case of finding no one at home, we left a door hanger with contact information (see Figure A12)
- Also, surveyors had the chance to re-schedule the interview if the respondent had time issues at the moment.

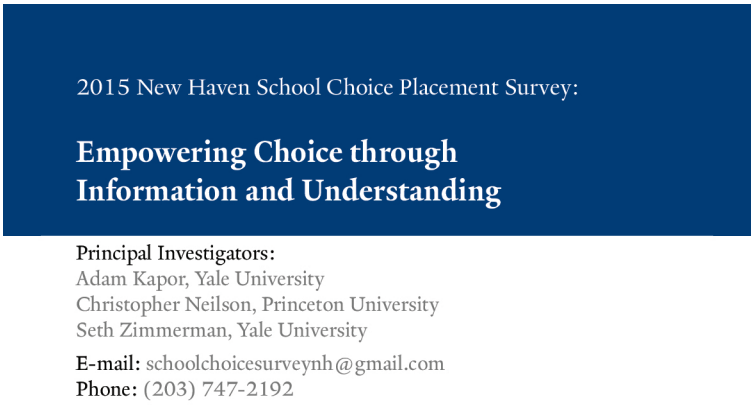


Figure A11: Business Card used during Fieldwork

2015 New Haven School Choice Placement Survey

Empowering Choice through Information and Understanding

We're sorry we missed you!

We are a team of researchers from Yale and Princeton Universities who are collaborating with New Haven Public Schools to conduct a survey about your experiences with school choice.

We want to hear from you! **Please call or text us at 203-747-2192** and we can find a time to talk that is convenient for you.

Thanks!



Figure A12: Door Hanger used during Fieldwork

B Appendix B: Survey Questionnaire