Exchange Rate Policies at the Zero Lower Bound*

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Abstract

This paper studies how the Central Bank of a small open economy (SOE) achieves an exchange rate objective in an environment that features a zero lower bound constraint on nominal interest rates and limits to arbitrage in international capital markets. If the nominal interest rate that is consistent with interest parity is positive, the Central Bank can achieve its exchange rate objectives at the cost of losing its monetary independence, a well know result in international finance. However, if the nominal interest rate consistent with interest rate parity is negative, the pursue of an exchange rate objective necessarely results in zero nominal interest rates, deviations from interest rate parity, capital inflows, and welfare costs associated to the accumulation of foreign reserves by the Central Bank. In this zero lower bound environment, reductions in the foreign interest rates and increases in the size of the capital flows unambiguously reduce welfare, the opposite of what happens when interest rates are positive. Negative nominal interest rates could help the Central Bank by restoring interest rate parity and hindering the capital flows. We use the framework to understand the experience of Central Banks that have accumulated massive amounts of foreign reserves while pursuing exchange rate policies at the zero lower bound.

Keywords: Fixed Exchange Rates, Currency Crises, Speculative Attacks, Zero lower bound

JEL classification codes: F31, F32

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1 Introduction

Following the global financial crisis of 2008, we have witnessed large international capital flows directed to assets denominated in strong currencies. With their economies operating close or at the zero lower bound in interest rates, such capital flows have led to appreciations of several major currencies. An illustrative example is the Swiss franc, whose value went from roughly 1.6 francs per euro prior to 2008 to 1.05 at the beginning of 2011, while the Swiss policy rate hovered around 25 basis points. Countries experiencing those inflows pursued exchange rate interventions to prevent their currencies from appreciating further. The Swiss National Bank (SNB), for example, established a currency floor with the objective of preventing the Swiss franc from going below 1.2 euros. This policy required massive interventions on foreign exchange markets by the SNB, and resulted in an accumulation of foreign reserves at unprecedented pace. Eventually, on January 15 2015, the SNB decided to abandon the floor, and let the Swiss franc appreciate relative to the euro. Similar experiences were observed for other advanced economies, such as Denmark and Sweden. These experiences are difficult to interpret from the point of view of standard speculative attacks models.[1]

In this paper, we study the effects of pursuing an exchange rate policy (i.e., a peg, or a floor) and analyze the behavior of capital inflows, foreign reserve accumulation, the role of zero lower bound constraint on nominal interest rates. Towards this goal, we study a set up where the Central Bank of a small open economy (SOE) tries to implement a particular exchange rate policy, which we take as given. A key assumption is the presence of limited international arbitrage, which we model as an upper bound on the amount of foreign wealth that can be invested in the SOE.

Our first set of results show that pursuing this exchange rate objective in an environment of zero interest rates necessarily entails deviations from interest parity, a result that follows from the well known trilemma of international macroeconomics.[2] These deviations from arbitrage create an incentive for foreign investors to accumulate assets of the SOE. As identified by Cavallino (2016) and Fanelli and Straub (2015), these capital inflows in the presence of interest parity deviations are *costly* for the SOE: the Central Bank needs to accumulate

[1]Most of the research on speculative attacks has focused on the feasibility and costs associated with policies that try to prevent a depreciation of the currency in the face of capital outflows, see for example the seminal contributions of Krugman (1979) and Obstfeld (1986). There are exceptions. Grilli (1986) studies reverse speculative attacks when the amount of foreign reserves that can be accumulated by the Central Bank face an ad-hoc upper bounded. In Amador et al. (2016), we study reverse speculative attacks when the upper bound arises because of balance sheet concerns by the Central Bank.

[2]There is a large literature that explores the trilemma, including the role of capital controls in escaping the trilemma. Recent important contributions include, among others, Rey (2013), Farhi and Werning (2014), and Devereux and Yetman (2014a).

foreign reserves and necessarily takes the opposite side of the arbitrage profit made by the foreigners. The presence of a zero lower bound (ZLB) constraint on nominal interest rates is critical for our argument. Away from the ZLB, the Central Bank can achieve its exchange rate objective with no arbitrage losses as the domestic interest rate can always adjust to prevent a deviation from interest rate parity. At the ZLB, instead, the nominal interest rate cannot be further reduced and deviations from interest rate parity are unavoidable, if the exchange rate policy were not to be abandoned.

Our second set of results discusses how a zero lower bound environment leads to the reversal of the welfare effects of changes in external conditions that would have been otherwise beneficial. For example, we show that a reduction on the limits of international arbitrage (i.e., an increase on the upper bound on wealth) is always beneficial when away from the zero bound, independently of the exchange rate policy. However, such a reduction is always detrimental when the economy operates at the zero bound and the welfare loss increases with the implied appreciation of the domestic currency. In other words, deeper financial integration with the outside world switches from being a virtue to being a curse when the economy moves from positive to zero nominal rates. The zero lower bound environment also changes the way the economy reacts to a reduction in the international interest rate. While away from the ZLB, a reduction in foreign interest rates is generally beneficial (or at least irrelevant) for a borrowing country; we show that, at the zero lower bound, it strictly makes the country worse off. The key reason behind these two reversals is the behavior of the Central Bank when off and on the ZLB constraint. Away from the ZLB, the Central Bank does not need to accumulate reserves. At the ZLB, the size of the required reserve accumulation increases with a reduction in either the limits of international abitrage or the foreign interest rate, magnifying the losses.

The ZLB also has implications for the ability of the Central Bank to exploit expectational mistakes by private agents. We show that, away from the ZLB, if private agents expect a higher appreciation of the currency than what the Central Bank will in effect implement, the Central Bank can always exploit this mistake and increase domestic welfare. The result again hinges on the ability of the Central Bank to lower the nominal interest rate, accumulate foreign assets, and take advantage of the foreign investor's mistaken beliefs. At the ZLB, the opposite holds. The Central Bank cannot anymore take advantage of the mistakes, and, because it cannot lower the nominal interest rate, it is forced to intervene and accumulate foreign reserves to maintain its exchange rate policy. This intervention at the ZLB unambiguously decrease domestic welfare. This resulting welfare loss however is lower than the one generated if the increase in the expectations of appreciation were to correspond to an actual change in the policy of the Central Bank. Although we do not pursue this in the

present paper, this result has implications for the possibility of self-fulfilling appreciation "runs" at the ZLB.

Another result focuses on the balance sheet and the degree of fiscal support of the Central Bank. We consider a situation where the Central Bank cannot issue interest paying liabilities (or has limited ability to do so). In addition, the Central Bank cannot receive transfers from the fiscal authority in the initial period. We show that, when the foreign wealth is sufficiently large, the Central Bank is fully constrained by the trilemma when the economy is away from the ZLB. That is, pursuing an exchange rate policy means giving up monetary indepedence, and in the unique equilibrium, *the Central Bank cannot raise the real domestic interest rate above the foreign one*. However, when operating at the ZLB, the Central Bank can defend its exchange rate policy and maintain the domestic real rate above the foreign on. There are also limits in this case: *at the ZLB, the Central Bank cannot raise the nominal interest rate above 0*. The intuition for these results is based on the ability of the Central Bank to expand its balance sheet at the level required with a violation in interest parity. When operating away from the ZLB, the Central Bank's balance sheet is limited by the utility services of the money that it issues. Because money is a dominated asset away from the ZLB, there will be a limit to any potential balance sheet expansion, and, when the foreign wealth is large enough, this limit will not allow the Central Bank to engineer a violation of interest parity. At the ZLB, however, the Central Bank is capable of expanding its balance sheet without limits, as money is now equivalent to government bonds. That is, the Central Bank can then undertake the foreign exchange rate interventions necessary to sustain the exchange rate policy.

One remaining question is how large these arbitrage losses are in reality. Although much of the recent literature on segmented markets shies away from deviations in covered interest parity (CIP), in this paper, we do not. In particular, in our model, the covered interest parity condition is violated. Recent work by Du et al. (2016) identifies deviations from CIP for major currencies since the onset of the Great Recession, a result that we confirm for the Swiss franc. Consistent with our model, these deviations from CIP are associated to massive exchange rate interventions of the SNB: we show that spikes in deviations from CIP predict most of the increase in foreign reserves throughout the 2010-2015 period. Using these deviations, as well as the time series on foreign reserve accumulation by the SNB, we calculate an estimate on the losses associated to the currency floor. We show that those losses were significant, reaching around 0.8 to 1 percent of GDP at certain points during the floor episode. We also show that the abandonment of the floor coincides with a point in time where the losses would have significantly increased if the parity would have been mantained (justifying the decision of the SNB). It is important to highlight that while performing

this exercise, we assume that the market expectations with regards to future exchange rate changes are correct. If these expectations were mistaken, the losses from defending the exchange rate (as we discussed above) could be significantly lower. As a result, we interpret this calculation as providing an upperbound on the potential losses.

This analysis provides a framework to understand the events leading to the abandonment of the Swiss currency floor in January 2015. Switzerland is a deeply financially integrated economy operating at the zero lower bound. The SNB decided to abandon its exchange rate floor at the time where the European Central Bank was just about to announce QE, which had the eventual effect of driving the international long-term rates even lower. Both of these raise the potential losses from foreign exchange interventions. In addition, the desire to contain the capital inflows that would have ensued also explains why Switzerland was among the first countries to experiment with negative interest rates. It is interesting to note that even though the SNB abandoned the floor in January 2015, it has not reduced its foreign reserve holdings, on the contrary, reserve accumulation is still ongoing. As a result, every period where these reserves positions are not liquidated, the SNB is still incurring a significant loss, as the CIP deviations are still present.

Our work is related to recent work on segmented international markets, such as Alvarez et al. (2009) abd Gabaix and Maggiori (2015). The contributions of Cavallino (2016) and Fanelli and Straub (2015) are particular relevant, specially the latter. These authors study the effects of foreign exchange interventions by a Central Bank in world with segments markets, just as we do here. In particular, Fanelli and Straub (2015) explicitly show how a deviation from interest parity generates a cost in the intertemporal resource constraint of the economy, which is proportional to size of the deviation. This is an insight that we exploit in our analysis, although in a slightly different model. A very related literature makes a similar point. Calvo (1991) first raised the warning about the potential costs of sterilizations by Central Banks in emerging markets.[3] Subsequent papers have discussed and estimated the "quasi-fiscal" costs of these operations, and similarly identified the costs of sterizalization as a loss in the intertemporal budget constraint of the government, proportional to the interest parity deviations and the size of the accumulated reserves (see Kletzer and Spiegel 2004, Devereux and Yetman 2014b, Liu and Spiegel 2015, and references therein). Differently from all of the above papers, our main objective is to study the effects that the zero lower bound constraint imposes for exchange rate management, as well as using the CIP deviations to quantify the potential losses. Other related recent work has also explored the international

[3]Backus and Kehoe (1989) is an earlier paper showing general conditions under which sterilization (i.e., a change in the composition of the currency of denomination of the government debt) is irrelevant; a situation that happens in our framework when the economy operates away from the zero bound and the foreign wealth is large enough.

implications of the zero lower bound on interet rates, in particular see Caballero et al. (2015) and Acharya and Bengui (2015).

The structure of the paper is as follows. Section 2 introduces the basic monetary setup. Section 3 discusses the equilibrium outcomes in a non-monetary version of the model. Section 4 applies these results to understand the equilibrium in the monetary economy and draw our results regarding the effects of the zero lower bound constraint in nominal interest rates. Section 5 studies how changes in foreign wealth and the foreign interest rate affect domestic welfare when on and off the zero lower bound constraint. Section 6 uses the simple setup to quantify the losses using Switzerland as a case study. Section 7 concludes.

2 The model

We consider a two-periods ($t = 1, 2$), two currencies (domestic and foreign), one-good, deterministic small open economy, inhabited by a continuum of domestic households, a monetary and a fiscal authority. The small open economy trades with a continuum of foreign investors and can potentially also access an international financial market. We now proceed to describe the economy in detail.

2.1 Exchange rates, and interest rates

Let s_t be the exchange rate in period t, i.e. the amount of domestic currency needed to purchase one unit of foreign currency in period t. We normalize the foreign price level (i.e. the amount of foreign currency needed to buy one unit of the good) to 1 in each period, and we assume that the law of one price holds. As a result, s_t is also the domestic price level, i.e. the units of domestic currency needed to purchase one unit of the consumption good.

There are two potential financial instruments. There is a nominal bond, which is traded in the domestic economy. This bond is denominated in domestic currency and has an interest rate which we denote by i.

In addition to the domestic financial market, domestic agents are also able to access the international financial markets and save in a foreign bond, denominated in foreign currency, and with an interest rate denoted by i^*. While the domestic interest rate will be determined endogenously on the domestic credit market, the foreign rate is exogenously given, in accord with the small open economy assumption.

2.2 Domestic households

Domestic households value consumption of the final good as well as from holding real money balances according to the following utility function:

$$U(c_1, c_2, m) = u(c_1) + h\left(\frac{m}{s_1}\right) + \beta u(c_2) \tag{1}$$

where $u(.)$ is a standard utility function, c_i is household consumption in period i, m is the nominal stock of money held by the household at the end of period 1 and $h(.)$ is an increasing and concave function, also displaying a satiation level $\bar{x}$ (i.e. there exists an $\bar{x}$ s.t. $h(x) = h(\bar{x})$, for all $x \geq \bar{x}$).

Domestic households are endowed with y_1 and y_2 units of the good in the two periods hence their budget constraints in periods 1 and 2 are

$$y_1 + T_1 = c_1 + \frac{m + a}{s_1} + f \tag{2}$$

$$y_2 + T_2 = c_2 - \frac{(1 + i)a + m}{s_2} - (1 + i^*)f \tag{3}$$

where a and f represent the domestic holdings of domestic and foreign bond and T_i represent the (real) transfers from the fiscal authority to the households. We assume that households cannot borrow directly in international financial markets, $f \geq 0$. The problem is thus:

$$\max_{m,a,f,c_1,c_2} U(c_1, c_2, m)$$

s.t.:

equations (2), (3)

$f \geq 0;\ m \geq 0$

2.3 Monetary authority

We impose throughout that the monetary authority has a given nominal exchange rate objective, which we denote by $s_1^*,\ s_2^*$. In general, an exchange rate objective would arise from the desire of achieving a particular inflation target or from the presence of nominal rigidities. In the current work we do not provide a particular justification for it. Instead, we simply assume that the monetary authority follows this objective, and proceed to evaluate the costs associated with it.

In period 1, the monetary authority issues monetary liabilities M and receives a transfer of τ_1 resources from the fiscal authority. It uses these resources to purchase foreign and

domestic bonds by amounts F and A, respectively. In the second period, the Central Bank uses the proceeds from these investments redeem the outstanding monetary liabilities at the exchange rate s_2, and to make a transfer τ_2 to the fiscal authority. Just as the domestic agents, we assume that the Central Bank cannot borrow in foreign bonds. As a result, the monetary authority faces the following constraints:

$$\begin{aligned} \frac{M}{s_1} + \tau_1 &= F + \frac{A}{s_1} \\ (1+i^\star)F + (1+i)\frac{A}{s_2} &= \frac{M}{s_2} + \tau_2 \\ M \geq 0;\ F &\geq 0 \end{aligned}$$

We will sometimes find it useful to analyze the case where the Central Bank cannot receive transfers from the fiscal authority in the first period, and cannot issue goverment bonds:

Assumption 1. *[Lack of Fiscal Support] The Central Bank does not receive a transfer in the first period, and cannot issue interest paying liabilities:* $\tau_1 \geq 0$ *and* $A \geq 0$.

2.4 Fiscal authority

The fiscal authority levies taxes/transfers (T_1, T_2) on households, collects transfers/losses (τ_1, τ_2) from the monetary authority, issues domestic nominal bonds B in period 1 and redeems them in period 2. The budget constraints are:

$$\frac{B}{s_1} = T_1 + \tau_1 \tag{4}$$

$$\tau_2 = T_2 + (1+i)\frac{B}{s_2} \tag{5}$$

Note that we assume that the fiscal authority does not borrow, nor invests in foreign markets. Although it is not key because of Ricardian equivalence, from now on we will treat the bond issuance of the fiscal authority as a fixed parameter, and a result, the fiscal authority problem is determined by the transfers it reiceves from the Central Bank.

2.5 Foreign Investors and the international financial markets

A key assumption is that domestic and foreign markets are not fully integrated. In particular, there is a limit to the resources that foreign investors can channel to the domestic economy.[4] We assume that the only foreign capital that can be invested in the domestic economy is in the hands of a continuum of foreign investors, and is limited by a total amount $\bar{w}$, denominated in foreign currency.[5]

We assume that the foreign investors only value consumption in the second period. These investors cannot borrow in any of the financial markets, but can purchase both domestic and foreign assets.[6] In period 1, they decide how to allocate their wealth between foreign assets $f^\star$, domestic assets $a^\star$, and domestic currency $m^\star$; while in the second period they use the proceeds from their investments to finance their second period consumption, c^*. The foreign investor's problem is

$$\max_{f^\star, a^\star, m^\star} c^* \tag{6}$$

$$\text{subject to:} \tag{7}$$

$$\bar{w} = f^\star + \frac{a^\star + m^\star}{s_1} \tag{8}$$

$$c^\star = (1 + i^\star) f^\star + (1 + i)\frac{a^\star}{s_2} + \frac{m^\star}{s_2} \tag{9}$$

$$f^\star \geq 0,\ a^\star \geq 0 \text{ and } m^\star \geq 0. \tag{10}$$

Notice that unlike domestic investors, foreign investors do not enjoy a utility flow from holding domestic currency, so optimally, they will not hold domestic currency when the domestic interest rate i is positive.

2.6 Market clearing and the monetary equilibrium

Recall that our objective is to study whether a particular exchange rate policy can be attained as an equilibrium by the monetary authority, and to compute the costs of pursuing such a policy. Towards this goal, we will define equilibrium *for a given exchange rate policy* (s_1, s_2):

[4] There is a recent literature on segmented international asset markets, see for example Alvarez et al. (2009) and Gabaix and Maggiori (2015).

[5] This way of modelling foreign inverstors is different from Fanelli and Straub (2015). In that paper, foreign demand for domestic assets ends up been a linear function of the arbitrage return, that crosses the origin. In our model, instead, the foreign demand will be a step function of the arbitrage return. That is, there is always a strictly positive amount of foreign wealth ready to arbitrage away any profits from investing in the SOE.

[6] An alternative interpretation is that $\bar{w}$ already represents the total wealth available for investing in period 0, inclusive of any amount that could be borrowed.

Definition 1. A monetary equilibrium, given an exchange rate policy (s_1, s_2), is a consumption profile for households, (c_1, c_2), and asset positions, (a, f, m); a consumption for investors, $c^\star$, and their asset positions $(a^\star, f^\star, m^\star)$; money supply, M; transfers from the monetary authority to the fiscal, (τ_1, τ_2); investments by the monetary authority, (A, F); transfers from the fiscal authority to the household, (T_1, T_2); and a domestic interest rate i, such that:

(i) The representative household chooses consumption and portfolio choices to maximize utility, subject to its budget and borrowing constraints.

(ii) Foreign investors choose consumption and porffolio choices to maximize their utility, subject to their budget and borrowing constraints.

(iii) The purchases of assets by the monetary authority, its decision about the money supply and its transfers to the fiscal authority satisfy its budget constraints, as well as $F \geq 0$.

(iv) The fiscal authority satisfies its budget constraints.

(v) And the domestic asset market clears for both money and bond

$$\begin{aligned} m + m^\star &= M \\ a + a^\star + A &= B \end{aligned}$$

It is helpful to write down, using the market clearing conditions, the foreign asset position of the small open economy in any equilibrium. In particular, using the household budget constraint in the first period, as well as the monetary authority and fiscal authority budget constraints, we have the following equality, linking the trade deficit to the net foreign asset position:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{m^\star + a^\star}{s_1}}_{\text{foreign liabilities}} - \underbrace{[f + F]}_{\text{foreign assets}} \tag{11}$$

Similarly, using the budget constraint in the second period, we have the following equality:

$$c_2 - y_2 = (1 + i^\star)(f + F) - \frac{m^\star + (1 + i)a^\star}{s_2} \text{ ; for all } s \in S \tag{12}$$

Before proceeding to fully characterize how equilibrium outcomes depends on the chosen exchange rate policy, it is useful to characterize equilibria in a simplified "real" version of the economy.

3 A non-monetary version of the model

We consider a real economy where there is no money, there is a real domestic bond, which pays a real interest rate r, as well as a real foreign bond which pays a real interest rate, $i^\star$. In this economy the central bank's only decision is how much foreign reserves F to accumulate in the first period. Since reserves (plus interest) are rebated back to households in the second period, reserve accumulation amounts to shift household resources from the first to the second period. We can then define

$$\begin{aligned}\tilde{y}_1 &= y_1 - F \\ \tilde{y}_2 &= y_2 + (1+i^\star)F\end{aligned}$$

That is, $(\tilde{y}_1, \tilde{y}_2)$ represent the endowment of the economy, after the Central Bank has made its foreign reserve decisions.

The domestic representative household maximizes utility $u(c_1) + \beta u(c_2)$ subject to the following budget constraints:

$$\begin{aligned}c_1 &= \tilde{y}_1 - f - a \\ c_2 &= \tilde{y}_2 + (1+i^\star)f + (1+r)a\end{aligned}$$

where f and a represent their purchases of foreign and domestic assets, respectively. As in the monetary economy, we impose that they cannot borrow abroad, so $f \geq 0$.

The foreign investors are willing to invest up to the maximum of their wealth, $\bar{w}$, to maximize their return. That is, their domestic asset demand, $a^\star$, satisfies:

$$\max_{0 \leq a^\star \leq \bar{w}} a^\star(r - i^\star) = \bar{w}(r - i^\star) \tag{13}$$

where the last equality follows from the maximization.

Market clearing in the domestic financial market imposes that the total demand of domestic assets be zero:

$$a^\star + a = 0$$

We define a non-monetary equilibrium as follows:

Definition 2. A *non-monetary equilibrium given* F is a consumption pair (c_1, c_2), and a domestic real interest rate, r; such that there exists a demand for domestic assets by foreign investors, $a^\star$, and bond holdings by domestic households, (a, f), with the properties that (i) (c_1, c_2) and (a, f) maximize the households' utility subject to the budget and borrowing

constraints, (ii) $a^\star$ maximize the foreign investor's utility; and (iii) the domestic asset market clears.

From the households' optimality, we obtain the following conditions:

$$u'(c_1) = (1+r)\beta u'(c_2) \tag{14}$$

$$r \geq i^\star \tag{15}$$

with $f = 0$ if the last inequality holds strictly. The first condition is just the Euler equation for the domestic households in the domestic asset. The second condition imposes that the real interest rate at home cannot be strictly lower than abroad, as the domestic asset demand will be minus infinity.

Using the budget constraints of the domestic households and solving out for domestic asset holdings, we obtain the following intertemporal budget constraint:

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1+r} - f\left[\frac{r - i^\star}{1+r}\right] = 0$$

From the household optimality condition state above, we know that $f = 0$ if $r > i^\star$, so it follows then that

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1+r} = 0 \tag{16}$$

Finally, we have one additional condition, related to the trade deficit. Recall that $c_1 = \tilde{y}_1 - f - a$, using that $-a = a^\star \leq \bar{w}$ and that $f \geq 0$, it follows that

$$c_1 \leq \tilde{y}_1 + \bar{w} \tag{17}$$

with equality if $r > i^\star$. This last follows from the optimal decisions when $r > i^\star$: households do not invest abroad, $f = 0$, and foreigners invest all of their wealth, $a^\star = \bar{w}$.

It turns out that conditions (14), (15), (16), and (17) fully characterize a non-monetary equilibrium. Let us define by the "first-best", the consumption allocation that would be attained if there were not borrowing nor wealth constraints. That is, c_1^{fb}, c_2^{fb} are defined to be

$$(c_1^{fb}, c_2^{fb}) \equiv \arg\max_{(c_1,c_2)} \{u(c_1) + \beta u(c_2)\}$$

subject to:

$$y_1 + \frac{y_2}{1+i^\star} = c_1 + \frac{c_2}{1+i^\star}$$

Note that the first best is independent of F. We have then following characterization of a non-monetary equilibrium:

Lemma 1. *Non-monetary equilibrium are characterized as follows, depending on F and $\bar{w}$:*

(i) If $y_1 - F + \bar{w} \geq c_1^{fb}$, then there is a unique non-monetary equilibrium given F, and it features $r = i^\star$; $c_1 = c_1^{fb}$; and $c_2 = c_2^{fb}$.

(ii) If $c_1^{fb} > y_1 - F + \bar{w} > 0$, then there is a unique non-monetary equilibrium given F that features $c_1 = y_1 - F + \bar{w} < c_1^{fb}$; c_2 is the only value that solves

$$c_2 = y_2 + (1 + i^\star)F - \frac{u'(c_1)}{\beta u'(c_2)}\bar{w} \tag{18}$$

and $r = \frac{u'(c_1)}{\beta u'(c_2)} - 1 > i^\star$.

(iii) There is no non-monetary equilibrium for $y_1 - F + \bar{w} < 0$.

This lemma shows that there are only two possible equilibrium outcomes. The cases are shown in Figure 1.

Panel (a) corresponds to the case when there is enough foreign wealth to cover the difference between the endowment $\tilde{y}_1 = y_1 - F$ and the first period level of first-best consumption, c_1^{fb}. In this case, the domestic real rate equals the foreign one, and the first-best consumption allocation, point A, is the unique equilibrium outcome.

Panel (b) illustrates the case when the wealth is not enough to reach the first period level of first-best consumption from the endowment $\tilde{y}_1 = y_1 - F$, that is $c_1^{fb} - \tilde{y}_1 > \bar{w}$. Competition for these limited external resources results in a higher domestic real interest, which is a rent for foreign investors.[7] Note that the equilibrium point in this case, point B, lies strictly within the feasibility frontier for the small open economy, as the domestic interest rate is higher than the foreign one, and the country is a borrower. Given point $(\tilde{y}_1, \tilde{y}_2)$, consumption in the first period is determined by $c_1 = \tilde{y}_1 + \bar{w}$, while consumption in the second period, c_2, is such that the budget line with slope $1 + r$ is tangent to the utility function. As we discussed above, there is only one such c_2.

As done in Fanelli and Straub (2015), another way of representing the losses faced by domestic households is to rewrite the intertemporal budget constraint solving out for foreign reserve holdings, using that $a^\star(r - i^\star) = \bar{w}(r - i^\star)$ together with the market clearing condition, which leads to:

$$y_1 - c_1 + \frac{y_2 - c_2}{1 + i^\star} - \bar{w}\left[\frac{1 + r}{1 + i^\star} - 1\right] = 0 \tag{19}$$

[7] This is related to Costinot et al. (2014), who emphasize the benefits of interest rate manipulation for a large country.

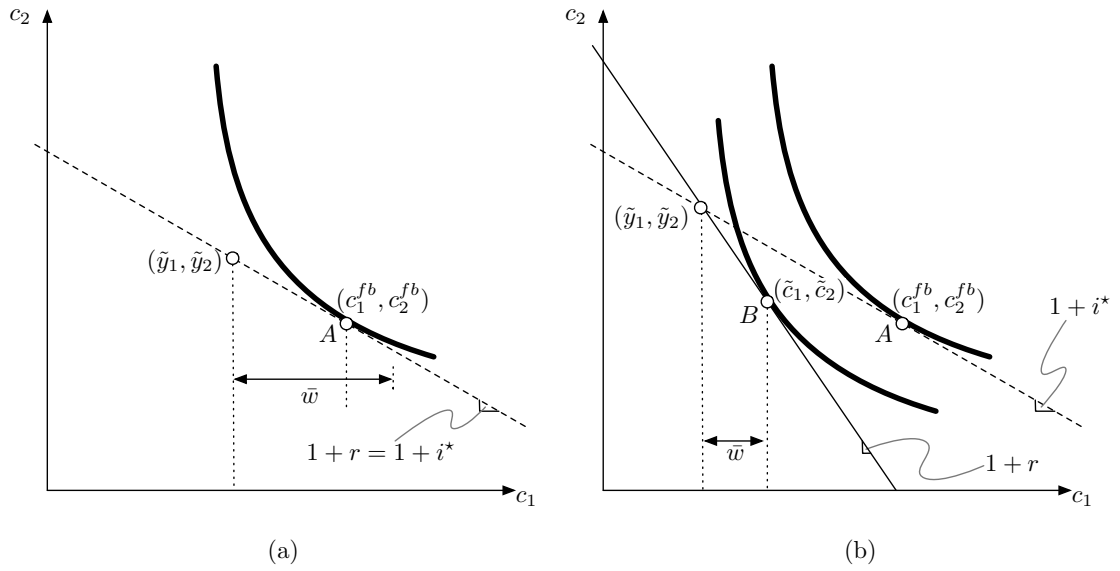


Figure 1: Non Monetary Equilibria

The first two terms represent the standard intertemporal resource constraint for an economy that could borrow and save freely at rate $i^\star$. But there is an additional term, which captures the reason why the equilibrium consumption outcome lies strictly within the feasibility frontier. As stressed by Fanelli and Straub (2015), this term represents a loss: as the foreigners invest when the domestic interest is above the foreign one, they gain a profit which is a loss to the country. As can be seen, the losses are proportional to the amount of wealth invested by the foreign investors, $a^\star$, and the differential interest rate, $r - i^\star$. Note that differently from Fanelli and Straub (2015), in our environment, these losses may arise even absent a Central Bank intervention, if the foreign wealth is not large enough to take the economy to the first best allocation. When studying the zero lower bound environment, we will find it more useful to work with a version of equation (16), rather than with (19).

We have the following corollary, detailing how the domestic real rate is affected by the foreign wealth $\bar{w}$ and F:

Corollary 1 (Comparative Statics)**.** *There is a continuous function $g(\cdot, \cdot)$ such that for any F and $\bar{w}$, in the non-monetary equilibrium given F, the domestic interest rate r is such that $r = g(F, \bar{w})$. In addition, the function g satisfies (i) $g(F, \bar{w}) = i^\star$ for all $y_1 - F + \bar{w} \geq c_1^{fb}$ and (ii) $g_1(F, \bar{w}) > 0$ and $g_2(F, \bar{w}) < 0$ for $0 < y_1 - F + \bar{w} < c_1^{fb}$; and where $\lim_{x \to -(y_1 + \bar{w})} g(x, \bar{w}) = i^\star + \frac{y_2 + (1+i^\star)y_1}{\bar{w}} = \overline{r} > i^\star$.*

Proof. For $y_1 - F + \bar{w} \geq c_1^{fb}$, the domestic interest rate equals the foreign rate, $r = i^\star$. Note

that for $y_1 - F + \bar{w} < c_1^{fb}$, r is determined by the following implicit equation:

$$1 + g(F, \bar{w}) = \frac{u'(y_1 - F + \bar{w})}{\beta u'(y_2 + (1 + i^\star)F - (1 + g(F, \bar{w}))\bar{w})}$$

There is a unique possible value of g. Note also that as $y_1 - F + \bar{w}$ approaches c_1^{fb} from below, the solution to the above equation approaches $i^\star$.

Using the implicit function theorem, the derivative of g with respect to $\bar{w}$ is

$$\frac{dg}{d\bar{w}} = \frac{u''(c_1)}{\beta u'(c_2)} + \frac{u'(c_1)u''(c_2)}{\beta(u'(c_2))^2}\left((1 + g) + \bar{w}\frac{dg}{d\bar{w}}\right)$$
$$\frac{dg}{d\bar{w}} = \frac{u''(c_1) + \left(\frac{u'(c_1)}{u'(c_2)}\right)^2 u''(c_2)}{\beta u'(c_2) - \frac{u'(c_1)}{u'(c_2)}u''(c_2)\bar{w}} < 0$$

where the second equality uses that $1 + g = u'(c_1)/(\beta u'(c_2))$, and the last inequality follows from strict concavity of the utility function.

The derivative with respect to F is

$$\frac{dg}{dF} = -\frac{u''(c_1)}{\beta u'(c_2)} - \frac{u'(c_1)u''(c_2)}{\beta(u'(c_2))^2}\left((1 + i^\star) - \bar{w}\frac{dg}{dF}\right)$$
$$\frac{dg}{dF} = -\frac{u''(c_1) + (1 + i^\star)\frac{u'(c_1)}{u'(c_2)}u''(c_2)}{\beta u'(c_2) - \frac{u'(c_1)}{u'(c_2)}u''(c_2)\bar{w}} > 0$$

where again, the last inequality follows from strict concavity of the utility function. □

This corollary states that the domestic interest rate is strictly decreasing in the foreign wealth, as long as $y_1 - F + \bar{w} \leq c_1^{fb}$. It also shows that there is a maximum possible domestic interest rate consistent with equilibrium for any given $\bar{w}$. This corollary is useful in addition because it shows that, given an foreign asset position by the Central Bank, increases in foreign wealth lead to a reduction in the domestic real interest rate. From 16, this will imply an unambiguos increase in domestic welfare.

How does the foreign asset accumulation of the Central Bank affect the equilibrium outcome? What about the role of foreign wealth? As we will see now, interventions by the Central Bank, are never beneficial in this real environment, and a higher value of $\bar{w}$ weakly increases welfare:

Lemma 2. *The welfare of the domestic households in the non-monetary equilibrium given F is strictly decreasing in F and strictly increasing in $\bar{w}$ for $0 < y_1 - F + \bar{w} < c_1^{fb}$, and is constant otherwise.*

Proof. The second part of the lemma is straigthforward, as the equilibrium is not affected by F or $\bar{w}$ for $y_1 - F + \bar{w} \geq c_1^{fb}$. The interesting part is then for $y_1 - F + \bar{w} < c_1^{fb}$.

Note that in this range, $c_1 = y_1 - F + \bar{w}$ and $c_2 = y_2 + (1 + i^\star)F - (1 + r)\bar{w}$, and domestic welfare is

$$W = u(y_1 - F + \bar{w}) + \beta u(y_2 + (1 + i^\star)F - (1 + r)\bar{w})$$

where r is the equilibrium domestic rate, which is of course a function of both F and $\bar{w}$. The derivative of welfare with respect to F (using the envelope) is

$$\frac{dW}{dF} = -u'(c_1) + (1 + i^\star)\beta u'(c_2) = -(r - i^\star)\beta u'(c_2) < 0$$

as $r > i^\star$ for $y_1 - F + \bar{w} < c_1^{fb}$.

Taken the derivative with respect to $\bar{w}$, we get

$$\begin{aligned}\frac{dW}{d\bar{w}} &= u'(c_1) - (1 + r)\beta u'(c_2) - \beta u'(c_2)\bar{w}\frac{dr}{d\bar{w}} \\ &= -\beta u'(c_2)\left(\bar{w}\frac{dr}{d\bar{w}}\right)\end{aligned}$$

where have used the equilibrium condition that $1 + r = \frac{u'(c_1)}{\beta u'(c_2)}$. So, to compute the derivative of the welfare in this region we just need to know how the real rate is affected by the increase in foreign wealth. But corollary 1 guarantees that r is strictly decreasing in $\bar{w}$ in the region of interest, and so welfare is strictly increasing in $\bar{w}$ for $\bar{w} \in (0, c_1^{fb} - y_1 + F)$. □

What this lemma shows is that when the economy is borrowing constrained, so that $y_1 - F + \bar{w} < c_1^{fb}$, an increase in the reserves held by the Central Bank strictly reduces welfare; and an increase in foreign wealth, strictly increases it.

From our discussion surrounding equation (19), one may have guessed that higher foreign wealth should lead to lower welfare for the domestic households, as the size of the losses increase. But this conclusion is not correct. The intution for why more foreign wealth is beneficial is as follows. When foreign wealth is limited, so that $y_1 - F + \bar{w} < c_1^{fb}$, the domestic interest rate lies strictly above the foreign one, and the households are borrowing the maximum possible from the foreign investors. An increase in the foreign wealth has two effects. First, it allows the households to borrow more at the given equilibrium interest rate. However, this has no first-order effect on welfare, as the households where already maximizing given that equilibrium interest rate. Second, an increase in foreign wealth reduces the equilibrium domestic interest rate; and this generates a first order benefit, as domestic households, who were borrowing $\bar{w}$, can now reduce their interest rate payments. This first

order gain of this in terms of period 2's consumption is $-\bar{w}\frac{dr}{d\bar{w}}$, the key expression that shows up in the proof of Lemma 2. It is immediate then that domestic welfare in the best monetary equilibrium when away from the zero lower bound is increasing in foreign wealth.

For the effect of F, the key is that the induced reduction in $\tilde{y}_1$ and increase in $\tilde{y}_2$ ends up increasing the real interest rate. Note that from the resource constraint, equation (16), we have that:

$$y_1 - \tilde{c}_1 + \frac{y_2 - \tilde{c}_2}{1+r} - F\left[\frac{r - i^\star}{1+r}\right] = 0 \tag{20}$$

The Central Bank intervention has two effects, first it increases the domestic real rate, whose adverse effects can be seen from the first two terms of the resource constraint above. But in addition, even if the intervention were not to affect the real rate, there are additional losses, captured by the last term. These losses appear because the Central Bank strategy consists of saving abroad at a low return, while the economy is in effect borrowing at a high one, and the more it saves, the higher these losses become. [8]

Figure 2 demonstrates this graphically. With no intervention, the equilibrium is denoted by point A, which in this case corresponds to the first best allocation. With a sufficiently large accumulation of foreign reserves, the Central Bank moves the economy to the income profile from (y_1,y_2) to $(\tilde{y}_1, \tilde{y}_2)$. As a result, the first best allocation cannot longer be attained, as foreign wealth is not large enough. In this case, the equilibrium domestic real rate exceeds $i^\star$, and the consumption allocation is now at point B. There are two losses from the intervention. First, is that the real rate is now higher than $i^\star$. Given that the country is a borrower, this generates a loss. This is represented in the figure by the movement from point A to the point (the gray dot in the figure) that will maximize utility subject to budget line BC_1. There is an additional loss generated by the Central Bank intervention, and that is the movement from BC_1 to BC_2, and this captures the last term of equation (20).

In this non-monetary world, Central Bank interventions are not desirable (at best they have no effect). It would be optimal for the Central Bank to set $F = 0$. We show below how, in a monetary environment, the Central Bank may be forced (because of its exchange rate objective and the zero lower bound) to engage on this type of costly interventions.

4 The monetary equilibrium

Let us return to the monetary economy. From the household first order conditions, we obtain the following result:

[8] This notion of the losses is the one we will use later on in section 7 to quantify the losses incurred by the Swiss National Bank.

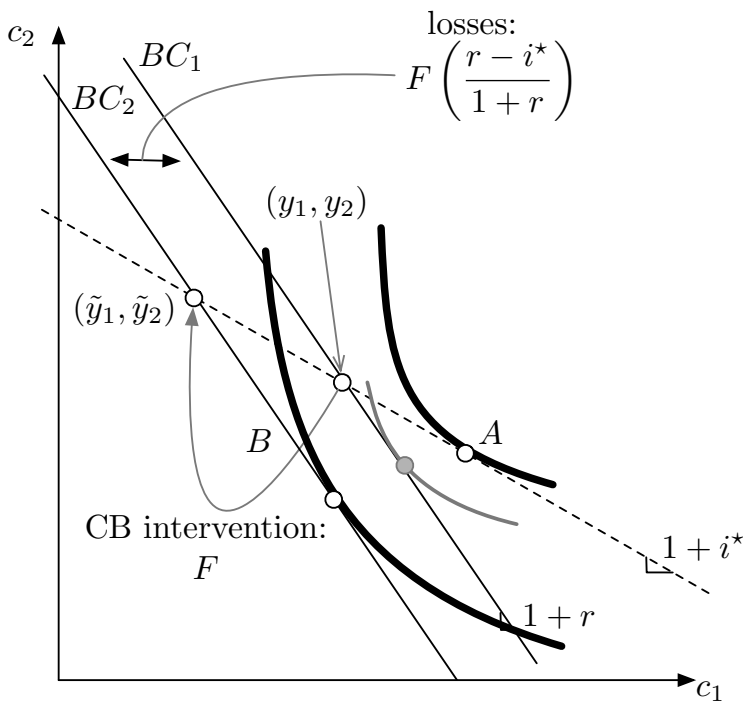


Figure 2: Central Bank Interventions

Lemma 3. *In a monetary equilibrium given* (s_1, s_2), $i \geq 0$ *and*

$$u'(c_1) = \beta(1+i)\frac{s_1}{s_2}u'(c_2) \tag{21}$$

$$(1+i)\frac{s_1}{s_2} \geq (1+i^\star) \tag{22}$$

$$h'\left(\frac{m}{s_1}\right) = \frac{i}{1+i}\frac{u'(c_1)}{s_1} \tag{23}$$

and $f = 0$ *if* $(1+i)\frac{s_1}{s_2} > (1+i^\star)$.

Using the budget constraints of the households, together with market clearing condition in the money market, we get the following equation:

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - f\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] - F\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] + \frac{i}{\frac{s_1}{s_2}(1+i)}\frac{m^\star}{s_2} = 0$$

Note however that $f = 0$ if $1 - \frac{s_2(1+i^\star)}{s_1(1+i)} > 0$ (from the above lemma). So we have that

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] + \frac{i}{\frac{s_1}{s_2}(1+i)}\frac{m^\star}{s_2} = 0$$

The first three terms correspond to the way we wrote the intertemporal resource constraint for the non-monetary economy, equation (20), as the domestic real interest rate in this monetary economy is $(1+i)\frac{s_1}{s_2}$. The interpretation of these terms is similar. There is however a new term, the last one. This captures the potential seigniorage collected from foreigners. Because foreigners do not receive liquidity services from holding money balances, they will never hold domestic money, unless the domestic nominal interest rate is 0. As a result, the following intertemporal resource constraint holds

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F\left[1 - \frac{s_2(1+i^\star)}{s_1(1+i)}\right] = 0 \tag{24}$$

The final equilibrium condition revolves around the Central Bank asset position. Recall from equation (11) that

$$c_1 - y_1 + F = \frac{m^\star + a^\star}{s_1} - f \leq \bar{w}$$

where the last inequality follows from $f \geq 0$ and $m^\star + a^\star \leq s_1\bar{w}$. In addition, if $\frac{1+i}{1+i^\star}\frac{s_1}{s_2} - 1 > 0$, then we know that $m^\star + a^\star = s_1\bar{w}$ and $f = 0$ (i.e., foreigners invest everything in the domestic assets, and households do no invest in the foreign asset). It follows then that, in a monetary equilibrium,

$$c_1 \leq y_1 - F + \bar{w}; \text{ with equality if } \frac{1+i}{1+i^\star}\frac{s_1}{s_2} - 1 > 0 \tag{25}$$

In other words, the foreign wealth must finance the trade deficit plus the reserve accumulation of the Central Bank.

Note that equations (21), (22), (24), and (25) are the same equations that characterize a non-monetary equilibrium, equations (14), (15), (16), and (17), with $r = (1+i)\frac{s_1}{s_2} - 1$ and $\tilde{y}_1 = y_1 - F$ and $\tilde{y}_2 = y_1 + (1+i^\star)F$. Thus, any monetary equilibrium must deliver an allocation consistent with a non-monetary equilibrium outcome. Note however that a monetary equilibrium imposes the additional restriction that the nominal interest rate must be non-negative (i.e., the zero lower bound), a key restriction that will play an important role in what follows.

As a result, given the central bank's exchange rate objective (s_1, s_2) there is, potentially, a continuum of monetary equilibria, indexed by the size of the Central Bank foreign exchange intervention, F. Let $\underline{r}$ denote the domestic real interest rate in the best non-monetary equilibrium, that one associated with $\tilde{y}_1 = y_1$ and $F = 0$. From section 3 we know that $\underline{r} \geq i^\star$. There are two cases two consider for the monetary economy. The cases are differentiated by whether the zero lower bound constraint is satisfied or not at the real interest rate $\underline{r}$.

4.1 Away from the ZLB

Consider first the situation where $(1+\underline{r})\frac{s_2}{s_1} \geq 1$. Then, any non-monetary equilibrium, and in particular, the best non-monetary equilibrium, is also a possible monetary outcome:

Lemma 4. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} \geq 1$. *Then, for all* $F \in [0, y_1 + \bar{w})$, *the non-monetary equilibrium* (c_1, c_2, r) *given* F *constitutes a monetary equilibrium outcome.*

The intuition behind this lemma is as follows. When the Central Bank does not intervene, so that $F = 0$, the non-monetary equilibrium will feature a real interest rate equal to $\underline{r}$. This real rate, together with the exchange rate policy, implies a nominal rate i which satisfies the zero lower bound constraint (give that $(1+\underline{r})\frac{s_2}{s_1} > 1$), and as a result, constitutes a monetary equilibrium. If the Central Bank were to increase the size of its reserves holdings, we know from Corollary 1 that this will imply an even higher real domestic rate, which will keep the zero-lower bound constraint from binding, generating another possible monetary equilibrium outcome.

Let us now consider the best monetary equilibrium outcome: that is, the monetary equilibrium outcome that maximizes household's utility. We know from Lemma 2 that in the non-monetary economy, household's welfare is decreasing in F, and a result, in that environment, the best equilibrium outcome will emerge under no intervention, $F = 0$.[9] In the monetary economy, in addition, we need to keep track of the utility generated from real money balances. However, as discussed above, increasing F, weakly increases the equilibrium real rate, and a result, the equilibrium nominal interest rate. This implies that the real money balances will (weakly) decrease with F, further reducing welfare. The following result is then immediate:

Lemma 5. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} \geq 1$. *Then, the best monetary equilibrium outcome can be attained by setting* $F = 0$ *and* $i = (1+\underline{r})\frac{s_2}{s_1} - 1$. *If, in addition,* $c_1^{fb} \leq y_1 + \bar{w}$ *then, interest parity holds:* $(1+i) = (1+i^\star)\frac{s_2}{s_1}$.

Just as in the non-monetary world, the best equilibrium outcome can be attained when the Central Bank does not intervene and accumulates foreign reserves.

Fiscal Support. It is interesting to note that, if the Central Bank cannot receive fiscal support in the first period, under certain conditions, the best monetary equilibrium is the unique equilibrium outome. That is, the Central Bank does not have the ability to increase the interest rate away from parity, as it is restricted by its balance sheet:

[9] Note that if $c_1^{fb} - y_1 < \bar{w}$, then all $F \geq 0$ such that $c_1^{fb} - y_1 - F < \bar{w}$ also generate non-monetary equilibria that attaing the best possible outcome. This will be true as well in the monetary environment.

Lemma 6 (A Central Bank without fiscal support away from the ZLB)**.** *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} > 1$ *and that assumption 1 holds. In addition, suppose that* $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$*. Then all monetary equilibra attain the first best consumption allocation, the same domestic welfare, and the interest parity holds.*

Lemma 6 tells us that a Central Bank that cannot issue interest rate paying liabilities will be constrained in its ability to raise the domestic real rate above the foreign one. In particular, suppose that the Central Bank tries to raise the domestic rate above the foreign one. This will lead to an immediate inflow of foreign capital, of size $\bar{w}$, which will push down the domestic interest rate. To keep the rate from falling, the Central Bank must purchase a large amount of the inflow and accumulate foreign reserves. But the purchasing power of the Central Bank is limited by its balance sheet. Under assumption 1, the Central Bank's liabilities are bounded by the satiation point of money, $\bar{x}$, as the nominal interest rate rate is always striclty positive in this case; which is not enough to counteract the capital inflow. To summarize, if the external wealth is sufficiently high then no matter what the size of the Central Bank foreign asset position is (as long as it is consistent with its limited balance sheet), the equilibrium consumption allocation is always the first-best one. In this environment, the foreign exchange interventions of the Central Bank are irrelevant, and do not affect equilibrium outcomes (a result related to the classic irrelevance result of Backus and Kehoe 1989).

4.2 At the ZLB

The second case is when $(1+\underline{r})\frac{s_2}{s_1} < 1$. In this case, the non-monetary equilibrium without intervention, $\tilde{y}_1 = y_1$, generates a domestic real rate that is inconsistent with the zero lower bound, and as a result, it cannot be attained as a monetary outcome given the exchange rate policy s_1, s_2. As a result, in a monetary equilibrium, the domestic real interest rate will need to lie strictly above the foreign one.[10]

So, for there to be a monetary equilibrium, the Central Bank will need to intervene and accumulate reserves of a magnitude sufficient to raise the real rate above the level consistent with interest parity. Recall that we have defined $\bar{r}$ to be the highest possible real interest rate in the non-monetary economy (that is, the interest rate associated with the maximum possible intervention). We have then the following result:

[10] In case this is not clear, this follows immediately from the following set of inequalities:

$$(1+i)\frac{s_1}{s_2} - 1 \geq \frac{s_1}{s_2} - 1 > \underline{r} \geq i^\star$$

where the first term is the domestic real rate, the first inequality follows from the ZLB, the second defines the case of interest, and the last one is the restriction that appears in any non-monetary equilibrium.

Lemma 7. *Suppose that* $1+\underline{r} < \frac{s_1}{s_2} < 1+\bar{r}$*, then there exists an* $\underline{F} > 0$ *such that for all* $F \in [\underline{F}, y_1 + \bar{w})$*, the non-monetary equilibrium* (c_1, c_2, r) *given* F *constitutes a monetary equilibrium outcome. In addition, the best monetary equilibrium outcome is attained when* $F = \underline{F}$ *and* $i = 0$*.*

Similarly to our previous case, the best monetary equilibrium is the one consistent with the lowest possible level of intervention by the Central Bank. In this case, however, the intervention has to be strictly positive. From equation (20), given the domestic real rate, we can identify the losses generated by the Central Bank intervention by the term $F\left(1-(1+i^{\star})\frac{s_2}{s_1}\right)$. That is, the losses are equal to the size of reserve holdings times the return differential between domestic and foreign assets. As discussed previously, when the Central Bank accumulates reserves it is effectively borrowing at the high rate $\frac{s_1}{s_2}$ and investing at the low rate $1+i^*$, generating a loss.

The mechanism at play here is related to the one highlighted in closed-economy New Keynesian models, such as in Eggertsson and Woodford (2003), Christiano et al. (2011) and Werning (2011). In both setups restoring equilibrium at the zero lower bound requires a reduction in the desired savings by domestic agents, and in both setups this adjustment is costly. In new Keynesian closed economy models the reduction in saving arises because of declines in current output, caused by nominal rigidities, and the cost is the output loss itself. In our setup the reduction in desired savings is generated through the Central Bank intervention, which transfers resources from the the present to the future, and the loss is driven by the fact that this intervention entail transfers of resources from domestic to foreign agents.

Fiscal Support. Recall that equilibrium at the ZLB is characterized by a real domestic interest rate that is strictly higher than the foreign one. Differently from the case away from the ZLB, a Central Bank without fiscal support can indeed do this. However, a Central Bank without fiscal support cannot raise the nominal interest rate above 0:

Lemma 8 (A Central Bank without fiscal support at the ZLB)**.** *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} < 1$ *and that assumption 1 holds. In addition, suppose that* $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$*. Then the unique monetary equilibrium outcome is the one where* $F = \underline{F}$ *and* $i = 0$*.*

Differently from when the economy operated away from the ZLB, Lemma 8 now tells us that an Central Bank without fiscal support is able to raise the domestic real rate above the foreign one, as long as the nominal interest rate remains at zero. In this case, by defending the exchange rate, the Central Bank is forced to issue currency to purchase the foreign assets necessary to maintain the domestic rate above the foreign one. The main difference from the

previous case is that now, the liabilities of the Central Bank (i.e., currency) are equivalent to government bonds (because the zero nominal rate) and can be expanded without limits. Thus, at the zero lower bound, Central Bank foreign exchange interventions are the mechanism that enables the economy to have an equilibrium in credit markets at an interest rate that is strictly above the international rate. Note that associated with the Central Bank defense of the exchange rate in this case, there is a similar expansion on its balance sheet.

5 Domestic welfare and external conditions

The impact of foreign wealth

Let us consider now how changes in $\bar{w}$ affect welfare in the best monetary equilibrium. As before, it will be important to distinguish the cases where the ZLB does not bind from the case where it does.

A first result is that, away from the zero lower bound, higher $\bar{w}$ weakly increases domestic welfare in the best monetary equilibrium. We obtain this result from the following argument. We know from Lemma 2, that the welfare of the domestic households in the non-monetary economy is (weakly) increasing in $\bar{w}$ (given F). So, in the best monetary equilibrium (when away from the ZLB), the utility generated from consumption is weakly increasing in $\bar{w}$. The only thing left to check is real money balances. But given that the real interest rate is (weakly) decreasing in $\bar{w}$, the nominal interest rate in the best monetary equilibrium will also be weakly decreasing in $\bar{w}$. As a result, the utility from real money balances also increases, and a higher $\bar{w}$ unambiguosly increase domestic welfare away from the ZLB.[11]

The second result concerns the behavior of the economy at the ZLB. In this case, an increase in foreign wealth will not lead to a reduction in the domestic real rate, as the zero bound impedes it. At the ZLB, we can characterize domestic welfare at the best monetary equilibrium, given the level of foreign reserves accumulated by the Central Bank, as follows:

$$W \equiv \max_{(c_1,c_2)} u(c_1) + \beta u(c_2) + h(\bar{x}) \tag{26}$$

$$\text{subject to:}$$

$$y_1 - c_1 + \frac{y_2 - c_2}{s_1/s_2} - \underline{F}\left[1 - \frac{s_2(1+i^\star)}{s_1}\right] = 0$$

This characterization is obtained from the consumer optimization, and the fact that $i = 0$.

[11] There is a subtle issue here as the increase in foreign wealth can lead to the ZLB to start binding (even if originally, it wasn't), as the nominal interest rate decreases. If that happens, then the behavior of additional foreign wealth follows the discussion next, when the ZLB binds.

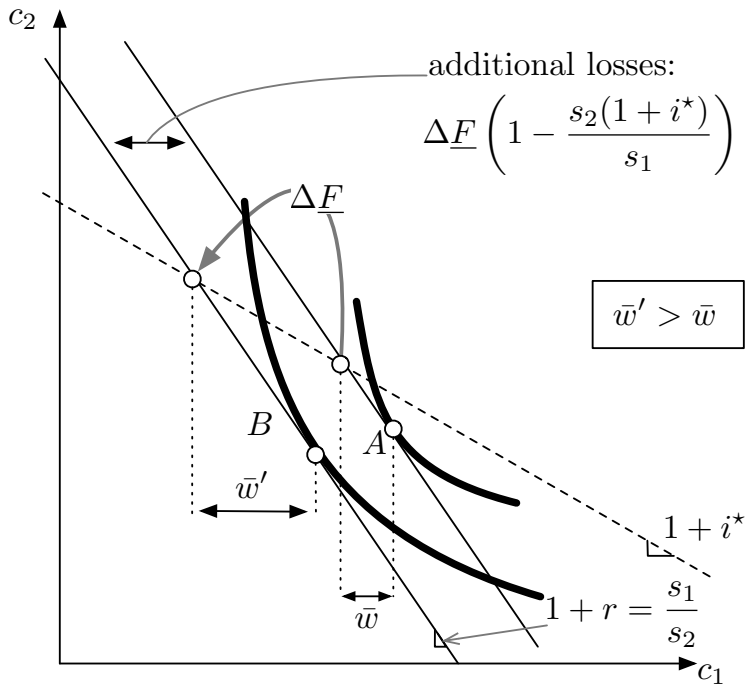


Figure 3: Changes in $\bar{w}$ at the ZLB

Note that an increase in $\bar{w}$ affects domestic welfare only through its effects on $\underline{F}$, as the real rate is determined (an equal to s_1/s_2). If higher $\bar{w}$ leads to higher $\underline{F}$, welfare unambigously decreases. By corollary 1, the equilibrium real rate must satisfy $s_1/s_2 = g(\underline{F}, \bar{w})$, and it follows that $d\underline{F}/d\bar{w} = -g_2/g_1 > 0$, where the last inequality holds given that $g_1 < 0$ and $g_2 < 0$ at the ZLB. As a result, welfare must strictly decrease with $\bar{w}$ at the ZLB.

Figure 3 displays how at the ZLB, a bigger foreign exchange intervention is required when foreign wealth increases. Point A in the figure depicts the equilibrium consumption allocation for an economy at the ZLB with a low $\bar{w}$. Note that the presence of the ZLB implies that interest parity is violated, which forces Central Bank intervention to keep the domestic interest rate above the foreign one. Point B represents the equilibrium that prevail when wealth moves to $\bar{w}' > \bar{w}$. The domestic real interest rate at equilibrium B is the same as at A, as the economy is at the ZLB in both points and domestic real rate is pinned down by $\frac{s_1}{s_2}$. Despite the fact the real rate has not changed, welfare at B is unambigoulsy lower than at A, as the higher $\bar{w}$ forces the Central Bank to intervene more ($\Delta\underline{F} > 0$). The losses generated by this bigger intervention are represented by the parallel shift in the budget lines from point A to point B.

The impact of foreign interest rates

If the economy is borrowing from abroad, we would expect that a decrease in the foreign interest rate will be beneficial. Such an intuition is indeed confirmed when the economy operates away from the ZLB, but with a small caveat. If the economy attains the first-best allocation, then whether a reduction in foreign rates is beneficial or not hinges on whether the economy is borrowing or saving from abroad. However, if the foreign wealth is not enough for the economy to attain the first best outcome, then domestic welfare is independent of the foreign interest rate.

Lemma 9. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} > 1$. *If* $\bar{w} > c_1^{fb} - y_1 > 0$ *, then a marginal reduction in* $i^\star$ *increases domestic welfare. If* $c_1^{fb} - y_1 > \bar{w}$*, then a marginal reduction in* $i^\star$ *does not affect domestic welfare.*

The second result follows because when $c_1^{fb} - y_1 > \bar{w}$, the domestic real interest rate is strictly higher than $i^\star$. Because of this, changes in $i^\star$ do not affect the domestic real rate, and as result, the consumption allocation that can be achieved by the households remains the same.[12]

At the zero lower bound, the behavior of the economy is different. Consider again, problem 26. Similarly to the case with foreign wealth, whether welfare increases or decreases with a reduction in $i^\star$ relies on whether $\underline{F} \times \left[1 - \frac{s_2(1+i^\star)}{s_1}\right]$ falls or increases. If $i^\star$ decreases, given $\underline{F}$, the losses generated by the foreign exchange intervention increase, as the term $\left[1 - \frac{s_2(1+i^\star)}{s_1}\right]$ is now bigger. That's the first effect. But in addition, $\underline{F}$ must now increase. The logic is as follows. A reduction in the foreign interest rate, diminishes the increase in second period consumption that a given a size of foreign reserve purchases generates. Because of this reduction in the power of its foreign exchange purchases, the Central Bank will be forced to increase the size of the intervention to maintain a high domestic real rate; driving domestic welfare to an even lower level. Summarizing this discussion, we have

Lemma 10. *Suppose that* $(1+\underline{r})\frac{s_2}{s_1} < 1$. *A marginal reduction in* $i^\star$ *strictly increases* $\underline{F}$ *and strictly reduces domestic welfare.*

The result of this Lemma is illustrated in Figure 4. We are considering a situation where the SOE is already at the zero lower bound, and its consumption lies at point A. The dashed budget line represents the resource constraint using an initial foreign rate equal to $i_0^\star$. We consider then a reduction in the international rate to $i_1^\star < i_0^\star$. Suppose that in response

[12] This indifference result relies on our assumption that domestic households cannot borrow any amount directly from international lenders at $i^\star$. If they could borrow up to some strictly positive level, then, when the economy is constrained away from the first best, a reduction in $i^\star$ will strictly increase welfare.

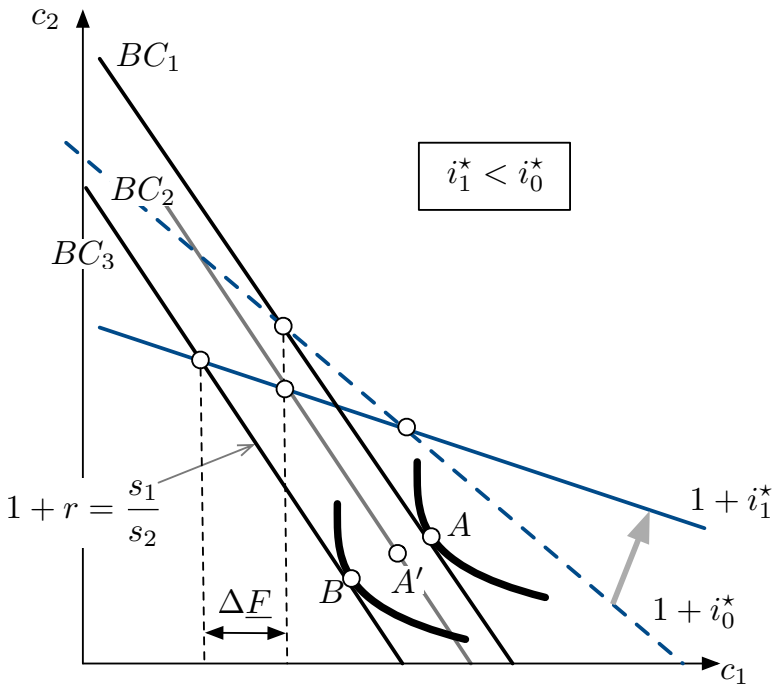


Figure 4: Changes in $i^\star$ at the ZLB

of this reduction, the Central Bank does not change the level of its foreign reserves, and domestic real rates were not to change. This will imply a shift in the budget constraint of the households from BC_1 to BC_2, and a movement to A'. Note that in this point, welfare is lower: the Central Bank intervention generates bigger losses as the interest parity deviation is larger. However, A' is not an equilibrium. The domestic households now would like to save, as their endowment in the second period is not as high as it used to be before the decrease in the foreign rate, which implies that the domestic asset market is not in equilibrium. The reduction in the foreign interest rate reduces the ability of the Central Bank to affect consumption tomorrow. As a result, the Central Bank must increase its foreign reserve accumulation, driving the economy to its final equilibrium point B, with an even higher reduction in welfare.[13]

[13]There is potentially another effect that we do not consider here. Suppose that the reduction in $i^\star$ were to allow the foreigners to borrow more from the international financial markets. This is equivalent to a larger amount foreign wealth $\bar{w}$ available for investment in the SOE in the first period. The additional effects generated by this will be similar to the already discussed exogenous increase in $\bar{w}$: it will increase the foreign reserve holdings by the Central Bank, magnifying the welfare losses.

6 Expectational mistakes at the ZLB

In the preceeding analysis, we have assumed rational expectations of the private agents. That is, the households and the foreign investors correctly anticipate the exchange rate that the Central Bank will set in the second period. In this section, we relax this assumption and asked what would happen if the private agents are mistaken and their expectations are wrong. We will show that, away from the ZLB, the Central Bank can exploit these mistakes and unambiguously increase the welfare of the domestic households. In particular, the Central Bank can convince foreign investors to invest at a real rate that is below the return in the Central Bank foreign reserves, generating a welfare gain to the SOE. We will show however, that at the ZLB, the Central Bank will be unable to exploit these mistakes. More dramatically, at the ZLB, the expectational mistakes will end up unambiguously reducing domestic welfare. As before, the key for these results is the inability of the Central Bank to reduce the nominal interest rate when at ZLB.

To make our point, we introduce the following simple change to the environment. We continue to let (s_1, s_2) denote the actual Central Bank exchange rate policy. We maintain the assumption that the Central Bank will indeed pursue this policy, just as before. The change is that market participants (i.e., domestic households and foreign investors) fully know that s_1 is the exchange rate in the first period, but they expect the exchange rate in the second period to be $\hat{s}_2$, potentially different from s_2. Keeping with our desire to maintain simplicity in this section, we assume that the private agents do not learn or infer any information from the actions of the Central Bank. After having defined the equilibrium outcomes, we will look for the best possible equilibrium with respect to household's welfare evaluated at the Central Bank true policy objective, s_1, s_2. Our goal is to perform comparative statics, and trace how domestic welfare changes in the best equilibrium outcome when $\hat{s}_2$ changes.

Definition 3. An *equilibrium given* (s_1, s_2) *and market beliefs* $\hat{s}_2$ consists of a domestic interest rate i, a consumption profile $(c_1, c_2, \hat{c}_2)$, asset positions for foreign investors $(a^\star, f^\star, m^\star)$, money M, investments by the monetary authority (A, F); transfers from the monetary authority to the fiscal, $(\tau_1, \tau_2, \hat{\tau}_2)$, and transfers from the fiscal authority to the households, $(T_1, T_2, \hat{T}_2)$ such that

(i) the allocation $(c_1, \hat{c}_2, a, f, m, a^\star, f^\star, m^\star, \tau_1, \hat{\tau}_2, A, F, T_1, \hat{T}_2)$ with nominal interest rate i constitutes a monetary equilibrium given the exchange rate $s_1, \hat{s}_2$.

(ii) The second-period consumption and transfers, (c_2, τ_2, T_2) satisfy

$$c_2 = y_2 + T_2 + \frac{(1+i)a + m}{s_2} + (1+i^\star)f$$
$$\tau_2 = (1+i^\star)F + (1+i)\frac{A}{s_2} - \frac{M}{s_2}$$
$$T_2 = \tau_2 - (1+i)\frac{B}{s_2}$$

Note that in the definition of equilibrium, part (i), we use the beliefs to define the monetary equilibrium. However, in period 2, the realization of the exchange rate will be s_2, and, as a result, the second period "true" allocations (c_2, τ_2, T_2) are calculated with respect to the true exchange rate (part (ii) of the definition). We will call $(c_1, \hat{c}_2)$ the *perceived* consumption allocation, and (c_1, c_2) the *true* consumption allocation. We will also say that $(1+i)\frac{s_1}{\hat{s}_2}$ is the *perceived* real interest rate, and we call $(1+i)\frac{s_1}{s_2}$ the *true* real rate interest rate.

When evaluating policies, the Central Bank computes the welfare of the domestic agents using the true consumption allocation, that is

$$u(c_1) + \beta u(c_2) + h(m/s_1)$$

We denote by the best equilibrium outcome the one that maximizes the domestic household's welfare under this criteria.

Clearly if $s_2 = \hat{s}_2$, the definition of equilibrium above is identical to our definition of a monetary equilibrium. We will consider the case where $s_2 > \hat{s}_2$, that is, the market agents expect the currency to be more appreciated next period as compared to the policy that will be chosen by the Central Bank.

We continue to let c_1^{fb}, c_2^{fb} defined the first best allocation given $1 + i^\star$. And denote by c_1^b, c_2^b, i^b and m^b the consumption allocation, nominal interest rate and money associated with the best monetary equilibrium given beliefs s_1, s_2. Our first result is that, away from the ZLB, the Central Bank can exploit the mistaken market beliefs and achieve a strictly higher welfare for the households:

Lemma 11 (Away from the ZLB)**.** *Suppose that $s_2 > \hat{s}_2$ and $(1+\underline{r})\frac{\hat{s}_2}{s_1} \geq 1$, then the best equilibrium outcome with distorted beliefs has $F = y_1 - c_1^b + \bar{w}$, $c_1 = c_1^b$, $\hat{c}_2 = c_2^b$, $c_2 > \hat{c}_2$, $1 + i = (1+i^b)\frac{\hat{s}_2}{s_2} < (1+i^b)$, and $h(m/s_1) > h(m^b/s_1)$. In particular, true domestic welfare strictly decreases with $\hat{s}_2$.*

Note that, in this case, the Central Bank guarantees that the maximum amount of capital

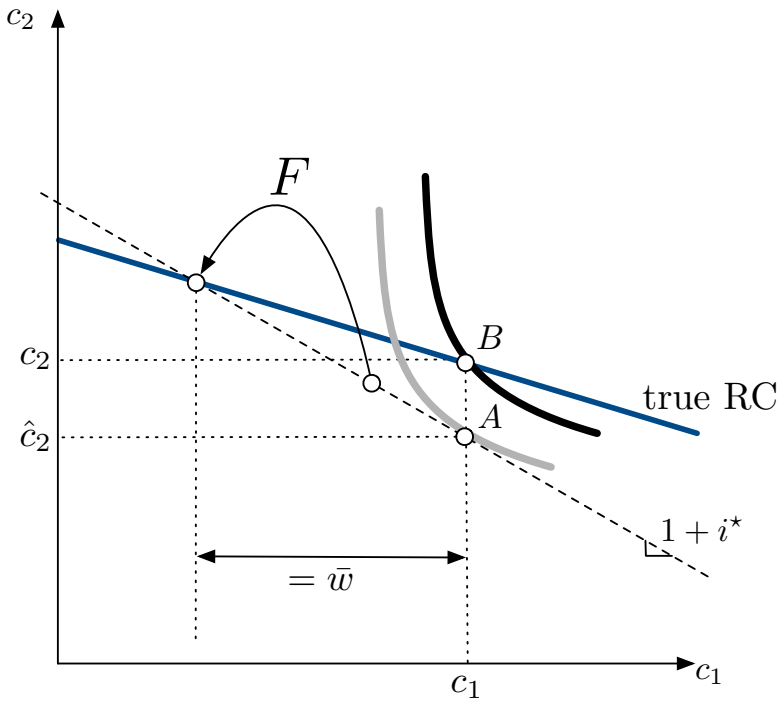


Figure 5: Exploiting expectational mistakes away from ZLB

enters the domestic economy in period 1, $\bar{w}$, while reducing the nominal interest rate away from the one that would have been in place without mistaken beliefs, that is $i < i^b$. The capital inflow at a lower nominal rate allows the domestic households to borrow at what effectively is a lower true real interest rate, increasing true second period consumption. As an added bonus, the lower nominal interest rate increases the utility generated from the monetary services. As a result, household's welfare, as evaluated by the Central Bank, increases. This increase in domestic household's utility comes at the expense of a reduction in the foreign investors welfare as compare to the case where expections are not mistaken.

An example of this is illustrated in Figure 5. Point A represents the original equilibrium when $\hat{s}_2 = s_2$. In this case, the first best consumption bundle is attained, and the CB does not need to intervene (that is, $F = 0$ is optimal). We consider then what happens if $\hat{s}_2 < s_2$. In this case, the CB reduces the nominal interest rate, such that the "perceived" real rate of return remains the same. As a result, the private agents behave as if the equilibrium remains at point A. In their eyes, the intervention of the Central Bank (denoted by the arrow) is irrelevant. However, ex-post, the exchange rate equals s_2, and the consumption of the domestic households will be at point B, rather than A (generating a welfare gain). The intervention by the Central Bank allowed the domestic economy to extract the maximum gain from the expectational mistakes.

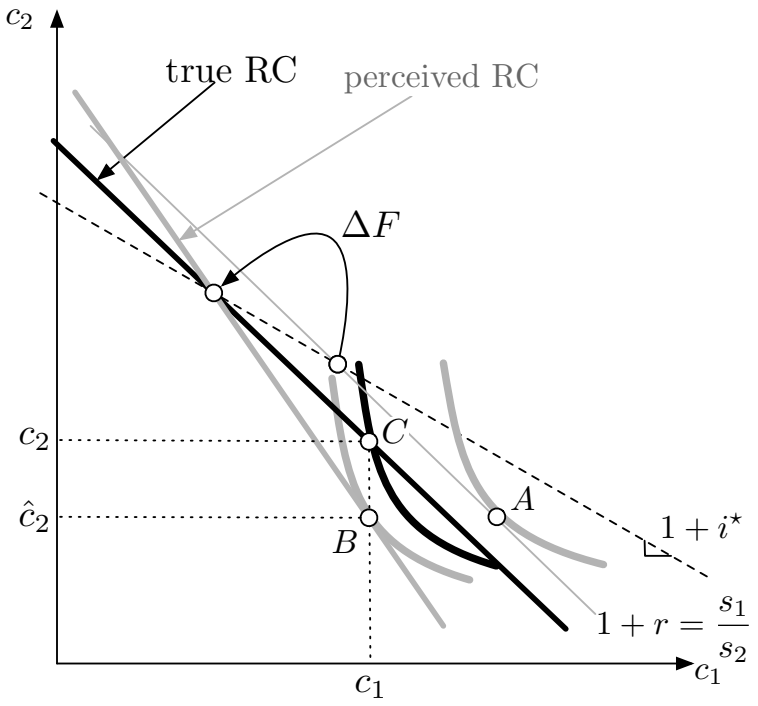


Figure 6: The inability to exploit expectational mistakes at from ZLB

Note that the key reason why the Central Bank can exploit the mistaken beliefs is based on its ability to lower the domestic nominal interest rate when the beliefs deviate from the true ones. At the ZLB, this is not possible. As a result, the expectation mistakes cannot be exploited and welfare cannot be increased. We will show an even more negative result: the mistaken beliefs at the ZLB will unambigously generate a reduction in welfare.

To see this, let us start from a situation where $(1+\bar{r})\frac{s_2}{s_1} < 1$, so that the ZLB will be binding in the best equilibrium if $\hat{s}_2 = s_2$. As discussed above, a reduction in $\hat{s}_2$ away from s_2 cannot anymore be accompanied by a reduction in the nominal rate. So as a result, the perceived real rate in the domestic economy has increased. We know from XX, that this will require an even bigger intervention by the Central Bank, so F will increase, increasing the amount of losses that the Central Bank generates, as well as distorting first period consumption.

Lemma 12 (At the ZLB)**.** *Suppose that $s_2 > \hat{s}_2$ and $(1+\underline{r})\frac{s_2}{s_1} < 1$, then the best equilibrium outcome with distorted beliefs has $i = 0$ and true domestic welfare that strictly increases in $\hat{s}_2$.*

The results of this Lemma are shown in Figure 6. Point A represents the original equilibrium, where the ZLB binds, and $\hat{s}_2 = s_2$. We consider then how the equilibrium changes

if $\hat{s}_2 < s_2$. In this case, the Central Bank cannot further reduce i, as it is already at zero. As a result, the perceived domestic rate of return is now higher, and the Central Bank needs to intervene to maintain the exchange rate. The perceived equilibrium consumption allocation shifts to point B. However, ex-post, the exchange rate remains at s_2, and the true rate of return is identical to the original. The true consumption allocation that is attained in equilibrium is given by point C, an improvement over point B, but definitely dominated by A.

The results of this section highlights that even if the Central Bank is commited to its exchange rate policy, if the markets do not believe it, then there will be costs to defending the policy when the economy operates at the ZLB. In addition, the bigger the expectational mistake, the bigger the required foreign exchange intervention by the Central Bank and the bigger the generated welfare losses. That is, at the ZLB, expectational mistakes will come accompanied with costly balance sheet expansions by the Central Bank. Note that those expantions could, by themselves, trigger an abandonment of the exchange rate policy if the Central Bank is worried about a large balance sheet.[14] The above discussion opens the door to the possibility of self-fulfilling "appreciation" runs at the ZLB. In particular, if the private agents erroneously anticipate an appreciation, when the Central Bank is at the ZLB, such mistaken expectations will be costly. This may force the Central Bank to abandon the defense of the current exchange rate regime, allowing the currency to appreciate, somewhat validating the mistaken expectations of the markets. The analysis in this section calls for a more detailed study of the game between the Central Bank and the private agents, but that is something that we leave for future work.

7 The 2011-2015 Swiss franc floor

In the previous sections we have seen that, in an our environment, keeping a currency temporarily depreciated when nominal interest rates are at zero is feasible for a Central Bank, but it comes with costs. From problem 26, we are going to approximate these losses by the following formula (a familiar formula from the sterilization literature):[15]

$$\text{cost}_t = \left[\frac{1+i_t}{1+i_t^*}\frac{s_t}{s_{t+1}} - 1\right] \times F_t. \tag{27}$$

In this section we show how equation (27) can be used in practice to measure the costs

[14] This is something that we do not analyze here, but is a point we studied in Amador et al. (2016).

[15] The literature that estimates the quasi-fiscal costs of sterilizations for emerging markets uses the same relationship. See, for example, Kletzer and Spiegel (2004) and references therein.

associated to such exchange rate policies.Specifically, we focus our analysis to the implementation of the currency floor by the Swiss National Bank between 2011 and 2015. We first proceed by constructing empirical counterparts to the right hand side of equation (27), and use them to generate a time series for the costs that the SNB incurred for maintaining the currency floor.

In our deterministic model, deviations from UIP are synonymous of arbitrage profits, an implication of the absence of risk in the model. More generally, however, deviations from UIP are commonly observed in the data and the consensus in the literature is that these deviations do not necessarily reflect arbitrage opportunities (see for references, Engel, 2014). For example, they may simply reflect fair compensation that investors demand for holding currency risk. For this reason, we will measure the first term in the right hand side of equation (27) as deviation from covered interest rate parity (CIP),

$$\left[\frac{1+i_t}{1+i_t^*}\frac{s_t}{f_{t+1}}-1\right],$$

where f_{t+1} is the forward rate on the domestic currency.[16]

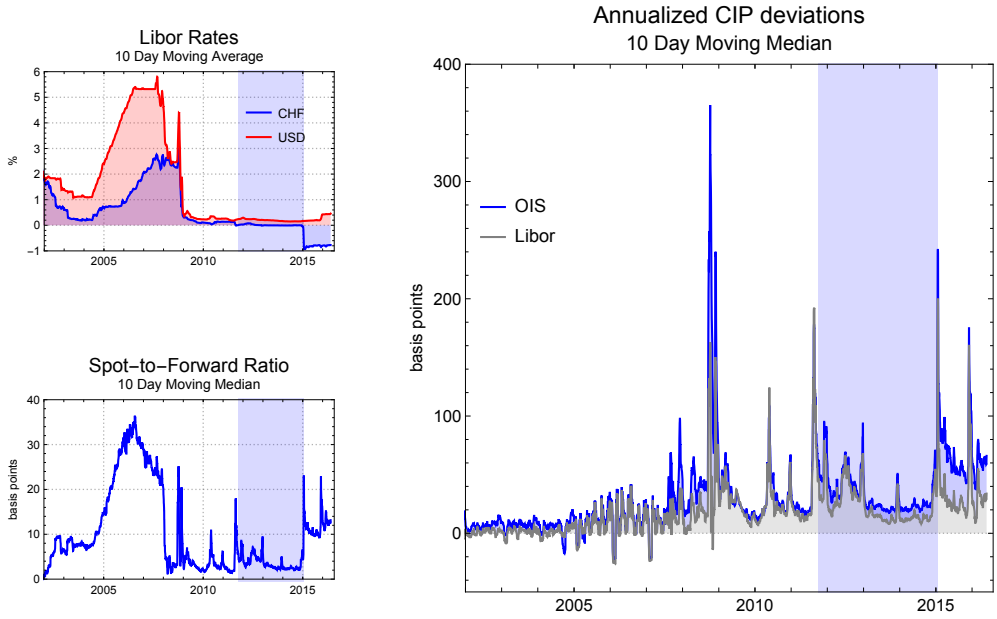
We calculate daily deviations from interest rate parity between the Swiss franc and the U.S. dollar for the period 2000-2015.[17] We map i_t to the interest rate on a one month loan in Swiss francs contracted in the London interbank market, while i_t^* is the interest rate on a loan in U.S. dollars with the same maturity. As an alternative measure, we also use the one month OIS interest rate for both Swiss francs and U.S. dollars. We measure s_t with the average between the bid and the ask spot exchange rate between the Swiss franc and the U.S. dollar, while f_{t+1} is the average between the bid and the ask one-month forward rate. All the data are obtained from Bloomberg.

Figure 7 reports domestic and foreign interest rates, the ratio between spots and forward exchange rates, and the measured deviations from CIP, expressed in annualized basis points (using Libor and OIS interest rates). There are some pattern that we wish to emphasize. First, and consistent with previous research, we can see that CIP holds fairly well in the data until the end of 2007: deviations from CIP (using Libor) averaged only 0.06 basis points in annualized terms during this period, with a standard deviation of 0.13%. Second, and starting with the financial crisis of 2007, we have observed persistent deviations from CIP. During the 2008-2015 period, deviations from interest rate parity averaged 30 basis

[16] In our model, the two conditions are equivalent because the spot exchange rate in period 2 equals the forward rate contracted in period 1 in absence of shocks.

[17] We compute deviations from CIP between the Swiss franc and the U.S. dollar, rather than using the euro, because interbank rates for borrowing in euro contained a significant credit risk component during the euro-area debt crisis, see Du et al. (2016).

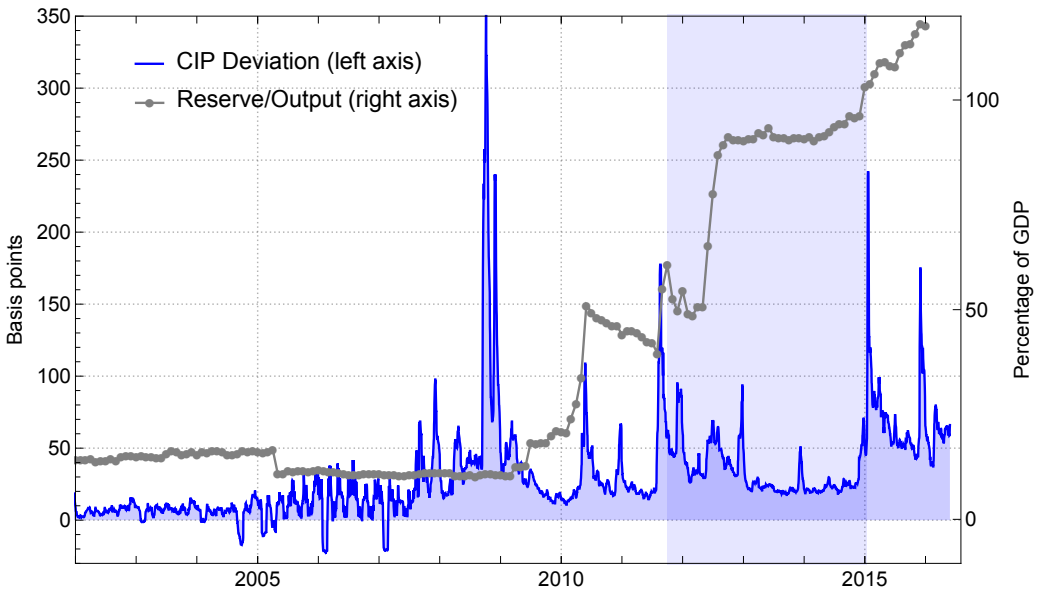
Figure 7: CIP Deviations: Swiss Franc-U.S. dollar



points, with peaks reaching 3%. Third, it is interesting to point out that these deviations are always one sided, with the Swiss interest rate being always above the interest rate in dollar denominated assets. Fourth, we can observe that starting from 2009, interest rate for borrowing in U.S. dollars and in Swiss francs were essentially at zero: from that point on, deviations from CIP emerged purely because the forward rates were consistently below spot rates. This suggests that deviations from CIP emerged because markets expected an appreciation of the Swiss franc (under the risk neutral measure).

One major concern is that these deviations from CIP may not necessarily reflect arbitrage profits. When investors buy forward cover against exchange rate risk, they enter in a bilateral relation with a counterparty, and as a result they become exposed to the risk that this latter will not honor its obligations. However, there is suggestive evidence that the CIP deviations reported in Figure 7 are not reflecting a rise in counterparty risk, but are indeed reflecting deviations from arbitrage. Du et al. (2016), in fact, show that these deviations from CIP are not systematically related to indicators of credit risk in the U.S. and the Swiss interbank market. Moreover, foreign demand for assets denominated in Swiss francs was extremely sensitive to these deviations from CIP, which is suggestive that these deviations indeed represented arbitrage opportunities. To verify this latter point, we can study the relation between measured CIP deviations and the accumulation of foreign reserves by the

Figure 8: CIP Deviations and Foreign Reserves



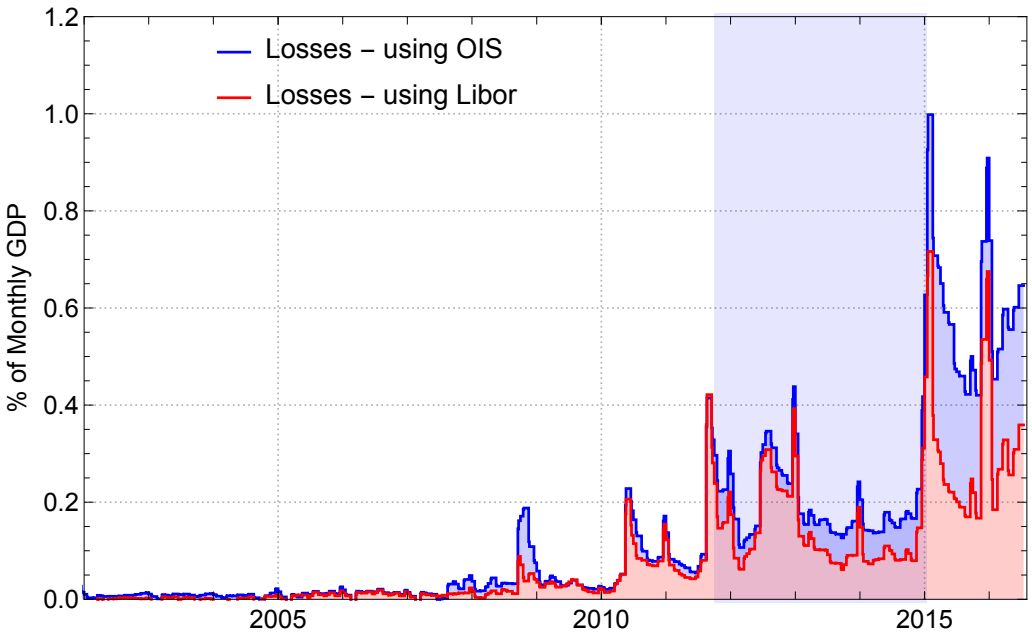
Swiss National Bank.[18] Indeed, the SNB tried to avoid excessive appreciation of the franc even prior of the currency floor of 2011. Therefore, changes in foreign reserves held by the SNB can be used as a proxy for the excess demand for Swiss francs, which would otherwise be unobservable. Figure 8 plots the measured CIP deviations with the foreign reserves accumulated by the SNB. The Figure shows a systematic relation between these two variables. Periods in which the CIP deviations are positive are also periods during which the SNB intervened to supply Swiss currency. These interventions were massive: throughout this period, foreign reserves went form being 5% of annualized GDP to levels of 80%.

Having measured deviations from interest rate parity, and argue that they indeed represented arbitrage profits, we can now make equation (27) operational and measure the costs implicit in maintaining the Swiss currency floor. Figure 9 shows the monthly losses as percentage of monthly GDP. During the period of the floor, and associated to the major foreign exchange rate intervention, the losses were significant. They reached a highest point of 0.8-1.0% of GDP around January 2015, when the SNB decided to abandon the floor and set negative interest rates. It is interesting to note that the losses have continued even after the floor was abandoned: as long as the SNB continues to maintain its foreign reserve position

[18]Foreign reserves are monthly, measured in Swiss francs, and they are downloaded from the SNB website. We exclude gold from our calculations.

when the CIP is violated, it is still incurring a loss.

Figure 9: Losses computed using equation (27)



An important caveat of the analysis in this section is that we have assumed that the covered interest parity deviations represent true arbitrage profits. That is, the underlying expections of exchange rates underlying the covered interest parity calculations are assumed to be correct and correspond to the equilibrium exchange rate policy followed by the Central Bank. As we discussed in Section 6, if these market expectations are incorrect, there could still be arbitrage losses at the ZLB given the increase in foreign reserves by the Central Bank, but these will be smaller than those computed here.

8 Conclusion

In a world with limited international arbirtrage, the zero lower bound makes the pursue of an exchange rate objective costly for a Central Bank. The welfare losses associated with a given exchange rate policy increase with both the level of foreign wealth available to be invested as well as with reductions in the foreign interest rate. The behavior of the Central Bank with respect to its foreign reserve holdings is key to generate these welfare losses, as at the zero lower bound, by accumulating foreign reserves the Central bank borrows at a high domestic rate and saves at a low international rate. We have constructed an estimate

of the costs for the Switzerland during the 2011-2015 period where a floor on the Swiss franc was imposed by the SNB. These costs were significant, reaching almost 1% of GDP before the Swiss National Bank decided to abandon the floor, providing a strong justification for such an action. The current work focuses on quantifying the costs of a given exchange rate policy, and it does not analyze the potential benefits of a such a policy. Expanding our setup so to include features that generate benefits from a exchange rate policies (for example in terms of higher output, or more favorable international terms of trade) would allow a more comprehnsive evaluation of the full impact of such policies, and we view it as a promising research direction.

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