Building the Auction Markets for the World's Premier Risk-Free Securities: A Structural Analysis of the Primary Dealer System in the United States Treasury Auctions.

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ABSTRACT

The goal of this paper is to quantify the contribution of the primary dealer system in the US Treasury auctions to the debt management objective of lowering the finance cost over time. The new idea of the paper is that the primary dealer system impacts the bidding behavior and the market outcome through two channels. The first channel is the "information pooling channel" where a primary dealer pools its information with that of indirect bidders when the primary dealer observes the indirect bidders' bids. The second channel is the "competition channel" where the auction system maintains its competitiveness by keeping bids by indirect bidders rather than the primary dealers

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internalizing them. Based on this idea, we build a new model of uniform price auctions of discrete units with the primary dealer system, nonparametically estimate the bidder private values, and partially identify the effect of policy counterfactual using the asymptotic approximation method. We find that the primary dealer system provides the lowest price volatilities while maintaining the equal level of auction prices through these two channels in comparison with the direct bidding system and the syndicate bidding system. These properties of the primary dealer system could have been valuable during the period of financial crisis.
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Table of Contents

1 Introduction. **2**
A Related Work. . . . 13

2 Structural Analysis of the Primary Dealer System in the US Treasury Auctions. **16**
A Institutional Features of the US Treasury Auctions. . . . 16
A.1 The Primary Dealer System. . . . 16
A.2 Single Price Auctions. . . . 17
B Modelling the US Treasury Auctions. . . . 18
B.1 The Primary Dealer System. . . . 19
B.2 The Single Price Auction Mechanism. . . . 20
B.3 Bayesian Game. . . . 21
B.4 Discussion of Modeling Choices. . . . 23
C Partial Identification of Bidder Values from the First Order Conditions. 25
C.1 Bidder First Order Conditions. . . . 25
C.2 Partial Identification of Bidder Private Values. . . . 27
D Nonparametric Estimation of Bidder Private Values. . . . 28
E Asymptotic Approximation of Bidder Strategic Responses to Auction Systems. . . . 29
E.1 Partial Identification of Equilibrium Bids. . . . 29
E.2 Asymptotic Behavior of Equilibrium Bids. . . . 30
E.3 Asymptotic Approximation of Equilibrium Bids in Finite Markets. 30

F Counterfactual Policy Experiments. 33

3 Results of the Analysis: the Primary Dealer System in the US Treasury Auctions. 33

A Primary Dealers Behavior After the Financial Crisis. 34

B The Contribution of the Primary Dealer System to the Debt Management Objective. 35

4 Conclusion. 36

5 Appendix. 51

A Example of Comparison of the Primary Dealer System, the Direct Bidding System, and the Syndicate Bidding System. 51

B Proof of Lemma 1. 53

C Proof of Proposition 2. 55

D Proof of Proposition 3. 57

E The Estimation Procedure. 64

F Proof of Proposition 4. 66

G The Statement and Proof of Proposition 8. 68

1. Introduction.

The Goal of The Paper. The United States Treasury market is the *crown jewel* of the US and the world economy. First, from the fiscal perspective, the Federal government issues debt securities in the primary market to finance its operations and to refinance its matured debts. Second, from the monetary perspective, the Federal Reserve conducts open market operations in the Treasury markets to affect the federal funds rate. Third, from the financial market perspective, the US Treasury yields are the benchmark for pricing fixed income domestic and foreign securities[1]. Furthermore, short term Treasury securities are the most widely accepted forms of collaterals for borrowing and repo financing. Therefore, it is of public importance to study the analysis and design of the US Treasury market structure.

[1] Duffie, Dworczak, and Zhu (2014) consider the role of benchmark in facilitating trade. Duffie and Dworczak (2014) study the design of the benchmark. In this sense, this paper study the formation of the benchmark in the US Treasury auctions.

In this paper we focus on the auction market design in the primary market. The US Treasury department is authorized under 31 U.S.C. Subtitle III of the United States Code to issue Treasury securities and to prescribe terms and conditions for their issuance and sale. The current market structure for the distribution of the security is the "single price auctions" with the "primary dealer system"[2]. Under the primary dealer system, bidders are classified into primary dealers, indirect bidders, and direct bidders. Indirect bidders bid through primary dealers. Primary dealers bid for their own house accounts and route bids by indirect bidders before submitting their own bids. Direct bidders bid for their own house accounts. The distribution of securities is determined by single-price (uniform price) auctions[3]. In the single price auction, the Treasury awards all noncompetitive bids and then accepts competitive bids in ascending order of their yields until the quantity of awarded bids reaches the offering amount. All competitive and noncompetitive bids are awarded at the highest possible yield. A key element of the primary dealer system is that primary dealers can strategically use information obtained from observing indirect bidders' bids[4].

But recently there have been significant changes and uncertainty in the economic environment concerning Treasury auctions. First, from the fiscal perspective, the Treasury has auctioned a large amount of securities since 2008 to deal with the Financial Crisis. But the deficit for the fiscal year 2014 was the lowest since 2008 and represents a decrease of 29 per cent from the previous year. The Treasury expects to gradually decrease the auction sizes for shorter-dated securities. Second, from the monetary perspective, the Federal Reserve recently announced the end of the Quantitative Easing. Furthermore, there have been expectations that the Federal Reserve will start raising short-term interest rates. Third, from the financial market perspective, Basel III requires banks to hold more cash in reserve for assets such as bonds they keep on their

[2] Bruno, Glosten, and Spatt (2005), Biais and Green (2007), Parlour and Seppi (2009), and Foucault, Pagano, and Röell (2013) provide the recent survery of the market microstructure.

[3] The US Treasury adopts auctions for bills since December 1929, for notes since November 1970, and for bonds since January 1973.

[4] In a recent report concerning the development of the primary dealer system, Harper and Kruger (2013) report an interview with Jason Evans, a former Treasuries trader at Goldman Sachs and Steve Rodosky of Treasury and derivatives trading at PIMCO. Concerning the role of the primary dealer, Evans said: "a lot of the value of being a dealer is being at the nexus of the information flow." Rodosky's comment shows that an indirect bidder is concerned that primary dealers may use the information contained in their bids: "one of the weaknesses in the old-fashioned model was the fact that with five or ten minutes to go you share information about size and price with somebody else in the market. And from time to time that information would be used to somebody else's advantage because they would know that people were eager to buy or that nobody was eager to buy."

balance sheets. In addition, the foreign official holding of US Treasury securities has increased from 400 billion USD in January 1994 to 3 trillion USD in June 2010 (The US Treasury (2013)).

Based on these changes, the US Treasury has asked the following questions about the market structure of the primary market to the Treasury Borrowing Advisory Committee (Treasury Borrowing Advisory Committee (2014)):

> *Treasury has used the Primary Dealer model for auctioning and distributing its debt for several decades. This highly efficient system has been a key feature for the effective functioning of Treasury auctions. Given the evolution of the financial service industry, market structure, regulation and technology over recent years, does the current structure for distributing Treasury securities remain optimal? Are there any modifications that could result in a lower cost of funding for Treasury and/or enhance secondary market liquidity?*

To answer this question, the aim of this paper is to quantify the significance of the primary dealer system in the US Treasury primary auction to achieve the debt management objective of lowering issuance costs over time after the financial crisis. This is achieved through two goals:

- The first goal is to identify the change in bidding behavior in the US Treasury auctions during the period of the financial crisis in comparison with the period before the financial crisis.
- The second goal is to quantify the significance of the primary dealer system in comparison with other policy alternatives such as the direct bidding system and the syndicate bidding system that have been used in other markets[5].

Previous Steps. Our starting point toward the goal is the previous academic studies on auctions among heterogeneous bidders[6].

[5] A "direct bidding system" is a system where the Treasury provides a direct link to every bidder bids directly to the seller. Such bidding system is adopted in electricity auctions and emission trading. A "syndicate bidding system" is a system where an underwriter or underwriting syndicate submits a bid and the securities are awarded to the underwriting syndicate with the best bid. An example of the syndicate bidding system is the competitive sale of municipal securities. Another possible example of the syndicate bidding system is bidding consortia in auctions for offshore leases where participating firms may reveal their bidding intentions on all the tracts under discussion (Porter (1995)).

[6] More generally, our approach is related to studies of auctions among heterogeneous bidders. Woods

The first step of previous research is comparison between the single price auction method and the multiple price auction method. US studies include Friedman (1962), Rieber (1965), Bolton (1973), Umlauf (1991), Simon (1994), and Malvey and Archibald (1998). International evidences include Umlauf (1993), Nyborg and Sundaresan (1996), Fevrier, Preget, and Visser (2004), Keloharju, Nyborg, and Rydqvist (2005), Preget and Waelbroeck (2005), Armantier and Sabai (2006), Brenner, Galai, and Sade (2009), Hortacsu and McAdams (2010), and Kastl (2012).

The second step conducts the structural analysis of auctions to Treasury auctions (see Athey (2005a), Athey and Haile (2005b), Hendricks and Porter (2007), and Porter (2015) for a survey). (1). Fevrier, Preget, and Visser (2004) consider a model of share auctions with the parametric assumption that the distribution of values is a gamma function and that the conditional distribution of signals is exponential and use the method of moments for the French Treasury auctions. Castellanos and Oviedo. (2004) employ Fevrier, Preget, and Visser (2004)'s model to the Mexican Treasury auction data. (2). Hortacsu (2002) and Hortacsu and McAdams (2010) extend nonparametric identification and estimation method ("the first order approach" of Porter (2015)) of Elyakime, Laffont, Loisel, and Vuong (1994) and Guerre, Perrigne,

(1965) is a pioneering study of organizational bidding in offshore oil auctions between informed bidders who have explored adjacent wildcat tracts and uninformed bidders. Since then, Wilson (1967), Weverbergh (1979), Engelbrecht-Wiggans, Milgrom, and Weber (1982), Milgrom and Weber (1982a), and Milgrom and Weber (1982b) develop a theory of first price auctions with common values among informed bidders and uninformed bidders. The key insight of these papers is that informed bidders bid more aggressively than uninformed bidders who are concerned with winners' curse. **?** conduct analysis in the private value environment.

In the second step, these theories have been applied to the study of real-world auction markets. In the drainage of the US offshore oil and gas lease by the Department of Interior, some firms have explored adjacent wildcat tracts and are likely to have better information than other bidders (Hendricks, Porter, and Boudreau (1987), Hendricks and Porter (1988), Hendricks, Porter, and Spady (1989), Hendricks and Porter (1992), Hendricks and Porter (1993), Hendricks, Porter, and Tan (1987), Hendricks, Porter, and Wilson (1994), and **?**). In timber auctions by the US forest service, large firms ("mills") have manufacturing capacities and smaller firms ("loggers") lack such capacities (Athey and Levin (2001), Athey, Levin, and Seila (2011), and Athey, Coey, and Levin (2013)). In electricity market, Hortacsu and Puller (2008) compare the bidding behavior of large firms performing closer to the benchmark and small firms who submit steep bid functions due to the presence of scale economies. In cookie-based online advertising, buyers use cookies placed on users' computers to collect information such as users' order history with the online retailer (McAfee (2011) and Abraham, Athey, Babaioff, and Grubb (2012)). Compte, Lambert and Verdier (2004) and Burguet and Perry (2007) are related papers in procurements.

Our approach is to apply the insights from these previous studies to the primary dealer system in the Treasury auctions where primary dealers are better informed than indirect bidders as a result of observing their bids.

and Vuong (2000) to multiple unit auctions using the Turkish Treasury auction data[7]. Kang and Puller (2008) apply Hortacsu (2002)'s framework to Korean Treasury auctions. Kastl (2011) consider uniform price auctions of divisible goods where bidders submit discrete endpoints. (3). Armantier and Sabai (2006) consider constrained strategic equilibrium of Wang and Zender (2002) auctions with risk aversion and supply uncertainty and estimate the parameters of the model using the French Treasury data. Armantier and Sabai (2009) conduct counterfactual comparisons using constrained strategic equilibrium of these auctions.

In the third step concerning the primary dealer system, (1). Smith (1966) studies the bidding problem of a dealer who faces uncertain resale prices in the secondary market. Chatterjee and Jarrow (1998) find that dealers can observe the order flow in the when-issued market and, when the short selling is large, the dealer has an incentive to corner and short squeeze in discriminatory auctions[8]. Bikchandani, Edsparr, and Huang (2000) find that dealers with large demands may forgo buying at low prices on the when-issued market in order not to reveal the information. (2). Fleming (2007) uses the TreasuryDirect investor class data and bidder category data and identifies the bidding methods used by various investor classes. Fleming and Garbade (2007) assess dealer behavior in the specials market. Fleming and Myers (2013) find that, in the post-Lehman period, the primary dealers' share of purchases of new securities has decreased, primary dealers' ownership of Treasury securities has actually increased, and both direct and indirect purchases have increased. (3). Hortacsu and Sareen (2006) document that dealers submit their own bids after they submit their customers' bids, and dealers who submit bids on behalf of a larger number of customers bid closer to the market clearing price of the auction in the Canadian Treasury auction. Hortacsu and Kastl (2012) estimate the economic benefits that dealers derive from observing indirect bidders' bids in the Canadian auctions. Hortacsu, Kastl, and Zhang (2014) examine US Treasury auctions and find that the primary dealer's ability to shade its bids is higher and bids yields are higher. (4) Boyarchenko, Lucca, and Veldkamp (2015) examine the effect of primary dealers sharing order flow information with indirect bidders ("the

[7]The structural estimation method follows a theory of inferences on partially identified models (Tamer (2003), Haile and Tamer (2003), Andrews, Berry, and Jie (2004), Chernozhukov, Hong, and Tamer (2007), Ciliberto and Tamer (2009), Hortacsu and McAdams (2010), Athey and Nekipelov (2012), Pakes, Porter, Ho, and Ishii (2014) and Somaini (2014)). Horowitz (2001) is a survey of bootstrap in econometrics.

[8]A short squeeze happened in the two-year note auctioned on April 1991. Salmon Brothers admitted falsifying customers' bids that lead to the concentration of auction awards to Salomon and some of its customers.

financial accelerator channel") on auction outcomes in the linear-nomral framework.

The one closest to our study is The US Treasury Office of Debt Management (2012) that finds that primary dealers place the most aggressive bids among the three investor classes after taking out bids placed to fulfill their pro-rata bidding requirement[9], consistent with Milgrom and Weber (1982b) and Hendricks and Porter (1988).

The Distance between the Previous Researches and the Goal of the Paper. Nevertheless, there are still three substantial distances from the previous researches to the goal of the paper.

The first distance is the analysis concerning the behavior of primary dealers during the period of financial crisis. Fleming (2007) study the primary dealer behavior between 2003 and 2005. Fleming and Myers (2013) consider the primary dealer behavior till 2012 but do not consider the breakdown for issue type nor consider the composition of direct and indirect bids. Thus we need to conduct further analysis to study the behavior of primary dealers, investment funds, and direct bidding.

The second distance is the economic mechanisms that the primary dealer system affects the bidder behavior and the market outcome. Previously, Hortacsu and Kastl (2012) consider a benefit for the primary dealer from observing the order flow. Hortacsu, Kastl, and Zhang (2014) consider the bidder behavior in US Treasury auctions. Boyarchenko, Lucca, and Veldkamp (2015) compare the impact of information sharing on the market outcome. Thus we need to identify the economic mechanism that the primary dealer system affects the bidding behavior and the market outcome in comparison with the direct bidding system and the syndicate bidding system.

The third distance is the policy counterfactuals. Previously, Hortacsu (2002), Chapman, McAdams, and Paarsch (2006), Hortacsu and McAdams (2010), and others compare discriminatory auctions with Vickrey auctions or "uniform price auctions with truthful bidding." Hortacsu, Kastl, and Zhang (2014) do not conduct counterfactual analysis with other auction system. Armantier and Sabai (2006) and Armantier and Sabai (2009) restrict attention to the constrained space of bidder strategies. Thus we need to develop a method to conduct counterfactual analysis to quantify the effect of policy changes.

The New Approaches of the Paper. Then our approach to close these distances toward the goal of the paper is to adopt the debt management perspective to evaluate the pri-

[9] At each Treasury auction, primary dealers are required to submit, at a minimum, their equally weighted share of the auction's total offering size to ensure coverage ("pro rata requirement").

mary dealer system not only in terms of the expected cost as in the previous research, but also on the dimension of risk and volatility of the auction. Risk and volatility of auctions are important since investors value liquidity and safety of the US Treasury securities (Krishnamurthy and Vissing-Jorgensen (2013)) and the Treasury market is a benchmark for risk-free interest rates and pricing for fixed income securities. Thus, reducing the volatility and risk will increase the demand for Treasury securities and contribute to the financial stability[10]. Nevertheless, this risk and volatility aspects of the auction have never been studied previously. Thus, our study complements previous studies that focus solely on the expected issuance costs[11]. We will then conduct counterfactuals to measure the impact of the auction design on bidding strategies and market outcome using the asymptotic approximation method[12].

The main thesis of the paper is that the primary dealer system affects the bidding behavior and the auction outcome through two channels. The first channel is "the information pooling channel" that the primary dealer can have more precise estimates of the order flow from observing indirect bidders' bids. This channel will reduce the volatility of bids and prices[13]. The second channel is "the competition channel" that the auction system will preserve competition, since the primary dealer needs to forward the bids by indirect bidders and indirect bidders bids are verified. As a result, the primary dealer system could reduce yield volatility further from the direct bidding

[10] The US Treasury debt management objective itself does not explicitly mention risk. But United Kingdom Treasury (2015) states the objective as "minimizing financing costs over the long term, taking account of risk, and to minimize the cost of offsetting the Government's net cash flows over time, while operating in a risk appetite approved by Ministers in both cases." IMF and World Bank (2014) also states the objective as "to raise the required amount of funding at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk."

[11] For example, Hortacsu, Kastl, and Zhang (2014) conclude that "...under the assumption that the mechanism is achieving an approximately efficient allocation, the total bidder surplus in these auctions amounts to about 3 basis point and thus the 3 basis point is the upper bound that one can hope from a redesign of the auction mechanism." But in reality, even if the primary dealer system has low expected bidder surplus, if it entails significant risks to the Treasury markets, then one would need to reconsider the effectiveness of the primary dealer system. That would imply a rationale for further studies beyond Hortacsu, Kastl, and Zhang (2014).

[12] Rothenberg (1984) is a survey of asymptotic approximation method in econometrics. Hong and Shum (2004) show that when information structure and economic enviroment are different, the rate at which the auction aggregates information will also be different. Manski (2007) and Manski (2014) conduct partial identification of counterfactual choice probabilities. Tang (2012) conduct counterfactuals between private values and common values. Menzel (2014) and Menzel (2015) conduct asymptotic approximations on large two-sided matching markets.

[13] DeBrock and Smith (1983), Krishna and Morgan (1997), and Levin (2002) consider the effect of joint bidding though merging of signals in common value auctions.

system without hurting the seller revenue[14]. Example 5.A presents a simple analytical example that compares the primary dealer system, the syndicate bidding system, and the syndicate bidding system

Steps of Analysis. Following the approach described above, we proceed in six steps toward the goal.

Step 1: We summarize the institutional features of the US Treasury auction system characterized by the primary dealer system and the single price auctions.

Step 2: To answer the first research question, we consider the latest Treasury Securities Offering Announcement, Treasury Auction Result, and Investor Class Auction Allotments data. We examine the behavior of primary dealers, investment funds, and direct bidding using the methodology of Fleming (2007).

Step 3: To answer the second research question, we first develop a multiple-unit auction model of uniform price auctions with the primary dealer system with the following features[15].

(1). The seller offers a fixed unit of indivisible security. The justification of the indivisibility assumption is that it reflects the economic reality that the security is offered with the minimum bid amount and multiples. Also this formulation allows to apply the asymptotic analysis developed in the indivisible goods models[16].

(2). The model has primary dealers, direct bidders, and indirect bidders. This as-

[14]Previously Hortacsu and Kastl (2012) quantify the information advantage of the primary dealer from observing the order flow of the indirect bidders. The difference between Hortacsu and Kastl (2012) and this paper is that this paper identifies the channels ("the information pooling channel" and "the competition channel") that the primary dealer system affects bidder behavior and the market outcome, and by quantifying these effects with couterfactual simulations.

[15]The model is based on Vickrey (1961), Reichert (1968), Milgrom (1981), Rustichini, Satterthwaite, and Williams (1994), Ausubel and Cramton (2002), Jackson and Swinkels (2005), Reny and Perry (2006), Fudenberg, Mobius, and Szeidl (2007), Kazumori (2013), and Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) that develop a model of multiple unit auctions. Wilson (1979), Back and Zender (1993), Kastl (2011), and Kastl (2012) consider an alternative divisible goods framework.

The model of this paper is different from Hortacsu and McAdams (2010) because this model incorporates a single price auction and the primary dealer system that are the institutional feature of the US Treasury auctions. This model is also different from Hortacsu and Kastl (2012) and Hortacsu, Kastl, and Zhang (2014) because the model adopts the indivisible goods framework to conduct asymptotic analysis.

[16]There are two other reasons that we adopt the indivisible good framework. (1) Kremer and Nyborg (2004) show that underpricing in the share auctions model will vanish with a discrete demand. Thus Kremer and Nyborg (2004) argue that the model with indivisible goods can be a better description of the auction since underpricing does not materialize in pracice. (2) Kastl (2011) develops a model of uniform auctions of a perfect divisible good in which the bidders submit discrete bid points. But Kastl (2012) notes that equilibria may not exist with uniform price auctions because of ties (also see Reguant (2013)).

sumption allows us to encompass the primary dealer system, the direct bidding system, and the syndicate bidding system within the single framework.

(3). Bidders have private values for the security. The justification of the private value assumption is that, although this assumption is more restrictive than the common value assumption, it allows us to nonparametrically estimate the bidder's value from the market outcome without parametric assumptions[17]. At the same time, the private value assumption still maintains the main economic conclusion of the common value model that informed bidders bid more aggressively than uninformed bidders.

(4). The price and the allocation are determined by a single price auction, as in the US Treasury auctions.

(5). We consider the trembling perfect equilibrium of bidder behavior[18]. A justification of the perfect equilibrium assumption is that we consider a practially reasonable equilibrium robust to noncredible threat and uncertainty in bidding behavior such as noncompetitive bids[19].

Step 4: We use the first order condition to partially identify bidder values. An idea is that, when a bidder places a bid, it must be that the bidder does not wish to bid higher, and thus the value cannot be too high. Also since the bidder does not wish to bid lower, the value should not be too low.

Step 5: We nonparametically estimate these bidder value bounds from the observed market outcome using the first order approach of Guerre, Perrigne, and Vuong (2000). We adopt a clustered bootstrap (Cameron, Gelbach, and Miller (2008)) to directly deal with possible correlations of bids among a primary dealer and indirect bidders who bid through the primary dealer. We also employ the smoothed boostrap (Tibshirani (1988) and Hall, DiCiccio, and Romano (1989)) motivated by a perfect equilibrium.

Step 6: We apply the asymptotic approximation method to the first order condition to derive the bounds on equilibrium bids under different market structures. In an equilibrium, a bidder shades the bid to lower the payment but not more than to

[17] This nonparametric estimation can be valuable since it allows fat-tail distributions.

[18] Hereafter we will call a 'trembling perfect equilibrium' as a 'perfect equilibrium." It should not be confused with another 'subgame perfect equilibrium' that will be applied to extensive form games.

[19] Technically, these perturbations and nonstrategic bids in perfect equilibrium serve as a smoothing kernel of the equilibrium bid distributions. Empirical examples of such non profit-maximizing bids could be foreign official investor's inelastic direct or indirect bids driven by these dollar reserves (Krishnamurthy and Vissing-Jorgensen (2013)) and financial institutions' bids to purchase Treasury securities to satisfy the regulatory requirements. Prevously, Wolak (2003a) and Wolak (2003b) introduce smoothing in the demand side of the model. Hortacsu and McAdams (2010), Hortacsu and Kastl (2012), and Hortacsu, Kastl, and Zhang (2014) introduce perturbations in the supply side.

lose the security. The exact amount of shading cannot be determined without knowing the bidding strategy of other bidders. But we can derive $O(1/n)$-bounds with the truthtelling bidding strategy given estimated bidder values. This asymptotic approximation method has not been employed in the previous literature.

The choice of the auction system affects the bidder behavior through the information pooling channel and the competition channel. First, the difference between the primary dealer system and the syndicate bidding system is the competition channel. The syndicate bidding system has the lower level of competition in comparison with the primary dealer system. Second, the difference between the primary dealer system and the direct bidding system comes from the information pooling channel. In the primary dealer system, the dealer observes indirect bidders' bids and mirrors them. In the direct bidding system, the primary dealer does not observe indirect bidders' bids. Thus the dealer reduces its bids to match (approximately) their expected bids (not the actual bids). When the realizations of indirect bidders' bids are higher than the expected value, the dealer bids less in the primary dealer system. When there are intense competitions ("the competition channel"), even when the realized indirect bidders' bids are low, the primary dealer would not be able to shade the bids much since shading the bids much means losing the security against other primary bidders' bids (and their indirect bidders' bids). In other words, an observation of indirect bidders' bids, that is, more accurate information on the residual supply, can limit the dealer's bid shading and makes its bid more aggressive with sufficient competitions, consistent with Milgrom and Weber (1982b), Hendricks and Porter (1988), and The US Treasury Office of Debt Management (2012).

Step 7: We partially identify counterfactual auction outcomes. We first sample bidder private values from the identified set of bidder values estimated from the observed market outcome. Then, for each bidder, for each sampled private value, and for each auction system, we derive the interval of possible equilibrium bids. By sampling bids from this interval, we can simulate the auction outcome[20].

The Main Findings of the Analysis. By following the above steps of the analysis, we find following answers to the goal of the paper:

For the first research question of the primary dealer behavior after the financial crisis, we find the following 3 points:

[20] Haile and Tamer (2003) calculate the bounds on optimal reserve price based on the estimated bounds on the bidder private values.

- Primary dealers: their allotment shares have decreased especially in securities with short term maturities. The economic mechanism is that, during the period of the financial crisis, the prime dealer's capacity and demand for short-term Treasuries for hedging has decreased because of the balance sheet effects and because dealers' demand for Treasury securities to hedge their inventories of the corporate bond and mortgage-backed securities have been reduced (Adrian, Burke, and McAndrews (2009), Fontaine and Garcia (2011), Graveline and McBrady (2011), Lou, Yan, and Zhang (2013), Cassola, Hortacsu, and Kastl (2012), and Adrian, Fleming, Stackman, and Vogt (2015)).
- Investment funds: their shares have increased in securities with longer term maturities and the share of the direct bidding has increased. An economic mechanism is that investment fund managers subject to withdrawal when fund performance falls below a threshold have preference for liquidity that increases with volatility (Vayanos (2004), and Beber, Brandt, and Kavajecz (2009)).
- Direct bidding: the shares of direct bidding have increased. Increases come from depository institutions, dealers and brokers other than primary dealers, pension and retirement funds, investment funds, and Foreign and international.

For the second research question of quantification of the significance of the primary dealer system to the debt management objective and to the financial stability in comparison with other policy alternatives using counterfactuals, we find the following 3 points:

- The primary dealer system: the primary dealer system has the lowest volatility in comparison with the same level of auction prices in comparison with other distribution systems (23.2% lower than the direct bidding system, and 64% in comparison with the syndicate bidding system). An economic mechanism is as follows: in the primary dealer system, the primary dealer mirrors indirect bidders' bids, and indirect bidders shade their bids further. Thus the volatility of bids is lower than the direct bidding system ("the information pooling channel"). Furthermore, the syndicate bidding system has a fewer number of bidders ("the competition channel")[21].

[21] An idea is that, start from the direct bidding system and consider allowing dealers to observe indirect bidders' bids. Then dealers will have more accurate estimate of the residual supply curve. Then dealers will bid more aggressively and have less uncertainty in bidding. Then, given dealer's aggressive bidding behavior, direct and indirect bidders, who have less information than the dealers,

- The primary dealer allocation: the dealer allocation in the primary dealer system is slightly higher than in the direct bidding system, and has significantly lower standard deviations than those in other systems. The economic mechanism is as follows: under the primary dealer system, a primary dealer has more precise information on the residual supply curve via observation of indirect bidders' bids ("the information pooling channel"). Thus, primary dealers will have a higher allocation and less volatility in the allocation in comparison with other class of bidders.
- The primary dealer profits: their profits are higher with the primary dealer system in comparison with the ones in the direct bidding system. But its magnitude is much smaller than the one in the Canadian Treasury auctions estimated by Hortacsu and Kastl (2012). The economic mechanism is that the dealer's additional knowledge of indirect bidders' bids ("the information pooling channel") will increase the dealer profits over the one in the direct bidding system, but the competition channel among primary dealers and limits dealer profits[22].

In conclusion, during the period of financial crisis where the primary dealer's capacities and demands are limited and investment funds looked for assets with safety and risklessness, the primary dealer system would have reduced the interest rate risks, enhanced the demand for Treasury securities, and contributed to the debt management objective[23].

The Contributions of the Paper. The contribution of this paper is that it analyzes the policy question asked by the US Treasury and quantifies the contribution of the primary dealer system to the debt management objective using economic theory and data analysis. The paper advances the previous research in the following 5 aspects. (1). This paper documents the change in the primary dealer, investment funds, and direct bidding behavior after the period of financial crisis going beyond the study of Fleming (2007) and Fleming and Myers (2013). (2) This paper considers the risks of

will reduce their bids, that will further reduce the volatility of bids and prices (the information pooling channel). But at the same time, the fact that many primary dealers place bids will limit the effect on seller's revenue (the competition channel).

[22]In the US market, the number of primary dealers in the US auctions is about the twice of the number of primary dealers in the Canadian auctions, leading to a higher level of competition among primary dealers. This result is consistent with a recent finding in Hortacsu, Kastl, and Zhang (2014).

[23]These results are consistent with the market microstructure literature that shows importance of the market making system (Huang and Stoll (1996), Tse and Zabotina (2004), and Venkataraman and Waisburd (2007)).

the auction prices as a metric to evaluate the performance among different auction systems to the debt management objective. This risk aspect is important since investors in Treasury securities value safety and liquidity of Treasury securities. It advances the previous research since none of the structural analysis of Treasury auctions consider the risk of the primary dealer system and Boyarchenko, Lucca, and Veldkamp (2015) compare the primary dealer system with and without the primary dealer feedback ("financial accelerator channel") but do not compare the risk of the primary dealer system in comparison with other auction systems. (3). This paper identifies the economic mechanisms ("the information pooling channel" and "the competition channel") that the primary dealer system can affect the market outcome including the risk of the auction. It advances the previous research that considers the information advantage of the primary dealers but does not consider its impact on the auction outcome. (4). This paper applies the asymptotic approximation method to estimate the bidding strategies in different auction systems and conduct counterfactual analysis. Previous counterfactual analysis of multiple unit auctions need to assume truthful bidding and does not allow the strategic feature of these auction systems. (5). The finding that the main difference among the auction systems is not in terms of the expected prices (yields) but rather in terms of risks is new and practically important for the Treasury auctions since investors of Treasury securities value their liquidity and safety (Krishnamurthy and Vissing-Jorgensen (2013))[24].

The Organization of the Paper. The organization of the paper is as follows. Section 2 develops a structural analysis of US Treasury auctions. Section 3 reports the results of the analysis of the primary dealer system of the US Treasury auctions. Section 4 concludes. Appendix 5 provides proofs of Propositions and additional details.

A. Related Work.

We now discuss the relationship between the related work concerning the auctions with heterogeneous bidders and this paper.

Hortacsu and Sareen (2006) find, using the data from Canadian Treasury auctions, that a vast majority of bid submissions are concentrated within the last 10 minutes

[24] See McAfee and McMilan (1987), Bajari and Hortacsu (2003), and Athey, Levin, and Seila (2011) for theoretical and empirical analysis of entry in auctions. Our results are in line with the practionners' comments such as Bartolini and Cottarelli (1994). Longstaff (2004) estimates importance of liquidity in Treasury markets.

preceding the bid submission deadline, dealers submit their own bids seconds after they submit their customers' bids, and that dealers who submit bids on behalf of a larger number of customers consistently bid closer to the market clearing price of the auction. This paper extends and strengthens their analysis in the following 2 points. First, this paper develops an economic mechanism that the primary dealer system affects the market outcome. Second, this paper conducts structural estimation and counterfactual analysis to compare the primary dealer system with other auction systems.

Fleming (2007) uses the TreasuryDirect investor class data and bidder category data and identifies the bidding methods used by various investor classes. This paper extends and strengthens their analysis in the following two points. First, this paper extends Fleming (2007)'s analysis using the latest data including the period of the financial crisis. Second, this paper identifies the economic mechanism that the primary dealer system affects the bidding behavior and the market outcome and conducts structural estimation and counterfactual analysis.

Hortacsu and McAdams (2010) consider a non-parametric model of discriminatory auctions among symmetric bidders with private values, estimate the bounds of private values with the resampling method using the data from Turkish Treasury auctions, and compare the outcome with hypothetical uniform price auctions with truthful bidding. This paper extends and strengthens their analysis in the following two points. First, this paper conducts structural estimations of bidder private values using a model of uniform price auctions with the primary dealer system. This extension is not trivial because of possible correlation of bids among the primary dealers and indirect bidders that is not present in Hortacsu and McAdams (2010) where bidders are symmetric. We employ a more general smoothed clustered boostrap for estimation. Second, we conduct counterfactual to compare auction systems. This extension is not trivial since we do not assume that bidders will bid truthfully in these auctions as in Hortacsu and McAdams (2010). Thus we develop the asymptotic approximation method.

Athey, Levin, and Seila (2011) consider open auctions and sealed bid auctions among "loggers" (small firms) and "mills" (larger firms) with private values, parametrically estimate bidder values and entry costs, and compare relative performances of open and sealed bid auctions using data from the U.S. Forest Service timber auctions. This paper contributes to the study of bidder heterogeneity using uniform price auctions with the primary dealer system, nonparametrically estimating bidder private values, and conducting counterfactual analysis among auction systems using the asymptotic

approximation method.

Hortacsu and Kastl (2012) consider the model of discriminatory auctions with primary dealers in the divisible good auction framework, and test where the dealers learn about competition (residual supply curve) or fundamentals, and estimate dealers' profits using the Canadian Treasury auction data. This paper extends and strengthens their analysis in the following two points. First, this paper focuses on the significance of the primary dealer system for the seller. Second, the paper further quantifies the significance of these channels by conducting counterfactual comparison using the asymptotic approximation method[25]. These extensions are important to answer the policy questions about the significance of the primary dealer system for the debt management objective.

Fleming and Myers (2013) find that, in the post-Lehman period, the primary dealers' share of purchases of new securities has decreased, primary dealers' ownership of Treasury securities has actually increased, and both direct and indirect purchases have increased. This paper extends and strengthens their analysis in the following 2 points. First, this paper develops a more through analysis of bidder behavior during the period based on the methodology developed by Fleming (2007). Second, this paper formulates an economic theory of the primary dealer system, and conducts an empirical analysis.

Hortacsu, Kastl, and Zhang (2014) analyze the US Treasury auction and finds that primary dealer's ability to bid-shade is higher, leading to higher yield bids. This paper extends and strengthens their analysis in the following three points. First, this paper identifies the economic channels that the primary dealer system can affect the bidder behavior and the auction outcome. Second, we conduct counterfactual simulations comparing auction systems. The result confirms their findings. More importantly, this paper focuses on the interest rate risk of the auction systems that Hortacsu, Kastl, and Zhang (2014) have not studied at all[26].

[25] But Hortacsu and Kastl (2012) do not conduct counterfactuals with the direct bidding system where dealers do not observe customer bids (Hortacsu and Kastl (2012), p.2533).

[26] Hortacsu, Kastl, and Zhang (2014) report that while primary dealer willingness-to-pay is similar to or even higher than direct and indirect bidders', their ability to bid-shade is higher, leading to higher yield bids." But in US Treasury auctions, primary dealers are required to always place a pro rata bids. The US Treasury Office of Debt Management (2012) point out that some bids by primary dealers are nonserious "throw-away bids," and after taking these throw-away bids out, "primary dealers usually place the most aggressive bids among the three investor classes."

In other words, the results of this paper can be summarized as follows: "Hortacsu, Kastl, and Zhang (2014) show that under the assumption that the mechanism is achieving an approximately

Boyarchenko, Lucca, and Veldkamp (2015) develop a theory of intermediaries where the primary dealers aggregate the information across clients, and use it to advise their clients. Boyarchenko, Lucca, and Veldkamp (2015) find that the primary dealer system increases expected auction revenue, but also make the revenue more sensitive to changes in asset value ex ante. This financial accelerator channel adds to the information pooling channel of this paper and both channels make the auction price better reflects the fundamental value of the security. An empirical quantification of the financial accelerator channel, which requires further data on the nature of information dissemination by the primary dealers, will be a topic of future research.

2. Structural Analysis of the Primary Dealer System in the US Treasury Auctions.

We now develop structural analysis of the uniform price auctions with the primary dealer system following the 6 steps described in the Introduction.

A. Institutional Features of the US Treasury Auctions.

We begin by summarizing the institutional features of the US Treasury auctions[27].

A.1. The Primary Dealer System.

The primary dealer system is a bidder management system where bidders are classified into primary dealers, indirect bidders, and direct bidders[28].

Primary Dealers. The primary dealers serve as trading counterparties of the Federal Reserve Bank of New York in its implementation of the monetary policy. In addition,

efficient allocation, the total bidder surplus in these auctions amounts to about 3 basis point and thus the 3 basis point is the upper bound that one can hope from a redesign of the auction mechanism. The counterfactual analysis of the paper result confirms their findings. Furthermore, we find that the key difference among these auction systems is in terms of interest rate risks, and that the primary dealer system reduces the risk of the Treasury auction system. Then, the conclusions of this paper are that the primary dealer system contributes to the debt management objective when the competition channel is ensured with the primary dealer participation and the verification of indirect bidders' bids."

[27] Cantor and Stabile (1989), The US Treasury et al. (1992), Garbade (2012), and Sunner (2012) document the evolution of the US Treasury auction system. The US Treasury (2015) defines the current rule of the US Treasury auctions.

[28] The Federal Reserve conducts its purchases and sales of securities chiefly through transactions with primary dealers ("market operations") rather than directly with the Treasury. As such, the Federal Reserve purchases most of the Treasury securities in the secondary market.

these primary dealers serve the following 4 crucial roles in the US Treasury auctions: (1). Bid in every auction, for, at a minimum, an amount of securities representing a pro rata share, based on the number of primary dealers at the time of the auction, of the offered amount. (2). Their bid prices should be reasonable when compared to the range of rates trading in the when-issued market, taking into account of market volatility and other risk factors. (3). Submit bids from indirect bidders directly to the Treasury. (4). Also in the quarterly refunding process, the Treasury solicits advice and views from the private sector through questions to primary dealers and interview half of the primary dealers each quarter through face-to-face meetings. There are 46 primary dealers in 1988 and there are 22 at this moment[29].

Indirect Bidders. Indirect bidders place bids through a direct submitter. Indirect bidders are investment funds (money-market funds and hedge funds) and Foreign and International Monetary Authorities (who bid through the Federal Reserve Bank of New York). Indirect bidders can also be corporations or individuals.

The Federal Reserve engages in spot-checking of customer bids in Treasury auctions to verify the legitimacy of large winning bids submitted for customer accounts. In addition, the Treasury and the FRBNY implement a formal system that requires customers who make large winning bids through dealers or depository institutions to verify their bids in writing to the Federal Reserve prior to the settlement date (The US Treasury et al. (1992))[30].

Direct Bidders. Direct bidding refers to a submission of bids by an entity directly to the Treasury or Federal Reserve rather than through an intermediary such as a bank or a securities dealer[31]. Direct bidders include primary dealers, other brokers and dealers various types of investment funds, insurance companies, depository institutions, foreign and international entities, the Federal Reserve (System Open Market Account)[32].

[29] Primary dealer list is available at the FRBNY website. In 1992 the Treasury announced that all government securities brokers and dealers registered with the SEC would be eligible to submit bids for customers in Treasury auctions. Previously, only primary dealers and depository institutions had this privilege. The Treasury is not imposing any limitation on the combined amount awarded to a dealer and the customers for whom the dealer has placed bids. The model of the paper allows a wider interpretation of "the primary dealer system" where bidders other than the primary dealer can route the bids from indirect bidders.

[30] This development is in response to Salmon Brothers submission of unauthorized customer bids in December 1990, February 1991, April 1991, and May 1991 auctions.

[31] Peter Fischer, then Under Secretary of the Treasury, introduced the direct bidding in 2002 (Fischer (2002)).

[32] Harper and Kruger (2013) report that current version of the direct-bidding system, introduced in 2008, is used by 266 investors. According to Harper and Kruger (2013), Jason Evans said: "the

A.2. Single Price Auctions.

A single price auction is a method according to which the Treasury determines the allocation and the price/yields based on bids submitted by bidders[33].

Public Announcements. The public announcement press releases provides information on the amount of the security being offered, auction date, issue date, maturity date, terms and conditions of the offering, and noncompetitive and competitive bidding close times. Bids may be submitted at any time from the opening of the bidding until the public deadline.

Noncompetitive Bids. Bidders can submit competitive bids or noncompetitive bids[34]. A noncompetitive bid that comply with auction rules will be accepted. A noncompetitive bid is capped to $5 million for individuals and entities, and $100 million for sovereign central banks. A noncompetitive bidder is not permitted to have a position in the security being auctioned in the when-issued, futures, or forward markets prior to the auction. The noncompetitive bidding closes before the close of the competitive bidding.

Competitive Bids. A competitive bid specifies the yield, or discount margin that is acceptable. The Treasury changed the rule to receiving bids on the basis of yields rather than prices in 1983. The minimum price increments are 0.5 basis points for thirteen, twenty-six, fifty-two weeks, and cash management bills, and 0.1 basis points for all other securities. Award is limited to 35 percent of the amount offered to the public[35]. A bidder is not permitted to submit both competitive and noncompetitive bids in a single auction (The US Treasury et al. (1992)). The competitive closing time for bills is normally 11:30 a.m and normally 1:00 p.m. for notes, bonds, FRNs, and TIPS.

Allocation. After the close of an auction, after it has been determined that bids have complied with the Treasury requirements, the Treasury awards all noncompetitive

primary-dealer community has a pretty good idea that all of the big accounts probably have the lines set up so that they can bid directly."

[33] The Treasury department conducts auctions at the Federal Reserve Bank of New York and the Bureau of Public Debt in Washington DC. The Federal Reserve Act of 1913 provides that the Federal Reserve Banks will act as fiscal agents and depositories when required to do so by the Secretary of the Treasury.

[34] A provision of noncompetitive bids was first introduced in 1967 for bill auctions.

[35] The Treasury adopts ceilings of the maximum award per bidder of 35 percent of the public offering since 1962. A bidder must report its net long postion along with its auction bids if the sum of its net long position in when-issued, forward, and futures contracts and its bids exceeds 35 percent of the offering.

bids and then accepts competitive bids in the ascending order of their yields (lowest to highest) until the quantity of awarded bids reaches the offering amount.

Price/Yields. All bidders will receive the same yield at the highest accepted bid[36].

Auction Results Announcement. After the conclusion of the auction, the auction results are posted on the TreasuryDirect website as a press release.

Settlement. On the day of issuance, the settlement (the exchange of securities for funds) takes place either on the books of a depository institution or between depository institutions through the Federal Reserve's Fedwire securities transfer system.

The previous section summarized the institutional features of the US Treasury auctions. We now proceed to conduct analysis of the primary dealer system according to the steps described in the Introduction.

B. Modelling the US Treasury Auctions.

In the previous step we have recorded the institutional features of the US Treasury auctions. In this step, we develop a mathematical model of the primary dealer system and the single price auctions for the statistical analysis. We formulate the primary dealer system based on models of Milgrom (1981), Rustichini, Satterthwaite, and Williams (1994), Jackson and Swinkels (2005), Cripps and Swinkels (2006), Athey, Levin, and Seila (2011), and Kazumori (2013), and we slightly extend the model to incorporate the primary dealer system. We discuss some specific modeling choices at Section 2.B.4.

B.1. The Primary Dealer System.

The Seller. The seller offers multiple units of the security. Let $M \in \mathbf{N}_{++}$ be the offering amount of the securities in the auction.

Bidders. In the primary dealer system, there are 3 classes of bidders: primary dealers, indirect bidders, and direct bidders[37]. Let $\mathbf{I}_{PD}$, $\mathbf{I}_{ID}$, and $\mathbf{I}_{D}$ be sets of primary dealers, indirect bidders, and direct bidders. Then $\mathbf{I} \equiv \mathbf{I}_{PD} \cup \mathbf{I}_{ID} \cup \mathbf{I}_{D}$ be the collection of all bidders. Let us further denote $N_{PD} \in \mathbf{N}_{+}$, $N_{ID} \in \mathbf{N}_{+}$, and $N_{D} \in \mathbf{N}_{+}$ as the number

[36] The UK gilt auctions have an additional Post Auction Option Facilty where all bidders are offered the right to purchase up to an additional percentage of the gilt they were allocated at the published prices.

[37] We simplify the model by not including noncomptitive bids explicitly in the model. A perfect equilibrium incorporates a bidder's response to random elements in the bidder demands. Empirically, the share of noncompetitive bids in the auction is small.

of primary dealers, indirect bidders, and direct bidders. Then there are $N_{PD}+N_D+N_{ID}$ bidders in the auction. For each indirect bidder $i \in \mathbf{I}_{ID}$, let $PD(i) \in \mathbf{I}_{PD}$ be the primary dealer through which indirect bidder i submits its bid in the auction. For each primary dealer $i \in \mathbf{I}_{PD}$, let $\{j \in \mathbf{I}_{ID} : PD(j) = i\}$ be a collection of indirect bidders connected to dealer i and let $N_{ID,i} \in \mathbf{N}_+$ be the number of indirect bidders connected to the primary dealer i.

Bidder Private Value. Let us denote bidder i's monetary value of an $m = 1, ..., M$th unit of the security offered by $v_{i,m} \in V_{i,m} = [0, \overline{v}] \subseteq \mathbf{R}_+$. Here $\overline{v} \in \mathbf{R}_+$ is the maximum value that any bidder can have for a unit of security. Then, bidder i's value profile is an M-dimensional vector $\overrightarrow{v}_i \equiv (v_{i,1}, ..., v_{i,M})$. We assume that a bidder's marginal values are nonincreasing: $v_{i,1} \geq ... \geq v_{i,M}$. Then the set of possible value profiles for a bidder is denoted by $V_i = \underbrace{V_{i,m} \times ... \times V_{i,m}}_{M \text{ units}} \subseteq \mathbf{R}_+^M$.

Bidder Allocation. Let $q_{i,m} \in \{0, 1\}$ be bidder i's allocation for the $m = 1, ..., M$th unit from the auction: $q_{i,m} = 1$ if bidder i wins the mth unit of the security and 0 otherwise. Then, bidder i's allotment is an M-dimensional vector $\overrightarrow{q}_i \equiv (q_{i,1}, ..., q_{i,M}) \in \{0, 1\}^M$.

Payment. In a single price auction, every bidder pays the same price p for each unit of the security.

Ex Post Payoff. We assume that a bidder's ex post payoff is additively separable across units and is quasi-linear. Suppose bidder $i \in \mathbf{I}$ has a private value $\overrightarrow{v}_i \in V_i$. Thus bidder i's ex post payoff function is given by $u(v_i, p, q_i) \equiv \sum_{m=1}^{M} q_{i,m}(v_{i,m} - p)$. We assume that a bidder is risk-neutral.

Information Structure. We assume that class $i = PD, D, ID$ bidder's value is an independent draw according to a distribution $F_i : \mathbf{R}_+^M \to [0, 1]$ with an atomless and uniformly bounded (from below and from above) density $f_i : \mathbf{R}_+^M \to \mathbf{R}_{++}$ with its common support V_i. They are common knowledge among the seller and bidders.

Auction Systems. In the primary dealer system, primary dealers route bids from indirect bidders and direct bidders who submit bids directly to auctioneer. In the direct bidding system, every bidder has a direct link to the auctioneer[38]. In the syndicate bidding system, we make a simplified assumption where a primary dealer and indirect bidders form a consortium where a primary dealer and indirect bidders share the private

[38] That is, the direct bidding system affects the bidders through the competition channel but the information pooling channel is absent.

values and submit competitive bids to maximize the profit for the consortium[39].

Scaling the Economic Environments. We conduct an asymptotic analysis as the number of bidders increase. In this process, indexed by n, the seller auctions $M \cdot n$ units of securities among $n \cdot N_{PD}$ primary dealers, $n \cdot N_D$ direct bidders, and $n \cdot N_{ID}$ indirect bidders. That is, in the nth environment, there are $n \cdot (N_{PD} + N_D + N_{ID})$ bidders in this auction. Each bidder demands a finite $M \in \mathbf{N}_{++}$ units of securities[40].

B.2. The Single Price Auction Mechanism.

Bids. Bidder i places a bid $b_{i,m} \in \mathbf{R}_+$ for each $m \in \{1, ..., M\}$th unit of the security[41]. Let $\overline{b} \in \mathbf{R}_{++}$ be the maximum recognized bid. Let $\underline{b} < 0$ be a nonparticipation bid (option) that ensures the zero payoff regardless of other bidders' bids[42]. The set of possible (unit) bids for the mth unit is denoted by $B_{i,m} \equiv \{\underline{b}\} \cup [0, \overline{b}] \subseteq \mathbf{R}$[43]. Bidder i's bid profile is an M-dimensional vector $\overrightarrow{b}_i \equiv (b_{i,1}, ..., b_{i,M}) \in \mathbf{R}^M$. The set of possible bids for bidder i is $B_i \equiv \underbrace{B_{i,m} \times ... \times B_{i,m}}_{M \text{ bids}}$.

Profile of Bids. A profile of bids by all bidders is an $M \cdot n \cdot (N_{PD} + N_D + N_{ID}) \in \mathbf{N}_{++}$ dimensional vector $\overrightarrow{b} \equiv (\overrightarrow{b}_1, ..., \overrightarrow{b}_{n \cdot (N_{PD}+N_D+N_{ID})})$. For each bidder i, let b_{-i} denote an $M \cdot \{n \cdot (N_{PD} + N_D + N_{ID}) - 1\}$ dimensional vector $b_{-i} = (\overrightarrow{b}_1, ..., \overrightarrow{b}_{i-1}, \overrightarrow{b}_{i+1}, ..., \overrightarrow{b}_{n \cdot (N_{PD}+N_D+N_{ID})})$ that are bids by bidders other than i.

Order Statistics of Bids. Given a collection of $M \cdot n \cdot (N_{PD} + N_D + N_{ID})$ bids $\overrightarrow{b}$, we can reorder them so that $b_{M \cdot n \cdot (N_{PD}+N_D+N_{ID}):M \cdot n \cdot (N_{PD}+N_D+N_{ID})} \leq ... \leq b_{M \cdot n \cdot (N_{PD}+N_D+N_{ID}):1}$. Let $b_{M \cdot n \cdot (N_{PD}+N_D+N_{ID}):t}$ be the tth highest order statistics out of $M \cdot n \cdot (N_{PD}+N_D+N_{ID})$ bids.

[39] That is, the direct bidding system affects the bidders through the information pooling channel but the competition channel is absent. The impact of the lack of the competition channel effect would be robust to a detail of interactions among bidders within the consortium.

[40] This formulation follows Pesendorfer and Swinkels (1997). It is not important that the bidder demands exactly M units as long as they demand only a finite amount of units in the auction. Furthermore, we further classify primary dealers and indirect bidders into following subcategories. Let $PD_l \subseteq \mathbf{I}_{PD}, l = 0, 1, ...,$ be the collection of primary dealers connected to l indirect bidders. Let $|PD_l| \in \mathbf{N}_+$ denote the number of primary dealers with l indirect bidders connected. For each $n = 1, 2, ...$th economic environment, we assume that there are $n \cdot |PD_l|$ primary dealers each connected to l indirect bidders.

[41] This paper assumes that the bid is in terms of the security price instead of yield to simplify the calculation of a bidder payoff. Since there is one-to-one relations between the bond price and the yield, this assumption is without loss of generality.

[42] Technically, this assumption ensures that a bidder will not play dominated strategies in the auction. This assumption is common in the literature (Athey (2001) and Reny and Zamir (2004), for example).

[43] The set of possible bids is continuum given the assumption that bidders have private values.

The Price. Let us denote $p(b) : \underbrace{B_i \times ... \times B_i}_{M \cdot n \cdot (N_{PD}+N_D+N_{ID}) \text{ bids}} \to B_{i,m}$ the market clearing price when bidders submitted bids $\overrightarrow{b}$. In a single price auction, the price is set to be the lowest accepted bid : $p(b) \equiv b_{M \cdot n \cdot (N_{PD}+N_D+N_{ID}):M \cdot n}$.

The Allocation. Let $q_m(\overrightarrow{b}_i, \overrightarrow{b}_{-i}) : \underbrace{B_i}_{\text{bidder } i\text{'s bids}} \times \underbrace{B_i \times ... \times B_i}_{n \cdot (N_{PD}+N_D+N_{ID})-1 \text{ times: bids by bidders other than } i} \to \{0, 1\}$ be the allocation of bidder i for the mth unit when bidder i's bid is $\overrightarrow{b}_i$ and other bidders' bids are $\overrightarrow{b}_{-i}$. In the single price auction, $q_m(\overrightarrow{b}_i, \overrightarrow{b}_{-i}) = 1$ if $b_{i,m} > p(b)$,or $q_m = 0$ if $b_{i,m} < p(b)$, and the ties are broken using the uniform tie-breaking rule[44].

B.3. Bayesian Game.

We now formulate bidders' bidding decisions using the framework of a Bayesian game (Harsanyi (1967)). Let $\Gamma(n)$ denote a Bayesian game among $n \cdot N_{PD}$ primary dealers, $n \cdot N_{ID}$ indirect bidders, and $n \cdot N_D$ direct bidders bid in the single price auction.

Bidder's Information Set. Bidder $i \in \mathbf{I}$ places a bid $b_i \in B_i$ based on its private value $\overrightarrow{v}_i \in V_i$. In addition, an auction system might allow some bidders to obtain additional information about other bidders before submitting their bids. Formally, let $\overrightarrow{z}_i$ be an information variable that represents the information available to bidder $i \in \mathbf{I}$ before the submission of bids. Let Z_i be the set of possible information that would be available to bidder i. Primary dealer i has indirect bidders $\{j \in \mathbf{I}_{ID} : PD(j) = i\}$ who will bid through primary dealer i. Then the primary dealer's information additional to its private value is $\overrightarrow{z}_i = \{\overrightarrow{b}_j\}_{j \in \mathbf{I}_{ID}:PD(j)=i}$. Neither a direct bidder nor an indirect bidder will observe other bidders' bids. For such bidder i, $\overrightarrow{z}_i = \phi$. A primary dealer in the syndicate bidding system are assumed to observe the private values of indirect bidders connected to the primary dealer.

Bidder's Strategy. Bidder i's (behavioral) strategy is an $M-$dimensional random variable $\overrightarrow{\beta}_i(\overrightarrow{v}_i, \overrightarrow{z}_i) : V_i \times Z_i \to B_i$ over the set of possible bid profiles. Let $g_i(\overrightarrow{b}_i | \overrightarrow{v}_i, \overrightarrow{z}_i) : B_i \times V_i \times Z_i \to \mathbf{R}_+$ be the probability density function of a bid schedule $\overrightarrow{b}_i \in B_i$ when bidder i bids according to the strategy $\overrightarrow{\beta}_i(\overrightarrow{v}_i, \overrightarrow{z}_i)$.

Bayesian Nash Equilibrium. A strategy profile $\overrightarrow{\beta}^*_n = (\overrightarrow{\beta}^*_{n,PD}, \overrightarrow{\beta}^*_{n,ID}, \overrightarrow{\beta}^*_{n,D})$ is a type-symmetric Bayesian Nash equilibrium (Athey, Levin, and Seila (2011)) of $\Gamma(n)$

[44] That is, we assume the uniform tie-breaking rule, but the details of the tie-breaking rules are not relevant in the private value auction (see Jackson and Swinkels (2005), Reny and Zamir (2004), and Reny and Perry (2006) for discussions on tie-breaking rules). We also assume that, when a bidder bids the same amounts for multiple units, a bidder is allocated from earlier units.

if (i) every primary dealer chooses a strategy $\overrightarrow{\beta}^*_{PD,n}$, every indirect bidder chooses a strategy $\overrightarrow{\beta}^*_{ID,n}$, every direct bidder chooses a strategy $\overrightarrow{\beta}^*_{D,n}$, and, (ii) for each bidder $i \in \mathbf{I}$, for its private value $\overrightarrow{v}_i \in V_i$ almost everywhere, for almost every realization of information $\overrightarrow{z}_i \in Z_i$, any bid $\overrightarrow{b}_i \in B_i$ that the bidder chooses with a positive probability according to $\overrightarrow{\beta}_i(\overrightarrow{v}_i, \overrightarrow{z}_i)$ maximizes the expected payoff of bidder i when other bidders submit bids according to $\overrightarrow{\beta}^*_n$.

Trembling-Hand Perfect Equilibrium. A perfect equilibrium (Selten (1975)) is a Bayesian Nash equilibrium that is robust to noncredible threats. To define a perfect equilibrium, first consider a "smoothed game" $\Gamma(\varepsilon, n)$ where each bidder, with a given positive probability $\varepsilon > 0$, chooses a bid for each unit nonstrategically according to the uniform distribution on $B_{i,m}$, independently from a bidder's private value, from bids for other units, and from other bidders[45]. Let $\overrightarrow{\beta}^*_{\varepsilon,n}$ be a type-symmetric Bayesian Nash equilibrium of $\Gamma(\varepsilon, n)$. A type-symmetric perfect equilibrium of $\Gamma(n)$ is a type-symmetric Bayesian Nash equilibrium of $\Gamma(n)$ that is also a limit of a sequence of type-symmetric Bayesian Nash equilibria $\{\overrightarrow{\beta}^*_{\varepsilon,n}\}_\varepsilon$ of smoothed games $\{\Gamma(\varepsilon, n)\}_\varepsilon$ as $\varepsilon \to 0$[46].

Equilibrium Existence. Given the private value of bidders, there exists a perfect equilibrium of $\Gamma(n)$.

Lemma 1: *For each size of the auction market $n \in \mathbf{N}_{++}$, there exists a perfect equilibrium $\overrightarrow{\beta}^*_n$ of the auction market game $\Gamma(n)$.*

The proof is in Appendix 5.B. The existence result ensures the validity of our analysis using a perfect equilibrium. We write down the first order condition for in Section 2.C.1 to characterize equilibrium bids.

B.4. Discussion of Modeling Choices.

We discuss several factors of potential importance in the modelling of Treasury auctions

Types of Securities. The Treasury employs auctions for Treasury Bills since 1929 but did not employ fixed-price offerings for notes till November 1970 and for bonds till January 1973 on the grounds that small banks, corporations, and individuals would not have the professional capability to bid in the auctions and thus would not participate in auctions for these long-term bonds(Garbade (2012)). These historical development

[45] We make assumptions of uniform distributions to simplify the presentation.

[46] The notion of convergence is convergence in distribution (i,e., weak convergence).

suggests the possibility that a different type of securities may have different distribution of bidders. This paper considers that these different bidder characteristics are encoded in the distributions of bidder private values. The benefit of this assumption is that we can apply the same model for auctions of securities of different characteristics.

Bidder Inventory, Capacity Considerations, and Secondary Market. Fleming and Garbade (2007) find that dealers regularly take into inventory a large share of Treasury issuance so that dealer positions increase during auction weeks. Such time-series patterns in bidding due to inventories or capacity constraints can be potentially important. Furthermore, bidders can be potentially concerned with the total payoffs over a series of auctions, when-issued market, secondary market, and the repo markets. This paper assumes that these considerations are encoded in the bidder valuations in the primary market auctions. The benefit of this assumption is that we can estimate bidder values and conduct counterfactuals without solving dynamic auction games. It would be reasonable to think that the information pooling channel and the competition channel found in this model would extend with dynamic considerations[47].

Indivisible Goods Assumption. We assume that the seller offers a fixed units of indivisible security instead of the share auction framework. The benefits of the indivisible goods assumptions are the following 4 points. (1) The indivisible good assumption is that it reflects the economic reality that security is offered with some fixed units. (2) The indivisible goods assumption avoids the criticism of the share auctions by Kremer and Nyborg (2004) that the share auctions model has an implausible underpricing equilibrium. Such underpricing equilibrium vanishes with an indivisible good environment. (3) The indivisible goods model always has a Bayesian Nash equilibrium, avoiding the nonexistence problem pointed out by Kastl (2011) and Kastl (2012)for a model of uniform auctions of a perfect divisible good in which the bidders submit discrete bid points. (4). The indivisible goods model allows us to apply the asymptotic analysis and approximation based on the results of Milgrom (1981), Rustichini, Satterthwaite, and Williams (1994), Jackson and Swinkels (2005), Cripps and Swinkels (2006), and Kazumori (2013).

Private Values. We assume that the bidders have private values. There are 4 benefits from the private value assumption. (1) The private value assumption captures the fact that bidders' demands are driven by idiosyncratic considerations such as dealers' short positions in when-issued markets, hedging, regulatory requirements, and portfolio

[47] Haile (2001) study the effect of resale and the secondary market in US Forest service timber sales.

diversification specific to each bidder. (2) The private value assumption is consistent with the findings by Hortacsu and Kastl (2012) that the estimated marginal valuation before and after the observation of the bids by indirect bidders does not change. (3) The private value assumption allows nonparametric estimation of bidder private values that accommodates fat tails distributions important in the risk analysis. On the other hand, the structural estimation is currently possible only with normal distributions (Somaini (2014)). (4) Information pooling channel and the competition channel are present in both of the common value model and the private value model.

Risk Aversion. We assume that bidders are risk-neutral although a bidder is usually assumed to be risk averse in the finance literature. The benefit of this assumption is that it is consistent with the observation by Bartolini and Cottarelli (1994) that risk aversion would not be much of a problem since no single auction is large relative to the wealth of auction participants for a realistic degree of risk aversion[48].

Budget Constraints. We assume that bidders do not fact the budget constraints in the model since no single auction would not be large relative to the wealth of large bidders[49].

Collusions. We do not consider bidder collusions in this model. Rieber (1965) finds that of allotment, unit premium, and profits make it unlikely that collusion exists in the auction market. Jegadeesh (1993) examines the issue and report variability that allocation to banks and dealers are negatively related to auction profits. Umlauf (1993) suggest the presence of collusion among large bidders in the Mexican discriminatory Treasury auctions and report that the bidder profits fell sharply after the adoption of uniform price auction. Athey, Levin, and Seila (2011) find that sealed-bid auctions (like the single price auctions in the US Treasury auctions) are robust to collusions than open auctions. A possibility of collusions is a topic of further research[50].

Perfect Equilibrium. We consider a trembling-hand perfect equilibrium in auction games. There are 3 benefits from this assumption. First, a trembling hand perfect equilibrium takes into account of real-world uncertainty about bidder behavior such as noncompetitive bidding. Second, a perfect equilibrium has been used in auction literature to rule out intuitively unappealing equilibria in which uninformed bidders bid aggressively (Milgrom (1981) and Abraham, Athey, Babaioff, and Grubb (2012),

[48] For US Forest Service Timber auctions, Athey and Levin (2001) find bidders behavior is consistent with risk aversion. It will be a future research question to consider the effect of risk aversion.

[49] This is in contrast to FCC's Advanced Wireless Spectrum auctions where bidder budget constraints play a crucial role in determining the bidding strategy (Bulow, Levin, and Milgrom (2009)).

[50] Marshall and Marx (2012) develop an economic analysis of cartels, bidding rings, and detection.

for example). Third, it provides a "smoothed" version of the demand function that allows more accurate estimations of quantiles (Wolak (2003a) and Wolak (2003b)).

C. *Partial Identification of Bidder Values from the First Order Conditions.*

Step 1 formulated the model of uniform price auctions with the primary dealers. In the next step, we analyze the bidder's decision problem and derive the first order condition that an equilibrium bid has to satisfy. Then, by solving the first order condition, given the bid, one can derive the identified set of private values.

C.1. Bidder First Order Conditions.

Take a Bayesian game $\Gamma(\varepsilon, n)$ and take a bidder $i \in I$ with a private value $\overrightarrow{v}_i \in V_i$[51]. We assume that bidding behavior are characterized by a Bayesian Nash equilibrium strategy profile $\overrightarrow{\beta}^*_{\varepsilon,n}$. Let $\overrightarrow{b}^*_{\varepsilon,i,n} \in B_i$ be a bid profile that bidder i places with a positive probability according to the equilibrium strategy $\overrightarrow{\beta}^*_{\varepsilon,i,n}(\overrightarrow{v}_i)$.

Bidder's Information. Bidder i will choose the bids to maximize his expected payoffs in competition with other bidders' bids. The bidder estimates other bidders' bids based on the information available to the bidder. A direct bidder does not observe bids by other bidders. A primary dealer observes the realization of its indirect bidders' bids. An indirect bidder also does not observe bids by other bidders but indirect bidder i is aware that its primary dealer observes the bids and estimates other bidders' bids based on the information.

The First Order Condition. We first consider an "upward deviation" where bidder $i \in \mathbf{I}$ increases the bid for an $m = 1, ..., M$th unit by $\Delta \in \mathbf{R}_{++}$ while keeping the bids for other units constant. Let $\overrightarrow{b}'_{\varepsilon,i,n} \in B_i$ be the new bid profile of bidder i after the

[51] To be exact, we will be considering three auction systems: the primary dealer system, the direct bidding system, and the joint bidding system. There are also three class of bidders: primary dealers, direct bidders, and indirect bidders. The argument below applies to all these class of bidders in these auctions when we adjust the notations appropriately. To be exact, in the primary dealer system, a primary dealer's expectations need to take into account of the bids by indirect bidders connected to the primary dealer. An indirect bidder's expectations take into account that the primary dealer chooses the bids based on their observation of indirect bidders' bids. A direct bidder's expectations take into account of these interactions between primary dealers and indirect bidders. In the syndicate bidding system, the primary dealer's expectations take into account of the fact that only primary dealers bid in the syndicate bidding system. Here we consider a perturbed game $\Gamma(\varepsilon, n)$ to introduce smoothed bootstrap.

change in the bids. The first order condition implies that bidder i prefers $\overrightarrow{b}^{*}_{\varepsilon,i,n}$ over $\overrightarrow{b}'_{\varepsilon,i,n}$. We formulate the first order condition in four steps. First, there are two cases that the change in bids affects the outcome of the auction for bidder i. The first case is that $\overrightarrow{b}^{*}_{\varepsilon,i,n}$ does not win the mth unit but $\overrightarrow{b}'_{\varepsilon,i,n}$ wins the mth unit. Let us denote this event by $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$. The second case is that $\overrightarrow{b}^{*}_{\varepsilon,i,n}$ is the pivotal bid, wins the mth unit, and an increase in the bid will increase the auction price. Let us denote this event by $\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$[52]. Second, let $\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})$ be the change in the market clearing price when other players' bids are $\overrightarrow{b}^{*}_{\varepsilon,-i,n}$. Third, calculate the expected gain from winning the mth unit obtained by increasing the bid at the event $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ and the expected cost from increasing the market clearing prices at the event $\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$. Finally, derive the condition that the expected gain from winning is less than the expected cost from higher prices[53].

Proposition 2: *Consider bidder $i \in \mathbf{I}$ with the private value $\overrightarrow{v}_i \in V_i$. Take $\overrightarrow{b}^{*}_{\varepsilon,i,n} \in B_i$ to be a bid that bidder i places with a positive probability according to the equilibrium bidding strategy $\overrightarrow{\beta}^{*}_{\varepsilon,i,n}(\overrightarrow{v}_i)$. Consider an "upward deviation" where bidder i increases the bid for an $m \in \{1, ..., M\}$th unit by $\Delta > 0$ while keeping the bids for other units constant. Let $\overrightarrow{b}'_{\varepsilon,i,n} \in B_i$ be the new bid profile of bidder i after the change in the bid.*

[52] These events depend on whether bidder i is a primary dealer, a direct bidder, or an indirect bidder. Thus these events have subcript i.

[53] The first order condition can be understood as an analogue of the pricing condition under imperfect competition (for example, Breshnahan (1989) and Klemperer and Meyer (1989)). The formal argument is in Appendix 5.C. The first order conditions in this model is different from the ones in McAdams (2008) and Hortacsu and McAdams (2010) since this paper considers single price auctions. Also they are different from the ones in Hortacsu, Kastl, and Zhang (2014) since this paper adopts the multiple unit auctions framework.

Then, $\overrightarrow{b}^{*}_{\varepsilon,i,n}$ *satisfies the following inequality:*

$$\begin{aligned}
&\Pr_i\Big(\underbrace{\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})}_{\text{win the } m\text{th unit as a result of increasing the bid}}\Big) \qquad (1)\\
&\underbrace{\cdot\Big(v_{i,m}-\mathbf{E}_i[P(\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]\Big)}_{\text{payoff from winning the } m\text{th unit}}\\
\leq\ &\Pr_i\Big(\underbrace{\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})}_{\text{win the } m\text{th unit as a result of increasing the bid}}\Big)\\
&\underbrace{\cdot(m-1)\cdot\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{the increase in the payment for units } 1,\ldots,m-1}\\
&+\Pr_i\Big(\underbrace{\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})}_{\text{already won the } m\text{th unit with the old bid and increases the price as a result of increasing the bid}}\Big)\\
&\underbrace{\cdot m\cdot\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{increase in the payment for units } 1,\ldots,m}
\end{aligned}$$

The first order condition for the "downward deviation" where bidder $i\in\mathbf{I}$ decreases the bid for the $m\in\{1,...,M\}$th unit by $\Delta>0$ while keeping the bids for other units constant can be derived following the similar steps (Proposition 8 in Appendix 5.G).

C.2. Partial Identification of Bidder Private Values.

Suppose that bidders bid according to the equilibrium bidding strategies $\overrightarrow{\beta}^{*}_{\varepsilon,n}$. Suppose we observe that bidder i bids $\overrightarrow{b}^{*}_{\varepsilon,i,m,n}$ for the mth unit. Then, from the first order condition for the "upward deviation" (1), we obtain an upper bound for bidder i's value for the mth unit $v_{i,m}$[54]:

$$\begin{aligned}
v_{i,m}\ \leq\ &\mathbf{E}_i[P(\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]\\
&+(m-1)\cdot\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]\\
&+\frac{\Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n}))}{\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n}))}\cdot m\cdot\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})].
\end{aligned}$$

[54]Since each player places a random bid with probability $\varepsilon>0$ in the perturbed game, $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})$ is a positive probability event.

Let us denote the upper bound (the right hand side of the inequality) by $\overline{G}_{\varepsilon,i,m,n}(\overrightarrow{b}^*_{\varepsilon,i,n}) \in \mathbf{R}_+$. An intuition is that, since bidder i does not wish to increase the bid beyond $\overrightarrow{b}^*_{\varepsilon,i,m,n}$, its private value cannot be too high and the amount that bidder i shades its bid depends on Similarly, the first order condition for the "downward deviation" provides a lower bound $\underline{G}_{\varepsilon,i,m,n}(\overrightarrow{b}^*_{\varepsilon,i,n}) \in \mathbf{R}_+$. Then, the identified set of private values for the mth unit consistent with the observed bid is $[\underline{G}_{\varepsilon,i,m,n}(\overrightarrow{b}^*_{\varepsilon,i,n}), \overline{G}_{\varepsilon,i,m,n}(\overrightarrow{b}^*_{\varepsilon,i,n})]$[55]. Proposition 3 ensures that these upper and lower bounds converge to $v_{i,m}$ as $n \to \infty$.

D. Nonparametric Estimation of Bidder Private Values.

Step 3 derived the identified set of bidder private values. The next step is to estimate the bounds of bidder private values from the observed market outcome. Since it would not be possible to derive these bounds and the upper bound analytically, we estimate these bounds based on the empirical analogue using the smoothed bootstrap in the following two steps

The first step is to estimate the distribution of other bidders' bids by resampling from the empirical distribution of observed bids. Since a primary dealer observes bids by indirect bidders, the primary dealer's bids and indirect bidders' bids are correlated. Thus, a simple resampling will not be able to replicate the correlation in the data. To deal with this problem, we consider a clustered bootstrap (Cameron, Gelbach, and Miller (2008)). We classify primary dealers and indirect bidders into the following clusters. The first cluster consists of primary dealers who are not connected to any indirect bidders. The second cluster consists of primary dealers and indirect bidders connected to these primary dealers where each primary dealer is connected to one indirect bidder, etc. Recall we assume that bidders in each class (i.e. primary dealers, indirect bidders, and direct bidders) have iid distribution of private values and follow symmetric equilibrium bidding strategies. Therefore, within each cluster, bids by groups of bids by a primary dealer and by connected indirect bidders are iid. For each cluster we conduct the standard bootstrap to estimate its distribution of bids. Then, by putting them together, we can estimate the distribution of bids by other bidders.

The second step is to estimate probabilities of the events that the change in the bid will affect the outcome and the price impacts using the estimated distribution of

[55] In case we cannot explicitly solve for the upper bound and the lower bound function from the moment conditions, we follow the criterion function approach of Chernozhukov, Hong, and Tamer (2007). Haile and Tamer (2003) and McAdams (2008) consider partial identification of bidder values in the auction models.

other bidders' bids. An ε-smoothing can reduce the variance of quantile estimators. Combining them, we obtains an estimate of the bounds. The bootstrap approximation to the $1-\alpha$ level critical points can be used in the construction of confidence intervals. The details of the estimation procedure are described in Appendix 5.E.

E. Asymptotic Approximation of Bidder Strategic Responses to Auction Systems.

In the previous step, we have estimated the identification set for bidder private values. The next step toward the goal of the paper is to estimate a bidder's equilibrium response to policy changes[56]. We proceed in the following 3 substeps. In the first substep, we derive, from the first order condition (1), the bounds that an equilibrium bid should satisfy. These bounds depends on equilibrium bids by other bidders. But one cannot derive equilibrium bids with a finite number of bidders analytically without parametric assumptions. Thus, we need to employ the asymptotic approximation method. In the second substep, we show that these equilibrium bids converge to truthtelling as the market becomes large. Finally, we estimate these bounds by using the distribution of bidder private values as an approximation of equilibrium bids.

E.1. Partial Identification of Equilibrium Bids.

Consider an auction market game $\Gamma(\varepsilon, n)$ and suppose that bidders bidding behavior are characterized by an equilibrium bidding strategy profile $\overrightarrow{\beta}^*_{\varepsilon,n}$. Consider bidder $i \in \mathbf{I}$ with the private value $\overrightarrow{v}_i \in V_i$. From the first order condition for the "upward deviation" (1) we have:

$$
\begin{aligned}
& v_{i,m} - (m-1) \cdot \mathbf{E}_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\
& -(\mathbf{E}_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] - b^*_{\varepsilon,i,m,n}) \\
& -\frac{\Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}{\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))} \cdot m \cdot \mathbf{E}_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\
\leq \; & b^*_{\varepsilon,i,m,n}.
\end{aligned}
$$

[56] Recently, Satterthwaite, Williams, and Zachariadis (2014) consider the buyer's bid double auctions and conduct the asymptotic analysis of the first order condition to obtain an approximate solution of the offset strategy. The formula is similar to the one in Proposition 3 and contained in the interval in Proposition 3. Thus one can conduct an alternative analysis using Satterthwaite, Williams, and Zachariadis (2014) and can derive the qualitatively same result.

Intuitively, a bidder bids below the true value of a security as long as lowering a bid reduces the price and payment without losing the security, but not more than that. Let us denote this lower bound (i.e., the left hand side of the inequality) by $\underline{H}_{\varepsilon,i,m,n}(\overrightarrow{v}_i) \in \mathbf{R}_+$. Given that a bidder does not employ a dominated bid, the identified set of bids consistent with the strategic behavior is $\{b_{\varepsilon,i,m}\colon \underline{H}_{\varepsilon,i,m,n}(\overrightarrow{v}_i) \leq b_{\varepsilon,i,m} \leq v_{i,m}\} \subseteq B_{i,m}$.

E.2. Asymptotic Behavior of Equilibrium Bids.

As a next step of asymptotic approximations, we show that, when there are a large number of bidders, as in the US Treasury auctions, the discretions that each bidder can have to strategically manipulate bids will be small, and bidders' bids will be close to the price taking behavior[57].

Proposition 3: *Consider a sequence of auction markets* $\{\Gamma(\varepsilon, n)\}_{n=1,2,...}$. *Then, for any sequence of trembling hand perfect equilibrium strategy profiles* $\{\overrightarrow{\beta}^*_n\}_{n=1,2,...}$ *of* $\{\Gamma(n)\}_{n=1,2,...}$, *there exists a subsequence* $\{n'\}$ *such that, for each bidder* $i \in \mathbf{I}$, *for each private value* $v_i \in V_i$, *for each* $m = 1, ..., M$*th unit, an equilibrium bidding strategy* $\{\overrightarrow{\beta}^*_{i,m,n'}(v_i)\}_{n'=1,2,...}$ *converges to the price taking behavior of bidding* $v_{i,m}$ *as* $n' \to \infty$, *and the rate of convergence is* $O(1/n)$.

The formal proof is in Appendix 5.D[58]. Proposition 3 applies to a primary dealer, a direct bidder, and an indirect bidder in each auction system.

E.3. Asymptotic Approximation of Equilibrium Bids in Finite Markets.

Then we now derive the asymptotic approximation of the lower bound $\underline{H}_{\varepsilon,i,m,n}(\overrightarrow{v}_i)$ with $\underline{\widetilde{H}}_{\varepsilon,i,m,n}(\overrightarrow{v}_i)$ by approximating the equilibrium bids by price taking behavior.

[57] We define that bidder i adopts the *price taking* behavior for the mth unit if bidder i bids the true value of the mth unit of the security $v_{i,m}$.

[58] This result does not seem to have a counterpart in the divisible goods auction model. Cripps and Swinkels (2006) show that every mixed equilibrium bids converge to the price taking behavior in private value uniform price double auctions (without primary dealers) with weaker conditions of distribution of bids and values using the large deviation theory. We consider trembling hand perfect equilibrium to consider robustness to uncertainty and to justify smoothed bootstrap.

That is,

$$\begin{aligned}\underline{\widetilde{H}}_{\varepsilon,i,m,n}(\overrightarrow{v}_i) = \; & v_{i,m} - (m-1)\cdot \mathbf{E}_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{v}_{-i})|\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\ & -(\mathbf{E}_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] - b^*_{\varepsilon,i,m,n}) \\ & -\frac{\Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}{\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}\cdot m\cdot \mathbf{E}_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{v}_{-i})|\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]\end{aligned}$$

where the expectation are taken in terms of other bidders' private values.

Economic Intuition of these Bounds. To understand the intuition about these bounds, we consider how the bound $\underline{\widetilde{H}}_{\varepsilon,i,m,n}(\overrightarrow{v}_i)$ will look like when it is determined by the bid placed by the perturbation rather than by truthtelling bids since it will provide a clear comparison of the bounds among different market structures.

Lemma 4: *Consider an auction market game* $\Gamma(\varepsilon, n)$. *Consider a bid increment* $\Delta \in \mathbf{R}_{++}$. *In the primary dealer system, bidder* $i \in \mathbf{I}$ *with the private value* $v_{i,m} \in V_{i,m}$ *for an* $m \in \{1, ..., M\}$*th unit has a bid from an interval* $[v_{i,m} - \Delta - (2m-1)\cdot \frac{\bar{b}}{\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}+1}, v_{i,m}] \subseteq B_{i,m}$ *if the bidder is a direct bidder or an indirect bidder, and an interval* $[v_{i,m} - \Delta - (2m-1)\cdot \min\{\frac{\bar{b}}{\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1-N_{ID,i}\}+1}, \min_{\{b_{\varepsilon,j,m}, i\in PD(j), m=1,...,M: b_{\varepsilon,j,m}\geq v_{i,m}\}}(b_{\varepsilon,j,m} - v_{i,m})\}, v_{i,m}]$ *if the bidder is the primary dealer. In the direct bidding system, bidder* $i \in I$ *will bid from an interval* $[v_{i,m} - \Delta - (2m-1)\frac{\bar{b}}{\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}+1}, v_{i,m}]$. *In the syndicate bidding system, primary dealer* $i \in I_{PD}$ *will bid, representing a consortium, from an interval* $[v_{i,m} - \Delta - (2m-1)\frac{\bar{b}}{\varepsilon\cdot M\cdot\{n\cdot N_{PD}-1\}+1}, v_{i,m}]$.

A formal proof of Lemma 4 is in Appendix 5.F. These lower bounds do not depend on equilibrium selections. Intuitively, the degree that a bidder will reduce the bid depends on the level of competition in the auction. For example, $\frac{\bar{b}}{\varepsilon\cdot M\cdot n\cdot\{(N_{PD}+N_D+N_{ID})-1\}+1}$ is the amount that a bidder can increase the bid to win over the smoothing bids while keeping the price impact minimum. This lower bound will decrease as the number of bidders n increases ("the competition channel"). The term $\min_{\{b_{\varepsilon,j,m}, i\in PD(j), m=1,...,M: b_{\varepsilon,j,m}\geq v_{i,m}\}}(b_{\varepsilon,j,m} - v_{i,m})$ is the amount that the primary bidder can increase the bid to win over the indirect bidders with the minium price impacts ("the information pooling channel")[59]. We would also take the view that these intervals are the set of bids that the

[59] When there are sufficiently many bidders in the auction, according to Proposition 3, bids by direct bidders, indirect bidders, and primary dealers bids are approximated by the price taking behavior with the rate $O(1/n)$. As such, including the bounds from these bids would not affect the comparison among

auctioneer can expect from a bidder given the private value. Thus the width of the interval represents the auctioneer's uncertainty about the bidder's bids, thus is some indication of a risk of an auction.

Comparison between the Primary Dealer System and the Direct Bidding System. The key difference between the primary dealer system and the direct bidding system comes from the primary dealer's observation of indirect bidders' bids ("the information pooling channel"). In the primary dealer system, the dealer observe indirect bidders' bids and mirror them. In direct bidding system, the primary dealer does not observe indirect bidders' bids and reduce bids to match (approximately) their expected bids. When the realizations of indirect bidders' bids are higher than the expected value, the dealer bids less in the primary dealer system. But if there are intense competitions ("the competition channel"), when the realized indirect bidders' bids are low, the primary dealer would not be able to shade the bids much since it will lose the security. In other words, an observation of indirect bidders' bids, that is, more accurate information on the residual supply, can limit the dealer's bid shading and make its bid more aggressive with sufficient competitions[60]. Empirically, The US Treasury Office of Debt

auction systems in the first order.

[60] We now illustrate this point using a very simple example. Suppose that the seller has two units to sell and the seller employs a uniform price auction. Consider a dealer with the private value 10 for the security. Let b_{PD} be the bid by the dealer. Suppose, in addition, that the dealer is on the margin, i.e., there is another bidder whose bid is above 10 (for sure) and that the dealer is pivotal. Suppose that there are n indirect bidders whose bids are iid from uniform $[0, 1]$. Then the primary dealer intends to reduce the bid till it matches the highest bid from indirect bidders. Let $b_{ID,\max}$ be the highest bid from indirect bidders.

Let us compare the amounts that the dealer would shade the bid under the primary dealer system and the direct bidding system. In the primary dealer system where the dealer can observe indirect bidders' bids, the dealer would shade the bid to match the highest bid among indirect bidders and the dealer can actually observe the highest bid. In the direct bidding system, the dealer does not observe any of indirect bidders' bids and has to make an inference. Note that the distribution of the maximum of n bids among indirect bidders is $F_{b_{ID,\max}}(b_{ID,\max}) = {b_{ID,\max}}^n$. The primary dealer wins only when its bid is higher than $b_{ID,\max}$ and in that case it pays b_{PD} for the security. Thus the dealer chooses the bid b_{PD} to maximize the expected profit $\Pr(b_{PD} > b_{ID,\max}) \cdot (1 - b_{PD}) = {b_{PD}}^n \cdot (1 - b_{PD})$. The first order condition is $n \cdot b_{PD}^{n-1} - (n+1) \cdot b_{PD}^n = 0$. Thus $b_{PD} = \frac{n}{n+1}$. In other words, in the direct bidding system, the dealer shades the bid to match the expected highest bids from indirect bidders due to the competition channel.

Now consider the probability that the amount that the dealer shades the bid in the primary dealer system is less than the amount that the dealer shades in the direct bidding system:

$$\begin{aligned} & \Pr\left(\begin{array}{c}\text{the primary dealer's bid under the primary dealer system} \\ \geq \text{the primary dealer's bid under the direct bidding system}\end{array}\right) \\ = \quad & \Pr\left(b_{ID,\max} \geq \frac{n}{n+1}\right) = 1 - (\frac{n}{n+1})^n \to 1 \text{ as } n \to \infty. \end{aligned}$$

Management (2012) finds that the primary dealers place the most aggressive bids over indirect and direct bidders (after taking out the bids submitted to fulfill their pro-rata bidding requirement) that matches the pattern of bids in Proposition 4[61].

Comparison between the Direct Bidding System and the Joint Bidding System. The difference between these two systems is the competition channel: the syndicate bidding system has the lowest level of competition among $n \cdot N_{PD}$ consortium[62].

F. Counterfactual Policy Experiments.

Based on previous steps, we proceed to conduct a counterfactual analysis among market structure in the following substeps. The first substep estimates, for each bidder, its bounds on bidder private values from the observed market outcome using the first order condition (2.C.2). The second substep samples bidder private values from the identification intervals. The third substep estimates, for each auction system, for each bidder, the interval of possible equilibrium bids using the asymptotic approximation (2.E.3). The fourth substep samples bids from this interval and calculate the possible market outcome for each auction system. The final step compares the market outcome among different market structures.

3. Results of the Analysis: the Primary Dealer System in the US Treasury Auctions.

The goal of this paper is to answer two research questions about the recent development on the primary dealer system and the contribution of the primary dealer system to the debt management objective. To proceed toward this goal, the previous section developed the structural analysis of the uniform price auctions under the pri-

In other words, given the bidding behavior of other bidders, as the number of indirect bidders increases who bid through the primary dealer, the primary dealer has a better information about other bidders, and, with the high probability, the primary dealer will shade less under the primary dealer system.

[61] Similarly, in OCS lease auctions, Hendricks, Porter, and Wilson (1994) that the distribution of the informed bidders' bids stochastically dominates the distribution of uninformed bidders' bids. In the timber auctions, Athey, Levin, and Seila (2011) find that "mills" (large firms with manufacturing capacity) submit high bids and win disproportionately.

[62] Cheng (2015) considers the model where the winner in the secondary market can resale the good to the buyers in the secondary market and shows that including a bidder from the secondary market to the primary market can increase the revenue for the seller since it would reduce the rents of the intermediary.

mary dealer system. This section reports the results of the analysis to answer the above two questions.

A. Primary Dealers Behavior After the Financial Crisis.

We first examine the first research question concerning the primary dealer behavior after the financial crisis.

Data. We use the data from Treasury Securities Offering Announcement, Treasury Auction Result, and Investor Class Auction Allotments. The Treasury Auction Results describe the distribution of securities among primary dealers, indirect bidders, direct bidders, and noncompetitive bids. The Investor Class Auction Allotment data show the distributions of securities among depository institutions, dealers and brokers, investor funds, foreign and international, individuals, and others[63]. We consider records of 1584 auctions from April 1, 2008 to December 19, 2013. We then compare the results with Fleming (2007) who studied Treasury auctions during the period between May 5, 2003 and December 28, 2005. There are three findings:

Decreases in the Primary Dealer Shares. In 2009-13, primary dealers account for 56.6% of securities, down from 70.9% in 2001-05 (see Table 1)[64]. Primary dealers have bought as little as 14.0%, in contrast to 33.6% of an issue in previous years. Figure 2 shows that the dealer shares for cash management bills are 69% in contrast to 93.1% in 2001-05, 61.7% in contrast to 84.7% for 4 week bills, 60.9% in contrast to 66.3% for thirteen-week bills, and 52.1% in contrast to 63.1% for 26-week bills. That is, primary dealers have higher shares in securities with short term maturity, and their shares have dropped more for these short maturity securities.

[63] "Depository Institutions" include banks, savings and loan associations, credit unions, and commercial bank investment accounts. "Individuals" include individuals, partnerships, personal trusts, estates, non-profit and tax-exempt organizations, and foundations. "Dealers and Brokers" include primary dealers, other commercial bank dealer departments, and other non-bank dealers and brokers. "Pension and Retirement Funds and Insurance Companies" include pension and retirement funds, state & local pension funds, life insurance companies, casualty and liability insurance companies, and other insurance companies. "Investment Funds" include mutual funds, money market funds, hedge funds, money managers, and investment advisors. "Foreign and International" include private foreign entities, non-private foreign entities placing tenders external of the FRBNY, and official foreign entities placing tenders through FRBNY.

[64] For an earlier period, The US Treasury et al. (1992) report that during the period of January 1990 through Setember 1991, primary dealers were awarded about 72 percent, customers were awarded 5 percent, and noncompetitive awards accounted 20 percent of privately awarded Treasury securities. The top 10 firms (out of primary dealers and customers) combined 50 percent of total private awards in bill auctions and 66 percent in note and bond auctions. This pattern is consistent with the one in Fleming (2007).

Increases in the Investment Fund Shares. Table 3 from the Investor Class Data shows that Investment funds now have 18.0%, up from 6.5%. Investment fund shares tend to increase with maturity, with an average of 16.2% (up from 3.2 %) for cash management bills and 27% (up from 13.7%) for ten-year notes.

Increases in the Direct Bidding. Direct bidders other than primary dealers buy 9.1%, an increase from 2.4%. Table 4 shows that increases in direct bidding come from: depository institutions who get the allotment from direct bidding and indirect bidding evenly, dealers and brokers other than primary dealers who get 80% of the allocation from direct bidding instead of 48% in Fleming (2007), pension and retirement funds who get 30% of allocation from direct bidding that is also an increase from Fleming (2007), investment funds who get 16% which is an increase from 3% in Fleming (2007), and Foreign and international who get about 10% of allocation[65]. Figure 6 finds that, first, for securities with short and medium term maturities, the dealer and brokers have majority of direct bidding, although the share of investment funds have increased[66]. Second, for securities with longer maturities, the investment funds are often the majority of direct bidding, in contrast to the previous periods when the dealers and brokers had majorities.

Effects of Financial Crisis. These changes could be explained as the bidder reactions to the changes in macroeconomic and financial market conditions during the financial crisis. The decrease in primary dealers share for securities with shorter maturities is consistent with the view that the primary dealers have less capability for hedging because of the balance sheet effect. The increase in the investment funds share would come from the investor behavior seeking safe and riskless Treasury securities[67].

[65] Figure 5 shows that, first, for securities with a short period of maturity, the share of investment funds have increased and the shares of dealers and brokers for indirect bidding have decreased. Second, for securities with the medium maturity, the foreign and international purchases player a large role as in the previous years, but the share of investment funds have increased. Finally, for securities with long maturity, investment funds play a large role in contrast to the previous period when Foreign and International were majorities.

[66] The chart is based on regressions of 3 bidder categories on 6 investor classes with other dealers and brokers excluding the primary dealer, and without individuals. We conduct regression of direct bidding shares on investor classes while excluding individuals. The qualitative results are robust to other specifications with different categories for bids and investors.

[67] Another possibility is that the Treasury provided direct bidding status to investors to enhance participation in Treasury auctions. In a press article, Fliter (2011) reports that the US Treasury provided the People' Bank of China a direct computer link to its auction system in 2011.

B. *The Contribution of the Primary Dealer System to the Debt Management Objective.*

We then report the result of the counterfactual analysis that compares comparing the primary dealer system, the direct bidding system, and the syndicate bidding system. We illusrate our findings using an example of a 10 year note auction held in 2012[68]. We set $\Delta = 5$ (in terms of prices) and $\varepsilon = 0.005$[69]. We generate bids based on the procedure explained in Section 3 and then calculate the distribution of market prices.

Prices. Figure 8 , 9 , and 10 show that the primary dealer system has the lowest volatility of the market prices with the equal level of prices. The primary dealer system has the lowest CaR (the maximum interest cost that can be realized in a given auction at a given confidence level[70]) among all the auction systems.

Allocations. The allotment for the primary dealer is 2% higher with the primary dealer system than with the direct bidding system. But the standard deviation of the primary dealer allotment under a primary dealer system is 27% lower than under the direct bidding system and 40% lower in comparison with the syndicate bidding system. This result is consistent with the view that the primary dealer has a better estimate of the residual supply under the primary dealer system that leads to less uncertainty ("the information pooling channel").

Information Advantage of the Primary Dealers. The information advantage of primary dealers is much lower in the US Treasury auctions than in Canadian Treasury auctions because of the competition channel in the US auctions. The standard deviation of the primary dealer profits is 22% lower than the direct bidding system because of the information pooling channel.

4. Conclusion.

The US Treasury securities markets play a crucial role in the US and world economy. In face of the recent economic development, the US Treasury asked the validity of the primary dealer system in the primary auctions. Then the goal of this paper is to understand the primary dealer behavior in Treasury auctions after the financial crisis

[68] This example is a representative one.

[69] It will correspond to about 10 basis point in terms of yields in these auctions. The quantiative results presented in this paper are robust to various parameter values.

[70] For example, Risbjerg and Holmlund (2004).

and to quantify the contribution of the primary dealer system to the auction objective of the lowering cost over time using economic theory.

Our new approach to this goal is to adopt the debt management perspective and to consider not only the expected issuance cost but also the volatility of the auction prices since the investors value safety and liquidity of Treasury securities. The main idea of the paper is that the primary dealer system contributes to the debt management objective of minimizing the cost of financing over time by realizing the balance between the information pooling and competition. The counterfactual analysis shows that the primary dealer system achieves the lowest price volatilities in comparison with the policy alternatives of the syndicate bidding system and the direct bidding system. At the time of the financial crisis where the primary dealers' capacity and demands are limited due to the balance sheet effects, the primary dealer system could have reduced the interest rate risk, enhanced the demand from investment funds who value riskelessness and safety of Treasury securities, and contributed to the debt management objective.

The contributions of the paper over previous research are the following 5 points. (1). This paper documents the change in the primary dealer, investment funds, and direct bidding behavior after the period of financial crisis. (2). This paper considers the risks of the auction prices as a metric to evaluate the performance among different auction systems to the debt management objective. (3) This paper identifies the economic mechanisms ("the information pooling channel" and "the competition channel") that the primary dealer system can affect the market outcome including the risk of the auction. (4). This paper applies the asymptotic approximation method to estimate the bidding strategies in different auction systems and conduct counterfactual analysis. (5). The finding that the main difference among the auction systems is not in terms of the expected prices (yields) but rather in terms of risks is new and practically important since investors of Treasury securities value their liquidity and safety (Krishnamurthy and Vissing-Jorgensen (2013)).

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5. Appendix.

This Appendix provides an example of the primary dealer system, proofs for Proposition 1-8, and the description of the bootstrap estimation procedure. This Appendix is not intended for publications.

A. *Example of Comparison of the Primary Dealer System, the Direct Bidding System, and the Syndicate Bidding System.*

We now explain the comparison between the primary dealer system, the syndicate bidding system, and the syndicate bidding system using a simple example. We proceed in following steps: (1). We first present the economic environment. (2). We then define the direct bidding system, the joint bidding system, and the syndicate bidding system. (3) We then derive an equilibrium of direct bidding system and joint bidding system. (4) We then consider an equilibrium of the primary dealer system where the primary dealer observes a bid by the indirect bidder and mirrors its bid. (5) We then compare equilibria of these three auction systems[71].

Economic Environment. Suppose that a seller auctions a unit of security to two bidders ("bidder 1" and "bidder 2") using the single price auction. With a single unit, the auction is the second price auction. Suppose there are 2 bidders. We assume

[71] After the paper was presented in the Winter Meeting of the Econometric Society in January 2014, an author has become aware of an example in Hortacsu and Sareen (2006) that examines discriminatory auctions. Recently, Fischer, Guth, Kaplan, and Zultan (2014) consider first- and second-price private value auctions with sequential bidding where second movers may discover the first movers bids. Fischer, Guth, Kaplan, and Zultan (2014) find that second-price auctions generate higher revenue with a higher probability of bid observation. In contrast, this example studies the primary dealer system in uniform price auctions. Furthermore this paper identifies the channels that affects the debt management objective.

that each bidder has a private value from iid uniform [0.1] and their preferences are quasi-linear and risk-neutral.

Auction Systems. In this setup, the direct bidding system corresponds to the standard second price auction. In the syndicate bidding system, these bidders bid as a single entity. In the primary dealer system, we assume that bidder 1 is the primary dealer, observes bidder 2's bid, routes the bid to the auctioneer, then submits its own bid. An economic interpretation is as follows. The direct bidding system has the competition among two bidders but there are no information pooling among bidders. The syndicate bidding system pools bidders' information but there is no competition channel. In the primary dealer system, information is pooled when bidder 1 observes bidder 2's bid. At the same time, there are competition between bidder 1 and bidder 2 since bidder 2's bid will be submitted to the auctioneer.

Equilibrium of Direct Bidding and Syndicate Bidding System. We now compare the equilibria of these three auction mechanisms. The direct bidding system is the standard second price auction. Thus the truthful bidding is a dominant strategy. The seller revenue is the second highest private value. The expected revenue is 1/3 and the variance is 1/12. In the syndicate bidding system, there is only one bidder and the seller revenue is 0.

Equilibrium of Primary Dealer System. We now study the equilibrium bidding behavior in the primary dealer system. Truthtelling is an equilibrium in this single unit setting. But it will not be the case in a more general and realistic setting with multiple units of supply where bidders have an incentive to shade the bids to reduce the payment for other units (for example, Ausubel and Cramton (2002) and Ausubel, Cramton, Pycia, Rostek, and Weretka (2014))[72].

Thus, we consider another class of equilibria with the feature that the primary dealer "mirrors" the bids by the indirect bidder, consistent with the empirical observation[73]. Now consider a best response of the primary dealer 1 with signal v_1 when the primary dealer observes indirect bidder 2's bid b_2. If $v_1 > b_2$, then, primary dealer 1's best response is to bid any amount above b_2. Suppose $v_1 < b_2$. What will be the primary dealer 1's best response? Suppose the primary dealer 1 bids 0. Then, given this primary dealer 1's response, indirect bidder 2 will always bid 1 and the primary dealer 1 will

[72] I thank Ron Siegel for helpful discussion on this issue.

[73] In Canadian Treasury auctions, Hortacsu and Sareen (2006) and Hortacsu and Kastl (2012) document that the dealer submits a new bid that is weakly lower below its original bid after observing a relatively low bid by a customer.

always bid 0. Thus bidding 0 does not seem to be a best response for the primary dealer. Indeed, suppose the primary dealer 1 chooses a following strategy:

$$b_1(v_1, b_2) = \begin{cases} b_2 + \varepsilon & \text{if } v_1 > b_2 \\ b_2 - \varepsilon & \text{if } v_1 < b_2. \end{cases}$$

In words, if the indirect bidder bids below the primary dealers's value, the primary dealer bids just above the bid by the indirect bidder. If an indirect bidder bids above the value, the primary dealer bids just below the bid to deter jump bids[74]. Given this response, what will be indirect bidder 2's best response? Indirect bidder 2 wins when $v_1 < b_2$ and the payment is (approximately) b_2. Then its expected payoff is $\Pr(v_1 < b_2) \cdot (v_2 - b_2) = b_2(v_2 - b_2)$. Thus its bidding strategy is $b_2(v_2) = v_2/2$. Then the seller revenue is always $v_2/2$.

Comparison among Auction Systems. In the primary dealer system, the seller expected revenue is 1/4 and the variance is 1/48. Although the revenue is 25% lower the variance is reduced to 1/4th compared with the direct bidding system[75]. We note that the variance reduction in the primary dealer system comes from two sources. First, after observing bids by an indirect bidder, the primary dealer bids to match the bids by the indirect bidder. Second, knowing this, the indirect bidder reduces its bids. This bid shading also reduces the variance.

B. Proof of Lemma 1.

The goal of Lemma 1 1 is to establish existence of a trembling perfect equilibrium of the Bayesian game $\Gamma(n)$. Toward this goal, we proceed in following four steps: (1). We consider a perturbed Bayesian game $\Gamma(\varepsilon, n)$ where $\varepsilon > 0$ is the probability of perturbation. (2) We then approximate $\Gamma(\varepsilon, n)$ by a finite game $\Gamma(\Delta, \varepsilon, n)$ where $\Delta > 0$ is the minimum bid grid size construct a Bayesian Nash equilibrium $\overrightarrow{\beta}^*_{\Delta,\varepsilon,n}$ of $\Gamma(\Delta, \varepsilon, n)$. (3) We take the bid grid size $\Delta \to 0$ and show existence of a Bayesian Nash equilibrium $\overrightarrow{\beta}^*_{\varepsilon,n}$ of a smoothed game $\Gamma(\varepsilon, n)$. (4). We then take $\varepsilon \to 0$ and construct a perfect equilibrium $\overrightarrow{\beta}^*_n$ of $\Gamma(n)$.

[74] This equilibrium strategy has the property that the distribution of the primary dealer's bids is identical to that of the uninformed bidder as in Engelbrecht-Wiggans, Milgrom, and Weber (1982).

[75] The loss of revenue from the primary dealer system in this example is due to the fact that there is only one primary dealer. The US Treasury auctions system has 21 primary dealers and competition among bidder ("the competition channel") will lead to more aggressive bids.

Step 1: *Perturbed Auction Game* $\Gamma(\varepsilon, n)$. We would like to prove existence of a perfect equilibrium $\overrightarrow{\beta}^*_n$ of $\Gamma(n)$. To apply the definition of a perfect equilibrium, we consider a sequence of perturbations $\{\varepsilon_l\}_{l=1,2,...} \in \mathbf{R}_{++}$ such that $\varepsilon_l \to 0$ as $l \to \infty$. For each $\varepsilon_l > 0$, let $\Gamma(\varepsilon_l, n)$ be a Bayesian game where, with probability ε_l, a bid is chosen according to the uniform distribution on B_i.

Step 2: *Finite Approximation of* $\Gamma(\varepsilon_l, n)$. Fix $\varepsilon_l \in \mathbf{R}_{++}$ and consider a smoothed game $\Gamma(\varepsilon_l, n)$. At $\Gamma(\varepsilon_l, n)$, a bidder's profit function is not continuous when a bidder's bid is tied with other bidders' bids. Thus we cannot apply the standard existence result (Glicksberg (1952)) to show existence of a mixed strategy Bayesian equilibrium in $\Gamma(\varepsilon_l, n)$. Therefore, we consider a finite Bayesian game $\Gamma(\Delta, \varepsilon_l, n)$ with the bid grid size $\Delta > 0$ where each bidder submits a bid for each unit from a finite set $B_{\Delta,i,m} = \underline{b} \cup \{0, \Delta, 2\Delta, ..., \bar{b}\}$. For each $\Gamma(\Delta, \varepsilon_l, n)$ with $\Delta > 0$, since each player has a finite set of strategies, there exists a mixed strategy Bayesian Nash equilibrium $\overrightarrow{\beta}^*_{\Delta,\varepsilon_l,n}$.

Step 3: *Take limit as* $\Delta \to 0$. Consider a sequence of perturbations $\{\Delta_k\}_{k=1,2,...} \in \mathbf{R}_{++}$ with $\Delta_k \to 0$ as $k \to 0$. For each $\Delta_k > 0$, consider a finite Bayesian game $\Gamma(\Delta_k, \varepsilon_l, n)$ and its Bayesian Nash equilibrium $\overrightarrow{\beta}^*_{\Delta_k,\varepsilon_l,n}$. Then, by Prohorov's theorem, there exists a subsequence $\{\Delta_{k'}\}_{k'=1,2,...}$ and a limit strategy profile $\overrightarrow{\beta}^*_{\varepsilon_l,n}$ such that $\overrightarrow{\beta}^*_{\Delta_{k'},\varepsilon_l,n} \to \overrightarrow{\beta}^*_{\varepsilon_l,n}$ weakly as $\Delta_{k'} \to 0$. Suppose $\overrightarrow{\beta}^*_{\varepsilon_l,n}$ involves a tie with a positive probability: that is, there exists a positive set of private values such that bidders with such private values places the same bid for some units with positive probabilities. Then, for any such bid, every bidder who places that bid will have a nonnegative payoff from winning the security at that price[76]. Thus, the bidder with the highest private value among such private values will have a strictly positive payoff from winning the security at that price. Then, that bidder will prefer to increase the bid to break tie to win the security for sure. This deviation will be a profitable deviation from $\overrightarrow{\beta}^*_{\Delta_k,\varepsilon_l,n,i}$ for $\Gamma(\Delta_k, \varepsilon_l, n)$ for sufficiently small Δ_k. Therefore, it cannot be that $\overrightarrow{\beta}^*_{\varepsilon_l,n}$ involves a tie with a positive probability. It implies that there would not be discontinuity as we take the limit of $\overrightarrow{\beta}^*_{\Delta_k,\varepsilon_l,n}$ as $\Delta_k \to 0$. Then, the limit strategy profile $\overrightarrow{\beta}^*_{\varepsilon_l,n}$ is a Bayesian Nash equilibrium of $\Gamma(\varepsilon_l, n)$[77].

Step 4: *Take limit as* $\varepsilon_l \to 0$. Now consider a sequence of perturbations $\{\varepsilon_l\}_{l=1,2,...} \in \mathbf{R}_{++}$ with $\varepsilon_l \to 0$ as $l \to \infty$. For each $\Gamma(\varepsilon_l, n)$, by Step 3, there exists a Bayesian

[76] Since every bidder has an option of nonparticipation $\underline{b}$, the bidder much have a nonnegative payoff from winning the security at the price.

[77] See Athey (2001), Reny and Zamir (2004), Jackson and Swinkels (2005), Kazumori (2013) for a complete argument regarding the resolution of ties in the private value model.

Nash equilibrium $\overrightarrow{\beta}^*_{\varepsilon_l,n}$. Then, by Prohorov's theorem, there exists a subsequence $\{\varepsilon_{l'}\}_{l'=1,...,}$ and a strategy profile $\overrightarrow{\beta}^*_n$ such that $\varepsilon_{l'} \to 0$ and that $\overrightarrow{\beta}^*_{\varepsilon'_l,n} \to \overrightarrow{\beta}^*_n$ weakly as $\varepsilon_{l'} \to 0$. By following the argument in Step 3, $\overrightarrow{\beta}^*_n$ does not involve ties with a positive probability. Therefore, the limit strategy profile $\overrightarrow{\beta}^*_n$ is a Bayesian Nash equilibrium of $\Gamma(n)$. Then, $\overrightarrow{\beta}^*_n$ satisfies all the requirement for a perfect equilibrium of $\Gamma(n)$. Thus we have constructed a perfect equilibrium of $\Gamma(n)$.

C. *Proof of Proposition 2.*

The goal of Proposition 2 is to formulate the first order condition of a Bayesian Nash equilbrium $\overrightarrow{\beta}^*_{\varepsilon,n}$ of $\Gamma(\varepsilon, n)$. We proceed in the following Consider an auction market game $\Gamma(\varepsilon, n)$ and suppose that bidders' behavior are described by a Bayesian Nash equilibrium strategy profile $\overrightarrow{\beta}^*_{\varepsilon,n}$. Let us consider a decision problem of bidder i with private value $\overrightarrow{v}_i$. Bidder i would be choosing a bid according to an equilibrium bidding strategy $\overrightarrow{\beta}^*_{\varepsilon,i,n}(\overrightarrow{v}_i)$. Let $\overrightarrow{b}^*_{\varepsilon,i,n}$ be a bid bidder i places with a positive probability according to a bidding strategy $\overrightarrow{\beta}^*_{\varepsilon,i,n}(\overrightarrow{v}_i)$. Then $\overrightarrow{b}^*_{\varepsilon,i,n} = (\overrightarrow{b}^*_{\varepsilon,i,1,n}, ..., \overrightarrow{b}^*_{\varepsilon,i,M,n})$ where $\overrightarrow{b}^*_{\varepsilon,i,m,n}$ would denote the bid for the mth unit.

Consider an "upward deviation" where bidder i increases the bid for the mth unit by $\Delta > 0$ while keeping the bids for other units constant. Let $\overrightarrow{b}'_{\varepsilon,i,n}$ be a new bid profile after the upward deviation. That is, $\overrightarrow{b}'_{\varepsilon,i,n} = (b^*_{\varepsilon,i,1,n}, ..., b^*_{\varepsilon,i,m-1,n}, b^*_{\varepsilon,i,m,n} + \Delta, b^*_{\varepsilon,i,m+1,n}, ..., b^*_{\varepsilon,i,M,n})$. Then there are 2 cases that increasing the bid to $\overrightarrow{b}'_{\varepsilon,i,n}$will affect the outcome for bidder i. The first case is the case that bidder i is able to win the mth unit of the security after the increase of the bid from $\overrightarrow{b}^*_{\varepsilon,i,n}$ to $\overrightarrow{b}'_{\varepsilon,i,n}$. Let $\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ be the event where other bidders' private values are such that the above situation will take place when bidder i has information $R_{\varepsilon,i,n}$. When the event $\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ takes place, the bidder obtains profits from winning the mth unit, but the bidder might need to pay a higher price for the security since the upward deviation may increase the market clearing price. We now calculate these terms one by one.

Payoff from Winning the mth Unit. First, let us denote $P_i(\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))$ as the probability of the event $\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$[78]. When bidder i wins the mth unit,

[78]We note that the probability $P_i(\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n}))$ depends on the bidder's identity (categories, to be exact) since the estimates of the residual supply curve will be different depending on whether the bidder is a primary dealer, direct bidder, or an indirect bidder. Furthermore, we would like to note that the probability does not depends on the realization of bidder i's private value. It

bidder i collects the value $v_{i,m}$ from the mth unit. In this case, the price that bidder i pays is, if other bidders' bids are $\overrightarrow{b}^{*}_{\varepsilon,-i,n}$, by definition, $P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})$ where the $P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})$ function, defined in Section 2.B, returns the market clearing price when the profile of the bids is $(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})$. But of course, bidder i does not observe the bids by other bidders, and the probability that the other bidders' bids is $\overrightarrow{b}^{*}_{\varepsilon,-i,n}$ depends on the distribution of the residual supply curve conditional on the event $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$. Consequently, the expected price that bidder i would pay is $E_i[P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]$. Then, given the assumption of quasi-linear preferences, bidder i's expected payoff from the mth unit is[79]

$$\begin{aligned} &P_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\ &\cdot(v_{i,m} - E_i[P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]). \end{aligned} \tag{2}$$

Higher Payment for Other $m-1$ Units of Security. When bidder i wins the mth unit as a result of placing a higher bid for the mth unit, bidder i may need to pay higher prices for the other $m-1$ units of security that bidder i would be allotted. We now calculate the change in bidder i's payment. First, the probability of the event is, as in the previous case, $P_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))$. Let us define the change in the market clearing price from increasing the bid from $\overrightarrow{b}^{*}_{\varepsilon,i,n}$ to $\overrightarrow{b}'_{\varepsilon,i,n}$ when other bidders' bids are $\overrightarrow{b}^{*}_{\varepsilon,-i,n}$ to be $\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n}) = P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n}) - P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})$. Since bidder i is not informed of the bids by other bidders at the time of bidding and bidder i is risk-neutral, bidder i estimates its expected change in payment conditional on the event $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))$ by $E_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]$. By the assumption of nonincreasing bids, bidder i would need to pay for the increases in the price for other $m-1$ units. Thus the change in the expected payment is

$$(m-1) \cdot E_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]. \tag{3}$$

The Payment Increase in the Second Case. We now consider the second case where increasing the bid to $\overrightarrow{b}'_{\varepsilon,i,n}$ changes the outcome of the auction in a following situation: bids by other bidders are such that bidder i already wins the mth unit with the bid

is because of the assumption that the distribution of private values among bidders is independent: a bidder's estimates of the other bidders' values do not depend on the specific realization of a bidder's private value.

[79] Here, again, given bidder i's bids, the expected value does not depend on the specific realization of bidder i's private value by the assumption of independent private values.

$\overrightarrow{b}^*_{\varepsilon,i,n}$, thus increasing the bid will not affect the allocation but will increase the market clearing price, thus bidder i has to pay more. Let $\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ be the event such that the other bidders' private values are such that the above case takes place given bidder i's information $R_{\varepsilon,i,n}$. Then, following the reasoning in the previous case, the change in the payment is

$$m \cdot E_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]. \tag{4}$$

Putting All Together. By combining (2), (3), and (4), we get the first order condition (1). We note that this formulation applies to bidders in all categories (because different information that a bidder in a different category has is encoded in the residual supply curve $R_{\varepsilon,i,n}$).

D. *Proof of Proposition 3.*

Outline of the Proof. Our objective is to show that, when bidders employ perfect equilibrium strategies, then, as the number of bidders in the market increases, the bid will converge to the price taking behavior where a bidder's bid is its private value for the security. Our approach is first to approximate a perfect equilibrium with Bayesian Nash equilibria of the smoothed games. The reason we introduce a perturbation is that it will smooth the residual supply curve. This smoothness, that is, absence of gaps in the residual supply curve, will allow us to bound the bidder strategic behavior. Then, as the number of market participants increases, the price impact will vanish and a bidder's bid will converge to the price taking behavior as the number of market participants increases. Then, since the limit strategy profile is the price taking behavior for each perturbation, as the probability of perturbation goes to zero, the equilibrium bids converge to the price taking behavior.

Interchanging the Order of Limits. In the above argument, we first increase the number of market participants with the fixed probability of perturbations and then take the probabilities of perturbations to zero. But in the perfect equilibrium, we first take the probability of perturbations to zero and then increase the number of bidders in the auction. Thus we need to make sure that the order of limits can be interchanged. The following condition (Pringsheim convergence) will provide a sufficient condition.

Definition 5: *A sequence* $\{a_{n,m}\} \in \mathbf{R}$ *indexed by* (n, m) *converges to* $L \in \mathbf{R}$ *in the sense of Pringsheim (in terms of* n*) if, for any* $\varepsilon > 0$ *and any* $m \in \mathbf{N}_{++}$*, there exists*

$N(\varepsilon, m) \in \mathbf{N}_{++}$ *such that, for any* $n > N(\varepsilon, m)$, $|a_{n,m} - L| < \varepsilon$.

This condition says, for every index $m \in \mathbf{N}_{++}$, for a sequence $\{a_{n,m}\}_{n=1,2,...}$ for m fixed, if we take n sufficiently large, then, $a_{n,m}$ will be sufficiently close to the same limit L[80]. In the context of the auction model, n is the number of bidders in the auction and m is an index for the probability of the perturbation. This condition would be satisfied if, for each positive probability of perturbation, equilibrium bids converge to the same price taking behavior as the number of bidders in the auction increases. Next, we show that, once this condition is satisfied, one can interchange the order of limits:

Lemma 6: *Suppose (1)* $\{a_{n,m}\} \in \mathbf{R}$ *converges to* $L \in \mathbf{R}$ *in the sense of Pringsheim (in terms of* n*), and (2) for each* $n \in \mathbf{N}_{++}$, $\{a_{n,m}\}_{n=1,2,...}$ *for fixed* n *converges to* $a_n \in \mathbf{R}$ *as* $m \to \infty$*, then,* $lim_{n\to\infty} \lim_{m\to\infty} a_{n,m} = L$.

Proof: By assumption (2), for each n, $\lim_{m\to\infty} a_{n,m}$ exists and $a_n = \lim_{m\to\infty} a_{n,m}$. Then we want to show that, for each $\varepsilon > 0$, there exists $N \in \mathbf{N}_{++}$ such that, for each $n > N$, $|a_n - L| < \varepsilon$. By approximating a_n by $a_{n,m}$, we have $|a_n - L| \leq |a_n - a_{n,m}| + |a_{n,m} - L|$. Then, since $\lim_{m\to\infty} a_{n,m} = a_n$, for each $\varepsilon/2$, there exists m such that $|a_n - a_{m,n}| < \varepsilon/2$. By assumption (1), for such m, for $\varepsilon/2$, there exists $N(\varepsilon/2, m)$ such that, for every $n > N(\varepsilon/2, m)$, $|a_{m,n} - L| < \varepsilon/2$. Then, we have $|a_n - L| < \varepsilon$ for $n > N(\varepsilon/2, m)$. □

Thus, if a double sequence $\{\beta^*_{\varepsilon,n}\}$ satisfies the condition in Definition 5, then, we can interchange the order of limits, which implies that a sequence of perfect equilibria will converge to the price taking behavior. Then, to prove Proposition 3, it remains to show that $\{\beta^*_{\varepsilon,n}\}$ satisfies the condition of Definition 5.

Approximation by Smoothed Games. Consider a sequence of perfect equilibria $\{\overrightarrow{\beta}^*_n\}_{n=1,2,...}$ of $\{\Gamma(n)\}_{n=1,2....}$. Then, by the definition of a perfect equilibrium, for each n, for each $\overrightarrow{\beta}^*_n$, there exists a sequence of Bayesian Nash equilibria $\{\overrightarrow{\beta}^*_{\varepsilon,n}\}_\varepsilon$ of $\{\Gamma(\varepsilon, n)\}_\varepsilon$ such that $\overrightarrow{\beta}^*_{\varepsilon,n} \to \overrightarrow{\beta}^*_n$ weakly as $\varepsilon \to 0$. Let us now fix $\varepsilon > 0$, and consider a sequence of equilibria $\{\overrightarrow{\beta}^*_{\varepsilon,n}\}_{n=1,2,...}$ of smoothed games $\{\Gamma(\varepsilon, n)\}_{n=1,2,...}$ as $n \to \infty$. By Prohorov's theorem, there exists a subsequence $\{\overrightarrow{\beta}^*_{\varepsilon,n'}\}_{n'}$ and a limit strategy profile $\overrightarrow{\beta}^*_\varepsilon$ such that $\overrightarrow{\beta}^*_{\varepsilon,n'} \to \overrightarrow{\beta}^*_\varepsilon$ as $n' \to \infty$. Then, by following the steps of the argument developed in Lemma 1, we can show that $\overrightarrow{\beta}^*_\varepsilon$ is an equilibrium of the limit game $\Gamma(\varepsilon)$. It remains to show that bidders bid their true values (price taking behavior) at $\overrightarrow{\beta}^*_\varepsilon$ for

[80] Pringsheim (1897) defines a condition for convergence in double sequences. This definition is a slight modification (where the limit as $n \to \infty$ is the same for all fixed m). I thank Gabriel Carroll for a conversation on this definition.

every $\varepsilon > 0$.

Asymptotic Analysis of the First Order Condition. From the first order condition (1), since the event $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ takes place with a positive probability given ε-smoothing, we have

$$\begin{aligned} & v_{i,m} - \underbrace{\mathbf{E}_i[P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{(A)}} \\ \leq \; & (m-1) \cdot \underbrace{\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{(B)}} \\ & + \underbrace{\frac{\Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}{\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}}_{\text{(C)}} \\ & \cdot m \cdot \underbrace{\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{(D)}}. \end{aligned}$$

We then evaluate each term one by one.

Evaluation of (A). Note that $E_i[P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]$ is the expected market clearing price when a previous bid $b^{*}_{\varepsilon,i,m,n}$ did not win the mth unit of security but a higher bid $b'_{\varepsilon,n,i,m}$ does. Therefore, the market clearing price conditional on $\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ is bounded below by $b^{*}_{\varepsilon,i,m,n}$ and bounded above by $b'_{\varepsilon,n,i,m}$. Consequently, as $\Delta \to 0$, $E_i[P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \to b^{*}_{\varepsilon,i,m,n}$.

Evaluation of (B). $E_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]$ is the expected price impact when bidder i increases the bid from $b^{*}_{\varepsilon,i,m,n}$ to $b'_{\varepsilon,n,i,m}$. Given the pricing rule, the market clearing price is $b_{M\cdot n\cdot(N_{PD}+N_D+N_{ID}):M\cdot n}$. Then, when bidder i increases the bid from $b^{*}_{\varepsilon,i,m,n}$ to $b'_{\varepsilon,n,i,m}$, the expected change in the market clearing prices is bounded by

$$\begin{aligned} & \mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\ \leq \; & \mathbf{E}_i[b_{M\cdot n\cdot\{(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)-1} - b_{M\cdot n\cdot\{(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)} \\ & |\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\ \leq \; & \mathbf{E}_i[\text{distance between 2 consecutive bids by perturbation}|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\ \approx \; & \frac{\overline{b}}{\varepsilon \cdot\cdot M \cdot \{n \cdot (N_{PD}+N_D+N_{ID}) - 1\} + 1}. \end{aligned}$$

For the first inequality, when a bid $b^*_{\varepsilon,i,m,n}$ does not win, it has to be that the bid is less than the pivotal bid $b_{\underbrace{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}}_{\text{total number of bids other than bidder } i}:\underbrace{M\cdot n-(m-1)}_{\text{available unit}}}$. Furthermore, since $b^*_{\varepsilon,i,m,n}+\Delta$ wins, $b^*_{\varepsilon,i,m,n}+\Delta$ is at least the pivotal bid. Thus, even if $b^*_{\varepsilon,i,m,n}+\Delta$ exceeds $b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)-1}$, it cannot be that the change in the price is more than $b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)-1}-b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)}$. For the second inequality, $b_{M\cdot n\cdot\{(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)-1}-b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)}$ is the distance between consecutive 2 order statistics among $M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}$ bids. Most of these bids are placed strategically, but some of them, approximately $\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}$, are placed by smoothed bids. Then, the distance $b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)-1}-b_{M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}:M\cdot n-(m-1)}$ is less than the distance between two consecutive order statistics among $\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}$ bids placed by perturbations. For the third equality, when there are $\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}$ bids placed independently according to the uniform distribution on $B_{i,m}$, according to the standard result on the distribution of order statistics from the uniform distribution, the expected value of the distance between two order statistics is $\overline{b}/(\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}+1)$.

Then, for fixed ε, as $n\to\infty$, $E_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n},\overrightarrow{b}^*_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})]\to 0$ for the rate at $O(1/n)$.

Evaluation of (C). The objective is to show that, for each fixed $\varepsilon>0$, the term (C) is bounded below and above by bounds independent of n[81]. Let us first consider an event $\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})$. When this event $\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})$ takes place, bidder i wins the mth unit as a result from increasing the bid to $b^*_{\varepsilon,i,m,n}+\Delta$. Then, consider the case where there are $M\cdot n+1$ bids above $b^*_{\varepsilon,i,m,n}$[82]. Second, there are m bids by bidder i above $b^*_{\varepsilon,i,m,n}$. Second, there is one bid between $b^*_{\varepsilon,i,m,n}$ and $b^*_{\varepsilon,i,m,n}+\Delta$ by bidders other than i. Thus,

bidder i	m bids above $b^*_{\varepsilon,i,m,n}$
bidder i'	1 bid between $b^*_{\varepsilon,i,m,n}$ and $b^*_{\varepsilon,i,m,n}+\Delta$ m' bids above $b^*_{\varepsilon,i,m,n}+\Delta$
bidder other than i and i'	$M\cdot n-m-m'+1$ bids above $b^*_{\varepsilon,i,m,n}$
total	$M\cdot n+1$ above $b^*_{\varepsilon,i,m,n}$

[81] This argument is related to the proof of Lemma 4 in Fudenberg, Mobius, and Szeidl (2007) that derives bounds on lower and upper bounds for the conditional densities of buyers and sellers.

[82] This is the case where there is 1 bid by bidders other than i between $b^*_{\varepsilon,n,i,m}$ and $b^*_{\varepsilon,n,i,m}+\Delta$. The case where there are more than 1 bid in this interval can be studied following a similar procedure to show that they are also bounded from below and from above uniformly for n.

where bidder i can have at most $M \cdot n - m + 1$ bids above $b^*_{\varepsilon,i,m,n}$. Thus this event can be represented by[83]

$$
\begin{aligned}
& \overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}) \\
= \; & (\text{bidder } i \text{ has } m \text{ bids above } b^*_{\varepsilon,i,n,m}) \\
& \cap \bigcup_{i' \neq i} \Big(\bigcup_{m'=1,\ldots,M-m+1} (\text{bidder } i' \text{ has an } m'\text{th bid between } b^*_{\varepsilon,i,n,m} \text{ and } b'_{\varepsilon,i,n,m}) \\
& \cap (\text{bidders other than } i \text{ and } i' \text{ has } M \cdot n - m - m' + 1 \text{ bids above } b^*_{\varepsilon,i,n,m})\Big).
\end{aligned}
$$

Next consider an event $\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$. $\overline{L}_{i,m}$ is the event that bidder i wins the mth unit with the bid by $b^*_{\varepsilon,i,n,m}$ and increasing the bid will increase the market clearing price. Thus $b^*_{\varepsilon,i,n,m}$ has to be the $M \cdot n$ th highest bid. As in the previous case, bidder i has m units of bids above $b^*_{\varepsilon,i,n,m}$. Thus,

bidder i	m bids above $b^*_{\varepsilon,i,m,n}$
bidder i'	m' bids in total above $b^*_{\varepsilon,i,m,n}$
bidder other than i and i'	$M \cdot n - m - m'$ bids above $b^*_{\varepsilon,i,m,n}$
total	$M \cdot n$ above $b^*_{\varepsilon,i,m,n}$

Then, this case can be represented by:

$$
\begin{aligned}
& \overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}) \\
= \; & (\text{bidder } i \text{ has } m \text{ bids above } b^*_{\varepsilon,i,n,m}) \\
& \cap \bigcup_{i' \neq i} \Big(\bigcup_{m''=0,\ldots,M-m} \\
& (\text{another } i' \text{ has a } m'' \text{ bid above } b^*_{\varepsilon,i,n,m}) \\
& \cap (\text{bidders other than } i \text{ and } i' \text{ has } M \cdot n - m - m'' \text{ bids above } b^*_{\varepsilon,i,n,m}).
\end{aligned}
$$

where bidder i' has at most $M \cdot n - m$ bids above $b^*_{\varepsilon,i,m,n}$. Note it is possible that bidder i' does not have any bid above $b^*_{\varepsilon,i,m,n}$ as long as other bidders "make up." Thus the index for m'' starts from 0. Then, since the distribution of private values is independent (and we consider independent smoothing), we have

[83] To be precise, since a primary dealer observes bids by indirect bidders, bids by the primary dealer and bids by indirect bidders are not independent. But our bounds only depend on ε-smoothing that take place independently across bidders. Thus this formulation is legitimate to calculate the bounds from such independent ε-smoothings.

$$\frac{\Pr_i\left(\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})\right)}{\Pr_i\left(\overline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n},\overrightarrow{b}'_{\varepsilon,i,n})\right)}$$

$$= \frac{\sum_{i'\neq i}\sum_{m'=1,\ldots,M-m+1}\left(\Pr_i\left(\begin{array}{c}\text{bidder } i' \text{ has 1 bid between } b^*_{\varepsilon,i,m,n} \text{ and } b^*_{\varepsilon,i,m,n}+\Delta\\ m' \text{ bids above } b^*_{\varepsilon,i,m,n}+\Delta\end{array}\right)\cdot\Pr_i(\text{bidders other than } i \text{ and } i' \text{ have } M\cdot n-m-m'+1 \text{ bids above } b^*_{\varepsilon,i,n,m})\right)}{\sum_{i'\neq i}\sum_{m''=0,\ldots,M-m}\left(\Pr_i(\text{bidder } i' \text{ has an } m'' \text{ bid above } b^*_{\varepsilon,i,n,m})\cdot\Pr_i(\text{bidders other than } i \text{ and } i' \text{ have } M\cdot n-m-m'' \text{ bids above } b^*_{\varepsilon,i,n,m})\right)}$$

To find the bounds, we use the following simple result from Milgrom (1979):

Lemma 7: *Let* $\{a_n\}$, $\{b_n\},\{c_n\}$, *and* $\{d_n\}$ *be sequences of nonnegative real numbers. Suppose that there exist some constants* $e,f,g,h\in\mathbf{R}_+$ *such that* $e\leq\frac{a_n}{b_n}\leq f$ *and* $g\leq\frac{c_n}{d_n}\leq h$ *for every* n. *Then we have* $eg\leq\frac{\sum a_nc_n}{\sum b_nd_n}\leq fh$.

Then we can understand the difference between $W_{i,m}$ and $L_{i,m}$ as follows. For each bidder i, when bidders other than i and i' have $M\cdot n-m-m'$ units of bids above $b^*_{\varepsilon,i,n,m}$, (1) if bidder i' has m' bids above $b^*_{\varepsilon,i,n,m}$, then, there will be $M\cdot n$ bids above $b^*_{\varepsilon,i,n,m}$ and $b^*_{\varepsilon,i,n,m}$ is pivotal, thus the event $L_{i,m}$ will take place, and (2) if bidder i' bids more aggressively and places one more bid between $b^*_{\varepsilon,i,n,m}$ and $b^*_{\varepsilon,i,n,m}+\Delta$, then $b^*_{\varepsilon,i,n,m}$ will not win and the event $W_{i,m}$ takes place[84].

Then, from the above Lemma 7, it will be suffice to show that, for every bidder $i,i'\in\mathbf{I}$ with $i\neq i'$, and for each $m\in\{1,...,M\}$,

$$\frac{\Pr_i\left(\begin{array}{c}\text{bidder } i\text{' places } m \text{ bids above } b^*_{\varepsilon,i,n,m}+\Delta\\ \text{and bidder } i' \text{ places a bid between } b^*_{\varepsilon,i,n,m} \text{ and } b^*_{\varepsilon,i,n,m}+\Delta\end{array}\right)}{\Pr_i\left(\text{bidder } i' \text{ places } m \text{ bids above } b^*_{\varepsilon,i,n,m}\right)}$$

[84] In other words, there have to be a bidder who bids more aggressively.

$$= \frac{\underbrace{\Pr_i(\text{bidder } i\text{' places } m \text{ bids above } b^*_{\varepsilon,i,n,m}+\Delta)}_{\text{(C1)}} \underbrace{\cdot \Pr_i\left(\begin{array}{c}\text{bidder } i' \text{ places a bid between } b^*_{\varepsilon,i,n,m} \text{ and } b^*_{\varepsilon,i,n,m}+\Delta \\ |\text{bidder } i\text{' places } m \text{ bids above } b^*_{\varepsilon,i,n,m}+\Delta\end{array}\right)}_{\text{(C2)}}}{\underbrace{\Pr\left(\text{bidder } i' \text{ places } m \text{ bids above } b^*_{\varepsilon,i,n,m}\right)}_{\text{(C3)}}}$$

is bounded from below and above by constants independent of n and m. Then we will show each of (C1), (C2), and (C3) is bounded from below and above by constants independent of n and m. An intuition is that, since a distribution of bids is uniformly bounded because of ε-smoothing, there are uniform lower and upper bounds of these probabilities.

Evaluating (C1). When bidder i' places m bids above $b^*_{\varepsilon,i,n,m}+\Delta$, the probability of this event is highest when bidder i's bidding strategy $\beta^*_{\varepsilon,n,i,m}$ puts all the weights above $b^*_{\varepsilon,i,n,m}+\Delta$, and lowest when $\beta^*_{\varepsilon,n,i,m}$ puts none of the weights above $b^*_{\varepsilon,i,n,m}+\Delta$. For the upper bound, then the probability that each bid has a bid above $b^*_{\varepsilon,i,n,m}+\Delta$ is all the probability except for smoothing. In this case, bidder i' places a unit bid above $b^*_{\varepsilon,i,n,m}+\Delta$ with the probability $(1-\varepsilon\frac{b^*_{\varepsilon,i,n,m}+\Delta}{\bar{b}})$. Thus an upper bound for (C1) is $(1-\varepsilon\frac{b^*_{\varepsilon,i,n,m}+\Delta}{\bar{b}})^m$. This bound is maximized at $m=1$ and the maximum is $(1-\varepsilon\frac{b^*_{\varepsilon,i,n,m}+\Delta}{\bar{b}})$. We note that this upper bound is effective for all n and m. Next, the lower bound is the case where bidder i places a bid above $b^*_{\varepsilon,i,n,m}+\Delta$ only through smoothing. In this case, bidder i places a bid above $b^*_{\varepsilon,i,n,m}+\Delta$ with the probability $\varepsilon\cdot(\frac{\bar{b}-b^*_{\varepsilon,i,n,m}-\Delta}{\bar{b}})$. Then a lower bound for (A) is given by $\left\{\varepsilon\cdot(\frac{\bar{b}-b^*_{\varepsilon,i,n,m}-\Delta}{\bar{b}})\right\}^M$. This lower bound is also independent of n and m.

Evaluating (C2). The bid for the $m+1$st unit is not independent from the bid for the 1st unit to the mth unit. But given the assumption that an ε-smoothing of bids for the $m+1$st unit takes place independent of bids for other units, the maximum probability that the $m+1$st unit takes place between $b^*_{\varepsilon,i,n,m}$ and $b^*_{\varepsilon,i,n,m}+\Delta$ is the case that the conditional distribution of bids puts all the weights on the interval, and in this case, the probability is $(1-\varepsilon)+\varepsilon\cdot\frac{\Delta}{\bar{b}}$. On the other hand, the minimum probability that the bid will be in the interval is the case where an equilibrium strategy puts none of the weights on the interval. In this case, the probability is $\varepsilon\cdot\frac{\Delta}{\bar{b}}$. In both cases, the lower and upper bounds are independent of n and m.

Evaluating (C3). The probability $\Pr\big(\text{bidder } i' \text{ places } m \text{ bids above } b^*_{\varepsilon,i,n,m}\big)$ can be bounded following the steps used in (C1) and these bounds are uniform and independent of n and m.

Then, since (C1)-(C3) will be uniformly bounded, from Lemma 2, (C) is also uniformly bounded. Then, Proposition 2 follows by combining (A), (B), and (C).

E. The Estimation Procedure.

Consider a smoothed auction market game $\Gamma(\varepsilon, n)$ where there are $n \cdot N_{PD}$ primary dealers, $n \cdot N_{ID}$ indirect bidders, and $n \cdot N_D$ direct bidders in the auction and also each bidder's bids will be smoothed with probability ε. Suppose bidders' bidding behavior are described by a Bayesian equilibrium $\overrightarrow{\beta}^*_{\varepsilon,n} = (\overrightarrow{\beta}^*_{\varepsilon,n,PD}, \overrightarrow{\beta}^*_{\varepsilon,n,D}, \overrightarrow{\beta}^*_{\varepsilon,n,ID})$ where each primary dealer uses a bidding strategy $\overrightarrow{\beta}^*_{\varepsilon,n,PD}$, a direct bidder employs a bidding strategy $\overrightarrow{\beta}^*_{\varepsilon,n,D}$, and each indirect bidder uses a bidding strategy $\overrightarrow{\beta}^*_{\varepsilon,n,ID}$. Then consider bidder i with a private value $\overrightarrow{v}_i$ and a bid $\overrightarrow{b}^*_{\varepsilon,i,n}$. Then the objective is to estimate bounds using the bootstrap.

Smoothing. In a smoothed game, bidder i chooses an equilibrium strategy $\overrightarrow{\beta}^*_{\varepsilon,n,i}$ with probability $1-\varepsilon$ and a bid is chosen nonstrategically from a uniform distribution on B_i with probability ε. In other words, each bid is additively augmented with independent random errors from the underlying kernel density. That is, these samples are obtained as $(1-\varepsilon)\overrightarrow{\beta}^*_{\varepsilon,i,n}+\varepsilon\overrightarrow{Z}_i$ with an independent iid sample $\overrightarrow{Z}_i$ from the uniform distribution.

Clustering. To deal with dependencies between indirct bidder bids and the primary dealer bids who routes these indirect bidders' bids, we now classify bids into following clusters: (1) bids by direct bidders (Category D). (2) bids by primary dealers who do not have any indirect bidder who bids through that indirect bidder (Category PD_0), (3) bids by primary dealers who have only one indirect bidder and its indirect bidder's bids (Category PD_1),.... Then, in each cluster that involves a primary dealer, we consider a profile of bids by a primary dealer and bids by indirect bidders who bid through the primary dealer instead of bids by each bidder. For example, if we consider a cluster with a primary dealer and one indirect bidder we consider a vector of $2M$ bids for each pair. Then for each category, bids (or a vector of bids) are iid.

Conduct Bootstrap within Each Cluster. As an example, consider a cluster PD_1. In this category, a pair of a primary dealer's bids and an indirect bidder's bids is a $2M$ dimensional bids. For each $i \in PD_1$, let $\widetilde{b}^*_{\varepsilon,i,n} = (\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,j,n:i=PD(j)})$ denote a vector

of primary dealer i's bids and an indirect bidder's bids. Let $|PD_1|$ be the number of primary dealers in PD_1. Let $F_{\varepsilon,PD(1)}(\widetilde{b}^*_{\varepsilon,i,n})$ be the distribution function of a $2M$ dimensional bids by a primary dealer and an indirect bidder. Then, $\widetilde{\beta}^*_{\varepsilon,1,n}, ..., \widetilde{\beta}^*_{\varepsilon,|PD(1)|,n}$ are independent random variables with a common distribution function $F_{\varepsilon,PD(1)}$. Let $F_{\varepsilon,PD(1),n}$ be the empirical distribution of $\widetilde{b}^*_{\varepsilon,1,n}, ..., \widetilde{b}^*_{\varepsilon,|PD(1)|,n}$, putting mass $1/|PD(1)|$ on each bid profile. Then, in the resampling, the data $\widetilde{b}^*_{\varepsilon,1,n}, ..., \widetilde{b}^*_{\varepsilon,|PD(1)|,n}$ are treated as a population with the distribution function $F_{\varepsilon,PD(1),n}$. Given $(\widetilde{b}^*_{\varepsilon,1,n}, ..., \widetilde{b}^*_{\varepsilon,|PD(1)|,n})$, let $\widehat{b}^*_{\varepsilon,1,n}, ..., \widehat{b}^*_{\varepsilon,n,n}$ be a conditionally independent sample with the common distribution $F_{\varepsilon,PD(1),n}$. Then the idea of bootstrap is that a statistics from a resample approximates the original distribution $F_{\varepsilon,PD(1)}$.

Convergence in the Mallows metric. Following Bickel and Freedman (1981), we consider the Mallows metric between 2 distributions. Let S_2 be the set of distributions functions F with the finite second moments. In the construction of this paper, when there are sufficiently many bidders in the auction, these bidding strategies will be close to the price taking behavior, thus the distribution of equilibrium bids will have finite second moments. Then, for two distributions F' and F'', consider the Mallows metric $\rho^2(F', F'')$ as in Mallows (1972). The convergence of a sequence of distribution function $\{F_n\}$ to F in terms of the Mallow metric is equivalent to $F_n \to F$ weakly and the convergence of the second moments. Then, Bickel and Freedman (1981) and De Martini and Rapallo (2008) show that, for multidimensional distributions, $F^*_{\varepsilon,PD(1),n}$ converges to $F_{\varepsilon,PD(1)}$ in the Mallows metric as $n \to \infty$.

Convergence of the Bid Distributions. The discussion in the above paragraph shows that, for each category of bids, bootstrap distributions of bids converge to the distribution of equilibrium bids in the Mallows metric. The distribution of bids in the auction is a mixture of distributions of bids from each category. Then, the bootstrap distributions of bids obtained above converge to the distributions of bids generated by $\overrightarrow{\beta}^*_{\varepsilon,n}$ in the Mallows metric.

Estimation of bounds. Since the convergence in the Mallows metric implies convergence in the distribution, the quantile of the bootstrap distribution converges to the quantile of the distribution of equilibrium bids according to $\overrightarrow{\beta}^*_{\varepsilon,n}$. This implies consistency of the bootstrap estimates of $\overline{W}$, $\underline{W}$, $\overline{L}$, and $\underline{L}$ as $n \to \infty$. Furthermore, convergence in distribution implies convergence of (A). Consequently, the above bootstrap estimates will lead to consistent estimates of bounds. Asymptotic normality follows from the argument of Bickel and Freedman (1981) Theorem 3.1.

F. *Proof of Proposition 4.*

Since a bidder can use a nonparticipation bid $\underline{b}$, the bidder will not bid more than its private value $v_{i,m}$. Thus it remains to derive the lower bounds.

Primary Dealer System. We first consider a primary dealer system, where primary dealers, direct bidders, and indirect bidders place bids. For direct bidders,

$$\begin{aligned}
&\underline{H}^{PD}_{\varepsilon,i,m,n}(v_{i,m}) = v_{i,m} \\
&-(m-1)\cdot \underbrace{\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{(A)}} \\
&-\underbrace{(\mathbf{E}_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] - b^{*}_{\varepsilon,n,i,,m})}_{\text{(B)}} \\
&-\underbrace{\frac{\Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})}{\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}))}}_{\text{(C)}} \cdot m \cdot \underbrace{\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{L}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]}_{\text{(D)}}.
\end{aligned}$$

We study each term one by one. For (A), we have

$$\begin{aligned}
&\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\
\leq\ & \frac{\overline{b}}{\varepsilon\cdot M\cdot\{n(N_{PD}+N_D+N_{ID})-1\}+1}
\end{aligned}$$

This bound is derived by nonstrategic bids that are independent of bidders' equilibrium strategies, a bidder's information set, and its private values. For (B), for any value of $b^{*}_{\varepsilon,n,i,,m}$, (B) is an expected increase in the market clearing price when bidder i increases the bid by Δ. As such, (B) is bounded above by Δ. For (C), we have $0 < \Pr_i(\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})/\Pr_i(\overline{W}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) < 1$. For (D), as in (A), we have

$$\begin{aligned}
&\mathbf{E}_i[\Delta P(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^{*}_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overrightarrow{b}^{*}_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\
\leq\ & \frac{\overline{b}}{\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}+1}.
\end{aligned}$$

Putting them together, we have

$$\underline{H}^{PD}_{\varepsilon,n,i,m}(v_{i,m}) \geq v_{i,m} - \Delta - (2m-1)\cdot\frac{\overline{b}}{\varepsilon\cdot M\cdot\{n\cdot(N_{PD}+N_D+N_{ID})-1\}+1}.$$

An indirect bidder chooses its best response using the first order condition. Proposition 2 shows that bids by direct bidders, indirect bidders, and primary dealers will be close to the price taking behavior as the auction markets become large. Thus we employ the same bound with the direct bidder.

For a primary dealer, his ability to shade its bid is limited by the bids by indirect bidders. That is, even if there are no nonstrategic bids that limit the primary dealers' ability to affect the price, if the primary dealer shades its bid below the indirect bidders' bids, then the primary dealer will lose its bids. Thus, the primary dealer takes into account of indirect bidders' bids when estimating the price impact. Primary dealer i has bids $\{b_{\varepsilon,n,j}\}_{j:PD(j)=i}$ from indirect bidders connected to the primary dealer i. Then a bid by the indirect biders closest to the primary dealer's bid $b^*_{\varepsilon,i,m,n}$ is denoted by $\min_{b_{\varepsilon,j,m,n}:\ \underbrace{PD(j)=i}_{\text{bid through bidder } i} \text{ and } \underbrace{b_{\varepsilon,n,j,m} \geq b^*_{\varepsilon,i,m,n}}_{\text{bid higher than } b^*_{\varepsilon,i,m,n}}} b_{\varepsilon,n,j,m}$. That is, when primary dealer i increases the bid from $b^*_{\varepsilon,n,j,m}$ to $b^*_{\varepsilon,n,j,m}+\Delta$, the increase in the market clearing price may come from surpassing the indirect bidders' bids or the nonstrategic bid. That is,

$$\begin{aligned} & \mathbf{E}_i[\Delta P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\ = \ & \min\{\min_{b_{\varepsilon,j,m,n}:\ \underbrace{PD(j)=i}_{\text{bid through bidder } i} \text{ and } \underbrace{b_{\varepsilon,n,j,m} \geq b^*_{\varepsilon,i,m,n}}_{\text{bid higher than } b^*_{\varepsilon,i,m,n}}} b_{\varepsilon,j,m,n} - b^*_{\varepsilon,i,m,n}, \\ & \frac{\overline{b}}{\varepsilon \cdot M \cdot \underbrace{\{n \cdot (N_{PD}+N_D+N_{ID}) - N_{I,i} - 1\}}_{\text{bidders excluding the primary bidder and the bidder connected to that bidder}} + 1}\}. \end{aligned}$$

Direct Bidding System. In the direct bidding system, every bidder has a direct link to its auctioneer. Thus a bidder's bound is the same with the one in the direct bidder's bid in the primary dealer system. In other words, in the direct bidding system, there is no information pooling channel involving primary dealers and indirect bidders.

Syndicate Bidding System. In the syndicate bidding system, we assume that a primary dealer submits bids that represents a consortium. In this competitive bidding process, the auction has only $n \cdot N_{PD}$ bidders. Thus its bid is bounded by

$$\underline{H}^{PD}_{\varepsilon,i,m,n}(v_{i,m}) \geq v_{i,m} - \Delta - (2m-1) \cdot \frac{\overline{b}}{\varepsilon \cdot M \cdot \{n \cdot N_{PD} - 1\} + 1}.$$

That is, since the syndicate bidding system, only the consortium bids in the auctions,

and there are no competition channel.

G. *The Statement and Proof of Proposition 8.*

We now consider the downward deviation where bidder i decreases the bids for the mth unit from $b^*_{\varepsilon,i,m,n}$ to $b^*_{\varepsilon,i,m,n} - \Delta$. We follow the steps similar to the ones used in the upward deviation case. Let $\overrightarrow{b}'_{\varepsilon,i,n}$ be the new bid profile after bidder i decreases the bid for the mth unit. That is, $\overrightarrow{b}'_{\varepsilon,i,n} = (\overrightarrow{b}^*_{\varepsilon,i,1,n}, ..., \overrightarrow{b}^*_{\varepsilon,i,m-1,n}, \overrightarrow{b}^*_{\varepsilon,i,m,n} - \Delta, \overrightarrow{b}^*_{\varepsilon,i,m+1,n}, ..., \overrightarrow{b}^*_{\varepsilon,i,M,n})$. Then there are 2 cases that reducing the bid will affect the outcome for bidder i. In the first case, after lowering the bid, bidder i will lose the bid for the mth unit of the security. Let $\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ be the event where other bidders bids are such that bidder i will no longer be allocated the mth unit of the security as a result of lowering the bids. In the second case, bidder i will still secure the mth unit after lowering the bids, and lowering the bid will save the payment for bidder i. Let $\underline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})$ be an event such that both bid $\overrightarrow{b}^*_{\varepsilon,i,m,n}$ and $\overrightarrow{b}^*_{\varepsilon,i,m,n} - \Delta$ would be allocated the mth unit.

Proposition 8: *Consider a "downward deviation" where bidder $i \in \mathbf{I}$ with the private value $v_i \in V_i$ decreases the bid for the $m = 1, ..., M$th unit by $\Delta > 0$ while keeping the bids for other units constant. Let $\overrightarrow{b}'_{\varepsilon,i,n}$ be the new bid profile of bidder i after the deviation. Then,*

$$
\begin{aligned}
& \Pr_i(\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\
& \cdot (v_{i,m} - E_i[P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]) \\
\geq\ & (m-1) \cdot \Pr_i(\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\
& \cdot E_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})] \\
& + \Pr_i(\underline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\
& \cdot m \cdot E_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n}) | \underline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})].
\end{aligned} \tag{5}
$$

We now provide the proof of Proposition 8 below.

The Payoff from the First Case. When the event $\underline{L}_{i,m}$ takes place, the bidder loses the payoffs that the bidder used to obtain from the mth unit, but the bidder would be able to pay less for other units of security by lowering the market clearing price. We now calculate the payoffs one by one. First, when bidder i used to employ the

bid $b^*_{\varepsilon,i,m,n}$, the bidder is allocated the mth unit. In this case, bidder i will derive value $v_{i,m}$ from obtaining the mth unit of the security. The payment in this case was $P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})$ when other bidders employ the bids $\overrightarrow{b}^*_{\varepsilon,-i,n}$. Then the payoff bidder i used to derive is

$$\begin{aligned} &\Pr_i(\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\ &\cdot(v_{i,m} - E_i[P(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]). \end{aligned} \tag{6}$$

Second, lowering the bid to $\overrightarrow{b}'_{\varepsilon,i,n}$ may affect the market clearing price, thus the payment for other $m-1$ units of the security that bidder i will be allocated. Let us denote the savings in the market clearing price by $\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})$. Then, the expected reduction in the payment is

$$\begin{aligned} &\Pr_i(\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\ &\cdot(m-1)\cdot E_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\underline{L}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]. \end{aligned} \tag{7}$$

The Payoff from the Second Case. We now consider the payoff changes when lowering the bid from $\overrightarrow{b}^*_{\varepsilon,i,n}$ to $\overrightarrow{b}'_{\varepsilon,i,n}$ will not affect the allocation but reduces the payment for bidder i. Let us denote the saving in the market clearing price in this case by $\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})$. Then, the expected change in the payment is

$$\begin{aligned} &\Pr_i(\underline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})) \\ &\cdot m\cdot E_i[\Delta P(\overrightarrow{b}'_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}^*_{\varepsilon,-i,n})|\underline{W}_{i,m}(\overrightarrow{b}^*_{\varepsilon,i,n}, \overrightarrow{b}'_{\varepsilon,i,n})]. \end{aligned} \tag{8}$$

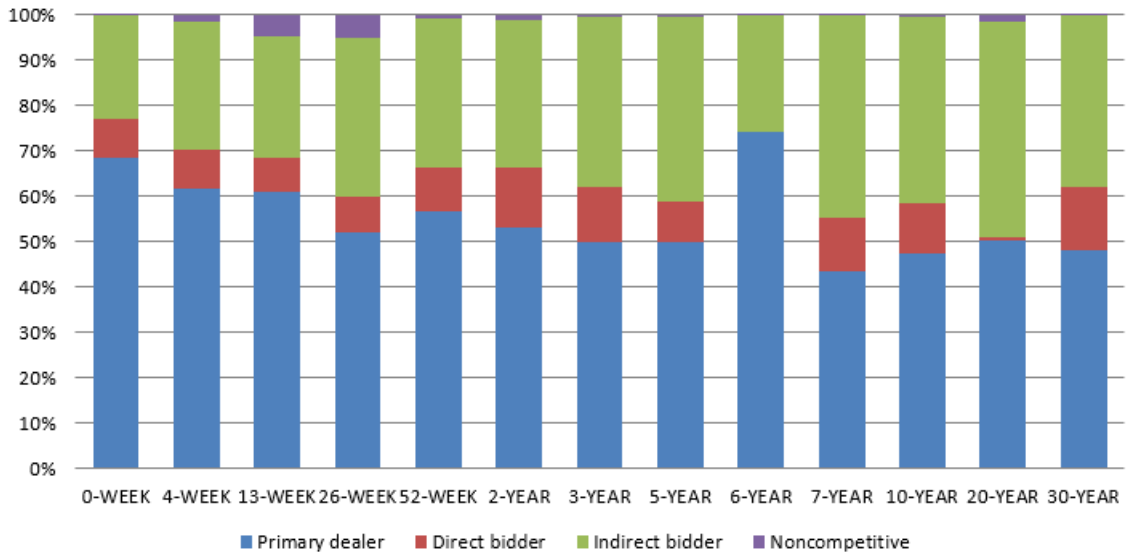
Putting It All Together. Since a bid $\overrightarrow{b}^*_{\varepsilon,i,n}$ is a best response for bidder i, bidder i will not benefit from lowering the bid from $\overrightarrow{b}^*_{\varepsilon,i,n}$ to $\overrightarrow{b}'_{\varepsilon,i,n}$. It implies that the expected monetary loss from the mth unit is larger than the possible saving from the lower market price. Then, combining (6)-(8), we get (5).

Table 1 Bidder Category Purchase Shares for All Treasury Secuirites, 2008-13.

Category	Mean	SD	Min	Max
Primary Dealer	56.6% (70.9%)	12.9% (14.6%)	14.0% (33.6%)	100.0% (100.0%)
Direct Bidder	9.1% (2.4%)	6.1% (3.6%)	0.0% (0.0%)	45.4% (31.6%)
Indirect Bidder	32.0% (21.6%)	12.3% (12.7%)	0.0% (0.0%)	70.9% (64.8%)
Noncompetitive	2.3% (5.1%)	2.4% (4.7%)	0.0% (0.0%)	12.4% (16.8%)
Sample Size	1584			

Source: Treasury Auction Results available at TreasuryDirect. Note: This table reports descriptive statistics of bidder category purchase shares for 1585 Treasury auctions April 1, 2008 and December 19, 2013. The numbers in the parenthesis are from Fleming (2007).

Table 2 Bidder Category Purchase Shares by Issue Type, 2008-13



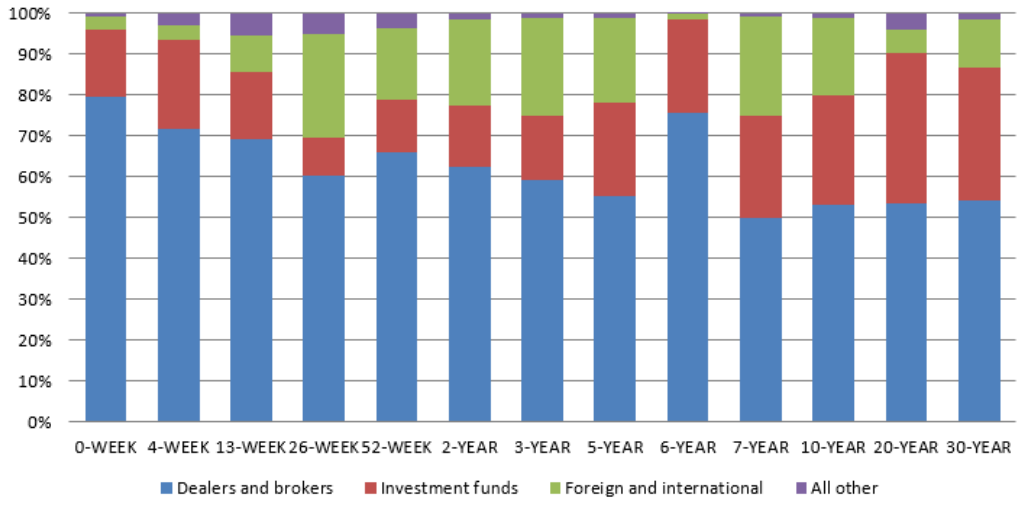
Source: Treasury Auction Results available at TreasuryDirect. Note: This chart plots average bidder category purchase shares by issue type for 1585 Treasury auctions between April 1, 2008 and December 19, 2013.

Table 3 Investor Class Allotment Shares for All Treasury Securityes, 2008-13

Investor Class	Mean	SD	Min	Max
Depository Institutions	0.5% (0.5%)	1.3% (2.6%).	0.0% (0.0%)	24.2% (31.6%)
Individuals	1.7% (0.5%)	1.8% (1.2%)	0.0% (0.0%)	10.2% (16.3%)
Dealers and Brokers	65.1% (75.4%)	13.6% (14.6%)	26.0% (34.5%)	100.0% (100.0%)
Private and Brokers	0.2% (0.0%)	0.8% (0.4%)	0.0% (0.0%)	11.0% (8.5%)
Investment Funds	18.0% (6.5%)	11.5% (7.6%)	0.0% (0.0%)	57.1% (46.1%)
FIMA	13.8% (12.5%)	11.6% (9.7%)	0.0% (0.0%)	61.1% (38.6%)
Other	0.8% (4.6%)	2.2% (4.2%)	0.0% (0.0%)	27.3% (20.3%)

Source: Investor Class Auction Allotment data. Note: This table reports descriptive statistics of bidder category purchase shares for 1567 Treasury auctions April 1, 2008 and December 19, 2013. The numbers in the parenthesis are from Fleming (2007).

Table 4 Bidder Category Purchase Shares in Issue Type, 2008-13.



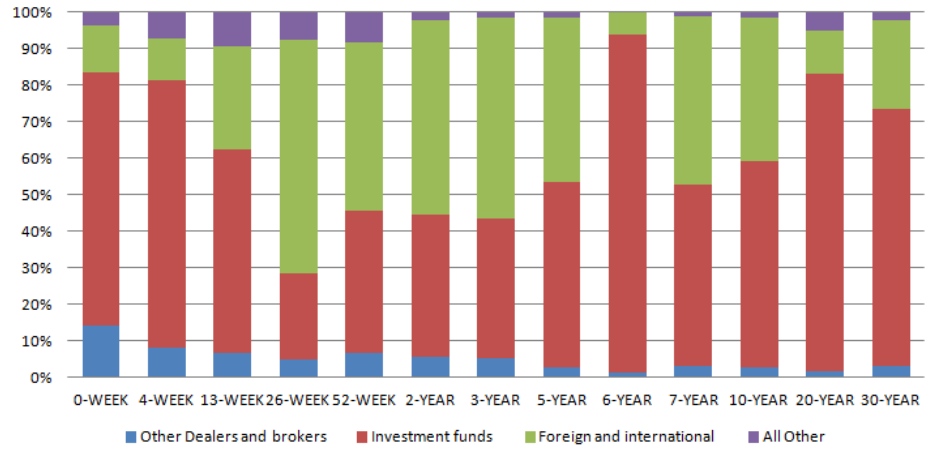
Source: Investor Class Auction Allotment data. Note: The chart plots average investor class allotment shares by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013.

Table 5 Results of Regressions of Bidder Category Purchase Shares on Investor Class Allotment Shares

Investor class	Bidder Category			
	Primary Dealer	Direct	Indirect	Noncompetitive
Depository Institutions	0.000	0.515	0.484	0.001
	(0.000232)	**(0.0941)	**(0.0963264)	(0.0202167)
Individuals	0.002	-0.647	0.415	1.230
	**(0.000166)	**(0.0675)	**(0.0690)	**(0.0145)
Primary Dealers	1.000	-0.021	0.019	0.002
	**(1.256e-05)	**(0.00509)	**(0.00522)	(0.00109)
Dealers Brokers other than PD	0.000	0.803	0.185	0.012
	(6.461e-05)	**(0.0262)	**(0.0268)	*(0.00563)
Pension Retirement fund	-0.001	0.305	0.718	-0.022
	**(0.000393)	(0.1593)	**(0.163)	(0.0342)
Investment Funds	0.000	0.156	0.858	-0.014
	(2.133e-05)	**(0.00866)	**(0.00886)	**(0.00186)
Foreign	0.000	0.096	0.883	0.021
	(2.293e-05)	**(0.00930)	**(0.00952)	**(0.002)
Other	0.000	0.026	0.926	0.049
	(0.000135)	(0.0546)	**(0.0559)	**(0.0117)
R^2 Adjusted	1.000	0.814	0.980	0.912

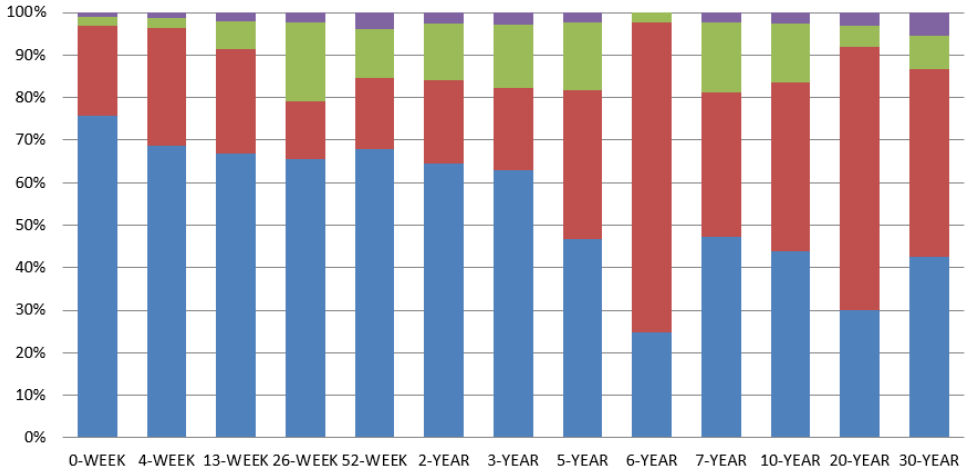
Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This table reports results of regressions of bidder category purchase shares on investor class allotment shares for 1567 Treasury auctions between April 1, 2008 and December 19, 2013. Table 5 is the regression of bidder category shares over 4 bidders - primary dealer, direct, indirect, and noncompetitive on the investor class allotment shares over 8 classes - depository, individual, primary dealers, other dealer and brokers (excluding PD), private pension funds, investment funds, foreign and international, and finally others. Note that this result is different from Fleming (2007) in two ways: (1) the regression includes primary dealers as an independent variable and (2) the estimated shares are based on the regressions run together for bills and coupon securities. Qualitative results of these regressions are robust to other specifications.

Table 6 Estimated Breakdown of Indirect Bid by Issue Type, 2008-13.



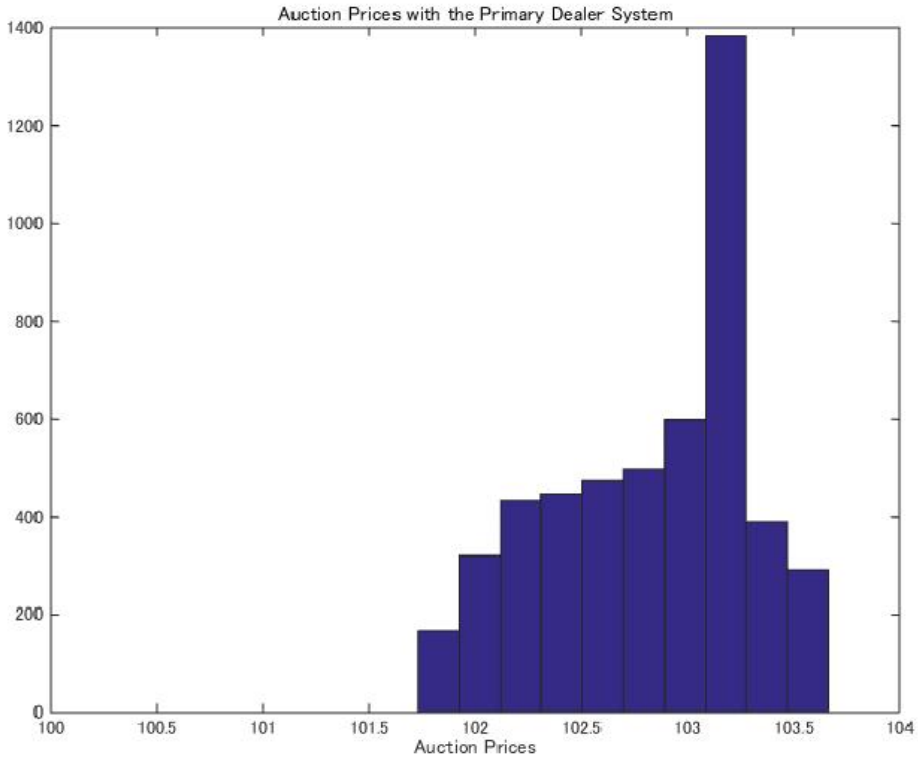
Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This chart plots the average estimated breakdown of indirect bidder purchases by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013 based on the previous regressions.

Table 7 Estimated Breakdown of Direct Bid by Issue Type, 2008-13



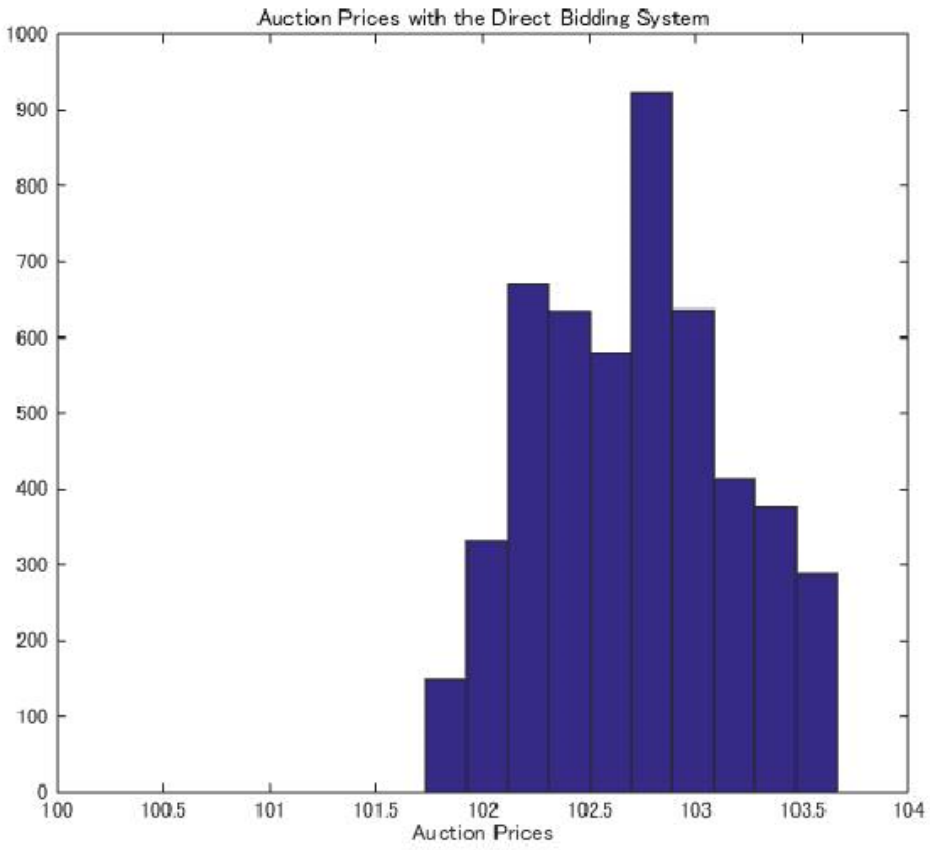
Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This chart plots the average estimated breakdown of indirect bidder purchases by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013.

Table 8 The Distribution of Simulated Market Prices under the Primary Dealer System



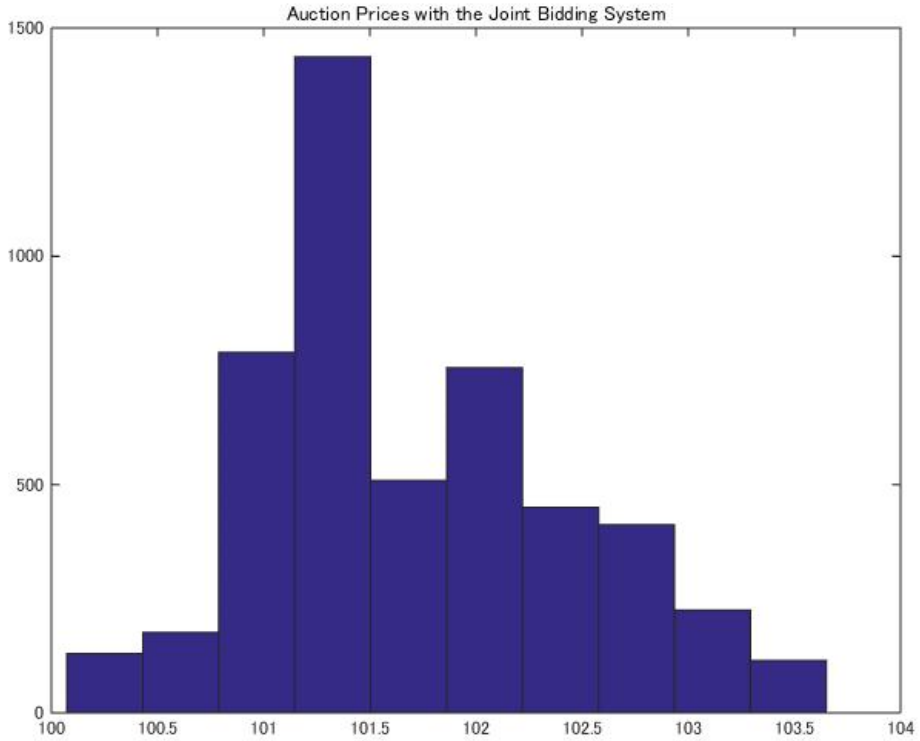
Note: This table is based on the simulation results for 5000 samples of auction results for the primary dealer system. The simulation results use only the estimated private values.

Table 9 The Distribution of Simulated Market Prices Under the Direct Bidding System



Note: This table is based on the simulation results for 5000 samples of auction results for the direct bidding system. The simulation results use only the estimated private values.

Table 10 The Distribution of Simulated Market Prices Under the Joint Bidding System



Note: This table is based on the simulation results for 5000 samples of auction results for the syndicate bidding system. The simulation results use only the estimated private values.