General Equilibrium Dampened

(i) from Micro to Macro (ii) Forward Guidance

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Motivation

- GE effects key to macroeconomics
  - limit usefulness of
    - ★ partial-equilibrium (PE) intuitions
    - ★ micro-based evidence a la Mian-Sufi
  - drive structural interpretations of data
  - crucial for policy predictions

Motivation

- **But:** GE hinges on
  - not only knowledge, but also common knowledge of
    - ★ state of economy
    - ★ structure of economy
    - ★ rationality
  - instantaneous adjustment of beliefs
  - immense coordination
- Above hardwired in
  - informational assumptions
  - solution concept
- Concern particularly relevant for non-stationary contexts
  - ZLB and forward guidance

This talk: Part I

- Formalize notion that

  "GE Adjustment Takes Times"

- Two ways:
  - relax solution concept $\rightarrow$ Tattonment ("off equilibrium")
  - relax info assumptions $\rightarrow$ imperfect coordination ("on equilibrium")
- Result: equivalence between two
  - similar equiv with "reflective equilibrium"

This talk: Part I

- Broader lesson:
  - lack of CK = relaxation of solution concept = dampen GE
- From Micro to Macro:
  - Reinforce PE intuitions
  - Empirical work a la Mian-Sufi

This talk: Part II

- Forward Guidance Puzzle
  - Disentangle PE and GE
  - Role of Horizon and HOB
  - Dampen power of FG relative to current policy
- Similar implications for deflation spirals, Fisherian effects, etc

Framework

- Simple economy
  - Many goods/markets
  - Representative competitive firm
  - Representative competitive household
  - Both idiosyncratic and aggregate shocks
- Helps formalize PE vs GE, and micro vs macro
  - To start: review predictions of standard paradigm
  - Later: variants with Tatonnment and Incomplete Info

Households and "Demand Shocks"

- Household preferences:

$$U = u(C, \xi) + x$$

$$u(C) = \xi^{\frac{1}{\sigma}} C^{1-1/\sigma}$$

$$C = \left\{ \int_0^1 \left(\delta_i^{\frac{1}{\varepsilon-1}} c_i \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

- $\xi, \delta_i =$ aggregate and good-specific "demand shocks"
- $x =$numeraire (leisure, consumption in the future, etc)

Firms and "Supply Shocks"

- Competitive firms
- Each differentiated good is produced from numeraire

$$q_i = f(A_i, l_i)$$

- q_i =quantity produced, l_i =input
- A_i = sum of aggregate and idiosyncratic "supply shocks"
- f features diminishing returns and power specification

Demand and Supply

- Henceforth all variables are in log deviations
- Demand for good i:

$$c_i = \xi_i - \varepsilon \underbrace{(p_i - P)}_{\text{relative price of } i} - \sigma \underbrace{P}_{\text{price index}}$$

$$c_i = D(p_i, P, \xi_i) \equiv \xi_i - \varepsilon p_i + (\varepsilon - \sigma) P$$

where $\xi_i \equiv \xi + \delta_i$

- Supply for good i :

$$q_i = S(p_i, P, A_i) \equiv A_i + \psi p_i$$

where $A_i = A + v_i$

Partial Equilibrium

- Impose market clearing in market i, for arbitrary P
- Solve for "local" or "PE outcomes" as

$$q_i = f^q(\theta_i, P) \equiv ...$$

$$p_i = f^p(\theta_i, P) \equiv ...$$

where

$$\theta_i \equiv (A_i, \xi_i) = \theta + z_i$$
$$= \text{sum of local and agg shocks}$$

Micro vs Macro

- How does the economy respond to shocks in θ_i?
- **Micro elasticity, or PE effect:**

$$\varepsilon_i^{micro} \equiv \left.\frac{dq_i}{dz_i}\right| = \frac{\partial f^q}{\partial \theta} = \left.\frac{\partial q_i}{\partial \theta_i}\right|_{P \text{ constant}}$$

- **GE Effect:** if aggregate shock, P changes too
  - total effect in market i:

$$\frac{dq_i}{d\theta} = \underbrace{\frac{\partial f^q}{\partial \theta}}_{PE_i} + \underbrace{\frac{\partial f^q}{\partial P}\frac{\partial P}{\partial \theta}}_{GE_i}$$

- **Macro effect:** average total effect

$$\varepsilon^{Macro} \equiv \frac{dQ}{d\theta} = \int \frac{dq_i}{d\theta} di = \text{avg PE} + \text{avg GE}$$

The GE Effect

- How does P respond to aggregate shock?
  - P such that *all* markets clear at once
- In our setting, this means the following.
  - Aggregating demand and supply gives

$$C \equiv \int c_i di = AD(P, \xi) \equiv D(P, P, \xi)$$

$$Q \equiv \int q_i di = AS(P, A) \equiv S(P, P, A)$$

  - It follows that P^* solves

$$AD(P^*, \xi) = AS(P^*, A)$$

    which can be solved for $P = P^*(\xi, A)$, a function of aggregate shocks

Interpretation

- Simple framework, but general ideas
- Flexible interpretation
  - P is a proxy for prices and/or quantities of all other markets
  - "markets" can mean zip codes, sectors, periods, etc
- Standard paradigm:
  - adjustment in P is instantaneous
- What we are after:
  - slow adjustment in P

Tâtonnement

- **Assumption 1:** market clearing at local level for *some* perception of P:

$$(q_i, p_i) = f(\theta_i, \hat{P}_i) \quad \forall i,$$

  - for simplicity, $\hat{P}_i = \hat{P}\ \forall i$.

- **Assumption 2:** "Walrasian auctioneer" uses a Tâtonnement process to adjust $\hat{P}$ from $P^*_{old} \equiv P^*(\theta_{old})$ to $P^*_{new} = P^*(\theta_{new})$

Tâtonnement

- Let t index round of iteration in Tatonnement process (soon to be reinterpreted as time)
- Process for $\hat{P}_t$ given by the following ODE:

$$\frac{d\hat{P}_t}{dt} = -b_t \cdot \left[AS\left(\hat{P}_t, \theta_{new}\right) - AD\left(\hat{P}_t, \theta_{new}\right) \right],$$

with initial condition

$$\hat{P}_0 = P^*_{old},$$

and for some exogenous $\{b_t\}$, with $b_t \geq \underline{b} > 0 \forall t$

Tâtonnement: outcomes

- Local outcomes at t:

$$q_{i,t} = f^q\left(\theta_i, \hat{P}_t\right) \qquad q_{i,t} = f^q\left(\theta_i, \hat{P}_t\right)$$

- Corresponding aggregates:

$$Q_t = \int q_{i,t} dj \qquad P_t = \int p_{i,t} dj$$

Tâtonnement: micro vs macro

- PE as in benchmark, but GE is dampened
- Macro elasticity:

$$\varepsilon_{T\hat{a}t}(t) \equiv \frac{dQ_t}{d\theta} = \varepsilon^{micro} + \underbrace{w(t) \cdot \left(\varepsilon^{Macro} - \varepsilon^{micro}\right)}_{GE_{T\hat{a}t}(t)}$$

where $w(t)$ is increasing in t, with $w(0) = 0$ and $w(\infty) = 1$.

Tâtonnement: micro vs macro

Proposition. $\varepsilon_{T\hat{a}t}$ is monotone and continuous in t, with

$$\lim_{t\to 0} \varepsilon_{T\hat{a}t}(t) = \varepsilon^{micro}$$

$$\lim_{t\to \infty} \varepsilon_{T\hat{a}t}(t) = \varepsilon^{Macro}$$

- That is, we can span the gap between the micro and the macro by varying the round t in Tâtonnement

Reinterpret t as time

- Economy is now dynamic

$$U = \int \{u(C_t, \xi) + x_t\}\, dt = \int u(C_t, \xi)\, dt + x$$

- Previous result $\rightarrow$ **"GE adjustment takes time"**
- Caveat: "off equilibrium" (relaxation of solution concept)
- What's next: "on equilibrium" (relaxation of informational assumptions)

Incomplete Information

- Similar environment, adding "islands" and dispersed info
  - island i = market i
- "Big family" with a consumer and a producer in each island i
  - capture disaggregated production and consumption decisions
- Information:
  - perfect knowledge of local conditions (θ_i, q_i, p_i)
  - lack common knowledge of global conditions (θ, Q, P)

Incomplete Information

- Local shocks/outcomes serve as signals of aggregates
  - however, if $\sigma_{idio} \ggg \sigma_{agg}$, little info in such signals
- Let $\theta \sim N(0,1)$ and summarize local information about aggregate conditions in a private signal of the form

$$s_i = \theta + \frac{v_i}{\sqrt{\omega}}$$

where $v_i \sim N(0,1)$ and ω is exogenous scalar

Equilibrium

- **Rational-Expectations Equil:** for all i,
  - decisions are optimal given p_i and given local belief $\hat{P}_i$ of P;
  - p_i clears market i
  - $\hat{P}_i$ is rational expectation of P conditional on local info
- In a nutshell, (q_i, p_i) solve

$$q_i = D(\xi_i, p_i, \hat{P}_i) = S(A_i, p_i, \hat{P}_i)$$

and belief satisfies

$$\hat{P}_i = E_i[P]$$

  - remark about D and S

Micro vs Macro

- PE same as in benchmark
  - due to perfect knowledge of local conditions
- But GE dampened
  - due to lack of common knowledge of global conditions
- Macro elasticity

$$\varepsilon_{inco}(t) \equiv \frac{dQ_t}{d\theta} = \varepsilon^{micro} + \underbrace{g(\omega) \cdot \left(\varepsilon^{Macro} - \varepsilon^{micro}\right)}_{GE_{inco}(\omega)}$$

where $g(\omega)$ is monotone in ω, with $g(0) = 0$ and $g(\infty) = 1$

GE and HOB

- First-order beliefs of exogenous shock:

$$\bar{E}^1\theta \equiv \int E_i\theta\, di = \lambda\theta$$

  where $\lambda \equiv \frac{\omega}{1+\omega} \in (0,1)$.
- Higher-order beliefs of exogenous shock:

$$\bar{E}^h\theta \equiv \bar{E}\left[\bar{E}^{h-1}[\theta]\right] = \lambda^h\theta$$

- HOB of $\theta \leftrightarrow$ equil beliefs of P
- GE effect:

$$\frac{GE_{inco}}{GE} = \frac{1}{1-\alpha}\sum_h \alpha^h\lambda^h$$

  where α is an increasing function of GE

Corollary. The stronger the GE effect in the standard model, the stronger the anchoring effect of higher-order beliefs in the incomplete-information variant of the model.

- In this sense, the stronger GE effect is, the more relevant our point is!

Adding Dynamics

- Continuous time, repeated version of benchmark economy

$$U = \int_0^T \{u(\xi, C_t) + x_t\}\, dt = \int_0^T u(\xi, C_t) dt + x$$

- Receive private signals continually over time

$$ds_{it} = \theta\, dt + \frac{dv_{it}}{\sqrt{\omega_t}}$$

  where v_{it} is a Brownian Motion and ω_t is the precision of private signal at time t
- Capture gradual adjustment of equil beliefs
  - Coibion and Gorodnichenko
- GE dampened in SR, but converges to benchmark over time

Equivalence Result

Proposition. For any $\{b_t\}$ governing the speed of Tâtonnement, there exists a sequence of precisions $\{\omega_t\}$ such that

1. Tâtonnement economy's Q_t and P_t same as inco-info economy's Q_t and P_t
2. Walrassian auctioneer's $\hat{P}_t$ same as $\bar{E}_t P_t$ in incomplete information economy

The converse is also true

- **lack of CK = microfoundation of Tâtonnement**

Complementary Result

- **Reflective Equilibrium** (Garcia-Schmidt & Woodford, 2015)
  - Walrassian auctioneer adjusts conjectured aggregate price $\hat{P}_t$
  - according to the difference between $\hat{P}_t$ and P_t
- Garcia-Schmidt & Woodford interprets $\hat{P}_t$ as perceived aggregate price
- Similar equivalence result
- Take-home lesson: **lack of CK = relaxation of solution concept = dampening of GE**

Connection to Empirical Work

- Variation in cross section = variation in z_i
- Empirical work such as Mian-Sufi helps identify ε^{micro}
  - the effect of local deleverage on local consumption, employment, etc
- Macro question: ε^{Macro}
- Difference between the two: GE effect
  - GE effect = "fixed effect" in regressions
  - can be either negative or positive (depending on micro-foundations/story)

Forward Guidance Puzzle

- Far future forward guidance in standard NK models
  - Huge effects on current outcome especially when ZLB binds
  - Effects grow with the horizon
  - Nakamura-Mckay-Steinsson; Del Negro-Giannoni-Patterson
- Puzzle is about strong GE effect of inflation and income
  - PE effect of future interest rate on demand bounded
- Hinge more on coordination than the effect of current monetary stimulus
  - need to form expectation of inflation and income in the far future
  - very high order of beliefs
- Adding info friction will significantly dampen these GE effects
  - Can also be thought as relaxation of solution concept

Deconstructing Forward Guidance

- Consumers:

$$\max_{\{c_{i,t},n_{i,t}\}} \sum \beta^t \left(\log c_{i,t} - \frac{n_{i,t}^{1+\varepsilon}}{1+\varepsilon} \right)$$

$$s.t.\ \forall t, \quad c_{i,t} + a_{i,t+1} = \frac{R_{i,t-1}}{1+\pi_{i,t}} a_{i,t-1} + w_{i,t} n_{i,t} + z_{i,t}$$

  $z_{i,t}$=share of profits, $w_{i,t}$=real wage; $R_{i,t-1}$=nominal interest rate, $\pi_{i,t}$=inflation

- allowed to vary across i for two reasons:
  - help disentangle GE from PE
  - idiosyncratic shocks $\Rightarrow$ avoid perfect revelation of aggregate shocks
- Always know *current* conditions: $z_{i,t}, \pi_{i,t}, w_{i,t}$
  - emphasize frictions of beliefs of *future* endogenous variables
  - lack of CK about them dampens forward guidance

Deconstructing Forward Guidance

- Change in future nominal interest rate
  - plays the role of exogenous θ shock in benchmark framework
- Disentangle PE and GE effects
  - PE: direct effect, holding constant inflation and income
  - GE: response of inflation and income (actions of others)

Consumer's Problem

- From now on, log linearization
- Euler equation:

$$c_{i,t} = E_{i,t}\left[c_{i,t+1}\right] - \left(R_{i,t} - E_{i,t}\left[\pi_{i,t+1}\right]\right)$$

- Labor supply:

$$n_{i,t} = \frac{1}{\varepsilon}\left(w_{i,t} - c_{i,t}\right)$$

- Optimal consumption:

$$\begin{aligned} c_{it} =& b \cdot \frac{R_{i,t-1}}{1+\pi_{i,t}} a_{i,t-1} \\ & - \sum_{k=0}^{+\infty} \beta^{k+1} E_{i,t}\left[R_{i,t+k} - \pi_{i,t+k+1}\right] \\ & + (1-\beta) \cdot \sum_{k=0}^{+\infty} \beta^{k} E_{i,t}\left[(1-\alpha) w_{i,t+k} + \alpha z_{i,t+k}\right] \end{aligned}$$

where b, λ are constants (b measure MPC, α measures income share of profits)

Consumer's Problem

- PE effect of forward guidance:

$$\frac{\partial c_{i,t}}{\partial E_{i,t}\left[R_{i,t+k-1}\right]} = \beta^k$$

  - bounded and decreases with horizon
- Forward guidance puzzle is mostly about GE effects from inflation and income
  - depend on what others do/believe (coordination)

Supply Side

- Same as 3 equation NK model
- Firm: monopolistic competition, linear technology
- Case I: Firms have access to all information at t

$$\pi_t = \kappa c_t + \beta E_t \pi_{t+1}$$

- Case II: Information friction on firm side:

$$\pi_t = \bar{E}_t \left[\kappa c_t + \beta \pi_{t+1} \right]$$

Dynamic Beauty Contest

- Let consumer's idiosyncratic shocks be i.i.d. over time, so that, for all $k \geq 1$, $E_{it}\pi_{i,t+k} = E_{it}\pi_{t+k}$, $E_{it}w_{i,t+k} = E_{it}w_{t+k}$ etc
- Use $(1-\alpha)w_t + \alpha z_t = c_t$
- Aggregating optimal consumption rules gives

$$c_t = \underbrace{-\sum_{k=1}^{+\infty} \beta^k \bar{E}_t \left[R_{i,t+k-1}\right]}_{PE}$$
$$\underbrace{+ (1-\beta)\, c_t + \sum_{k=1}^{+\infty} (1-\beta)\, \beta^k \bar{E}_t \left[c_{t+k}\right]}_{GE:income}$$
$$\underbrace{+ \sum_{k=1}^{+\infty} \beta^k \bar{E}_t \left[\pi_{t+k}\right]}_{GE:inflation}$$

- GE terms: expectation of endogenous future outcomes
  - depend on **HOB** of future interest rates

Dynamic Beauty Contest

- In Case I (complete info on supply side)
  - Substitute $\pi_t = \kappa \left\{ c_t + E_t \sum_{k=1}^{+\infty} \beta^k c_{t+k} \right\}$
  - Using law of iterated expectation
- A dynamic consumption beauty contest

$$c_t = \underbrace{-\sum_{k=1}^{+\infty} \beta^k \bar{E}_t \left[R_{i,t+k-1} \right]}_{PE}$$

$$\underbrace{+ (1-\beta) c_t + \sum_{k=1}^{+\infty} (1-\beta) \beta^k \bar{E}_t \left[c_{t+k} \right]}_{GE:income}$$

$$\underbrace{+ \sum_{k=1}^{+\infty} k \kappa \beta^k \bar{E}_t \left[c_{t+k} \right]}_{GE:inflation}$$

Example

- Consider the effect of changing R at $t = T$
  - at $t < T$, interest rate constant (ZLB)
  - at $t > T$, replicate flexible-price outcomes ($c_{T+1} = 0$)
- Consumer information
  - each consumer gets a private signal of R_T at $t = 0$: $s_i = R_T + \frac{v_i}{\sqrt{\omega}}$
  - no further info at t, so that $E_{it} = E_{i0}$ for all i, t
- Firm information:
  - Case I: complete info
  - Case II: like the consumer

Equilibrium outcomes

- Case I: standard PC, adjusted Euler

$$\begin{aligned} c_T &= -\bar{E}[R_T] \\ c_{T-1} &= -\beta \bar{E}[R_T] - [\kappa + (1-\beta)] \bar{E}[c_T] \\ &= -\beta \bar{E}[R_T] - [\kappa + (1-\beta)] \bar{E}^2[R_T] \end{aligned}$$

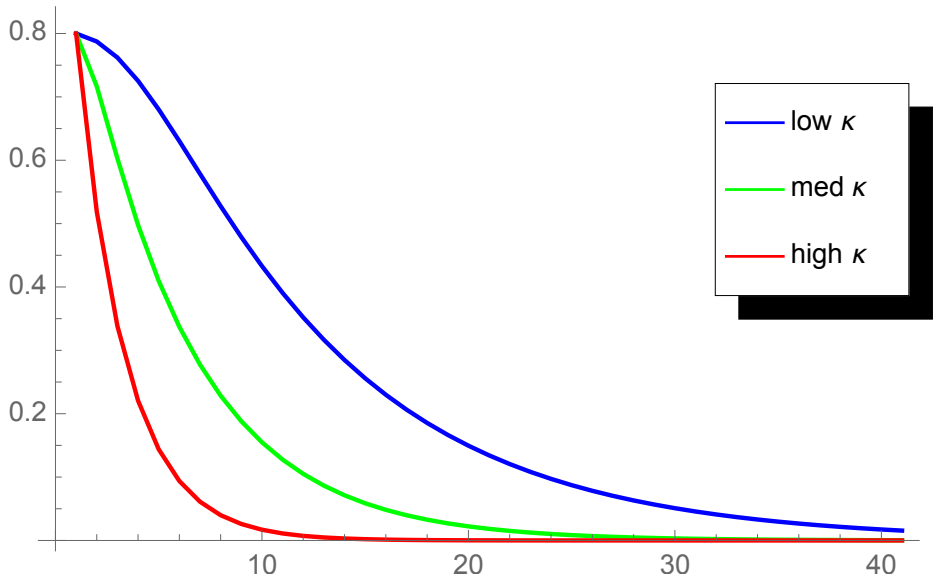
$$c_{T-j} = -\beta^j \bar{E}[R_T] - f\left(\bar{E}^2[R_T], ..., \bar{E}^{j+1}[R_T]\right)$$

- Case 2: adjusted PC
  - now, *actual* inflation itself depends on HOB
  - shift weight towards beliefs of higher order

Forward Guidance Dampened

Proposition. In either case,

$$\frac{\left.\frac{\partial c_0}{\partial R_T}\right|_{variant}}{\left.\frac{\partial c_0}{\partial R_T}\right|_{standard}} \to 0 \quad \text{as } T \to \infty$$



- also, dampening is stronger when prices are more flexible

Forward Guidance Dampened

Proposition. At least in case II, when λ is small enough,

$$\left.\frac{\partial c_0}{\partial R_T}\right|_{variant} \to 0 \quad \text{as } T \to \infty,$$

whereas $\left.\frac{\partial c_0}{\partial R_T}\right|_{standard} \to \infty$.

- This is relevant also for
  - shocks at ZLB, deflationary spirals
  - eq. selection, Fisherian debate

Discussion

- Is this just about information, or inattention? NO
- It's more about robustness and plausibility of solution concepts and predictions
  - lack of CK = relaxation of solution concept = imperfect coordination
- What matters most is
  - not beliefs of future MP
  - rather beliefs about current and future response of economy
    - ★ no obvious reason why such beliefs must "jump" in the way standard model assumes
- HOB = belief anchor = nominal anchor?

Discussion

- Forward guidance vs acting now
  - suppose MP changes R_0 rather than R_T
  - direct/PE effect is stronger
  - preceding considerations are less relevant
- Compare this *relative* effect to
  - adjustment frictions, inattention, sparsity
    - ★ above designed to dampen PE, not GE
  - incomplete markets
    - ★ observational equiv. with "discounted Euler conditions"
    - ★ but rests on beliefs and coordination, not financial frictions

Conclusion

- Worth questioning solution concept in macro
  - even if we maintain individual rationality
- Lack of CK = relaxation of solution concept = GE dampened
  - formalization of "GE takes time"
  - in short run, "Macro is (close) to Micro"
  - topical application: Forward Guidance
- Other applications...
  - aggregate demand and keynesian multipliers
  - fiscal policy