The Forward Guidance Puzzle

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Abstract

With short-term interest rates at the zero lower bound, forward guidance has become a key tool for central bankers, and yet we know little about its effectiveness. This paper first empirically documents the impact of forward guidance announcements on a broad cross section of financial markets data and professional forecasts. We find that FOMC announcements containing forward guidance had heterogenous effects depending on the other content of the statement. We show that once we control for these other elements, forward guidance had, on average, positive and sizeable effects on output and inflation. Using this benchmark, we then show that standard medium-scale DSGE models tend to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon we call the "forward guidance puzzle." We explain why this is the case and show that incorporating a perpetual youth structure into the benchmark provides a possible resolution to the puzzle.

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1 Introduction

For decades, macroeconomists have attempted to quantify the effects of monetary policy actions on the economy. By now, a large literature has documented the transmission mechanism of surprise changes in short-term interest rates onto the economy, using either VARs or DSGE models (e.g., Sims (1980), Christiano et al. (1999), Christiano et al. (2005)). While we thus have some understanding of the effects of short-term interest rates, this tool has been constrained by the zero lower bound (ZLB) during the financial crisis in most developed economies. Instead, many central banks have used other tools such as announcements about the future path of the policy rate ("forward guidance"), or "quantitative easing" measures involving a change in the size and the composition of the central bank balance sheet. Forward guidance has been used extensively and explicitly by the Federal Reserve since the FOMC meeting of December 16, 2008, so as to affect long-term bond yields and stimulate aggregate expenditures (see Woodford (2012) and Campbell et al. (2012a)).[1] Moreover, Woodford (2012), building on results by Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2011), emphasizes the "signaling channel" of the Fed's asset purchases – that is, he argues that quantitative easing itself can at least in part be interpreted as implicit forward guidance.

While the literature has provided strong theoretical justifications for the use of such forward guidance (e.g., Eggertsson and Woodford (2003)), evidence on the quantitative effects of this policy tool on the macroeconomy is still limited. This paper is intended to help fill that gap. The lack of evidence in the literature may not be surprising in light of the fact that there are significant identification challenges in the case of shocks that are anticipated. An announcement by policymakers that they will maintain the policy rate at the ZLB for longer than initially anticipated by market participants may have two types of effects. On the one hand, it could be interpreted as more monetary stimulus: it should lower the market's expectation of future federal funds rate (FFR), which contributes to lower longer term

[1] At that meeting, the FOMC's statement mentioned that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate for some time." Three months later, the FOMC reinforced its forward guidance by stating that the exceptionally low levels of the federal funds rate would likely be warranted "for an extended period." This sentence was reiterated in each subsequent FOMC statement until August 9, 2011, when the FOMC argued that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate at least through mid-2013." That sentence was maintained in subsequent statements until January 25, 2012, when the date was pushed forward to "late 2014."

yields, hence stimulates economic activity and puts upward pressure on inflation. On the other hand, such an announcement could be interpreted by market participants as revealing negative news about the state of the economy, if they believe that the FOMC has access to information not shared by market participants. In this case, such an announcement would be associated with lower long-term yields and lower projections of economic activity. The interpretation chosen by the market participants could thus depend in very subtle ways on the FOMC communication.[2]

Empirically, Gürkaynak et al. (2005b) and more recently Campbell et al. (2012a) find strong evidence that FOMC announcements move asset prices. However, Campbell et al. (2012a) illuminate the challenge in using time series to identify the effect of forward guidance. Specifically, when they assess the impact of exogenous anticipated changes in monetary policy on private expectations, they find that the opposite sign from what theory predicts, showing that the public's interpretation of the policy depends crucially on other components of the communication. Moreover, even if it were possible to identify the impact of, say, four quarters-ahead forward guidance, its effect would not necessarily be the same as, say, that of eight-quarters ahead forward guidance (Campbell et al. (2012a) consider one through four quarters ahead forward guidance; forward guidance communicated in early 2012 was going through the end of 2014, and hence amounted to approximately eight quarters). Given that policymakers have seldom experimented with forward guidance this far in the future, there is little data to guide them.

New Keynesian DSGE models following the work of Christiano et al. (2005) and Smets and Wouters (2007) are in principle well suited to study the effects of forward guidance. Such models have been found to fit the data reasonably well and to provide a good forecasting performance relative to reduced form models such as VARs, private forecasters, or the Greenbook (see Smets and Wouters (2007), Del Negro et al. (2007), Edge and Gürkaynak (2010), and Del Negro and Schorfheide (2013)). Most importantly, being laboratory economies, they can be used to study the impact of policy experiments never performed before. As shown by Laseen and Svensson (2011), forward guidance can be captured in DSGE models using anticipated policy shocks. Such shocks reflect deviations of the short-term interest rate from

[2]Woodford (2012) argues that several announcements made prior to August 2012 about the future path of policy rates did not indicate a clear commitment to maintaining short-term rates low, so that they ran the risk of being interpreted as reflecting a deteriorating forecast for output and or inflation.

the historical policy rule that are anticipated by the public. They can be affected by policy-makers' announcements about their intentions regarding the future path of the policy rate. Milani and Treadwell (2011) study the impulse responses to anticipated policy shocks using a simple three-equations New Keynesian DSGE model. Campbell et al. (2012b) go even further and investigate the impact of forward guidance on the macroeconomy by estimating a medium scale DSGE model broadly similar to the one in this paper using data on market expectations for the federal funds rate, in addition to a standard set of macro variables, for the sample 1987-2007. They find that forward guidance explains about 9 percent of output and hours fluctuations at the business cycle frequency, and more than 50 percent of the movements in the federal funds rate. Their results indicate that even in the pre-Great Recession period, forward guidance played a large role in monetary policy – a finding that echoes that of Gürkaynak et al. (2005b) – and a significant role in terms of business cycle fluctuations.

The problem with DSGE models, however, is that they appear to deliver unreasonably large responses of key macroeconomic variables to central bank announcements about future interest rates – a phenomenon we can call the "forward guidance puzzle". Carlstrom et al. (2012) show that the Smets and Wouters model would predict an explosive inflation and output if the short-term interest rate were pegged at the ZLB between eight and nine quarters. This is an unsettling finding given that the horizon of forward guidance by the FOMC has at times been of at least eight quarters.

This paper makes three contributions. First, we document empirically the response of financial market variables and forecasts of key macroeconomic variables to actual FOMC communications involving changes in calendar-based forward guidance. We both provide a detailed description of the heterogeneity in past forward guidance episodes, as well as a realistic benchmark against which our model's predictions can be compared. We find that the estimated effect of forward guidance on output and inflation expectations are positive but not quantitatively large. Second, we characterize the quantitative implications of forward guidance in a setting that is arguably more realistic than that adopted by Carlstrom et al. (2012). In their experiment, these authors assess the impact of fixing the interest rate to the zero lower bound relative to the steady state baseline. Given the current state of the economy, we view the assumption that interest rates would be at steady state in absence of forward guidance as unrealistic. We instead incorporate current market expectations for the

short rate in our baseline forecast using the approach described in Del Negro and Schorfheide (2013). Specifically, we use the FFR expected path through mid-2015 implied by OIS rates as of August 28, 2012. Doing so allows us to incorporate valuable information for the estimation of the state of the economy. We then investigate the effect of extending the forward guidance by two quarters, from the end of 2014 to mid-2015. Using the FRBNY-DSGE model we show that even for this much more modest (relative to Carlstrom et al. (2012)) experiment, the same findings are confirmed: the model-implied response of macroeconomic variables is unrealistically large.

The third contribution of the paper is to point to the source of the problem and suggest a solution. Credible announcements about future short-term policy rates should affect the current long-term bond yields, and these in turn affect economic activity and inflation. However, the model predicts an excessive response of the long-term bond yield to policy announcements, compared to what is observed in the data. In order to address this issue, we adopt Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Nisticò (2012) and Piergallini (2006). We assume, as in these papers, that agents face each period a constant probability of dying and being replaced by a new agent, an assumption that we believe adds realism. While each individual in this model behaves similar to the representative agent, the cohort structure results in an aggregate economy that discounts the future more heavily. This implies that announcements of policy changes in the future generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents. This discounting bears some similarities with the mechanism proposed by McKay et al. (2014). In their paper, current conditions also matter more than conditions far into the future. The mechanism is, however, different, as these authors emphasize the role of precautionary savings and borrowing constraints, whereas we focus on the effect of overlapping cohorts.

The paper proceeds as follows. Section 2 provides some empirical evidence on the effects of forward guidance in the US. It documents the responses of financial market data as well as private sector forecasts of key macroeconomic variables to FOMC announcements. Section 3 briefly describes the DSGE model used, its estimation, and how we formalize the introduction of a fixed interest-rate path. It then presents the implications of interest rate announcements in this model, and reports that the model generates an excessive response to announcements about changes in the expected future interest rate path, a phenomenon we call the forward

guidance puzzle. Section 4 proposes incorporating a perpetual-youth structure into a DSGE model as a potential solution to this puzzle. Section 5 concludes.

2 Empirical Evidence on the Effects of Forward Guidance

In this section, we provide empirical evidence on the effect of forward guidance announcements. Our goal is twofold – first, we aim to provide a comprehensive description of forward guidance episodes, and second, we aim to provide a benchmark against which we can compare the predictions of our model. Empirically documenting the effects of forward guidance on the macroeconomy is challenging both due to data constraints and due to the nature of forward guidance announcements, which confound negative news about the economy, forward guidance and in some cases quantitative easing. Since we model forward guidance as purely stimulative in our theoretical exercises, it is important for our benchmark to disentangle the direct effect of forward guidance from these other elements.

2.1 Descriptive Evidence of Forward Guidance Episodes

We begin by documenting the impact of the three FOMC announcements in which there were calendar-based announcements about future interest rate paths: August 2011, January 2012, and September 2012.[3] We first document the impact of these announcements on a broad cross section of financial markets data, following the approach of Krishnamurthy and Vissing-Jorgensen (2011).[4] Since real economic variables are not available as frequently, we complement this evidence by documenting the impact of FOMC announcements on Blue Chip forecasters' expectations, after controlling for other news about the state of the economy. Table 1 summarizes the important elements of each of these three events.[5] While all statements are similar in their forward guidance announcements, the table below shows that

[3]Note that for simplicity, we do not look at December 2008, as this was also the date at which the interest rate hit the zero lower bound.

[4]A subset of these data has already been analyzed by Filardo and Hoffman (2014), who limit their analysis to forward rates and nominal long term Treasuries, and Femia et al. (2013).

[5]See Table B1 and B2 in the Appendix for additional detail on the FOMC statement language

they differ significantly in their language about the economy as well as their QE announcements. As we will discuss further in Section 2.1.2, the various components of the statement each play important roles.

Table 1: Summary of Forward Guidance FOMC Statement Language

	Economic Conditions		Policy Action	
	Growth	Inflation	Forward Guidance	Quantitative Easing
August 2011	"considerably slower"	"picked up"	"low levels warranted at least through mid 2013"	–
January 2012	"expanding moderately"	"at or below mandate"	"low levels warranted at least through late 2014"	–
September 2012	"moderate pace"	"subdued"	"until after the recovery strengthens... at least through mid 2015"	announce QE3

2.1.1 Evidence from Financial Markets

Table 2 reports changes in several asset prices (measured in basis points, unless otherwise noted) in the two day window following each forward guidance event, as in Krishnamurthy and Vissing-Jorgensen (2011). Most of these asset prices coincide with those reported in Krishnamurthy and Vissing-Jorgensen (2011), and are constructed using the same methodology and data sources. The Table provides a full description of the data, though we urge the reader to note several important elements.

First, the top panel shows that nominal rates declined substantially at most horizons following the January 2012, and especially the August 2011, announcements. In the two days following the August 2011 meeting, constant maturity Treasury yields fell by 23 basis points. Similarly, Figure 1 reports changes in federal funds rate futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve. The futures and forward interest rate curves flattened substantially not only at the short horizons, consistent with the announcements, but also at longer horizons. The flattening at shorter horizons

seems to indicate that explanations for the forward guidance puzzle based on imperfect credibility of the central bank (e.g., Bodenstein et al. (2012)) or on the notion that financial market participants don't pay attention to FOMC announcements are not consistent with the evidence.

Table 2: Evidence from Financial Markets

	Treasury Yields (constant maturity)					**Agency Yields** (Fannie/Freddie)				**MBS Yields**	
Maturity (years)	30	10	5	3	1	30	10	5	3	30	15
8/9/2011	-14	-23	-18	-12	-3	-19	-23	-27	-25	-24	-26
1/25/2012	-5	-12	-15	-8	0	-10	-13	-18	-14	-16	-18
9/13/2012	17	11	2	2	0	10	5	0	1	-13	-11

	TIPS (constant maturity)					**Implied Vol.**	**SP 500**	**DJ IA**	**FX USD/EUR**
Maturity (years)	30	20	10	7	5		(% change)		(% change)
8/9/2011	-26	-16	-33	-52	-39	-8.11	0.12	-0.83	-0.01
1/25/2012	-8	-11	-15	-18	-20	-4.21	0.29	0.46	0.56
9/13/2012	-9	-8	-15	-19	-25	-1.13	2.03	1.95	1.78

	Breakevens			**Inflation Swaps**					**TIPS Spread**		
Maturity (years)	20	10	5	30	20	10	5	1	20	10	5
8/9/2011	-7	10	21	8	9	14	13	-3	16	4	-8
1/25/2012	3	3	5	3	3	4	8	12	0	1	3
9/13/2012	24	26	27	26	27	21	28	23	3	-5	1

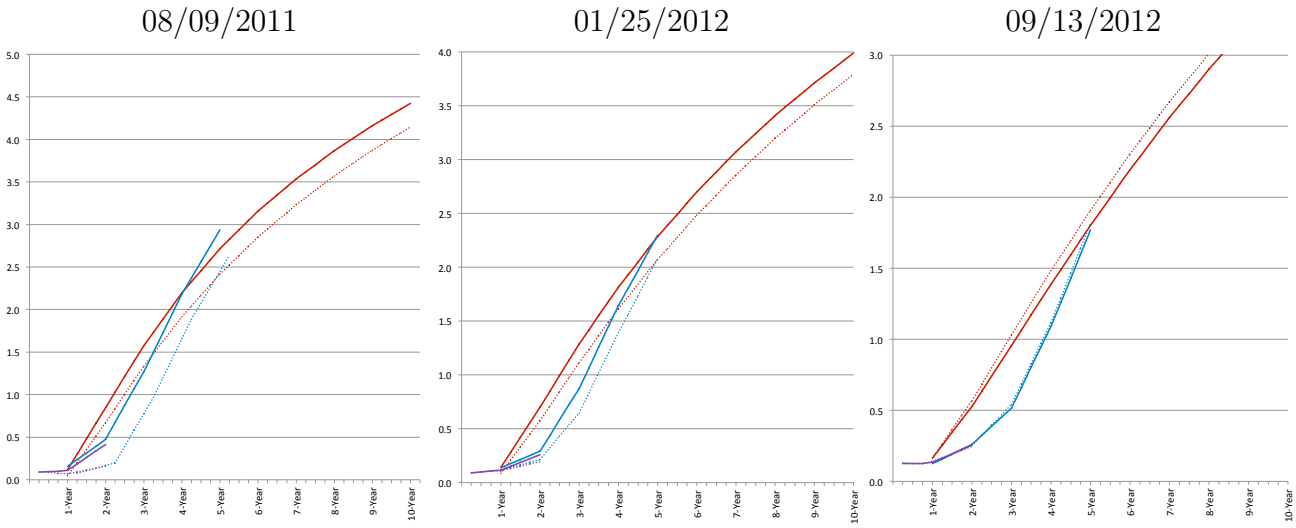
	Corporate Yields											
	Intermediate term						Long term					
	Aaa	Aa	A	Baa	Ba	B	Aaa	Aa	A	Baa	Ba	B
8/9/2011	-8	-6	-8	-8	2	16	-11	-9	-5	-5	26	33
1/25/2012	-10	-13	-11	-16	-9	-13	-12	-15	-17	-13	-16	-10
9/13/2012	11	10	7	-2	-8	-15	0	-1	-1	5	-12	-18

Notes: All figures are in basis points unless otherwise noted.

Second, in August 2011, corporate yields fell less than Treasuries for high credit quality bonds, but actually rise substantially for low credit quality bonds, suggesting an increase in

the safety premium. Caballero and Farhi (2014) provide a model where forward guidance has no effect on economic activity but results in increased safety premia, and this evidence appears to be consistent with such theory. Evidence shown in the next section however shows that the August 2011 statement may have been interpreted by forecasters as conveying bad news about the economy, and many of the asset price movements shown here (the decrease in real rate, the stall in the stock market in spite of significant lower real rates, and the increase in safety premia) are also consistent with this explanation. Conversely, in January 2012, yields on corporate bonds fell by roughly the same amount as Treasuries, regardless of the credit rating, suggesting little variation in the safety premium following the announcement.

Figure 1: Change in the Nominal Futures and Forward Curve following FOMC Announcements



Notes: Federal funds futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve are in purple, blue, and red, respectively. Pre- and post-announcement curves are solid and dotted, respectively.

Third, the response of financial markets to the September 2012 announcement was altogether different from the others. Nominal yields on government securities rose, instead of falling. Agency debt yield also rose, but less than for corresponding Treasuries, while agency MBS yields fell. Real yields fell substantially. Note that the rise in nominal yields makes the yield search explanation for the fall in real yields arguably less plausible. Breakeven and inflation swap rates rose at both short and long horizons, and the dollar depreciated. The stock market rose by about two percent. Most notably, while high credit quality corporate bond yields rose in line with Treasuries, the yield on low credit quality fell, indicating a

sizable decrease in the safety premium.

One explanation for the heterogeneity across episodes is related to the start of QE3: as suggested in Table 1, the central bank announced in September 2012 that it intended to "increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month." According to this explanation, the announced acquisition of agency MBS lowered safety premia, affecting the yield of low credit quality corporate bonds, while the increase in nominal Treasury was then the result of disappointed expectations that QE3 would also include long-term Treasuries. A complementary explanation is that this announcement, together with other parts of the statement also signaled that monetary policy was going to be more accommodative than previously thought, resulting in a decline in real rates, and increased inflation and growth expectations. Evidence presented in the next section, based on the revision of Blue Chip forecasts, is consistent with this explanation. The more optimistic outlook, in turn, resulted in a flattening of the safety premium. We will explore this heterogeneity further in Section 2.2.

2.1.2 Evidence from Surveys of Forecasts

In order to quantify the effect of forward guidance on other aspects of economic activity, we use an event study framework and the Blue Chip Financial Forecasts. If information on expectations for economic activity, inflation, and financial market variables were available the day before and two days after the event, we would simply report the two day-window change in expectations and attribute it to the news about monetary policy. Unfortunately, the expectations survey is available only once a month. Instead, using the exact collection dates of the survey, we control for any macroeconomic news and changes in asset prices in between surveys and attribute the residual change in expectations during the (approximate) one-month window to the FOMC announcement.[6] Of course, the macroeconomic news and asset price changes are not exogenous, and may themselves capture some of the effect of forward guidance. Our construction of the economic news over the time frame is intended

[6] Altavilla and Giannone (2014) follow a related approach to assess the impact of unconventional monetary policy on forecasters' projections for bond yields. These authors use the Survey of Professional Forecasters however, which is available only once a quarter, which makes it harder to control for other developments in the economy.

to deal with this to the best of our ability.[7] Specifically, we use the panel of forecasters i to run the following panel regression separately for each predicted variable (k) and h-quarter ahead forecast (h):

$$\Delta f_{(k,h)_{i,t}} = \gamma_{(k,h)_0} + \gamma'_{(k,h)_1}\ \Delta M_t + \gamma'_{(k,h)_2}\ \Delta AP_t + \gamma'_{(k,h)_3}\ Z_{(k,h)_{t,i}} + \beta_{(k,h)^e}\ D^e_t + \varepsilon_{(k,h)_{i,t}},\ \text{for } t = 1,..,T,\ i = 1,..,n, \quad (1)$$

where $\Delta f_{(k,h)_{i,t}}$ is the change in the h-quarters ahead forecast of participant i for variable k (e.g., GDP growth) between periods t and $t-1$, ΔM_t and ΔAP_t are vectors of macroeconomic surprises (e.g., payroll report) and changes in asset prices (e.g., stock prices) in the one-month window, respectively, $Z_{(k,h)_{t,i}}$ is a vector of participant-specific variables (e.g., the lagged change in the forecast), and D^e_t is the event dummy which is equal to one if the forward guidance event takes place in the one-month window (that is, $t = t^e$), and is zero otherwise. The γ and β coefficients are indexed by $_{(k,h)}$ to stress the fact that these vary across variables k and forecast horizons h. They are therefore estimated by running separate regressions for each h and k.[8]

The vector ΔM_t contains surprises in macroeconomic releases occurring between $t-1$ and t, where t is the date of the forecast collection.[9] These surprises, which are listed in Table B3 in the Appendix, are computed as the difference between the actual release and the median forecast in the Bloomberg survey and are expressed in the units of the release.[10] The vector ΔAP_t contains changes in asset prices that occurred between the time the $t-1$ and t surveys were collected.[11] These are a subset of the variables discussed in the previous section, one for each category. Appendix Table B3 lists these variables, along with the units in which the change is measured (basis points or log change). Of course, part of these changes

[7] Our approach bears similarities with that of Campbell et al. (2012a) in that it looks at the effect of policy announcements on private forecases, but it differs in two important ways. First, we use the individual-level forecast, rather than the census. Second, we adopt an event study framework rather than the identified factor analysis from Gürkaynak et al. (2005b).

[8] Of course, this implies that we ignore the potential correlation of $\varepsilon_{(k,h)_{i,t}}$ across k and h.

[9] We assume that forecasters incorporate news occurring in the two-day collection period, as data is usually released in the morning, allowing forecasters to adjust their Blue Chip submission in light of relevant information released on the last day of collection of the survey.

[10] This specification is inspired by the literature computing the effect of economic news on financial variables (e.g., Fleming and Remolona (1999) and Gürkaynak et al. (2005a))

[11] Specifically, this window ranges from the day after the last collection day of the $t-1$ release and the last collection day of the t release.

may be due to the forward guidance announcement itself. In computing AP_t we therefore subtract from the overall change the change in asset prices that occurred in the two-day window after the FOMC announcement. Finally, the vector of participant-specific variables $Z_{(k,h)_{t,i}}$ includes the lagged value of the change in forecast $\Delta f_{(k,h)_{i,t-1}}$, the previous period's forecast itself $f_{(k,h)_{i,t-1}}$, and participant specific fixed-effects. These variables are included to control for mean reversion and the possibility that forecasters are not rational, and hence do not respond to current period news only (see Coibion and Gorodnichenko (2012)).

We use OLS to obtain the estimates of all coefficients in the spirit of White (1982), but we take into account both heteroskedasticity and correlation across i in computing the standard errors.[12] Specifically, we assume a factor structure for $\varepsilon_{(k,h)_{i,t}}$:

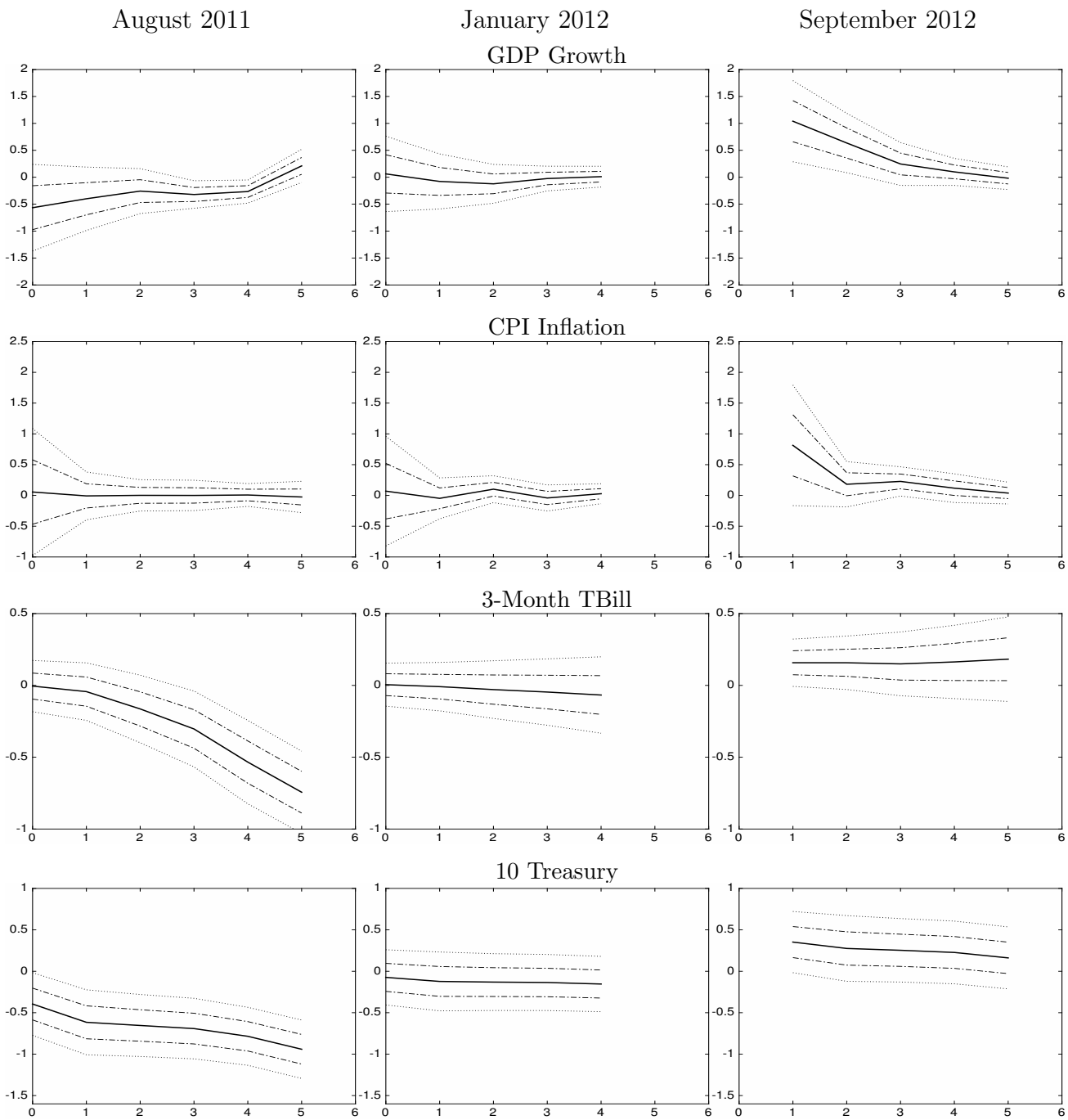
$$\begin{aligned}
\varepsilon_{(k,h)_{i,t}} &= e_{(k,h)_t} + \nu_{(k,h)_{i,t}}, \\
E[e_{(k,h)_t^2}] &= {\sigma^2}_{(k,h)_0}, \\
E[\nu_{(k,h)_{i,t}} \nu_{(k,h)_{j,t}}] &= \begin{cases} {\sigma^2}_{(k,h)_i} & \text{for } i = j \\ 0 & \text{otherwise,} \end{cases} \\
E[e_{(k,h)_t} e_{(k,h)_s}] &= E[\nu_{(k,h)_{i,t}} \nu_{(k,h)_{i,s}}] = 0, \text{ for } t \neq s. \quad (2)
\end{aligned}$$

We estimate ${\sigma^2}_{(k,h)_0}$ as the sample variance of $\hat{e}_{(k,h)_t} = \frac{1}{n} \sum_i \hat{\varepsilon}_{(k,h)_{i,t}}$, and ${\sigma^2}_{(k,h)_i}$ as the sample variance of $\hat{\nu}_{(k,h)_{i,t}} = \hat{\varepsilon}_{(k,h)_{i,t}} - \hat{e}_{(k,h)_t}$. We then use the $n \times n$ variance covariance matrix $\hat{\Sigma}_{(k,h)}$, whose diagonal and off-diagonal elements are $\hat{\sigma}^2{}_{(k,h)_0} + \hat{\sigma}^2{}_{(k,h)_i}$ and $\hat{\sigma}^2{}_{(k,h)_0}$, respectively, to construct the White (1980)-robust standard errors.

We can properly recover the effect of the announcement on expectations under the following assumptions: 1) the γ coefficients are stable over time and are consistently estimated using the panel regression, 2) the regressors capture all public (common) news over the one-month event window, 3) the effect of the announcement on financial markets is fully captured by the two-day window change in asset prices. Assumption (1) is standard, and we use only post-2008 data (that is, data that are mostly in the zero lower bound regime) to account for the fact that the γs may have been different in the pre-zero lower bound period. Assumption (2) states that the average residual $\varepsilon_{(k,h)_{i,t^e}}$ only captures the reaction to the event. This

[12] This is particularly important in this setting as forecasters likely speak to each other over the forecast period. Indeed, as we show in Table B4, standard errors adjusted in this way are much larger than those only taking into account heteroskedasticity.

Figure 2: The Effect of Forward Guidance Announcements on Expectations



Notes: The panels in Figure 2 show the estimates of $\beta(k,h)^e$ for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot. For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

assumption can be defended by including all possible sources of common information, especially asset price movements. Despite our many macroeconomic controls, it is unlikely that we cover all public information, but asset prices should be a sufficient statistic for all relevant public information, and therefore, their inclusion should help cover gaps. Finally, assumption (3) is taken from the event-study literature (Krishnamurthy and Vissing-Jorgensen (2011), Krishnamurthy and Vissing-Jorgensen (2013), Gagnon et al. (2010)). If it is violated because the impact of the event on asset prices extends beyond the two-day window, then some of the movements in ΔAP_{t^e} are attributable to the event. Vice versa, if the movements in the two-day window reflect information other than the event, then this information is not adequately captured in ΔAP_{t^e}.[13]

Under our assumptions, the mapping between the forecast horizon h and $\beta_{(k,h)}{}^e$ can be interpreted as an impulse response, as it shows the change in the projection for variable k following the FOMC announcement, $E[f(h,k)|\Omega_{t-1} \cup e] - E[f(h,k)|\Omega_{t-1}]$, over horizon h, where Ω_{t-1} represents the time $t-1$ information set of survey forecasters. This is a strong interpretation as it presumes we are able to perfectly control for all other factors affecting the forecast, but it provides a clear exposition and a good approximation.

The panels in Figure 2 show the estimates of $\beta_{(k,h)}{}^e$ for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. The sample for each regression is $t = 2008.06, .., 2015.02$. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.[14] For each triplet (e, k, h) we report the OLS estimate of $\beta_{(k,h)}{}^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using the previously described robust standard errors.[15]

As with the financial variables, Figure 2 shows that response of expectations are very different across events. In August 2011, the response of the expected 3-month TBill is muted

[13]We check the robustness of this assumption in Table B4.

[14]Note that for August 2011 and January 2012 we report the change in the nowcast ($h = 0$), that is, the forecast for the quarter in which the event takes place, for September 2012 this information is not available given that the first survey after the event is the October survey, for which the nowcast is the first quarter after the event. Hence the estimate for $\beta_{(k,h)}{}^e$ begin with the first quarter ($h = 1$).

[15]For the results in Figure 2 we run separate regressions for each event (that is, we use one dummy at the time), but the results are nearly identical if we use all dummies at the same time. See Table B4.

at short horizons, since rates were already expected to stay at the zero lower bound, but is very strong at longer horizon. This response has the same sign and pattern as those in section 2.1.1, although the magnitude is larger in terms of the point estimates.[16] Similarly, the response of the nominal 10-year Treasury rate is significantly negative and very persistent. Alternatively, the January 2012 announcement seems to produce little significant movement in expectations for either the 3-month TBill or the 10-year Treasury rates.[17] Lastly, in September 2012, expectations for the short-term rate increase by a small amount (less than 20 basis points) while those for the 10-year rate increase more substantially, in line with the reaction of financial markets to the announcement, but the change in expectations is quite imprecisely estimated.

The fact that both short and long-term nominal rates *increased* in September 2012 may seem puzzling: How can policy accommodation lead to an increase in rates? To understand this, it is useful to look at that response of inflation and output growth forecasts following the FOMC announcements. The August 2011 event lowers the GDP growth nowcast by about 0.6 percent, although the estimates are quite uncertain and the 90 percent bands include zero while the September 2012 event raised expectations by one percentage point and the January 2012 episode had no effect.[18] The effects in August 2011 and September 2012 were largest in the beginning, but the effect is quite persistent.[19] The response of CPI inflation to the August 2011 and January 2012 announcements was close to zero and insignificant, but the September 2012 episode increased inflation expectations by almost 1 percentage point.

One likely explanation for the large difference in the reaction of expectations and asset prices to the announcements has to do with the language used in the FOMC statements. This interpretation is similar to that in Campbell et al. (2012a), who distinguish between

[16]We should note that the actual forecasts revisions $\Delta f_{(k,h)_{i,t}}$, 5-quarters ahead, are larger than the adjusted revisions $\beta_{(k,h)}{}^{e} + \varepsilon_{(k,h)_{i,t}}$. As we would expect, the controls attribute some of the downward revisions to factors other than the forward guidance announcement.

[17]This is slightly at odds with the evidence on forward rates and nominal long term rates presented in the previous section, which show both as declining, although the extent of this decline is indeed much smaller than in August 2011.

[18]Note that the September 2012 response implies that the impulse response on the *level* of output is hump-shaped, with a peak at least 5 quarters after the shock, in line with the VAR evidence on the impact of policy shocks.

[19]Note that the first forecast pertains to the first quarter ($h = 1$) *after* the announcement in September 2012 because we do not have information about the change in the nowcast from September to October.

types of forward guidance based on the public's interpretation of the language within the statement. Table 1 shows a substantial change in the language associated with the interest rate announcement. The August 2011 FOMC statement emphasized negative news about economic activity but was more nuanced about inflation, while the September 2012 the language on economic conditions was more positive on economic progress but slightly more nuanced on current inflation.[20] The September 2012 statement was also more clearly stimulative by emphasizing that monetary policy would need to remain highly accommodative "for a considerable time after the economy strengthens." The statement also included an announcement of QE3, which would likely have contributed to the positive market response.[21] Additionally, as we discussed in Section 2.1.1, the safety premium rose following the August 2011 announcement while it fell after September 2012, again suggesting that news played a different role across these episodes. Together, using the language of Campbell et al. (2012a), these observations suggest that forward guidance in August 2011 was interpreted as "Delphic," in that is was interpreted as conveying bad news about the economy, while forward guidance in September 2012 was interpreted as "Odyssean," in that it was interpreted as a commitment to more monetary policy stimulus.[22]

2.2 Benchmarking Forward Guidance in Our Model

While the heterogeneity in the effect of various statements containing forward guidance is descriptively interesting, in order to move to an estimate that we can more directly compare to the impulse responses generated by our models in Section 3 and 4, we need to isolate the effect of stimulative forward guidance from the news that is associated with it. In theory, it could be the case that the modest estimates for the effect of forward guidance in the previous

[20] See Table B1 and B2 in the Appendix for the precise language

[21] Note however that as we discussed in the previous section, the reaction of financial markets to that language was quite different from that reported for other QE episodes, suggesting that forward guidance also played an important role.

[22] This response is – at least qualitatively – consistent with the one implied by DSGE models of the kind presented below. For instance, this pattern could be generated with change in the policy rule that would increase the response coefficient on the output gap. The announced change in the reaction function increases inflationary expectations more than nominal interest rates increase, thereby reducing the real rate and stimulating the economy. The increase in nominal rates is then simply the equilibrium response to a much improved state of the economy. However, as discussed below, such rational expectations DSGE models tend to exaggerate the impact of the change in the policy, as the expectational channel is very strong.

section are masking large and offsetting negative news shocks and policy stimulus. Therefore, our aim in this section is to estimate the effect of purely Odyssean forward guidance. In order to do this, we make more rigorous use of the language in the entire set of FOMC statements. We do this by imposing more structure on the error term and use that to decompose the overall statement effect into its various components. While this method relies on additional assumptions and therefore does not provide fully satisfactory estimates, it suggests that there are not large offsetting effects, and that we should expect the effect of purely stimulative forward guidance to have positive, yet not very large, effects on the economy.

Rather than comparing calendar dates in which there was a forward guidance announcement (D_t^e), we instead include a set of dummies that control for different aspects of the FOMC statement that have the potential to move expectations. Specifically, similar to equation (1), we estimate the following panel regression:

$$\Delta f_{(k,h)_{i,t}} = \gamma_{(k,h)_0} + \gamma_{(k,h)_1}' \, \Delta M_t + \gamma_{(k,h)_2}' \, \Delta AP_t + \gamma_{(k,h)_3}' \, Z_{(k,h)_{t,i}} + \beta_{(k,h)^e} \, POLICY_t^e + \varepsilon_{(k,h)_{i,t}}, \text{ for } t = 1, .., T, \; i = 1, .., n, \quad (3)$$

where $POLICY_t$ includes the following series of dummies: a dummy for the announcement of a forward guidance episode, a dummy for a quantitative easing announcement, a dummy for an announced continuation of quantitative easing, a measure of the language for output conditions, and a measure of the language for inflation conditions.[23] By utilizing other FOMC statements in which similar language was used or policies were announced, we can separately estimate the effect of the components of the statements and decompose the heterogeneity in the forward guidance episodes studied in Section 2.1. In addition to the necessary modeling assumptions discussed in section 2.1.2, this will accurately characterize the effect of Odyssean forward guidance as long as 1) the constructed statement dummies capture all other relevant aspects of the statement and 2) the statement effects of the statements are additive. We can attempt to check the validity of the first assumption by exploring alternate specifications. The second assumption is more heroic – if it is the case that quantitative easing is seen as a commitment device for forward guidance, then forward guidance that is announced with quantitative easing may be more effective than when it is announced alone. Similarly, if forward guidance that is announced with bad news is discounted more than forward guidance

[23] See the appendix for the details on how we constructed these variables. In the future, we can improve this by using machine learning techniques that may better quantify these categories.

that is announced with better news, then this equation would be misspecified. However, we do not have sufficient data to explore these interactions empirically, and the benchmark estimate for a homogenous effect is still informative.

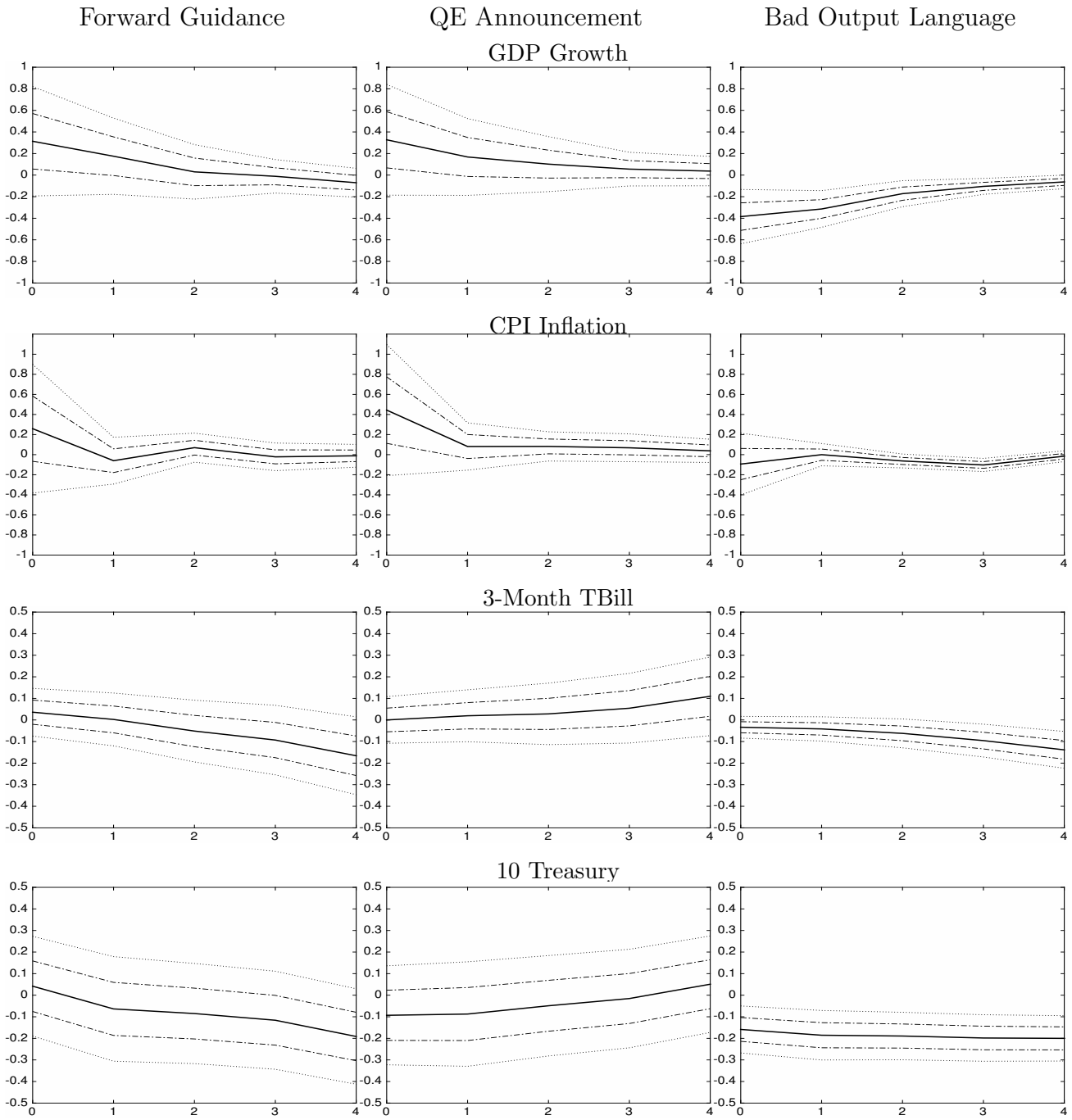
Figure 3 shows the impulse response functions implied by the β coefficients on the forward guidance dummy, the quantitative easing announcement dummy and the negative news about GDP dummy.[24] We see here that while the estimates are noisy, the coefficient relating both forward guidance and quantitative easing to GDP growth are slightly positive though statistically insignificant, with the point estimate indicating a 0.3 percent increase in output growth.[25] The coefficient for bad news about GDP is instead significantly negative. The coefficient relating both forward guidance and quantitative easing to inflation are also slightly positive while the coefficient for bad news about GDP is almost zero, perhaps reflecting the fact that news about inflation enters separately.

Although forward guidance announcements have little effect on the short-term rate, given that the announcements considered have been made while near the zero lower bound, the typical forward guidance announcement lowers the short-term rate four quarters ahead by about 15 basis points. While the long-term rate does not move much on impact either, it is also expected to be nearly 20 basis points lower after four quarters. These estimates suggest that forward guidance announcements have had economically meaningful effects, with GDP growth and inflation increasing by about 0.3 percentage points in response to a reduction of about 15 basis points in the expectation of short-term rates one year ahead. QE announcements have similar positive effects on GDP and inflation. One key difference with respect ot forward guidance though, is that QE announcements tend to lower the long-term rate on impact. Language suggesting bad news on GDP has negative and persistent effects on the 3-month and 10-year Treasury rate. This perhaps suggests that forecasters anticipate policy action when the statements are negative, leaving little room for further movements in expectations when policies are actually announced.

[24] See the Appendix Table B1 for the estimated coefficents on all policy dummies.

[25] Note that the forward guidance dummy here can be interpreted as the average residual change in the forecase in the three forward guidance episodes explored in Section 2.1 *after* controlling for the other aspects of the statement.

Figure 3: Decomposing FOMC Statements: The Effect of Forward Guidance, QE Announcements and Bad GDP Language



Notes: The panels in Figure 3 show the estimates of $\beta(k,h)^e$ for three different elements of the FOMC statement – forward guidance announcements, QE announcements and bad GDP language – and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

This analysis lends additional support for the "Delphic" and "Odyssean" interpretation of the heterogeneity in the effect of forward guidance across episodes as discussed in Campbell et al. (2012a). We find that the news about the economy do matter, with negative news inducing lower forecasters' expectations. However, the results also suggest that the effect of news, while important, is unlikely to be masking very large stimulative effects of purely Odyssean forward guidance. Indeed, even taking our point estimates seriously and translating market expectations to economic outcomes, we can only reasonably expect forward guidance involving a 15 basis point decrease in short-term rates 4 quarters ahead to produce meaningful positive effects on output growth and inflation, of about 0.3 percentage point. Indeed, we can rule out output growth and inflation responses that are above 1 percentage point.

3 The macroeconomic implications of interest rate announcements

We now proceed with an evaluation of the effects of extending the forward guidance focusing on the stimulative effects of policy, and abstracting from the possible effects of information conveyed by the FOMC regarding the assessment the state of the economy. In this section, we first briefly describe the DSGE model, its estimation, and the baseline forecasts. In particular we discuss the modification of the standard feedback rule describing monetary policy to allow for anticipated policy shocks, and how we incorporate current FFR market expectations into the forecast. Next, we show that when we condition the forecast on a specific interest-rate path the model produces results that are hardly credible, and explain why this is the case.

3.1 Model and baseline forecasts

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano et al. (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by

Bernanke et al. (1999). The actual implementation of the credit frictions closely follows Christiano et al. (2009). Detailed information about the equilibrium, the data, and the priors used in the Bayesian estimation of this model are contained in Del Negro et al. (2013). The appendix to this paper also includes the list of log linearized equilibrium conditions, as well as the priors and posteriors for the estimated parameters. In this section we focus on the features of the model that are needed to properly describe this exercise. In particular, we discuss: i) the state-space representation of the linearized DSGE model, ii) anticipated policy shocks, iii) incorporating market's FFR expectations into the baseline forecast.

The solution to the log-linear approximation of the model's equilibrium conditions around the deterministic steady state (obtained using the method in Sims (2002)) yields the following transition equation:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \tag{4}$$

where s_t is the model's vector of "state" variables, the matrices Φ_1 and Φ_ϵ are functions of the vector of all model parameters θ, and ϵ_t is the vector of structural shocks. The vector of observables y_t described below is in turn related to the states according to the system of measurement equations:

$$y_t = \Psi_1(\theta) + \Psi_2(\theta)s_t. \tag{5}$$

The variables included in y_t are: 1) annualized real GDP per capita growth, where the real gross domestic product is computed as the ratio of nominal GDP (SAAR) to the chain-type price index from the BEA;[26] 2) the log of labor hours, measured as per capita hours in non-farm payroll; 3) the log of labor share, computed as the ratio of compensation of employees to nominal GDP, from the BEA; 4) the annualized rate of change of the core PCE deflator (PCE excluding food and energy, but including purchased meals and beverages), seasonally adjusted; 5) the effective federal funds rate, percent annualized, computed from daily data; and 6) the spread between the Baa rate and the rate on 10 year Treasuries. We estimate the vector of model parameters θ using data from 1984Q1 to 2012Q3 using Bayesian methods as described in Del Negro and Schorfheide (2010), applied to the state-space representation of the linearized DSGE model provided by equations (4) and (5).

Starting in 2008Q3 (one period before the implementation of the zero lower bound) we incorporate FFR market expectations, as measured by OIS rates, into our outlook following

[26]Per capita variables are obtained by dividing through the civilian non-institutionalized population over 16. We HP-filter the population series in order to smooth out the impact of Census revisions.

the approach described in Section 5.4 of Del Negro and Schorfheide (2013).[27] These data contain valuable information for the estimation of the state of the economy, as the market expectations of continued low interest rates reflect both a relatively weak economy as well as an accommodative monetary policy.

These market expectations are assumed to be driven by the policy rule that the Cental Bank is expected to follow as well as on the deviations from that rule that the Central Bank has already communicated in its forward guidance. Specifically, we assume that the Central Bank sets the short-term interest rate according to the following feedback rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \quad (6)$$

where $\sum_{j=0}^{3} \hat{\pi}_{t-j}$ is 4-quarter inflation expressed in deviation from the Central Bank's objective π_* (which corresponds to steady state inflation), $\sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$ is 4-quarter growth rate in real GDP expressed in deviation from steady state growth, and $\epsilon_{R,t}$ is the standard contemporaneous policy shock, where $\epsilon_t^R \sim N(0, \sigma_r^2),\ i.i.d.$.[28] The last term captures forward guidance following Laseen and Svensson (2011), where $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume that $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2),\ i.i.d.$ Note that we make the – arguably counterfactual – assumption that the anticipated shocks are independent from one another. Campbell et al. (2012b) argue, based on their own findings as well as Gürkaynak et al. (2005b)'s, that anticipated shocks follow a factor structure. It would be important to relax the independence assumption if we were to estimate the model with forward guidance shocks. However, this assumption bears no implications in the policy exercise described in sections 3.2 and 3.3.[29]

For simplicity we estimate the model parameters assuming no forward guidance – that

[27] Specifically, we take FFR expectations up to K quarters ahead into account by augmenting the measurement equation (5) with the expectations for the policy rate: $FFR_{t,t+k}^e = 400\left(\mathbb{E}_t \widehat{R}_{t+k} + \ln R_*\right)$, where $FFR_{t,t+k}^e$ denotes data on market's expectations for the FFR k quarters ahead.

[28] The economy displays a stochastic trend, so if $\hat{y}_{t-j}$ is output in deviation from this trend and $\hat{z}_t$ corresponds to the growth rate of technology in deviations from steady state, then the growth rate of output in period t is $\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$.

[29] In this log-linearized model the variance-covariance matrix of the shocks does not affect the equilibrium conditions.

Table 3: The model-implied consequences of forward guidance

	2012 (Q4/Q4)	2013 (Q4/Q4)	2014 (Q4/Q4)	2015 (Q4/Q4)
GDP growth				
Baseline	1.9	2.2	1.7	1.3
Forward guidance	3.5	4.9	1.5	0.3
Core PCE inflation				
Baseline	1.6	1.2	1.5	1.6
Forward guidance	1.8	1.9	1.8	1.7
Federal funds rate				
Baseline	0.10	0.20	0.40	1.32
Forward guidance	0.25	0.25	0.25	1.09

Notes: The table reports the model's predictions conditional on alternative assumptions regarding the federal funds rate: the baseline forecast, and a counterfactual policy experiment in which the federal funds rate is maintained at 25 basis points until 2015Q2.

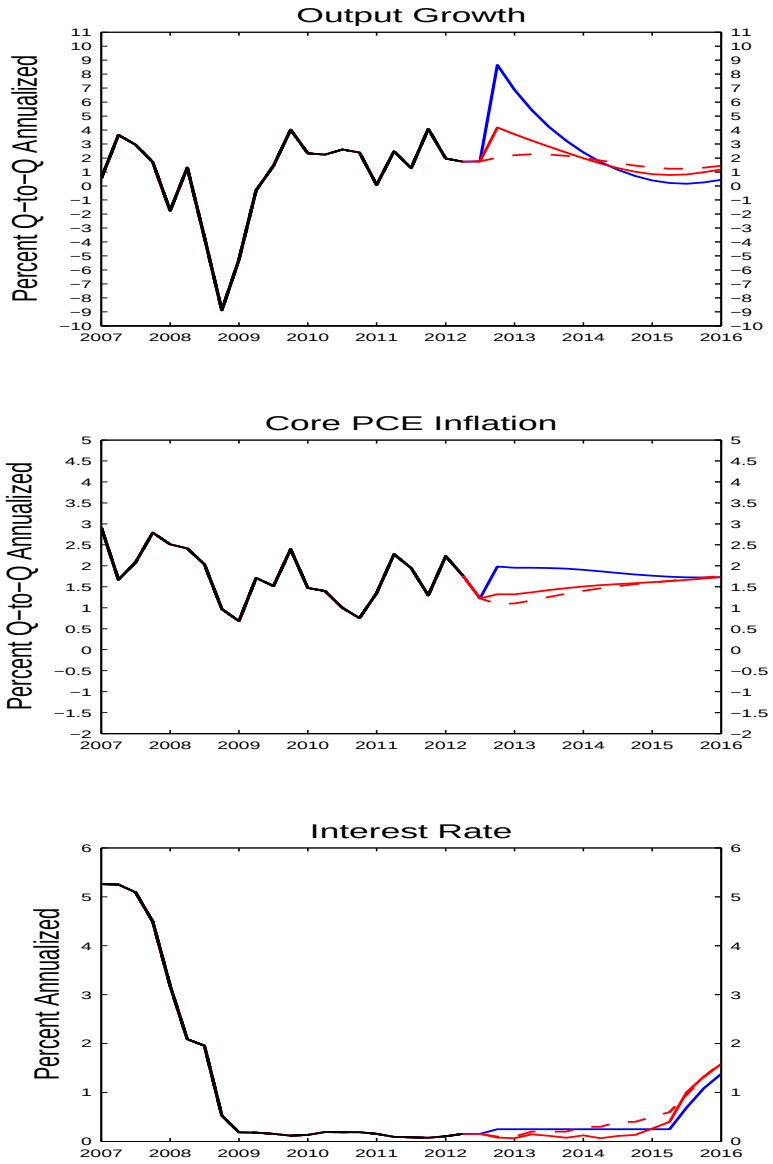
is setting the last term in (6) to zero –, and without data on expected future policy rates. Implicitly we are assuming that forward guidance has little impact on the estimated model parameters. We are however recognizing that it has a potentially large impact on our inference about the state of the economy s_t in the 2008Q3-2012Q3 period (conditional on the estimated parameters), and hence on the model's forecasts. We are therefore re-estimating the state s_t in that period using expectations of future federal funds rates.[30] Our baseline forecast, which is described in Table 3 and Figure 4, is therefore obtained using data released through 2012Q2 augmented for 2012Q3 with observations on the federal funds rate and the Baa corporate bond spread, and with market's FFR expectations through mid-2015 (hence $K = 11$ in equation (6)) as measured by OIS rates on August 28, 2012.[31]

Figure 4 shows the model's predictions for real GDP growth, core PCE inflation and

[30]The only extra parameters introduced by the forward guidance are the standard deviations $\sigma_{k,r}$ of the anticipated shocks. Since we do not have estimates for these parameters, we assume that these shocks have the same standard deviation as the contemporaneous shock: $\sigma_{k,r} = \sigma_r$. Importantly, note that the parameters $\sigma_{k,r}$ do not enter *any* of the policy experiments described below.

[31]As 2012Q3 observations for the the FFR and the Baa corporate bond spread we are using the average of daily rates during the quarter up to this date.

Figure 4: The model-implied consequences of forward guidance



Notes: The figure shows the model's predictions conditional on alternative assumptions regarding the federal funds rate. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. The solid blue lines show in turn the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red lines show the model's predictions in a counterfactual policy experiment in which more forward guidance is provided about the federal funds rate such that the 10-year bond yield falls by 10 basis points.

the federal funds rate, conditional on alternative assumptions regarding the federal funds rate. These forecasts are obtained using the mode of the posterior distribution for θ and s_t, although these modal forecasts in the baseline case essentially coincide with the mean of the

forecast distribution obtained by drawing from the full posterior of θ and s_t. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. In this forecast, GDP growth is 1.9 percent in 2012 (Q4/Q4), rises to 2.2 percent in 2013 but remains mostly below 2 percent throughout the rest of the forecast horizon (see the first row in each of the three panels of Table 3). Core PCE inflation is predicted to be at 1.6 percent in 2012 and is also expected to remain below 2 percent throughout the forecast horizon.

3.2 Using anticipated shocks to condition on an interest-rate path

We now proceed with our counterfactual policy experiment in which the federal funds rate is set to 25 basis points (the current rate paid on excess reserves held at the central bank, or IOR) until 2015Q2, and that it follows the historical policy rule after that.[32] We first summarize the procedure used to condition the model's predictions on a given interest-rate path, which is taken from section 6.3 of Del Negro and Schorfheide (2013), and then describe the outcome of the experiment.

Suppose that at the end of period T, after time T shocks are realized, the central bank announces its intention to commit to a given interest-rate path: $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$. For the agents, the announcement is a one-time surprise in period $T+1$. This corresponds to the realization of a single unanticipated monetary policy shock ϵ^R_{T+1} and a sequence of anticipated shocks $\{\epsilon^R_{1,T+1}, \epsilon^R_{2,T+1}, \ \ldots, \ \epsilon^R_{K,T+1}\}$ where $K = \bar{H} - 1$. Notice that all policy shocks that are used to implement the interest rate path are dated $T+1$. The Appendix describes an algorithm that determines the time $T+1$ monetary policy shocks needed to to generate predictions conditional on a given interest rate path, so that the announced interest rate path will be attained in expectation.

The solid blue lines in Figures 4 show in turn the model's predictions in our counterfactual policy experiment. Such a policy change would imply a reduction in the expected federal funds rate of 15 basis points at the end of 2014 compared to the baseline forecast. According

[32] At the time we wrote the paper, this was one policy option discussed by market commentary for the upcoming FOMC meeting, see the September 10, 2012 WSJ "MarketBeat" Blog at `blogs.wsj.com/marketbeat//qe3-what-everybody-that-matters-on-wall-street-expects/`. We chose 25 basis points for simplicity as it coincides with the IOR, but of course choosing any lower rate would make the results even stronger as the policy would be even more accommodative.

to the model, this alternative policy assumption generates a massive stimulus in 2012 and 2013. Indeed, in this alternative scenario, real GDP growth is forecast to jump to 3.5 percent in 2012 (Q4/Q4), and to 4.9 percent in 2013. GDP growth is however lower than under the baseline scenario in 2014 and 2015, as the effects of the policy stimulus fade over time and the GDP *level* returns to the level it would have had without the stimulus (see the second row in each of the three panels of Table 3). The stimulative effect of policy also raises inflation in 2012 and 2013 to respectively 1.8 percent (Q4/Q4) and 1.9 percent, but inflation is also forecast to remain below 2 percent in 2014 and 2015. The model seems to be generating an implausibly large response of real GDP growth and inflation to an apparently small change in the federal funds rate.

3.3 What is the excessive response due to?

To understand this, consider a simplified version of the FRBNY DSGE model in which there is no habit persistence and no shocks other than monetary policy shocks. In this case, the consumption Euler equation reduces to the conventional expression

$$\hat{c}_t = I\!E_t[\hat{c}_{t+1}] - (\hat{R}_t \ - \ I\!E_t[\hat{\pi}_{t+1}]), \tag{7}$$

where $\hat{c}_t$ denotes consumption deviations from steady state. Iterating this equation forward to eliminate expected future consumption, we obtain

$$\hat{c}_t = -\sum_{j=0}^{\infty} I\!E_t[\hat{R}_{t+j} \ - \ \hat{\pi}_{t+1+j}], \tag{8}$$

so that contemporaneous consumption is directly negatively related to the long-term real interest rate (at infinite maturity), which is given by $\hat{r}_t^L = \sum_{j=0}^{\infty} I\!E_t[\hat{R}_{t+j} \ - \ \hat{\pi}_{t+1+j}]$. It follows that anticipated changes in the short-term real rate in the future affect consumption both today and in the future. To see this, suppose for now that prices are fixed so that the nominal and real interest rates move by the same amount, and suppose that the short-term rates declines by $\hat{R}_t = -\Delta$ contemporaneously but reverts to steady state $\hat{R}_{t+j} = 0$ after that. In this case, the long run real rate would also decline by Δ in period t and revert to steady state after that, so that consumption would increase temporarily by Δ in period t. Consider now an the announcement of a temporary decline in the short-term rate at date

$t+j$ of $\hat{R}_{t+j} = -\Delta$. In that case, the long-term real rate $\hat{r}_t^L$ would decline from periods t to $t+j$, before reverting to steady state from $t+j+1$ onward. This implies that consumption would increase by Δ in period t and remain at that level until $t+j$, before reverting to steady state.

Now letting prices adjust, a New Keynesian Phillips curve, would imply that inflation also increases on impact as demand for goods and services is expected to surpass the potential output of the economy from periods t to $t+j$. The persistent increase in inflation induces the real interest to fall by more than the nominal interest rate, which reinforces the effect of the initial policy shock. Countervailing this force however is the effect of monetary policy which tends to raise the real rate as inflation increases. All these effects are playing out in the medium scale model.

While we have focused here on the response of consumption, real investment is also related to the long-term real interest rate in the medium scale model. A natural question is then whether the strong response of economic activity in the model to changes in the near-term path of the short-term interest rate is due to too strong a response of consumption and inflation to given changes in the long-term interest rate, or alternatively to too strong a response of the long-term interest rate.

Looking at the model's interest-rate projections farther into the future provides valuable insights. Figure 5 shows the paths of short-term interest rates under the baseline projection (red dashed lines), and the counterfactual policy (blue solid line) until 2027Q4. This figure reveals that while the expected short-term rate is only 15 basis points lower in the counterfactual than in the baseline at the end of 2014, the difference between the two interest-rate paths is expected to be much larger farther in the future, in particular between 5 and 10 years following the current policy announcement. These large drop far in the future of the expected future short-term rate compared to the baseline path is in turn resulting in a large drop of the long-term interest rate.

To see this more clearly, we compute the long-run interest rate response, proceeding as follows. At the end of period T, after the realization of all period-T shocks, the pre-intervention interest rate with maturity L at date $T+1$ is computed as the average of future short-term rate over the relevant horizon, and is given by the following expression:

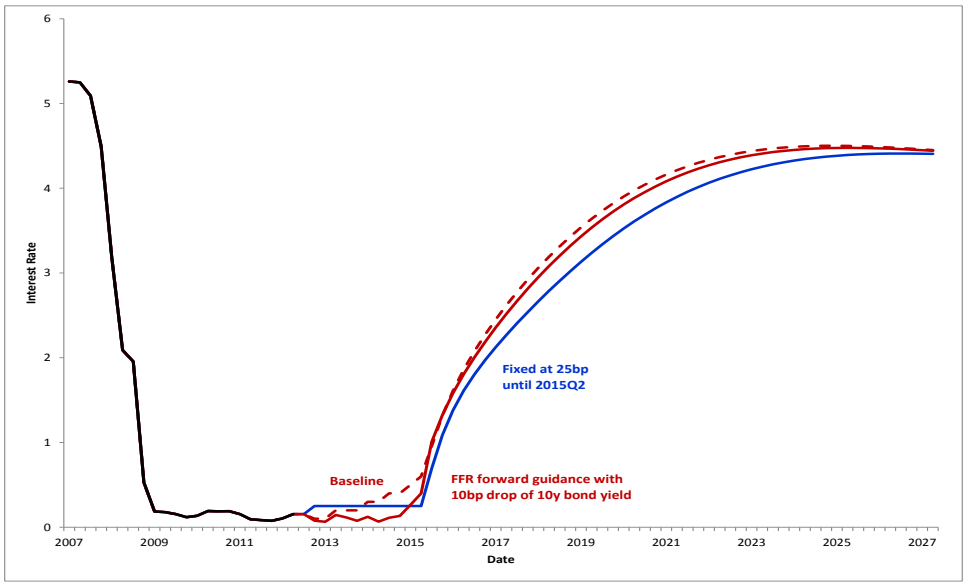
$$R_{T+1}^L = \frac{1}{L}\sum_{j=1}^{L} I\!E_T[R_{T+j}] \tag{9}$$

The post-intervention long-term rate R_T^{L*}, i.e., the rate obtained after the announcement of period-$T+1$ policy shocks $(\bar{\epsilon}_{T+1}^R, \bar{\epsilon}_{1:K,T+1}^{R\,\prime})$ is given by:

$$R_{T+1}^{L,*} = \frac{1}{L}\sum_{j=1}^{L} I\!E_T[R_{T+j}|\bar{\epsilon}_{T+1}^R, \bar{\epsilon}_{1:K,T+1}^{R\,\prime}] \tag{10}$$

In the counterfactual experiment, the 5-year yield falls by 16 basis points upon the announcement, compared to the baseline scenario, and the 10-year yield falls by as much as 25 basis points. The fact that the 10-year yields falls by more than the 5-year yield is simply a reflection, again, that the short-term interest rate is expected to deviate more from the baseline at long horizons than in the near term, as shown in Figure 5.

Figure 5: Interest-rate projections farther into the future



Notes: The figure shows the model's predictions for the federal funds rate farther into the future. The black solid line shows the historical data. The dashed red line shows the FRBNY DSGE model's baseline forecast. The solid blue line shows the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red line shows the model's predictions in a counterfactual policy experiment in which the 10-year bond yield falls by 10 basis points.

The model-implied responses for the long-term rate do not seem to match the 5 and 10-year yield responses observed in the data, however. While the movement in the long rates was similarly large in September 2011, as we discussed in Section 2, this episode likely contained bad news beyond the announcement of forward guidance, and therefore is not the relevant benchmark for forward guidance as we have modeled it here. Rather, it is more

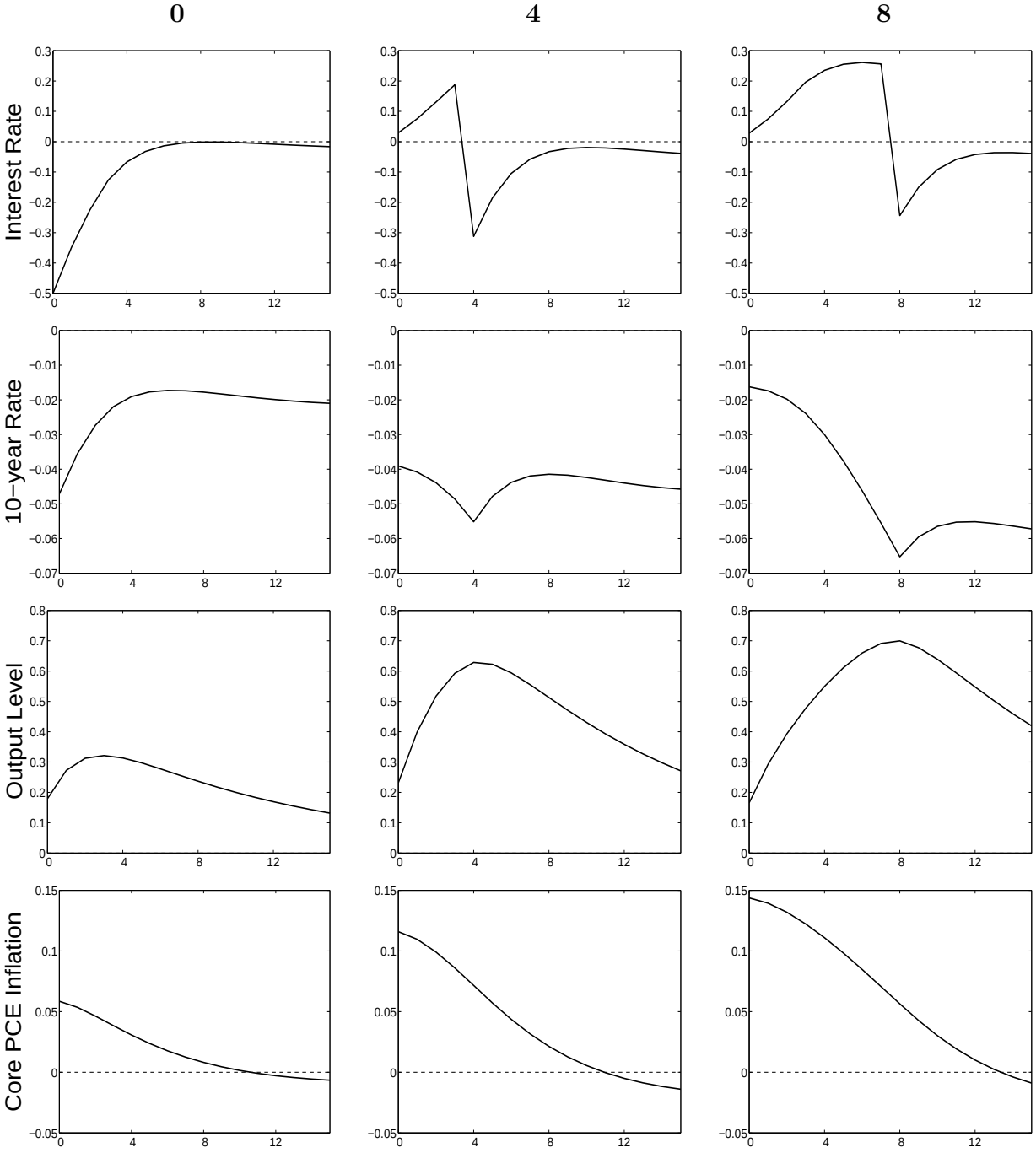
accurate to compare this to the estimated effect that forward guidance had on forecasters expectations of long rates. In Section 2.2, where we controlled for other element of the statements, we found that forward guidance had a much smaller effect on the long rate.[33]

Figure 6 shows the impulse response functions to contemporaneous and anticipated policy shocks in order to provide some more intuition for what is happening. Specifically, the figure shows the response of the short term interest rate, the 10-year nominal rate, the level of output, and inflation to expansionary 50 basis points shocks. The difference between the three columns in Figure 6 is that this 50 basis points shock is either contemporaneous (left column) or anticipated 4 and 8 periods ahead (middle and right columns, respectively). We want to highlight four features of Figure 6: i) Since the anticipated expansionary shock leads to higher inflation and output *before the shock takes place*, and since the policy authorities are bound to follow the rule before that date, the interest rate follows a zig-zag pattern, where it first rises and then falls. If this pattern of interest rates appears awkward, bear in mind that we are unlikely to see an eight periods-ahead shock *in isolation* (e.g., Campbell et al. (2012b)). ii) The response of the 10-year rate, quite understandably, reaches its lowest point at the time the shock takes place, and the trough decreases monotonically with the anticipation horizon. iii) The peaks in the response of output roughly coincide (with a slight delay) with the peaks in the response of the 10-year rate, in agreement with equation (8). In addition, the effect on output increases monotonically with the horizon. The delay is due to features like habit persistence. iv) The impact on inflation also increases monotonically with the horizon – not a surprising finding given the output responses.

The responses in Figure 6 provide some economic intuition behind the finding of Carlstrom et al. (2012) that the response of macroeconomic variables to an interest rate peg

[33]As mentioned in the introduction, Carlstrom et al. (2012) assess the impact of pegging the policy rate in three variants of the New Keynesian model. They start from the steady-state equilibrium and analyze the effect of lowering the policy rate to the ZLB for K quarters. In their calibration, this amounts to lowering the policy rate by 4 percentage points for K quarters. In the simple version of the New Keynesian model, it can be shown that the response of inflation or output is directly proportional the distance between the steady state interest rate and the level at the ZLB, and the response of inflation and output grows exponentially with the number of periods that the policy rate is expected to be maintained at the ZLB. Since the short-term rate is assumed to return back to steady state $K + 1$ periods after the announcement, the 10-year long-run nominal rate is assumed to fall by $400K/40 = 10K$ basis points in their experiment. Concretely, their experiment assumes that a forward guidance of 8 quarters would imply a drop of 80 basis points in the 10-year bond yield.

Figure 6: Impulse response functions to contemporaneous and anticipated policy shocks



Notes: The figure shows the percent change over a 12 quarter horizon of the short term interest rate, the 10-year nominal rate, the level of output, and Core PCE inflation in response to a contemporaneous, 4 quarter and 8 quarter ahead negative 50 basis points policy shock.

is a convex function of the horizon of the peg. Imagine the policymakers want to lower interest rates by 50 basis points for 7 periods. This can be implemented with a sequence of contemporaneous and anticipated shocks up to 7 periods ahead. Now imagine they decide to extend the peg one extra period. Because of the zig-zag feature of the 8th period impulse response, that decision will tend to lift the short-term rate in quarters 0 to 7 and so requires a cascade of shocks over that period to push the interest rate back down. In light of these impulse responses (and the related impact on the long rate) it is not surprising that even a modest amount of forward guidance produces large effects, as long as it extends far enough into the future.

4 A Proposed Resolution

We propose here a potential resolution to the "forward guidance puzzle" described above, by introducing an overlapping generations structure. More specifically, we adopt Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Nisticò (2012) and Piergallini (2006). We assume, as in these papers, that the economy is populated by an indefinite number of cohorts that each period face a constant probability of dying and being replaced before the next period begins. While each individual agent satisfies their Euler equation, the cohort structure implies that the aggregate economy discounts future consumption more heavily. Compared to standard models, a positive death probability also induces higher discounting of future expected dividends in a linearized asset pricing condition, and leads price setters to discount more future marginal costs, in setting their prices. An implication of this is that announcements of policy changes far in the future tend to generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents.[34]

[34]While our explanation of the forward guidance puzzle retains a rational expectations framework, other authors have recently proposed a resolution to this puzzle by assuming boundedly rational agents. In Gabaix (2015), agents use a simplified model of the world as a default and consider parsimonious enrichments to this model paying attention only to those enrichments that matter enough for their decisions. In making consumption and saving decisions, these agents pay limited attention to the interest rate and income. This yields a more muted response of agents' consumption and hence of inflation to announcements about future interest rate changes. Gabaix's application to a simple New Keynesian model implies a discounted Euler equation similar to the one presented here, where the rate of discounting depends on the degree of

In order to explore the effect that this additional feature has on the implied effect of forward guidance, we first present a simple model in which we can clearly discuss the intuition. Then, in order to quantify the empirical relevance of this mechanism, we calibrate a DSGE model and show that for reasonable calibrations, the perpetual-youth modification yields more reasonable estimates for the effect of forward guidance.

4.1 Intuition

We begin with a simple description of the demand side, which is intended to clearly illustrate the effect that a perpetual youth structure has on forward guidance. We assume that in every period j, a new cohort is born with mass p, and each cohort has a constant probability p of dying which does not depend on j. Hence, in period t, the mass of survivors from cohort j is $p\,(1-p)^{t-j}$.[35]

Household's utility is given by:

$$\sum_{s=0}^{\infty}(\beta(1-p))^{s}\log(C_{j,t+s}) \tag{11}$$

as the future is discounted at the rate $\beta(1-p)$ to take into account the probability of dying. Since death is random, households will generally die with positive wealth. We follow Blanchard (1985) and assume that households buy fair annuities, which they will want to do because there is no bequest motive. Under this contract, agents receive a fraction p of their current wealth in each period in which they are alive in exchange from paying their entire wealth upon their death. Therefore, the budget constraint for cohort j is:

$$S_{j,t+1} \leq \frac{R_t}{1-p}(S_{j,t}+Y_t-C_{j,t}), \tag{12}$$

attention that households give to movements in interest rates and income, instead of the death probability. García-Schmidt and Woodford (2015) also propose a resolution to the forward guidance puzzle by considering deviations from rational expectations. They assume an explicit cognitive process for the formation of expectations according to which agents beliefs are adjusted in the direction of the discrepancy between the model prediction and the conjectured beliefs. They show that in the simple New Keynesian model, a commitment to keep interest rates low should raise inflation and output but by less than under perfect foresight as long as agents' expectations havent fully adjusted to the perfect foresight expectations.

[35]Note that the total population is normalized to one, since $\sum_{j=-\infty}^{t} p\,(1-p)^{t-j} = p\sum_{k=0}^{\infty}(1-p)^{k} = 1$, where $k = t-j$.

where $S_{j,t}$ is the period t wealth for cohort j, $C_{j,t}$ is consumption, Y_t is income (which is the same across cohorts), and importantly, $R_t/(1-p)$ is the rate of return for the household, where the $1/(1-p)$ term comes from the annuity contract.[36]

As usual, the individual maximizes her utility subject to her budget constraint. The first order condition yields the familiar consumption Euler equation:

$$\beta \frac{C_{j,t}}{C_{j,t+1}} R_t = 1. \tag{13}$$

Note that this is the same Euler equation from a standard representative agent model. The probability p does not show up directly in this equation, as the increased discounting at the individual level is cancelled out by the increased wealth from the annuity contract.

Importantly, the aggregate counterpart of the individual budget constraint (12) is

$$S_{t+1} = R_t(S_t + Y_t - C_t). \tag{14}$$

Aggregate wealth accumulates at the rate R_t, as opposed to $R_t/(1-p)$, because the annuity is just a cross-cohort transfer, which nets out for the aggregate economy.

Assume that we are in a small open economy where $R_t = R = \beta^{-1}$, and where income is constant over time and across cohorts and equal to Y. In this economy – unlike in the representative agent version – wealth has to be zero and $C_{j,t} = Y$ (as opposed to $C_{j,t} = Y + (R-1)S$ for some indeterminate level of wealth as in the representative agent model).[37]

Imagine that in this economy, the path for R_t is altered so that R_T is reduced to $R\Delta$, where $\Delta < 1$, at date T and otherwise $R_t = R$ at all other dates. Figure 7 shows the simulated impulse responses to this future change in the interest rate for both the individual and for the aggregate economy, when $T = 10$. The blue line shows the impulse responses for $p = 0.15$ and the red for $p = 0.0001$. As the bottom-left panel of the figure illustrates, the individual Euler equation (13) implies that each cohort's consumption level rises immediately, is constant until T and decreases to a constant lower level after T. When interest rates are

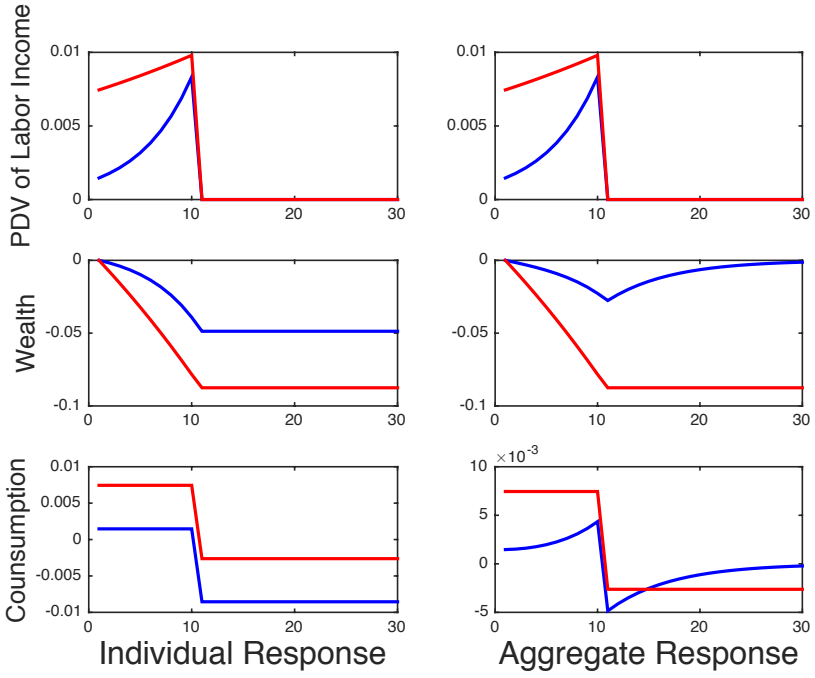
[36] To understand why, note that $1/(1-p) = 1 + p/(1-p)$ where for any unit of wealth carried over from the previous period, agent j receives in addition $p/(1-p)$ units from the life insurance which collects a fraction p of the financial wealth each cohort (as a fraction p of the agents in each cohort dies) and divides it up among the $(1-p)$ remaining agents.

[37] Intuitively, new cohorts are born with no wealth but a steady stream of income, and if they are to keep consumption constant over time, they have to consume Y in each period.

anticipated to be lowered in the future, agents who want to smooth their consumption over time, want to start enjoying higher consumption immediately, and reduce their wealth accordingly. However, a lower future interest rate tends to increase the household's present discounted value of future labor earnings, as shown in the top-left panels of Figure 7. In the following period, the effect on consumption of the lower wealth cancels out with the fact that the present discounted value of labor earnings is larger. Together, this implies that consumption stays the same until T. After period T, when the interest rate rises back to R, the effect on labor income disappears, and the lower wealth leads to lower consumption.

Comparing the blue and the red lines, we see clearly that the change in consumption today is a decreasing function of p. When p is higher, the payout from the annuity contract is higher providing greater incentives to save. Similarly, with a higher p, households discount future income at a higher rate so that the effect on the present value of income is smaller the farther into the future is the change in R. Households thus tend to raise their consumption by less, the higher p.

Figure 7: Forward Guidance in Blanchard-Yaari Models: Individual vs. Aggregate Response



Notes: The figure shows impulse response functions to an anticipated drop in interest rates 10 quarters in the future. The red line shows the simulation with $p = 0.0001$ and the blue line shows the impulse responses with $p = 0.15$.

The red lines in the right panel of Figure 7 shows that with positive p, the aggregate response in consumption differs from the individual response. Intuitively, while each individual increases their consumption, a fraction p dies and is replaced by a new cohort, who was not able to increase their consumption by reducing their wealth in the first period. Therefore, they feel only the effect of the higher present discounted value of labor income, due to the lower discounting, but not the reduction in wealth. Their consumption is thus higher than that of the older cohorts. This generational effect continues growing until T, leading to a progressively large aggregate consumption response as time approaches T.

This simple model illustrates the mechanisms behind this perpetual youth structure that help mediate the forward guidance puzzle. The mechanism is not driven only by the increased discounting at the individual level, but rather the compositional effect plays a major role. While each existing cohort is on the standard Euler equation, some of the total wealth increase stemming from the future increase in the interest rate accrues to cohorts who are not yet born, and therefore who cannot drop their current consumption as they would have wanted. This leads to a dampening of the initial consumption response.

4.2 The Smets and Wouters (2007) Model with Perpetual Youth

We now insert the perpetual youth households into the framework of Smets and Wouters (2007) and Christiano et al. (2005). For convenience we simplify the household's problem by separating the consumption/saving decision from the portfolio allocation decision. The latter is made by "mutual funds" on behalf of the households. We describe the problem of the households, that of the mutual funds managers and, since this model is non-Ricardian, tax policy. The remainder of the model is identical to Smets and Wouters (2007) or, in the case where we have financial frictions, to Del Negro et al. (2015) and Del Negro and Schorfheide (2013), and is described in the Appendix.

4.2.1 Households

In every period j a new cohort is born with mass p, and each cohort has a constant probability of dying p which does not depend on j.

The budget constraint for members of cohort j

$$C_{j,t} + Q_t S_{j,t} \leq \frac{(Q_t + D_t)}{1-p} S_{j,t-1} + \frac{W_t^N}{P_t} L_{j,t} - \mathcal{T}_t, \tag{15}$$

where $S_{j,t}$ represents shares of cohort j of a "mutual fund" offering dividends D_t, Q_t is the value of the shares in terms of consumption goods; $C_{j,t}$ and $L_{j,t}$ are consumption and labor supply for cohort j, respectively; W_t^N, $\mathcal{T}_t$, and P_t are nominal wages, lump sum taxes (which are the same across cohorts), and the price level in period t. The return on the mutual fund for each member of cohort j still alive is given by $\frac{(Q_t + D_t)}{(1-p)Q_{t-1}}$ because, as in Blanchard (1985), they have entered an annuity contract by which the fraction p of members dying in each periods leaves its wealth to those who remain alive.

Household's utility is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1-p))^s \left[\log(C_{j,t+s} - hC_{t+s-1}) + \nu \log(1 - L_{j,t+s})\right] \tag{16}$$

where the future is discounted at the rate $\beta(1-p)$ taking into account the probability of dying, and where the parameter h captures (external) habits. The individual Euler equation (for all $s = 0, ...$) is

$$\mathbb{E}_t \left[\frac{Q_t}{C_{j,t} - hC_{t-1}} - \beta \frac{(Q_{t+1} + D_{t+1})}{C_{j,t+1} - hC_t} \right] = 0 \tag{17}$$

and the intratemporal condition is

$$\frac{W_t^h}{C_{j,t} - hC_{t-1}} = \nu \frac{1}{1 - L_{j,t}}. \tag{18}$$

where $W_t^h = \frac{W_t^N}{P_t}$ is the real wage paid to households. Define cohort j's stochastic discount factor, conditional on surviving, as:

$$\mathcal{F}_{j,t,t+s} = \beta^s \frac{C_{j,t} - hC_{t-1}}{C_{j,t+s} - hC_{t+s-1}}. \tag{19}$$

Following Piergallini (2006), Nisticò (2012), and Castelnuovo and Nistico (2010), one can show that the policy function for consumption amounts to

$$C_{j,t} - hC_{t-1} = \frac{1 - \beta(1-p)}{(1+\nu)} \left(\frac{(Q_t + D_t)}{1-p} S_{j,t-1} + H_{j,t} \right), \tag{20}$$

where

$$H_{j,t} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} (1-p)^s \mathcal{F}_{j,t,t+s} (W_{t+s}^h - \mathcal{T}_{t+s} - hC_{t+s-1}) \right]. \tag{21}$$

Aggregation

Consider any variable $X_{j,t}$ for cohort born at time j and define the aggregate variable

$$X_t \equiv \sum_{j=-\infty}^{t} p\,(1-p)^{t-j}\, X_{j,t}. \tag{22}$$

Define $\mathcal{F}_{t,t+s}$ as the population weighted average of the cohort-specific stochastic discount factors:

$$\mathcal{F}_{t,t+s} = \sum_{j=-\infty}^{t} p\,(1-p)^{t-j}\, \mathcal{F}_{j,t,t+s}, \tag{23}$$

and note that the Euler equation (17) can be rewritten as

$$I\!E_t\left[\mathcal{F}_{t,t+1}\frac{(Q_{t+1}+D_{t+1})}{Q_t}\right] = 1. \tag{24}$$

Since (20) is linear in wealth, we obtain

$$C_t - hC_{t-1} = \frac{1-\beta(1-p)}{(1+\nu)}\left(\mathcal{S}_t + H_t\right). \tag{25}$$

where total financial wealth at the beginning of period t is defined as

$$\mathcal{S}_t = (Q_t + D_t)\, S_{t-1} \tag{26}$$

and

$$H_t = I\!E_t\left[\sum_{s=0}^{\infty}(1-p)^s \mathcal{F}_{t,t+s}(W^h_{t+s} - \mathcal{T}_{t+s} - hC_{t+s-1})\right]. \tag{27}$$

Aggregating the household's budget constraint and combining it with the Euler condition (24) and the intratemporal condition (18) we obtain a law of motion for financial wealth

$$(1+\nu)C_t - \nu h C_{t-1} + I\!E_t\left[\mathcal{F}_{t,t+1}\mathcal{S}_{t+1}\right] = \mathcal{S}_t + W^h_t - \mathcal{T}_t, \tag{28}$$

Equations (25) and (28) describe the evolution of aggregate consumption and wealth in this economy for given W^h_t and $\mathcal{T}_t$, together with the Euler equation (24), and the law of motion for H_t, namely

$$H_t = W^h_t - \mathcal{T}_t - hC_{t-1} + (1-p)I\!E_t\left[\mathcal{F}_{t,t+1}H_{t+1}\right]. \tag{29}$$

In fact, H_t can be eliminated from the equilibrium conditions by replacing (25) with

$$C_t - hC_{t-1} = \frac{p}{1-p}\frac{(1-\beta(1-p))}{\beta(1+\nu)}I\!E_t\left[\mathcal{F}_{t,t+1}\mathcal{S}_{t+1}\right] + I\!E_t\left[\beta^{-1}\mathcal{F}_{t,t+1}(C_{t+1} - hC_t)\right], \tag{30}$$

which obtains by combining (25), (28), and (29). The aggregate version of the intratemporal condition

$$W_t^h(1 - L_t) = \nu(C_t - hC_{t-1}), \tag{31}$$

yields labor supply.

4.2.2 Mutual Funds

There is a continuum of competitive, identical mutual funds, who are placeholders for the households' portfolio problem. This portfolio problem differs depending on whether we are considering the Smets and Wouters (2007) version of the model, or the version with financial frictions. In the Smets and Wouters (2007) version of the model the mutual funds' budget constraint is:

$$D_t S_{t-1} = \frac{B_{t-1}}{P_t} + \left(\frac{R_t^k}{P_t} u_t \bar{K}_{t-1} - a(u_t) \Upsilon^{-t} \bar{K}_{t-1} \right) + \Pi_t - I_t - \frac{B_t}{b_t R_t P_t} + Q_t(S_t - S_{t-1}), \tag{32}$$

where I_t is investment, B_t is holdings of government bonds, R_t is the gross nominal interest rate paid on government bonds, b_t is a "risk premium shock", and Π_t is the profit the mutual fund gets from owning firms. The term within parenthesis represents the return to owning $\bar{K}_t$ units of capital. Mutual funds managers choose the utilization rate of their own capital, u_t, and end up renting to firms in period t an amount of "effective" capital equal to:

$$K_t = u_t \bar{K}_{t-1}, \tag{33}$$

and getting $R_t^k u_t \bar{K}_{t-1}$ in return. They however have to pay a cost of utilization in terms of the consumption good which is equal to $a(u_t)\Upsilon^{-t}\bar{K}_{t-1}$. Mutual funds accumulate capital according to the equation:

$$\bar{K}_t = (1 - \delta)\bar{K}_{t-1} + \Upsilon^t \mu_t \left(1 - \phi(\frac{I_t}{I_{t-1}}) \right) I_t, \tag{34}$$

where δ is the rate of depreciation, and $\phi(\cdot)$ is the cost of adjusting investment, with $\phi'(\cdot) > 0, \phi''(\cdot) > 0$. The term μ_t is a stochastic disturbance to the price of investment relative to consumption. Note that expression (32) comes from the aggregation of the transaction between the mutual funds and households of cohort j, which is inclusive of the annuity:

$$\frac{(D_t + Q_t)}{1 - p} S_{j,t-1} = \frac{B_{t-1}}{P_t} + \left(\frac{R_t^k}{P_t} u_t \bar{K}_{t-1} - a(u_t) \Upsilon^{-t} \bar{K}_{t-1} \right) + \Pi_t - I_t - \frac{B_t}{b_t R_t P_t} + Q_t S_{j,t}. \tag{35}$$

Each mutual fund supplies a constant number of shares equal to one, which is also the aggregate number of shares in the economy since the continuum of mutual funds has mass one. The fund's manager objective function is to maximize the present discounted value of dividends:

$$I\!E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} D_{t+s}. \tag{36}$$

The first-order conditions for bonds imply:

$$1 = R_t b_t I\!E_t \left[\frac{\mathcal{F}_{t,t+1}}{\pi_{t+1}} \right]. \tag{37}$$

Call q_t^k the Lagrange multiplier associated with constraint (34) expressed in units of the multiplier on the resource constraint (32). q_t^k has the interpretation of the value of installed capital relative to consumption goods (i.e., Tobin's Q). The FOC with respect to investment, capital, and capital utilization are:

$$q_t^k \mu_t \left(1 - \phi(\frac{I_t}{I_{t-1}}) - \phi'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} \right) + I\!E_t[q_{t+1}^k \mathcal{F}_{t,t+1} \mu_{t+1} \phi'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2] = 1, \tag{38}$$

$$q_t^k = I\!E_t \left[\mathcal{F}_{t,t+1} \left(\frac{R_{t+1}^k}{P_{t+1}} u_{t+1} - a(u_{t+1}) + q_{t+1}^k (1-\delta) \right) \right], \tag{39}$$

$$\frac{R_t^k}{P_t} = a'(u_t). \tag{40}$$

In the financial frictions version of the model the portfolio choice of the mutual funds amounts to investing in government bonds and deposit to banks (B_t^d) at the safe nominal interest rate R_t:

$$D_t S_{t-1} = \frac{B_{t-1}}{P_t} - \frac{B_t}{b_t R_t P_t} + \frac{B_{t-1}^d}{P_t} - \frac{B_t^d}{R_t P_t} + Q_t(S_t - S_{t-1}). \tag{41}$$

Banks simply lend to entrepreneurs and make zero profits in equilibrium.

4.2.3 Other Private Agents

The problems of the other private agents are identical to those in Smets and Wouters (2007). There are final goods producers who buy intermediate goods, package them into a final good Y_t which they resell to consumers, the mutual funds, and the government, on a perfectly

competitive market. Intermediate goods producers use labor and capital to produce intermediate goods on a monopolistic competitive market, facing price stickiness as in Calvo (1983) and partial indexation. Whenever they can reoptimize their price, they do so by maximizing their discounted future profits, where they discount those using the stochastic discount factor $\mathcal{F}_{t,t+s}$. Labor unions combine the homogeneous labor supplied by households and differentiate the labor services to firms, setting wages according to a Calvo (1983) mechanism with indexation, and discounting the future using again the stochastic discount factor $\mathcal{F}_{t,t+s}$.

4.2.4 Government Policies

The government budget constraint is of the form

$$P_t G_t + B_{t-1} = P_t \mathcal{T}_t + \frac{B_t}{b_t R_t}, \tag{42}$$

where $\mathcal{T}_t$ are nominal lump-sum taxes (or subsidies) that also appear in household's budget constraint. This model being non-Ricardian, we need to specify a tax policy. For simplicity, we will assume that government debt is constant (in detrended real terms), and therefore the transfer policy equals government spending plus interest payments.

The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_2}\right]^{1-\rho_R} \left(\frac{Y_t}{Y_{t-1}} \frac{Y_{t-1}^f}{Y_t^f}\right)^{\psi_3} e^{r_t^m} \tag{43}$$

where the parameter ρ_R determines the degree of interest rate smoothing, R^* is the steady state nominal rate and Y_t^f is output under flexible/prices and wages. Note that policy reacts to both level differences between Y_t and Y_t^f ($\psi_2(1-\rho_R)$ coefficient) as well as growth differences (ψ_2 coefficient). Note also that the exogenous part of monetary policy is captured by the process r_t^m. This follows the process:

$$\hat{r}_t^m = \rho_{r^m} \hat{r}_{t-1}^m + \sigma_r \varepsilon_{r^m,t} + \sum_{k=1}^{K} \sigma_{k,r} \varepsilon_{k,t-k}^R, \tag{44}$$

where $\varepsilon_{R,t}$ is the usual contemporaneous policy shock and $\varepsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume as usual that $\varepsilon_{k,t-k}^R \sim N(0,1),\ i.i.d.$.

4.2.5 Equilibrium

Adding up the budget constraint of the households, mutual funds, and the government, we obtain the resource constraint

$$C_t + I_t + a(u_t)\Upsilon^{-t}\bar{K}_{t-1} + G_t = Y_t \tag{45}$$

where Y_t denotes the aggregate level of output.

We detrend all non-stationary variables by $Z_t = e^{\gamma t + \frac{1}{1-\alpha}\tilde{z}_t}$, where γ is the steady-state growth rate of the economy, and $\tilde{z}_t$ be the linearly detrended log productivity process that follows the autoregressive law of motion

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{46}$$

The growth rate of Z_t in deviations from γ, denoted by z_t, follows the process:

$$z_t = \ln(Z_t/Z_{t-1}) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t}. \tag{47}$$

We determine the steady state and perform log-linear approximations of all equilibrium conditions around that steady state.

4.2.6 Log-Linear Equilibrium Conditions of the SWBY Model

All variables in the following equations are expressed in log deviations from their non-stochastic steady state. Steady-state values are denoted by $*$-subscripts and steady-state formulas will be provided in the Appendix.

The consumption Euler equation is given by:

$$\begin{aligned} c_t = &-\frac{(1-\tilde{h})}{(1+\tilde{h}\eta)}\left(R_t - I\!E_t[\pi_{t+1} + z_{t+1}] + b_t\right) + \frac{\tilde{h}}{(1+\tilde{h}\eta)}\left(c_{t-1} - z_t\right) \\ &+ \frac{\eta}{(1+\tilde{h}\eta)} I\!E_t\left[c_{t+1} + \tilde{h} z_{t+1}\right] + \frac{(1-\tilde{h})(1-\eta)}{(1+\tilde{h}\eta)} I\!E_t[s_{t+1}], \end{aligned} \tag{48}$$

where c_t is consumption, R_t is the nominal interest rate, π_t is inflation, s_t is total household financial wealth at the beginning of period t. The exogenous process b_t drives a wedge between the intertemporal ratio of the marginal utility of consumption and the riskless real return $R_t - I\!E_t[\pi_{t+1}]$, and follows an AR(1) process with parameters ρ_b and σ_b. The

parameter $\tilde{h} \equiv he^{-\gamma}$ measures the habit persistence adjusted for the economy's steady state growth rate. The parameter

$$\eta \equiv \left(1 + \frac{p}{1-p}\frac{(1-\beta(1-p))}{(1+\nu)(1-\tilde{h})}\frac{s_*}{c_*}\right)^{-1} \tag{49}$$

plays an important role.

In the conventional case that $p = 0$, or if households have no net wealth in steady state ($s_*/c_* = 0$), then we have $\eta = 1$. When $p > 0$, and household net wealth is positive, we have $\eta < 1$. Furthermore, η is a decreasing function of the degree of habit persistence $\tilde{h}$. Looking at the linearized consumption Euler equation, we note that current consumption depends on expected future consumption with a coefficient of $\eta/(1+\tilde{h}\eta)$. Therefore the higher the death probability p, the lower η and thus the less future conditions affect current consumption.

This is particularly clear in the case of no habit persistence ($\tilde{h} = 0$); indeed, by integrating forward (48), we obtain

$$\begin{aligned} c_t &= -\left(R_t - I\!E_t[\pi_{t+1} + z_{t+1}] + b_t\right) + \eta I\!E_t\left[c_{t+1}\right] + (1-\eta) I\!E_t[s_{t+1}] && (50) \\ &= I\!E_t \sum_{k=0}^{\infty} \eta^k \left[(1-\eta)\, s_{t+k+1} - (R_{t+k} - \pi_{t+1+k} - z_{t+k+1} + b_{t+k})\right]. && (51) \end{aligned}$$

The integrated Euler equation (51) shows that consumption depends negatively on all future real interest rates $(R_{t+j} - \pi_{t+1+j})$, however with weights η^j that are declining with the horizon. This will be the key mechanism to address the forward guidance puzzle. With infinitely lived agents, i.e., when $p = 0$, these weights are constant at $\eta = 1$, so that current output depends on the sum of all future deviations of the short-term real interest rate $(R_{t+j} - \pi_{t+1+j})$ from its steady state. In the hypothetical case that the real interest rate would be brought permanently below its steady state, consumption, and hence output, would jump to $+\infty$. When $p > 0$ instead, the response of output to a similar permanent drop in the real interest rate is finite.

Turning to investment, the following condition expresses the optimal choice of investment i_t measured in terms of consumption goods:

$$i_t = \frac{1}{\phi'' e^{2\gamma}(1+\tilde{\beta})} q_t^k + \frac{1}{1+\tilde{\beta}}\left(i_{t-1} - z_t\right) + \frac{\tilde{\beta}}{1+\tilde{\beta}} I\!E_t\left[i_{t+1} + z_{t+1}\right] + \mu_t, \tag{52}$$

which depends on the value of capital in terms of consumption q_t^k, the investment adjustment cost (ϕ'' is the second derivative of the adjustment cost function) and by μ_t, an exogenous process called the "marginal efficiency of investment," which affects the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The exogenous process μ_t follows an AR(1) process with parameters ρ_μ and σ_μ.

The parameter $\tilde{\beta} = f_*$ is the steady-state value of the detrended stochastic discount factor. It relates to the steady-state gross real interest rate $r_* \equiv R_*/\pi_*$ and the economy's steady state growth rate according to $r_* = \tilde{\beta}e^{\gamma}$. As we show in the Appendix, $\tilde{\beta}$ relates to the underlying time preference parameter β according to

$$\tilde{\beta} = \beta\eta. \tag{53}$$

This implies that for positive death probability p, the steady-state value of the stochastic discount factor $\tilde{\beta}$ is smaller than the time preference parameter β, reflecting the fact that household discount future streams of income at a higher rate than given by their time preference. It follows that whenever β appears in the linearized equilibrium conditions of the SW model, it will be replaced by the lower value $\tilde{\beta}$ when p is positive. For instance, the response of current investment to expected future investment (which is given by $\tilde{\beta}/(1+\tilde{\beta})$) in (52) is less than with infinitely lived households ($p = 0$). This is another channel through which a positive p mitigates the effect of future movements in interest rates.

The capital stock, $\bar{k}_t$, evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*}\phi'' e^{2\gamma}(1+\tilde{\beta})\mu_t, \tag{54}$$

where $i_*/\bar{k}_*$ is the steady-state ratio of investment to capital. The arbitrage condition between the return to capital and the riskless rate is:

$$\frac{r_*^k}{r_*^k + (1-\delta)} I\!E_t[r_{t+1}^k] + \frac{1-\delta}{r_*^k + (1-\delta)} I\!E_t[q_{t+1}^k] - q_t^k = R_t + b_t - I\!E_t[\pi_{t+1}], \tag{55}$$

where r_t^k is the rental rate of capital, r_*^k its steady-state value, and δ the depreciation rate. Given that capital is subject to variable capacity utilization u_t, the relationship between $\bar{k}_t$ and the amount of capital effectively rented out to firms k_t is

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \tag{56}$$

The optimality condition determining the rate of utilization is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{57}$$

where ψ captures the utilization costs in terms of forgone consumption. Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha l_t - \alpha k_t, \tag{58}$$

where w_t is the real wage, l_t denotes hours worked, and α is the income share of capital (after paying markups and fixed costs) in the production function. From the optimality conditions of goods producers, it follows that all firms have the same capital-labor ratio:

$$k_t \;=\; w_t - r_t^k + l_t. \tag{59}$$

The production function is:

$$y_t \;=\; \Phi_p \left(\alpha k_t + (1-\alpha) l_t\right) + \mathcal{I}\{\rho_z < 1\}(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t, \tag{60}$$

if the log productivity is trend stationary. The last term $(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t$ drops out if technology has a stochastic trend, because in this case one has to assume that the fixed costs are proportional to the trend. Similarly, the resource constraint is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r_*^k k_*}{y_*} u_t - g_* \mathcal{I}\{\rho_z < 1\}\frac{1}{1-\alpha}\tilde{z}_t, \tag{61}$$

where the last term disappears if technology follows a unit root process. Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz}\sigma_z \varepsilon_{z,t}.$$

The price Phillips curve takes the conventional form

$$\pi_t = \kappa\; mc_t + \frac{\iota_p}{1+\iota_p\tilde{\beta}}\pi_{t-1} + \frac{\tilde{\beta}}{1+\iota_p\tilde{\beta}}\mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \tag{62}$$

where

$$\kappa = \frac{(1-\zeta_p\tilde{\beta})(1-\zeta_p)}{(1+\iota_p\tilde{\beta})\zeta_p((\Phi_p-1)\epsilon_p+1)},$$

the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices. Again, the key difference with

respect to the Smets and Wouters (2007) is that the coefficient on expected future inflation is replaced by a smaller coefficient ($\tilde{\beta}/(1+\iota_p\tilde{\beta})$) when p is positive. Integrating this equation forward, we note that this mechanism contributes to attenuate the response of current inflation to fluctuations in expected future marginal costs.

The wage Phillips curve is in turn of the usual form

$$\begin{aligned} w_t &= \frac{(1-\zeta_w\tilde{\beta})(1-\zeta_w)}{(1+\tilde{\beta})\zeta_w((\lambda_w-1)\epsilon_w+1)}\left(w_t^h - w_t\right) - \frac{1+\iota_w\tilde{\beta}}{1+\tilde{\beta}}\pi_t \\ &\quad + \frac{1}{1+\tilde{\beta}}\left(w_{t-1} - z_t + \iota_w\pi_{t-1}\right) + \frac{\tilde{\beta}}{1+\tilde{\beta}}I\!E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \end{aligned} \tag{63}$$

where ζ_w, ι_w, and ϵ_w are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for wages. Similarly to the price Phillips curve, the coefficient on expected future real wages in the SW model is replaced by a smaller coefficient ($\tilde{\beta}/(1+\tilde{\beta})$) when p is positive. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{\nu c_*}{(1-L_*)w_*}\left(c_t - \tilde{h}c_{t-1} + \tilde{h}z_t\right) + \frac{L_*}{1-L_*}l_t, \tag{64}$$

where $L_*/(1-L_*)$ would equal the inverse of the Frisch elasticity in the absence of wage rigidities. The markups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} + \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1}, \text{ and}$$

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} + \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1},$$

respectively.

Given that household wealth enters the consumption Euler equation, we need to take into account its law of motion, which is given by

$$\begin{aligned} \tilde{\beta}\frac{s_*}{c_*}I\!E_t\left[\hat{s}_{t+1}\right] = \tilde{\beta}\frac{s_*}{c_*}\left(\hat{R}_t + \hat{b}_t - I\!E_t[\hat{\pi}_{t+1} + \hat{z}_{t+1}]\right) + \frac{s_*}{c_*}\hat{s}_t \\ + \frac{w_*}{c_*}L_*\hat{w}_t^h + \frac{w_*}{c_*}L_*\hat{L}_t - \frac{\mathtt{t}_*}{c_*}\hat{\mathtt{t}}_t - \hat{c}_t. \end{aligned} \tag{65}$$

Such an equation is not needed in the standard SW model.

Since the model is non-Ricardian, we further need to include the linearized government budget constraint

$$\mathtt{t}_*\mathtt{t}_t = y_*g_*g_t - y_*g_*\mathcal{I}\{\rho_z < 1\}\frac{1}{1-\alpha}\tilde{z}_t - \frac{b_*^g e^{-\gamma}}{\pi_*}(z_{t-1}^* + \pi_t) + \frac{b_*^g}{R_*}(b_t^g + R_t), \tag{66}$$

where $\mathtt{t}_t$ denotes taxes, and b_t^g represents government debt. Finally, the monetary authority follows a generalized feedback rule:

$$\begin{aligned} R_t &= \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1 \pi_t + \psi_2 (y_t - y_t^f)\right) \\ &\quad + \psi_3 \left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\right) + r_t^m, \end{aligned} \tag{67}$$

where the flexible price/wage output y_t^f is obtained from solving the version of the model without nominal rigidities (that is, Equations (48) through (61) and (64)), and the residual r_t^m follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m}.

4.3 Quantitative Assessment in an Estimated SWBY Model

As discussed in the previous section, introducing a positive death probability p in a the SW model results in a very similar model expect that consumption tends to respond less to fluctuations in expected future variables such as interest rates, investment depends less on future investment, price inflation depends less on expected future marginal costs, and the real wage is less responsive to expected future real wages. To assess whether such a SW model with Blanchard-Yaari households (i.e., the SWBY model) includes mechanisms that are sufficiently relevant too address the forward guidance puzzle, we take the SW model, reestimated with data up to 201xQx, and analyze the effects of changing the parameter p. The parameters of the model estimation are reported in Table XXX of the Appendix.

4.3.1 Calibration of p

To have a sense the magnitude of the death probability p, we use the actuarial life table from the Social Security Administration and construct the death probability for individuals of ages 20 and above, weighting by the population at each age. This yields an average probability of dying before the next quarter of $p = 0.0042$. These calculations treat the wealth to consumption ratio and the probability of death as independent from each other. In the data, however, both the probability of death and wealth are rising with age. As an alternative calibration, we weight the probability of death at each age by both the fraction

of population at each age and by the share of wealth in each age category.[38] The resulting weighted average probability of dying before the next quarter is $p = 0.0076$.

While our model refers formally to the probability of dying, such an event amounts to re-setting the initial wealth of the household to zero. In this light, one can think of death in the model as representing also default on the part of households. Based on the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York, the fraction of consumers with new bankruptcies in a given quarter has been on average 0.2 percent from 2003 to 2014. Focusing on mortgage owners, the probability of transitioning from being current on mortgage payments to being 90+ days late on that payment within a quarter has been on average 0.0034.[39] Taking into account both the probability of a new bankruptcy and the probability of being 90+ days late on a mortgage payment, and noting that the two events can be correlated, we set the probability of defaulting in a given quarter to at least 0.005. Adding this to the probability of dying mentioned above, suggests that our calibrated value for p should be at least 0.0126.

We raise the value of p to 0.03 and 0.06 to include other forms of "re-setting" the agent's initial wealth that we might have omitted. Such values are still well below the posterior mean of $p = 0.1292$. obtained by Castelnuovo and Nistico (2010) based on a Bayesian estimation using only aggregate data.

4.3.2 Results

Table 2 shows how different assumptions about the death probability p affect some of the key parameters. The implied value for the discounting coefficient entering equations (51), (52), (62), and (63) varies significantly with changes in p. Our lower bound $p = 0$ implies no discounting in the consumption Euler equation as $\eta = 1$. This coefficient is lowered to 0.96 when $p = 0.06$. Similarly, with such a value of p, the fluctuations in expected future

[38]For wealth, we use data from the Federal Reserve's Survey of Consumer Finances for 2010, and use median wealth by age category.

[39]In addition, the probability of transitioning from being current to being 30-60 days late on the mortgage payment in substantially larger, averaging 1.7 percent per quarter. This provides a signal of incoming default as the probability of transitioning form this category to being 90+ days late on a mortgage payment is 24 percent. It follows that the probability in any given quarter of being 90+ days late on the mortgage payment within the next 6 months is $0.0034 + 0.017 * 0.24 = 0.00748$.

variables are also discounted more heavily in the equations determining investment, price inflation, and real wages. The slope of the Phillips curve κ is also affected, rising with the death probability p.

Table 2. Implied Coefficients for Alternative Death Probabilities

Death probability	p	0	0.03	0.06
Implied coefficients				
Discounting in consumption	η	1	0.987	0.960
Discounting in investment	$\tilde{\beta}/(1+\tilde{\beta})$	$0.xxx$	$0.xxx$	$0.xxx$
Slope of Phillips Curve	κ	$0.xxx$	$0.xxx$	$0.xxx$
Discounting in price inflation	$\tilde{\beta}/(1+\iota_p\tilde{\beta})$	$0.xxx$	$0.xxx$	$0.xxx$
Discounting in real wage	$\tilde{\beta}/(1+\tilde{\beta})$	$0.xxx$	$0.xxx$	$0.xxx$

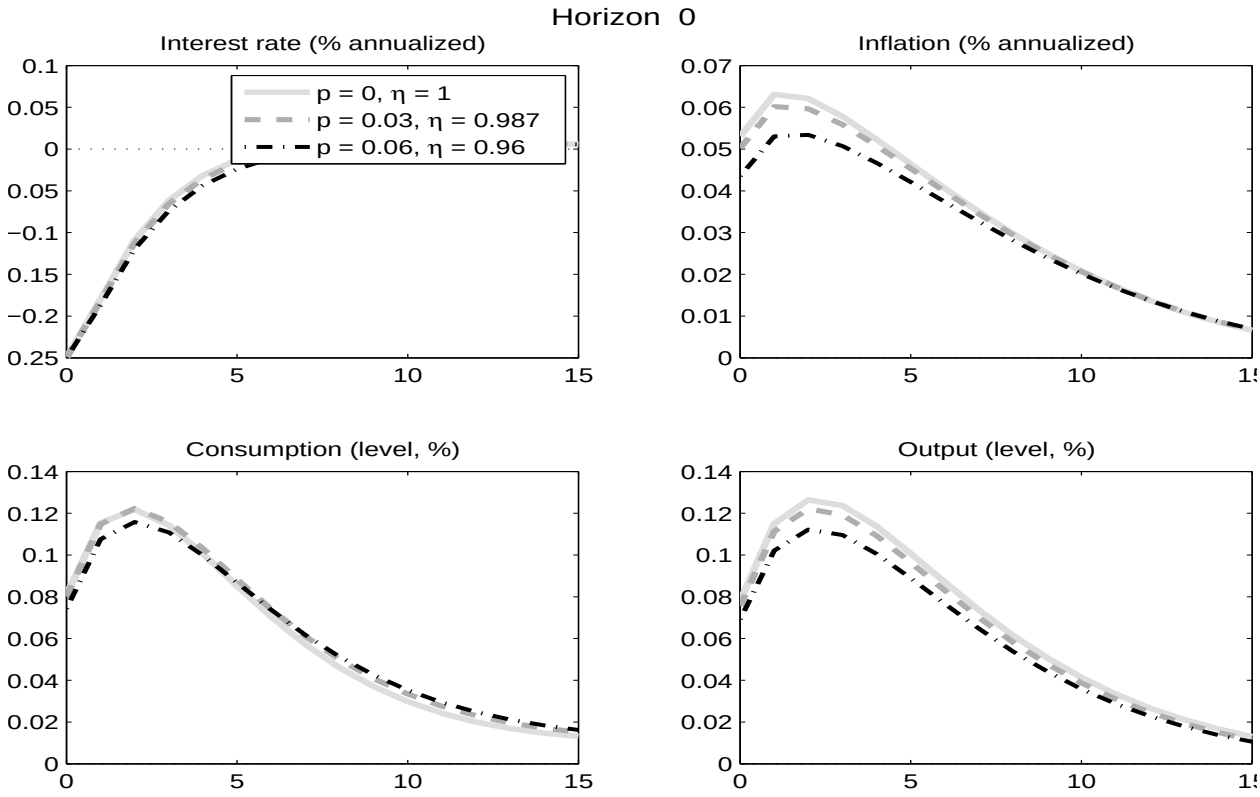
Prior to evaluating the effects of a positive p, we need to make sure that the SWBY model with p set to 0 approximates quite well the original SW model. Indeed, even when $p = 0$, our SWBY model differs slightly from the original SW model in that it assumes a different utility function (SW assume a utility function that is nonseparable in consumption and hours worked.) Figure B2 in the Appendix shows for both models the effect of a contemporaneous monetary policy shock, that is, an unanticipated decline of 25 basis points in the short-term nominal interest rate. Clearly both models reveal very similar responses, so that the difference in utility functions does not appear to play an important role, at least in terms of the responses to monetary shocks.

As shown in Figure 8, increasing the death probability does not affect much the economy's response to contemporaneous monetary shocks. This suggests that the model has standard implications in response to contemporaneous shocks.

However, in response to *anticipated* shocks, the effects of a positive p become much more significant. Figure 9 shows the effect of an an announcement that the short-term nominal interest rate will we lowered by 25 basis points 4 quarter into the future. This experiment also assumes that this interest rate will remain constant at its initial value in all periods until the announced drop takes place.[40] Figure 10, shows the same, but for an announcement that

[40]Specifically, this is modeled by having and unanticipated monetary shock as well as anticipated monetary policy shocks in periods 1 through 4 that implement that path of the short-term policy rate.

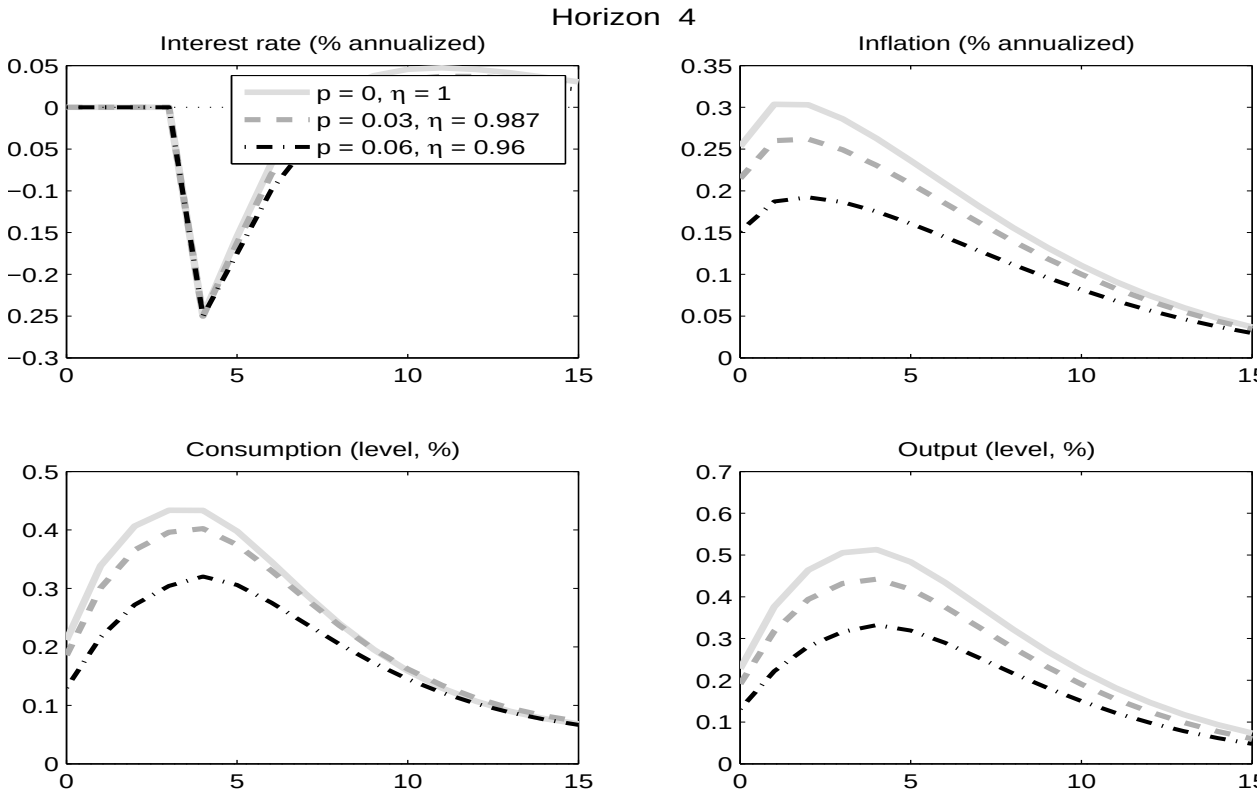
Figure 8: Effect of a Contemporaneous Nominal Interest Rate Cut in SWBY Model for Different Values of p



the nominal interest rate will be lowered by 25 basis points 8 quarters in to the future. The latter experiment relates closely to the one presented in Section 3.2.

For $p = 0$, the model-implied effects of such announcements are again very large. Indeed, as shown in Figure 10, the announcement of a interest-rate drop 8 quarters ahead causes output to increase by almost 4 percent on impact and by about 7 percent after a year. Inflation also jumps by more than 4 percentage points over the subsequent year. This experiment reproduces again the forward guidance puzzle documented in Section 3.2. However, when $p = 0.03$, the effects are significantly muted with the dynamic responses of consumption, output and inflation rising by less than half: output increases by about 1.5 percent on impact and reaches a peak of 2.5 percent about a year after the announcement. Inflation also increases, but by less than 2 percentage points. This confirms that with realistic values for p, the model implies much more reasonable responses of inflation, output and real interest rates

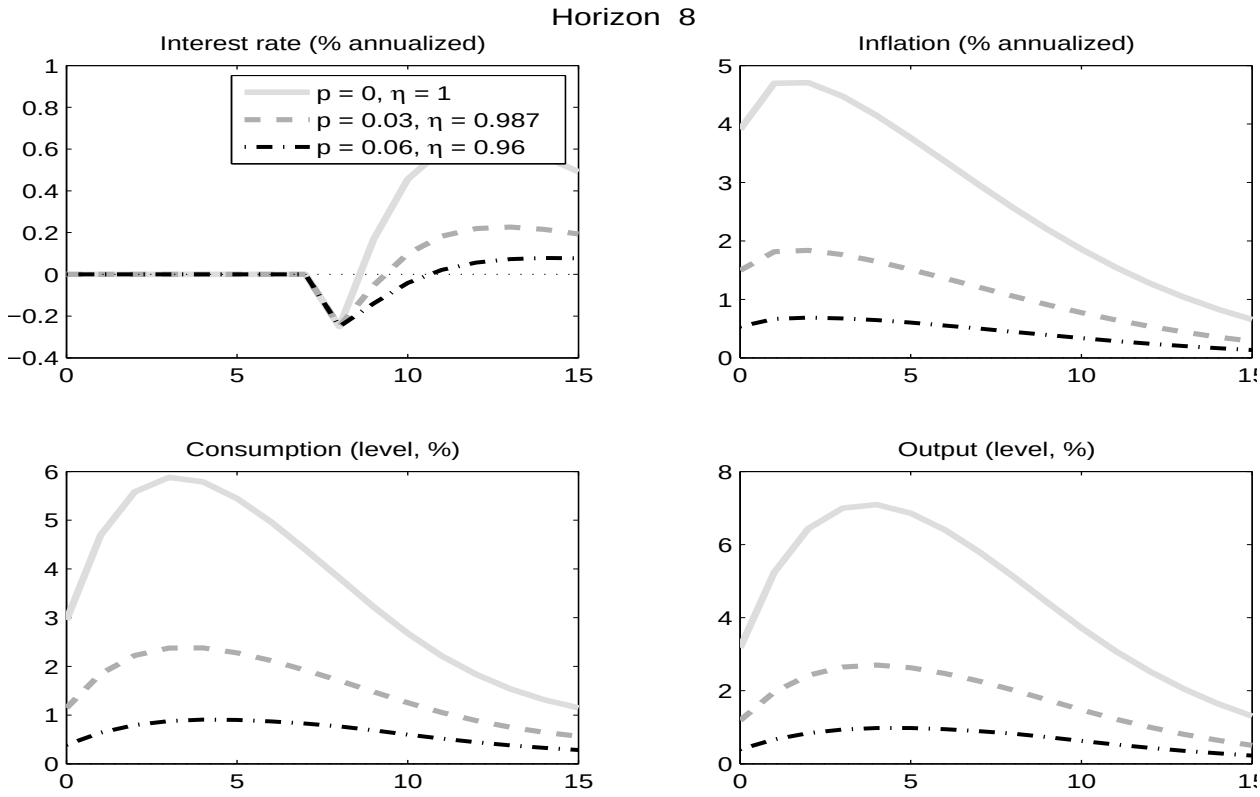
Figure 9: Effect of an Announced Nominal Interest Rate Cut 4 Periods Ahead in the SWBY Model for Different Values of p



to forward guidance announcements. When we consider a death probability of $p = 0.06$, the dampening effects of the perpetual youth model are even stronger, with consumption, output and inflation responding by less than 1 percent to the announcement, as indicated by black dashed-dotted lines in Figure 10. The latter responses seem quite realistic when compared to the evidence reported in Section 2.2.

We have focused here on the effect of announcements about changes in the nominal interest rate at various horizons, in order to capture the kind of announcements made by the FOMC. However to insure that the results obtained are not merely due to an unusual response of the inflation rate in the model, we report in Figures B3–B5 of the Appendix the effects of announcements about changes in the *real* short-term interest rate at various horizons. Such experiments are also more closely related to those performed by McKay et al. (2014). The results are overall qualitatively similar to those reported above with the responses being

Figure 10: Effect of an Announced Nominal Interest Rate Cut 8 Periods Ahead in the SWBY Model for Different Values of p



more muted with positive death probability p than when $p = 0$. The magnitudes of the responses are however smaller when we consider announcements about real interest rate changes than is the case when the announcements pertain to nominal interest rate changes. This difference is due to the fact that the inflation response tends to amplify the effects of an announcement about the nominal rate onto the real rate: the announcement of a drop in the nominal rate stimulates the economy, pushes inflation up, hence lowers the real rate by more than the nominal rate. Given that the responses are smaller, the attenuation of these responses provided by positive death probabilities, while still considerable is also smaller.

5 Conclusion

Forward guidance has become an essential tool for monetary policy in many industrial economies confronted with the effective lower bound on interest rates. Yet, little is know about the quantitative effects of forward guidance announcements on the economy. In this paper, we document the impact of forward guidance announcements on broad cross section of financial markets data, and on forecasts of key economic variables reported in the panel of Blue Chip forecasts. We find that these impacts have been very heterogenous and depend in very subtle ways on the type forward guidance announcements. We show that once we control for these other elements, forward guidance had, on average, positive and sizeable effects on output and inflation.

The paper analyzes the effect of forward guidance announcements in the context of a fairly standard medium scale DSGE model, and shows that this model tends to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon we call the "forward guidance puzzle." We argue that the model's excess response to announcements about future policy changes is likely due to the lack of discounting of future economic outcomes. We propose a tentative resolution to this puzzle based on the fact that life is finite. We show that incorporating a perpetual youth structure into a DSGE model leads to an aggregate economy that discounts the future more heavily. This implies that announcements of policy changes far in the future generate significantly smaller effects on current aggregate variables than is the case in models with infinitely lived agents.

While the paper has focused on effects of forward guidance announcements, it illustrates the fact that conventional DSGE models tend to generate more generally excessively strong responses of key macroeconomic variables to news about conditions far in the future. We have argued that incorporating a Blanchard-Yaari perpetual youth structure in medium-scale DSGE models is desirable, as it is feasible – given its tractability – and likely to generate more realistic model dynamics.

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A Appendix

A.1 Additional Empirical Results

A.1.1 Description of Policy Decompositions in 2.2

In order to assign numerical values to the FOMC language, we used the followings rules. In the future, we hope to use better machine learning techniques to improve on this albeit rough codification:

- If the the statement said that inflation was expected to be "at or below target", we coded it as have neutral inflation language. Otherwise, the statement was said to have "concerned inflation language"
- If the statement used the word "moderate" to describe output growth, then the statement was coded to neutral GDP language. If the words "slowed", "contracted" or "paused" was used, it received a 1 for "Bad GDP language" and conversely, if the words "picked up" or "rebounded" were used, then it received a 1 for "Good GDP language"
- If calendar based forward guidance was newly announced on that date, it received a 1 for "Forward Guidance".
- If the statement contained the first announcement of an asset purchase program, the date received a 1 for "QE announcement"
- If the statement contained an announcement that it was going to continue its previously announced asset purchases, it received a 1 for "QE continuation." Note that we did not separately code the announcements for the tapering of QE. This choice would be something to check the robustness of in future specifications.
- If the statement announced that it would continue "reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction," it also received a 1 for the "maturity" dummy.

A.1.2 Additional Tables and Figures

Table B1: Forward Guidance FOMC Statements Language – Economic Conditions

August 2011:
Information received ... indicates that economic growth so far this year has been considerably slower than the Committee had expected. ... Inflation picked up earlier in the year, ... More recently, inflation has moderated as prices of energy and some commodities have declined from their earlier peaks. The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting ... Moreover, downside risks to the economic outlook have increased...
January 2012:
Information received ... suggests that the economy has been expanding moderately, notwithstanding some slowing in global growth ...The Committee expects economic growth over coming quarters to be modest ...The Committee also anticipates that over coming quarters, inflation will run at levels at or below those consistent with the Committee's dual mandate.
September 2012:
Information received ... suggests that economic activity has continued to expand at a moderate pace in recent months. ... Inflation has been subdued, although the prices of some key commodities have increased recently.

Table B2: Forward Guidance FOMC Statements Language – Policy

August 2011:

...The Committee currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. ...

January 2012:

...the Committee expects to maintain a highly accommodative stance for monetary policy. In particular, the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. ... The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September.

September 2012:

The Committee is concerned that, without further policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. ... the Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month... The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities ... These actions ... will increase the ... holdings of longer-term securities by about $85 billion each month through the end of the year, should put downward pressure on longer-term interest rates, ... and help to make broader financial conditions more accommodative ... If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability ...

To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

Table B3: List of Economic News and Asset Prices Regressors in Equation (1)

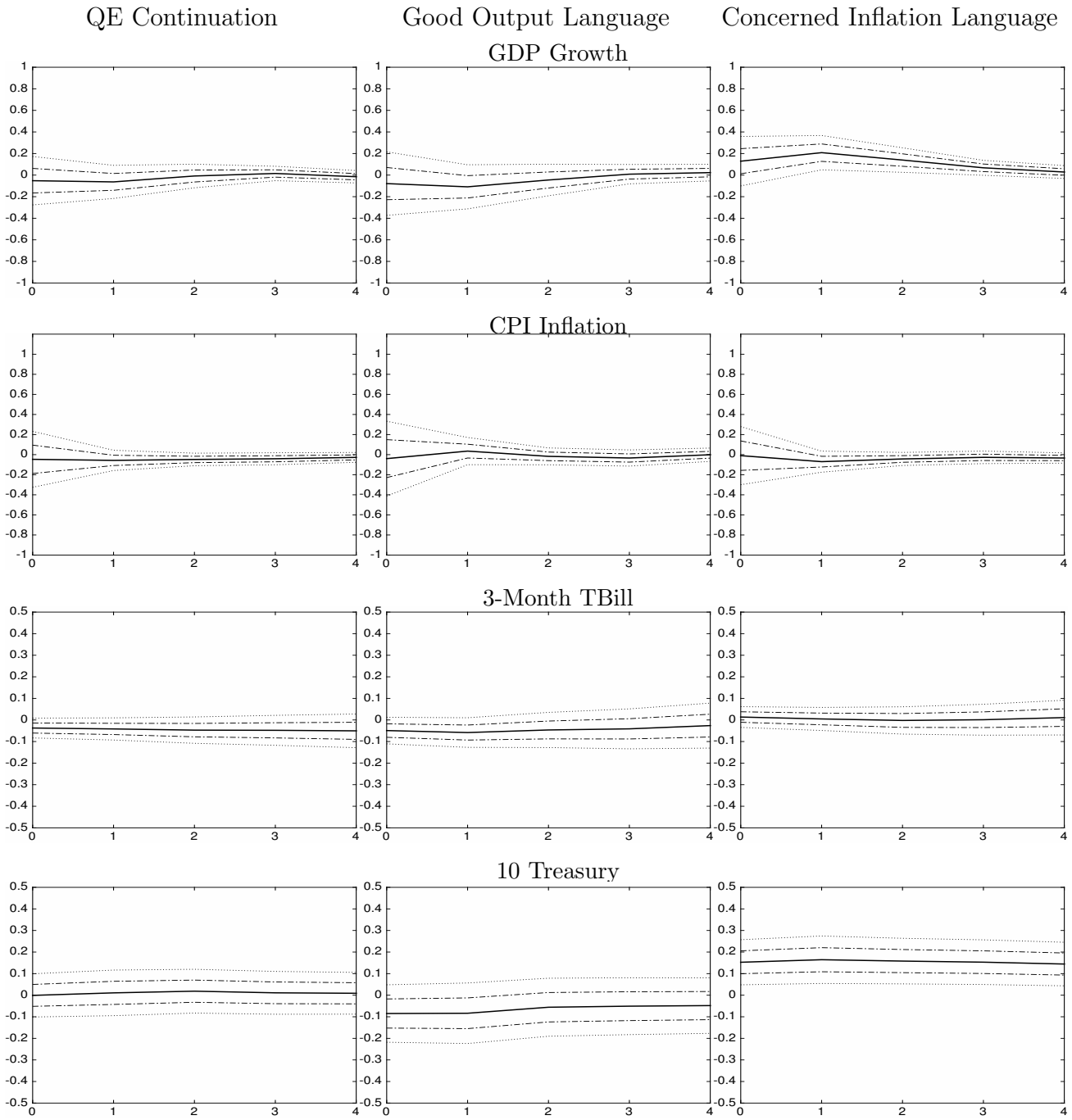
Name	Description	Unit
Economic News (ΔM)		
GDP	real GDP QoQ: Advanced, Preliminary, Final	% change
NAPMPMI	ISM Manufacturing PMI SA	level
NFP	US Employees on Nonfarm Payrolls Total MoM Net Change SA	level
USURTOT	U-3 US Unemployment Rate	
	Total in Labor Force Seasonally Adjusted	level
RSTAMOM	Adjusted Retail & Food Services Sales SA Total Monthly % Change	% change
CPI	US CPI Urban Consumers MoM SA	% change
CPTI	US Capacity Utilization % of Total Capacity SA	level
INJCJC	US Initial Jobless Claims SA	level
CONCCONF	Conference Board Consumer Confidence SA 1985=100	level
LEI	Conference Board US Leading Index MoM	% change
NHSLTOT	US New One Family Houses Sold Annual Total SAAR	level
Asset Prices (ΔAP)		
BBOX	Barclays Swaption Volatility	log change
MBS30	Average of Mortgage Backed Securities	
	(GNMA, FMNMA, GOLD), 30 Year Constant Maturity	bp difference
SP500	S&P 500 Index	log change
FXEDUS	EURUSD Spot Exchange Rate - Price of 1 EUR in USD	log change
DGS10	US Treasury Yield Curve Rate	
	T Note Constant Maturity 10 Year	bp difference
TIPS10	S&P 10 Year US TIPS Index Average Yield	bp difference

Table B4: Robustnes Checks for Figure 2

	GDP	CPI	3-Month TBill	10 Treasury
Grouped Dummies				
August 2011	-0.426	0.170	0.023	-0.350
Baseline s.e.	(0.40)	(0.52)	(0.09)	(0.19)
Heteroskedastic s.e.	(0.12)	(0.16)	(0.02)	(0.04)
Factor s.e.	(0.36)	(0.44)	(0.07)	(0.175)
January 2012	0.182	0.1662	0.025	-0.035
Baseline s.e.	(0.34)	(0.45)	(0.08)	(0.17)
Heteroskedastic s.e.	(0.10)	(0.14)	(0.02)	(0.04)
Factor s.e.	(0.31)	(0.40)	(0.06)	(0.15)
September 2012	1.013	0.857	0.1651	0.296
Baseline s.e.	(0.39)	(0.51)	(0.09)	(0.19)
Heteroskedastic s.e.	(0.11)	(0.15)	(0.02)	(0.042)
Factor s.e.	(0.34)	(0.45)	(0.07)	(0.17)
Not-Excluding 2-Day Window of Asset Prices				
August 2011	-0.581	0.116	0.026	-0.120
	(0.40)	(0.52)	(0.06)	(0.14)
January 2012	0.185	0.264	-0.018	0.050
	(0.36)	(0.47)	(0.06)	(0.13)
September 2012	0.773	0.864	0.024	0.062
	(0.45)	(0.60)	(0.08)	(0.17)

Notes: The coefficient reported in the table is the $\beta^{e}_{(k,1)}$ coefficient from Equation 1 for GDP, inflation, 3-Month TBill and the 10-year TBill.

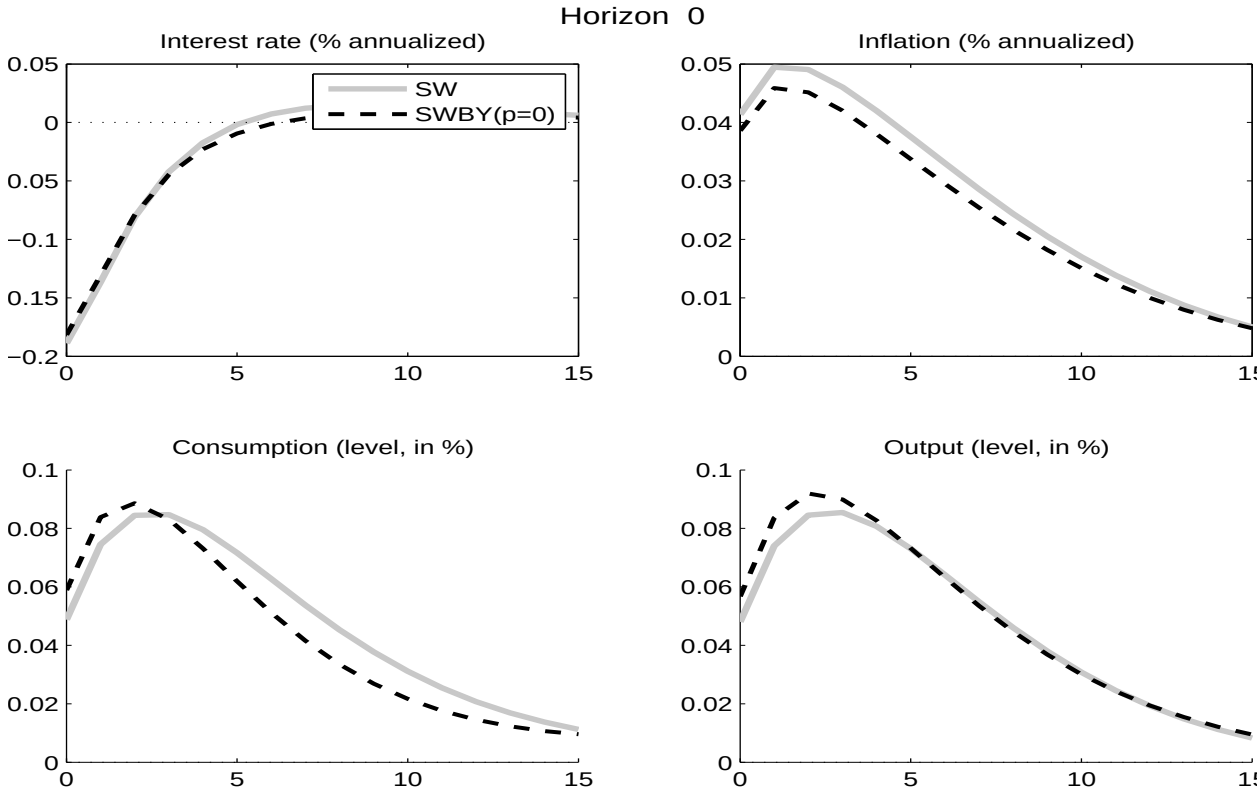
Figure B1: Decomposing FOMC Statements: The Effect of QE Continuation, Good Output Language and Concerned Inflation Language



Notes: The panel in Figure **??** shows the estimates of $\beta(k,h)^e$ for three different elements of the FOMC statement – forward guidance announcements, QE announcements and bad GDP language – and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

A.1.3 Impulse Responses to Unanticipated and Anticipated Shocks to Interest Rates

Figure B2: Effect of a Contemporaneous Monetary Policy Shock: SW vs SWBY($p = 0$)



Notes: The figure compares the impulse response functions to anticipated monetary shocks at horizons in the model of Smets and Wouters (2007) (gray solid lines) with those of the SWBY model in which the death probability p is set to 0 (black dashed lines). Both models differ in their specification of the household utility function.

Figure B3: Effect of a Contemporaneous Real Interest Rate Cut in the SWBY Model for Different Values of p

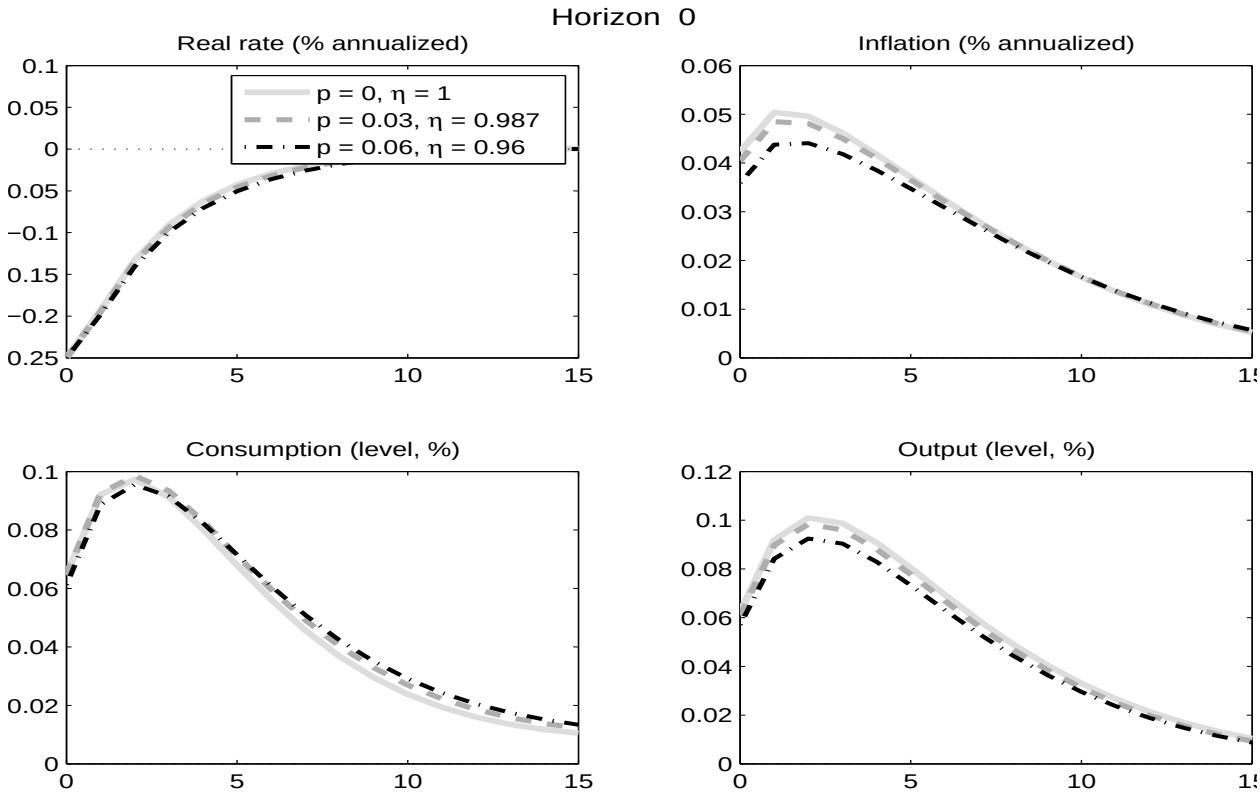


Figure B4: Effect of an Announced Real Interest Rate Cut 4 Periods Ahead in the SWBY Model for Different Values of p

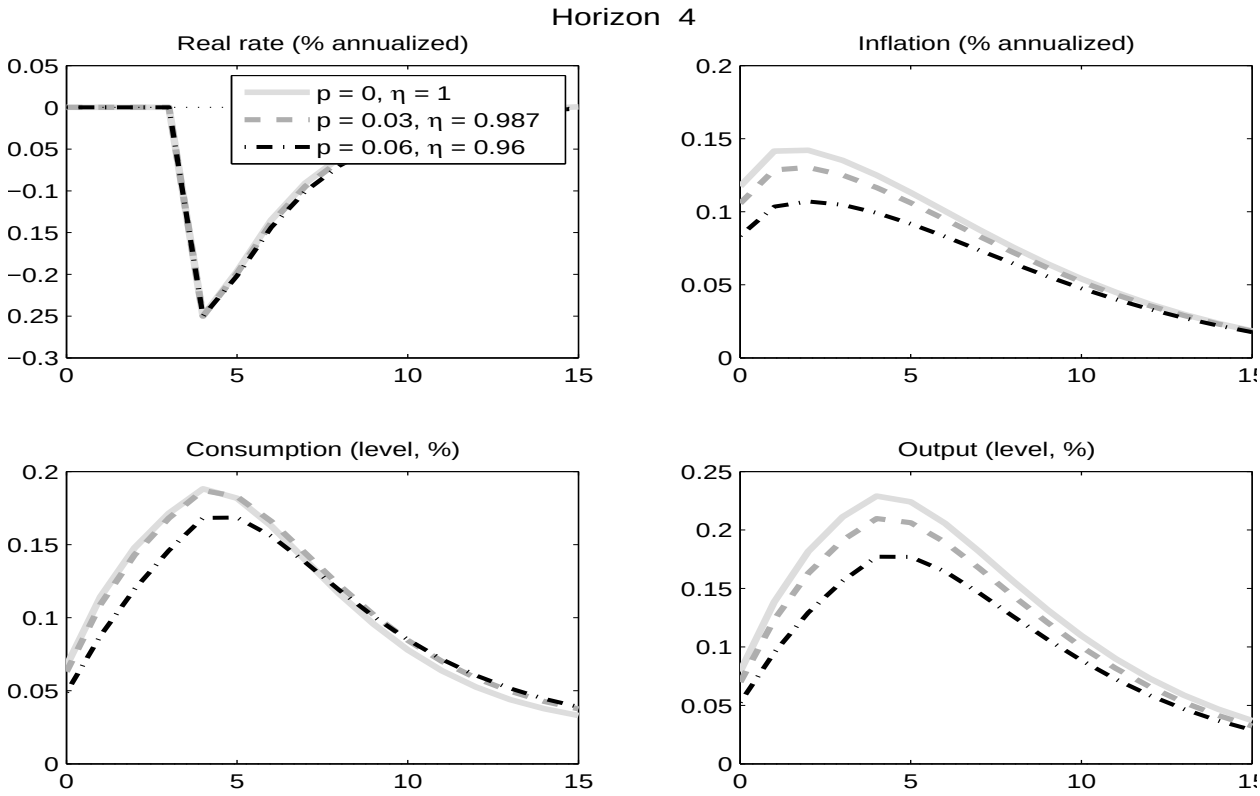
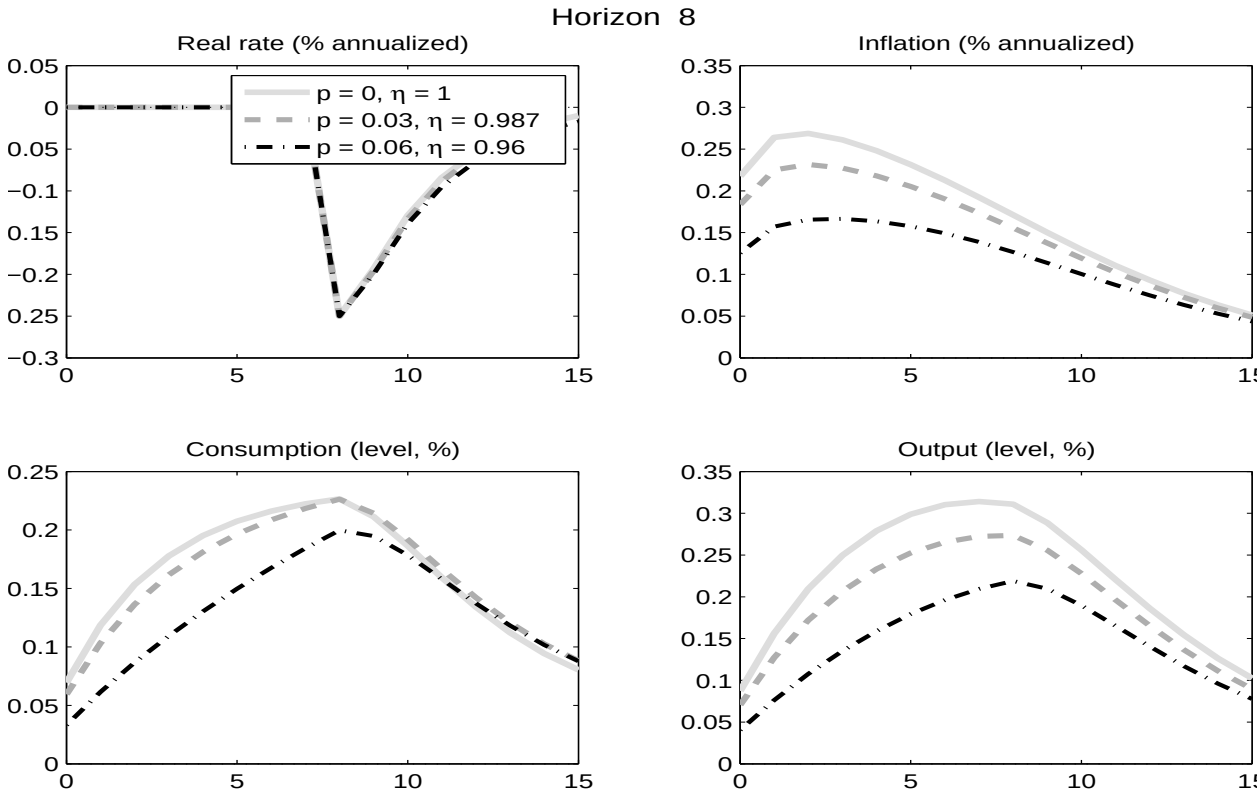


Figure B5: Effect of an Announced Real Interest Rate Cut 8 Periods Ahead in the SWBY Model for Different Values of p



A.2 Model Description

In this Appendix, we summarize the log-linear equations that characterize the FRBNY DSGE model used in Section 3. The microeconomic foundations of the model are described in Del Negro et al. (2013). Because the model has a source of non-stationarity in the process for technology Z_t, to solve the model we first rewrite its equilibrium conditions in terms of stationary variables, and then solve for the non-stochastic steady state of the transformed model. Finally we take a log-linear approximation of the transformed model around its steady state. This approximation generates a set of log-linear equations, which we solve to obtain the model's state-space representation, using the method of Sims (2002). We then use the state-space representation in the estimation procedure.

Below we list the log-linear equations of the model. We follow the usual convention of denoting log-deviations from steady state with hatted variables: for any stationary variable x_t, $\widehat{x}_t \equiv \log(x_t/x_*)$, where x_* denotes its steady state value. The steady state itself is a function of the model's parameters. Equations describing the mapping between parameters and steady state variables are available upon request.

The *Consumption Euler Equation* that characterizes the optimal allocation of consumption over time is given by

$$\hat{\xi}_t = \hat{R}_t + I\!E_t[\hat{\xi}_{t+1}] - I\!E_t[\hat{z}_{t+1}] - I\!E_t[\hat{\pi}_{t+1}], \tag{68}$$

where $\hat{R}_t$ is the gross nominal interest rate on government bonds, and $\hat{\xi}_t$ is the marginal utility of consumption.

The *Marginal Utility of Consumption* ξ_t evolves according to

$$\begin{aligned}(e^\gamma - h\beta)(e^\gamma - h)\hat{\xi}_t = &- (e^{2\gamma} + \beta h^2)\hat{c}_t + he^\gamma \hat{c}_{t-1} \; - \; he^\gamma \hat{z}_t \\ &+ \beta h e^\gamma I\!E_t[\hat{c}_{t+1}] \; + \beta h e^\gamma I\!E_t[\hat{z}_{t+1}],\end{aligned}$$

where $\hat{c}_t$ is consumption, e^γ is the steady-state (gross) growth rate of the economy and h captures habit persistence in consumption.

The *Capital Stock* follows

$$\hat{\bar{k}}_t = -(1 - \frac{i_*}{k_*})\hat{z}_t + (1 - \frac{i_*}{k_*})\hat{\bar{k}}_{t-1} + \frac{i_*}{k_*}\hat{\mu}_t + \frac{i_*}{k_*}\hat{\imath}_t, \tag{69}$$

where $\hat{\bar{k}}_t$ is installed capital, $\hat{z}_t$ is the growth rate of productivity, i_* and k_* are steady state investment and the level of capital, respectively, and $\hat{\mu}_t$ is the exogenous process that affects the efficiency by which a foregone unit of consumption contributes to capital utilization.

The *Effective Capital* $\hat{k}_t$ is in turn given by

$$\hat{k}_t = \hat{u}_t - \hat{z}_t + \hat{\bar{k}}_{t-1}, \tag{70}$$

where $\hat{u}_t$ is the level of capital utilization.

Capital Utilization is given by

$$r_*^k \hat{r}_t^k = a''(u)\ \hat{u}_t, \tag{71}$$

where r_*^k is the steady state rental rate of capital and the function $a(u)$ captures the utilization cost.

The *Optimal Investment* decision satisfies the Euler equation

$$\hat{i}_t = \frac{1}{1+\beta} I\!E_t[\hat{i}_{t-1} - \hat{z}_t] + \frac{\beta}{1+\beta} I\!E_t[\hat{i}_{t+1} + \hat{z}_{t+1}] + \frac{1}{(1+\beta)S''e^{2\gamma}} \hat{q}_t^k + \frac{1}{(1+\beta)S''e^{2\gamma}} \hat{\mu}_t, \tag{72}$$

where $\hat{i}_t$ is investment, $S(.)$ is the cost of adjusting capital, with S' and $S'' > 0$, and $\hat{q}_t^k$ is the price of capital.

The *Realized Return on Capital* is given by:

$$\hat{\bar{R}}_t^k - \hat{\pi}_t = \frac{r_*^k}{r_*^k + (1-\delta)} \hat{r}_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} \hat{q}_t^k - \hat{q}_{t-1}^k, \tag{73}$$

where δ is the rate of capital depreciation, π_t is the inflation rate, whose evolution is described below, $\hat{r}_t^k$ is the capital rental rate and $\hat{q}_t^k$ is the price of capital.

The *Expected Excess Return on Capital* (or 'spread')

$$I\!E_t\left[\hat{\bar{R}}_{t+1}^k - \hat{R}_t\right] = \zeta_{sp,b}\left(\hat{q}_t^k + \hat{\bar{k}}_t - \hat{n}_t\right) + \tilde{\sigma}_{\omega,t} \tag{74}$$

can be expressed as a function of the entrepreneurs' leverage (i.e., the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of the entrepreneurs' idiosyncratic productivity, $\tilde{\sigma}_{\omega,t} \equiv \zeta_{sp,\sigma_\omega} \hat{\sigma}_{\omega,t}$. The parameter $\zeta_{sp,b}$ is the elasticity of the spread with respect to leverage, and ζ_{sp,σ_ω} is the elasticity of the spread with respect to the volatility of the spread shock.

The *Entrepreneurs' Net Worth,* $\hat{n}_t$, evolves according to

$$\hat{n}_t = \zeta_{n,\tilde{R}^k}\left(\widehat{\tilde{R}}^k_t - \hat{\pi}_t\right) - \zeta_{n,R}\left(\hat{R}_{t-1} - \hat{\pi}_t\right) + \zeta_{n,qK}\left(\hat{q}^k_{t-1} + \widehat{\bar{k}}_{t-1}\right) + \zeta_{n,n}\hat{n}_{t-1} - \gamma^e\frac{v_*}{n_*}\hat{z}_t - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\tilde{\sigma}_{\omega,t-1}, \quad (75)$$

where $\zeta_{n,\tilde{R}^k}$, $\zeta_{n,R}$, ζ_{n,q^K}, $\zeta_{n,n}$, and ζ_{n,σ_ω} are the elasticities of net worth to the return on capital, the nominal interest rate, the cost of capital, net worth itself and the volatility σ_ω, respectively, and γ^e is the fraction of entrepreneurs who survive each period.

The evolution of the *Aggregate Nominal Wage* is then given by

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \frac{1-\zeta_w}{\zeta_w}\hat{\tilde{w}}_t, \quad (76)$$

where ζ_w is the fraction of workers who cannot adjust their wages in a given period and $\hat{\tilde{w}}_t$ is the optimal wage chosen by workers that can freely set it, or optimal reset wage.

The *Optimal Reset Wage* follows

$$(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t \;+\; (1+\zeta_w\beta\nu_l(\frac{1+\lambda_w}{\lambda_w}))\hat{w}_t = \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})I\!E_t[\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}] \;+\; \hat{\varphi}_t \;+\; (1-\zeta_w\beta)(\nu_l\hat{L}_t - \hat{\xi}_t) + \;\zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})I\!E_t[\hat{\pi}_{t+1}+\hat{z}_{t+1}],$$

where $\hat{\varphi}_t$ is a stochastic preference shifter affecting the marginal utility of leisure and λ_w is the parameter that determines the elasticity of substitution between differentiated labor services.

The optimal price-setting decision yields a *Phillips Curve* equation

$$\hat{\pi}_t = \beta I\!E_t[\hat{\pi}_{t+1}] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p}\hat{mc}_t + \frac{1}{\zeta_p}\tilde{\lambda}_{f,t}, \quad (77)$$

where $\hat{\pi}_t$ is inflation, $\hat{mc}_t$ is nominal marginal cost, β is the discount factor, and ζ_p is the Calvo parameter, representing the fraction of firms that cannot adjust their prices each period. $\tilde{\lambda_{f,t}}$ is the following re-parametrization of the cost-push shock $\lambda_{f,t}$: $\tilde{\lambda_{f,t}} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)]\lambda_{f,t}$, where λ_f is the steady state value of the markup shock.

The *Marginal Cost (or labor share)* satisfies

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}^k_t, \quad (78)$$

where α is the output elasticity to capital and $\hat{r}_t^k$ is the capital rental rate.

The *Production Function* is given by

$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{L}_t, \tag{79}$$

where the *Capital-Labor Ratio* satisfies

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t. \tag{80}$$

The *Resource Constraint* is

$$\hat{y}_t = \hat{g}_t + \frac{c_*}{c_* + i_*}\hat{c}_t + \frac{i_*}{c_* + i_*}\hat{i}_t + \frac{r_*^k k_*}{c_* + i_*}\hat{u}_t, \tag{81}$$

where $\hat{y}_t$ is output and $\hat{g}_t$ is government spending.

Finally, the *Policy Rule* is

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \tag{82}$$

where $\sum_{j=0}^{3} \hat{\pi}_{t-j}$ is 4-quarter inflation expressed in deviation from the Central Bank's objective π_* (which corresponds to steady state inflation), $\sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$ is the 4-quarter growth rate of real GDP expressed in deviation from steady state growth, ϵ_t^R is the standard contemporaneous policy shock, and the terms $\epsilon_{k,t-k}^R$ are anticipated policy shocks, known to agents at time $t-k$.

A.3 The Exogenous Processes

The exogenous processes $\widehat{z}_t, \widehat{\varphi}_t, \tilde{\lambda}_{f,t}, \hat{\mu}_t, \tilde{\sigma}_{\omega,t}$ and $\hat{g}_t$ are assumed to follow AR(1) processes with autocorrelation parameters denoted by $\rho_z, \rho_\varphi, \rho_{\lambda_f}, \rho_\mu, \rho_{\sigma_\omega}$, and ρ_g, respectively. The innovations to these processes are structural shocks driving the model dynamics. They are assumed to be normally distributed with mean zero and a standard deviation denoted by $\sigma_z, \sigma_\varphi, \sigma_{\lambda_f}, \sigma_\mu, \sigma_{\sigma_\omega}$, and σ_g, respectively. The remaining structural shocks are the monetary policy shocks, both unanticipated, ϵ_t^R, and anticipated, $\epsilon_{k,t-k}^R$, all assumed *i.i.d.*

A.4 Measurement Equations

Real output growth (%, annualized)	$400(\ln Y_t - \ln Y_{t-1})$	$=$	$400(\widehat{y}_t - \widehat{y}_{t-1} + z_t)$
Hours (%)	$100 \ln L_t$	$=$	$100(L_t + \ln L^{adj})$
Labor Share (%)	$100 \ln LS_t$	$=$	$100(\widehat{L}_t + \widehat{w}_t - \widehat{y}_t + \ln LS_*)$
Inflation (%,annualized)	π_t^{Core}	$=$	$400(\widehat{\pi}_t + \ln \pi_*)$
Interest Rate (%,annualized)	FFR_t	$=$	$400(\widehat{R}_t + \ln R_*)$,
Spread (%,annualized)	SP_t	$=$	$400(\mathbb{E}_t \left[\widehat{\widehat{R}}^k_{t+1} - \hat{R}_t\right] + SP_*)$,

where the parameter L^{adj} captures the units of measured hours.

A.5 Prior and Posterior

Parameters	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
Policy Parameters					
ψ_1	2.000	0.250	2.016	1.782	2.257
ψ_2	0.200	0.100	0.273	0.175	0.369
ρ_r	0.500	0.200	0.762	0.721	0.804
σ_r	0.200	4.000	0.152	0.132	0.170
Nominal Rigidities Parameters					
ζ_p	0.750	0.100	0.879	0.852	0.908
ζ_w	0.750	0.100	0.904	0.872	0.938
Other "Endogenous Propagation and Steady State" Parameters					
α	0.330	0.020	0.350	0.345	0.355
a''	0.200	0.100	0.294	0.129	0.444
h	0.700	0.050	0.704	0.636	0.770
S''	4.000	1.500	3.121	2.247	3.994
ν_l	2.000	0.750	1.273	0.465	2.017
r^*	1.500	1.000	0.288	0.038	0.525
π^*	2.000	0.250	2.382	2.115	2.655
γ	2.750	0.500	1.687	1.307	2.063
g^*	0.300	0.100	0.195	0.090	0.300
ζ_{sp}	0.050	0.020	0.070	0.041	0.100
spr_*	2.000	0.500	1.163	0.750	1.556
ρs and σs					
ρ_z	0.400	0.250	0.487	0.369	0.605
ρ_ϕ	0.750	0.150	0.284	0.165	0.397
ρ_{λ_f}	0.750	0.150	0.470	0.364	0.572
ρ_μ	0.750	0.150	0.991	0.982	1.000
ρ_g	0.750	0.150	0.927	0.847	0.999
ρ_{sigw}	0.750	0.150	0.965	0.938	0.995
σ_z	0.300	4.000	0.788	0.695	0.877
σ_ϕ	3.000	4.000	29.403	12.878	45.636
σ_{λ_f}	0.200	4.000	0.087	0.072	0.101
σ_μ	0.750	4.000	0.358	0.271	0.450
σ_g	0.500	4.000	0.224	0.180	0.266
σ_{sigw}	0.050	4.000	0.085	0.076	0.095

Note: The following parameters are fixed: $\delta = 0.025$, $\nu_m = 2$, $\lambda_w = 0.3$, $\chi = 0.1$, $\lambda_f = 0.15$, $F(\omega) = 0.15$, $\gamma_* = 0.99$. L^{adj} has a prior mean of 253.500, with standard deviation at 5.

A.6 Anticipated Shocks to Condition on Interest-Rate Path

The following algorithm determines the time $T+1$ monetary policy shocks as a function of the desired interest rate sequence $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$ to generate predictions conditional on an announced interest rate path. The announced interest rate path will be attained in expectation. We denote by ϵ_t the vector that collects the innovations of the unanticipated shocks (both policy and non policy shocks), and by $\epsilon^R_{1:K,t}$ the vector of anticipated policy shocks.

Algorithm 1. Drawing Counterfactual Forecasts via Anticipated Shocks.[41]

1. Use the Kalman filter to compute the mean $s_{T|T}$ of the distribution $p(s_T|\theta, Y_{1:T})$.

2. Consider the following system of equations, omitting the θ argument of the system matrices:

$$\begin{aligned}
\bar{R}_{T+1} &= \Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T + \Psi_{R,2}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]' \\
\bar{R}_{T+2} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^2 s_T + \Psi_{R,2}\Phi_1\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]' \\
&\vdots \\
\bar{R}_{T+\bar{H}} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T + \Psi_{R,2}(\Phi_1)^{\bar{H}-1}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]'
\end{aligned} \tag{83}$$

This linear system of $\bar{H}$ equations with $\bar{H}$ unknowns can be solved for for the vector of policy shocks $\bar{\epsilon}^R = [\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\prime}_{1:K,T+1}]'$. Specifically, rewrite the system (83) as

$$b = M_{\bar{H}}\bar{\epsilon}^R, \tag{84}$$

where

$$\begin{aligned}
b &= [\bar{R}_{T+1}, ..., \bar{R}_{T+\bar{H}}]' - [\Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T, ..., \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T]', \\
M_{\bar{H}} &= [\Psi_{R,2}, \Psi_{R,2}\Phi_1, ..., \Psi_{R,2}(\Phi_1)^{\bar{H}-1}]\Phi_{\epsilon,R},
\end{aligned} \tag{85}$$

and $\Phi_{\epsilon,R}$ collects the columns of the matrix Φ_ϵ corresponding to the vector of policy shocks $\bar{\epsilon}^R$. The solution of (83) is then

$$\bar{\epsilon}^R = M^{-1}_{\bar{H}} b. \tag{86}$$

[41] The algorithm in Del Negro and Schorfheide (2013) describes how to draw from the entire counterfactual predictive distribution, conditional on draws of θ from the posterior density. Here we focus on the mode of the posterior density for θ.

3. Starting from $s_{T|T}$, iterate the state transition equation (4) forward to obtain a sequence $s_{T+1:T+H|T}$:

$$s_{t|T} = \Phi_1(\theta^{(j)})s_{t-1|T} + \Phi_\epsilon(\theta^{(j)})[\epsilon_t^R, 0, \ldots, 0, \epsilon_{1:K,t}^{R\,\prime}]', \quad t = T+1, \ldots, T+H,$$

where (i) $\epsilon_{T+1}^R = \bar{\epsilon}_{T+1}^R$ and $\epsilon_t^R = 0$ for $t = T+2, \ldots, T+\bar{H}$; (ii) $\epsilon_{1:K,T+1}^R = \bar{\epsilon}_{1:K,T+1}^R$ and $\epsilon_{1:K,t}^R = 0$ for $t = T+2, \ldots, T+\bar{H}$ (that is, in both cases use solved-for values in period $T+1$ and zeros thereafter).

4. Use the measurement equation (5) to compute $y_{T+1:T+H}$ based on $s_{T+1:T+H|T}$. □