Measuring the Natural Rate of Interest: International Trends and Determinants*

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Abstract

U.S. estimates of the natural rate of interest–the real short-term interest rate that would prevail absent transitory disturbances–have declined dramatically since the start of the global financial crisis. For example, estimates using the Laubach-Williams (2003) model indicate the natural rate in the United States fell to around zero during the crisis and has remained there through the end of 2015. Explanations for this decline include shifts in demographics, trend productivity growth, and global factors affecting real interest rates. This paper applies the Laubach-Williams methodology to a number of other advanced economies–including the Euro Area, the United Kingdom, and Canada. The country-by-country results are then analyzed to distinguish between global and idiosyncratic factors that influence the natural rate across countries and time.

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1 Introduction

Since Wicksell (1936), the natural rate of interest – which we define to be the real short-term interest rate consistent with output equaling its natural rate and constant inflation – has played a central role in macroeconomic and monetary theory. The natural or "equilibrium" real interest rate provides a benchmark for measuring the stance of monetary policy, with policy expansionary (contractionary) if the the short-term real interest rate lies below (above) the natural rate. This role is illustrated clearly in monetary policy rules such as the Taylor (1993) rule, according to which the real interest rate exceeds the natural rate when inflation exceeds its target rate and vice versa, all else equal. It also provides an anchor for the behavior of interest rates more generally, once cyclical or other factors have died out.

Although economic theory provides insights into the various factors affecting the natural rate of interest (see, for example, Congressional Budget Office, 2014; International Monetary Fund, 2014; Council of Economic Advisers, 2015; Eggertsson et al. 2015; Pescatori and Turunen 2015; Hall 2016), measurement of the natural rate of interest has proven more challenging. This arises because the natural rate, like other latent variables, must be inferred from the data rather than directly observed. This is a particularly relevant topic today. Sustained extraordinarily low interest rates in most advanced economies since the global financial crisis have heightened interest in the question of whether the natural rate of interest has permanently declined and why.

Analysis based on U.S. data suggests that the natural rate of interest has moved significantly lower over the past quarter century, with a sharp drop occurring over the past decade (Williams 2015). This finding is robust to alternative methodologies (Hamilton et al. 2015; Kiley 2015; Laubach and Williams 2015; Lubik and Matthes 2015). In this paper, we extend this analysis to other advanced economies.[1] We estimate a version of the Laubach and Williams (2003) model of the natural rate of interest, originally developed for the U.S. economy, using data from four economies: the United States, Canada, the Euro Area, and the United Kingdom. This model applies the Kalman filter to data on real GDP, inflation, and the short-term interest rate to extract highly persistent components of the natural rate of output, its trend growth rate, and the natural rate of interest.

Our approach contrasts with other research that has focused on short-term fluctuations

[1] For earlier studies on the natural rate of interest in the Euro Area, see Giammariou and Valla (2003), and Mésonnier and Renne (2007).

in the natural rate of interest, assuming the longer-run value is constant (Neiss and Nelson 2003, Woodford 2003, Andres et al., 2009, Barsky et al., 2014). Recent analysis using dynamic stochastic general equilibrium (DSGE) models also finds evidence of a large decline in the natural rate of interest since the onset of the global financial crisis (Cúrdia 2015; Cúrdia et al., 2015; Goldby et al., 2015). Although the definitions and methods differ across studies, we view the shorter-term perspective in these studies to be complementary to our longer-run approach.

This analysis yields four key results. First, we find evidence of time-variation in the natural rate of interest in all four economies. Second, there is a downward trend in the estimated natural rates of interest in all four economies. At the end of our sample, the estimated natural rates of interest in all have all fallen to historically very low levels. This is in large part explained by a significant decline in the estimated trend growth rates found in all four economies, but other highly-persistent factors also appear to be at work. We find no evidence that the natural rate has moved back up recently. Third, although estimation is done on a economy-by-economy basis, there is substantial comovement in the estimates of the natural rates of interest and trend GDP growth across economies. This suggests an important role for so-called global factors influencing the natural rates. Finally, the estimates of the natural rate of interest are highly imprecise, reinforcing a key finding of the original Laubach and Williams (2003) paper. In fact, the natural rate estimates for the three countries are much more imprecise than those for the United States.

The paper is organized as follows. The first section lays out the theoretical model and its empirical counterpart. The second section reports the estimation results. The third section analyzes the movements in the natural rates of interest over time and the comovement across economies. The final section concludes.

2 An Empirical Framework for Estimating the Natural Rate

Our empirical approach to estimating the natural rate of interest draws on two separate strands of literature. As discussed in the introduction, we follow Wicksell in defining the natural rate as the real rate of interest consistent with stable inflation and output being at its natural rate. This definition is akin to the one developed in the DSGE literature (e.g. Woodford 2003), and hence we build on the New Keynesian framework of a Phillips curve

relationship and an intertemporal IS equation to describe the dynamics governing the output gap and inflation as a function of the real rate gap. However, we relax the assumptions about the steady state that most DSGE models use to derive log-linear approximations of the dynamics of inflation and the output gap. In particular, key parameters that are being treated as fixed in that literature, such as the growth rate of technology and the rate of time preference of the representative household, may in fact be subject to highly persistent, but difficult-to-detect fluctuations (e.g. Stock and Watson 1998). Thus, whereas in the DSGE literature the natural rate of interest is a stationary linear combination of transitory shocks to preferences and technology, in our framework we explicitly allow for the natural rate to be affected by low-frequency nonstationary processes.[2]

2.1 Description of the model

A useful starting point for modeling the natural rate of interest is the neoclassical growth model. For a representative household with CES preferences, that model implies that the natural rate of interest varies over time in response to shifts in preferences and the growth rate of output. In a non-stochastic steady state, household intertemporal utility maximization yields the relationship between the steady-state real one-period interest rate r^* and steady-state growth:

$$r^* = \frac{1}{\sigma} g_c + \theta, \tag{1}$$

where σ denotes the intertemporal elasticity of substitution in consumption, g_c is the growth rate of per capita consumption, and θ is the rate of time preference. This steady-state relationship is shared by standard monetary DSGE models, and hence the value given by (1) provides the appropriate intercept for simple interest rate rules in such models.

The purpose of our study is to model the natural rate of interest as a time-varying process that is subject to potentially permanent changes and to provide estimates of this process for several major economies. Roberts (2001), Edge et al. (2007), and Kahn and Rich (2007) provide evidence of shifts in the trend growth rate of labor productivity in the United States, suggesting one source of persistent movements in the natural rate of interest;

[2] As discussed below, a further difference between our measure of the natural rate and that from DSGE models is that we allow for shocks that have transitory effects on the output gap and inflation but that we do not measure as part of the natural rate, so as to focus on the low-frequency determinants of that rate. By contrast, the natural rate concept estimated in DSGE models is defined as the rate that achieves price stability period-by-period. As discussed in the introduction [check], our concept of the natural rate is intended to be useful as a potential time-varying intercept in an interest rate rule.

other potential sources of highly persistent changes in r^* could be related to shifts in fiscal policy (Laubach 2009) or changes in the global supply of savings (Bernanke 2005, Council of Economic Advisers 2015).

Given our focus on the international dimension of influences and determinants of the natural rate, we start from the inflation and output gap dynamics described by the open-economy version of the New Keynesian model (Galí 2008). These dynamics are summarized by a Phillips curve

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \kappa \tilde{y}_t \tag{2}$$

and an IS equation of the form

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{H,t+1}] - r_t^n) \tag{3}$$

In these equations, $\pi_{H,t}$ denotes the growth rate of prices of *domestically produced* goods, $\tilde{y}_t$ the output gap, and i_t the safe one-period nominal interest rate. The parameters κ and σ in these equations are functions of underlying parameters describing households' preferences and technology, such as the intertemporal elasticity of substitution in consumption, the substitutability of home and foreign goods in households' preferences, the price adjustment frictions etc.

It is worth noting two properties of the variable r_t^n in (3). In the simple model (2)-(3), this variable summarizes all shocks affecting inflation and the output gap, and setting the real rate gap $i_t - E_t[\pi_{H,t+1}] - r_t^n$ to zero period-by-period completely stabilizes these two variables. Second, in contrast to a closed economy, the variable r_t^n depends not only on expected domestic output growth but in general also on expected world output growth. While we will depart in the following from the first property, the dependence of the natural rate of interest on global as well as domestic conditions is central to our analysis.

The equations we use for estimating the natural rate of interest relax the restrictions imposed by the New Keynesian model along two dimensions. First, we work with reduced-form equations that are somewhat agnostic about the precise lead-lag relationships among the endogenous variables. Doing so reduces the risk that our natural rate estimates are unduly affected by estimates of structural parameters based on potentially misspecified output gap and inflation dynamics. Second, we allow for the presence of shocks that affect the output gap and inflation but not the natural rate of interest, which we define as a

low-frequency concept. In particular, we estimate the following equations:

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r^*_{t-j}) + \epsilon_{\tilde{y},t} \quad (4)$$

$$\pi_t = B_\pi(L)\pi_{t-1} + b_y\tilde{y}_{t-1} + \epsilon_{\pi,t}. \quad (5)$$

where $\tilde{y}_t = 100 * (y_t - y^*_t)$, y_t and y^*_t are the logarithms of real GDP and the unobserved natural rate of output, respectively, r_t is the real short-term interest rate, and π_t denotes consumer price inflation.[3] The presence of the stochastic terms $\epsilon_{\tilde{y},t}$ and $\epsilon_{\pi,t}$ captures transitory shocks to the output gap and inflation, while movements in r^*_t reflect persistent shifts in the relationship between the real short-term interest rate and the output gap (Williams 2003). The distinction between shocks of different persistence reflects our focus on a concept of the natural rate of interest as reflected in the intercept of simple interest rate rules; more transient shocks would leave this intercept unchanged, as the monetary policy response to such shocks would be governed by the response to output and inflation gaps prescribed by such a rule.

Based on the theoretical link between the natural rate of interest and output (or consumption) growth noted above, we assume that the law of motion for the natural rate of interest is given by:

$$r^*_t = g_t + z_t, \quad (6)$$

where g_t is the trend growth rate of the natural rate of output and z_t captures other determinants of r^*. Note that we assume a one-for-one relationship between the trend growth rate of output and the natural rate of interest, which corresponds to assuming $\sigma = 1$ in equation 1. In Laubach and Williams (2003), we estimated this relationship and found a coefficient of close to unity. Because this relationship is not well identified in the data, we chose to impose a coefficient of unity.[4]

We specify log potential output as a random walk with a stochastic drift g that itself

[3]The open-economy version of the New Keynesian Phillips curve (2) describes a relationship for home-goods inflation $\pi_{H,t}$. In the absence of data for such a concept, we use measures of consumer price inflation described in the Appendix.

[4]See Hamilton et al. (2015) and Laubach and Williams (2016) for further discussion.

follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t}, \tag{7}$$

$$g_t = g_{t-1} + \epsilon_{g,t}. \tag{8}$$

This specification implies that potential output is integrated of order 2. In particular, an innovation $\epsilon_{y^*,t}$ has a permanent effect on the level of potential output, but only a one-period effect on its rate of change, while an innovation $\epsilon_{g,t}$ permanently affects the growth rate of potential output. Stock and Watson (1998) examine the hypothesis that U.S. log real GDP over the post-WW II period is integrated of order 2 and find evidence in support of a slow-moving and apparently nonstationary trend growth rate. We view the I(2) specification as agnostic for the purpose of estimation, as it avoids having to estimate the unconditional mean of real GDP growth. Finally, we assume that z also follows a random walk,

$$z_t = z_{t-1} + \epsilon_{z,t}. \tag{9}$$

We assume $\epsilon_{y^*,t}, \epsilon_{g,t}$, and $\epsilon_{z,t}$ are normally distributed with standard deviations σ_{y^*}, σ_g, and σ_z respectively and are serially and contemporaneously uncorrelated. Equations (7)–(9) constitute the transition equations of our state-space model.

2.2 Empirical implementation

Given that the model is linear in the unobserved state variables, we apply the Kalman filter to estimate, for each economy, the natural rate of output, its trend growth rate, and the natural rate of interest (or alternatively the component z unrelated to trend growth). The data and specifics of model variables are described in the Appendix.

The time series and survey evidence suggests that real GDP growth, labor productivity growth, and real interest rates are subject to highly persistent changes masked by volatile transitory shocks. Maximum likelihood estimates of the standard deviations of the innovations to z and the trend growth rate, σ_g and σ_z, are therefore likely to be biased towards zero owing to the so-called "pile-up problem" discussed in Stock (1994). We therefore use Stock and Watson's (1998) median unbiased estimator to obtain estimates of the ratio $\lambda_g \equiv \frac{\sigma_g}{\sigma_{y^*}}$

and $\lambda_z \equiv \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}$.[5] We impose these ratios when estimating the remaining model parameters (including $\sigma_{\tilde{y}}$ and σ_{y^*}) by maximum likelihood.

Our estimation method proceeds in sequential steps. In the first step, we follow Kuttner (1994) and apply the Kalman filter to estimate the natural rate of output, omitting the real rate gap term from equation (4) and assuming that the trend growth rate, g, is constant. We compute the exponential Wald statistic of Andrews and Ploberger (1994) for a structural break with unknown break date from the first difference of this preliminary estimate of the natural rate of output to obtain the median unbiased estimate of λ_g. In the second step, we impose the estimated value of λ_g from the first step and include the real interest rate gap in the output gap equation under the assumption that z is constant. We estimate the five model equations and apply the exponential Wald test for an intercept shift in the IS equation at an unknown date to obtain an estimate of λ_z.

In the final step, we impose the estimated values of λ_g from the first step and λ_z from the second step and estimate the remaining model parameters by maximum likelihood as described by Harvey (1989).[6] During all three stages of the estimation, we impose the constraints that the slope a_r of the IS equation is negative and the slope b_y of the Phillips curve is positive. We view these as minimal priors on the structure of the model that, in the event, facilitate the convergence of the numerical optimization during estimation. We compute confidence intervals and corresponding standard errors for the estimates of the states using Hamilton's (1986) Monte Carlo procedure that accounts for both filter and parameter uncertainty.[7]

[5] Substituting (6) into (4), the latter can be written as

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}(r_{t-1} + r_{t-2} - (g_{t-1} + g_{t-2})) - \frac{a_r}{2}(z_{t-1} + z_{t-2}) + \epsilon_{\tilde{y},t}$$

The ratio λ_z based on the test statistic for an intercept shift at an unknown date in this equation is thus the ratio of $a_r \sigma_z$ to $\sigma_{\tilde{y}}$.

[6] The vector of unobserved states in the first-stage estimation includes three lags of potential output; in the second stage it also includes one lag of g; and in the third stage, a second lag of g plus two lags of z. We compute the conditional expectation and covariance matrix of the initial state, key inputs for the Kalman filter, as follows. For the lags of potential output, we apply the HP filter to log real GDP data starting three quarters prior to our estimation sample and take the values of the HP trend component for the three quarters prior to the sample start. For lags of g, we use the first difference of the HP trend. The component z is initialized at 0. We compute the covariance matrix of these states from the gradients of the likelihood function.

[7] We draw the parameter vectors from a normal distribution with the covariance matrix of the parameter vector computed as the outer product of the gradients. During these draws, we impose in addition to the constraints $a_r \leq -0.0025$ and $b_y \geq 0.025$ the constraint that $a_{y,1} + a_{y,2} < 1$. Discarding draws that violate either of these constraints reduces the width of the standard errors, substantially so for the Euro Area and the UK. The initial value of the state vector is drawn from a normal distribution with the mean

3 Estimation Results

Table 1 reports estimation results for the four economies. For three of the economies, the sample spans 220 quarters from 1961:I to 2015:IV, while data availability for the Euro Area dictates a sample start in 1972:I.

Focusing first on the estimates for the United States, the values for λ_g and λ_z indicate substantial time variation in trend growth and the natural rate of interest over the 55-year sample. For example, the estimates of the standard deviation of ϵ_g and the first difference of r^* imply that the standard deviations of $g_{2015:IV} - g_{1961:I}$ and $r^*_{2015:IV} - r^*_{1961:I}$ are on the order of 1.8 percent and 2.8 percent, respectively (the results for both variables are expressed at annual rate). The 90 percent confidence interval for λ_g, computed by Monte Carlo simulations, ranges from 0.001 to 0.129.[8]

The slope coefficients a_r and b_y for the U.S. are reasonably large and precisely estimated, suggesting that both the output gap and the real rate gap are well identified. Nonetheless, even with hindsight, the natural rate of interest is estimated imprecisely, with a sample average standard error of 1.1 percentage points, as shown in the section of Table 1 labeled "S.E. (sample ave.)." The imprecision in estimates of the natural rate of interest is even greater when examining the one-sided estimates of r^* that correspond more closely to "real-time" estimates, in that only current and past observations are used in estimating the state. (Because the full sample is used in estimating the model parameters, the analogy, however, is not exact.) The final observation standard errors shown at the bottom of the table, which provide a measure of the imprecision of one-sided estimates assuming the true values of λ_g and λ_z are known, are even wider.

Comparing parameter estimates across countries, the estimates of λ_g indicate time variation in the trend growth rate, albeit at a smaller scale in the euro area and the UK. Even though the estimates of λ_z are of similar magnitude across countries, the time variation of r^* is magnified for the euro area and the UK by the smaller values of the slope parameter a_r for these economies. Moreover, it is evident that the uncertainty around the natural

and covariance given by the smoothed estimate of the initial state and its covariance matrix. Note that these estimated confidence intervals hold the imposed values of λ_g and λ_z fixed but take into account the uncertainty regarding $\sigma_{\tilde{y}}$, σ_π, and σ_{y^*}.

[8] To compute this confidence interval, we use parameter estimates based on the second-stage model with a time-varying trend growth and a constant natural rate of interest to construct 10,000 simulated series of $\{y^*_t, g_t\}$. For each pair of simulated series we compute the exponential Wald statistic for an intercept shift in the first difference of y^*_t.

rate estimates is closely related to the statistical significance of the slope parameters a_r and b_y. For Canada, although a_r is of similar magnitude as in the U.S. and is statistically significant, the fact that b_y is both smaller and less precisely estimated implies that the average standard errors around y^* and r^* are larger than in the U.S. For the euro area, the slope of the IS equation is considerably flatter, and the average standard error for r^* is very large, primarily driven by draws of the parameter vector in which a_r is very close to zero and hence r^* barely identified.[9]

The parameter estimates for the UK are in several respects quite different from those for the other economies. Notably, the output gap is estimated to be much less persistent, as shown by the smaller value for $\sum a_y$, but inflation responds very strongly to these more transitory output gap fluctuations, with b_y being about ten times the size compared to its value for the other three economies. By contrast, this output gap estimate seems to bear little relation to the real rate gap, with a_r estimated to be close to zero. Inflation also seems to be less well explained by its own lags and the lagged output gap, as shown by the very large standard deviation σ_π.

The upper panels of Figure 1–4 display the filtered (one-sided) estimates of the output gap for the four economies. The shaded regions indicate the periods of recessions for each economy.[10] With a few exceptions, large downward movements in estimated output gaps generally conform to the timing of recessions. All four economies experience negative output gaps following the global financial crisis. However, output gaps after the crisis are generally less negative (or more positive) than some other estimates (for example, the International Monetary Fund 2015). The subdued declines in estimated output gaps in our model likely reflects the relatively modest decline in core inflation in the four economies, which in the context of the model is at odds with large negative output gaps (Laubach and Williams 2016).

A striking finding common to all four economies is the secular downward trend in the estimated trend growth rates of output. This process appeared to accelerate in the final part

[9]The fact that more probability mass of the distributions of a_r and b_y is located close to zero is also reflected in the number of draws during the computation of the standard errors that are being rejected because they violate the constraints $a_r \leq -0.0025$ and $b_y \geq 0.025$. While hardly any draws are being rejected for the US, about 1/4 of all draws are being rejected for Canada and the euro area, and about 1/3 for the UK.

[10]The following sources are used for recession dating in the figures United States: National Bureau of Economic Research; Canada and the United Kingdom: Economic Cycle Research Institute; Euro Area: CEPR Euro Area Business Cycle Dating Committee.

of the sample, with trend potential output growth slowing by a percentage point on average over 2007–2015. Except for the Euro Area, these declines are concentrated in the 2008-09 period of the global financial crisis and its aftermath, as shown by the dashed lines in the lower panels of Figures 1–4. In these three regions, trend growth has remained relatively stable at its new lower level since 2010. In contrast, estimated trend growth in the Euro Area fell relatively modestly during the global financial crisis, but has been on a steady downward trajectory following the onset of the Euro crisis.

This pattern of declining trend GDP growth in advanced economies is broadly consistent with alternative estimates based on methodologies that decompose potential output into its component parts (see Fleischman and Roberts, 2011; Congressional Budget Office, 2016; and International Monetary Fund 2015). These analyses highlight the roles of slowing labor force growth due to demographics and a slowdown in trend productivity growth. The productivity slowdown in the United States is documented by Byrne et al. (2016) and Gordon (2016), and the European productivity slowdown is analyzed in Cette et al. (2016).

4 Global and Local Determinants of the Natural Rate

In this section, we investigate the interdependence of natural rates of interest across the four economies we study. As discussed earlier, economic theory implies that movements in natural rate of interest should be correlated across countries. These cross-country linkages were not imposed in estimation. However, as shown here, there is evidence of substantial comovement of the estimates of both the natural rate of interest and the trend growth rate of output over time. We start with a discussion of the overall trends in natural rates over the past 25 years and then turn to an exploration of interdependence using a vector error-correction (VECM) model.

4.1 Declining Natural Rates of Interest

Although estimated natural rates differ across economies owing to idiosyncratic influences, all exhibit two phases of significant and sustained declines over the past quarter century: First, a moderate secular decline over 1990-2007, and second, a more substantial decline since 2007. The upper portion of Table 2 reports the annual averages of the natural rate

estimates for three years: 1990, 2007, and 2015. In 1990, the estimates range between about 2-1/2 and 3-1/2. In each case, the estimates are lower in 2007, on the eve of the global financial crisis, ranging from 2 to 2-1/2 percent. In Canada and the Euro Area, this decline is more than fully accounted for by the estimated decline in the trend growth rate of potential output, as indicated in the lower portion of the table. For the United States and the United Kingdom, declines in both trend growth of potential output and the random walk factor, z, contribute to the decline in the estimated natural rate over this period.

Estimated natural rates plummeted for all four regions in the immediate aftermath of the global financial crisis (Figures 1-4). Estimates for Canada and the United States then edged down further over 2010-2015, and are close to all-time lows at the end of our sample. In both of these countries, the majority of the overall decline in the natural rate from 2007 to 2015 is accounted for by the drop in the estimated trend growth rate. In contrast to the results for Canada and the United States, natural rate estimates for the Euro Area and the United Kingdom rebounded in 2010, but then fell sharply again following the Euro crisis. Over the entire period of 2007 through 2015, estimated natural rates in the UK and the Euro Area fell by about 1-1/2 and 2-1/2 percentage points, respectively, with about half of that decline accounted for by lower estimated trend growth.

4.2 Correlations of Natural Rates Across Regions

The preceding discussion highlighted the downward movements in natural rate estimates over the past 25 years. In this section, we explore more formally the comovement of natural rate estimates over the full sample for which we have estimates for all four economies (1972-2015). As discussed previously, natural rates are assumed to be nonstationary in our model, and standard augmented Dickey-Fuller tests confirm this, as shown in the upper part of Table 3. Given this nonstationarity, we cannot use methods that require stationarity, such as correlations or principal components, to describe natural rate comovement over time. Instead, we characterize the joint movement in natural rate estimates using a simple VECM model. Because the natural rates are themselves generated regressors from the estimated model, care should be taken in interpreting these results and their statistical significance. Instead, this analysis should be viewed as a simple way to describe the comovement evident in the estimates, without putting much credence to the specific numerical results or inference.

With that caveat in mind, there is evidence of a single cointegrating vector linking the four natural rates series based on a standard Johansen test. The estimated cointegrating vector is reported in Table 3. Based on lag tests, a VECM model with eight lags is estimated and the key results are reported in the table. The VECM estimates suggest that natural rates comove over time, but are also subject to idiosyncratic influences. To summarize the interdependence in natural rates, Figure 5 shows variance decompositions for each natural rate process in terms of shocks to the four natural rates, using a Cholesky decomposition with ordering: Canada, UK, Euro Area, and the United States. Although the specific values of the variance decompositions are sensitive to the ordering of the regions, the overall finding of substantial comovement in the estimated natural rates of interest is robust.

The variance decompositions indicate the presence of a great deal of interdependence in natural rates across economies. For example, shocks to the U.S. natural rate contribute about 40 percent to natural rate variation in Canada at a 10-year horizon. Shocks to natural rates in the Euro Area and the UK play a smaller role, but still have a nontrivial effect on the natural rate in Canada at long horizons. A similar pattern is seen in the Euro Area, where U.S. natural rate shocks are a major source of natural rate variance at long horizons, while shocks in Canada and the UK have a smaller, but still noticeable effect. Estimated natural rates in the UK and US are less dependent on those in other regions; however, they are not completely independent, with between 20 and 40 percent of the long-horizon variance emanating from other economies.

We also find evidence of comovement in the estimates of trend output growth across the four economies. Table 4 reports the results of augmented Dickey-Fuller and cointegration tests for the model estimates of trend GDP growth. The results from the estimated VECM model for trend growth rates indicate the presence of interdependence across countries. The resulting patterns in variance decompositions shown in Figure 6 are similar to those for the natural rates of interest. This finding is not entirely surprising since movements in trend output growth are a major source of time variation in natural rates of interest in our model.

Overall, these results point to the potential for formally incorporating global factor in the analysis of trend output growth and the natural rate of interests across countries. Such an approach is also supported by the evidence of a global factor influencing asset prices across countries, described by Miranda-Agrippino and Rey (2015). We leave the explicit modeling of global and local factors for trend growth rates and natural rates of interest to

future work.

5 Conclusion

In this paper, we estimated the natural rates of interest and output and the trend growth rate in four economies – Canada, the Euro Area, the United Kingdom, and the United States–using a version of the Laubach-Williams (2003) model. Our findings mirror those for the United States from earlier research: estimated natural rates of interest exhibit significant variation over time, movements in the trend growth rate are an important determinant of changes in the natural rate, and natural rates of interest have declined over the past quarter century, reaching historically low levels in the most recent past. In addition, and again consistent with past research, natural rates estimates tend to be highly imprecise.

These findings suggest that declining natural rates of interest are an international phenomenon and therefore stem in large part from developments common to many countries, rather than idiosyncratic national factors. This argues for more research that takes an international perspective in analyzing natural rates, by including data from economies besides the United States and by examining structural changes that influence natural rates across the globe. In addition, research using alternative model specifications more suited for each economy's features may yield more precise estimates and insights into the robustness of our results to alternative approaches.

Finally, as discussed in Laubach and Williams (2015), very low natural rate of interest, if sustained into the future, have profound implications for monetary policy (see also Blanchard, Dell'Ariccia, and Mauro, 2010; Summers 2014). All else equal, a lower average real interest rate in turn implies that episodes of monetary policy being constrained at the effective zero lower bound are likely to be more frequent and longer. Moreover, our finding that natural rates have declined in a number of major advanced economies suggests that this is not a problem unique to the United States, but has broader consequences globally. For example, as discussed in Eggertsson et al. (2015), in an environment of a very low natural rate of interest, the effects of the lower bound on interest rates are amplified and international spillovers and the benefits from international policy coordination may increase.

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Table 1: Parameter Estimates

Parameter	United States	Canada	Euro Area	United Kingdom
Sample	1961-2015	1961-2015	1972-2015	1961-2015
λ_g	0.052	0.052	0.031	0.024
λ_z	0.030	0.026	0.042	0.026
$\sum a_y$	0.943	0.954	0.949	0.915
a_r	−0.071	−0.068	−0.037	−0.004
	(4.002)	(2.891)	(1.803)	(0.919)
b_y	0.075	0.040	0.055	0.562
	(3.045)	(1.497)	(1.572)	(2.577)
$\sigma_{\tilde{y}}$	0.354	0.357	0.294	0.110
σ_π	0.792	1.399	0.995	2.705
σ_{y^*}	0.581	0.630	0.403	0.884
σ_g	0.120	0.130	0.050	0.083
σ_z	0.151	0.138	0.336	0.652
$\sigma_{r^*} = \sqrt{\sigma_g^2 + \sigma_z^2}$	0.193	0.190	0.340	0.658
S.E. (sample ave.)				
r^*	1.086	1.677	4.061	4.722
g	0.401	0.458	0.244	0.423
y^*	1.549	2.524	1.762	0.865
S.E. (final obs.)				
r^*	1.547	2.382	6.359	6.763
g	0.546	0.608	0.346	0.545
y^*	2.052	3.117	2.801	1.011

Table 2: Change in r^* Estimates over Time

	United States	Canada	Euro Area	United Kingdom
r^* estimates				
1990	3.5	3.2	2.4	2.5
2007	2.3	2.4	2.0	2.1
2015	0.4	1.4	−0.4	0.7
Change in estimates				
1990-2007				
Δr^*	−1.1	−0.8	−0.4	−0.3
Δg	*−0.6*	*−1.0*	*−0.7*	*−0.1*
2007-2015				
Δr^*	−1.9	−1.0	−2.4	−1.5
Δg	*−1.3*	*−0.9*	*−1.0*	*−0.8*

Table 3: Test Results: Natural Rate Estimates

	United States	Canada	Euro Area	United Kingdom
ADF test (p-value)				
1961-2015	0.75	0.81	*n/a*	0.18
1972-2015	0.92	0.67	0.71	0.17
VECM model				
Cointegrating vector	1.00	−0.75	−0.67	0.28
(S.E.)	(–)	(0.10)	(0.12)	(0.11)
Error-correction coef.	−0.01	0.08	0.14	0.17
(S.E.)	(0.05)	(0.03)	(0.05)	(0.07)
Granger lag test (p-value)	0.20	0.06	0.98	< 0.01
(8 lags)				

Table 4: Test Results: Trend Growth Estimates

	United States	Canada	Euro Area	United Kingdom
ADF test (p-value)				
1961-2015	0.55	0.81	*n/a*	0.10
1972-2015	0.90	0.68	0.56	0.41
VECM model				
Cointegrating vector	1.00	0.10	−1.52	0.01
(S.E.)	(–)	(0.17)	(0.37)	(0.24)
Error-correction coef.	−0.02	0.04	0.05	0.01
(S.E.)	(0.03)	(0.02)	(0.02)	(0.02)
Granger lag test (p-value) (8 lags)	0.02	< 0.01	< 0.01	< 0.01

Figure 1: Estimation Results for the US

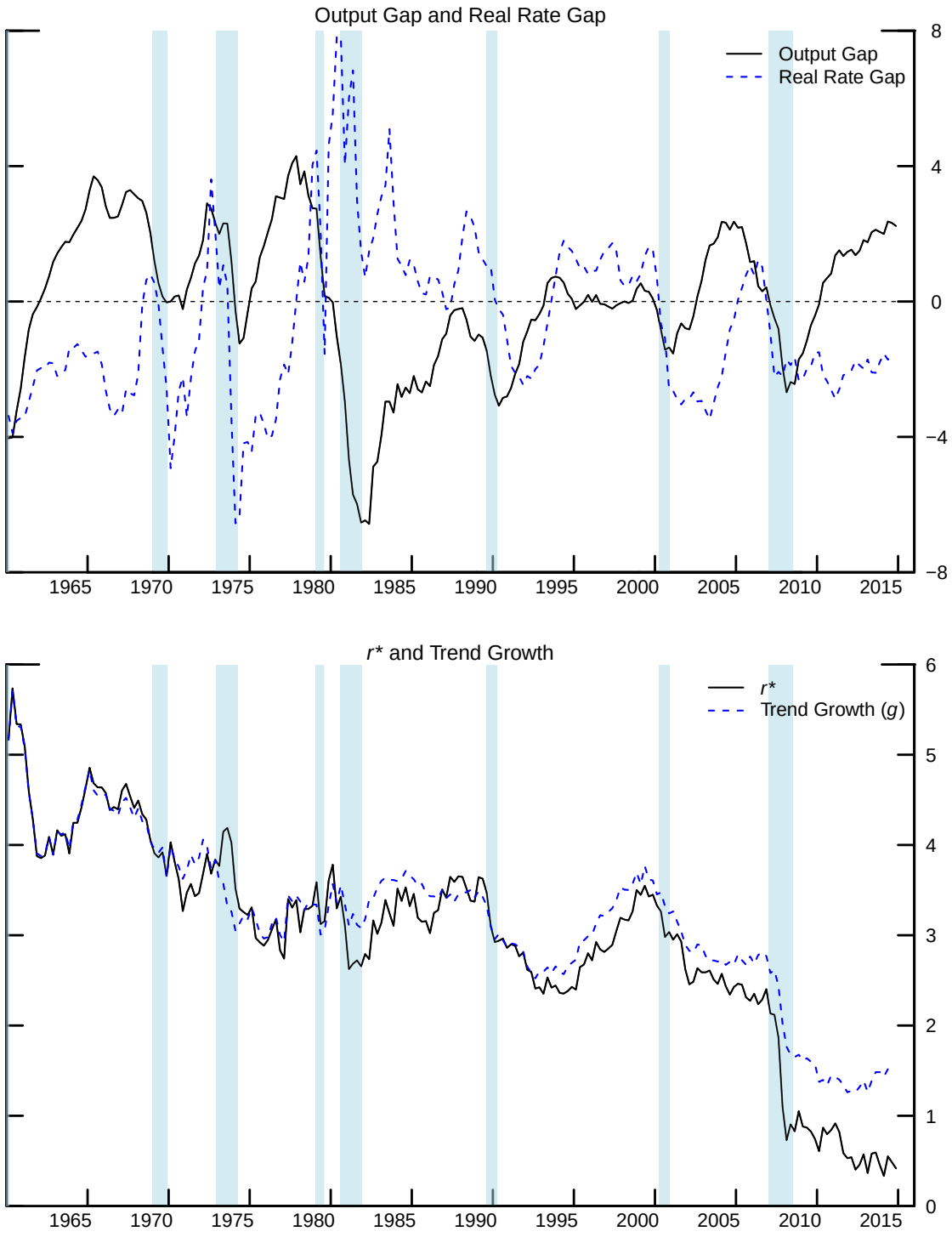


Figure 2: Estimation Results for Canada

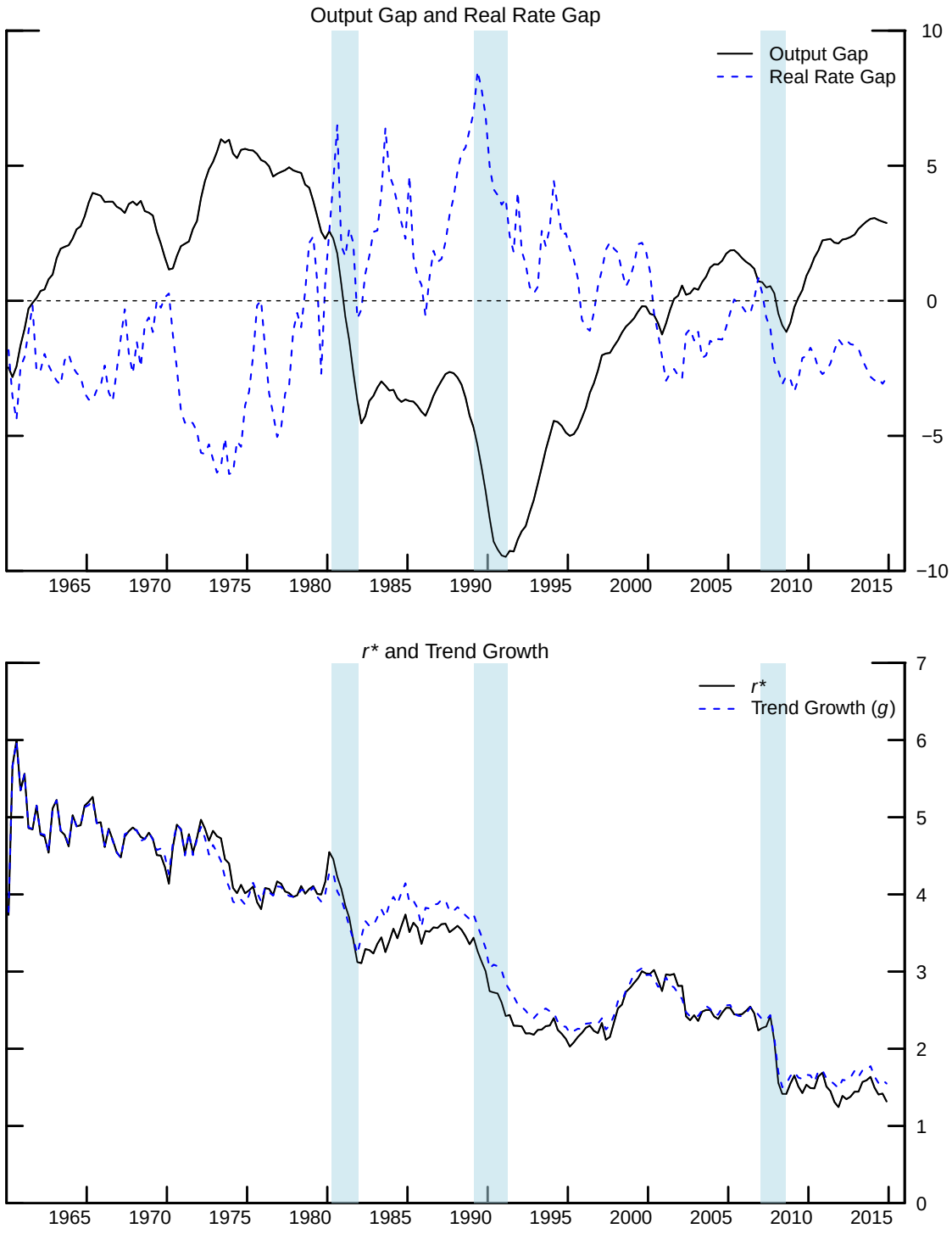


Figure 3: Estimation Results for the Euro Area

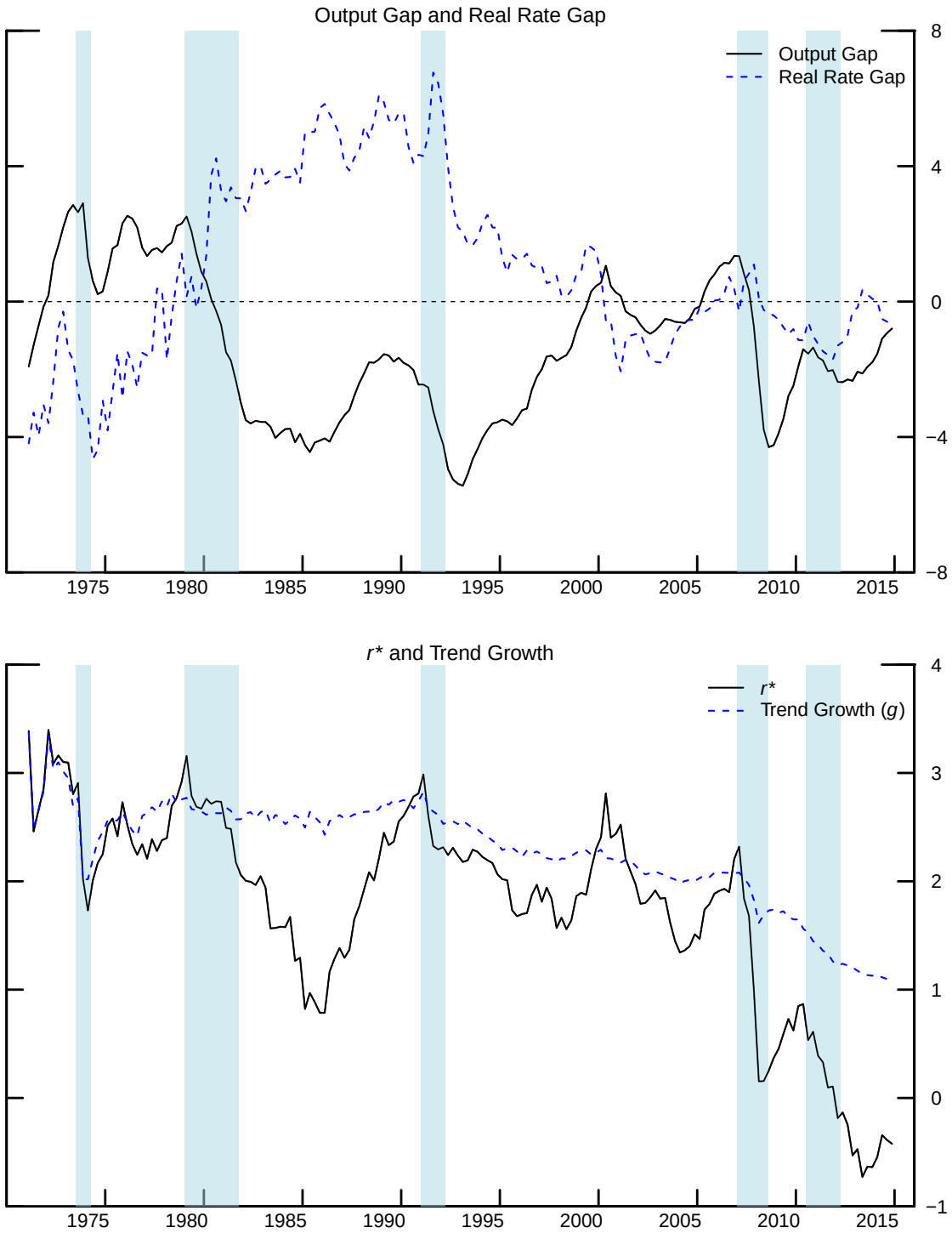


Figure 4: Estimation Results for the UK

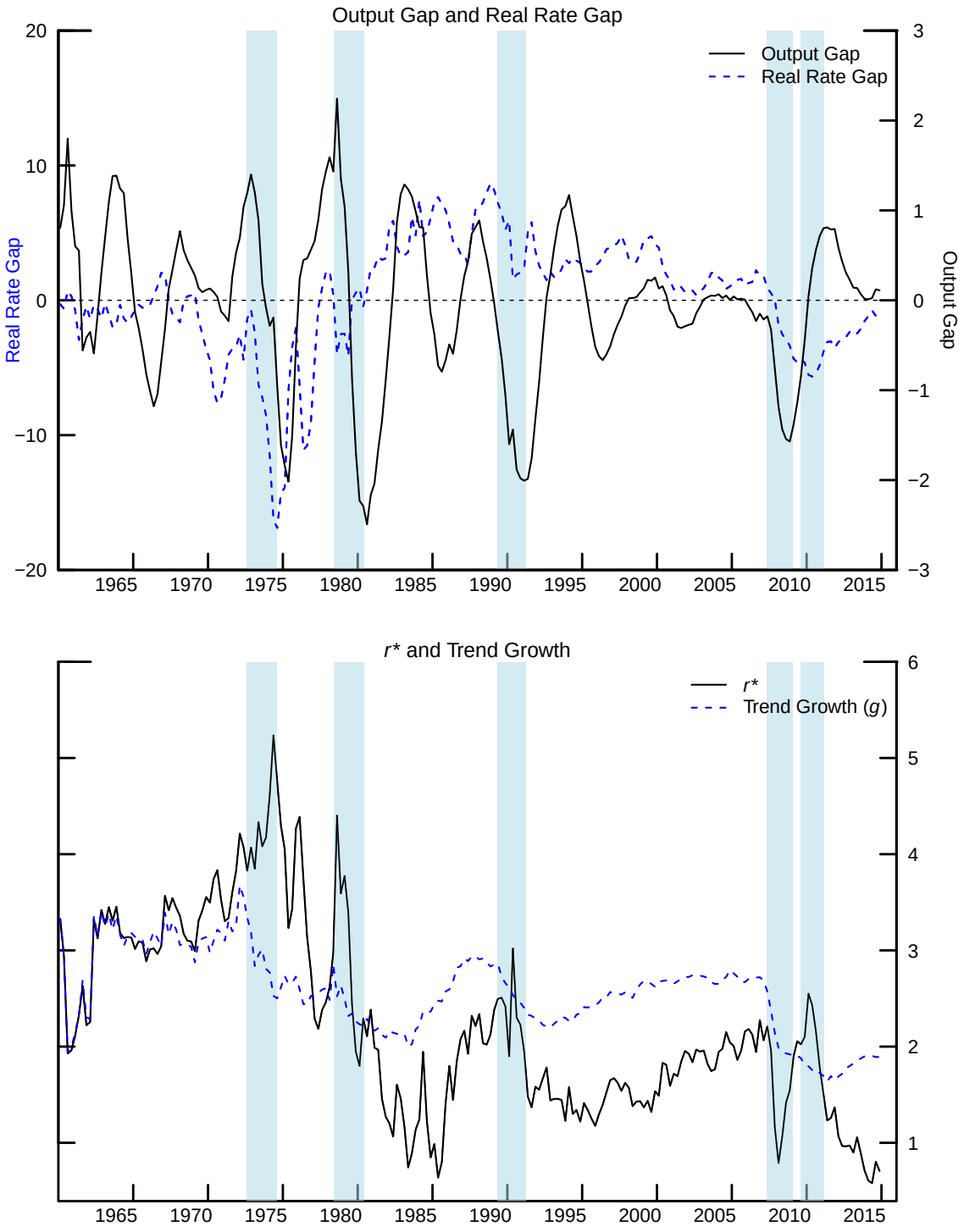


Figure 5: Natural Rate Estimates Variance Decompositions

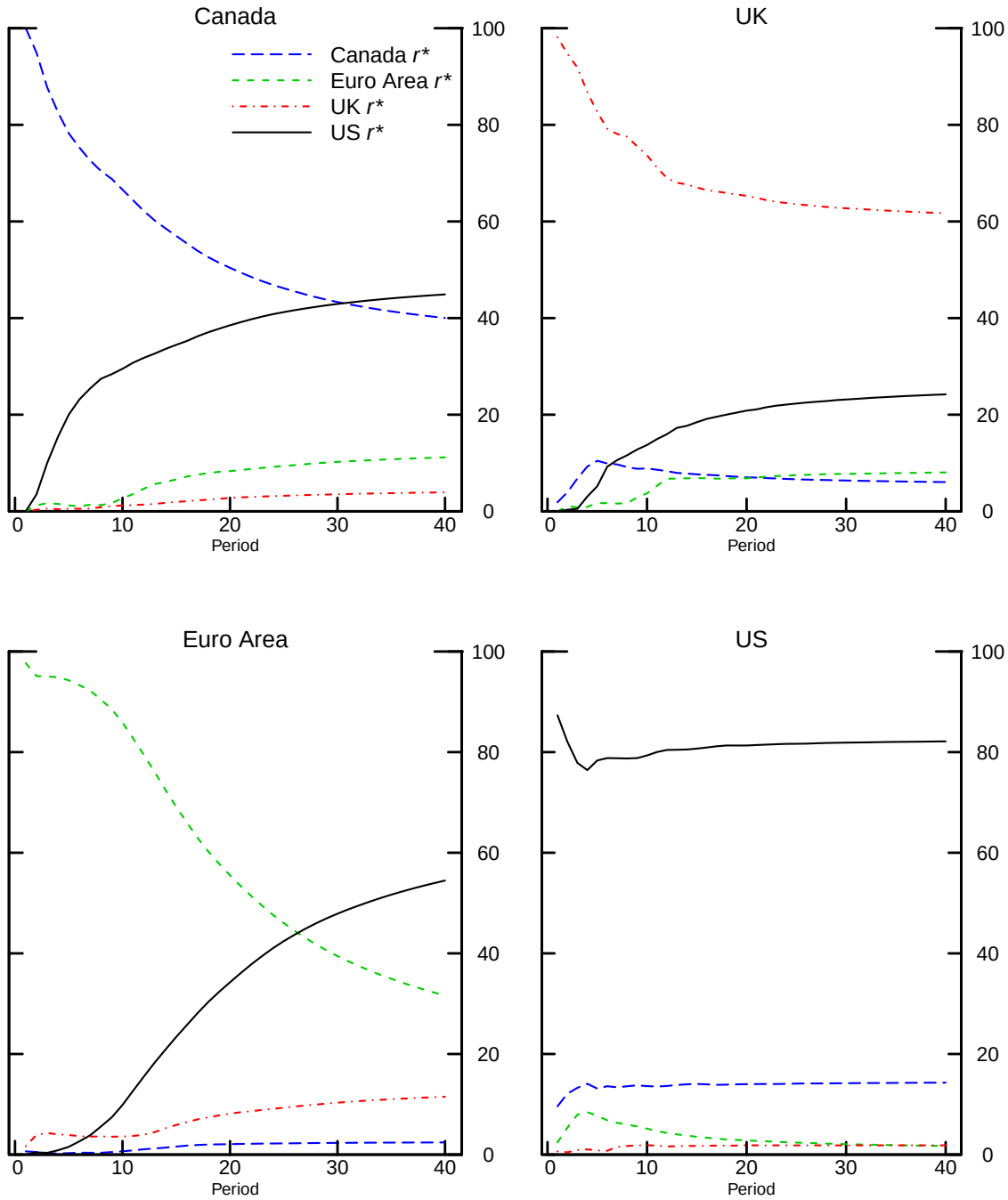
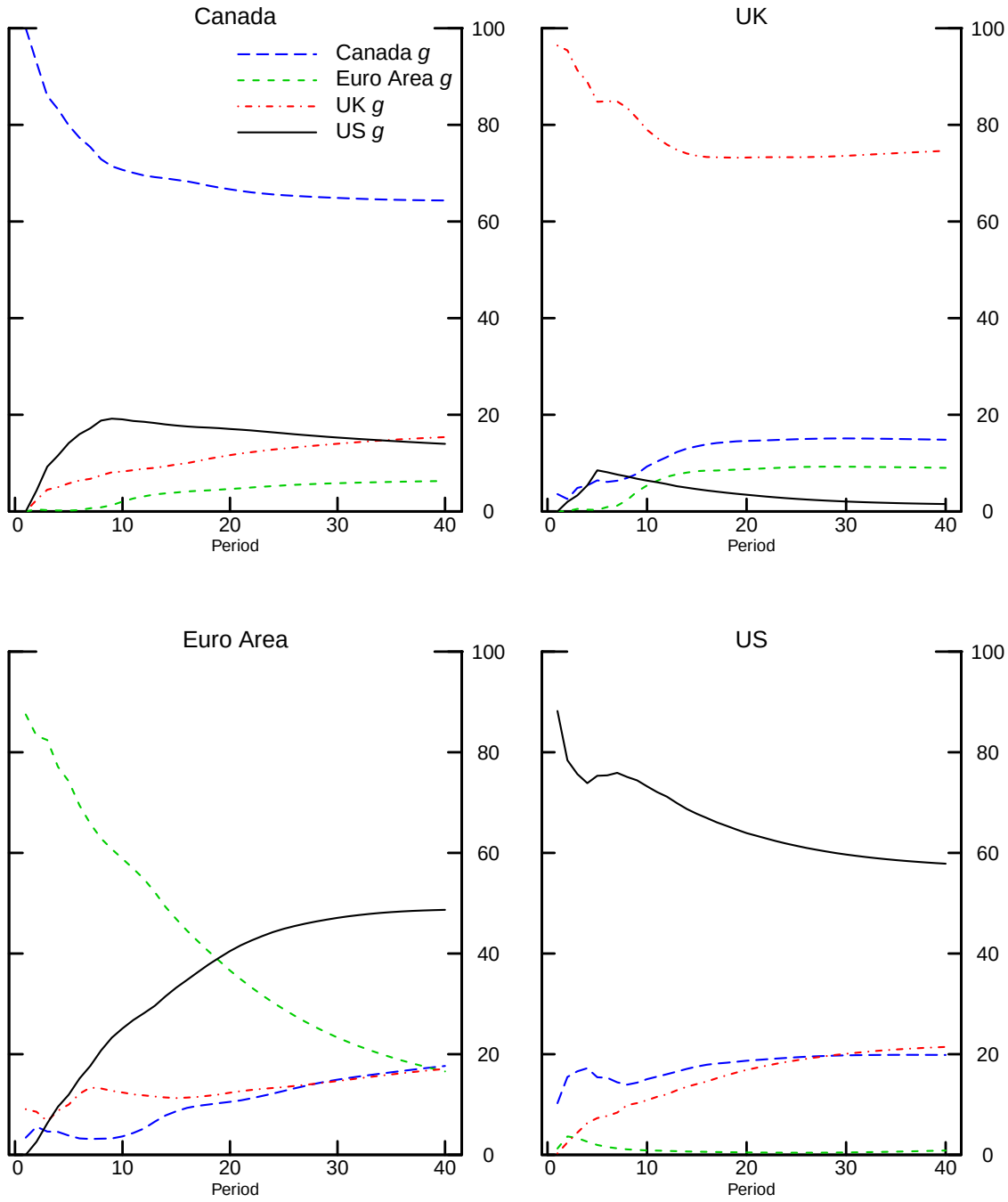


Figure 6: Trend Growth Estimates Variance Decompositions



Appendix: Data

In this appendix, we describe the data used in this paper. For each country for which the model consisting of equations (4)-(9) is being estimated, we require data for real GDP, inflation, and the short-term nominal interest rate, as well as a procedure to compute inflation expectations to calculate the real short-term interest rate r_t.

For each country, the variable y refers to the log of real GDP. The inflation measure is the annualized quarterly growth rate of the specified consumer price series. With the exception of the United States, for which core personal consumption expenditure price data are available over the entire sample, the inflation series is constructed by splicing the core price index with an all-items price index. We use a four-quarter moving average of past inflation as a proxy for inflation expectations.

For the United States, our measure of inflation is constructed using the price index for personal consumption expenditures excluding food and energy, referred to as core PCE inflation. Real GDP and core PCE data are published by the Bureau of Economic Analysis. The federal funds rate is the annualized nominal funds rate, available from the Board of Governors, with the quarterly figure computed as the average of the monthly values. Because the federal funds rate frequently fell below the discount rate prior to 1965, we use the Federal Reserve Bank of New York's discount rate prior to 1965, part of the IMF's International Financial Statistics (IFS) Yearbooks.

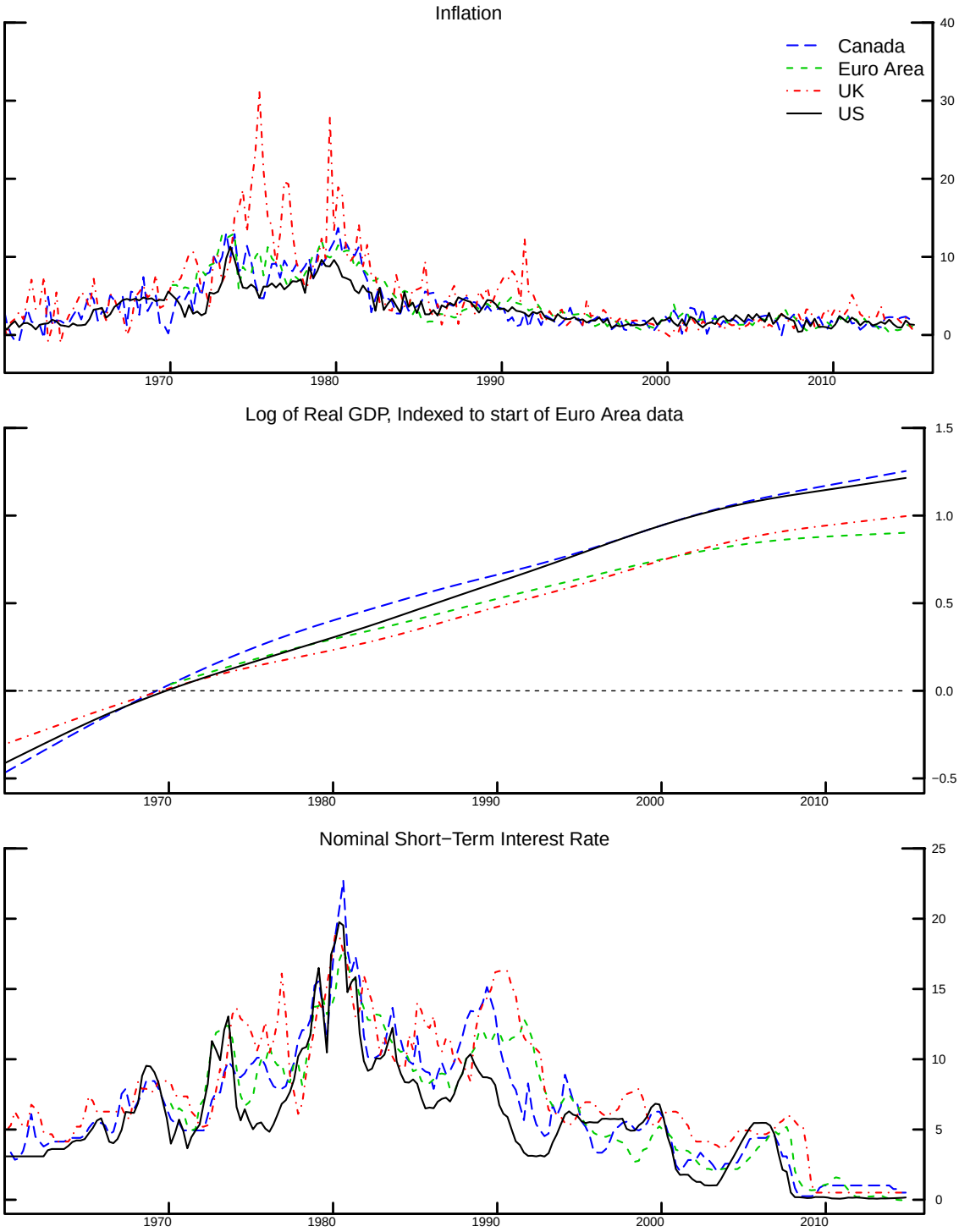
Canadian real GDP data is available from IFS. The short-term nominal interest rate is the Bank of Canada's target for the overnight rate, taken as the end-of-period value for each month and aggregated to quarterly frequency. Since the Bank of Canada began treating the target rate as its key interest rate in May 2001, we use the bank rate as the short-term interest rate prior to that date. We use the Bank of Canada's core Consumer Price Index to construct our inflation series. Prior to 1984, we use CPI containing all items. With the exception of real GDP, all data is from Statistics Canada.

UK GDP and inflation data are published by the Office of National Statistics. The inflation series is the CPI excluding energy and unprocessed food beginning in 1996, and all items CPI prior to 1996. The short-term nominal interest rate is the Bank of England's Official Lending Rate, published by the Bank of England.

Euro Area data is from the Area Wide Model. The inflation measure is HICP excluding energy; prior to 1988 we use overall HICP. The nominal short-term interest rate is the

three-month rate.

Figure 7: Comparison of Data used across Economies



Appendix: Comparison to Laubach and Williams (2003)

In this appendix, we compare the estimates for the United States reported in this paper with real-time estimates generated using the Laubach-Williams (2003) model, which are reported on the Federal Reserve Bank of San Francisco's website. Figure 8 displays the filtered (one-sided) estimates of the output gap, trend growth of potential output, and the natural rate of interest, as well as the real rate gap generated using the ex-ante real short-term interest rate and the filtered natural rate estimates.

As shown in the upper panel, estimates of the output gap and real rate gap in this paper largely resemble estimates from the LW (2003) model. In general, the trend growth and natural rate estimates exhibit similar movements across both models, with the recent decline in the natural rate occurring simultaneously across both sets of estimates. More of the recent decline in the natural rate is accounted for by a decline in trend growth in the estimates from this paper than in estimates from the Laubach-Williams model.

One departure in this paper from the Laubach-Williams (2003) methodology is the method for calculating a measure of inflation expectations, which we use to determine the real short-term interest rate as described in the Data Appendix. While the Laubach-Williams (2003) model uses a univariate AR(3) of inflation estimated over the past 40 quarters to generate a forecast of four-quarter-ahead inflation, in this paper we proxy inflation expectations using an average of inflation over the prior four quarters. This change is driven in part by data constraints; for the Euro Area, the estimation sample could not start prior to 1983q1 if the former method were used. In Figure 9, we compare the two measures of inflation expectations and find that they are close for the US.

Table 5: Comparison of HLW and LW (2003) Parameter Estimates

Parameter	HLW: US	LW (2003): Current	LW (2003): Vintage
Sample	1961q1-2015q4	1961q1-2015q4	1961q1-2002q2
λ_g	0.052	0.017	0.042
λ_z	0.030	0.044	0.058
$\sum a_y$	0.943	0.956	0.945
a_r	−0.071	−0.058	−0.098
	(4.002)	(3.551)	(3.81)
b_y	0.075	0.039	0.043
	(3.045)	(1.973)	(1.63)
c	1.000	1.340	1.068
$\sigma_{\tilde{y}}$	0.354	0.361	0.387
σ_π	0.792	0.768	0.731
σ_{y^*}	0.581	0.598	0.605
σ_g	0.120	0.041	0.102
σ_z	0.151	0.270	0.323
$\sigma_{r^*} = \sqrt{c^2\sigma_g^2 + \sigma_z^2}$	0.193	0.273	0.340
S.E. (sample ave.)			
r^*	1.086	1.415	1.88
g	0.401	0.230	0.48
y^*	1.549	1.701	3.02
S.E. (final obs.)			
r^*	1.547	1.745	2.61
g	0.546	0.298	0.63
y^*	2.052	2.224	4.23

Figure 8: Comparison of HLW and LW (2003) Estimates

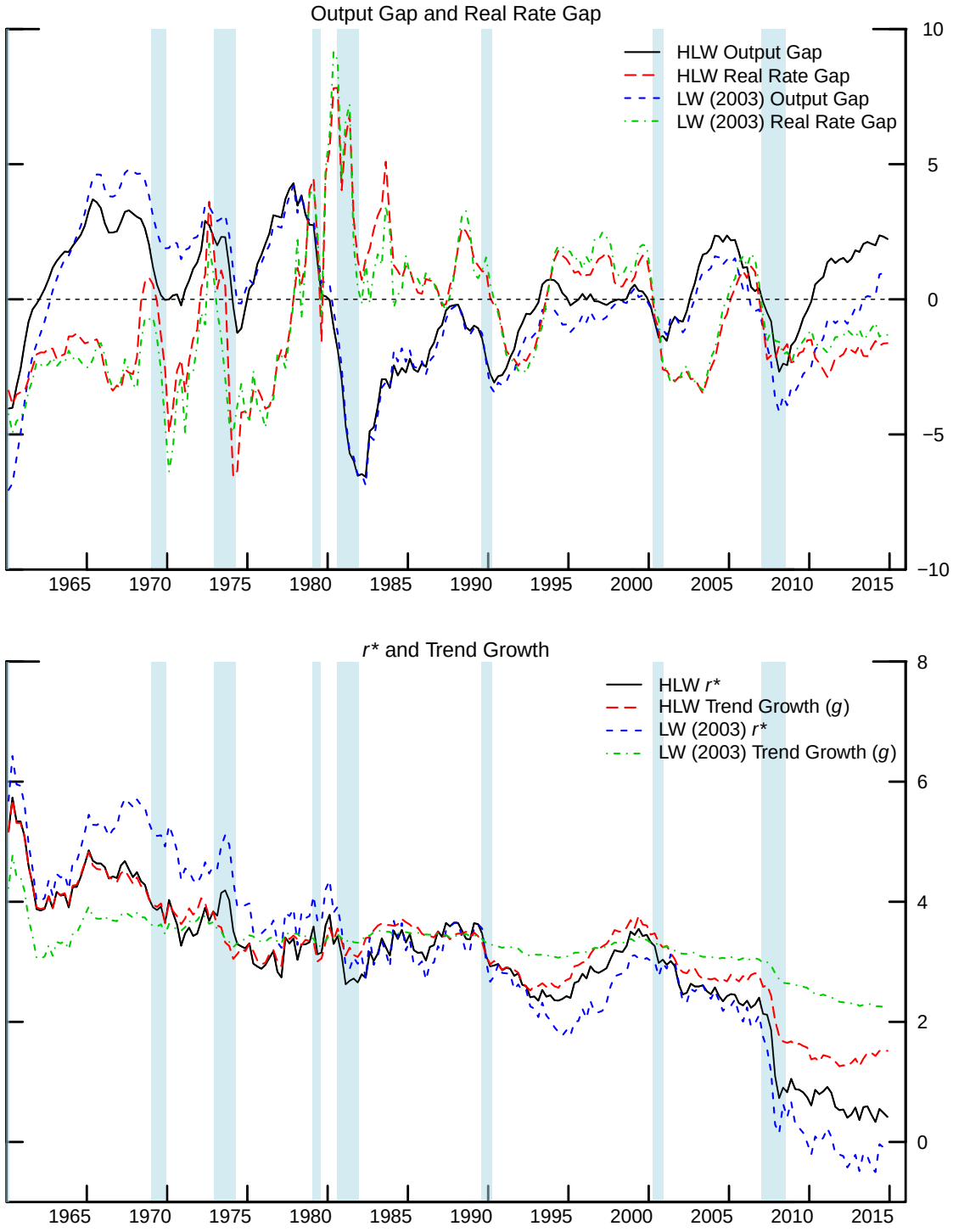


Figure 9: Comparison of HLW and LW (2003) Measures of Inflation Expectations

