Can Redistribution Lead to Economic Development?

Sang Yoon (Tim) Lee
Toulose School of Economics

Nicolas Roys
Federal Reserve Bank of St. Louis

Ananth Seshadri
University of Wisconsin-Madison

March 24, 2016

Abstract

How effective are redistributive policies in generating subsequent economic development? This paper presents a quantitative economic model to answer this question in the context of the twenteth century United States. Over the course of the last century, several redistributive policies were enacted. To evaluate the contribution of these policies, we present a model in which the main reason for a government to intervene is imperfection in credit markets to finance human capital. A parent cannot borrow against a child's income to invest in him or purchase insurance against his ability. These deviations from the fricitonless Neoclassical benchmark are considered in a model in which altruistic parents decide upon the number of children and how much to invest in them. Preliminary results suggest that redistributive policies have a large effect on the increase in GDP per capita between 1900 and 2000, accounting for roughly one quarter of the rise. The interaction between human capital and fertility choice plays a critical role. Subsidies to education have a large effect especially when parents are credit constrained. They respond to universal primary education by reducing fertility and increasing investments in children.

1 Introduction

Can redistribution lead to economic development? Theoretically, there are two related schools of thought. Standard Neoclassical theory with Arrow-Debreu securities essentially ignores any need for redistribution under the assumption that redistribution can be achieved independently from growth (Bourguignon, 2000). In contrast, development economists have long argued for redistribution as a means of accelerating growth Perotti (1993), Saint-Paul and Verdier (1993). Subsequent developments in neoclassical theory has incorporated incomplete markets, but since Aiyagari's (1991) study of the quantitative importance of market incompleteness, there is a widely held view that the welfare costs from completing missing markets are small and hence there is little room left for redistribution to be a central force behind economic development by completing missing markets. Furthermore, while some studies argue that the evidence suggests that inequality is harmful for growth others argue that inequality is essential for incentives (see Benabou 1996).

In this paper, we posit a rich model of intergenerational human capital accumulation with endogenous fertility to quantify the effect of redistribution on income per capita. We investigate whether redistribution can lead to significant gains in income per capita, and characterize under which circumstances initial inequality hampers subsequent growth. We also analyze how redistribution interacts with inequality—is redistribution more effective when inequality is low or when inequality is high?

There are staggering differences in income per capita across countries. The richest countries of the world produce and consume over 30 times the level achieved by the poorest nations in Africa. Economists are divided on what causes this large disparity across countries and which policies can improve the plight of poor countries. Should we focus on increasing the pie so there is more to share? Or can redistribution itself lead to large gains in output per person and lead poor societies out of poverty? These questions have plagued development economists as well as macroeconomists. While many development economists believe that redistribution had the potential to remove a society out of poverty, this view is by no means universally accepted. For instance, in describing the effect that the industrial revolution had, Robert Lucas ends his essay with the following:

"Of the tendencies that are harmful to sound economics, the most seductive, and in my opinion the most poisonous, is to focus on questions of distribution... But of the vast increase in the well-being of hundreds of millions of people that has occurred in the 200-year course of the industrial revolution to date, virtually none of it can be attributed to the direct redistribution of resources from rich to poor. The potential for improving the lives of poor people by finding different ways of distributing current production is nothing compared to the apparently limitless potential of increasing production."

Lucas (2004)

What leads Lucas to this conclusion is that despite the theoretical possibilities of redistribution leading to subsequent growth, there have been no successful attempts at demonstratring that such channels could have quantitative significant effects. Clearly one needs to deviate from a frictionless neoclassical framework in order to have the potential for redistribution to affect aggregate outcomes. While there are numerous possible deviations, the friction we turn our attention to is the inability of parents to invest efficiently in human capital. The reasons we do so are straightforward. First, it has long been recognized by both development economists as well as macroeconomists that human capital is a very important factor central to any discussion of the wealth of nations. Second, while it has been shown that individual borrowing constraints on physical capital assets have little effect on income per capita levels, this is because the borrowing constraints we can identify from the data are not severe enough to have quantitative effects. In contrast, the difficulty of using human capital as collateral in financial markets has been well recognized since Okun (1975), Loury (1981) and Becker and Tomes (1986). It seems natural, therefore, to investigate whether redistribution policies focusing on human capital can successfully generate large welfare gains, where redistribution of physical capital fails.

We apply our model to the United States over the 20th century, during which it emerged as a global economic powerhouse. There are multiple of reasons why the United States emerged as a leader, but a previously unexplored candidate is the emphasis it placed on human capital. At the turn of the 20th century during what some call the "Progressive

Era," there was an enormous redistribution from the rich to the poor in trying to get large segments of the population educated post-literacy. This was also a time during which the U.S. made huge strides in terms of technological progress, and there was a dramatic rise in secondary schooling enrollment rates. The crux of our analysis is to separately quantify how much redistributive policies and exogenous technological progress contribute to the increase in enrollment rates and economic growth.

Our paper is related to previous work on the relationship between growth and inequality with imperfect credit markets (see Banerjee and Newman (1991), Aghion and Bolton (1997) and Piketty (1997)), in particular those studies that specifically focus on human capital (see Benabou, 1996, Durlauf, 1996 and Galor and Zeira, 1993. However, these works were purely theoretical, while the only quantitative exploration we are aware of is Fernandez and Rogerson (1998) who study the quantitative impact of redistributing educational expenditures in a model calibrated to the U.S. Perhaps most closely related is Benabou (2002) who who studies tax and educational policies in a dynamic economy and concludes that the efficiency costs and benefits of redistribution are roughly of the same order of magnitude. We incorporate new developments in two dimensions. The first is an explicit mechanism for human capital accumulation at the individual level, which allows us to use micro-level data to discipline the model. The second is the incomplete markets literature that deals with the implications of imperfect insurance markets for savings behavior and wealth inequality (see Laitner, 1992; Huggett, 1993 and Aiyagari, 1994), which forms the backbone of our numerical solution.

The rest of the paper is organized as follows. Section 2 posits a general equilibrium model of human capital accumulation and adds to it a parental spillover. The model features multiple sources of heterogeneity and government policies. In section 3, we discuss how we use data on schooling and earnings across generations to calibrate our human capital parameters, while using aggregate data to calibrate macro parameters. Section 4 examines the impact of redistribution policies. Section 5 concludes.

2 The Model Economy

Our model economy consists of a continuum of individuals, a representative firm and a government. In addition to consumption-savings decisions, individuals also make fertility decisions and education investments in their children. The timing of fertility is exogenous, parents can only decide on the number of children they have.

We split the life of an individual into periods, where each period comprises 6 years. An individual lives for T periods beginning with period 0. During the first I periods of life, he is attached to his parent and does not make any decisions on his own. Instead, his parent makes consumption decisions for him, and decides how much to invest in his human capital. Furthermore, we assume that the longevity of the child is also endogenously determined by parental investments.

At the end of period I, he becomes independent and begins making his own decisions. He begins the period with a level of human capital and intergenerational asset transfers received from his parents. Human capital at period I is determined by an individual's innate ability, which is partially transmitted from his parent, his parent's human capital, and how much his parent invested in his human capital prior to period I. At period B, he gives birth to his own children, whose innate abilities are drawn from a distribution that depends on his own. He cares about his own utility as well as the discounted utility of his offspring. Retirement is exogenous at period R, and starting from period $R+1$ the individual faces stochastic death with certain death in period T. In summary, the life cycle is 10+ periods:

Period 0 (age 0-5):

Early childhood - different kind of investment, see above.

Period 1 (age 6-11):

Primary education (by parent)

Period 2 (age 12-17):

Secondary education (by child)

Period 3 (period I, age 18-23):

Independence and Ben-Porath

Period 4 (period B, age 24-29):

Childbirth - early childhood investments, Ben-Porath

Period 5: (age 30-35):

Child primary school investments, Ben-Porath

Period 6: (age 36-41):

Child secondary school investments (may also work), Ben-Porath

Period 7-10 (-period R, age 42-65):

Ben-Porath then retirement

Period 11-15 (-period T, age 66-95):

Retired and stochastic survival, with certain death at end of period $T = 15$ (age 95).

We assume a stationary environment, so the number of individuals of any given age is constant.

2.1 Human Capital Acquisition

Abilities are fixed when born and drawn according to the AR(1) process

$$\log z_k = (1 - \rho_z)\mu_z + \rho_z \log z + \eta, \qquad \eta \sim \mathcal{N}(0, \sigma_\eta^2).$$

where z and z_k are the abilities of the parent and child, respectively. Here, μ_z is the average level of abilities while ρ_z measures the persistence of innate abilities across generations, while η is an uninsurable shock—there is no market in which parents can fully insure themselves against the ability of their children prior to their birth. This is the only source of individual uncertainty in this economy. Since we assume a stationary environment, we assume that the initial distribution of z is the stationary distribution associated with the AR(1) process.

Let a index the age of an independent individual and a_k the child. The human capital of a child at the beginning of period 1 (age 6 human capital) is determined by

$$h_{1_k} = n_p h_B,$$

where n_p is parental time and h_B is the parent's human capital level at the time of childbirth. The introduction of parental human capital in the child's initial human capital is meant to parsimoniously capture some advantages high human capital parents may have when educating their children. First, it can mean that children of high human capital parents are literally endowed with a larger level of human capital, which is different from the transmission of learning abilities that determines the speed at which children learn. Another interpretation is that it reflects innate ability that is not genetically transmitted but rather augments learning during the formative early years of life.

The second interpretation, in turn, could be interpreted as capturing several effects as well. Since the human capital of the parent and childcare time enter multiplicatively, this formulation may also be thought as capturing "quality time" in the sense that a parent with higher human capital is better at augmenting the learning of his child. Also, a parent with higher human capital could provide a better learning environment for the child by living in better neighborhoods and surroundings which would also augment his child's human capital.

Given his human capital at period 1, we assume the human capital of the child at the beginning of period 2 is determined by

$$h_{2_k} = \left[\gamma_k^{\frac{1}{\sigma}}(m_{1_k} + d_1)^{\frac{\sigma-1}{\sigma}} + (1-\gamma_k)^{\frac{1}{\sigma}} h_{1_k}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

where m_{1_k} and d_1 is private and public goods investments in the child's human capital in primary school. Human capital at period 3 is determined by:

$$h_{3_k} = z_k \left(n_{2_k} h_{2_k}\right)^{\gamma_{s_1}} \left(m_{2_k} + d_2\right)^{\gamma_{s_2}} + h_{2_k}$$

where z_k is the innate ability of the child, n_{2_k} the child's time spent in secondary school, and m_{2_k}, d_2 are private and public good investments in secondary education.

Among other things, this formulation captures several desirable features of what we know about human capital formation prior to entering the labor market:

1. Complementarity between early and later investments (between period 0 and later periods) as measured by σ

2. Parental spillovers as measured by γ_p

3. Parental time investments are important in early period of life, goods investments more important later

4. Child's own time becomes more important as he matures

5. Government subsidies during primary and secondary schooling. Note that we ignore subsidies during early childhood (age 0-5). Although childcare subsidies have been rising as of recent, it was not a major public expenditure item during the period we focus on (early to mid-late 20th century).

Once independent, in the first stage of independence we allow college subsidies:

$$h_4 = z(n_3 h_3)^{\gamma_{c_1}} (m_3 + d_3)^{\gamma_{c_2}} + h_3$$

where z is own ability and n_3 time spent on human capital investment during period 3—this is meant to capture time spend in college. Similarly, m_3 and d_3 represent college expenses and tuition subsidies, if any. For the rest of life we assume that human capital evolves as Ben-Porath with only time investments:

$$h_{a+1} = z(n_a h_a)^{\gamma} + h_a.$$

2.2 Individual's problem

We write the problem recursively; there are only two nodes at which an individual has to make decisions:

1. Age I: upon becoming independent, an individual needs to make a fertility choice (that is realized at age B) in addition to consumption-savings-own human capital investment decisions.

2. Age B: an individual observes the realization of the abilities of his children. In addition to the standard consumption-savings decisions, he also makes human capital investments in both himself and his children, investments in his children's longevities, and decides on intergenerational transfers.

Additionally, starting from age $R+1$ (after retirement), the individual makes no more decisions but faces stochastic death. Note that we are ignoring that grandparents make decisions after observing their grandchildren.

Stage I problem Upon becoming independent, an age I (high school graduate age) individual solves (remember that $B = I + 1$)

$$V_I(s_I, h_I; z, L) = \max_{c_I, n_I, s_B, f} u(c_I) + \mathbb{E}_z V_B(s_B, h_B; z, L; z_k, f)$$

subject to the budget constraint

$$c_I + s_B = w h_I (1 - n_I) + m_I + (1 + r) s_I + \phi_I(\cdot),$$

where $\phi_a(\cdot)$ is a period-dependent government tax-transfer program applied at every period in life, to be specified below, and

- $u(\cdot)$: period utility
- L: longevity post-retirement
- h_I, z: human capital at period I, and innate ability
- c_I, s_I: consumption and transfers from parent
- n_I, m_I: time and goods investments in own human capital
- f, z_k: fertility choice and (unrealized) innate ability of children.

Stage B problem Upon bearing a child, the value of the new parent is determined by

$$\begin{aligned}
&V_B(s_B, h_B; z, L; z_k, f) \\
&= \max_{\substack{\{c_a, n_a, s_{a+1}\}_{a=B}^{R}, n_p, m_p, \\ \{c_{a_k}\}_{a_k=0}^{I-1}, s_{I_k}, n_k, l_k}} \sum_{a=B}^{R} \beta^{a-B} u(c_a) + \beta^{R-B+1} L \pi_{R+1} \cdot V_{R+1}(s_{R+1}, p; L) \\
&\quad + \theta(f) \cdot \left[\sum_{a_k=0}^{I-1} \beta^{a_k} u(c_{a_k}) + \beta^I V_I\left(s_{I_k}, h_{I_k}; z_k, L_k(l_k)\right) \right]
\end{aligned}$$

subject to the budget constraint

$$\begin{aligned}
&\sum_{a=B}^{R} \frac{c_a}{(1+r)^{a-B}} + f\left[\sum_{a_k=0}^{I-1} \frac{c_{a_k}}{(1+r)^{a_k}} + l_k + \frac{m_{1_k}}{1+r} + \frac{m_{2_k}+s_{I_k}}{(1+r)^2}\right] \\
=&wh_B(1-n_B-fn_p) + \sum_{a=B+1}^{R} \frac{wh_a(1-n_a)}{(1+r)^{a-B}} + (1+r)^{B-I}s_B + \sum_{a=B}^{R} \frac{\phi_a(\cdot)}{(1+r)^{a-B}} \\
&+ \frac{f}{(1+r)^2} \cdot wh_{2_k}(1-n_{2_k}),
\end{aligned}$$

where

- $\theta(f)$: altruism factor that depends on fertility f, chosen earlier
- h_B: human capital at period B
- c_{a_k}: child's consumption at period a_k
- s_{I_k}: transfer to child (decided one-period ahead) when child becomes independent at age I_k
- n_p: parental time and goods investments into child's human capital (pre-secondary school)
- n_{2_k}: child's time and household goods investments into secondary school
- m_{a_k}, l_k: goods investments in child's human capital and longevity
- π_{R+1}: survival probability from period R to $R+1$
- p: social security benefits

Retirement value function After retirement (age $a \geq R+1$), an individual just eats his pensions p facing stochastic death:

$$\begin{aligned}
V_a(s_a, p; L) =& \max_{s_{a+1}} \{u(c_a) + \beta L \pi_{a+1} \cdot V_{a+1}(s_{a+1}, p; L)\} \\
\text{s.t.} \qquad & c_a + s_{a+1} = p + (1+r)s_a + \phi_a(\cdot),
\end{aligned}$$

and where π_{a+1} are exogenously assumed survival probabilities, which are shifted by an individual's longevity L. We assume $\pi_{T+1} = 1$ so death is certain after period T.

2.3 Government

The crux of our paper is to investigate whether and how redistributive government policies interact with development. Three major changes in the role of government when comparing the early and late 20th century are

1. Introduction of income taxes (progressive)
2. Introduction of social security and other safety nets
3. Expansion of public education provision

While we mainly focus on educational transfers, we also model money transfers and progressive taxes, the latter from which the transfers are financed. The social security system is assumed to be self-financing. Educational transfers have already been specified in section as additively augmenting the goods expenditures parents spend on their children's schooling. Here we specify the tax policy, social security system and money transfer, embedded in the function $\phi_a(\cdot)$.

Taxation In every period of life a, individuals pay taxes according to a progressive tax schedule on their combined capital and labor income which we will denote as $y_a(e_a, s_a)$, where e_a is the labor earnings and s_a the asset holdings of an individual of age a. Specifically, at period 2 (or period $I-1$), although still attached to their parents, an individual generates an earning of

$$e_2 = wh_2(1-n_2).$$

For periods $a \in \{I, \ldots, B-1, B+1, B+3, \ldots, R\}$, the income of an individual is

$$e_a = wh_a(1-n_a),$$

and at age B, since a parent has to spend additional time with the child,

$$e_B = wh_B(1-n_B-fn_p)$$

For ages $a \in \{R+1, \ldots, T\}$ the individual has no earnings ($e_a = 0$) and lives only on social security benefits. Taxable income for ages $a \in \{I, \ldots, B+1, B+3, \ldots, T\}$ is earnings plus

interest income:

$$y_a = e_a + rs_a$$

while at age $B+2$ there is also the earnings of the child:

$$y_{B+2} = e_{B+2} + e_{2_k} + rs_{B+2}$$

which is taxed according to a progressive schedule of the following parametric form:

$$\tau(y_a) = \tau_0 \left(\frac{y_a}{\bar{y}} \right)^{\tau_1},$$

where $\tau(\cdot)$ is the average tax rate and $\bar{y}$ denotes the average income in the economy. This form is commonly used in both theoretical and quantitative explorations of taxation, and captures progressive marginal tax rates in a convenient yet realistic way.

We also assume that that intergenerational transfers are taxed above a certain level, $\bar{s}$, per child. Unlike income taxes, inheritance taxes (or taxation on wealth) have been levied even before the expansion of federal redistribution programs. If a parent decides to leave s_{I_k} per child, the taxes paid at the moment of transfer is

$$f \cdot \max\left\{\tau_b(s_{I_k} - \bar{s}), 0\right\}.$$

Social Security Another significant redistribution program implemented in the mid-1900s is the social security program. In contrast to progressive taxation or transfers of which main purpose is to redistribute across individuals at the same point in time, a PAYGO social security program primarily redistributes across generations. We model the social security system as a flat payroll tax τ_s on labor earnings during working age (periods I to R), and an affine payout $p(\cdot)$ after retirement:

$$p(\bar{y}) = p_0 + p_1 \bar{e} \quad \text{where} \quad \bar{e} = \frac{\sum_{a=5}^{10} e_a}{6},$$

i.e., the payout consists of a lumpsum component p_0 and a linear component $p_1 \bar{y}$, where $\bar{y}$ is the average of the 36 years of earnings before retirement.

Money Transfers Money transfers are formulated as a negative income tax (NIT). These money transfers can be used by recipients for consumption, bequests or educational expenditures. The NIT guarantees that every individual possesses a net income of at least g, but

that the transfer amount decreases as wealth increases, and above a certain level becomes zero. Denote the rate at which after tax income is reduced by t. The net amount that a parent who does not live with a child receives is then given by

$$\max\{g - tz_a, 0\}$$

where z_a is after-tax income:

$$z_a \equiv \begin{cases} [1-\tau(y_a)]\, y_a - \tau_s e_a & \text{if } a \in \{I, \ldots, B-1, B+3, \ldots, R\} \\ p(\bar{y}) + (1-\tau(y_a))y_a & \text{if } a \in \{R+1, \ldots, T\}. \end{cases}$$

Individuals with $z_a \leq g/t$ level qualify for the NIT, while those with higher z_a receive less in transfers. Observe that as g increases or as t decreases, more individuals participate in the program. Also note that the deduction rate t lies between 0 and 1. Lump-sum transfers and income maintenance programs are special cases of the NIT when t takes on the extreme values of 0 and 1, respectively.

Parents live with their children from periods B to $B+2$ (children aged 0 to 2), the last period in which children can also generate earnings. For these families, we multiply the OECD equivalence scale to the transfers. Specifically, we give a weight of $q_0 = q_1 = 0.5$ to children in periods 0 and 1, and $q_2 = 0.7$ in period 2. The transfer received is then

$$\max\{(1 + q_{a-B}f)g - tz_a, 0\}$$

where

$$\begin{aligned} z_a &\equiv [1-\tau(y_a)]\, y_a - \tau_s e_a && \text{for } a \in \{B, B+1\} \text{ and} \\ z_{B+2} &\equiv [1-\tau(y_{B+2})]\, y_{B+2} - \tau_s e_{B+2} + (1-\tau(y_2))y_2. \end{aligned}$$

In summary, $\phi_a(\cdot)$ can be written as

$$\phi_a = \begin{cases} \max\{g - tz_a, 0\} - \tau(y_a)y_a - \tau_s e_a & \text{if } a \in \{I, \ldots, B-1, B+I, \ldots, R\} \\ \max\{(1+q_{a-B}f)g - tz_a, 0\} - \tau(y_a)y_a - \tau_s e_a - \mathbb{1}_{a=B+2} \cdot f \cdot \max\{\tau_b(s_{I_k} - \bar{s}), 0\} & \\ \qquad\qquad \text{if } a \in \{B, B+1, B+2\} & \\ \max\{g - tz_a, 0\} - \tau(y_a)y_a + p_0 + p_1 \cdot \frac{\sum_{j=5}^{10} y_j}{6} & \text{if } a \in \{R+1, \ldots, T\} \end{cases} \quad \text{where}$$

$$z_a = \begin{cases} a \in \{I, \ldots, R\}: & [1 - \tau(y_a)]\, y_a - \tau_s e_a \\ a \in \{R, \ldots, T\}: & [1 - \tau(y_a)]\, y_a + p_0 + p_1 \cdot \frac{\sum_{j=5}^{10} y_j}{6} \end{cases}$$

and

$$y_a = rs_a + \begin{cases} e_a = wh_a(1 - n_a) \text{if } a \in \{I, \ldots, B-1, B+1, B+3, \ldots, R\} \\ e_B = wh_B(1 - n_B - fn_p) \\ e_{B+2} + e_{2_k} = wh_{B+2}(1 - n_{B+2}) + fwh_{2_k}(1 - n_{2_k}) \\ 0 \qquad\qquad \text{if } a \in \{R+1, \ldots, T\}. \end{cases}$$

We assume that both money and educational transfers are subject to a government budget constraint such that total transfers must equal a fixed share of the total government budget. Let $(\mathbf{x}_a, \mu_a)$ denote the state and mass of age $a \geq I$ individuals, and Ψ_a the age-dependent distribution over $\mathbf{x}_a$. Clearly,

$$\mathbf{x}_a = \begin{cases} (s_a, h_a; z, T) & \text{for individuals aged } I \leq a \leq B-1, \text{and} \\ (s_a, h_a; z, T; z_k, f) & \text{for individuals aged } B \leq a \leq T, \end{cases}$$

since abilities, lifespans and number of children do not change with age, and the children's abilities are only revealed at time of birth (period B). Then the government budget raised from the progressive income tax can be expressed as

$$B = \sum_{a \geq I} \mu_a \int \tau\left(y_a\right) d\Psi_a + \mu_I \int\limits_{s_I \geq \bar{s}} \tau_b(s_I - \bar{s}) d\Psi_I.$$

Now let (b_{NIT}, b_1, b_2, b_3) denote the shares of the government budget devoted to the NIT, primary school subsidies and secondary school subsidies, respectively. Then the government

budget constraints are

$$b_{NIT} \cdot B = \sum_{a=I;B+3}^{\bar{T}} \int_{z_a \leq g/t} [g - tz_a] \, d\Psi_a + \sum_{a=B}^{B+2} \int_{z_a \leq (1+q_{a-B}f)g/t} [(1 + q_{a-B}f)g - tz_a(h_a, s_a)] \, d\Psi_a, \tag{1a}$$

$$b_1 \cdot B = d_1 \cdot \mu_1 = d_1 \cdot \mu_{B+1} \int f(\mathbf{x}_{B+1}) d\Psi_{B+1}, \tag{1b}$$

$$b_2 \cdot B = d_2 \cdot \mu_2 = d_2 \cdot \mu_{B+2} \int f(\mathbf{x}_{B+2}) d\Psi_{B+2}, \tag{1c}$$

$$b_3 \cdot B = d_3 \cdot \mu_3 = d_3 \cdot \mu_{B+3} \int f(\mathbf{x}_{B+3}) d\Psi_{B+3}. \tag{1d}$$

We assume that the social security program balances itself (remember we are assuming a steady state):

$$\tau_s \sum_{a=I}^{R} \mu_a \int e_a d\Psi_a = \sum_{a=R+1}^{\bar{T}} \mu_a \left[p_0 + \frac{p_1}{6} \cdot \sum_{a'=5}^{R} \frac{\int e_{a'} d\Psi_{a'}}{\mu(a')} \right].$$

2.4 Equilibrium

To close the model and define an equilibrium, we assume a standard neoclassical firm which rents physical and human capital from individuals using a a constant-returns-to-scale technology w.r.t. factor inputs, and solves

$$\max_{K,H} \{ZF(K, H) - RK - wH\}$$

where K and H denote aggregate quantities of physical and human capital, and Z is the TFP residual. Letting δ denote the depreciation rate of physical capital and assuming this cost is borne by the firm, $R = r + \delta$. Assuming $F(K, H) = K^\alpha H^{1-\alpha}$, optimization yields the standard result that

$$\begin{aligned} &RK = \alpha ZF(K, H), \quad wH = (1 - \alpha)ZF(K, H), \qquad \text{and} \\ &Y = ZF(\kappa, 1)H, \end{aligned} \tag{2}$$

where Y is aggregate output and $\kappa \equiv K/H$ is the units of physical capital per one unit of human capital.

Recall that the only source of uncertainty in the model is the innate ability of the individuals' offspring. Hence individuals choose different numbers of children (different choices

of f) according to their state vector at period I. In a steady state, the fertility choices aggregate up to imply a constant population growth rate, π:

$$\pi = \frac{\int f(\mathbf{x}_I) d\Psi(\mathbf{x}_I)}{\int d\Psi(\mathbf{x}_I)} = \frac{\mu_{B+I}}{\mu_I}.$$

Definition 1 *A stationary, recursive competitive equilibrium consists of the set of individual decision rules; the representative firms' decision of K and H; the government's tax and transfer policies; prices (R, r, w); and (stationary) value functions (V_I, V_B) such that*

1. *Individuals optimize (decision rules solve V_I and V_B);*

2. *Firms optimize (equation (2) holds);*

3. *Markets clear:*

$$K = \sum_{a=3}^{\bar{T}} \int s_a d\Psi_a$$

$$H = \sum_{a=3}^{R} \int h_a(1 - n_a) d\Psi_a - \mu_B \int f n_P h_B d\Psi_B - \mu_{B+2} \int f h_{2_k}(1 - n_k) d\Psi_{B+2},$$

   and goods markets clears by Walras' Law;

4. *Government balances budget (equations (1) and (**??**) hold)*

5. *The aggregate distribution remains stationary and the population growth rate π is constant. For this we only need*

$$\Gamma(\Psi_I) = \Psi_I,$$

   *where Γ is the aggregate law of motion induced by the exogenous intergenerational process for abilities (**??**) and the policy functions associated with the individual value functions (V_I, V_B). Then all policy functions including the fertility choice $f(\mathbf{x}_I)$ and health investment $l(\mathbf{x}_B)$ remain constant in aggregate, so all other age-dependent distributions and the population growth rate also remains constant.*

3 Calibration

We assume that period utility is CRRA with parameter χ, and that the aggregate production is Cobb-Douglas with parameter α. The functional forms for ability transmission and human capital acquisition were specified in section, and for government policy in section. All that remains to be specified are the altruism and longevity functions. For this we assume

$$\theta(f) = \theta_0 f^{\theta_1}$$
$$L(l) = 1 - \exp(-\lambda l)$$

so for $\theta_0 > 0$, $\theta_1 \in (0, 1)$, we have an altruism function along the lines of (Barro and Becker, 1989) and for $\lambda \geq 0$, the survival probability is convex in l and strictly below 1.

Parameters are chosen from existing empirical work if available. Otherwise they are set so that model moments equal empirical moments from the U.S. economy in 1900. Whenever 1900 data is unavailable, we target a 2000 empirical moment instead. Before proceeding to a discussion of the calibration procedure and results, we clarify the precise manner in which comparisons across various redistributive policies are made and spell out the criteria used for comparisons.

It is important to note that the analysis focuses exclusively on steady states. That is, comparisons are being made across long-run equilibrium resulting from different policies, which are taken as exogenous by the agents within the model. The paper thus remains silent on the interim cost or benefits that arise from transitioning from one policy to another. An analysis of a transitional path is greatly complicated by the fact that the distributions of physical and human capital, and in particular fertility choices, would be changing along a transition to the new steady state. The agents would have to keep track of not only the distribution of assets and human capital but also demographics in order to predict prices and solve their optimization problem. We defer this problem to future research and in the current paper focus on showing whether redistributive policies can result in a long-run equilibrium with higher output and welfare.

In comparing policies, we will use the 1900 equilibrium as a benchmark with a TFP of $Z = 1$. Given the model parameters, we alter only the policy parameters in accordance with empirical evidence from 2000—this includes education subsidies, social security, etc.

Then we adjust Z so that output per capita in the new steady-state equilibrium relative to the benchmark is exactly equal to the GDP per capita ratio between 2000 and 1900. So Z simply accounts for output differences that cannot be explained by our model. We call the new resulting steady state equilibrium the "counterfactual 2000 economy." By comparing aggregate statistics such as Gini coefficients and intergenerational elasticities of income in the counterfactual 2000 economy to what is actually observed in the data, we can quantitatively account for how much of these changes are due to different policies and how much due to TFP; what we cannot account for is beyond the scope of our model.

Of course, not all policy changes were purely redistributive nor revenue-neutral. To address this, we adjust the median tax rate in the counterfactual 2000 economy so that total tax revenues are equal to that in 1900. This experiment answers how much of the observed differences can be explained by the implementation of a redistributive policy, such as progressive taxes or education/money transfers, as opposed to a larger government.

In our overlapping generations setup with endogenous fertility, in principle, when considering social welfare, we would need to take a stand on how to account for people who may no longer be born due to the implementation of different policies. Since it is not clear how we would account for this, we only consider the people in existence in any given equilibrium, but ensure that we include compensation for more or less utility derived from children when computing consumption equivalent welfare.

We conclude our main analysis by finding the optimal allocation of education subsidies as follows. In the model, we allow for 3 stages of education subsidies: primary, secondary and tertiary. First, beginning from the counterfactual 2000 economy, we compute steady state equilibrium statistics when we shift the entire education budget to only 1 out of the 3 stages of education, holding the aggregate budget for total education subsidies fixed, i.e.,

1. given the total education budget is $B_e = B \cdot \sum_{i=1}^{3} b_i$,

2. we consecutively set $d_i = B_e/\mu_i$, for $i = 1, 2, 3$, while keeping $d_{j \neq i} = 0$.

Having found the total impact of subsidizing different stages of education separately, we then numerically seek the convex combination of subsidies that maximizes output and/or welfare.

4 Methodology

Before proceeding to a discussion of the calibration and results, it is important to set out the precise manner in which comparisons across various redistributive policies are made and to spell out the criteria used in attempting such comparisons. This helps to clarify the scope of the present analysis.

First, it is important to note that the analysis focuses exclusively on steady-states. That is, comparisons are made across steady-states with different policies. The paper thus remains silent on transitions from one steady-state to another resulting from a policy change. An analysis of a transitional path is greatly complicated by the fact that the distributions of physical and human capital would be changing along a transition to the new steady-state and that the consumer would have to keep track of these distributions in order to predict prices and solve his optimization problem.

Different redistributive polices are contrasted in the following manner. First, within each of money transfers and educational transfers, the effects of equalization of transfers are examined. That is, comparisons are made among negative income taxes with different deduction rates and among educational transfers with different shares spent on the lump-sum transfer relative to the tuition subsidy. After the comparisons within each type of transfers are made, the relative desirability of educational transfers over money transfers is examined by changing the allocations of revenues to each transfer. In comparing policies, tax rates are held fixed. Hence, governmental revenues could be different depending on redistributive policies because the policies alter the distributions of human and physical capital and hence their aggregates. Since the analysis focuses on long-run effects, it appears more relevant to fix tax rates rather than tax revenues while making comparisons.

4.1 Calibration

The model we presented has 22 parameters, not including 12 policy parameters. Out of these, 20 are fixed á priori to standard numbers found in the literature, or set to an empirical counterpart that directly corresponds to the parameter. These parameters are presented in table 1. The remaining 14 parameters are chosen numerically so that equilibrium model

moments are equal to empirical moments, as summarized in table 2. We discuss how the parameters are chosen in more detail below.

Preferences and Production The discount factor, β is chosen in equilibrium so that the equilibrium interest rate is 4%. The CRRA coefficient χ is set to 2, and capital share of income α to 0.33. Both these values are standard in the literature. Steady state TFP, Z, is normalized to 1 in 1900. In 2000, we adjust Z to account for the change in output (GDP) per capita that cannot be explained by the change in policies.

Less standard are the two altruism parameters, (θ_0, θ_1). In models without fertility choice, the constant altruism factor is typically targeted to match aggregate levels of intergenerational transfers. Since we have two parameters, the natural additional target is the fertility rate. We choose θ_1 to replicate the fertility rate in 1900, but θ_0 to match the aggregate level of intergenerational transfers in 2000 since this data is not available for 1900.

Ability Transmission Average abilities μ_z is normalized to 1, since different values of μ_z would be reflected in the TFP, Z. Intergenerational persistence of abilities ρ_z is chosen to match the intergenerational persistence of earnings (IGE). We do not have data going as far back as the 1900s, but recent work by Chetty et al (2014) present empirical evidence that at least in the latter two generations of the 20th century, there has not been much change in intergenerational persistence. Hence we choose ρ_z so that the 2000 IGE equals 0.4, the level of persistence in their study (which is also statistically equal to the well-known number attributed to Solon (1992)).

The only fundamental source of heterogeneity in the model is ability, more precisely the variance of the idiosyncratic shock σ_η^2. Hence we use this parameter to target the Gini coefficient of income in 1900.

Health Investments and Longevity The survival probabilities post-retirement are given by $L\pi_a$, for $a \geq R+1$. Since all probabilities are shifted equally for an given individual, the relative survival probabilities from ages a to $a+1$ (for $a \geq R+1$) are the same across the population. Given that $\pi_{T+1} = 1$, we fix the ratios π_a/π_{R+1} for $a \geq R+2$ to match the demographic structure of the elderly in 1900. We then choosing π_{R+1} to match the relative

size of the working and retired populations in 1900, and λ to match the relative lifespans between the rich and the poor in 2000.

Human Capital Acquisition The share of time investments in the production of periods 2 and 3 human capital, $(\gamma_{s_1}, \gamma_{c_1})$, is calibrated to match the average years of schooling and college enrollment rate in 1900. Given these parameters, the share of goods investments in the production of period 2 human capital, γ_k, is calibrated to the expenditure share of primary school investments as a fraction of median earnings, $\bar{y}$, in 1900. Note that secondary school was not compulsory in 1900, so the share of goods investments in secondary school and college, γ_{s_2} and γ_{c_2}, are calibrated to the expenditure shares of secondary and college investments as a fraction of median earnings. The adulthood human capital parameter γ is calibrated to average earnings growth.

The elasticity of substitution parameter σ determines complementarity between primary and secondary school investments, lower values of which make early investments in children more important. Since this parameter has a large effect on the initial level of human capital, while we fix its value to its estimated counterpart in Cunha et al (2010) for model year 2000, we conduct a robustness check where we vary its value.

Taxation and Social Security Since there was no income tax in 1900, we calibrate (τ_0, τ_1) to the tax rates faced by the median and highest income bracket taxpayer in 2000. In particular, since τ_0 can be fixed to the median tax rate we find in the data since $\tau(\bar{y}) = \tau_0$. Inheritances were not taxed up to a level of **XXX** dollars in 1900 and 2000, respectively, facing average tax rates of XXX and XXX, which we can use to fix $\bar{s}$ and τ_b separately for each year—specifically, we set $\bar{s} = \tilde{s} \cdot \bar{y}$, where $\tilde{s}$ is fixed to the inheritance tax threshold as a multiple of median earnings, and calibrate $\bar{y}$ within the model.

The social security tax rate τ_s is set to 12% following XXX, while p_1 is set to XXX% (the median replacement rate). p_0 is then set to balance the social security budget.

Money and Education Transfers The money transfers correspond to means-tested transfers to poor households, such AFDC or TANF. Such policies were only established in XXX, with deduction rates of 1 except for the years 1967-1982, so we set $t = 1$. The

threshold g is fixed at $g = \tilde{g}\bar{y}$, where $\tilde{g}$=40%, since XXXX, while the government budget share of the transfers, s_{NIT}, is taken from XXX.

The main policy experiment we focus on is the lump-sum education subsidies. Such subsidies did not exist in 1900, and for 2000 we fix the level of subsidies as multiples of median earnings, i.e., $d_i = \tilde{d}_i\bar{y}$, $i \in \{1, 2, 3\}$.

Parameter
(α, δ)
χ
μ_z
σ
π_a/π_{R+1}
τ_0, t
$\tilde{g}, \tilde{d}_i$
$\tau_b, \tilde{s}$
(τ_s, p_0, p_1)

Table 1: Fixed Parameters (a priori)

5 The Effect of Redistributive Policies

What is the impact of redistributive policies?

We now proceed to the results of the numerical simulations. The effects of changes in (i) redistributive policies (ii) the effectiveness of educational transfers versus money transfers, and (iii) the impact of endogenous human capital and fertility choice are examined.

Baseline Economy: First, the statistics of the baseline economy calibrated to match data on the U.S. economy are presented in Table **??**. The match between the baseline economy and the U.S. data seems reasonable in terms of the presented statistics with the notable exception of the skewness in earnings, which is much lower in the model economy. The distribution of assets exhibits the greatest dispersion, followed by the distribution of earnings, and finally the distribution of consumption (see Table **??**). This is in accord with the data.

Parameter
β
(ρ_z, σ_η^2)
(θ_0, θ_1)
(π_{R+1}, λ)
γ_k
$(\gamma_{s_1}, \gamma_{s_2})$
$(\gamma_{c_1}, \gamma_{c_2})$
γ
τ_1

Table 2: Calibrated Parameters

The intergenerational correlation of consumption is higher than that of earnings (again, see Table **??**). This is a direct result of intergenerational consumption smoothing and is also supported empirically. Before turning to an examination of the effects of various policies, it is instructive to compare the baseline economy to an economy without any redistributive policies to see how effective current redistributive policies are.

6 Conclusion

References

[1] Aghion, P. and Bolton, P. (1997) "A Trickle-Down Theory of Growth and Development with Debt Overhang," Review of Economics Studies, 64(2), 151—172.

[2] Aiyagari, R. (1993) "Optimal Income Taxation With Incomplete Markets, Borrowing Constraints, and Constant Discounting," Journal of Political Economy, 32,

[3] Atkinson, A. (1970) "On the Measurement of Inequality," Journal of Economic Theory, 2, 244—263.

[4] Atkinson, A. and Stiglitz, J. (1980) "Lectures on Public Economics," Mc—Graw Hill, New York.

[5] Banerjee, A. and Newman, A. (1991) "Risk-Bearing and the Theory of Income Distribution," Review of Economic Studies, 58, 211-236.

[6] Banerjee, A. and Newman, A. (1993) "Occupational Choice and the Process of Development," Journal of Political Economy, 101, 274-298.

[7] Becker, Gary S. and Tomes, Nigel, (1986) "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics* 4: S 1 -S39.

[8] Ben Porath, Y., 1967, "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy* 75, pt. 1, 352-365.

[9] Bénabou, R. (1996a) "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and Education Finance," American Economic Review, 86(3), 584—609.

[10] Bénabou, R. (1996b) "Unequal Societies: Income Distribution and the Social Contract," NBER Working Paper 5583, May.

[11] Bénabou, R. (1996c) "Inequality and Growth," NBER Macroeconomics Annual, Ben Bernanke and Julio Rotemberg eds., MIT Press, 11—74.

[12] Benabou, R., 2002. Tax and education policy in a heterogeneous agent economy: what levels of redistribution maximize growth and efficiency? Econometrica 70, 481–517

[13] Browning, M., L. P. Hansen and J. J. Heckman, 1999, "Micro Data and General Equilibrium Models," in J. B. Taylor and M. Woodford (eds), **Handbook of Macroeconomics**, Vol. 1, North Holland, pp: 544-633.

[14] Chenery, H., M. Ahluwalia, C. Bell, J. Duloy and R. Jolly (1994), Redistribution with Growth, Oxford University Press

[15] Durlauf, S. (1996) "A Theory of Persistent Income Inequality," Journal of Economic Growth, 1, '75—94.

[16] Galor, O. and Zeira, J. (1993) "Income Distribution and Macroeconomics," Review of Economic Studies, 60, 35-52

[17] Heckman, J., L. Lochner, and C. Taber, 1998, "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," *Review of Economic Dynamics* 1, 1–58.

[18] Kanbur, S.M. (1979) "Of Risk Taking and the Personal Distribution of Income," Journal of Political Economy 87(4), 769—797.

[19] Laitner, J. (1992) "Random Earnings Differences, Lifetime Liquidity Constraints and Altruistic Intergenerational Transfers," Journal of Economic Theory, 58, 113—170.

[20] Loury, G. (1981) "Intergenerational Transfers and the Distribution of Earnings," Econometrica, 49, 843-867.

[21] Mookherjee, D. and D. Ray (2003), "Persistent Inequality," Review of Economic Studies, 70(2), April 2003, 369-394.

[22] Okun, Arthur: Equity and Efficiency: The Big Tradeoff. Washington, D.C: The Brookings Institution, 1975

[23] Piketty, T. (1997), The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing, Review of Economic Studies, 64(2), 173-89.