The Power of Forward Guidance with Bounded Rationality and Incomplete Markets

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This paper considers a setting with bounded rational consumers, who form expectations according to level-k thinking. We explores the implications for the response of output to interest rates and, in particular, the power forward guidance—defined as the sensitivity of spending to future interest rates relative to current interest rates. We show that departures from rational expectations qualitatively moderate the power forward guidance. However, the quantitative effects are quite modest. These effects are larger when bounded rationality is combined with incomplete markets, even in cases where incomplete markets have no effect on forward guidance under rational expectations. We conclude that the interaction of bounded rationality and market frictions is important, even if each element has modest effects. This suggests a more general point: that departures from benchmark models should not just be evaluated in isolation.

1 Introduction

Expectations about the future play a key role in macroeconomic phenomena in the real world. Households need to predict future prices, including interest rates, as well as the state of the economy, including the level of activity which may affect their future income and employment. This importance is also reflected in our models. It is now commonplace to discipline the formation of expectations by imposing rational expectations, expectations that are consistent with the predictions of the model. This, of course, strains belief, but no other obvious criterion has been put forth. In our view, it is nevertheless important to explore deviations from rational expectations.

One problem with rational expectations is that it may require agents to solve complicated fixed point problems. Indeed, the way we solve our models is the same way we may expect agents to come to their expectations. An alternative interpretation is that agents learn from experience, from repeatedly seeing the same situation, even if they do not understand how this situation comes about (i.e. they do not know the "model"). While this notion may be a good approximation in a number of scenarios, common situations that have been observed repeatedly, it may be a terrible one in other settings, such as uncommon situations or in the face of new policy interventions.

Policymakers may attempt to lessen the burden of such a monumental task by announcing their intended policies or even committing to a certain plan or contingent plan. For example, in the monetary policy arena, forward guidance has emerged as an important component of monetary policy actions. Central Banks discuss not just their current policy interventions but also their future ones. This may be especially important when current policy levers are constrained, for example, if interest rates are zero or near zero, leaving little or no room for further reductions.

When future policy is laid out in this way and is sufficiently well communicated, this may help agents anticipate some important variables, such as interest rates. Other variables, however, that are not under the direct control of the policymaker, remain important, such as the state of the future economy. Households still need to form expectations about their future income. Federal Reserve announcements do not make promises about these important outcomes.

In this paper we consider a specific deviation from rational expectations, introduced by Woodford (2013) and Garcia-Schmidt and Woodford (2015), and which we refer to as level-k thinking. We focus on the effects of monetary policy (changes in the future path of interest rates) in models with nominal rigidities. For simplicity, we consider an extreme form of nominal rigidity, in which prices are completely rigid. Then the only equilibrium variable households need to form expectations on, over and above the announced path of interest rates, is the level of output, which influences their income. Under level-k thinking, households go through iterated rounds of thoughts to predict future output. In the first round, they take the path of output to be fixed at the level that prevailed before the new path of interest rates. Their consumption decisions then only reflect the partial equilibrium effects of changes in interest rates. Equilibrium output is equal to consumption and changes in responses to the associated changes in consumption. In the second round, households take the path of future output to be the equilibrium path of output that obtains in the first round, etc. ad infinitum. The solutions converges to the rational expectations solution when the number of rounds goes to infinity. This process interpolates smoothly between a first round where the output effects monetary policy only reflect partial equilibrium effects, and rational expectations where the output effects of monetary policy also fully incorporates general equilibrium effects.

This specific form of bounded rationality introduces both mitigation—the output effects of interest rate changes—and horizon effects—the output effects of interest rate changes are more mitigated, the more in the future these interest rate changes take place (horizon). This provides a possible rationalization of the "forward-guidance puzzle", a term which refers to the extreme effectiveness of forward guidance (interest rate changes in the future) in standard rational expectations New-Keynesian models compared to its apparently more limited effectiveness in the data. However, these mitigation and horizon effects introduced by bounded rationality per se are relatively modest.

But we show that the mitigation and horizon effects introduced by bounded rationality become much stronger in the presence of incomplete markets, even in case where incomplete markets per se do not introduce any mitigation or horizon effect. We conclude that the interaction of bounded rationality and incomplete markets is important, even if each element has modest effects in isolation.

We unpack the separate contributions of the different features of incomplete markets:

binding borrowing constraints and precautionary savings with uninsurable idiosyncratic income risk. To analyze the former, we consider first in Section 2 a perpetual youth OLG model with annuities which we show can be reinterpreted as a model with occasionally binding borrowing constraints but without precautionary savings. To analyze the latter, we consider next in Section 3 a CARA economy with uninsurable idiosyncratic income risk but without borrowing constraints. These two models are solved in closed form, both under rational expectations with bounded rationality. Finally, we put these features together in Section 2 in an Aiyagari-Bewley economy with occasionally binding borrowing constraints, uninsurable idiosyncratic income risk, and precautionary savings. We show theoretically and quantiatively that a key determinant of the strength of the mitigation and horizon effects induced by bounded rationality is the frequency at which borrowing constraints bind. We also show that in the presence of positive-net-supply assets, the illiquidity of assets is an important separate determinant. Detailed intuitions for these results are provided in the paper.

2 Borrowing Constraints

We lay down an OLG perpetual youth model a la Blanchard-Yaari which we reinterpret as an economy where infinitely-lived agents are randomly hit by large permanent discount factor shocks and are subject to borrowing constraints. The horizon is infinite and time is continuous indexed by $t \geq 0$. There is unit mass of ex-ante identical atomistic agents indexed by $i \in [0, 1]$. We assume an extreme form of nominal rigidities with prices fixed in nominal terms and normalized to one. We allow for positive liquidity in the form of Lucas trees in positive-net-supply. This setup allows us to capture the impact of borrowing constraints but neutralizes precautionary savings because of the existence of annuity markets.

We consider two different expectations formation paradigms, rational expectations and a particular form of bounded rational expectations, level-k thinking. Rational expectations can be obtained as the limit of level-k thinking when the number of rounds of thoughts increases to infinity. We draw a stark contrast between the properties of monetary policy in the model under these two different expectations formation paradigms. Equivalently, we show that the properties of monetary policy depend critically on the sophistication, captured by the number of rounds of thoughts, of level-k agents. We first establish the rational expectations benchmark in Section 2.1, and we treat bounded rational expectations second in Section 2.2.

2.1 Rational Expectations

2.1.1 Setup

Preferences. Instantaneous utility over consumption is logarithmic. Agents are hit by idiosyncratic Poisson shocks with intensity π. The life of an agent i is divided into "periods" by the successive realizations n of his idiosyncratic Poisson process indexed by the stopping times τ_n^i with $\tau_0^i = 0$ by convention. Agent i has an important discount

factor$\beta < 1$ between the different periods, and an instantaneous discount rate ρ within each period. The lifetime utility of agent i is therefore

$$\mathbb{E}_0 \sum_{n=0}^{\infty} \beta^n \int_{\tau_n^i}^{\tau_{n+1}^i} \log(c_t^i) e^{-\rho t} dt.$$

Incomes, assets, and borrowing constraints. Real and nominal income in the economy is Y_t—both are equal since prices are fixed at one. A part $(1-\delta)Y_t$ of this income accrues in the form of labor income which is equally divided across agents. The remainder δY_t accrues in the form of dividends on a unit mass of identical Lucas trees with value V_t which are equally distributed to all agents at time 0.

The Lucas trees are the only assets in positive-net-supply in the economy. In addition, agents can sign actuarially faire annuity contracts with instantaneous premium π that allow them to insure against the realizations of their idiosyncratic Poisson shocks.

Importantly, we assume that agents cannot borrow against their future labor income or human wealth accruing in any future "period". In other words, for $\tau_n^i \leq t < \tau_{n+1}^i$, agent i cannot borrow against any future labor income or human wealth accruing after τ_{n+1}^i. We assume that the discount factor $\beta < 1$ is sufficiently low that agents are up against their borrowing constraints between two "periods", so that in equilibrium, agents always choose not to bring in any financial wealth from one "period" to the next and hence $a^i_{\tau_{n+1}^i} = 0$ for all $n \geq 0$ and $i \in [0,1]$, where a_t^i denotes the financial wealth of agent i at time t.

Budget constraints. Taking into account the behavior of agents across "periods" described above, the budget constraint of agent i can be written as

$$\frac{da_t^i}{dt} = (r_t + \pi)a_t^i + Y_t - c_t^i \quad \text{for} \quad \tau_n^i \leq t < \tau_{n+1}^i,$$

with

$$a_0^i = V_t, \quad \text{and} \quad a_{\tau_{n+1}^i} = 0,$$

where r_t is the nominal and real interest rate.

Individual consumption problem. The solution to the individual consumption problem is

$$c_t^i = (\rho + \pi)(a_t^i + H_t),$$

where H_t is the expected human wealth of agent i in the current "period", which is the same H_t for all agents because of our Poisson assumptions

$$H_t = \int_t^{\infty} e^{-\int_t^s (r_u + \pi) du} (1-\delta) Y_s ds.$$

No arbitrage pricing of Lucas trees. No arbitrage requires that Lucas trees be priced according to

$$V_t = \int_t^\infty e^{-\int_t^s r_u du} \delta Y_s ds.$$

Monetary policy. The path for the nominal interest $\{r_t\}_{t\geq 0}$ is determined by monetary policy at time 0. It coincides with the path for the real interest rate given our assumption of perfectly rigid prices.

2.1.2 Equilibrium

Equilibrium is defined equivalently either by goods clearing or by asset market clearing (the two are equivalent by Walras law)

$$\int_0^1 c_t^i di = Y_t,$$

$$\int_0^1 a_t^i di = V_t.$$

Equilibrium output satisfies

$$Y_t = (\rho + \pi)(V_t + H_t),$$

which we can rewrite as the following integral fixed-point equation

$$Y_t = (\rho + \pi) \int_t^\infty [\delta e^{-\int_t^s r_u du} + (1-\delta) e^{-\int_t^s (r_u + \pi) du}] Y_s ds.$$

As if familiar in New-Keynesian models, this equation has exactly one degree of indeterminacy, which we can index by long-run output $\lim_{t\to\infty} Y_t$. Given long-run output $\lim_{t\to\infty} Y_t = Y$, there is a unique solution to this integral fixed-point equation, and hence a unique equilibrium. In what follows, we always keep constant the long run level of output $\lim_{t\to\infty} Y_t = Y$.

2.1.3 Effects of Monetary Policy at Different Horizons

We now characterize the effects of monetary policy at different horizons.

One-time unexpected monetary policy shock at a steady state. We assume that the economy is initially in steady state with $Y_t = Y$. The corresponding interest rate is given by $r_t = r$ defined implicitly by the following equation

$$1 = (1-\delta)\frac{\rho + \pi}{r + \pi} + \delta \frac{\rho + \pi}{r}.$$

We then imagine that at time 0, a new path for the interest rate $\{r_t\}_{t\geq 0}$ is announced which is a small perturbation over the constant steady state interest rate path r, and we characterize the associated perturbation to equilibrium output Y_t over the constant steady state output path Y.

Comparative statics. We will later be interested in comparative statics with respect to four parameters: the amount of liquidity captured by δ, the amount of liquidity as captured by the parameter δ, the frequency of borrowing constraints as captured by the parameter π, and the instantaneous discount factor ρ. Changes in these parameters lead to changes in the steady state interest rate r, which will be reflected in our comparative static results.

It is important to understand what economic mechanisms are governed by these parameters. The amount of liquidity δ is simply the share of output that accrues as dividends on Lucas trees. The frequency of borrowing constraints π increases the Marginal Propensity to Consume (MPC) out of wealth $\rho + \pi$, as well as the amount of discounting $r + \pi$ which applies to future labor income or human wealth, but not the amount of discounting r which applies to future dividends or financial wealth.

In the important benchmark of no liquidity $\delta = 0$, the interest rate $r = \rho$ is independent of the frequency of borrowing constraints π. In that case, the frequency of borrowing constraints increases the amount of discounting $r + \pi$ which applies to human wealth—which in this case coincides with aggregate total wealth since aggregate financial wealth is zero—and the MPC out of wealth $\rho + \pi$ in ways that exactly offset each other. When $\delta > 0$ instead, this exact offsetting result disappears since the amount of discounting r that applies to financial wealth (future dividends) is unaffected by π, and as a result, r increases with π.

Localized cumulated interest rate changes. More precisely, we consider interest rate paths $\{r_t\}_{t\geq 0}$ is given by $r_t = r$ if $t \notin [\tau, \tau + d\tau]$ and $r_t = r + \Delta r_\tau \delta_\tau$ if $t = \tau$, where δ_τ is the Dirac function. This implies that $\int_0^t (r_u - r)du = 0$ for $t < \tau$ and $\int_0^t (r_u - r)du = \Delta r_\tau$ for $t > \tau$. This captures a very localized cumulated interest rate change of Δr_τ at date τ. We then characterize the derivative of the whole path of $\{Y_t\}_{t\geq 0}$ with respect to Δr_τ at $\Delta r_\tau = 0$, which we capture with the semi-elasticities $\epsilon_{\tau,t}$ of output at time t to an interest rate change at time τ:[1]

$$\epsilon_{\tau,t} = -\frac{1}{Y}\frac{dY_t}{d\Delta r_\tau}|_{\Delta r_\tau = 0}.$$

We can then compare these localized cumulated interest rate changes at different dates τ to characterize the effects of monetary policy at different horizons.

Forward-looking nature of aggregated model. Note that because that since the wealth distribution is the only endogenous state variable in the model, and since the model ag-

[1]Since any interest rate path can be decomposed into a combination of such purely localized interest changes at different dates, we will easily be able to compute derivatives for the whole of $\{Y_t\}_{t\geq 0}$ with respect to any interest rate path. For example consider the following smoother change in the interest rate $r_t = r + \Delta r e^{-\phi(t-t_0)}$ for $t \geq t_0$ and $r_t = r$ for $t < t_0$. We can recover the derivative of the whole path of $\{Y_t\}$ with respect to Δr at $\Delta r = 0$ using the localized cumulative responses at different dates:

$$-\frac{1}{Y}\frac{dY_t}{d\Delta r}|_{\Delta r = 0} = \int_{t_0}^{\infty} \Delta r e^{-\phi(\tau - t_0)}\epsilon_{t,\tau} d\tau.$$

gregates linearly because of logarithmic utility and of the presence of annuities, the aggregate equilibrium is entirely forward looking. This has a number of important implications. First, it implies that $\epsilon_{t,\tau}$ depends only on $\tau - t$. Second, it implies that past interest rate changes only affect the present and future wealth distribution but not the present and future aggregate equilibrium. As a result, for $t > \tau$, we always have $\epsilon_{\tau,t} = 0$. From now on, we focus on the case $t \leq \tau$.

Decomposition: PE and GE. We can decompose the effect of a localized change in interest rates at date τ on output at date $t \leq \tau$ into a general equilibrium effect and a partial equilibrium effect

$$dY_t = -(\rho + \pi) \int_\tau^\infty [\delta e^{-r(s-t)} + (1-\delta) e^{-(r+\pi)(s-t)}] Y ds \Delta r_\tau + (\rho + \pi) \int_t^\tau [\delta e^{-r(s-t)} + (1-\delta) e^{-(r+\pi)(s-t)}] dY_s ds,$$

with $dY_t = dY_s = -Y \Delta r_\tau$ for $t \leq s \leq \tau$. The first term on the right-hand-side corresponds to the Partial Equilibrium (PE) effect of a change in interest rates on consumption for an unchanged path of output—each agent i increases his consumption in response to a reduction in interest rates because the value of financial and human wealth both increase for an unchanged path of output because future labor income and future dividends accruing after τ are discounted at a lower rate. The second term on the right-hand-side captures the General Equilibrium (GE) effect of the change in output for a given interest rate—each agent i increases his consumption in response to a reduction in interest rates because the value of financial and human wealth both increase because future labor income and future dividends increase before τ.

Monetary policy at different horizons. This expression immediately implies that for $t \leq \tau$, the semi-elasticity of output at time t to an interest rate change at time τ is given by

$$\epsilon_{t,\tau} = 1.$$

We can decompose it into a PE effect and a GE effect

$$1 = \epsilon_{t,\tau} = \epsilon^{PE}_{t,\tau} + \epsilon^{GE}_{t,\tau},$$

where

$$\epsilon^{PE}_{t,\tau} = -\frac{1}{Y} \frac{dY_t}{d\Delta r_\tau} |^{PE}_{\Delta r_\tau = 0} = (1-\delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau - t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau - t)},$$

$$\epsilon^{GE}_{t,\tau} = -\frac{1}{Y} \frac{dY_t}{d\Delta r_\tau} |^{GE}_{\Delta r_\tau = 0} = (1-\delta) \frac{\rho + \pi}{r + \pi} [1 - e^{-(r+\pi)(\tau - t)}] + \delta \frac{\rho + \pi}{r} [1 - e^{-r(\tau - t)}].$$

The PE semi-elasticity $\epsilon^{PE}_{t,\tau}$ and the GE semi-elasticity $\epsilon^{GE}_{t,\tau}$ always sum up to the total semi-elasticity $\epsilon_{t,\tau} = 1$, but the relative importance of these effects depends on a number of parameters.

2.1.4 Independence w.r.t. Horizon and Borrowing Constraints

Therefore, the effects of future changes in interest rates on current output Y_t at time t are completely *independent* of the horizon $\tau \geq t$ at which they take place when agents have rational expectations and are characterized by a unitary semi-elasticity of output to future interest rate changes.[2] We refer to the independence of the effects of monetary policy on its horizon as a lack of *horizon* effect. This is a version of the "forward guidance puzzle", which refers to the extreme effectiveness of forward guidance (interest rate changes in the future) in standard New-Keynesian models compared to its apparently more limited effectiveness in the data. The effects of future changes in interest rates on current output Y_t at time t are also completely *independent* of the amount of liquidity δ, of the frequency of borrowing constraints π, and of the instantaneous discount factor ρ. This sets up a very stark rational expectations *irrelevance* benchmark, against which we will compare our level-k expectations results.

To understand these results, it it useful to go back to the decomposition into a PE effect and a GE effect. For example, the lack of horizon effect

$$\frac{\partial \epsilon_{t,\tau}}{\partial \tau} = 0$$

can be understood as follows. The PE effect does feature a horizon effect so that $\epsilon^{PE}_{t,\tau}$ is decreasing with the horizon $\tau - t$ with

$$\frac{\partial \epsilon^{PE}_{t,\tau}}{\partial \tau} = -(\rho + \pi)[(1-\delta)e^{-(r+\pi)(\tau - t)} + \delta e^{-r(\tau - t)}] < 0.$$

This is because for a given path of output, a cut in interest rates is more discounted, and hence leads to a smaller partial equilibrium consumption increase, the further into the future the interest rate cut takes place. But the GE effect features an exactly offsetting anti-horizon effect so that $\epsilon^{GE}_{t,\tau}$ increases with the horizon $\tau - t$ with

$$\frac{\partial \epsilon^{GE}_{t,\tau}}{\partial \tau} = -\frac{\partial \epsilon^{PE}_{t,\tau}}{\partial \tau} => 0.$$

This is because in general equilibrium, output increases for a longer time (until the horizon of the interest rate cut), leading to a higher increase in human wealth, the further into the future the interest rate cut takes place, and hence leads to a larger consumption increase.

Similarly, the independence of the effects of monetary policy on the frequency π of borrowing constraints,

$$\frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0$$

which is a version of the irrelevance-of-incomplete-markets result in Werning (2015), can be understood as follows. We focus on the case where there is no liquidity $\delta = 0$ where

[2]The fact that the interest semi-elasticity is exactly equal to one relies because of our assumption of logarithmic utility.

the intuition is most transparent because in this case $r = \rho$ is independent of π. The PE effect is weaker the higher is π so that

$$\frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} = -(\tau - t)e^{-(\rho+\pi)(\tau-t)} < 0.$$

This is because for a given path of output, a higher frequency π of borrowing constraints leads to more discounting of future interest rate cuts, and hence to a lower response of partial equilibrium consumption to a future interest rate cut. But the GE effect is stronger, the higher is π, in a completely offsetting way

$$\frac{\partial \epsilon_{t,\tau}^{GE}}{\partial \pi} = -\frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} > 0.$$

This is because the MPC $\rho + \pi$ and hence the general equilibrium Keynesian multiplier increase with the frequency π of borrowing constraints. Note that this property holds despite the existence of a countervailing effect that arises because the increase in human wealth associated with the general equilibrium increase in output is lower when π is higher because human wealth is more discounted.

It is also interesting to unpack why the lack of horizon effect is independent of the frequency π of borrowing constraints

$$\frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} = 0.$$

We focus again on the case with no liquidity $\delta = 0$ for simplicity. The horizon effect of the PE effect is then stronger, the higher is π so that

$$\frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} = -[1 - (\rho + \pi)(\tau - t)]e^{-(\rho+\pi)(\tau-t)} < 0 \quad \text{as long as} \quad (\rho + \pi)(\tau - t) < 1.$$

This is because for given path of output, a higher frequency π of borrowing constraints leads to a faster increase of the discounting of interest rate cuts with their horizon. The fact that this property holds only for $(\rho + \pi)(\tau - t) < 1$ is due to a countervailing effect through an increase in the MPC $\rho + \pi$, which becomes dominant when the horizon $\tau - t$ is large enough. But the anti-horizon effect of the GE effect is weaker, the higher is π so that

$$\frac{\partial^2 \epsilon_{t,\tau}^{GE}}{\partial \pi \partial \tau} = -\frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} > 0 \quad \text{as long as} \quad (\rho + \pi)(\tau - t) < 1.$$

This is because the increase in the MPC $\rho + \pi$ with the frequency π of borrowing constraints multiplies the general equilibrium increase in human wealth responsible for the increase in consumption is larger, the further into the future the interest rate cut takes place. The fact that this property holds only for $(\rho + \pi)(\tau - t) < 1$ is due to a countervailing effect through higher discounting of human wealth, which becomes dominant when the horizon $\tau - t$ is large enough.

2.2 Bounded Rational Expectations: Level-k Thinking

We now depart from rational expectations and assume instead that agents form bounded rational expectations according to level-k thinking as proposed by Woodford (2013) and Garcia-Schmidt and Woodford (2015).

2.2.1 Equilibrium: level-k recursion

Specifically, we assume that when responding to a change in the path of interest rate $\{r_t\}_{t\geq 0}$ announced at time 0, agents go through the following mental rounds of thought.

Define the path of output in round 0 to be constant at the steady state $Y_t^0 = Y$. In round $n \geq 1$, at every date t, agents expect the future path of output for $s \geq t$ to be given by the path of output Y_s^{n-1} at round $n-1 = Y$ for $s \geq t$, and form their time t consumption and asset holding decisions accordingly. Agent i's consumption rule is then given by

$$c_t^{i,n} = (\rho + \pi)(a_t^{i,n} + H_t^n),$$

where

$$H_t^n = \int_t^\infty e^{-\int_t^s (r_u + \pi)du}(1-\delta)Y_s^{n-1}ds,$$

and the no arbitrage condition is given by

$$V_t^n = \int_t^\infty e^{-\int_t^s r_u du}(1-\delta)Y_s^{n-1}ds.$$

The equilibrium path of output in round n is defined defined equivalently either by goods clearing or by asset market clearing (the two are equivalent by Walras law)

$$\int_0^1 c_t^{i,n}di = Y_t^n,$$

$$\int_0^1 a_t^{i,n}di = V_t^n.$$

The solution is

$$Y_t^n = (\rho + \pi)(V_t^n + H_t^n).$$

This leads to the following recursion equation for equilibrium output

$$Y_t^n = (\rho + \pi)\int_t^\infty [\delta e^{-\int_t^s r_u du} + (1-\delta)e^{-\int_t^s (r_u+\pi)du}]Y_s^{n-1}ds,$$

with initial condition $Y_t^0 = Y$.

We can think of n as indexing the degree of rationality of agents. As n goes to infinity, the solution converges to the rational expectations solution.

With level-k thinking and by contrast with rational expectations, output at round n is not the solution of a fixed-point equation anymore, but instead the solution of a recursion over the number n of rounds of thinking. In particular, this implies that the one degree of indeterminacy that was present with rational expectations disappears with level-k thinking and instead becomes an initial condition in the recursion: For a given initial steady state Y at round 0, there is a unique equilibrium solution at round n.

2.2.2 Effects of Monetary Policy at Different Horizons

We now characterize the effects of cumulated localized change in interest rates at τ. We use the concept of the semi-elasticity of output at time t to an interest rate change at time τ at round n, defined by

$$\epsilon^n_{t,\tau} = -\frac{1}{Y}\frac{dY^n_t}{d\Delta r_\tau}|_{\Delta r_\tau = 0}.$$

Forward-looking nature of aggregated model. For the same reasons as in the rational expectations case, the aggregate equilibrium is entirely forward looking. This implies that $\epsilon^n_{t,\tau}$ depends only on $\tau - t$. This also implies that past interest rate changes only affect the present and future wealth distribution but not the present and future aggregate equilibrium. As a result, for $t > \tau$, we always have $\epsilon^n_{\tau,t} = 0$. From now on, we focus on the case $t \leq \tau$.

Decomposition: PE and GE. For $t \leq \tau$, we have

$$dY^n_t = -(\rho+\pi)\int_\tau^\infty [\delta e^{-\int_t^s r_u du} + (1-\delta)e^{-\int_t^s (r_u+\pi)du}]Y ds \Delta r_\tau + (\rho+\pi)\int_t^\tau [\delta e^{-\int_t^s r_u du} + (1-\delta)e^{-\int_t^s (r_u+\pi)du}]dY^{n-1}_s ds.$$

In a sense, the decomposition into a PE and a GE effect that we obtained with rational expectations still applies, but while the PE effect (the first term on the right-hand-side) is unchanged, the GE effect (the second term on the right-hand-side) now takes the form of a recursion on the level of thinking n instead of that of a fixed point.

The PE effect is independent of the level of thinking n. By contrast, the GE effect depends heavily on it: For $n = 1$, the GE effect is zero, in the limit $n \to \infty$, the GE effect converges to its rational expectations benchmark, and intermediate values of n interpolate smoothly and monotonically between these two extremes.

Monetary policy at different horizons. This immediately implies for $t \leq \tau$ the following expression for the semi-elasticity of output

$$\epsilon^n_{t,\tau} = \delta e^{-\rho(\tau-t)}\left[\sum_{k=1}^n \frac{\rho^{k-1}(\tau-t)^{k-1}}{(k-1)!}\right] + (1-\delta)e^{-(\rho+\pi)(\tau-t)}\left[\sum_{k=1}^n \frac{(\rho+\pi)^{k-1}(\tau-t)^{k-1}}{(k-1)!}\right].$$

We can again provide a decomposition in terms of a PE effect and a GE effect

$$\epsilon^n_{t,\tau} = \epsilon^{PE}_{t,\tau} + \epsilon^{n,GE}_{t,\tau},$$

where

$$\epsilon_{t,\tau}^{PE} = (1-\delta)\frac{\rho+\pi}{r+\pi}e^{-(r+\pi)(\tau-t)} + \delta\frac{\rho+\pi}{r}e^{-r(\tau-t)},$$

$$\epsilon_{t,\tau}^{n,GE} = (\rho+\pi)\int_t^\tau [\delta e^{-r(s-t)} + (1-\delta)e^{-(r+\pi)(s-t)}]\epsilon_{s,\tau}^{n-1}ds,$$

with the convention $\epsilon_{t,\tau}^0 = 0$. The PE semi-elasticity $\epsilon_{t,\tau}^{PE}$ is the same as in the rational expectations model, but the GE semi-elasticity is different and is now the solution of a functional recursion over the number n of rounds. We can now tease out the economic content of these formulas in a series of comparative statics results.

2.2.3 Mitigation, Horizon, and the Importance of Borrowing Constraints

These expressions have a number of interesting consequences, where from now on we focus on the interesting case $t \leq \tau$.

Convergence. The effects of monetary policy converge towards their rational expectations counterparts as the number of rounds n goes to infinity

$$\epsilon_{t,\tau}^{\infty} = \epsilon_{t,\tau} = 1,$$

where $\epsilon_{t,\tau}^{\infty} = \lim_{n\to\infty}\epsilon_{t,\tau}^n$. This convergence is entirely due to the convergence of the GE effect from $\epsilon_{t,\tau}^{1,GE} = 0$ to $\epsilon_{t,\tau}^{GE,\infty} = \epsilon_{t,\tau}^{GE} > 0$ since the PE effect $\epsilon_{t,\tau}^{PE}$ is independent of τ.

Mitigation. For any finite number or rounds n, the effects of monetary policy are mitigated compared to the rational expectations benchmark

$$\epsilon_{t,\tau}^n < \epsilon_{t,\tau} = 1.$$

the more so, the lower is n so that $\frac{\partial\epsilon_{t,\tau}^n}{\partial n} > 0$,where with a slight abuse of notation we write $\frac{\partial\epsilon_{t,\tau}^n}{\partial n}$ for $\epsilon_{t,\tau}^{n+1} - \epsilon_{t,\tau}^n$. This mitigation effect is entirely due to a mitigating of the GE effect $\epsilon_{t,\tau}^{GE,n} < \epsilon_{t,\tau}^{GE}$ since the PE effect is independent of τ.

Horizon. For any finite number of rounds n, there is a horizon effect of monetary policy

$$\frac{\partial\epsilon_{t,\tau}^n}{\partial\tau} = -\delta e^{-\rho(\tau-t)}\frac{\rho^n(\tau-t)^{n-1}}{(n-1)!} - (1-\delta)e^{-(\rho+\pi)(\tau-t)}\frac{(\rho+\pi)^n(\tau-t)^{n-1}}{(n-1)!} < 0,$$

an effect which disappears in the limit $n \to \infty$so that $\frac{\partial\epsilon_{t,\tau}^{\infty}}{\partial\tau} = \frac{\partial\epsilon_{t,\tau}}{\partial\tau} = 0$.[3]

[3]The effect is stronger, the lower is n so that $\frac{\partial^2\epsilon_{t,\tau}^n}{\partial n\partial\tau} > 0$ as long as $(\rho+\pi)(\tau-t) < n$.

Frequency of borrowing constraints, mitigation, and horizon. To characterize the effects of borrowing constraints we focus on the case with no liquidity $\delta = 0$ where $r = \rho$, which leads to very transparent formulas. For any finite number of rounds n, more frequent borrowing constraints lead to more mitigation of monetary policy

$$\frac{\partial \epsilon^n_{t,\tau}}{\partial \pi} = -e^{-(\rho+\pi)(\tau-t)} \frac{(\rho+\pi)^{n-1}(\tau-t)^n}{(n-1)!} < 0,$$

an effect which disappears in the limit $n \to \infty$where $\frac{\partial \epsilon^\infty_{t,\tau}}{\partial \pi} = \frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0$.[4] Hence, the mitigation of the effects of monetary policy with level-k thinking are stronger when the frequency of borrowing constraints is higher.

For any finite number of rounds n, more frequent borrowing constraints lead to a stronger horizon effect of monetary policy

$$\frac{\partial^2 \epsilon^n_{t,\tau}}{\partial \pi \partial \tau} = e^{-(\rho+\pi)(\tau-t)} \frac{(\rho+\pi)^{n-1}(\tau-t)^{n-1}}{(n-1)!} [(\rho+\pi)(\tau-t) - n] < 0,$$

for $(\rho+\pi)(\tau-t) < n$, an effect which disappears in the limit $n \to \infty$where $\frac{\partial^2 \epsilon^\infty_{t,\tau}}{\partial \pi \partial \tau} = \frac{\partial^2 \epsilon_{t,\tau}}{\partial \pi \partial \tau} = 0$.

Bounded rational level-k expectations vs. rational expectations. The results for a finite number of rounds n are in striking contrast to the rational expectations benchmark, which obtains in the limit $n \to \infty$. Level-k thinking leads to a *mitigation* of the effects of monetary policy so that interest rate changes have less of an effect on output. Level-k thinking also leads to a *horizon* effect of monetary policy so that interest rate changes have less of an effect on output, the further in the future they take place. The mitigation and horizon effects that arise with level-k thinking are stronger, the more frequent are borrowing constraints. This illustrates a profound *interaction* between level-k thinking and incomplete markets.

There is a simple intuition for all these results in terms of the decomposition of the effects of monetary policy into PE and GE effects. As explained in Section 2.1.4, the PE effect features mitigation—the effect of interest rate changes is lower than the full effect under rational expectations because the latter is the sum of the GE and the PE effect. It also features horizon—for a fixed path of output, interest rate changes affect partial equilibrium consumption less, the further in the future they are. Under rational expectations, the GE effect eliminates the mitigation feature by adding to the PE effect, and eliminates the horizon effect because the GE effect features an anti-horizon effect. At round $n = 1$, monetary policy coincides with the PE effect and features mitigation and horizon, and in the rational expectations limit $n \to \infty$, the mitigation and horizon effects disappear. Intermediate values of n interpolate smoothly and monotonically between these two extremes.

We can also understand the effects of the frequency π of which borrowing constraints. Indeed, as already explained in Section 2.1.4, the horizon and mitigation effects of the

[4]The effect is stronger, the lower is n so that $\frac{\partial^2 \epsilon^n_{t,\tau}}{\partial n \partial \pi} > 0$ as long as $(\rho+\pi)(\tau-t) < n$.

PE effect are stronger, the higher is π because of higher discounting of human wealth. Under rational expectations, the GE effect offsets this dependence on π because the MPC $\rho + \pi$ and hence the Keynesian multiplier increase with π. At round $n = 1$, monetary policy coincides with the PE effect and the horizon and mitigation features are stronger, the higher is π. In the rational expectations limit $n \to \infty$, the dependence of the mitigation and horizon features on π disappear. Intermediate values of n interpolate smoothly and monotonically between these two extremes.

2.2.4 Asset Illiquidity

We now focus on the liquidity of assets. An extreme way of capturing asset illiquidity is to assume that only a fraction χ of the Lucas trees can actively be traded. The remaining fraction $1 - \chi$ cannot be traded. Then from an aggregate perspective, an increase in asset illiquidity $1 - \chi$ acts exactly like a reduction in δ.

And for any finite number of rounds n, and for any positive frequency of borrowing constraints $\pi > 0$, the effects of monetary policy are less mitigated where liquidity δ is higher

$$\frac{\partial \epsilon^n_{t,\tau}}{\partial \delta} = e^{-\rho(\tau-t)} \left[\sum_{k=1}^{n} \frac{\rho^{k-1}(\tau-t)^{k-1}}{(k-1)!} \right] - e^{-(\rho+\pi)(\tau-t)} \left[\sum_{k=1}^{n} \frac{(\rho+\pi)^{k-1}(\tau-t)^{k-1}}{(k-1)!} \right] > 0,$$

and in addition

$$\frac{\partial^2 \epsilon^n_{t,\tau}}{\partial \delta \partial \tau} < 0.$$

These effects are stronger, the lower is n, and disappear in the limit $n \to \infty$ where $\frac{\partial \epsilon^\infty_{t,\tau}}{\partial \delta} = \frac{\partial \epsilon_{t,\tau}}{\partial \delta} = 0$. This demonstrates that asset market illiquidity interacts with bounded rationality to deliver stronger mitigation and horizon effects.

3 Precautionary Savings

We lay down a model where a unit-mass continuum of ex-ante identical agents indexed by $i \in [0, 1]$ face i.i.d. idiosyncratic income shocks, can only borrow and save in a risk-free asset, without borrowing constraints, and have Constant Absolute Risk Aversion (CARA). Time is discrete indexed by $t \geq 0$ and the horizon is infinite. We allow for positive liquidity in the form of a positive-net-supply of Lucas trees. We assume an extreme form of nominal rigidities with prices fixed in nominal terms and normalized to one. This setup allows us to capture precautionary savings but neutralizes borrowing constraints.

Once again, we consider two different expectations formation paradigms, rational expectations and a particular form of bounded rational expectations, level-k thinking We contrast the properties of monetary policy in the model under these two different expectations formation paradigms. We first establish the rational expectations benchmark in Section 3.1, and we treat bounded rational expectations second in Section 3.2.

3.1 Rational Expectations

3.1.1 Setup

Preferences. Agents value streams of consumption according to

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^i) \right],$$

where $u(c) = -e^{-\gamma c}$ is a CARA utility with absolute risk aversion $\gamma > 0$ and where $\beta < 1$ is the discount factor.

Incomes, assets, and idiosyncratic income shocks. Real and nominal income in the economy is Y_t—both are equal since prices are fixed at one. A part $(1-\delta)Y_t$ of this income accrues in the form of labor income which is equally divided across agents. The remainder δY_t accrues in the form of dividends on a unit mass of identical Lucas trees with value V_t which are equally distributed to all agents at time 0. The Lucas trees are the only assets in positive-net-supply in the economy. In addition, agents can borrow and save in the form of zero-net-supply short-term risk-free bonds with an interest rate R_t.

Agents are ex-ante identical and are subject to uninsurable i.i.d. idiosyncratic labor income shocks y_t^i so that their labor income is $(1-\delta)Y_t + y_t^i$. The labor income shocks y_t^i are drawn from a Normal distribution with mean 0 and variance σ^2.

Budget constraints. We denote by $\hat{a}_t^i$ the asset position of agents of agent i in period t. The budget constraints are

$$c_t^i + \frac{\hat{a}_{t+1}^i}{R_{t+1}} = (1-\delta)Y_t + y_t^i + \hat{a}_t^i,$$

with

$$\hat{a}_0^i = V_t.$$

We can rewrite this more conveniently using the asset position net of the per-agent supply of Lucas trees $a_t^i = \hat{a}_t^i - V_t$, by making use of the no-arbitrage pricing equation for Lucas trees derived below. We get

$$c_t^i + \frac{a_{t+1}^i}{R_{t+1}} = Y_t + y_t^i + a_t^i,$$

with

$$a_0^i = 0.$$

Individual consumption problem. The solution of the individual consumption problem is characterized by the budget constraints and the individual Euler equation

$$u'(c_t^i) = \mathbb{E}_t[\beta R_{t+1} u'(c_{t+1}^i)].$$

The solution can be represented with consumption policy functions

$$c_t^i = \alpha_t^0 + \alpha_t^1 (Y_t + y_t^i + a_t),$$

where α_t^0 and α_t^1 are defined recursively by

$$\frac{1}{\alpha_t^1} = 1 + \frac{1}{R_{t+1}} \frac{1}{\alpha_{t+1}^1},$$

$$\alpha_t^0 = \frac{\alpha_{t+1}^1 Y_{t+1} - (\alpha_{t+1}^1)^2 \frac{\gamma\sigma^2}{2} - \frac{1}{\gamma}\log(\beta R_{t+1})}{1 + \alpha_{t+1}^1 R_{t+1}} + \frac{\alpha_{t+1}^0}{1 + \alpha_{t+1}^1 R_{t+1}}.$$

Pricing of Lucas trees. No arbitrage requires that Lucas trees be priced according to

$$V_t = \delta Y_t + \frac{V_{t+1}}{R_{t+1}}.$$

Monetary policy. The path for the nominal interest $\{R_t\}_{t \geq 0}$ is determined by monetary policy at time 0. It coincides with the path for the real interest rate given our assumption of perfectly rigid prices.

3.1.2 Equilibrium

Equilibrium is defined equivalently either by goods market clearing

$$\int c_t^i di = Y_t,$$

or by asset market clearing

$$\int \hat{a}_t^i di = V_t,$$

which can be re-expressed using net positions as

$$\int a_t^i di = 0.$$

Equilibrium output satisfies the following fixed-point equation

$$Y_t = \frac{\alpha_t^0}{1 - \alpha_t^1},$$

where

$$\frac{1}{\alpha_t^1} = \sum_{s=t}^{\infty} \frac{1}{\Pi_{u=t+1}^s R_s},$$

$$\alpha_t^0 = \sum_{s=t+1}^{\infty} \frac{\alpha_s^1 Y_{t+1} - (\alpha_s^1)^2 \frac{\gamma\sigma^2}{2} - \frac{1}{\gamma}\log(\beta R_s)}{\Pi_{u=t+1}^s (1 + \alpha_u^1 R_u)}.$$

Hereα_t^1 is the MPC, $1 - \alpha_t^1$ is the traditional Keynesian multiplier, and α_t^0 is an endogenous aggregate demand shifter which depends on the future path of output.

3.1.3 Effects of Monetary Policy at Different Horizons

We now investigate the effects of monetary policy at different horizons.

One-time unexpected monetary policy shock at a steady state. We assume that the economy is initially in steady state with $Y_t = Y$. The corresponding interest rate is given by $R_t = R$ defined implicitly by the following equation

$$0 = (\frac{R-1}{R})^2 \frac{\gamma^2\sigma^2}{2} + \log(\beta R).$$

In this case, $\alpha_t^0 = \alpha^0$ and $\alpha_t^1 = \alpha^1$ with

$$\alpha^1 = \frac{R-1}{R},$$

$$\alpha^0 = \frac{Y}{R} - \frac{(\frac{R-1}{R})^2 \frac{\gamma\sigma^2}{2} + \frac{1}{\gamma}\log(\beta R)}{R-1}.$$

We then imagine that at time 0, a new path for the interest rate $\{R_t\}_{t\geq 0}$ is announced which is a small perturbation over the constant steady state interest rate path R, and we characterize the associated perturbation to equilibrium output Y_t over the constant steady state output path Y. Exactly as in Section 2, we will focus on the case of localized interest rate changes whereby only the interest rate in period τ is perturbed to $R_{\tau+1} = Re^{\Delta R_{\tau+1}}$ while the interest rates in all other periods are unchanged. We then characterize the derivative of the whole path of $\{Y_t\}_{t\geq 0}$ with respect to $\Delta R_{\tau+1}$ at $\Delta R_{\tau+1} = 0$, which we capture with the semi-elasticities $\epsilon_{t,\tau}$ of output in period t to an interest rate change in period τ

$$\epsilon_{\tau,t} = -\frac{1}{Y}\frac{dY_t}{d\Delta R_{\tau+1}}|_{\Delta R_{\tau+1}=0}.$$

We can then compare these localized cumulated interest rate changes at different dates τ to characterize the effects of monetary policy at different horizons.

Forward-looking nature of aggregated model. Exactly as in Section 2, since the wealth distribution is the only endogenous state variable in the model, and since the model aggregates linearly because of CARA utility, the aggregate equilibrium is entirely forward looking. This implies that $\epsilon_{t,\tau}$ depends only on $\tau - t$. This also implies that past interest rate changes only affect the present and future wealth distribution but not the present and future aggregate equilibrium. As a result, for $t > \tau$, we always have $\epsilon_{\tau,t} = 0$. From now on, we focus on the case $t \leq \tau$.

Comparative statics. We will later be interested in comparative statics with respect to three parameters: the standard deviation of idiosyncratic income shocks σ, the coefficient of absolute risk aversion γ, and the discount rate β. Changes in these parameters lead to changes in the steady state interest rate R, which will be reflected in our comparative static results.

3.1.4 Anti-Horizon with Precautionary Savings

In the absence of precautionary savings when $\sigma = 0$, we have that for $t \leq \tau$,

$$\epsilon_{\tau,t} = \frac{1}{\gamma RY}.$$

Hence monetary policy does not feature any horizon effect

$$\frac{\partial \epsilon_{\tau,t}}{\partial \tau} = 0,$$

exactly as in Section 2.1, and for similar reasons.

But in the presence of precautionary savings when $\sigma > 0$, monetary policy features an *anti-horizon* effect

$$\frac{\partial \epsilon_{\tau,t}}{\partial \tau} > 0,$$

an effect which is stronger, the higher the variance of idiosyncratic income shocks

$$\frac{\partial^2 \epsilon_{\tau,t}}{\partial \sigma \partial \tau} > 0.$$

The intuition for this anti-horizon effect of monetary policy in the presence of precautionary savings is the following. An interest rate cut at τ reduces the MPC at all date $t \leq \tau$. As a result, the contemporaneous impact of idiosyncratic income shocks on consumption at all dates $t \leq \tau$ is mitigated, reducing precautionary savings and increasing consumption at date $t-1$. The associated increase in output Y_{t-1} is then propagated to earlier periods $s < t-1$ through a mix of PE and GE effects exactly as discussed in Section 2. The cumulated effect at a given date t is the sum of all these effects of reductions in precautionary savings at dates $s \in [t+1, \tau]$. This explains why the increase in output is higher, the earlier the period t, and equivalently why the effect of an interest rate cut at date τ on output at date t increases with the horizon τ of the interest rate cut.

Figure 1 provides a numerical illustration of the effects of 1% interest rate cut at $\tau = 5$ on output at date t for a yearly calibration with $Y = 1$, $R = 1.04$ (and β chosen accordingly), $\gamma = 1$ (corresponding to a steady state coefficient of relative risk aversion of 1), and either $\sigma = 0$ (no idiosyncratic income shocks) or $\sigma = 3.3$. This is meant to be a very rough numerical illustration, and not by any means a serious calibration. The most controversial number is σ, because in practice, income shocks seem are very persistent while in our model, they are purely transitory. To calibrate $\sigma = 3.3$, we perform a crude adjustment and match the present value of the standard deviation of permanent income as estimated by Storelestten, Telmer, and Yaron (2004), and which we assume here coincides with the standard deviation of our transitory income shocks. The figure shows the absence of horizon effect with $\sigma = 0$, and the presence of a very weak anti-horizon effect with $\sigma = 3.3$. This rough calibration shows that for reasonable amounts of risk and risk aversion, the no-precautionary savings benchmark with no horizon effect is a good approximation.

3.2 Bounded Rational Expectations: Level-k Thinking

We now treat the case where agents form bounded rational expectations according to level-k thinking.

3.2.1 Equilibrium: level-k recursion

Define the path of output in round 0 to be constant at the steady state $Y_t^0 = Y$. In round $n \geq 1$, at every date t, agents expect the future path of output for $s \geq t$ to be given by the path of output Y_s^{n-1} at round $n-1 = Y$ for $s \geq t$, and form their time t consumption and asset holding decisions accordingly. Agent i's consumption rule can then shown to be given by

$$c_t^i = \alpha_t^{0,n} + \alpha_t^{1,n}(Y_t + y_t^i + a_t),$$

where $\alpha_t^{0,n}$ and $\alpha_t^{1,n}$ are given by

$$\alpha_t^{1,n} = \alpha_t^1,$$

$$\alpha_t^{0,n} = \sum_{s=t+1}^{\infty} \frac{\alpha_s^1 Y_{t+1}^{n-1} - (\alpha_s^1)^2 \frac{\gamma\sigma^2}{2} - \frac{1}{\gamma}\log(\beta R_s)}{\Pi_{u=t+1}^{s}(1+\alpha_u^1 R_u)},$$

and equilibrium output is given by

$$Y_t^n = \frac{\alpha_t^{0,n}}{1-\alpha_t^{1,n}}.$$

As n goes to infinity, the solution converges to the rational expectations solution.

3.2.2 Effects of Monetary Policy at Different Horizons

We now characterize the effects of cumulated localized change in interest rates at τ. We use again the concept of the semi-elasticity of output at time t to an interest rate change at time τ at round n, defined by

$$\epsilon_{t,\tau}^n = -\frac{1}{Y}\frac{dY_t^n}{d\Delta R_\tau}|_{\Delta R_\tau = 0}.$$

Forward-looking nature of aggregated model. For the same reasons as in the rational expectations case, the aggregate equilibrium is entirely forward looking. This implies that $\epsilon_{t,\tau}^n$ depends only on $\tau - t$. This also implies that past interest rate changes only affect the present and future wealth distribution but not the present and future aggregate equilibrium. As a result, for $t > \tau$, we always have $\epsilon_{\tau,t}^n = 0$. From now on, we focus on the case $t \leq \tau$.

3.2.3 Mitigation and Horizon with Precautionary Savings

We now have

$$\epsilon_{\tau,t}^n - \epsilon_{\tau,t} < 0,$$

so bounded rationality leads to mitigation (compared to the rational expectations solution). Moreover, we can show that

$$\frac{\partial \epsilon_{\tau,t}^n}{\partial \sigma} - \frac{\partial \epsilon_{\tau,t}}{\partial \sigma} < 0,$$

so that for a given n, there is more mitigation (compared to the rational expectations solution), the more precautionary savings there is. We also have

$$\frac{\partial^2 \epsilon^n_{\tau,t}}{\partial\sigma\partial\tau} - \frac{\partial^2 \epsilon_{\tau,t}}{\partial\sigma\partial\tau} < 0,$$

indicating that for a given n, there is more of a horizon effect (compared to the rational expectations solution), the more precautionary savings there is. Once again, as in Section 2, these effects indicate a strong interaction between precautionary savings and bounded rationality.

The intuition for the mitigation effect and the horizon effect can be traced back to the GE and PE decomposition discussed in Section 2. With $n = 1$, the total effect of monetary policy coincides with the PE effect. With $n = \infty$, it adds the full GE effect and coincides with the rational expectations solution. Intermediate values of n interpolate smoothly between these two extremes. This explains the mitigation effect. This also explains the horizon effect since the PE effect has a horizon effect for similar reasons to those discussed in Section 2. The presence of precautionary savings only strengthens this results.

Figure 2 provides a numerical illustration of the effects of 1% interest rate cut at $\tau = 5$ on output at date t using the same calibration as in Section 3.1.4 for $n \in \{1, 2, 3\}$. It shows that any given finite value of n, there is mitigation and horizon, in absolute terms or in comparison to the rational expectations solution. However, these effects are relatively weak, and for realistic amounts of risk, they are not very different from those that would arise in the absence of precautionary savings with $\sigma = 0$. For realistic amounts of risk, precautionary savings forces are much less powerful than potentially those arising from binding borrowing constraints discussed in Section 2.

4 Precautionary Savings and Borrowing Constraints in an Aiyagari-Bewley Economy

We have analyzed separately the interactions between bounded rationality and the main features of incomplete markets: borrowing constraints and (Section 2) and precautionary savings (Section 3) on the effects of monetary policy at different horizons. In this section, we put all these different features of incomplete markets together in an Aiyagari-Bewley economy with bounded rational agents and nominal rigidities, and we characterize the effects of monetary policy at different horizons. See figure 3 for an illustration.

Our simulations indicate in such economies, bounded rationality leads to significant mitigation and horizon effects of monetary policy. The strength of these effects seems to be determined mostly by a key sufficient statistic: the frequency at which borrowing constraints bind, in line with the intuitions from Section 2.

5 Conclusion

In this paper, we show that the interaction of bounded rationality and incomplete markets can strongly mitigate the effects of future interest rate changes, the more so, the further

in the future they take place. This potentially provides a powerful rationalization of the "forward guidance" puzzle. This suggests a more general point that behavioral and market frictions should be evaluated in conjunction and not just in isolation.

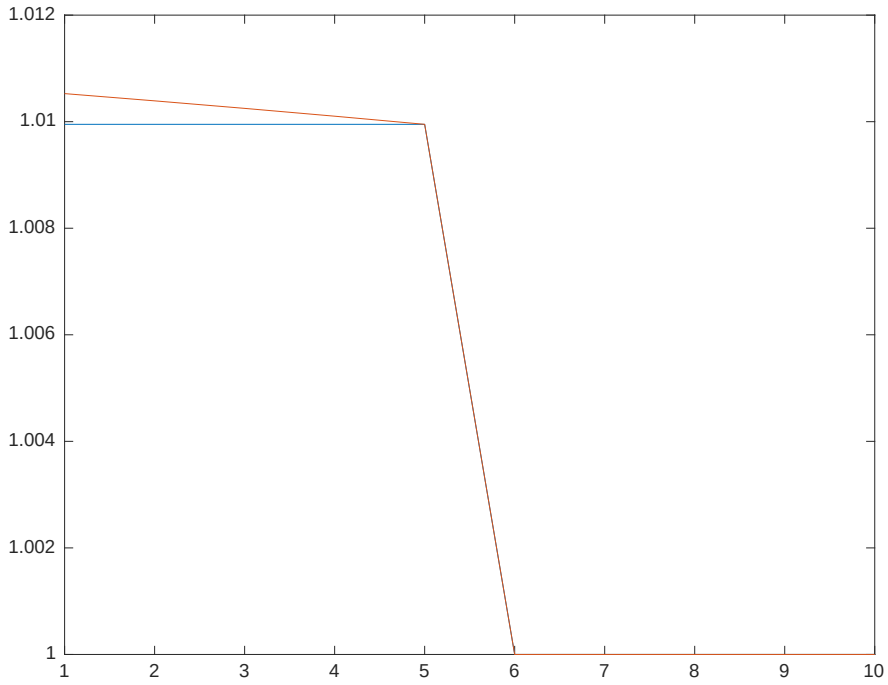


Figure 1: Output response to a 1% interest rate cut at date 5 under rational expectations in the precautionary savings model of Section 3. Yearly calibration with $Y = 1$, $R = 1.04$, $\gamma = 1$, and $\sigma = 0$ (blue line) and $\sigma = 3.3$ (red line).

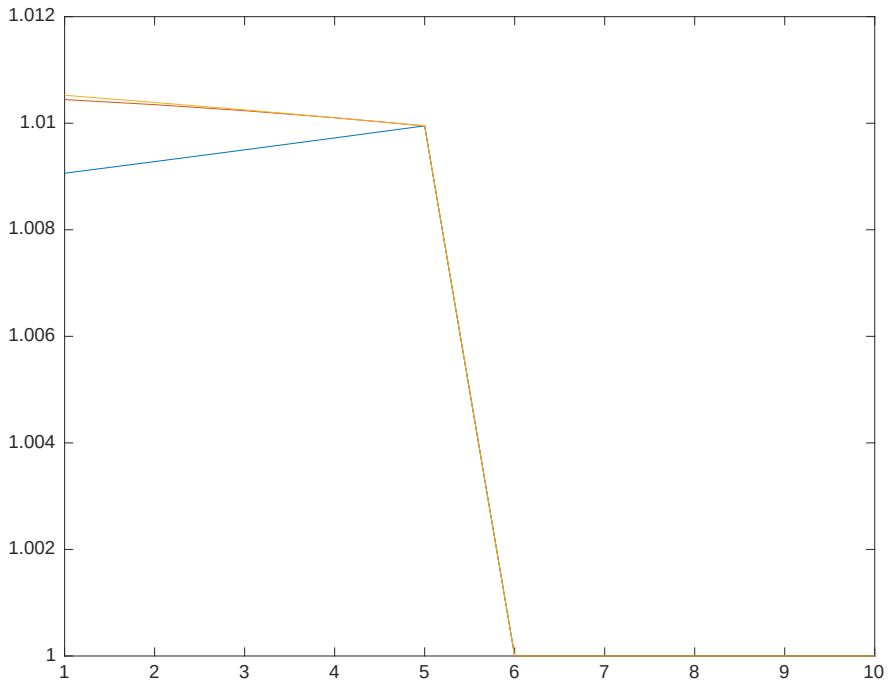


Figure 2: Output response to a 1% interest rate cut at date 5 under bounded rational expectations in the precautionary savings model of Section 3. Yearly calibration with $Y = 1$, $R = 1.04$, $\gamma = 1$, and $\sigma = 3.3$ for $n \in \{1, 2, 3\}$.

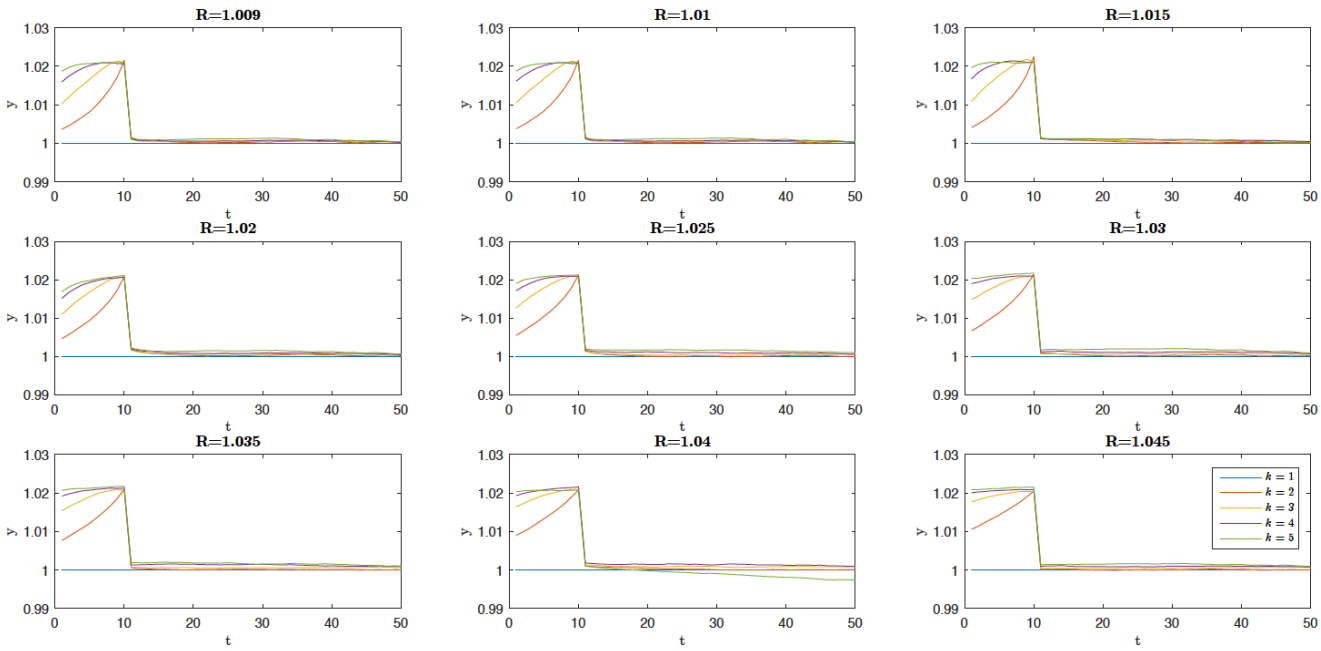


Figure 3: Output response to a 2% interest rate cut at date 10 under bounded rational expectations in the Aiyagari-Bewley model of Section 4. Different lines represent equilibrium output under level-k thinking with more and more rounds of thought.