Financial Shocks and Job Flows*

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Abstract

We argue that the creation and destruction margins of employment (job flows) at the aggregate level and disaggregated across firm age and size can be used to measure the employment effects of disruptions to firm credit. Using a firm dynamics model, we establish that a tightening of credit to firms reduces employment primarily by reducing gross job creation, exhibiting stronger effects at new/young firms and middle-sized firms (20-99 employees). We find that 15% of the decline in US employment during the Great Recession is due to the firm credit channel. Using MSA-level job flows data, we show that the behavior of job flows overall and across firm size and age categories in response to identified credit shocks is consistent with our model's predictions and hold within tradable and non-tradable industries.

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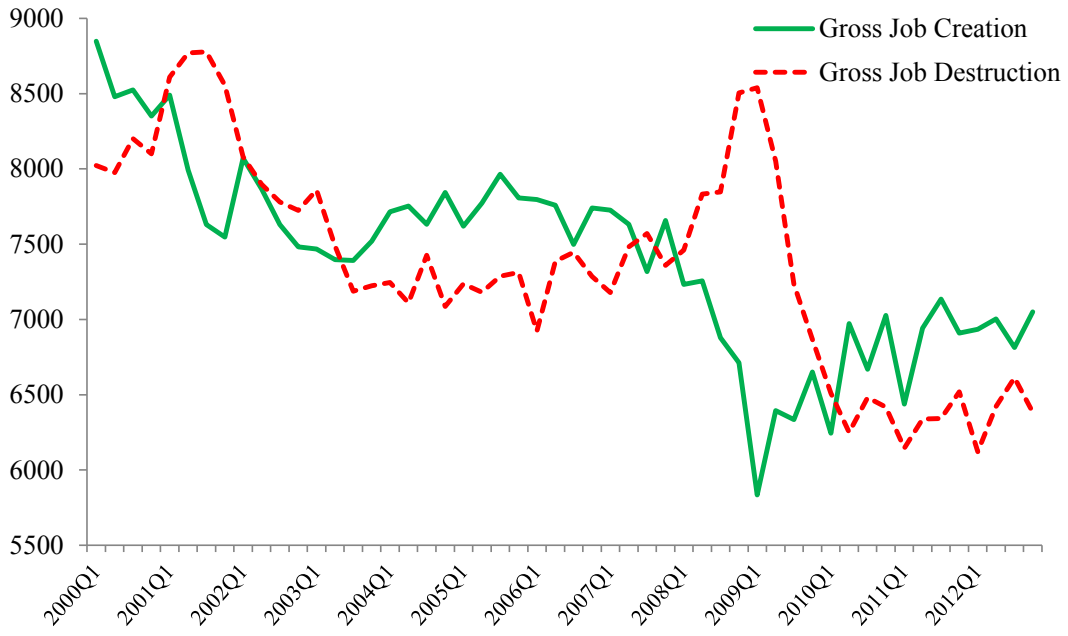
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1 Introduction

During the Great Recession, employment decreased approximately 6% from its peak in late 2007 to its trough in early 2010.[1] The sharp decline in US employment and its slow recovery have prompted an extensive debate on the underlying causes of the Great Recession and the channels through which a financial crisis reduces employment. In particular, the relative importance of competing channels through which the financial crisis reduced employment remains an open question. Existing empirical evidence typically falls into two categories: disruptions to household credit or disruptions to firm credit. Most prominently, Mian and Sufi (2014) offer a household credit explanation for the decline in employment: falling house prices induce deleveraging, reduce consumer demand, and interact with price or wage rigidity to decrease employment. Alternatively, Chodorow-Reich (2014) highlights a firm credit explanation: disruptions in the US financial system raised the cost of external financing for credit-constrained firms leading these firms to shed workers.[2]

Figure 1: US job flows



The figure shows aggregate US job flows for 2000Q1-2012Q4 from the Business Employment Dynamics.

Following on these contributions, an extensive empirical literature has sought to separately identify various channels through which financial frictions diminish household credit (and consumer demand) or inhibit the flow of credit to businesses. These studies typically focus on identification of

[1] Seasonally adjusted total nonfarm employment is taken from the US Bureau of Labor Statistics - the peak is in December 2007 and the trough is in January 2010.

[2] Models without nominal frictions generate declines in employment from a financial disruption by raising the effective cost of capital or labor (increases in credit spreads or rising costs of working capital) or increasing misallocation (lowering aggregate TFP).

exogenous shocks to household credit or firm credit using firm, bank, or geographic variation.[3] However, these empirical approaches, by measuring relative elasticities and holding aggregate factors constant, cannot quantify the contribution of firm credit disruptions relative to household credit disruptions to the overall reduction in employment during the Great Recession. A model is needed to disentangle these channels and determine their relative contribution to the fall in employment.

In this paper, we argue that gross job flows can help separate and quantify the contribution of the firm credit channel to declines in employment.[4] First, we establish theoretically and empirically that a firm-side credit disruption decreases employment primarily by reducing gross job creation. In contrast, we show theoretically and empirically that a household credit disruption that lowers consumer demand or operates indirectly through elevated risk premia decreases employment primarily by raising job destruction.

Secondly, overall job flows and flows by firm age/size can disentangle the relative contribution of these channels. We use a calibrated firm dynamics model to quantify the contribution of each channel to the overall decline in US employment. Our calibrated model matches key moments of the distribution of employment by firm age and size. Importantly, only a small fraction of firms in our model are financially constrained in accordance with evidence that the aggregate balance sheet of US firms was quite healthy throughout the Great Recession.

To model job flows, we must move away from a representative firm to an environment with expanding firms (contributing to gross job creation) and contracting firms (contributing to gross job destruction). The difference between gross job creation and gross job destruction is the change in employment. Figure 1 shows the behavior of gross job flows during the Great Recession. To our knowledge, our paper is the first to theoretically model how gross job flows respond to different business cycle shocks, and how these differences help identify the type of shocks at work.

We model a financial shock as a tightening of financial constraints in a firm dynamics model with financial frictions and decreasing returns to scale production.[5] Newly born firms and young firms accumulate assets and expand towards their optimal scale. Mature firms are more likely to be financially unconstrained and are free to expand or contract subject to idiosyncratic shocks to firm productivity or changes in factor prices. Firms differ in productivity levels so that some businesses remain small without any binding financial constraint. In our calibration, most financially constrained firms are small/medium-sized firms, but the vast majority of small/medium-sized firms

[3] On the household (demand) side, see, for example, Mondragon (2014) and Di Maggio and Kermani (2015). On the firm (supply) side, see Ivashina and Scharfstein (2010), Becker and Ivashina (2014), and Greenstone, Mas and Nguyen (2014).

[4] Cyclical movements in job flows may also be of independent interest given the importance of labor reallocation for productivity growth (see Davis and Haltiwanger (1999) and Haltiwanger (2012)).

[5] We model the financial constraint as in Buera and Shin (2011) and the mechanism we study is closest to the models of Buera, Fattal Jaef and Shin (2015) and Khan and Thomas (2013). Siemer (2013) and Schott (2013) also build firm dynamics models to study the financial crisis and firm entry, focusing on the slow labor market recovery. In contrast to this work, we emphasize the ability of job flows to determine the contribution of the firm credit channel to the overall decline in US employment.

are not constrained.[6] In addition to a financial shock, our model includes a productivity shock and a discount factor shock. We argue in the paper (and show formally in Appendix C) that the productivity shock captures the effect of a shock to household credit that induces consumer deleveraging, while the discount factor shock parsimoniously captures rising risk premia, increased pessimism, or heightened uncertainty that may be indirectly related to the financial crisis.[7] Intuitively, the equivalence between a productivity shock and a consumer deleveraging shock or other demand shocks *for employment and job flows* stems from the fact that these shocks all impact the same margin - firms reduce employment because revenues fall.

Using our firm dynamics model, we show that financial shocks diminish aggregate job creation and destruction along the transition path. In marked contrast to productivity or discount rate shocks, total reallocation (sum of creation and destruction) falls in response to a financial shock consistent with the findings of Foster, Grim and Haltiwanger (2014) who show that total reallocation typically rises in recessions but fell in the Great Recession. Our model also generates predictions for the behavior of job flows across firm age and size. In particular, a financial shock disproportionately reduces job creation at new and young firms (1-5 years) relative to mature firms (6+ years) and reduces job destruction at young firms relative to mature firms. This shock also reduces job creation at middle (20-99 employees) and large sized firms (100+ employees) while job creation actually rises at small firms (1-19 employees). By contrast, the productivity and discount shock delivers much more uniform effects across firm age or size categories.

The fact that financial shocks reduce employment via the job creation margin and exert differential effects across age allows us to estimate its contribution to the decline in US employment experienced in the Great Recession. We choose financial, productivity and discount rate shocks to match the initial movement of job flows at the onset of the Great Recession.[8] To our knowledge, we are the first to use disaggregate job flows data to estimate business cycle shocks in a fully nonlinear firm dynamics model. We find that a sizable financial shock (-21.5% fall in collateral values), a -1.4% productivity/deleveraging shock, and 3 percentage point increase in the equity premium (discount factor shock) are needed to generate a 6% decline in employment. Despite its large magnitude, the financial shock accounts for only 15% of the decline in employment. Much of the decline in employment is instead attributable to the discount rate shock (57%). Our findings indicate a modest role for the firm credit channel in the decline in overall employment and in explaining the sharp decline in job creation and suggest that the financial crisis reduced employment primarily

[6] In this dimension, our calibration fits the stylized fact in Hurst and Pugsley (2011) that most small business owners do not wish their business to grow large.

[7] The discount rate shock captures the mechanisms emphasized in Hall (2014), Baker, Bloom and Davis (2013), and Caballero and Farhi (2014) which may account for employment losses. This shock also captures the mechanism emphasized in Kehoe, Midrigan and Pastorino (2014) where firms are not directly financially constrained, but a housing credit shock raises household discount rates and lowers incentives for hiring.

[8] Productivity and discount rate shocks both reduce employment primarily via the job destruction margin, but have differential effects on job flows by firm age that allow us to separate their contribution.

via channels that impacts not only financially constrained, but unconstrained firms as well.[9]

In our empirical work, we validate our model by providing direct evidence that a decline in collateral values diminishes job creation and job destruction using MSA-level data from the Business Dynamics Statistics (BDS). We exploit MSA-level variation in job flows and housing prices to examine the effects of movements in MSA housing prices on job flows. House prices are taken as a proxy for credit conditions in the banking system but may have a direct effect on firm formation and expansion given the reliance of entrepreneurs on their personal wealth and the value of business real estate to secure lending.[10]

To address issues of causality, we include MSA and time fixed effects and add direct controls for the business cycle. We also utilize an IV approach based on differences across MSAs in their sensitivity to movements in aggregate US house prices. This land supply elasticity approach – used widely in the literature to examine the effect of collateral shocks on real variables – is applied here to examine the effect of housing prices on job flows.

Our empirical results show that a shock to housing prices reduces job creation persistently and reduces job destruction with a lag. These results hold under both the OLS and IV specifications and are robust to alternative controls for the MSA business cycle. Moreover, we document differences across firm age and size categories in the sensitivity of job flows to housing price shocks.[11] In particular, we find that job creation for middle-sized firms and new and young firms exhibit greater sensitivity to housing prices relative to small firms and mature firms respectively. Similar patterns hold for job destruction with middle-size firms and young firms exhibiting a fall in job destruction when house prices fall.[12] These patterns are consistent with the cross-sectional predictions of our model and related findings in state level data from Fort et al. (2013). Importantly, in our model, a productivity/deleveraging and discount rate shocks generate cross-sectional patterns at odds with the patterns we find.

Using state level data, we also document systematic differences in the sensitivity of employment at new firms versus employment at new establishments of existing firms (for example, a local inde-

[9] Christiano, Eichenbaum and Trabandt (2015) estimates a medium-scale DSGE model to determine the contribution of different shocks to the Great Recession, attributing much of the decline to a rising financial wedge. Recent work by Jermann and Quadrini (2012) and Liu, Wang and Zha (2013) examine the role of financial or collateral shocks as a source of business cycles.

[10] A subset of the empirical literature on the firm credit channel has emphasized the particular importance of house prices and real estate collateral values for employment and investment. Papers by Gan (2007) and Chaney, Sraer and Thesmar (2012) examine the effect of collateral shocks on firm investment. In the latter, authors use firm-level financial data to show that a decline in the value of real estate for a firm's headquarters has a statistically significant effect of firm investment. Adelino, Schoar and Severino (2013) documents that small business starts and employment levels showed a strong sensitivity to increases in housing prices during the boom years from 2002-2007. Chaney et al. (2015) explores the effect of real estate collateral on employment using firm level data in France.

[11] Our work is similar in spirit to Chari, Christiano and Kehoe (2013) who examine differences in the response of employment across firm sizein recessions and in response to monetary shocks.

[12] Furthermore, we show that our results are not driven by the direct effect of house prices on job flows within the construction industry. We show that age patterns remain when restricted to job flows ex construction at the state level. These findings are consistent with Fort et al. (2013). In addition, our empirical results hold within traded (manufacturing) and non-traded sectors (services), alleviating concerns that these patterns are solely driven by the housing net worth/deleveraging channel emphasized by Mian and Sufi (2014).

pendent coffee shop versus a new Starbucks location) to house price changes. Employment at new firms falls when housing prices decline, while employment at new establishments of existing firms is unchanged. This differential response is consistent with the view that house prices (after controlling for the local business cycle) are proxying for credit conditions given that new establishments from existing firms are likely to belong to older and larger firms with better access to external financing.

It is worth explaining the implications of our findings for the housing net worth/deleveraging channel (Mian and Sufi (2014)). As we noted in our structural estimation, 85% of the decline is due to factors other than the collateral shock, with 30% of the decline in employment due to the productivity shock. To the extent that this shock reflects household deleveraging, this channel accounts for a larger portion of the decline in employment than the firm credit channel. Additionally, we cannot rule out that the dominant shock - the discount factor shock - is itself driven by household deleveraging (as we noted, this is the mechanism in Kehoe, Midrigan and Pastorino (2014)) meaning that direct and indirect effects of deleveraging could account for most of the fall in employment. On the empirical side, our IV strategy does not rule out that the housing price shocks used in this paper and Mian and Sufi (2014) can have both a household and firm credit component. Our empirical findings are fully consistent with the patterns in Mian and Sufi (2014), where the household deleveraging component of a fall in house prices has a more uniform effect that reduces employment across all ages, while exerting a differential effect across traded and nontraded industries.[13] The firm component, by contrasts, manifests itself as differential effects across firm age.

The paper is organized as follows: Section 2 presents a simplified continuous time firm dynamics model and characterizes firm behavior. Section 3 outlines the benchmark model while Section 4 describes our calibration strategy, investigates the quantitative implications of financial shocks, and presents our estimation results. Section 5 discusses our data and presents empirical results on the link between financial shocks and job flows. Section 6 concludes.

2 Simple Model

We begin by presenting a simple continuous time firm dynamics model to analyze the effects of a change in financial constraints on asset accumulation, employment, and job flows in a stationary equilibrium. This simple model illustrates the basic mechanism at work that causes a tighter financial constraint to reduce employment via the job creation margin and exert a stronger effect on young firms and middle-sized firms. This environment also allows us to derive analytical results before turning to a richer model to make our quantitative statements.

We start with a real business cycle model and add (i) a financial friction that limits the amount of

[13] We also emphasize that a substantial empirical literature finds evidence of effects on investment and employment of collateral fluctuations induced by shocks to house prices.

firm borrowing, (ii) firm heterogeneity, (iii) and a decreasing returns to scale production technology. The economy consists of three types of agents: identical households, heterogeneous firms, and identical intermediaries. Each household consumes, supplies labor, and trades on asset markets. The household consists of a measure n workers. Workers supply labor to firms and return their wages to the household. Each firm hires workers from households and borrows capital from intermediaries to produce. Intermediaries own the capital stock, issue one-period real risk-free bonds, and rent capital to firms. Every period a fraction σ of firms exit and transfer their assets to the household while an equivalent measure of new firms are born; these firms receive an initial transfer of assets from the households. A single consumption good in the economy serves as the numeraire good, and there are two types of assets: capital and the risk-free one period real bonds. There is no aggregate uncertainty and the only idiosyncratic uncertainty is the risk of exit for individual firms. We assume that real interest rate on one-period safe bonds r is constant.[14]

2.1 Households

Let c_t be consumption and n_t be labor supply of a typical household in instant t. Household preferences are given by:

$$\int_0^\infty e^{-\rho t} U\left(c_t - v\left(n_t\right)\right) dt. \tag{1}$$

where ρ is the rate of time preference. We follow Greenwood, Hercowitz and Huffman (1988) and define instantaneous utility in terms of consumption in excess of disutility of labor, thereby eliminating wealth effects on labor supply.

The household faces a flow budget constraint as follows:

$$\dot{a}_t = w_t n_t + r a_t + \Pi_t - c_t, \tag{2}$$

where the dot above a variable denotes the derivative with respect to time, r is the one-period return on household assets a_t, Π_t is net payout to the household from the ownership of firms and w_t is real wage. The return on bonds r is exogenous, while wages w_t are endogenously determined.

Households start with initial holding of risk-free bond a_0^H and we impose the natural debt limit constraint:

$$a_t \geq -\int_t^\infty \left[w_s n_s + \Pi_s\right] e^{-r(s-t)} ds. \tag{3}$$

2.2 Firms

The economy is composed of a unit measure of firms which produce homogeneous output. Firms act competitively on output, asset, and labor markets. Each firm faces an exogenous rate of exit σ and

[14] We assume a constant real interest rate since the small open economy assumption applies in our empirical setting. See, for example, Mendoza (2010) for a similar assumption.

transfers its assets to the household at exit. Between t and $t+\Delta$, with Δ being sufficiently small, a measure $\sigma\Delta$ of firms exit and $\sigma\Delta$ of new firms enter. Every new firm enters with a predetermined level of initial assets a^F that is identical across firms.

Productivity of every firm $A \cdot z_i$ consists of two components: a common component A (aggregate productivity) and firm-specific productivity z_i where i indexes the firm. In our simple model, firm-specific productivity z_i can take on two values $\{z_L, z_H\}$ with $z_L < z_H$ that are constant over the firm's life. The probability of being born with a high firm-specific productivity is μ.

Firms apply $\Lambda_{t,t+\tau} = e^{-\rho\tau} U'\left[c_{t+\tau} - v(n_{t+\tau})\right] / U'\left[c_t - v(n_t)\right]$ as their discount factor between periods t and $t+\tau$ and each firm maximizes the present discounted value of its terminal wealth. Formally, firms choose capital and labor to maximize:

$$\max_{\{n_{i,t+\tau}, k_{i,t+\tau}\}_{\tau=0}^{\infty}} \int_0^{\infty} e^{-\sigma\tau} \Lambda_{t,t+\tau} a_{i,t+\tau} d\tau, \tag{4}$$

where $a_{i,t+\tau}$ are holdings of risk-free bonds by firm i at time $t+\tau$ and $k_{i,t}$ is capital rented by the firm in period t.[15] Firms face both a wealth accumulation constraint and a financial constraint. Their wealth accumulation constraint is given by:

$$\dot{a}_{i,t} = \pi_{i,t} + r a_{i,t}, \tag{5}$$

where the firms' profits are given by:

$$\pi_{i,t} = A z_i \left(k_{i,t}^{\alpha} n_{i,t}^{1-\alpha}\right)^{\phi} - r_{k,t} k_{i,t} - w_t n_{i,t}, \tag{6}$$

Firm output, $A z_i \left(k_{i,t}^{\alpha} n_{i,t}^{1-\alpha}\right)^{\phi}$, is given by a decreasing-returns-to-scale production function.

The firm faces a financial constraint of the following form:

$$k_{i,t} \leq \chi a_{i,t}, \tag{7}$$

where $\chi \geq 1$ denotes the leverage ratio which is common across firms. This constraint states that the firm cannot rent more capital than the amount of the firm's holdings of risk-free bonds times χ. Parameter χ indexes the severity of financial frictions: $\chi = \infty$ corresponds to a frictionless rental market and $\chi = 1$ corresponds to self-financing. This specification parsimoniously incorporates the frictions emphasized in corporate finance models with limited contract enforcement.[16]

[15] The assumption of no dividend payouts before exiting is similar to assuming that firms maximize the discounted stream of positive payouts to the household. In this alternative case, because of the binding financial constraint, firms would prefer to retain earnings until they grow out of the financial constraint. Once firms become unconstrained, the timing of payouts is irrelevant, and we can assume that all payouts occur when firms exit.

[16] See Evans and Jovanovic (1989) for an early use of this specification of the financial constraint. Buera and Shin (2011) show that this type of financial constraint can be derived by assuming limited liability on the side of the firms and one-period punishment for not honoring repayment.

2.3 Intermediaries

Competitive intermediaries issue one-period risk-free real bonds and rent out capital at rate $r_{k,t}$ to firms. Because the consumption good can be freely transformed to capital, the zero-profit condition of the intermediaries requires:

$$r_{k,t} = r + \delta, \tag{8}$$

where δ is the depreciation rate of capital. The zero-profit condition and the absence of capital adjustment costs imply that rental rate of capital is constant and pinned down by return on bonds and depreciation rate in equilibrium.[17]

2.4 Competitive Equilibrium

A competitive equilibrium is allocation $\left\{c_t, a_t, n_t, \{a_{i,t}, n_{i,t}, k_{i,t}\}_{i\in[0,1]}\right\}_{t\geq 0}$ and prices $\{w_t, r_{k,t}\}_{t\geq 0}$, r such that:

1. Households solve (1)-(3) given initial level of assets a_0^H taking prices $r, r_k, \{w_t\}_{t\geq 0}$ as given;
2. Firms solve (4) - (6) given initial level of assets a^F taking prices $r, r_k, \{w_t\}_{t\geq 0}$ as given;
3. Intermediaries optimize so that equation (8) is satisfied;
4. Firms and representative household choices clear the labor market: $n_t = \int n_{i,t} di$.

2.5 Characterization of the Firm's Problem

We now consider a stationary equilibrium in which prices are constant over time. Household's optimal labor choice leads to the following labor supply curve in the stationary equilibrium, equating the real wage with the marginal disutility of working[18]:

$$w = v'(n). \tag{9}$$

Firm maximization of its expected terminal wealth is equivalent to static optimization of the profit function conditional on the financial constraint. Optimal capital and labor choices imply the following labor and capital demand conditions:

$$Az_i \alpha\phi k_{i,t}^{\alpha\phi-1} n_{i,t}^{(1-\alpha)\phi} = r_k + \frac{\eta_{i,t}}{\lambda_{i,t}^F}, \tag{10}$$

$$Az_i (1-\alpha)\phi k_{i,t}^{\alpha\phi} n_{i,t}^{(1-\alpha)\phi-1} = w, \tag{11}$$

[17] The absence of capital adjustment costs is a strong assumption. However, Liu, Wang and Zha (2013) argue that in a model with financial frictions, capital adjustment costs are estimated to be close to zero and much smaller than in models without financial frictions such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007).

[18] There is also a standard Euler equation for households. However, its form does not affect the results that follow.

where $\lambda^F_{i,t}$ is firm's marginal value of additional unit of safe debt holdings $a_{i,t}$ and $\eta_{i,t} \geq 0$ is the Lagrange multiplier on the financial constraint. Equation (10) states that, at the optimum, a firm equates its marginal product of capital to rental rate of capital plus the cost of making the collateral constraint tighter. Equation (11) is the standard labor demand condition equating the marginal product of labor to the real wage.

When the financial constraint binds, $\eta_{i,t} > 0$ and $k_{i,t} = \chi a_{i,t}$. The labor demand condition - equation (11) - can be rewritten as follows:

$$n_{i,t} = \left[\frac{z_i A \phi (1-\alpha)}{w}\right]^{\frac{1}{1-\phi(1-\alpha)}} \cdot (\chi a_{i,t})^{\frac{\alpha\phi}{1-\phi(1-\alpha)}}. \tag{12}$$

Substituting optimal employment (12) and capital $k_{i,t} = \chi a_{i,t}$ into the profit function (6), we obtain a law of motion for assets as shown in Appendix A.2 along with the solution to this differential equation.

As shown in Appendix A.2, the path of assets is increasing in t since otherwise profits would be negative. The asset path is convex in t, increasing with χ at all t before the firm becomes unconstrained, and increasing in firm-specific productivity z_i.[19] We also show in Appendix A.3 that labor demand $n_{i,t}$ is increasing in t, may be convex or concave in t, is increasing in χ, and is increasing in z_i because labor demand (12) is a concave and increasing function of asset holdings $a_{i,t}$.

If the financial constraint does not bind, then $\eta_{i,t} = 0$. Optimality with respect to labor (11) and capital (10) allow us to express labor and capital demand in terms of prices:

$$n^*_i = (z_i A\phi)^{1/(1-\phi)} \left(\frac{\alpha}{r_k}\right)^{\alpha\phi/(1-\phi)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha\phi)/(1-\phi)},$$
$$k^*_i = (z_i A\phi)^{1/(1-\phi)} \left(\frac{\alpha}{r_k}\right)^{[1-(1-\alpha)\phi]/(1-\phi)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)\phi/(1-\phi)}.$$

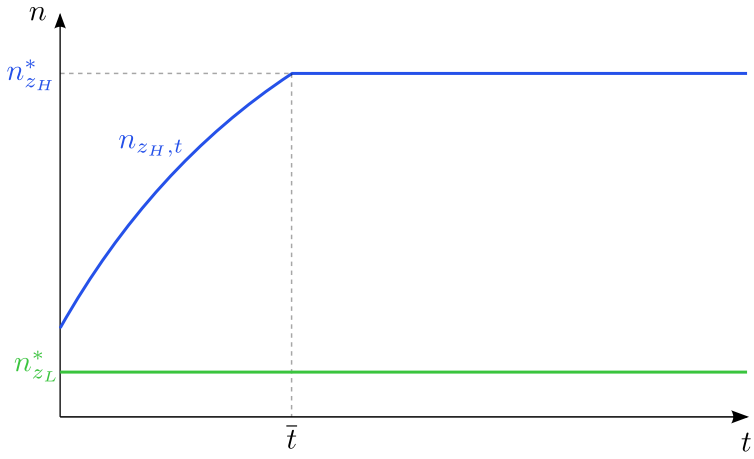
The optimal capital and labor choices are $k_{i,t} = \min\left\{\overline{k}_{i,t}, k^*_i\right\}$ and $n_{i,t} = \min\left\{\overline{n}_{i,t}, n^*_i\right\}$, where $\overline{k}_{i,t}, \overline{n}_{i,t}$ are the constrained optimal choice of capital and labor.

2.6 Comparative statics with fixed wages

With analytical solutions for capital and employment, we can now consider the effect of changes in aggregate productivity A and the financial constraint χ while holding wages w fixed. We first consider the life-cycle behavior of firms with differing permanent productivities to demonstrate the effect of productivity and financial shocks, and then analyze how these shocks aggregate over the age distribution of firms to determine overall employment and job flows.

[19] The fact that $a_{i,t}$ increases with χ before the firm becomes unconstrained is nontrivial. There are two competing forces. Firstly, higher values of χ imply that the firm can rent more capital which acts to increase assets $a_{i,t}$. Secondly, the higher capital and diminishing returns to capital imply that the growth rate of $a_{i,t}$ is lower for a given level of $a_{i,t}$. We show that the second effect does not reverse the first effect before the firm grows out of its financial constraint.

Figure 2: Firm employment dynamics



The figure shows the employment dynamics of two firms with different levels of firm-specific permanent productivity z conditional on surviving to certain age. The figure is plotted for fixed prices.

Figure 2 shows the firm-level employment dynamics conditional on survival for two firms with different levels of firm-specific productivity. The more productive firm has insufficient assets to jump to its optimal employment level $n^*(z_H)$ and has to accumulate assets over time. By contrast, the low-productivity firm has sufficient capital initially to jump to its optimal level of employment $n^*(z_L)$. Figure 2 starkly demonstrates how a financial shock has a stronger effect an young and medium sized firms. The financial constraint is irrelevant for the small low productivity firms at any age. By contrast, for growing high-productivity firms, the financial constraint impacts their rate of growth while leaving the unconstrained optimal level of employment unchanged.

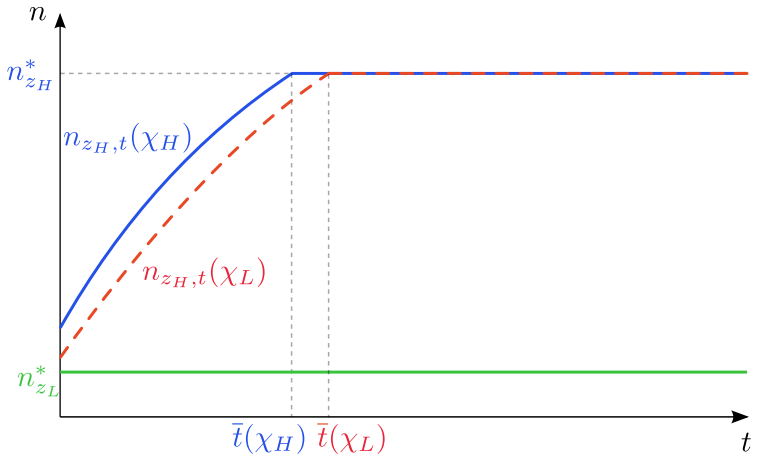
Let $\bar{t}$ denote the moment in time when a firm grows out of its financial constraint (assuming that the firm was financially constrained at the beginning of its life).[20] We can now compare two stationary equilibria with different levels of financial constraint parameter $\chi_L < \chi_H$. Based on our previous discussion, we can show that $n_{i,t}(\chi_L) \leq n_{i,t}(\chi_H)$ and the inequality is strict when financial constraint binds. Morevoer, we can show that it takes more time to grow out of financial constraints for a firm in an economy with tighter financial conditions, i.e. $\bar{t}(\chi_H) < \bar{t}(\chi_L)$. These results are shown in Figure 3. Because the optimal unconstrained size of the firm is unchanged, job creation at any given firm is unchanged over its lifecycle conditional on surviving long enough to reach its optimal size.

The individual firms' behavior under different χ's implies a straightforward aggregation across firms.[21] First, employment at the unconstrained firms does not depend on χ. Because it takes more time to reach the optimal employment level with a lower χ, the average constrained firm is smaller. Aggregation across all constrained and unconstrained firms immediately implies $n(\chi_L) < n(\chi_H)$,

[20] $\bar{t}$ solves equation $\overline{k}_{i,\bar{t}} = k^*$ which equates optimal unconstrained level of capital to optimal constrained level of capital.

[21] The details are summarized in Appendix A.4.

Figure 3: Firm employment dynamics: comparative statics with respect to χ



The figure shows how the employment paths for two firms with different levels of firm-specific permanent productivity z depend on the level of financial constraint parameter χ.

where $n(\cdot)$ denotes aggregate employment. Second, job destruction in the simple model only occurs when a firm exits. Job destruction is lower in a stationary equilibrium with a lower χ because the typical exiting firm is smaller and firms exits are i.i.d. Finally, in a stationary equilibrium, job creation must equal job destruction, implying that a decline in χ also lowers job creation.

Aggregate productivity A has a qualitatively different effect on employment paths of firms relative to χ. While the unconstrained optimal size of firms is independent of the financial constraint, productivity directly affects the optimal size. So, lower aggregate productivity depresses employment at all ages. Moreover, changes in aggregate productivity will affect both high and low-type firms, and constrained and unconstrained firms. However, like Khan and Thomas (2013), changes in productivity A may interact with the financial constraints to generate asymmetric effects on firm employment across age and size categories.

2.7 Comparative statics with flexible wages

Allowing for wage adjustment does not offset the partial equilibrium effects of a tighter financial constraint. Denote $n^d(w,\chi)$ aggregate demand for labor and $n^s(w)$ aggregate supply of labor. In the absence of wealth effects, labor supply only depends on the real wage. Labor market clearing requires $n(\chi) = n^d\left(w(\chi),\chi\right) = n^s\left(w(\chi)\right)$. This relation holds for any value of χ. Taking the full derivative with respect to χ, we obtain:

$$\frac{dn}{d\chi} = \frac{dn^s(w)}{dw}\cdot\frac{dw}{d\chi} = n_2^d\left(w(\chi),\chi\right)\frac{dn^s(w)/dw}{dn^s(w)/dw - n_1^d\left(w(\chi),\chi\right)} > 0,$$

where $n_1^d[w(\chi),\chi] < 0, n_2^d[w(\chi),\chi] > 0$ are partial derivatives of labor demand with respect to the first and second arguments and $dn^s(w)/dw > 0$. The above formula shows that positive elasticity of labor supply will lead to an increase in the wage w which will reduce labor demand. However,

this effect is not large enough to undo the direct effect of an increase in labor demand when χ increases.

Because firm exits are exogenous and i.i.d. across firms, aggregate job destruction is proportional to employment: $JD = \sigma n(\chi)$. This implies that tighter financial constraints lower job destruction. In stationary equilibrium, job creation equals job destruction and therefore job creation also falls when financial constraints tighten. While the financial constraint operates in a fairly stark and perhaps obvious manner in our simple model, idiosyncratic productivity shocks ensure that a substantial portion of job creation and destruction in our quantitative model comes from unconstrained firms. Moreover, general equilibrium and age-specific exit rates could, in principle, even lead a tightening of collateral constraints to *increase* job creation and destruction even in absence of idiosyncratic shocks. We must turn to a richer model to examine whether jobs flows can be useful in identifying aggregate shocks.

3 Benchmark Model

A quantitative examination of the effect of financial shocks on job flows requires us to extend our model along several dimensions. To facilitate these extensions, we shift to discrete time, and build on the simple model by adding firm-specific transitory productivity shocks to allow us to match the level of aggregate job flows in the data.[22] Firm exit rates are now made age dependent for young firms to match the declining hazard rate of firm exit. We also add a discount rate shock ω that increases the required return on capital. Like aggregate productivity shocks, this shock affects both constrained and unconstrained firms.[23] We study the transitional dynamics of job flows and employment following unexpected one-time permanent shocks to aggregate productivity, the financial constraint, or discount rate parameter ω.

Time is discrete and is indexed by t. We introduce a transitory firm-specific component of productivity $\epsilon_{i,t}$ that follows a Markov process which takes values in $\{\epsilon_1, \epsilon_2, \ldots, \epsilon_l\}$ and has conditional distribution $G(\epsilon_{i,t+1}|\epsilon_{i,t})$ with newly-born firms drawing from productivity distribution $G_0(\epsilon_{i,0})$. The permanent, firm-specific component of productivity z_i takes values in $\{z_1, z_2, \ldots, z_m\}$.

[22] See **?** and Cooley and Quadrini (2001) for early examples of firm dynamics models that examine implications of financial frictions for firm growth.

[23] See related literature by Hall (2014) and Kehoe, Midrigan and Pastorino (2014) for models that emphasize the role of discount rate shocks in explaining the Great Recession.

3.1 Households

The household problem is a discrete time analogue to (1)-(3) and can be summarized as follows:

$$\max_{c_t, n_t, \widetilde{a}_{t+1}} \sum_{t=0}^{\infty} \beta^t u\left[c_t - v(n_t)\right],$$
$$\text{s.t.} \quad c_t + \widetilde{a}_{t+1} = w_t n_t + (1+r)\widetilde{a}_t + \Pi_t.$$

Households choose consumption c_t, labor supply n_t, and next period assets $\widetilde{a}_{t+1}$ subject to the natural debt limit constraint and a standard household budget constraint. In our quantitative experiments, we assume that the real interest rate r is constant (small open economy assumption).[24] Denote the optimal solution to household problem as $c_t(\widetilde{a}), n_t$ - labor supply does not depend on the initial level of assets in the case of the GHH preferences.

3.2 Firms

The firm's problem is the discrete time analogue to (4)-(7). In contrast to the simple model, firms face transitory firm-specific productivity shocks $\epsilon_{i,t}$, and the firm exit rate σ_τ depends on age τ. Formally, firms solve:

$$\max_{k_{i,t}, n_{i,t}, a_{i,t+1}} \mathbb{E}_0 \left[\Lambda_{i,t} a_{i,t} \sigma_t \prod_{\tau=1}^{t-1}(1-\sigma_\tau)\right],$$
$$\text{s.t.} \quad a_{i,t+1} = A z_i \epsilon_{i,t} \left(k_{i,t}^{\alpha} n_{i,t}^{1-\alpha}\right)^{\phi} - r_{k,t} k_{i,t} - w_{i,t} n_{i,t} + (1 + r_{i,t}) a_{i,t},$$
$$k_{i,t} \leq \chi a_{i,t}.$$

Firms maximize the expected value of their terminal wealth with firm i choosing capital $k_{i,t}$, next period assets $a_{i,t+1}$, and employment $n_{i,t}$ subject to an accumulation equation for assets and the same financial constraint on renting capital as described in the previous section. Firms enter and exit exogenously, and we assume the number of newly born firms M_0 is equal to the number of exiting firms so that the total number of firms is constant. Let $n_t(a, \epsilon, z, \tau), k_t(a, \epsilon, z, \tau)$ be labor and capital of firms with assets a, temporary and permanent idiosyncratic components of productivity ϵ, z, and age τ.

The way in which the model captures consumer deleveraging deserve further discussion. In Appendix B, we provide a simple extension to the supply side of our model that demonstrates that markup shocks are isomorphic to productivity shocks. While our model does not feature any nominal rigidities, demand shocks in a model with nominal frictions impact firm's incentives to hire capital and labor by raising or lowering the markup. We also consider a more elaborate extension of our model in Appendix B that explicitly incorporates consumer deleveraging as in

[24] The constant real interest rate can alternatively be motivated by the zero lower bound on the monetary policy rate together with tightly anchored inflation expectations.

Midrigan and Phillippon (2016). We demonstrate that a shock to household credit that induces consumer deleveraging is isomorphic to the markup or productivity shock. Given this equivalence, the productivity shock captures the relevant effects on the firm of a shock to household credit.

Additionally, one may be concerned that the way the shock to firm credit is modeled may understate its effects on job destruction. Following the failure of Lehman Brothers in 2008, short-term credit markets including commercial paper experienced acute disruptions. However, these markets normalized quite rapidly and are unlikely to account for the multiyear decline in job flows in which we are interested. A recent literature (for example, **?**) has drawn a distinction between financial shocks we consider here and these shorter term liquidity shocks. In Appendix E, we present an extension to the baseline model that shows that these shocks are distinct in their effects on employment across firm size and age.

3.3 Intermediaries

Financial intermediaries are perfectly competitive and operate as described in the previous section. The zero-profit condition for intermediaries requires:

$$r_{k,t} = r + \delta + \omega$$

where ω is a discount rate shock that widens the wedge between the return on deposits r and the return on capital r_k. Equivalently, this wedge could be considered a "flight to safety" shock as investors pay a premium for safe assets.[25] Excess returns on capital earned by financial intermediaries are rebated lump sum to the representative household.

3.4 Equilibrium

Denote $\Psi_t(a, \epsilon, z, \tau)$ the distribution of firms over their assets a, firm-specific transitory and permanent productivity ϵ, z, and age τ. The firms optimal transition of bond holdings, and the Markov process for transition of temporary idiosyncratic component of productivity ϵ give the transition probability of states (a, ϵ). This transition probability together with Ψ_t yields Ψ_{t+1}.

An equilibrium is a sequence of prices $\{w_t, r_{k,t}\}$, a sequence of consumption $\{c_t(\widetilde{a})\}$, labor supply $\{n_t\}$, labor demand $\{n_t(a, \epsilon, z, \tau)\}$ and capital demand $\{k_t(a, \epsilon, z, \tau)\}$ and sequence of distributions $\{\Psi_t\}$ such that, given initial distribution Ψ_0:

1. $\{c_t(\widetilde{a})\}, \{n_t(\widetilde{a})\}, \{n_t(a, \epsilon, z, \tau)\}$, $\{k_t(a, \epsilon, z, \tau)\}$ are optimal given $\{w_t, r_{k,t}\}$,
2. intermediaries optimize $r_{k,t} = r + \delta + \omega$,
3. labor market clears: $n_t = \int n_t(a, \epsilon, z, \tau) d\Psi_t(a, \epsilon, z, \tau)$,
4. Ψ_t is consistent with the optimal behavior of firms.

[25] See, for example, Christiano, Eichenbaum and Trabandt (2015) who introduce a financial wedge in the household's capital Euler equation.

4 Calibration and Quantitative Predictions of the Model

We calibrate our benchmark model and examine the effect of one-time unanticipated and permanent financial and aggregate productivity shocks in our model on overall job flows and the distribution of job creation and job destruction across firm size and firm age categories along the transition path. We show how these shock operate on distinct margins of employment and impact job flows differentially across firm age and size. Using these differences, we estimate the set of shocks that best matches the behavior of overall job flows and job flows by age in the Great Recession and discuss the implications for US employment.

4.1 Calibration Strategy and Targets

Our calibration strategy chooses several common parameters from the literature. Given that our empirical evidence on job flows is observed in annual data, we use annual values for several common parameters. As shown in Table 1, the household's discount rate β and the capital share α are standard. The depreciation rate of capital δ is set to match the depreciation rate for equipment. The parameter ϕ governing the degree of decreasing returns to scale is set at 0.95, comparable to values chosen in Cooley and Quadrini (2001) and Khan and Thomas (2013). We experiment with several different values for the Frisch elasticity ν to gauge the importance of labor supply response in our quantitative analysis. A Frisch elasticity of zero conforms to the case of a vertical labor supply curve, while an infinite Frisch elasticity conforms to the case of a horizontal labor supply curve. In the former case, wages adjust so that total employment is unaffected by the collateral shock, while, in the latter case, wages are unchanged so employment is demand determined. In effect, this case illustrates the partial equilibrium effect of the collateral shock. In our preferred calibration, we choose a Frisch elasticity of $\nu = 1$ - within the range of typical Frisch elasticities in the macro literature.

Table 1: Calibration values

Aggregate Parameters		Value
Discount rate	β	0.99
Depreciation rate	δ	0.07
Capital share	α	0.3
Decreasing returns	φ	0.95
Frisch elasticity	ν	0;1;∞
Initial assets	a_0	8
Collateral constraint	χ	8
Transitory shock (size)	ε	0.025
Transitory shock (persistence)	ρ_ε	0.6

This table describes the model parameters and the values chosen in our calibration. The calibration strategy and targets are described in the text.

Table 2: Idiosyncratic shock calibration

Size Bins (# empl.)	Employment distribution in % (Data)	Firm distribution in % (Model)
1-4	2.5	44.0
5-9	3.6	22.5
10-19	5.1	15.3
20-49	8.5	9.8
50-99	7.1	4.1
100-249	9.8	2.5
250-499	7.0	0.8
500-999	6.4	0.4
1000-2499	8.6	0.2
>2499	41.4	0.5

The middle column of this table presents the distribution of employment across firm size bins for firms between 21-25 years of age in the BDS (2000-2006 averages). This distribution of the permanent component of productivity required to match this distribution of employment is given in the right-hand column.

It remains to choose an initial level of assets a_0, the collateral constraint parameter χ, firm exit rates σ_τ, and a support and distribution of idiosyncratic productivity levels ϵ. We select the distribution of the idiosyncratic productivity levels to target the distribution of employment by firm size of mature firms in the data. In our model, firms that survive long enough converge towards their optimal level of employment. We take averages of employment share by firm size categories for firms over 21 years of age in the Business Dynamics Statistics (BDS) from 2000-2006, and we back out the implied level of idiosyncratic productivity so that the optimal size of the firm is at the midpoint of the employment bin range. We choose the probability distribution of firms over idiosyncratic productivity levels to target the share of employment by firm size in the data. Table 2 shows the size bins used and the employment shares that our calibration targets. The last column shows the implied distribution of firms that matches the employment shares we are targeting, showing that most firms are small, low-productivity firms.

We choose time-dependent exit rates for the first five years and a constant exit rate for firms older than five years to capture the declining hazard of firm exit.[26] We choose entry and exit rates to match the empirical age distribution of firms using 2000-2006 averages from the BDS. Table 3 provides the age distribution of firms and the distribution implied by our calibration. By construction, the empirical distribution and model distribution match exactly for firms aged 0-5, but differs for older ages when a constant exit rate is assumed. The exit rate for firms older than age 5 is $\sigma = 0.069$ and implies a model age distribution that closely matches the empirical distribution.

[26] With an endogenous firm exit margin, selection effects would generate this declining hazard for young firms without the need for exogenous differences in exit rates.

Table 3: Exit rate calibration

Firm Age (years)	Firm distribution in % (Data)	Firm distribution in % (Model)
0	10.2	10.2
1	7.7	7.7
2	6.6	6.6
3	5.8	5.8
4	5.2	5.2
5	4.7	4.7
6-10	17.9	18.0
11-15	12.6	12.6
16-20	9.7	8.8
21-25	6.4	6.2
>25	13.4	14.3

This table displays the distribution of firms in the data and the model. The empirical age distribution of firms was computed using 2000-2006 averages from the BDS. Age-specific exit rates are chosen to exactly match the exit rates in the data for firms 5 years old or younger. A constant exit rate is assumed for firms older than five years. The implied distribution of employment for firms older than 5 years is given in the right-hand column of the table.

The final parameters that we choose are the initial level of assets a_0 and the collateral constraint parameter χ. We jointly choose these parameters (shown in Table 1) to best match the distribution of employment by firm age and size. The empirical and model distributions are given in Table 4. Our calibration closely matches the age distribution of employment and does a reasonable job matching the size distribution of employment. Our calibration has somewhat larger share of employment at middle sized firms (20-99 emps.) and, consequently, too low employment at large firms. In our baseline calibration, 97% of firms have less than 100 employees and 12% of firms are credit-constrained. Most constrained firms are small/medium-sized firms (91%), but the vast majority of small and medium-sized firms are financially unconstrained (89%).

For tractability and simplicity, we assume that firms face transitory shocks around their permanent level of idiosyncratic productivity. That is, firms are born with a permanent productivity level that determines the firm's optimal size and experience small shocks around this permanent productivity level. Transitory shocks evolve according to a symmetric three state Markov chain: transitory productivity can be high, neutral, or low. This specification ensures that adding transitory shocks only results in two additional parameters: the size of the shock and the persistence of the shock.

The size of the transitory shock is set at 2.5% to target job flows of 15% of employment, matching averages in the BDS from 2000-2006. The persistence of the idiosyncratic shock is set at 0.6 (the diagonal elements of the matrix of transition probabilities); this value is in line with

estimates for the annual persistence of idiosyncratic productivity shocks used in Khan and Thomas (2013) and Clementi and Palazzo (2016). For example, if current productivity is at its neutral level, the firm remains at the same level of productivity with probability 0.6 and transitions (annually) to either high or low productivity with probability 0.2 respectively.

Table 4: Distribution of employment by firm size and age

	Employ-ment	Job Creation	Job Destruction		Employ-ment	Job Creation	Job Destruction
Panel A: data							
Births	2.8	18.5	0.0	1-19 emps	19.3	28.0	23.0
1-5 years	13.2	16.5	15.7	20-99 emps	17.8	17.5	18.1
6+ years	84.0	65.0	84.3	100+ emps	62.9	54.4	58.9
Panel B: model							
Births	3.0	20.3	0.0	1-19 emps	18.9	33.8	23.4
1-5 years	13.1	15.2	16.5	20-99 emps	22.1	19.7	21.6
6+ years	84.0	64.5	83.5	100+ emps	59.0	46.5	55.0

Panel A of the table presents the distribution of employment and job flows over firm size and age categories in % in Business Dynamics Statistics database over 2000-2006. Panel B shows the distribution of employment and job flows over firms size and age categories in % in steady state of the baseline calibration of our model.

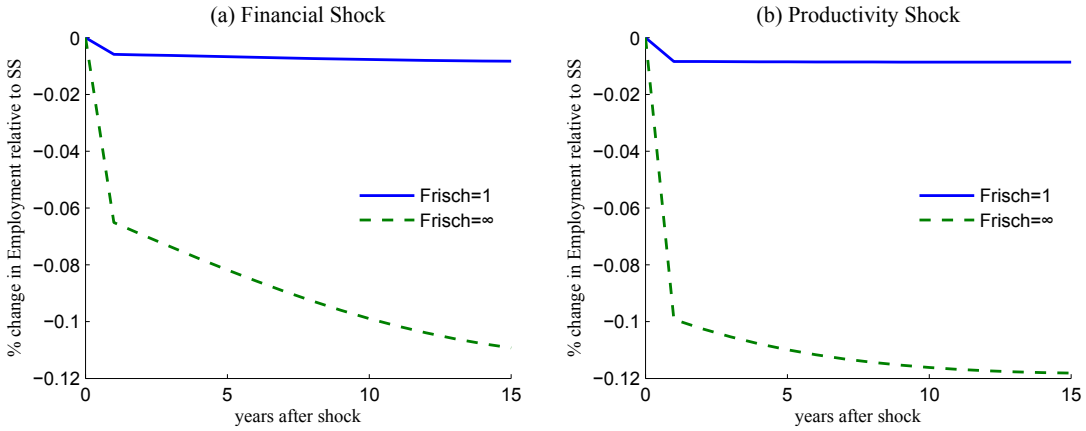
Table 4 summarizes the fit of our model with data in terms of the distribution of employment and job flows. The left panel compares the fit across firm age categories while the right panel compares the fit across firm size categories. In should be emphasized that these job flows moments are untargeted, and our calibration demonstrates that a parsimoniously parameterized model can a good job fitting the data for both employment and job flows across age and size categories.The model does an excellent job of matching the distribution of employment and job creation across firm age categories. Our calibration also performs fairly well in matching the distribution of employment and job flows across firm size categories. Job creation is a somewhat too high for small and middle-sized firms and job destruction is also too high at middle-sized firms.

4.2 Effect of Collateral and Productivity Shocks

We first consider the transition path after a permanent 20% tightening of the collateral constraint parameter from $\chi = 8$ to $\chi = 6.4$. This tightening conforms to the magnitude of the drop experienced in US housing prices during the Great Recession. The collateral shock is modeled as a permanent shock given the persistence of the drop in nominal US housing prices, with prices five years since the start of the recession still 20-25% below their peak. The results we present are unchanged for persistent shocks that last five years or longer and then gradually normalize.

The left panel of Figure 4 displays the transition paths for employment under a financial shock, while the right panel illustrates the same path for a productivity shock that generates a similar long-

Figure 4: Employment transition paths with permanent financial and productivity shocks



The figure displays the transition paths for employment under financial and productivity shocks. The size of productivity shock is chosen to deliver a similar decline in employment as under the financial shock for the case of Frisch elasticity of 1.

run decline in employment. We display transition paths holding wages constant and incorporating wage adjustment with a Frisch elasticity of $\nu = 1$.

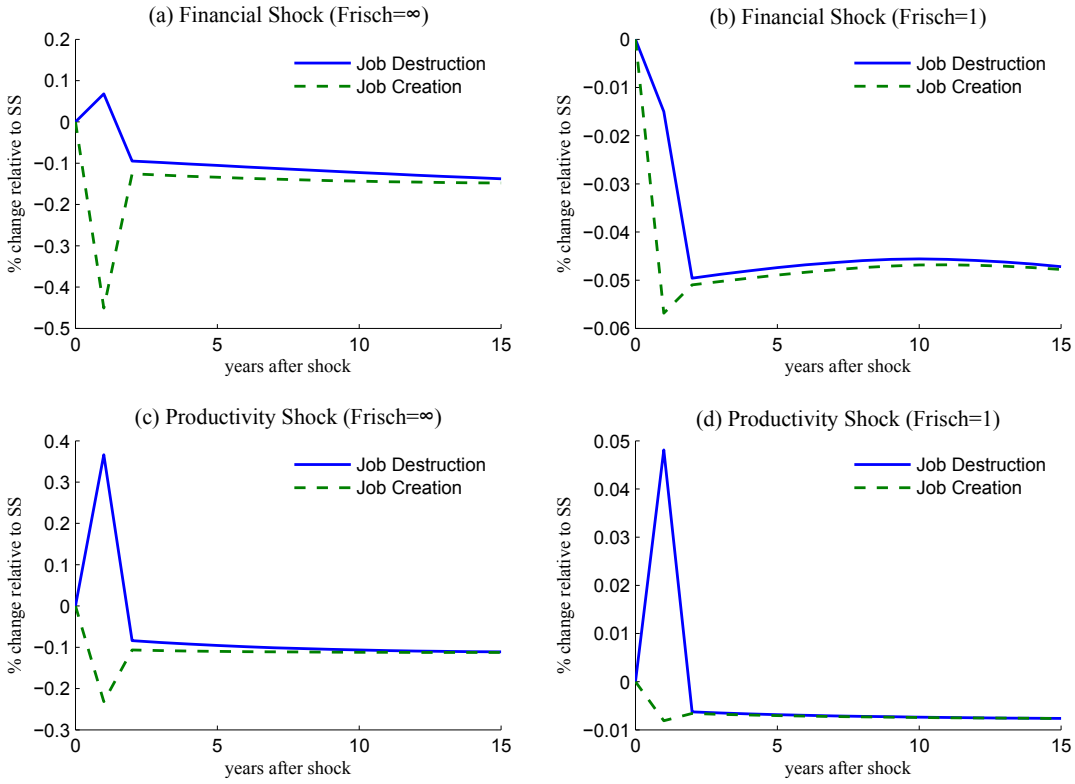
As the transition paths illustrate, both permanent financial and productivity shock generate similar effects on employment on impact. The firm dynamics model generates some endogenous propagation in subsequent periods as the financial shock reduces employment in subsequent periods as effects filter through the age distribution. Due to the tightened collateral constraint, large firms that exit are replaced by smaller firms reducing overall employment over the transition. With wage adjustment, this effect is somewhat offset.

Figure 5 displays the transition paths for gross job creation and job destruction under the financial and productivity shocks. The financial shock, shown in panels (a) and (b), reduces employment by sharply reducing job creation, while the productivity shock, shown in panels (c) and (d) reduces employment through a sharp increase in job destruction. The partial equilibrium effect of the financial shock on job creation in panel (a) is particularly stark with job creation dropping sharply relative to steady state while job destruction increases only slightly. Wage adjustment in panel (b) reduces the large effect of a financial shock on job creation to a plausible magnitude but preserves the relatively larger effect of financial shocks on gross job creation.[27] In Appendix D, we also consider the transition path for job flows for a discount rate shock to proxy for the effect of various types of demand shocks.[28]

[27] If both wages and interest rates adjust along the transition path, the fall in factor prices may be large enough to cause aggregate job creation to rise. However, the patterns across age and size that we emphasize below are preserved - job creation falls at financially constrained firms.

[28] In a model with sticky prices, a monetary policy shock or other demand shock would affect firm markups and the real interest rate. Our productivity shock is isomorphic to a markup shock if retailers costless differentiate a homogenous intermediate good and sell to consumers. We also derive explicitly in the appendix an equivalence between a shock to household credit that induces consumer deleveraging to the productivity shock considered here.

Figure 5: Job flows transition paths with permanent financial and productivity shocks



The figure displays the transition paths for gross job creation and job destruction under the financial and productivity shocks. The numbers plotted display changes relative to the initial (steady state) levels. For example, job creation declines by 45% on impact after the financial shock in case of infinite Frisch elasticity. The effects of the financial shock are shown in panels (a) and (b), while the productivity shock effects are shown in panels (c) and (d).

A financial shock reduces employment by reducing job creation as both new firms are smaller and existing firms grow less in the next period. Job destruction also increases because a tighter financial constraint leads some growing firms to reduce employment in the next period. A productivity shock reduces employment by increasing job destruction since all unconstrained firms contract in size, while exerting a smaller effect on the path of employment growth of those firms still growing out of their financial constraint. As we show in Appendix D, a discount rate shock has similar effects on aggregate job flows to a productivity shock, reducing employment primarily via the job destruction margin.

This pattern fits the response of job flows in the last two recessions as seen in Figure 1 - the 2001 recession was characterized by a relatively sharp response of job destruction, while the 2008 recession was characterized by a strong response of job creation. Indeed, in 2008, job destruction did not exceed the levels reached in 2001 in a much milder recession. These findings are also consistent with the conclusions reached by Foster, Grim and Haltiwanger (2014) who use state level data to observe that total reallocation (sum of job creation and job destruction) fell in the Great Recession in contrast to the the three other recessions since 1980 in their data. As Figure 1

illustrates, TFP shocks raise total reallocation while financial shocks show an outright decline. This decline in total reallocation reduces aggregate productivity growth as employment is reallocated to smaller, low-productivity firms who are financially unconstrained.

After the impact period, job creation from a financial shock falls below its steady state level and converges towards a lower level. Job destruction behaves similarly converging towards the lower level of job creation. While permanent productivity shocks also reduce job flows in the long-run, financial shocks result in a larger reduction in job flows; in the case of constant wages, job flows fall by 11% of steady state levels under a permanent financial shock while job flows fall by 9% of steady state levels - somewhat less - under a productivity shock.

Table 5: Effect of shocks on job flows

		Permanent Financial Shock			Permanent Productivity Shock		
		Frisch = ∞	Frisch = 0	Frisch = 1	Frisch = ∞	Frisch = 0	Frisch = 1
Panel A: Job Creation							
	Aggregate	-0.24	-0.04	-0.05	-0.15	0.00	-0.01
Age	Births	-0.12	-0.07	-0.07	-0.07	0.00	0.00
Age	1-5 years	-0.47	-0.31	-0.32	-0.12	0.01	0.00
Age	6+ years	-0.23	0.02	0.00	-0.18	0.00	-0.01
Size	1-19 emps	-0.07	0.05	0.04	-0.06	-0.01	-0.01
Size	20-99 emps	-0.33	-0.11	-0.12	-0.17	0.02	0.00
Size	100+ emps	-0.35	-0.08	-0.10	-0.21	0.01	-0.01
Panel B: Job Destruction							
	Aggregate	-0.04	-0.04	-0.04	0.06	0.00	0.01
Age	1-5 years	-0.07	-0.06	-0.07	0.02	0.00	0.01
Age	6+ years	-0.04	-0.03	-0.03	0.07	-0.01	0.01
Size	1-19 emps	0.00	0.02	0.02	0.07	-0.01	0.01
Size	20-99 emps	-0.07	-0.07	-0.07	0.06	0.00	0.01
Size	100+ emps	-0.05	-0.05	-0.05	0.07	-0.01	0.01

The table displays the effect of permanent negative financial shock (a 20% decline in collateral constraint parameter) and permanent negative productivity shocks (the size of the shock produces the same long-run decline in aggregate employment level) on the distribution of job creation and job destruction by firm age and size categories. The table shows the average effect over the first three years after the shock with job flows expressed as changes relative to the initial (steady state) level. For example, the aggregate job creation declines by 24% on average over the first three years after the shock. The first three columns display the job flows effects of a permanent financial shock, while the last three columns display the job flows effects of a permanent productivity shock. Each column conforms to different values for the Frisch elasticity: an infinite Frisch elasticity (rigid wages), zero Frisch elasticity (vertical labor supply), and the last column is our preferred specification.

Table 5 displays the effect of financial and productivity shocks on the distribution of job creation and job destruction by firm age and size categories. The table shows the average effect over the first three years after the shock where job flows are expressed as percentage changes from their initial (steady state) level. The left-hand side displays the job flows effects of a permanent financial

shock, while the right-hand side displays the job flows effects of a permanent productivity shock. The three columns in each panel conform to different values for the Frisch elasticity: first column is the case of a infinite Frisch elasticity (rigid wages), the second column is the case of a zero Frisch elasticity (vertical labor supply), and the last column is our preferred specification.

4.2.1 Age and Size Effects

The effect of a collateral shock on job creation is strongest at young firms, followed by new firms, with mature firms exhibiting the weakest response (as long as wages adjust). The job creation response is relatively stronger at young firms as opposed to new firms due to the absence of any extensive margin response. An endogenous entry decision would likely amplify the fall in job creation at new firms. General equilibrium effects are crucial for the finding that job creation at new and young firms falls more than mature firms. In partial equilibrium, the financial shock has a large effect on job creation at mature firms since the highest productivity firms remain financially constrained even after 5 years. However, when wages adjust, the unconstrained mature firms create jobs thereby offsetting the decline in job creation at financially constrained, mature firms. Productivity shocks, by contrast, have their largest effects on job creation at mature firms but overall effects are fairly uniform.

Across firm size categories, our model predicts that financial shocks will have the largest effect on job creation at medium-size firms (20-99 employees) followed by large firms (100+ employees) and small firms (1-19 employees). This perhaps counterintuitive result stems from the fact that the collateral constraint is most important for firms with relatively higher levels of productivity. Low productivity firms with a small optimal size are largely unaffected by a tightening of financial constraints. Rather, when wages fall, small firms create job since their optimal size expands with lower wages. Relatively high productivity firms that start small transit through the medium-sized category; this size category is the best proxy for financially constrained firms. Job creation falls less at large firms because unconstrained large firms create jobs that offset the decline in job creation at the large constrained firms. Once again, productivity shocks generate largely uniform effects on job creation across size.

Our model predicts that job destruction falls at both young and mature firms with a relatively larger response at young firms. On impact, job destruction rises at both young and mature firms since tighter financial constraints lower capital and labor demand. After impact, destruction falls at young firms because they become smaller after the collateral shock. Therefore, the jobs destroyed by these firms when they exit also fall. By contrast, for mature firms, there are two competing effects after impact: given exogenous exit rates, fewer firms survive to their optimal size reducing job destruction, however, as wages fall, optimal size increases for unconstrained firms leading to greater job destruction when these firms exit. The job destruction patterns for a financial shock by

firm age largely mirror the job creation patterns. Job destruction falls at medium sized firms since these firms are smaller after the financial shock and destroy fewer jobs during exit. By contrast, productivity shocks generate much more uniform effects across firm size with both the sign and ordering contrasting with a financial shock. As we will show in the next section, these patterns (signs and orderings) are consistent with the empirical response of job flows by firm age and size in response to shocks in housing prices.

4.2.2 Net Employment Effect

Financial and productivity shocks also generate disparate effects across age and size on employment. As Table 6 shows, a financial shock generates differential effects on employment across firm age - employment falls most at new and young firms relative to mature firms after a financial shock. By contrast, a TFP shock has the strongest effect on mature firms with relatively weaker effects on new and young firms. The employment effects of a financial shock by firm age are consistent with the findings of Siemer (2013) who documents a decline in employment growth at young firms during the Great Recession.

Table 6: Effect of shocks on net employment by firm age and size

		Financial Shock			Productivity Shock		
		Frisch = ∞	Frisch = 0	Frisch = 1	Frisch = ∞	Frisch = 0	Frisch = 1
Employment							
	Aggregate	-0.07	0.00	-0.01	-0.10	0.00	-0.01
Age	Births	-0.12	-0.07	-0.07	-0.07	0.00	0.00
	1-5 years	-0.14	-0.09	-0.09	-0.07	0.00	0.00
	6+ years	-0.06	0.02	0.01	-0.11	0.00	-0.01
Size	1-19 emps	0.02	0.09	0.09	-0.03	-0.01	-0.01
	20-99 emps	-0.10	-0.03	-0.03	-0.11	0.00	-0.01
	100+ emps	-0.09	-0.02	-0.03	-0.12	0.00	-0.01

The table displays the effect of permanent negative financial shock (a 20% decline in collateral constraint parameter) and permanent negative productivity shocks (the size of the shock produces the same long-run decline in aggregate employment level) on the distribution of employment by firm age and size categories. The table shows the average effect over the first three years after the shock with employment expressed as changes relative to the initial (steady state) level. For example, the aggregate employment declines by 7% on average over the first three years after the shock. The first three columns display the employemnt effects of a permanent financial shock, while the last three columns display the employment effects of a permanent productivity shock. Each column conforms to different values for the Frisch elasticity: an infinite Frisch elasticity (rigid wages), zero Frisch elasticity (vertical labor supply), and the last column is our preferred specification.

The differences between financial shocks and TFP shocks across firm size categories are harder to discern. As Table 6 shows, both types of shocks reduce employment the most at relatively large firms. In the case of a financial shock, employment rises slightly at the smallest firms while falling at middle-sized and large firms. A productivity shocks results in a similar ordering of employment responses across firm size categories showing that firm size is a less robust indicator of financial

constraints than firm age and highlighting the importance of decomposing employment into the creation and destruction margins to distinguish disruptions in credit supply from other types of shocks. It is worth noting that our employment effects by firm size in response to a TFP shocks are consistent with the findings of Moscarini and Postel-Vinay (2012) who find that employment responds more strongly at large versus small firms in recessions.

4.3 Shocks Decomposition

Given that financial shocks and productivity shocks impact employment via distinct margins, we can use aggregate job flows to decompose the effect of each of those factors on the decline in employment experienced in the US during the Great Recession. As we saw in Figure 5, financial and productivity shocks that lead to the same long-run decline in employment have dramatically different effects on aggregate job flows. This differential behavior of the two shocks on job flows allows us to identify their magnitudes.

In this decomposition, we go one step further by also estimating the contribution of a discount rate shock. We consider the effect of a one-time unexpected permanent increase in ω from 0 to some positive value. This shock increases the rental rate of capital, making it costlier for firms to rent capital. In aggregate, this shock has a similar effect on job flows as the negative productivity shock: it reduces employment by mostly increasing job destruction relative to job creation. However, there are differences in the effect of productivity and discount factor shocks on job flows across firm age and size.

A negative productivity shock has a larger negative effect on job creation by young firms relative to discount rate shock. A productivity shock directly affects all of the firms in the economy while discount factor shocks has a direct effect *only* on unconstrained firms. The constrained firms optimal capital demand is $k_{i,t} = \chi a_{i,t}$, and it does not directly depend on the rental rate of capital: firms are willing to rent more capital at the prevailing rental rate but are unable to do so. The difference in the effects of productivity and discount rate shocks on job flows across different categories will allow us to separately identify these two shocks.

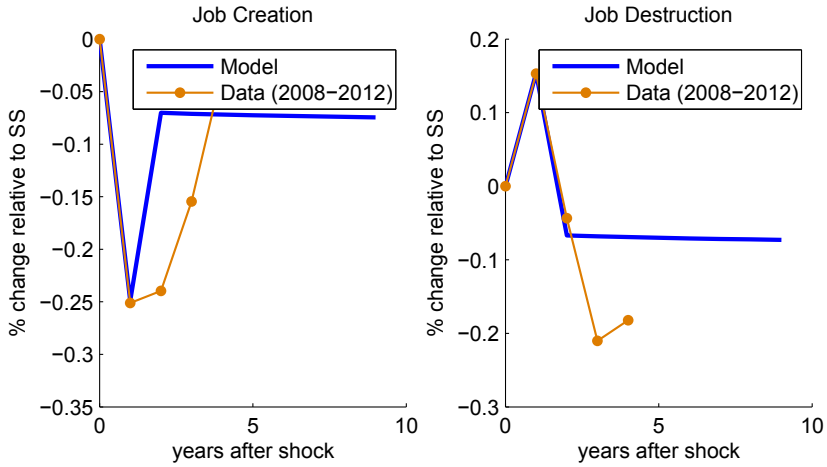
We nonlinearly estimate a financial, productivity, and discount rate shock on a grid to best match initial changes in aggregate job flows and job flows across firm age categories in the Great Recession in the US.[29] It is clear from Figure 1 that a sharp decline in aggregate job creation and

[29] Formally, we minimize the following objective by choosing the three shocks on a grid:

$$O = \left(\Delta \log JC_{model} - \Delta \log JC_{data}\right)^2 + \left(\Delta \log JD_{model} - \Delta \log JD_{data}\right)^2 + \Omega \sum_i \mu_i \left(\Delta \log JC^i_{model} - \Delta \log JC^i_{data}\right)^2 + \Omega \sum_j \mu_j \left(\Delta \log JD^j_{model} - \Delta \log JD^j_{data}\right)^2,$$

where $i \in \{new, young, mature\}$ and $j \in \{young, mature\}$, μ_i is the number of people employed by firms belonging to category i in stationary equilibrium relative to aggregate employment, $\Omega = 0.05$ indexes the importance of matching category specific job flows relative to aggregate job flows. As a result, we pick three shocks to best match seven moments. The small value of Ω ensures that we accurately match the on-impact behavior of aggregate job flows.

Figure 6: Job flows paths in the model and in the data.



The figure displays the paths for gross job creation and job destruction after the financial and productivity shocks in the model and in the data. The numbers plotted display changes relative to the initial (steady state) levels. The two shocks that drive model dynamics are chosen to match the initial changes (between 2008 and 2009) in job flows in the data.

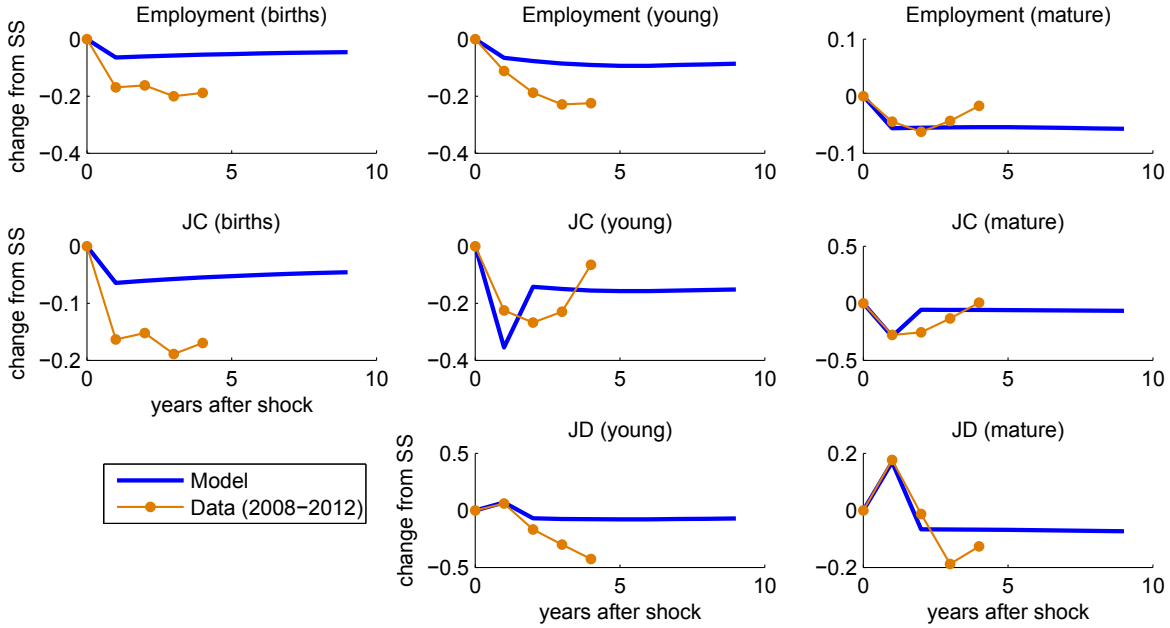
sharp increase in job destruction occurred between 2007Q4 and 2009Q1. In the annualized data coming from the BDS, similar magnitudes of changes in job flows are observed between March 2008 and March 2009. We focus on the initial difference between these two years because we only have annual data for the job flows behavior by firm categories.

To match the initial changes in job flows we estimate a negative productivity shock of 1.4%, a negative financial shock of 21.5%, and a discount rate shock of $\omega = 0.03$. Figure 6 compares the behavior of aggregate job flows in the model and the data. Job flows in the model match the behavior of job flows in the data on impact. Our model captures the initial dynamics of aggregate job flows but underestimates more recent job creation given the assumption of a permanent shock. US job creation recovers somewhat by 2012 as financial conditions have normalized and employment growth accelerated.

Figure 7 compares employment and job flows in our model across different firm age categories to their counterparts in the data. Our model matches the behavior of employment and job flows fairly well. Our model slightly overpredicts the fall on impact in job creation at young firms, but this discrepancy may be related to the particularly stark nature of the financial shock that impacts all young firms at the same time instead of more slowly impacting these firms over time as they seek to secure fresh financing for expansion. The model underpredicts the fall in job creation by new firms. This is likely due to the exogenous entry assumption in the model.

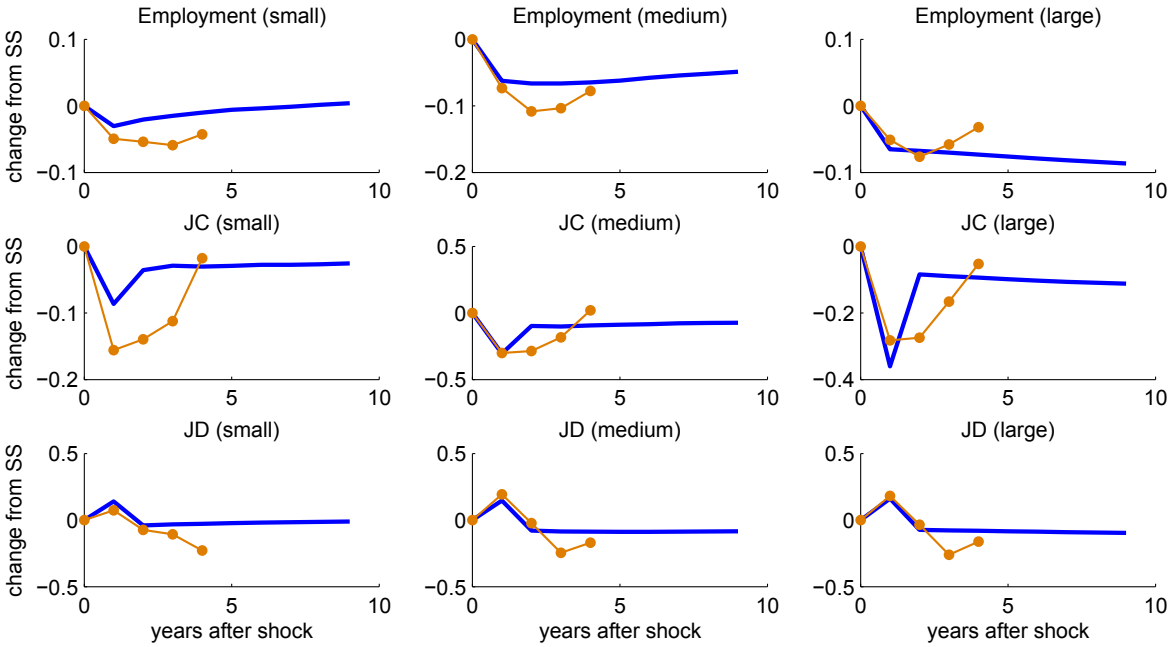
Figure 8 shows the model predictions about employment and job flows dynamics across firm size categories. The model matches the data for employment and job flows dynamics for medium and large firms fairly well. The model somewhat misses the employment and job creation dynamics for small firms. Here, the impact of wage adjustment on small firms is particularly important. As

Figure 7: Job flows paths in the model and in the data across firm age categories.



The figure displays the paths for gross job creation and job destruction after the financial and productivity shocks in the model and in the data across new, young and mature firms. The numbers plotted display changes relative to the initial (steady state) levels.

Figure 8: Job flows paths in the model and in the data across firm size categories.



The figure displays the paths for gross job creation and job destruction after the financial and productivity shocks in the model and in the data across small, medium and large firms. The numbers plotted display changes relative to the initial (steady state) levels.

wages fall, small unconstrained firm increase their optimal size. To the extent that these firms face any fixed costs, our model likely overpredicts the increase in employment and job flows at these firms from falling wages.

Finally, because we use financial, productivity, and discount rate shocks in the model to match the initial changes in the job flows in the recent US recession, we can study the relative importance of these shocks for explaining the employment decline. The ratio of the long-run decline in employment driven only by financial shock to the decline driven by all three shocks is 0.15, the same ratio for productivity shock is 0.3 and for discount rate shock is 0.57.[30] Our decomposition indicates that, despite being fairly large, the disruption to firm credit can explain only about 15% of the decline in employment in the Great Recession.[31] Significant aggregate shocks including the discount factor shock are needed to explain the overall decline in employment. The productivity and discount rate shock used here may themselves be driven by financial factors (productivity may proxy for declines in demand due to consumer deleveraging or the discount rate shock may proxy for a flight to safety), but our estimation concludes that these shocks that broadly impact all firms are needed for explaining the overall decline in US employment.

5 Empirical Strategy and Results

5.1 Empirical Strategy

Our model delivers testable implications for effect of a financial shock on job flows overall and across firm age and size categories. We can validate our model and build confidence the quantitative decomposition by examining whether these relationships hold in cross-city data on job flows in the US. Any test of the hypothesis that an increase in financial frictions diminishes job flows must overcome several challenges of both measurement and causality. Our empirical strategy addresses these issues by using MSA-level variation in job flows and financial conditions to determine the causal effect of firm credit shocks on job flows.

The first issue we confront is finding suitable proxy for financial conditions at the MSA level. Since we lack detailed data on firm balance sheets, we use data on the growth rate of MSA house prices as a proxy for financing conditions. House prices are an appropriate proxy for firm financing conditions for two reasons. First, movements in housing prices can directly affect the financing capacity of firms by either impacting the net worth of entrepreneurs.[32] Second, falling house prices

[30] Observe that the three numbers may not sum up to one because of the nonlinear interaction effects.

[31] In this respect, our findings are closer to the findings of Mian and Sufi (2014) or Greenstone, Mas and Nguyen (2014) who argue that the credit supply channel explains a relatively small part of the decline in US employment.

[32] Fairlie and Krashinsky (2012) provide direct evidence for changes in housing equity on entrepreneurship using data from the Current Population Survey, while Schmalz, Sraer and Thesmar (2013) show in French data that higher house prices increase the probability of becoming an entrepreneur and, conditional on starting a business, increase the initial scale of the firm. Adelino, Schoar and Severino (2013) also document the importance of the collateral channel in the employment at small establishments.

may impair the ability of younger and medium-sized firms to post collateral for bank loans. Recent empirical work by Chaney, Sraer and Thesmar (2012) shows that fluctuations in local house prices even affect larger public companies in Compustat by reducing collateral values.[33] Lastly, changes in housing prices can affect the availability of local bank credit to firms via losses/defaults on a bank's mortgage portfolio. These losses, whether via the household or commercial loan portfolio would raise the cost of bank credit to firms.

In addition to finding a suitable proxy for financial frictions, the relative dearth of job flows data in the time series limits any analysis of the effect of financial frictions on job flows in the aggregate data. Instead, we exploit MSA-level variation in job flows and housing prices to improve the power of our estimates and increase useful variation from state and regional housing booms.

The most significant challenge in establishing a causal effect of housing price movements on job flows is ruling out an aggregate demand channel that drives a correlation between job flows and housing prices. We address this concern in several ways. Firstly, we include location and time fixed effects to account for the business cycle and differences across MSAs in job flows. Secondly, to control for MSA-specific demand shocks, we include controls for the *local* business cycle. Our baseline regression takes the following form:

$$y_{it} = \alpha_i + \delta_t + \gamma(L)\,\Delta GSP_{it} + \beta(L)\,\Delta hp_{it} + \epsilon_{it}$$

where y_{it} is job creation or job destruction for MSA i at time t. ΔGSP_{it} represents the growth rate of the MSA-level business cycle variable, while Δhp_{it} is the growth rate of MSA housing prices. Our coefficient of interest is the sum of the coefficients $\beta(1)$ on MSA housing prices. A positive coefficient indicates that falling house prices decrease job flows over a three-year period (to facilitate comparison with the model responses).

Alternatively, we also consider an IV strategy following the methodology laid out in the empirical literature on the effects of housing price shocks. In our IV estimates, we use both a Bartik approach and the land supply elasticity approach, using elasticities computed in Saiz (2010). Under the Bartik approach, MSA-level house price growth is instrumented with US house price growth interacted with an MSA dummy. This IV strategy is similar to the methodology used in Nakamura and Steinsson (2014) in their study of government spending multipliers.[34]

Our other IV approach interacts the MSA-level land supply elasticities computed in Saiz (2010) with national house prices. In both cases, the identifying assumption is that whatever causes movements in national house prices are uncorrelated with MSA-specific aggregate demand shocks.

[33] One may be concerned that residential real estate prices are a poor proxy for commercial real estate. These authors use residential real estate prices and show that their results hold in a subset of their data for which commercial real estate prices are available.

[34] The authors use movements in national government defense spending as an instrument for state-level government spending by exploiting differences in state sensitivity to government defense expenditures.

Our IV regression takes the following form:

$$y_{it} = \alpha_i + \delta_t + \beta(L)\,\Delta\widetilde{hp}_{it} + \epsilon_{it} \qquad \text{(2nd stage)}$$
$$\Delta hp_{it} = \alpha_i + \delta_t + \rho_i(L)\Delta hp_t + u_{it} \qquad \text{(1st stage)}$$

where $\Delta\widetilde{hp}_{it}$ is the fitted value for MSA house prices obtained from the first-stage regression of MSA house prices on national house prices interacted with an MSA dummy or with the Saiz land supply elasticity. As before, the coefficient of interest is the sum of coefficients $\beta(1)$ measuring the effect of housing prices on job flows.

We further decompose the effect of housing prices on job flows by firm size and firm age categories, utilizing both OLS and IV specifications. Our OLS specification is a generalization of the MSA-level job flows regression:

$$y_{iht} = \alpha_i + \delta_t + \kappa_h + \gamma_h(L)\,\Delta GSP_{it} + \beta_h(L)\,\Delta hp_{it} + \epsilon_{iht}$$

where y_{iht} is job creation or job destruction for MSA i, in year t and category h. In addition to MSA and time fixed effects, we include category fixed effects. In these regressions, we allow both the MSA business cycle variable and MSA house prices to have differential effects on job flows across categories, and our coefficient of interest is $\beta_h(1)$ - the sum of coefficients of MSA house prices by category. The IV specification is analogous to the IV specification for aggregate job flows, where the instrument is now national house price growth interacted with a MSA-category dummy (Bartik approach) or the MSA land supply elasticity (Saiz approach):

$$y_{iht} = \alpha_i + \delta_t + \kappa_h + \beta_h(L)\,\Delta\widetilde{hp}_{it} + \epsilon_{iht} \qquad \text{(2nd stage)}$$

Importantly, it is worth stressing that our empirical strategy cannot rule out effects on job flows through the home-equity channel emphasized by Mian and Sufi (2014). Even if our IV approach successfully identifies exogenous housing price shocks, the effect of these shocks on job creation and job destruction may be driven by a decline in consumer demand due to a decline in household wealth. However, in the last set of regressions, we compare the behavior of employment at new firms versus employment at new establishments of existing firms; we argue that the fact that new establishments of existing firms do not respond to house price shocks provides evidence in favor of the credit supply channel. We also show that the patterns we find hold within tradeable and non-tradeable industry classifications and hold once job flows from the construction industry are excluded. Finally, the age and size patterns that we document for job flows are the in line with the patterns predicted by our firm dynamics model in response to a negative shock to firm credit.

5.2 Data

We draw on several distinct data sources for measures of job flows, house prices, and MSA measures of the business cycle. Data on job flows comes from the Business Dynamics Statistics compiled

by the US Census Bureau. The Business Dynamics Statistics is drawn from the Census Bureau's Longitudinal Database (LBD), a confidential database that tracks employment at the establishment and firm level over time. The Business Dynamics Statistics report job creation and job destruction by firm age and size categories at the state and MSA level. The job flows data in the BDS is drawn from Census Bureau's Business Register, which consists of the population of firms and establishments with employees covered by unemployment insurance or filing taxes with the Internal Revenue Service.[35]

Specifically, we use data on gross job creation and job destruction at the MSA level from 1982-2012. Firm level employment is recorded in March of each year and job flows are measured with respect to employment in the previous year. Our data set includes job flows from 366 MSAs resulting in a panel of 31 x 366 observations.

Our house price data comes from the Federal Housing Finance Agency's MSA level house price indices. We use the all-transactions indexes which provide a quarterly time series of housing prices from 1975 to present. These data are not seasonally adjusted, but we use year-over-year changes in the log of the house price index as our measure of MSA housing price growth. National housing prices are measured in the same way using the national house price index.

MSA-level business cycle measures come from the Bureau of Economic Analysis (BEA). Our baseline measure for the MSA business cycle is the growth rate of MSA personal income. We use measures of annual personal income and compute the growth rate as the change in the log of MSA personal income. Since job flows are measured as of March in a given year, we use the growth rate of MSA personal income in the previous year. For example, an observation of job creation for a given MSA in 2010 is matched with the growth rate of MSA personal income in 2009. Since housing prices are reported quarterly, no similar lag is required for house price growth. In addition to personal income, we also use real MSA gross product growth and employment growth as alternative proxies for the business cycle from BEA regional data.

5.3 Empirical Results

5.3.1 Aggregate Job Flows

Table 7 displays the coefficients of MSA housing price growth on job creation and job destruction at the MSA-level. MSA job creation and job destruction are converted to logs and detrended using a linear MSA-specific time trend.[36] As Table 7 shows, both the OLS and IV specifications give statistically significant coefficients for MSA house prices on job creation on impact and with a lag. For job destruction, the impact effect of house prices is negative, but the second lagged coefficient

[35] A more complete description of the BDS and access to job flows data is available at http://www.census.gov/cesdataproducts/bds/.

[36] By detrending, we are abstracting for the well known declining trend in job flows and reallocation. See Decker et al. (2014) and Karahan, Pugsley and Sahin (2015) for a discussion.

Table 7: Effect of housing prices on aggregate job flows

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Current housing price growth	0.34**	0.31**	0.71**	-0.34**	-0.21	-0.21
	(0.04)	(0.15)	(0.23)	(0.05)	(0.17)	(0.45)
1 year lagged housing price growth	0.18**	0.06	-0.81**	0.13**	-0.48**	-0.16
	(0.03)	(0.21)	(0.31)	(0.05)	(0.21)	(0.58)
2 years lagged housing price growth	0.00	0.20**	0.53**	0.29**	0.64**	0.48**
	(0.03)	(0.08)	(0.13)	(0.04)	(0.09)	(0.19)
Num. Obs.	9343	2653	2653	9343	2653	2653
First stage F-test		6.0	24.0		6.0	24.0
		3.8	23.0		3.8	23.0
Panel B						
Sum of coefficients	0.53**	0.57**	0.43**	0.09*	-0.05	0.11
	(0.05)	(0.08)	(0.10)	(0.05)	(0.08)	(0.12)

Panel A of the table presents coefficient estimates relating job flows to housing price growth at the MSA level. Panel B presents the sum of the coefficient estimates on current, 1 year and 2 years lagged housing price growth. Each column of the table reports results from a different regression. The dependent variable is MSA-level job creation in the first three columns and MSA-level job destruction in the last three columns. ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

is positive implying that a decline in house prices *reduces* job destruction in subsequent years. It is worth noting that since the sample ends in March 2012, our estimates for the effect of house prices on job flows are exploiting variation that does not fully include the weak recovery after the Great Recession.[37]

Panel B in Table 7 also displays the sum of the coefficients on housing prices. For job creation, the sum of the coefficients is positive and statistically significant indicating that housing price movements have a persistent effect on job creation. For job destruction, the sum of the coefficients under the baseline OLS and IV specifications is not statistically different from zero. However, excluding the impact effect, the sum of the lagged coefficients of housing prices on job destruction is positive and statistically significant across both the OLS and IV specifications. For the IV regressions, current and lagged house prices are instrumented with F-statistics above 10 under the Saiz approach.[38] The Bartik-type instrument delivers first-stage F statistics below 10, but partial r-squareds around 10%.[39] Our OLS results are robust to using either real MSA gross product growth

[37] Figure 1 uses a different data set, the Business Employment Dynamics, maintained by the Bureau of Labor Statistics that is available with a shorter delay and at quarterly frequencies, but is not available at the MSA-level.

[38] The Saiz land supply elasticities are only available for a subset of our MSAs. Therefore, both the Bartik and Saiz IV regressions are subsamples of the data used for the OLS regressions.

[39] Similar to the issues discussed in footnote 30 of Nakamura and Steinsson (2014), instrumenting local house prices with national house prices results in a large number of instruments for each endogenous regressor (MSA house

Table 8: Effect of housing prices on job flow by firm age

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Births	0.88**	0.66**	0.68**			
	(0.07)	(0.09)	(0.13)			
Young Firms,	0.48**	0.63**	0.55**	0.36**	0.20**	0.47**
1-5 years	(0.07)	(0.11)	(0.14)	(0.07)	(0.09)	(0.13)
Mature Firms,	0.33**	0.31**	-0.01	-0.06	-0.19**	-0.03
6+ years	(0.05)	(0.09)	(0.15)	(0.06)	(0.08)	(0.12)
Num. Obs.	28029	7959	7959	18686	5306	5306
Panel B						
H = Births - Mature	0.55**	0.36**	0.69**	0.43**	0.40**	0.50**
or Young - Mature	(0.08)	(0.08)	(0.14)	(0.07)	(0.07)	(0.13)

The table presents the effect of housing price growth at the MSA level on job flows by firm age categories (births, 1-5 and 6+ years old). Each column in the table reports results from a different regression. The dependent variable is job creation in the first three columns and job destruction in the last three columns. The numbers reported are the sum of the effects of current, 1 year and 2 years lagged changes in house price growth on job flows by firms age. The first three columns of panel B reports the difference in the effect of housing price changes on job flows between new and mature firms. The last three columns of panel B present the difference in the effect of housing price between young and mature firms. ** - coefficient estimate significant at the 5% level, *- coefficient estimate significant at the 10%. Standard errors are in parentheses.

or MSA employment growth as cyclical controls and are robust to using first-differenced job flows instead of linearly detrended job flows. Additionally, our results continue to hold in state-level data instead of MSA-level data.

These findings are consistent with our model's predictions for the effects of a financial shock. In our model, the impact effect of a financial shock is to lower job creation and raise job destruction but with a relatively stronger effect on the job creation margin. When we average over the three-year effect of a financial shock in our model, job destruction falls consistent with our point estimates in Table 7.

5.3.2 Category-Specific Job Flows

We first consider job flows by firm age categories: new firms, young firms (1-5 years of age), and mature firms (6+ years of age). Table 8 shows the sum of coefficients on MSA housing prices, $\beta_h(1)$ under the OLS and IV specifications. The table shows that job creation at new firms exhibit the strongest response to housing prices followed by job creation at young firms. Job creation at mature firms exhibits the least sensitivity to house prices and, in the case of column (3), is not

price growth has 88 instruments - each MSA dummy interacted with national house price growth) that results in F-statistics below 10. However, like Nakamura and Steinsson (2014), our instruments deliver similar magnitudes in terms of partial r-squareds.

Table 9: Effects on housing prices on job flows by firm size

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
1-19 employees	0.37**	0.25**	-0.13	-0.10**	-0.34**	-0.43**
	(0.04)	(0.06)	(0.11)	(0.04)	(0.07)	(0.11)
20-99 employees	0.75**	0.73**	0.66**	0.28**	0.01	0.18
	(0.06)	(0.06)	(0.09)	(0.06)	(0.07)	(0.11)
100+ employees	0.58**	0.81**	0.83**	0.13*	0.23**	0.56**
	(0.06)	(0.09)	(0.13)	(0.08)	0.09	(0.12)
Num. Obs.	28029	7959	7959	28029	7959	7959
Panel B						
H = (20-99 emp)	0.38**	0.49**	0.79**	0.38**	0.35**	0.60**
- (1-19 emp)	(0.04)	(0.04)	(0.09)	(0.04)	(0.05)	(0.09)

The table presents the effect of housing price growth at the MSA level on job flows by firms size categories (1-19, 20-99 and 100+ employees). Each column in the table reports results from a different regression. The dependent variable is job creation in the first three columns and job destruction in the last three columns. The numbers reported are the sum of the effects of current, 1 year and 2 years lagged changes in house price growth on job flows by firms size. Panel B reports the difference in the effect of housing price changes on job flows between medium (20-99 employees) and small firms (1-19 employees). ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

statistically different from zero. Our model predicts that job creation should fall for new and young firms, and that young firms should fall relatively more than mature firms. These predictions are verified in Panels A and B respectively of Table 8.

Job destruction at young firms shows a positive and statistically significant coefficient on housing prices, while job destruction at mature firms moves inversely to housing prices (though statistically significantly negative only in column (5)). The last row of Table 8 shows that the difference in the sensitivity of job destruction to house prices between young firms and mature firms is statistically significant under all specifications. The fall in job destruction at young firms and differential effect on job destruction between young and mature firms is in line with our model predictions.

In addition to age, we examine job flows by firm size, and consider three categories.[40]

Table 9 displays the results from the firm size job flows regressions. For job creation, middle and large sized firms exhibit the highest sensitivity to housing prices, followed by small firms consistent with our model predictions. In the case of the IV specification, the coefficient of housing prices on job creation for small firms is negative meaning a decrease in house price growth *raises* job creation at small firms. Our model predicts this behavior for job creation at small firms due the effect of falling wages that induces unconstrained firms to expand.

[40] Firm size assigns size categories based on an average of employment in the previous year and employment in the current year raising potential issues of reclassification bias (see Moscarini and Postel-Vinay (2012) for a discussion). However, initial firm size data is not available at the MSA level, and our results are unchanged in state level data using size categories based on initial firm size.

Table 10: Effect of housing prices on job creation within industry

	All Industries		Construction		All Industries ex-Construction		Manufacturing		Services	
	OLS (1)	IV (Bartik) (2)	OLS (3)	IV (Bartik) (4)	OLS (5)	IV (Bartik) (6)	OLS (7)	IV (Bartik) (8)	OLS (9)	IV (Bartik) (10)
Panel A										
Sum of coefficients	0.24**	0.54**	1.67**	1.57**	0.11	0.47**	-0.09	0.25	0.19	0.60**
	(0.10)	(0.12)	(0.18)	(0.25)	(0.10)	(0.12)	(0.18)	(0.29)	(0.13)	(0.14)
Num. Obs.	1479	1479	1479	1479	1479	1479	1479	1479	1479	1479
Panel B										
New and young firms	0.51**	0.75**	2.37**	1.71**	0.32**	0.71**	0.26	0.60*	0.51**	0.98**
< 5 years	(0.10)	(0.12)	(0.24)	(0.30)	(0.11)	(0.12)	(0.25)	(0.32)	(0.18)	(0.18)
Mature firms	0.11	0.41**	1.01**	1.51**	0.02	0.29**	-0.09	0.07	-0.02	0.24
> 5 years	(0.10)	(0.13)	(0.21)	(0.28)	(0.10)	(0.12)	(0.20)	(0.25)	(0.11)	(0.15)
Num. Obs.	2958	2958	2958	2958	2958	2958	2958	2958	2958	2958
Panel C										
New and young	0.39**	0.34**	1.36**	0.20	0.29**	0.42**	0.35	0.53	0.53**	0.74**
-mature firms	(0.09)	(0.08)	(0.28)	(0.24)	(0.09)	(0.09)	(0.33)	(0.35)	(0.14)	(0.15)

Panel A of the table presents the sum of coefficient estimates on current, 1 year, and 2 year lagged state housing price growth. Panel B presents the sum of the coefficient estimates on state house price growth by firm age categories. Panel C reports the difference in the sum of coefficients for new and young versus mature firms. Each column of the table reports results from a different regression. The dependent variable is state-level job creation by industry in all columns. ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

Job destruction for middle-sized firms display a positive coefficient on housing prices under all specifications also in line with our model, though the coefficients are not statistically significant under IV specifications. However, our model does miss on job destruction for large firms - our model predicts a positive coefficient but lower than the coefficient for medium-sized firms. However, the large positive coefficient for large firms is influenced by a large positive impact coefficient.[41] Table 9 also shows that the difference in coefficients between middle-sized firms and small firms is statistically significant across all specifications for both job creation and job destruction. In contrast, the difference for middle and large sized firms for job creation is generally not significant. Our model predicts the right signs and relative ordering for both creation and destruction at small and middle-sized firms.

5.3.3 Job Flows by Industry

Firms within the construction industry tend to be smaller and younger than firms in the economy overall. To rule out whether the size and age patterns we document are driven by industry composition, we use state level data that disaggregates job flows by industry.[42] Panel A in Table 10 reports the effect of current and lagged state house price growth on job creation. The OLS specification includes current and 2 lags of state GDP growth as controls, while the IV specification instruments

[41] If we restrict our attention to the lagged response, the empirical pattern across firm size categories is consistent with our model. Also, large firms are more likely to operate across multiple MSAs somewhat complicating comparison of model and data for large firms. Our state level regressions (not shown) find a coefficient of housing price shocks on job destruction at large firms that is lower than middle-sized firms consistent with model predictions.

[42] MSA level data disaggregated by industry and age is not publicly available from the Business Dynamics Statistics due to disclosure concerns. Indeed, state x industry x age is not available either. However, we use data from Fort et al. (2013) who obtained from the Census Bureau a customized state x industry x age disaggregation of job flows. Their data ends in 2010, therefore, our industry regressions utilize a panel of 51 states (including DC) from 1982-2010.

Table 11: Effect of housing prices on job destruction within industry

	All Industries		Construction		All Industries ex-Construction		Manufacturing		Services	
	OLS (1)	IV (Bartik) (2)	OLS (3)	IV (Bartik) (4)	OLS (5)	IV (Bartik) (6)	OLS (7)	IV (Bartik) (8)	OLS (9)	IV (Bartik) (10)
Panel A										
2 years lagged housing	0.30**	0.34**	1.11**	1.00**	0.21**	0.28**	0.37**	0.54**	0.24**	0.36**
price growth	(0.08)	(0.09)	(0.18)	(0.17)	(0.07)	(0.09)	(0.09)	(0.24)	(0.08)	(0.17)
Num. Obs.	1479	1479	1479	1479	1479	1479	1479	1479	1479	1479
Panel B										
New and young firms	0.24**	0.49**	1.63**	0.84**	0.07	0.47**	0.63**	0.98**	0.02	0.51**
< 5 years	(0.11)	(0.11)	(0.25)	(0.28)	(0.12)	(0.11)	(0.24)	(0.25)	(0.16)	(0.15)
Mature firms	-0.17	-0.30	-0.58**	-0.80**	-0.15	-0.27**	0.21	0.06	-0.05	-0.10
> 5 years	(0.13)	(0.10)	(0.20)	(0.27)	(0.12)	(0.10)	(0.25)	(0.22)	(0.11)	(0.14)
Num. Obs.	2958	2958	2958	2958	2958	2958	2958	2958	2958	2958
Panel C										
New and young	0.42**	0.79**	2.20**	1.64**	0.23**	0.74**	0.43	0.92**	0.07	0.62**
-mature firms	(0.10)	(0.10)	(0.26)	(0.22)	(0.11)	(0.11)	(0.28)	(0.22)	(0.14)	(0.14)

Panel A of the table presents the sum of coefficient estimates on current, 1 year, and 2 year lagged state housing price growth. Panel B presents the sum of the coefficient estimates on state house price growth by firm age categories. Panel C reports the difference in the sum of coefficients for new and young versus mature firms. Each column of the table reports results from a different regression. The dependent variable is state-level job destruction by industry in all columns. ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

current and both lags of state house prices using the Bartik approach.[43] Industry job flows are detrended with a state/age specific linear trends in the same manner as the MSA-level regressions.

As Panel A shows, job creation is more sensitive within construction, but the coefficient remains positive when job creation from construction is excluded in columns (5) and (6). Panel B shows that both within construction and excluding construction, new and young firms are more sensitive to house price growth than mature firms.[44] Construction displays higher elasticities, but the difference between young and mature firms shown in Panel C is generally statistically significant in both the OLS and IV specifications.

Columns (7)-(10) examine the effect of house prices on job creation within tradable (manufacturing) and non-tradable industries (services) along the lines of Mian and Sufi (2014). As the point estimates reveal in Panels B and C, job creation at new/young firms is more sensitive to house prices than job creation at mature firms within both manufacturing and services. However, for manufacturing, these coefficients are less precisely estimated. For services, a house price decline has statistically significant effects on overall job creation and a strong differential effect on job creation at young firms (relative to mature firms) consistent with the effects of a financial shock.[45]

We also find similar patterns for the effect of house price growth on job destruction after controlling for the contribution of construction. The first six columns in Table 11 displays the response of job destruction to house price growth for all industries, within construction, and all

[43] No state level land supply elasticities are available, but the Bartik coefficients appear similar to the elasticities of major MSAs within a state.

[44] The age categories in our industry regressions differ from the age categories in our MSA-level regressions due to the categories chosen by the authors in Fort et al. (2013). In our MSA-level age regressions, firms 5 years of age are included as young firms.

[45] The job flows industry breakdown in the Business Dynamics Statistics are relatively coarse (NAICS supersectors) in comparison with the highly disaggregated industry data used in Mian and Sufi (2014); however, the BDS has several advantages: job creation/destruction margin, firm level v. establishment level, and firm age breakdowns.

Table 12: Effect of housing prices on new firms and establishments

	Employment at New Firms		Employment at New Establishments of Existing Firms		Ratio of New Firm Employment to New Establishment Employment	
	OLS	IV (Bartik)	OLS	IV (Bartik)	OLS	IV (Bartik)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Sum of coefficients	0.50**	0.46**	-0.12	0.07	0.15**	0.09
	(0.16)	(0.15)	(0.16)	(0.23)	(0.05)	(0.06)
Num. Obs.	1479	1479	1479	1479	1479	1479

Panel A reports the sum of the coefficient estimates on current, 1 year and 2 years lagged housing price growth. Each column of the table reports results from a different regression. The dependent variable is detrended log employment at new firms in columns (1)-(2), detrended log employment at new establishments of *existing* firms in columns (3)-(4), and the fraction of employment at new firms to employment at all new establishments in columns (5)-(6). ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

industries excluding construction. Panel A displays the 2-year delayed response of job destruction to house prices. The positive coefficient is consistent with the coefficients reported in Table 7 and shows that a decline in house price growth depresses job destruction in subsequent years. Panel B display differences in the sum of coefficients between young and mature firms - job destruction falls more at young firms than mature firms after a decline in house prices, consistent with our MSA-level firm age regressions. While the response coefficients are stronger in construction, the differential effect between young and mature firms within construction is consistent with the presence of a firm credit channel in addition to the direct effects of a decline in house prices on demand for construction services. The differential effect of house prices on job destruction by age hold across both OLS and IV specifications.

Columns (7)-(10) show that the response of job destruction to house prices and the differential response between young and mature firms also holds within tradable and non-tradable industries. Point estimates are consistent with the MSA firm age findings and typically statistically significant. Overall, these industry specific regressions show that the job flows patterns we emphasize are not driven solely by the response of the construction industry to a fall in house prices and are not driven by differences in the job flows effects on tradable v. non-tradable industries. Our evidence on differences between young and mature firms within industries is also consistent with the findings in Fort et al. (2013) who adopt a different empirical specification.

5.3.4 Employment at New Establishments

The state level data on job flows from the Business Dynamics Statistics also allow us to distinguish the response of new firms versus new establishments of existing firms. If a decline in house prices operates primarily by reducing household demand, new firms and new establishments of existing

firms should respond similarly.[46] More concretely, if a decline in house prices proxies for a shock to credit conditions, then new independent coffee shops would not open but established chains like Starbucks that are unlikely to face binding credit constraints would still open new locations. By contrast, if a decline in house prices diminishes demand for all goods like coffee, both the independent coffee shop and Starbucks would be impacted.

Table 12 finds evidence consistent with the former supporting the firm credit channel. Panel A gives the cumulative effect of house price growth on employment at new firms in the left two columns, on employment at new establishments of existing firms in the middle columns, and on the fraction of employment at new firms relative to employment at all new establishments in the right two columns. As columns (1) and (2) show, a decline in house prices reduce employment at new firms consistent with our findings for job creation at new firms. However, as columns (3) and (4) show, a decline in house prices has no statistically significant effect on employment at new establishments of existing firms. Moreover, a decline in house prices lowers the share of employment at new firms relative to the share of employment at all new establishments, meaning that the differential response to house prices is statistically significant as seen in columns (5) and (6). In each case, employment is measured at the state level from 1982-2010 (MSA level data is not available), and log employment is detrended with state-specific linear trends. Importantly, in the OLS specification, employment at new establishments of existing firms is quite cyclical; existing firms respond strongly to the state business cycle in opening new establishments. In short, it is not the case that new employment at existing establishments is simply less sensitive to both the state business cycles and house price growth.

6 Conclusion

The US financial crisis has raised concerns that depressed collateral values and tighter lending may inhibit firm formation and expansion, disrupting the process of innovation and labor market turnover essential for a healthy economy. An extensive literature has documented the importance of real estate collateral for new firms to obtain lending and for businesses to obtain financing for expansion. Recent work also documents the disproportionate contribution of new and young firms to overall labor market turnover. Given these facts, it stands to reason that job flows may be particularly sensitive to a tightening of credit supply.

In this paper, we provide support for this hypothesis illustrating the theoretical and empirical link between job flows and credit supply shocks. We build a firm dynamics model with collateral constraints and examine the effect of a collateral shock on overall job flows and job flows by firm size and age. We show analytically in a simple version of our model that a collateral shock must

[46] An establishment is a single physical location where work takes place. The vast majority of new firms form a single new establishment. By contrast, large mature firms typically have multiple establishments (consider retailers like Walmart or Starbucks).

reduce employment, job creation and job destruction, and demonstrate why a collateral shock has stronger effects for young firms and medium-sized firms.

Our benchmark model is calibrated to match the distribution of employment by firm size and age observed in the data. Given the observed movements in US job flows in the Great Recession, we estimate the contribution of financial, productivity, and discount rate shocks to the decline in overall employment. We find evidence of a relatively large shock to firm credit that, despite its magnitude, accounts for a modest 15% of the decline in employment. Importantly, our results leave room for other factors, including household deleveraging (as in Mian and Sufi (2014)), complications from a binding zero lower bound, or increased uncertainty, to explain the remaining 85% of the decline in employment. A shock to household's discount factor that raises the required return on capital accounts for 60% of the decline in employment consistent with shocks emphasized in Hall (2014) and Kehoe, Midrigan and Pastorino (2014).

We validate the predictions of our model by using MSA-level variation in job flows and housing prices, and we show that both job creation and lagged job destruction decline in response to a fall in housing prices. We control for aggregate demand effects by introducing direct controls for the business cycle and by using a land supply elasticity approach common in the empirical literature on the real effects of collateral shocks. We also document size and age patterns in the sensitivity of job flows to housing prices, showing that job flows for new and young firms (0-5 years of age) are most sensitive to housing price shocks as are job flows for medium-sized firms (20-99 employees). These patterns are consistent with the age and size patterns predicted by our quantitative model. Moreover, the differential effects of house price shocks across firm age continues to hold after excluding job flows from the construction industry and looking within tradable and non-tradable industries. Further, we provide further evidence that movements in house prices proxy for firm credit disruptions given that new firm employment responds to house price shocks while employment at new establishments of existing firms remains unaffected.

Future work may explicitly model the housing market to further disentangle the effects of a housing crisis on household consumption versus the effect of a housing crisis on the banking system and firm collateral. An explicit model of the housing sector can also measure the direct effect of a housing crisis on construction employment while allowing for the evaluation of monetary and credit policies pursued in the Great Recession.

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A Simple Model: Characterization

A.1 Firm Optimality

To describe the solution to the firm's problem, we specify the Hamiltonian:

$$\mathcal{H} = e^{-\sigma t}\Lambda_{0,t}a + \lambda_F\left[z\epsilon\left(k^{\alpha}n^{1-\alpha}\right)^{\phi} - r_k k - wn + ra\right] - \eta[k - \chi a].$$

The maximum principle implies the following optimality conditions:

$$\begin{aligned}
\mathcal{H}_k &= \lambda_F[z\epsilon\alpha\phi k^{\alpha\phi-1}n^{(1-\alpha)\phi} - r_k] - \eta = 0,\\
\mathcal{H}_n &= \lambda_F[z\epsilon(1-\alpha)\phi k^{\alpha\phi}n^{(1-\alpha)\phi-1} - w] = 0,\\
\dot{\lambda}_F &= -\left\{e^{-\sigma t}\Lambda_{0,t} + \lambda_F r + \eta\chi\right\},\\
k &\leq \chi a,\ \eta \geq 0, \eta[k - \chi a] = 0.
\end{aligned}$$

A.2 Constrained Firm Asset Path

The law of motion for assets is given by:

$$\dot{a}_{i,t} = D_i a_{i,t}^{\psi} - Ba_{i,t},$$

where $D_i \equiv \left(z_i A\chi^{\alpha\phi}\right)^{\frac{1}{1-\phi(1-\alpha)}}\left[\frac{\phi(1-\alpha)}{w}\right]^{\frac{\phi(1-\alpha)}{1-\phi(1-\alpha)}}[1-\phi(1-\alpha)], B \equiv r_k\chi - r > 0; \psi \equiv \phi\alpha/(1-\phi(1-\alpha)) < 1$. To solve this differential equation:

$$\dot{a} = Da^{\psi} - Ba,$$

where we omitted time and firm subscripts, we introduce the following change of variables $y = \log a$.

$$\dot{y} = De^{(\psi-1)y} - B.$$

We can rewrite this equation as follows:

$$\frac{dy}{De^{(\psi-1)y} - B} = dt.$$

Rearranging this equation, we obtain:

$$\frac{1}{B(\psi-1)}\left(\frac{d\left(De^{(\psi-1)y} - B\right)}{De^{(\psi-1)y} - B} - d[(\psi-1)y]\right) = dt.$$

Integrating this equation, we obtain:

$$\log\left[De^{(\psi-1)y} - B\right] - (\psi-1)y = B(\psi-1)t + const.$$

Transforming back to original variable:

$$\log[D - Ba^{1-\psi}] = B(\psi-1)t + const.$$

Since initial level of assets is a^F, we have:

$$\log\left[\frac{D - Ba^{1-\psi}}{D - B(a^F)^{1-\psi}}\right] = B(\psi-1)t.$$

This can be expressed as:

$$a = \left\{\frac{D - (D - B(a^F)^{1-\psi})e^{-B(1-\psi)t}}{B}\right\}^{1/(1-\psi)}.$$

A.3 Properties of Asset Path for Financially Constrained Firms

Monotonicity in t. For an asset path of the firm facing the financial constraint to be increasing over time, it must be that $a_{i,t}^{1-\psi} < D_i/B$. We show that this inequality is satisfied for a financially constrained firm. If the financial constraint binds, then $\eta_{i,t} > 0$ and hence the optimality with respect to capital implies $Az_i\alpha\phi k_{i,t}^{\alpha\phi-1}n_{i,t}^{(1-\alpha)\phi} > r_k$. Taking into account $k_{i,t} = \chi a_{i,t}$ and the optimal choice of labor (12), the previous inequality can be rewritten as

$$a_{i,t}^{1-\psi} < \frac{\psi D_i}{\chi r_k} < \psi\frac{D_i}{\chi r_k - r} = \psi\frac{D_i}{B} < \frac{D_i}{B}. \tag{A.1}$$

This verifies that asset path is increasing over time for financially constrained firm.

Convexity in t. $\dot{a}_{i,t} = D_i a_{i,t}^{\psi} - Ba_{i,t}$ implies $\ddot{a}_{i,t} = \left(\psi D_i a_{i,t}^{\psi-1} - B\right)\dot{a}_{i,t} = \left(\psi D_i a_{i,t}^{\psi-1} - B\right)(D_i a_{i,t}^{\psi} - Ba_{i,t})$. From (A.1) we know that $\psi D_i a_{i,t}^{\psi-1} > B$ implying that the path is convex.

Monotonicity in χ.

$$\begin{aligned}\frac{da_{i,t}^{1-\psi}}{d\chi} &= \frac{\partial a_{i,t}^{1-\psi}}{\partial D_i}\frac{dD_i}{d\chi} + \frac{\partial a_{i,t}^{1-\psi}}{\partial B}\frac{dB}{d\chi} \\ &= \frac{D_i}{B}\left(1 - e^{-B(1-\psi)t}\right)\left[\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B}\right] + \left[\frac{D_i}{B} - (a^F)^{1-\psi}\right]e^{-B(1-\psi)t}(1-\psi)tB_\chi \end{aligned}\tag{A.2}$$

where we used notation $D_{i,\chi} = dD_i/d\chi, B_\chi = dB/d\chi$. The second term of this expression is positive while the first one is negative because

$$\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B} = \frac{\alpha\phi}{1-\phi(1-\alpha)}\frac{1}{\chi} - \frac{r_k}{r_k\chi - r} < 0.$$

We now show that the second term dominates. Let's denote $g(t) \equiv da_{i,t}^{1-\psi}/d\chi$ and write (A.2) in more concise form as

$$g(t) = -C_1(1 - e^{-\omega t}) + C_2 e^{-\omega t}\omega t,$$

where $C_1 = -\left[D_{i,\chi}/D_i - B_\chi/B\right]D_i/B > 0, C_2 = \left[D_i/B - (a^F)^{1-\psi}\right]B_\chi/B > 0$ and $\omega = B(1-\psi)$. It is easy to check that $C_2 > C_1$. Observe that $g(0) = 0$ and $g'(t) = \omega e^{-\omega t}(C_2 - C_1 - C_2\omega t)$. This implies that there exists $0 < T_1 < \infty$ such that $g'(t) > 0$ for $t \in [0, T_1)$ and $g'(t) \leq 0$ for $t \geq T_1$.

If function $g(t)$ is positive in instant T_2 when the firm grows out of its financial constraint then function $g(t)$ is positive in every instant before this happens. We now show that $g(T_2) > 0$. T_2 is defined by the following two equations

$$\begin{aligned} Az_i\alpha\phi k_{i,T_2}^{\alpha\phi-1}n_{i,T_2}^{(1-\alpha)\phi} &= r_k, \\ k_{i,T_2} &= \chi a_{i,T_2}. \end{aligned}$$

The first relation states that there is no longer a wedge in the optimal capital choice due to financial constraint. The second equation states that the optimal capital choice is still equal to assets times financial parameter. Combining these two equations:

$$a_{i,T_2}^{1-\psi} = \frac{\psi D_i}{r_k\chi} = \frac{D_{i,\chi}}{B_\chi}.$$

Next, taking into account the last expression and (A.2), we obtain:

$$e^{-B(1-\psi)T_2} = \frac{\frac{D_i}{B} - \frac{D_{i,\chi}}{B_\chi}}{\frac{D_i}{B} - (a^F)^{1-\psi}}.$$

From this we get:

$$\begin{aligned}
g(T_2) =& \frac{D_i}{B} \cdot \frac{\frac{D_{i,\chi}}{B_\chi} - (a^F)^{1-\psi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} \left[\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B}\right] \\
&+ \frac{D_i}{B}\left[\frac{D_i}{B} - (a^F)^{1-\psi}\right] \frac{\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B}}{\frac{D_i}{B} - (a^F)^{1-\psi}} \log \frac{\frac{D_i}{B} - \frac{D_{i,\chi}}{B_\chi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} \\
=& \frac{D_i}{B}\left[\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B}\right] \left\{ \frac{\frac{D_{i,\chi}}{B_\chi} - (a^F)^{1-\psi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} + \log \frac{\frac{D_i}{B} - \frac{D_{i,\chi}}{B_\chi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} \right\} \\
=& \frac{D_i}{B}\left[\frac{D_{i,\chi}}{D_i} - \frac{B_\chi}{B}\right] \left\{ \frac{\frac{D_{i,\chi}}{B_\chi} - (a^F)^{1-\psi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} + \log \left(1 - \frac{\frac{D_{i,\chi}}{B_\chi} - (a^F)^{1-\psi}}{\frac{D_i}{B} - (a^F)^{1-\psi}}\right) \right\}.
\end{aligned}$$

Note that $x \equiv \frac{\frac{D_{i,\chi}}{B_\chi} - (a^F)^{1-\psi}}{\frac{D_i}{B} - (a^F)^{1-\psi}} \in (0,1)$ and function $p(x) \equiv x + \log(1-x) < 0$ for $x \in [0,1)$ because $p(0) = 0$ and $p'(x) < 0$ for $x \in (0,1)$. This implies that $g(T_2) > 0$ and hence the derivative of asset path $a_{i,t}$ with respect to financial constraint parameter χ is positive at any time before a financially constrained firm becomes unconstrained.

Monotonicity in z_i.

$$\frac{da_{i,t}^{1-\psi}}{dz_i} = \frac{\partial a_{i,t}^{1-\psi}}{\partial D_i}\frac{dD_i}{dz_i} = \frac{D_i}{B}\left(1 - e^{-B(1-\psi)t}\right)\frac{\psi}{z_i} > 0.$$

A.4 Aggregation Across Firms when Wage is Fixed

Total labor demand is given by:

$$n(\chi) = \mu\left[\sigma\int_0^{\bar{t}(\chi)} n_{z_H,t}(\chi)e^{-\sigma t}dt + \sigma\int_{\bar{t}(\chi)}^{\infty} n^*_{z_H}e^{-\sigma t}dt\right] + (1-\mu)n^*_{z_L},$$

where we assumed that the low-productivity firms are not constrained. Next, the high-productivity firms labor demand under χ_L and χ_H satisfies

$$\begin{aligned}
&\int_0^{\bar{t}(\chi_L)} n_{z_H,t}(\chi_L)e^{-\sigma t}dt + \int_{\bar{t}(\chi_L)}^{\infty} n^*_{z_H}e^{-\sigma t}dt < \int_0^{\bar{t}(\chi_H)} n_{z_H,t}(\chi_L)e^{-\sigma t}dt + \int_{\bar{t}(\chi_H)}^{\infty} n^*_{z_H}e^{-\sigma t}dt \\
&< \int_0^{\bar{t}(\chi_H)} n_{z_H,t}(\chi_H)e^{-\sigma t}dt + \int_{\bar{t}(\chi_H)}^{\infty} n^*_{z_H}e^{-\sigma t}dt.
\end{aligned}$$

This leads to $n(\chi_L) < n(\chi_H)$.

Because firm exits are i.i.d., it must be that $JD(\chi) = \sigma n(\chi)$. So, $JD(\chi_L) < JD(\chi_H)$. Because in stationary equilibrium $JC(\chi) = JD(\chi)$, aggregate job creation also increases with χ.

B Equivalence of Productivity and Markup Shocks

In this section, we extend the benchmark model to include a monopolistically competitive retail sector that allows for price-setting and shock to firms' market power. We show that markup shocks are isomorphic to productivity shocks in this extension. Any friction in price-setting would ensure that monetary policy shocks, discount rate, or other demand shocks would lead to a temporary increase in the markup. In this way, markup shocks are a proxy for demand shocks.

Relative to the benchmark model, we now introduce a retailer sector populated by monopolistically competitive firms that costlessly differentiate the intermediate good produced by firms. Cost minimization on the part of households implies that retailers face the following demand schedules for good l:

$$y_t(l) = Y_t \left(\frac{p_t(l)}{P_t} \right)^{\theta_t} \tag{B.1}$$

$$P_t = \left(\int p_t^{1-\theta_t dl} \right)^{\frac{1}{1-\theta_t}} \tag{B.2}$$

where θ_t is a time-varying elasticity of substitution in the Dixit-Stiglitz aggregator and P_t is the price level of the consumption bundle consumed by households.

Retailers maximize profits taking the real price of the final good, $\frac{P_t^{int}}{P_t}$, as given:

$$\max_{p_t(l)} \quad \Pi_t^{ret} = \frac{p_t(l)}{P_t} y_t(l) - \frac{P_t^{int}}{P_t} y_t(l) \tag{B.3}$$

$$\text{s.t.} \quad y_t(l) = Y_t \left(\frac{p_t(l)}{P_t} \right)^{\theta_t} \tag{B.4}$$

where Π_t^{ret} are retailer profits. The optimality condition for retailers is a time-varying markup over real marginal costs:

$$\frac{p_t(l)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{P_t^{int}}{P_t} \tag{B.5}$$

The profit maximization problem for heterogenous producers is nearly unchanged. Instead of maximizing (6), the price of intermediate goods enters into the heterogenous firms profit function:

$$\max \quad \pi_{i,t} = \frac{P_t^{int}}{P_t} A_t z_{i,t} \left(k_{i,t}^{\alpha} n_{i,t}^{1-\alpha} \right)^{\phi} - r_{k,t} k_{i,t} - w_t n_{i,t} \tag{B.6}$$

subject to the financial constraint (7) and where A_t is aggregate productivity, and $z_{i,t}$ is idiosyncratic productivity. Firms choose capital and labor to maximize profits. Optimality conditions are unchanged except that real marginal cost now enters along with aggregate productivity as a determinant of the marginal product of capital and marginal product of labor.

In a symmetric equilibrium, $p_t(l) = P_t$ and, therefore, real marginal cost is equal to the inverse of the markup $\frac{\theta_t}{\theta_t - 1}$. We can define productivity inclusive of the markup shock as follows:

$$\tilde{A}_t = A_t \frac{\theta_t - 1}{\theta_t}$$

A decrease in θ_t will increase markups, depress real marginal costs, and lower $\tilde{A}_t$. Thus, a markup shock is isomorphic to a productivity shock.

As emphasize, the equivalence between a markup shock and a demand shock is not exact but approximate. With sticky prices and no markup shocks, firms fail to reset prices in periods of lower demand leading to an increase in markups (countercyclical markups). Prices gradually adjust to return markups to their optimal level - falling in periods of low demand (deflation) and rising in periods of high demand (inflation).Thus, demand shocks will induce variation in $\tilde{A}_t$, and markup shocks approximate the effects of various demand shocks.

C Equivalence of Productivity and Deleveraging Shocks

In this section, we establish a partial equivalence between a shock to household credit that induces deleveraging and aggregate productivity shocks. To introduce a deleveraging shock we use a version of a model presented in Midrigan and Phillippon (2016). Their model introduces a precautionary motive for household savings in an analytically tractable manner. A decline in the borrowing limits, a deleveraging shock, induces a fall in the real interest rate.

We proceed in two steps. First, we present a simple model without capital. We demonstrate that a deleveraging shock puts a downward pressure on the real interest rate. Then we introduce capital, heterogenous firms, and nominal frictions. With sticky prices and the zero lower bound on the nominal interest rate, the real interest rate cannot adjust, as a result output drops to restore the equilibrium in the goods market after a deleveraging shock. A decline in output reduces the demand for intermediate goods which are produced with capital and labor. As a result, the price for intermediate goods drops. We then show that this is equivalent (see below our exact definition of equivalence) to a decline in aggregate productivity in the intermediate goods production function.

C.1 A Simple Model of Deleveraging

Time. Time is discrete and infinite. Each period has two sub-periods. In the first subperiod, most of the household choices are made. In the second subperiod, the idiosyncratic taste shocks are realized and all of the unspent funds of shoppers invested in riskless bonds (see below).

Preferences. The representative household preferences are represented by the following expression:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[log \left(c_t - \frac{1}{1+\nu} n_t^{1+\nu} \right) + \int_0^1 v_{it} \log \underline{c}_{it} di \right]$$

where c_t is consumption bought on credit, $\underline{c}_{it}$ consumption bought with liquid assets, n_t is the labor supply. The representative household consists of a unit continuum of shoppers who buy goods with liquid assets in the second sup-period of every period. Each shopper experience iid preference shock $v_{it} > 1$. These shocks have a Pareto distribution with the cumulative distribution function $\mathbb{P}(v_{it} \leq v) = F(v) = 1 - v^{-\zeta}$, where $\zeta > 1$.

Technology. Goods are produced out of labor according to linear production function $y_t = n_t$. Note that there are no productivity shocks so far. They will be introduced later when we add capital to the model.

Securities. There are two types of assets in the economy. First, there is riskless one-period real bond: b_{t+1}. The return on bonds is defined as R_t. Note that representative household both

borrows and saves at the same time. Second, there are *intraperiod* liquid assets in infinite supply. Shoppers invest in liquid assets in first sub-period of every period and use them to pay in the second sub-period.

Household constraints. Each shopper in the household gets x_t in liquid assets for purchases of goods that can only be bought with liquid assets. This investment is made before shocks $\{v_{it}\}$ are realized. Thus, the shoppers face the following constraint:

$$\underline{c}_{it} \leq x_t.$$

The budget constraint of the household is:

$$x_t + c_t + b_t R_{t-1} \leq w_t n_t + b_{t+1} + a_t R_{t-1},$$

where b_t is gross liabilities of the household, and a_t is gross assets of the household. Household gross savings are the unspent funds of the shoppers:

$$a_{t+1} = \int_0^1 (x_t - \underline{c}_{it})\, di.$$

The household faces the borrowing constraint of the form:

$$b_{t+1} \leq B_t,$$

where B_t is the exogenous debt limit.

C.2 Solution

Households. Denote the Lagrange multiplier on the flow budget constraint as μ_t, on liquidity constraint as ξ_{it}, and the borrowing constraint as λ_t. Optimization with respect to consumption c_t leads to

$$\frac{1}{c_t - \frac{1}{1+\nu} n_t^{1+\nu}} = \mu_t.$$

The first order conditions with respect to $\underline{c}_{it}$ imply:

$$\underline{c}_{it} = \min\left\{ \frac{v_{it}}{\beta R_t \mathbb{E}_t \mu_{t+1}}, x_t \right\},$$

and

$$\xi_{it} = \max\left\{ \frac{v_{it}}{\underline{c}_{it}} - \beta R_t \mathbb{E}_t \mu_{t+1}, 0 \right\}.$$

The optimal choice of liquid assets x_t implies:

$$\mu_t = \beta R_t \mathbb{E}_t \mu_{t+1} + \int_0^1 \xi_{it} di.$$

Optimality with respect to debt holdings b_{t+1} lead to

$$\mu_t = \beta R_t \mathbb{E}_t \mu_{t+1} + \lambda_t.$$

Finally, the labor supply curve is given by:

$$n_t^{\nu} = w_t.$$

Firms. Firm's produce by hiring labor and productivity is normalized to unity:

$$\max_{n_t} n_t - w_t n_t$$

Optimal labor demand implies $w_t = 1$.

C.3 Equilibrium

Definition 1. *An equilibrium is an allocation* $\{c_t, \{\underline{c}_{it}\}, n_t, b_{t+1}, a_{t+1}\}_{t=0}^{\infty}$ *and prices* $\{R_t, w_t\}_{t=0}^{\infty}$ *such that households optimize, firms optimize, the labor market clears* ($n_t^{\nu} = 1$)*, the goods market clears* ($c_t + \int_0^1 \underline{c}_{it} di = n_t$)*, and assets markets clear* ($a_{t+1} = b_{t+1}$)*.*

Denote $\underline{c}_t^*$ to be consumption of a shopper with the lowest realization of the preference shock $v_{it} = 1$:

$$\underline{c}_t^* \equiv \frac{1}{\beta R_t \mathbb{E}_t \left(c_{t+1} - \frac{1}{1+\nu}\right)^{-1}}. \tag{C.1}$$

Using this notation, consumption of a shopper is:

$$\underline{c}_{it} = \min\left\{v_{it}\underline{c}_t^*, x_t\right\}.$$

Next, let's define the Euler equation wedge as:

$$\Delta_t \equiv \left(\beta R_t \mathbb{E}_t \frac{c_t - \frac{1}{1+\nu}}{c_{t+1} - \frac{1}{1+\nu}}\right)^{-1} - 1 \tag{C.2}$$

Optimality conditions with respect to x_t and $\underline{c}_{it}$ imply the following characterization of the wedge:

Lemma 1. *The Euler equation wedge is:*

$$\Delta_t = \frac{1}{\zeta - 1}\left(\frac{x_t}{\underline{c}_t^*}\right)^{-\zeta}, \tag{C.3}$$

The proof is in an appendix available from the authors. Note that $\Delta_t > 0$. The left-hand side is proportional to liquidity constrained shoppers, i.e., $1 - F(x_t/\underline{c}_t^*)$. The right-hand side of the expression would equal zero absent liquidity constraints. However, with the liquidity constraints this expression is positive. Thus, the real interest rate R_t is lower than $\left(\beta \mathbb{E}_t \frac{c_t - \frac{1}{1+\nu}}{c_{t+1} - \frac{1}{1+\nu}}\right)^{-1}$. As a result, in steady state, the interest rate can fall below $1/\beta$.

Note that $\lambda_t = \int_0^1 \xi_{it} di > 0$, implying that the borrowing constraint always binds:

$$b_{t+1} = B_{t+1}. \tag{C.4}$$

The next Lemma expresses aggregate consumption and savings of the shoppers:

Lemma 2. *Total consumption by shoppers is:*

$$\underline{c}_t \equiv \int_0^1 \underline{c}_{it} di = \frac{\zeta}{\zeta - 1}\left(c_t - \frac{1}{1+\nu}\right)(1 + \Delta_t)\left[1 - \frac{1}{\zeta}\left((\zeta - 1)\Delta_t\right)^{\frac{\zeta - 1}{\zeta}}\right]. \tag{C.5}$$

Total savings by the household are:

$$a_{t+1} = \underline{c}_t \left(\left[\frac{\zeta}{\zeta - 1} \left((\zeta - 1)\Delta_t \right)^{1/\zeta} - \Delta_t \right]^{-1} - 1 \right) \tag{C.6}$$

Household savings are positive for $\Delta_t \in [0, \alpha^{\zeta/(\zeta-1)}/(\zeta - 1)]$ *and decreasing in* Δ_t *for* $\Delta_t \in [0; \frac{1}{\zeta-1}]$ *and increasing when* $\Delta_t > \frac{1}{\zeta-1}$.

The proof is in an appendix available by request from the authors. Equilibrium on bonds market is described by the following relation:

$$B = \underline{c}_t \left(\left[\frac{\zeta}{\zeta - 1} \left((\zeta - 1)\Delta_t \right)^{1/\zeta} - \Delta_t \right]^{-1} - 1 \right). \tag{C.7}$$

The labor and goods markets clearing conditions lead to:

$$(c_t + \underline{c}_t)^{\nu} = 1. \tag{C.8}$$

Proposition 1. *Equilibrium is described by* $\{c_t, \underline{c}_t, \Delta_t, R_t\}$ *that solve equations* (C.2), (C.5), (C.7), (C.8).

C.4 Stationary Equilibrium

Consider a stationary equilibrium in which all aggregate variables are constant: $a_t = a$, $b_t = b$, $n_t = n$,$c_t = c$, $\underline{c}_{it} = \underline{c}_i$, $\underline{c}^*_t = \underline{c}^*$, $\underline{c}_t = \underline{c}$, $w_t = w$, and also $R_t = R$. The stationary version of equations (C.2), (C.5), (C.7), (C.8) is:

$$\Delta = (\beta R)^{-1} - 1, \tag{C.9}$$

$$\underline{c} = \frac{\zeta}{\zeta - 1} \left(c - \frac{1}{1 + \nu} \right) (1 + \Delta) \left[1 - \frac{1}{\zeta} \left((\zeta - 1)\Delta \right)^{\frac{\alpha - 1}{\alpha}} \right] \tag{C.10}$$

$$B = \underline{c} \left(\left[\frac{\zeta}{\zeta - 1} \left((\zeta - 1)\Delta \right)^{1/\zeta} - \Delta \right]^{-1} - 1 \right) \tag{C.11}$$

$$(c + \underline{c})^{\nu} = 1. \tag{C.12}$$

Proposition 2. *Stationary equilibrium is described by* $\{c, \underline{c}, \Delta, R\}$ *that solve equations* (C.9)-(C.12).

Equations (C.10) and (C.12) can be used to express $\underline{c}$ as function of Δ only.

Lemma 3. *Total liquid consumption is:*

$$\underline{c} = \frac{\nu}{1 + \nu} \cdot \frac{\frac{\zeta}{\zeta - 1}(1 + \Delta) \left[1 - \frac{1}{\zeta} \left((\zeta - 1)\Delta \right)^{\frac{\zeta - 1}{\zeta}} \right]}{1 + \frac{\zeta}{\zeta - 1}(1 + \Delta) \left[1 - \frac{1}{\zeta} \left((\zeta - 1)\Delta \right)^{\frac{\zeta - 1}{\zeta}} \right]}. \tag{C.13}$$

Combining equations (C.11) and (C.13) leads to the following asset market equilibrium conditions.

Lemma 4. *Bonds market equilibrium*

$$B = \frac{\nu}{1+\nu}(\zeta-1)^{-\frac{1}{\zeta}} \frac{(1+\Delta)\left(\frac{1+\Delta}{\Delta^{\frac{1}{\zeta}}} - \zeta(\zeta-1)^{\frac{1-\zeta}{\zeta}}\right)}{1+\frac{\zeta}{\zeta-1}(1+\Delta)\left[1-\frac{1}{\zeta}\left((\zeta-1)\Delta\right)^{\frac{\zeta-1}{\zeta}}\right]}.$$

The right-hand side of this expression satisfies $\lim_{\Delta\to+0}\left(\frac{1}{RHS}\frac{dRHS}{d\Delta}\right) = -\infty$.

The proof is in an appendix available by request from the authors. As a result, a small enough decline in B increases Δ and hence reduces R in steady state.

Proposition 3. *In a stationary equilibrium with small* Δ*, a sufficiently small decline in* B *reduces real interest rate* R*.*

C.5 Incorporating Firm Dynamics

To introduce capital, firm heterogeneity and nominal frictions, we assume that there are four types of firms in the economy instead of just one: (i) competitive intermediate goods producing firms that combine labor and capital to produce intermediate goods; (ii) monopolistically competitive retail firms that take intermediate goods and produce differentiated goods; (iii) competitive final goods producing firms who take differentiated goods and produce single final good, and (iv) intermediaries that own all of the capital and issue real risk-free one period bonds. In addition, there is a central bank that sets monetary policy.

The household problem is identical to the one present before except that the profits of new firms are rebated to the households. We present the optimization problems of new players in the economy.

Final goods producing firms.

$$\max_{\{y_{it}\}} P_t y_t - \int_0^1 P_{it} y_{it} di,$$
$$s.t.: y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}.$$

Optimal choice of the intermediate input $\{y_{it}\}$ implies:

$$y_{it} = y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta}.$$

The aggregate price index is $P_t = \left(\int P_{it}^{1-\theta} di\right)^{1/(1-\theta)}$.

Retail firms. The retail firms repackage intermediate goods: it takes one unit of intermediate good to produce a differentiated good. These firms set prices for their output conditional on the demand for differentiated goods and the price of inputs. To simplify the analysis we assume that at some point in time in the past all of the firms set identical prices that they cannot change in the future. As a result, $P_{it} = P_t = P$, we normalize the constant to be 1.

Note that because retail firms only disaggregate intermediate goods using linear technology and final goods producing firms are identical, in equilibrium, it must be that the total output of final goods equals the total output of intermediate goods.

Intermediate goods firms. There is a continuum of intermediate goods producing firms indexed by $j \in [0,1]$. The use capital and labor to produce according to a decreasing-returns-to-scale production function $m_{jt} = A_t z_j \epsilon_{jt} \left(k_{jt}^{\alpha} n_{jt}^{1-\alpha}\right)^{\phi}$. Firm j's problem is:

$$\begin{aligned}\max_{k_{jt},n_{jt},a_{jt+1}} \quad & \mathbb{E}_0 \left[\mu_t a_{jt} \sigma_t \prod_{\tau=1}^{t-1}(1-\sigma_\tau)\right], \\ \text{s.t.:} \quad & a_{jt+1}^F = \frac{P_{mt}}{P_t} A_t z_j \epsilon_{jt} \left(k_{jt}^{\alpha} n_{jt}^{1-\alpha}\right)^{\phi} - r_t^k k_{jt} - w_t n_{jt} + R_{t-1} a_{jt}^F, \\ & k_{jt} \leq \chi_t a_{jt}^F.\end{aligned}$$

where P_{mt} is the nominal price of intermediate goods. Denote the real price of intermediate goods in terms of final goods as $p_{mt} = P_{mt}/P_t$. The problem is identical to the problem considered in the text except for the fact that the real price of intermediate goods enter the evolution of firm's assets. The solution to this problem is:

$$n_{jt} = \min\left\{\left[\phi(1-\alpha)\frac{p_{mt} A_t z_j \epsilon_{jt}}{w_t}\right]^{\frac{1}{1-\phi(1-\alpha)}} \cdot \left(\chi a_t^F\right)^{\frac{\alpha\phi}{1-\phi(1-\alpha)}}, \left(\phi p_{mt} A_t z_j \epsilon_{jt}\right)^{\frac{1}{1-\phi}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha\phi}{1-\phi}} \left(\frac{\alpha}{r_t^k}\right)^{\frac{\alpha\phi}{1-\phi}}\right\}, \tag{C.14}$$

$$k_{jt} = \min\left\{\chi_t a_{jt}^F, \left(\phi p_{mt} A_t z_j \epsilon_{jt}\right)^{\frac{1}{1-\phi}} \left(\frac{1-\alpha}{w_t}\right)^{\phi\frac{1-\alpha}{1-\phi}} \left(\frac{\alpha}{r_t^k}\right)^{\frac{1-\phi(1-\alpha)}{1-\phi}}\right\}. \tag{C.15}$$

Because both constrained and unconstrained firms choose their own level of labor, the output of every firm must satisfy $m_{jt} = \frac{1}{p_{mt}} \cdot \frac{w_t n_{jt}}{\phi(1-\alpha)}$. As a result, aggregate supply of intermediate goods equals $m_t = \frac{1}{p_{mt}} \cdot \frac{w_t n_t}{\phi(1-\alpha)}$, where $n_t = \int n_{jt} dj$.

Intermediaries. Identical intermediaries own all capital in the economy k_t, issue one-period riskless real bonds a_t, rent capital to firms and provide loans to households. The zero profit condition ensures:

$$R_t - 1 = r_t^k - \delta.$$

The law of motion of intermediaries (physical) capital is:

$$k_{t+1} = (1-\delta)k_t + i_t.$$

Monetary policy. The second additional element is an explicit specification of monetary policy rule. Because we assume completely sticky prices, we have $R_t = 1 + i_t^{nom}$. We assume that monetary policy is characterized by the following rule subject to the zero lower bound:

$$R_t = \max\left\{R\left(\frac{y_t}{y}\right)^{\phi_y}, 1\right\},$$

where R is a real interest rate in a stationary equilibrium with fully flexible prices.

C.6 Equilibrium

Definition 2. *An equilibrium is an allocation* $\left\{c_t, \underline{c}_t, y_t, n_t, b_{t+1}, a_{t+1}, \left\{n_{jt}, m_{jt}, a^F_{jt+1}, k_{jt}\right\}\right\}_{t=0}^{\infty}$ *and prices* $\{p_{mt}, R_t, r^k_t, w_t\}_{t=0}^{\infty}$ *and the Euler equation wedge* $\{\Delta_t\}_{t=0}^{\infty}$ *such that households optimize, firms optimize, labor market clears* ($\int n_{jt}dj = n_t$)*, final goods market clears* ($c_t + i_t + \underline{c}_t = y_t$)*, intermediate goods market clears* ($y_t = m_t$)*, bonds market clears* ($\int a^F_{jt+1}dj + a_{t+1} = b_{t+1} + k_{t+1}$)*, and rental market for capital clears* ($\int k_{jt}dj = k_t$)

We can reduce unknowns to $\left\{c_t, \underline{c}_t, k_t, \left\{k_{jt}, n_{jt}, a^F_{jt+1}\right\}, \Delta_t, R_t, w_t, r^k_t, p_{mt}\right\}_{t=0}^{\infty}$ and equilibrium equations to:

$$\begin{aligned}
\Delta_t &= \left(\beta R_t \mathbb{E}_t \frac{c_t - \frac{1}{1+\nu} w_t^{\frac{1+\nu}{\nu}}}{c_{t+1} - \frac{1}{1+\nu} w_{t+1}^{\frac{1+\nu}{\nu}}}\right)^{-1} - 1, \\
\underline{c}_t &= \frac{\zeta}{\zeta - 1}\left(c_t - \frac{1}{1+\nu} w_t^{\frac{1+\nu}{\nu}}\right)(1+\Delta_t)\left[1 - \frac{1}{\zeta}\left((\zeta-1)\Delta_t\right)^{\frac{\zeta-1}{\zeta}}\right], \\
\int n_{jt}dj &= w_t^{1/\nu}, \qquad \text{(C.16)} \\
B_t + k_{t+1} &= \int a^F_{jt+1}dj + \underline{c}_t\left(\left[\frac{\zeta}{\zeta-1}\left((\zeta-1)\Delta_t\right)^{1/\zeta} - \Delta_t\right]^{-1} - 1\right), \qquad \text{(C.17)} \\
\int k_{jt}dj &= k_t, \\
a^F_{jt+1} &= p_{mt} A_t z_i \epsilon_{jt}\left(k_{jt}^{\alpha} n_{jt}^{1-\alpha}\right)^{\phi} - r^k_t k_{jt} - w_t n_{jt} + R_{t-1} a^F_{jt}, \\
\frac{w_t^{\frac{1+\nu}{\nu}}}{\phi\,(1-\alpha)\,p_{mt}} &= c_t + \underline{c}_t + k_{t+1} - (1-\delta)k_t, \\
R_t &= \max\left\{R\left(\frac{y_t}{y}\right)^{\phi_y}, 1\right\}, \\
r^k_t &= R_t - 1 + \delta,
\end{aligned}$$

as well as two equations (C.14) and (C.15) expressing the optimal choice of labor and capital by firms. Note that first four equations are almost identical to the equilibrium relations of the model without capital and nominal rigidities. The first difference is due to the fact that the real wage is no longer equals $w_t = 1$, because the production function is no longer linear in labor. Instead, the labor market clearing condition, equation (C.16), determines the wage w_t. The second difference is that the assets market clearing condition (C.17) not only involves the supply and demand for savings by households but also the supply of savings by the firms ($\int a^F_{jt+1}dj$) and the stock of physical capital by the intermediaries.

C.7 Equivalence Result

Here, we show that the evolution of the economy when the exogenous debt limit changes according to some process B_t is "partially" equivalent to the evolution of the economy when a sequence of aggregate productivity shocks A_t is chosen appropriately. The following definition summarizes our notion of partial equivalence. In words, we show how a shock to household credit B_t can generate the same factor allocations as a shock to productivity A_t.

Definition 3. *An equilibrium described by sequence of variables* $\left\{c_t, \underline{c}_t, k_t, \left\{k_{jt}, n_{jt}, a^F_{jt+1}\right\}, \Delta_t, R_t, w_t, r^k_t, p_{mt}\right\}_{t=0}^{\infty}$ *is partially equivalent to an equilibrium described by sequence of variables* $\left\{\widetilde{c}_t, \widetilde{\underline{c}}_t, \widetilde{k}_t, \left\{\widetilde{k}_{jt}, \widetilde{n}_{jt}, \widetilde{a}^F_{jt+1}\right\}, \widetilde{\Delta}_t, \widetilde{R}_t, \widetilde{w}_t, \widetilde{r}^k_t, \widetilde{p}_{mt}\right\}_{t=0}^{\infty}$ *if* $a^F_{jt} = \widetilde{a}^F_{jt}$, $k_{jt} = \widetilde{k}_{jt}$, $k_t = \widetilde{k}_t$ $n_{jt} = \widetilde{n}_{jt}$, $w_t = \widetilde{w}_t, R_t = \widetilde{R}_t, r^k_t = \widetilde{r}^k_t$ *for all* t.

This definition states that two equilibria are partially equivalent if the block of variables that describes the behavior of firms and prices that are relevant for firms decisions (except for p_{mt}) are identical in the two equilibria. Next, we state and prove our equivalence results.

Proposition 4. *If the real interest rate* R_t *is fixed, then for any sequence of exogenous debt limits* $\{B_t\}$ *and fixed aggregate productivity, there exists a sequence of aggregate productivity* $\{A_t\}$ *and fixed debt limit so that the two equilibria corresponding to these sequences are partially equivalent.*

Proof. When R_t is fixed at some constant level $\overline{R}$, for example, because the zero lower bound on the nominal interest rate binds, i.e., $\overline{R} = 1$. Under this assumption the rental rate of capital is also fixed at $r^k = R - 1 + \delta$. Denote an equilibrium in which exogenous debt limit varies according to $\{B_t\}$ and aggregate productivity is constant $A_t = A$ as $\left\{c_t, \underline{c}_t, n_t, i_t, k_t, a^F_{t+1}, \Delta_t, R_t, w_t, r^k_t, p_{mt}\right\}_{t=0}^{\infty}$ and equilibrium in which the exogenous debt limit is constant $B_t = B$ and aggregate productivity varies according to $\{A_t\}$ as $\left\{\widetilde{c}_t, \widetilde{\underline{c}}_t, \widetilde{n}_t, \widetilde{i}_t, \widetilde{k}_t, \widetilde{a}^F_{t+1}, \widetilde{\Delta}_t, \widetilde{R}_t, \widetilde{w}_t, \widetilde{r}^k_t, \widetilde{p}_{mt}\right\}_{t=0}^{\infty}$. We next show how to pick a sequence of A_t so that the two equilibria are partially equivalent. If the two equilibria are equivalent, then $a^F_{jt} = \widetilde{a}^F_{jt}$, $k_{jt} = \widetilde{k}_{jt}$, $k_t = \widetilde{k}_t$ $n_{jt} = \widetilde{n}_{jt}$, $w_t = \widetilde{w}_t, R_t = \widetilde{R}_t, r^k_t = \widetilde{r}^k_t$ for all t. Hence, we can write equilibrium conditions for the equilibrium with constant debt limit and varying aggregate productivity as follows

$$\widetilde{\Delta}_t = \left(\beta\overline{R}\mathbb{E}_t \frac{\widetilde{c}_t - \frac{1}{1+\nu} w_t^{\frac{1+\nu}{\nu}}}{\widetilde{c}_{t+1} - \frac{1}{1+\nu} w_{t+1}^{\frac{1+\nu}{\nu}}}\right)^{-1} - 1, \tag{C.18}$$

$$\widetilde{\underline{c}}_t = \frac{\zeta}{\zeta - 1}\left(\widetilde{c}_t - \frac{1}{1+\nu} w_t^{\frac{1+\nu}{\nu}}\right)\left(1 + \widetilde{\Delta}_t\right)\left[1 - \frac{1}{\zeta}\left((\zeta-1)\widetilde{\Delta}_t\right)^{\frac{\zeta-1}{\zeta}}\right], \tag{C.19}$$

$$B + k_{t+1} = \int a^F_{jt+1} dj + \widetilde{\underline{c}}_t \left(\left[\frac{\zeta}{\zeta-1}\left((\zeta-1)\widetilde{\Delta}_t\right)^{1/\zeta} - \widetilde{\Delta}_t\right]^{-1} - 1\right), \tag{C.20}$$

$$\frac{w_t n_t}{\phi\,(1-\alpha)\,\widetilde{p}_{mt}} = \widetilde{c}_t + \widetilde{\underline{c}}_t + k_{t+1} - (1-\delta)k_t, \tag{C.21}$$

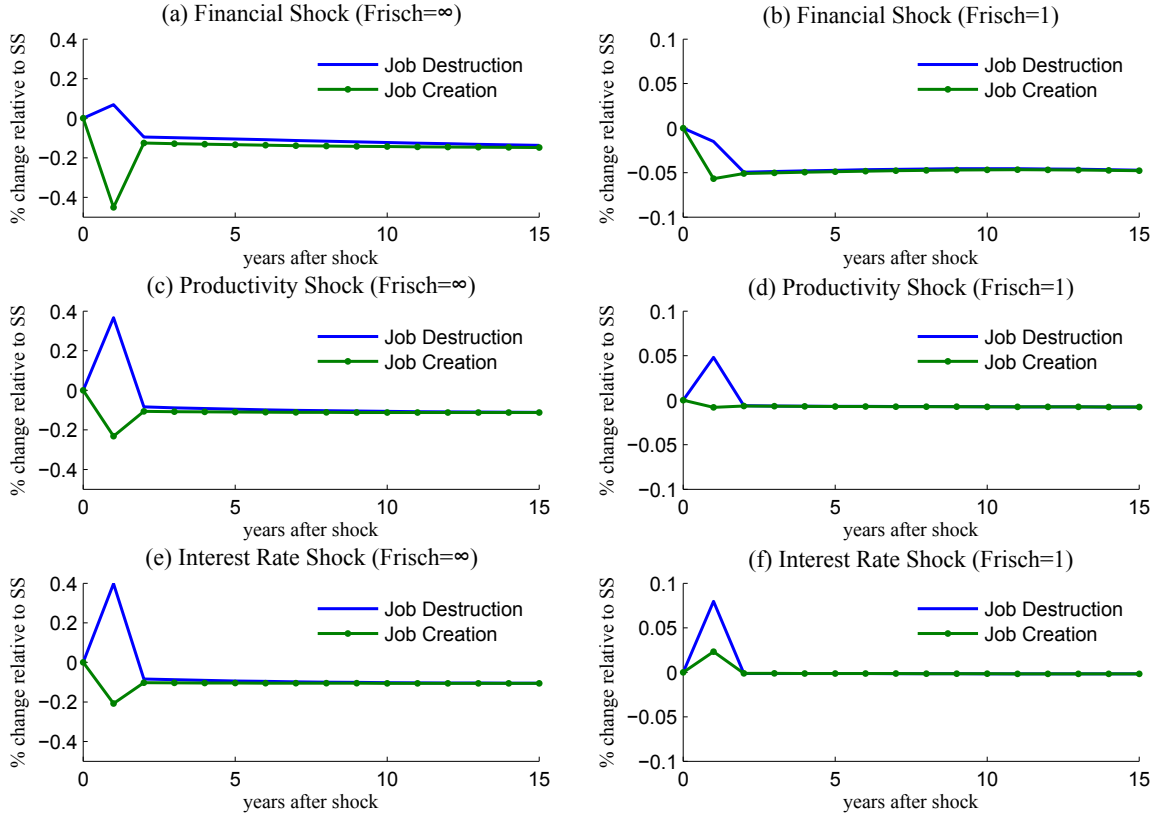
$$\int n_{jt} dj = w_t^{1/\nu}, \tag{C.22}$$

$$a^F_{jt+1} = \frac{1 - \phi(1-\alpha)}{\phi(1-\alpha)} w_t n_{jt} - r^k_t k_{jt} + \overline{R} a^F_{jt}, \tag{C.23}$$

$$n_{jt} = n_t\left(\widetilde{p}_{mt} A_t z_j \epsilon_{jt}, a^F_{jt}, w_t\right),$$
$$k_{jt} = k_t\left(\widetilde{p}_{mt} A_t z_j \epsilon_{jt}, a^F_{jt}, w_t\right),$$

where we replaced $\left(\widetilde{a}^F_{jt}, \widetilde{k}_{jt}, \widetilde{n}_{jt}, \widetilde{k}_t, \widetilde{w}_t, \widetilde{R}_t, \widetilde{r}^k_t\right)$ with $\left(a^F_{jt}, k_{jt}, n_{jt}, k_t, w_t, R_t, r^k_t\right)$ and dropped equations for R_t and r^k_t because they are fixed. We also omitted dependence of the optimal choice of labor and capital on the rental rate of capital in the last two equations. Equations (C.18)-(C.21) can be used to express $\left(\widetilde{\Delta}_t, \widetilde{c}_t, \widetilde{\underline{c}}_t, \widetilde{p}_{mt}\right)$ through $\left(n_t, n_{t+1}, k_{t+1}, \int a^F_{jt+1} dj, w_t\right)$. Equations (C.22) and (C.23) are automatically satisfied because they are identical to equations for the equilibrium

Figure 9: Job flows transition paths



The figure displays the transition paths for gross job creation and job destruction under the financial, productivity and discount rate shocks. The numbers plotted display changes relative to the initial (steady state) levels. For example, job creation declines by 45% on impact after the financial shock in case of infinite Frisch elasticity. The effects of the financial shock are shown in panels (a) and (b), the productivity shock effects are shown in panels (c) and (d), and the responses after the interest rate shock are in panels (e) and (f).

with changing debt limit and fixed aggregate productivity. If we set $A_t = p_{mt}/\widetilde{p}_{mt}$ then the last two equations are satisfied. As a result, there is a sequence of productivity shocks that generates the same values of variables related to firm behavior. □

Note that we assumed that the real interest rate, and hence, the rental rate of capital is fixed. This assumption can represent a situation in which the zero lower bound on the nominal interest rate binds forever. In this case, $\overline{R} = R$.

D Discount Rate Shocks

In this section, we consider the effect of discount rate shock on aggregate job flows and the distribution of job flows across age and size categories. We consider this shock a proxy for a variety of demand shocks such as monetary policy shocks, uncertainty/flight to safety shocks, or disaster shocks that could generate business cycles. A discount shock may also proxy for effects emphasized in Mian and Sufi (2014) and Eggertsson and Krugman (2012) by driving up the real interest rate relative to the flexible price natural rate of interest due to a binding zero lower bound. Like the productivity shock, a discount rate shock impacts all firms, raising the rental rate on capital. For unconstrained firms, this shock lowers capital and labor demand, while for financially constrained

firms, this shock raises payments to capital, lowers firm profitability, and reduces the growth rate of assets.

As Figure 9 shows, a discount rate rate shock delivers similar effects to a productivity shock on job creation and job destruction. We choose a rate shock (roughly 2.5%) that delivers the same decrease in employment as the financial and productivity shocks considered in the main text. Like a productivity shock, this shock lowers employment by operating on the job destruction margin. Job creation is nearly unchanged or even slightly. The shock, by reducing employment at unconstrained firms and lowering wages, increases job creation at new and young firms since a decrease in wages raises labor demand at these firms. A discount rate shock has similar effects on job flows across age and size categories as a productivity shock. However, this shock raises job creation at young and middle-sized firms, while raising job destruction across all size categories. The disparate effects of the productivity and discount rate shock across firm size allows our estimation to distinguish between these two shocks despite their similar effect on overall job flows.

E Liquidity Shocks

In this section, we introduce an extension to our simple model to incorporate short-term liquidity shocks that impair a firm's ability to finance working capital. In the weeks following the failure of Lehman Brothers, short-term credit markets including commercial paper markets experienced significant disruptions. To incorporate a liquidity shock, we follow Bacchetta, Benhima and Poilly (2014). As we show in this section, a liquidity shock will lower employment at mature firms that depend on a short-term borrowings, but will actually raise job creation at young and growing firms. The intuition is that smaller and younger firms do not participate in the liquid debt market and either hold cash or maintain slack in their working capital constraint by not fully pledging their assets to rent illiquid capital. Overall, the differential effects of a liquidity shock across a firm's lifecycle look like the discount rate shock considered in the previous section.

Each period is split in two subperiods. In the first subperiod, the firm can borrow in the illiquid debt market, it hires labor and rents capital. In the second subperiod, the firm can borrow at the liquid debt market and has to pay its labor force wages using liquid securities. The firm must repay its liquid debt borrowing at the beginning of the next period.

E.1 Discrete Time Constraints

To facilitate understanding, we first present the discrete time constraints of a typical firm. Define the firm's net worth in instant t by a_t. Let k_t be a firm's capital, d_t a firm's illiquid debt borrowing, l_t a firm's liquid debt borrowing and m_t a firm's cash holdings. The period t budget constraint is:

$$k_t + m_t \leq d_t + a_t.$$

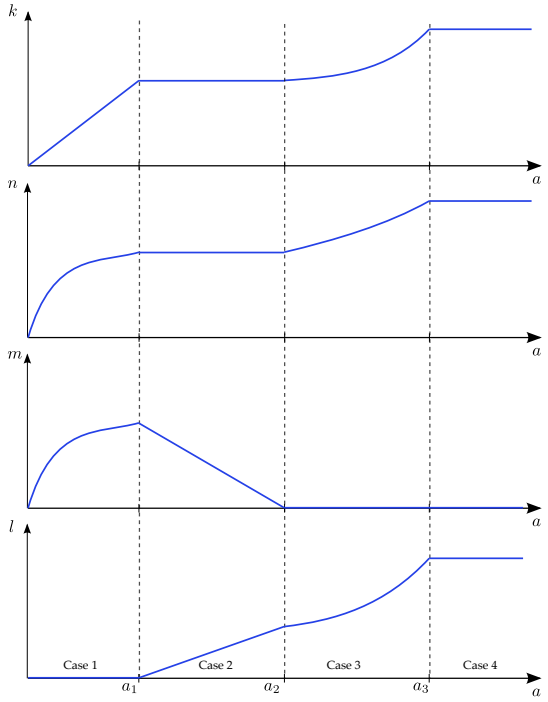
The law of motion of a firm's net worth is:

$$a_{t+1} = y_t + (1-\delta)k_t - (1+r_t)d_t - (1+r_t^l)l_t + (m_t + l_t - w_t n_t)(1+r_t^m),$$

where r_t, r_t^l, r_t^m are real net returns on illiquid debt, liquid debt and cash. This law of motion states that firms net worth in the next period equals revenues from production, plus undepreciated capital, net of gross illiquid and liquid debt repayments, plus gross returns on cash not used to pay wages. The law of motion of a firm's net worth can be rewritten as:

$$a_{t+1} - a_t = r_t a_t + y_t - (r_t + \delta)k_t - w_t n_t(1+r_t^m) - (r_t - r_t^m)m_t - (r_t^l - r_t^m)l_t.$$

Figure 10: Solution to the firm problem



Illiquid debt borrowing is constrained

$$d_t \leq \theta k_t + m_t,$$

where $\theta \in [0, 1]$. This constraint states that the firm cannot borrow more than fraction θ of its current level of capital plus the full value of its cash holdings. Note that the variables on the right-hand side of the constraint are period t variables. We make the same assumption in the paper. This constraint can be rewritten as follows:

$$k_t \leq \chi a_t,$$

where $\chi = 1/(1-\theta) \geq 1$. Liquid debt borrowing is also constrained:

$$0 \leq l_t \leq \theta k_t + m_t - d_t.$$

The first inequality states that the firm cannot lend on the liquid debt market. The second inequality states that the firm cannot borrow more than the left-over debt capacity after borrowing on the illiquid debt market. The liquid debt borrowing constraints can alternatively be written as

$$0 \leq l_t \leq a_t - \frac{k_t}{\chi}.$$

We motivate demand for liquid assets by assuming that firms have to pay wages with liquid assets:

$$w_t n_t \leq l_t + m_t.$$

Finally, to insure that the firm cannot issue cash, we assume:

$$m_t \geq 0.$$

E.2 Continuous Time Model

The above discrete time representation of the constraints lead to the following continuous time statement of the problem of the firm:

$$\max_{\{n_{t+\tau},k_{t+\tau}\}_{\tau=0}^{\infty}} \int_0^{\infty} e^{-\sigma\tau}\Lambda_{t,t+\tau}a_{t+\tau}d\tau,$$

$$\begin{aligned}
\text{s.t.: } \dot{a}_t &= r_t a_t + Az\left(k_t^{\alpha}n_t^{1-\alpha}\right)^{\phi} - (r_t+\delta)k_t - w_t n_t(1+r_t^m) \\
&\quad - (r_t - r_t^m)m_t - (r_t^l - r_t^m)l_t, \qquad [\lambda] \\
k_t &\leq \chi a_t, \qquad [\eta_k] \\
l_t &\leq a_t - \frac{k_t}{\chi}, \qquad [\eta_l] \\
w_t n_t &\leq l_t + m_t, \qquad [\xi] \\
-l_t &\leq 0, \qquad [\psi_l] \\
-m_t &\leq 0. \qquad [\psi_m]
\end{aligned}$$

The variables in square brackets denote Lagrange multipliers. The solution to this problem can be summarized by the following proposition:

Proposition 5. *Depending on the level of assets firm's optimal choice of capital, labor, cash and liquid debt is:*
Case 1: $a < a_1$

$$\begin{aligned}
k &= \chi a, \quad n = \left[\frac{(1-\alpha)\phi Az}{w(1+r)}\right]^{\frac{1}{1-(1-\alpha)\phi}} \cdot (\chi a)^{\frac{\alpha\phi}{1-(1-\alpha)\phi}}, \\
l &= 0, \quad m = w\left[\frac{(1-\alpha)\phi Az}{w(1+r)}\right]^{\frac{1}{1-(1-\alpha)\phi}} \cdot (\chi a)^{\frac{\alpha\phi}{1-(1-\alpha)\phi}}.
\end{aligned}$$

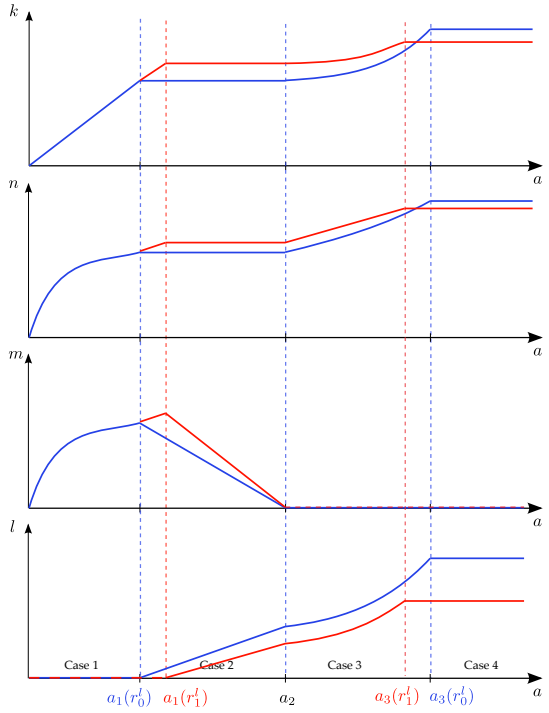
Case 2: $a \in [a_1, a_2]$

$$\begin{aligned}
k &= (\phi Az)^{1/(1-\phi)} \cdot \left[\frac{1-\alpha}{w(1+r)}\right]^{\phi(1-\alpha)/(1-\phi)} \cdot \left(\frac{\alpha}{r^k + (r-r^l)/\chi}\right)^{[1-(1-\alpha)\phi]/(1-\phi)}, \\
n &= (\phi Az)^{1/(1-\phi)} \cdot \left[\frac{1-\alpha}{w(1+r)}\right]^{(1-\alpha\phi)/(1-\phi)} \cdot \left(\frac{\alpha}{r^k + (r-r^l)/\chi}\right)^{\alpha\phi/(1-\phi)}, \\
l &= a - k/\chi, \quad m = wn - a + \frac{k}{\chi}.
\end{aligned}$$

Case 3: $a \in (a_2, a_3]$

$$\begin{aligned}
k: &\frac{\alpha\phi Az\chi}{w^{(1-\alpha)\phi}}\left(\frac{k}{\alpha\chi} - a\right)\left(a - \frac{k}{\chi}\right)^{(1-\alpha)\phi-1} k^{\alpha\phi-1} = 1 + r^l - \chi r^k, \\
n &= (a - k/\chi)/w, \\
l &= a - k/\chi, \quad m = 0.
\end{aligned}$$

Figure 11: Consequences of negative liquidity shock



Case 4: $a > a_3$

$$n = (\phi Az)^{1/(1-\phi)} \cdot \left[\frac{1-\alpha}{w(1+r^l)}\right]^{(1-\alpha\phi)/(1-\phi)} \cdot \left(\frac{\alpha}{r^k}\right)^{\alpha\phi/(1-\phi)},$$

$$k = (\phi Az)^{1/(1-\phi)} \cdot \left[\frac{1-\alpha}{w(1+r^l)}\right]^{\phi(1-\alpha)/(1-\phi)} \cdot \left(\frac{\alpha}{r^k}\right)^{[1-(1-\alpha)\phi]/(1-\phi)},$$

$$l = wn, \quad m = 0,$$

where a_1, a_2, a_3, a_4 are taken as given by the firm and are defined in the proof.

The proof of the proposition is available by request from the authors. Figure 10 summarizes the solution. There are four cases. When the firm is asset poor ($a < a_1$), it is so hungry for capital that it uses all its net worth as collateral to borrow on the illiquid debt market to invest in capital (and to buy some cash). Because it exhausts its borrowing capacity on the illiquid debt market, it cannot borrow on liquid debt market. As a result, it must set aside some cash in order to cover its wage expenses.

Once a firm's assets reach a_1, the firm does not exhaust its borrowing capacity on the illiquid debt market any more. Hence it can borrow on the liquid debt market. As a result it reduces the optimal amount of cash holdings and increases issuance of liquid debt. When $a \in [a_1, a_2]$ the firm substitutes cash for liquid debt keeping optimal labor and capital constant.

In the region $a \in (a_2, a_3]$, the firm has no cash. It does not jump to its optimal unconstrained level of capital and labor because it remains liquidity constrained. As a grows it can borrow more on the liquid debt market expanding its scale. When $a > a_3$ the firm is neither illiquid debt constrained nor liquid debt constrained. So, it can jump to its optimal size.

E.3 Effect of a Liquidity Shock

We represent a negative liquidity shock as an increase in r^l from r_0^l to $r_1^l > r_0^l$: firms have to repay more on the liquid debt market when "liquidity dries up." The effect of an increase in r_l is represented as the red line in Figure 11.

There are several effects of deteriorating liquidity. First, the net worth cutoffs for the four cases in Proposition 1 change as depicted in Figure 11. The sign of the effect on a_2 is ambiguous, so we leave it unchanged in the figure. Consider firms that have all the same characteristics except their net worth. Very high net worth firms, which correspond to case 4, are hit negatively by a liquidity shock because these are the firms who access liquid funds on the market. At the same time, the smallest net worth firms are not impacted by the shock at all because they are absent from the liquid debt market. Finally, firms with an intermediate level of net worth react positively to the shock. The approximate intuition for this result is as follows: in relative terms cash becomes more attractive than liquid debt (however, cash is still inferior to liquid debt) which implies that firms substitute away from liquid debt for "longer" (until a reaches $a_2(r_1^l)$) which makes them grow with a in $[a_2(r_0^l), a_2(r_1^l)]$.