Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime Switching Approach

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Abstract This paper develops an endogenous regime switching approach to modeling financial crises. In the model there are two regimes, one a crisis regime, the second a regime for normal economic times. The switch between regimes is based on a probability determined by economic variables in the economy. Agents in the economy know how economic fundamentals affect the probability of moving in or out of the crisis state. That is, it is a rational expectations solution of the model. The solution then ensures that decisions made in the normal state fully incorporate how those decision affect the probability of moving into the crisis state as well as how the economy will operate in a crisis. The model developed captures all of the salient features one would want in an empirical model of financial crises. First, it captures the non-linear nature of a crisis. Second, the regime switching model is solved using perturbation methods and a second order solution. This allows the solution to capture the impact of risk on decision rules due both in an out of the crisis. Third, since the solution method is perturbation based it can handle a number of state variables and many shocks. That is, we are less constrained than current non-linear methods in terms of the size of the model. Fourth, the speed of the solution method means that non-linear filters (e.g. particle filter) can be used to calculate the likelihood function of the model for a full Bayesian estimation of the relevant shocks and frictions that are fundamental to models of financial crises. Fifth, the fully rational expectations nature of the solution allows one to ask key counterfactual policy questions. For example, what policies, if any, would have mitigated or eliminated the Great Recession? We adopt this approach to study sudden stop episodes in Mexico. Our model is an adaption of Mendoza (2010). In particular, we rewrite his occasionally binding collateral contraint model as a multi regime model and take it to the data using our new estimation procedures.

1 Introduction

This paper develops a new approach to estimating structural macroeconomic models of financial crises. Financial crises are rare but large events, implying that any reasonable model for analyzing crises must be non-linear. This non-linearity of course poses problems computationally, particularly when one wants to take these models to the data formally using likelihood based empirical methods. In the proposed research we combine two key types of non-linearity. The first writes models of financial crises as a two (or more) regime model. By viewing a financial crisis as a discrete regime we can capture the large and significant change in an economy in crisis. The second is to solve our model to a second order solution, allowing us to capture the impact of risk on decision making both in and out of the the crisis.

The core of the new methodology develops an endogenous regime switching approach to modeling financial crises. In the model there are two regimes, one a crisis regime, the second a regime for normal economic times. in our model a crisis regime is a regime where an occasionally binding borrowing constraint binds, (Mendoza 2010). The switch between regimes is based on a probability determined by economic variables in the economy. Likewise, the switch back to normal times is based on economic fundamentals. In our model the probability of moving to the crisis regime where the borrowing constraint binds is a logistic function of the debt to output ratio. Agents in the economy know of this probability and how debt, output and other choices map into the probability of moving in or out of the crisis state. That is, it is a rational expectations solution of the model. Our solution then ensures that decisions made in the normal state fully incorporate how those decision affect the probability of moving into the crisis state as well how the economy will operate in a crisis (i.e the decision rules in this crisis).

The approach we develop allows us to capture all of the salient features one would want in an empirical model of financial crises. First, it captures the non-linear nature of a crisis: the crisis state can have very different properties/parameters from the normal state. Second, we solve the regime switching model using perturbation methods and a second order solution. This means that we can capture the change in decision rules as risk changes in a crisis. Third, since our solution method is perturbation based we can handle a number of state variables and many shocks. That is, we are less constrained than current non-linear methods in terms of the size of the model. Fourth, the speed of the solution method means that we can use non-linear filters (e.g. particle filter) to calculate the likelihood function of the model for a full Bayesian estimation of the relevant shocks and frictions that are fundamental to models of financial crises. Fifth, the fully rational expectations nature of the solution allows us to ask key policy questions. For example, what policies, if any, would have mitigated or eliminated the Great Recession?

We apply the method to a Mendoza (2010) style small open economy model with occasionally binding borrowing constraints. For this model we have written the model into a regime switching model and solved the model to the second order solution with endogenous regime switching. We are currently working on the estimation part of the paper.

1.1 Related Literature

The literature on Markov-switching linear rational expectations (MSLRE) is now well established (e.g Leeper and Zha (2003), Davig and Leeper (2007), and Farmer, Waggoner, and Zha (2009)). The MSLRE approach introduces an important nonlinearity into the standard linear rational expectations models. That nonlinearity is a discrete change in the parameters across regimes. The MSLRE approach has been widely used to model shifts in monetary and fiscal policy (e.g Bianchi 2014 or Davig, Leeper, and Walker 2010). Markov-switching models in general are useful because they provide a tractable way to model how agents form expectations over discrete changes in policies. Foerster et al (2014) argue that the MSLRE solution techniques, which impose Markov Switching after linearizing the model, may differ from the solution to a model where switching is present in the original model before linearization. In addition, the MSLRE model of course cannot be solved to higher orders, which may matter in economies where risk is important. To address these issues Foerster et al (2014) develop a perturbation methodology for constructing first or second order solutions of markov-switching DSGE (MSDSGE) models. A key innovation of their approach is to work with the original MSDSGE model directly, rather than starting with a system of linear rational expectations equations. The work in this proposal builds on their approach, but solves a class of model when the probability of regime switch is an endogenous function of the economy. This allows regime switching to be used for models of financial crises. An exogenous switching model would not be interesting for this class of models as choices (for debt, production, policy) would all be unrelated to whether or not a crisis occurs

The advantage of writing the occasionally binding constraint model as an endogenous regime switching model is that we can take the model to the data using likelihood based estimation methods. Since we are using a higher order solution this suggests that we use the particle filter similiar to that of Fernandez-Villaverde and Rubio-Ramirez (2007). Our approach builds on the literature on estimating exogenous regime switching models. Bianchi (2014) develops a Gibbs sampler to estimate a model with exogenous probabilities in the case of a first order solution of the model. Foerster(2014) estimates a similar regime switching model, but uses a second order solution of the model and adapts the particle filter to efficiently deal with multiple regimes. Our contribution here is to extend the estimation of the model to the class of models where the probability of a regime switch is endogenous. This project then makes two major methodological contributions in terms of solving and estimating regime switching models.[1] This contribution can be applied to many classes of models.

The application of the methodology is most closely related to the literature that has built on the seminal work of Mendoza (2010). This literature has studied the normative properties of model economies with endogenous financial crises (also labelled sudden stops or

[1]Dan Waggoner and Tao Zha have presented an endogenous regime switching model of the zero-bound at conferences, but no paper appears available yet. Their approach is a MSV approach, that employs first order solutions. Lind (2014) develops an endogenous switching approach that allows for analytic results as a special case. His work is applied to the zero lower bound. The first contribution to the endogenous switching literature was Davig and Leeper 2006 who allowed the regime to change with the econonomy in a prespecified manner.

credit crunches). Some examples include Bianchi (2011) who uses an endowment version of such an economy and finds that the competitive equilibrium always entails more borrowing relative to the constrained social planner allocation, and that a prudential tax on debt (i.e., a prudential capital control) can replicate the social planner allocation. Benigno et al (2012) show that in a production economy agents can actually borrow too little relative to what is socially optimal. Benigno et al (2014) compare alternative tax instruments chosen by a Ramsey planner in the same economy analyzed by Bianchi (2011) and find that taxes on consumption (i.e., real exchange rate interventions) dominate capital controls as a policy tool because they can achieve the unconstrained allocation while capital controls can achieve only the constrained efficient one. Cespedes, Chang and Velasco (2012) compare the transmission mechanism of alternative policy interventions in a similar model. Jeanne and Korinek (2011) and Bianchi and Mendoza (2010) analyze models in which the price externality arises because agents fail to internalize the effect of their decisions on an asset price. Korinek and Mendoza (2013) provide a thorough review of the models, questions and results from this large literature. They conclude by stating that an important future step for this literature is the "development of numerical methods that combine the strenghts of global solution methods in describing non-linear dynamics with the power of perturbation methods in dealing with a large number of variables so as to analyze sudden stops in even richer macroeconomic models". This is exactly what the methodology developed in this paper delivers.

The economic innovation of the paper is then to take the lessons from this normative literature to the data and develop ways to implement these policies. Since the model will be estimated on the data we can test, in a counterfactual sense, how various policies would have worked in different historical episodes. This is an important innovation to the literature, as the current papers (including my own) largely discuss policies effectiviness while abstracting away from the historical sequence of shocks that occured in any given crisis period. The paper will also contribute by documenting the sources of financial crises by identifying the key shocks that drive us into a crisis.

2 The Model and the Competitive Equilibrium

In this section, we describe our model set-up. The model is largely from Mendoza (2010). An exception is that we use a interest elastic debt function to pin down debt instead of an endogenous rate of time preference.

2.1 Representative Household-Firm

As in Mendoza (2010) there is a representative household that also makes decisions about production and capital accumulation. This structure ensures that the working capital constraint affects the borrowing constraint, a key feature of Mendoza's paper The households maximizes the utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}, \tag{1}$$

with C_t denoting the individual consumption and H_t the individual supply of labor. The elasticity of labor supply is ω, while ρ is the coefficient of relative risk aversion.

Households maximize utility subject to their budget constraint. The constraint each household faces is:

$$C_t + I_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t \left(W_t H_t + P_t V_t\right) - \frac{1}{(1+r_t)} B_t + B_{t-1} \tag{2}$$

V_t are imported intermediate goods. P_t is the price of these imports, which will be a stochastic process specified below. This shock is interpreted as a terms of trade shock. B_t are one period international bonds with price net interest rate r_t. The interest rate is exogenous and equal to a stochastic process specified below. The ϕr_t term reflects a working capital constraint, both wages and intermediate goods must be paid for with borrowed funds. The price of labor and capital are given by w_t and q_t, both of which are endogenous variables, but taken as given by the household. Investment incurs an adjustment cost in terms of net investment:

$$I_t = \delta K_{t-1} + \left(K_t - K_{t-1}\right)\left(1 + \frac{\iota}{2}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)\right) \tag{3}$$

I_t is gross investment.

The agents faces a collateral constraint:

$$\frac{1}{(1+r_t)} B_t - \phi\left(1+r_t\right)\left(W_t H_t + P_t V_t\right) \geq -\kappa q_t K_t \tag{4}$$

Households maximize (1) subject to (2) and (4) by choosing C_t, B_t, K_t, V_t and H_t. The first-order conditions of this problem are the following:

$$C_t : \left(C_t - \frac{H_t^{\omega}}{\omega}\right)^{-\rho} - \mu_t = 0 \tag{5}$$

$$V_t : \left(1-\alpha-\eta\right) A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{-\alpha-\eta} = P_t\left(1 + \phi r_t + \frac{\lambda_t}{\mu_t}\phi\left(1+r_t\right)\right) \tag{6}$$

$$H_t : \alpha A_t K_{t-1}^{\eta} H_t^{\alpha-1} V_t^{1-\alpha-\eta} = \phi W_t\left(r_t + \frac{\lambda_t}{\mu_t}\left(1+r_t\right)\right) + H_t^{\omega-1} \tag{7}$$

$$B_t : \mu_t = \lambda_t + \beta\left(1+r_t\right)\mathbb{E}_t \mu_{t+1} \tag{8}$$

$$K_t : \mathbb{E}_t \mu_{t+1} \beta \left(\begin{array}{c} 1 - \delta + \left(\frac{\iota}{2}\left(\frac{K_{t+1}}{K_t}\right)^2 - \frac{\iota}{2}\right) \\ +\eta A_{t+1} K_t^{\eta-1} H_{t+1}^{\alpha} V_{t+1}^{1-\eta-\alpha} \end{array}\right) = \mu_t\left(1 - \iota + \iota\left(\frac{K_t}{K_{t-1}}\right)\right) - \lambda_t \kappa q_t \tag{9}$$

Market optimal prices for capital and labor are

$$q_t = 1 + \iota\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{10}$$

$$W_t = H_t^{\omega-1} \tag{11}$$

The last two conditions are the budget and complementary slackness conditions is

$$C_t + I_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t \left(W_t H_t + P_t V_t\right) - \frac{1}{(1+r_t)} B_t + B_{t-1} \tag{12}$$

$$\left(\frac{1}{(1+r_t)} B_t - \phi \left(1+r_t\right)\left(w_t H_t + P_t V_t\right) + \kappa q_t K_t\right) \lambda_t = 0 \tag{13}$$

where μ_t is the multiplier on the period budget constraint, and λ_t is the multiplier on the international borrowing constraint.

We have three exogenous processes and 4 shocks. The interest rate has a debt elastic component as well as a stochastic shock to the elasticity (country specific risk premium) as well as to world interest rates.

$$r_t = r^* + \left(\psi_r + \sigma_r \varepsilon_{r,t}\right)\left(e^{\overline{B} - B_t} - 1\right) + \sigma_w \varepsilon_{w,t}$$

laws of motion for the exogenous variables

$$\log A_t = a + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t} \tag{14}$$

$$\log P_t = p + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t} \tag{15}$$

The borrowing cushion is given by the amount of borrowing over the debt limit

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi \left(1+r_t\right)\left(W_t H_t + P_t V_t\right) + \kappa q_t K_t \tag{16}$$

where $B_t^* \geq 0$.

2.2 Regime Switching

Given the complementary slackness condition (13), there are two regimes: one in which the constraint binds, and one in which it does not. The variable $s_t \in \{0, 1\}$ denotes the regime, and whether the constraint binds ($s_t = 1$) or does not bind ($s_t = 1$). The regime switching variable $\varphi\left(s_t\right) = s_t$ turns "on" or "off" the relevant portions of the slackness condition In this setup, the slackness condition becomes

$$\varphi\left(s_t\right) B_t^* = \left(1 - \varphi\left(s_t\right)\right) \lambda_t. \tag{17}$$

As a consequence, when $s_t = 0$, we have

$$\lambda_t = 0$$

and when $s_t = 1$,

$$B_t^* = 0.$$

In order to ensure the regime switching setup maps as closely to the original framework as possible, the transition probabilities are logistic functions. When the constraint is not

binding, the probability that it binds the next period depends on the value of its debt relative to the credit limit (**??**):

$$\Pr\left(s_{t+1}=1|s_t=0\right)=\frac{\exp\left(\gamma_{0,0}-\gamma_{0,1}B_t^*\right)}{1+\exp\left(\gamma_{0,0}-\gamma_{0,1}B_t^*\right)}. \tag{18}$$

When the constraint is binding, the probability that it does not bind the next period depends on the slackness multiplier

$$\Pr\left(s_{t+1}=0|s_t=1\right)=\frac{\exp\left(\gamma_{1,0}+\gamma_{1,1}\lambda_t\right)}{1+\exp\left(\gamma_{1,0}+\gamma_{1,1}\lambda_t\right)}. \tag{19}$$

The transition matrix is then

$$P_t=\left[\begin{array}{cc} p_{00,t} & p_{01,t} \\ p_{10,t} & p_{11,t} \end{array}\right]=\left[\begin{array}{cc} 1-\frac{\exp(\gamma_{0,0}-\gamma_{0,1}B_t^*)}{1+\exp(\gamma_{0,0}-\gamma_{0,1}B_t^*)} & \frac{\exp(\gamma_{0,0}-\gamma_{0,1}B_t^*)}{1+\exp(\gamma_{0,0}-\gamma_{0,1}B_t^*)} \\ \frac{\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_t)}{1+\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_t)} & 1-\frac{\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_t)}{1+\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_t)} \end{array}\right].$$

Note that as $\gamma_{0,0},\gamma_{0,1},\gamma_{1,0},\gamma_{1,1}\to\infty$ the probabilities achieve the theshhold behavior of the original constraint.

In addition to the borrowing constraint, several other parameters change at the same time as the regime, which allows for capturing important aspects of crises. In particular, the discount factor switches, so (8) and (9) become

$$\mu_t=\lambda_t+\beta\left(s_t\right)\left(1+r_t\right)\mathbb{E}_t\mu_{t+1} \tag{20}$$

$$\mathbb{E}_t\mu_{t+1}\beta\left(s_t\right)\left(\begin{array}{c} 1-\delta+\left(\frac{\iota}{2}\left(\frac{K_{t+1}}{K_t}\right)^2-\frac{\iota}{2}\right) \\ +\eta A_{t+1}K_t^{\eta-1}H_{t+1}^{\alpha}V_{t+1}^{1-\eta-\alpha} \end{array}\right)=\mu_t\left(1-\iota+\iota\left(\frac{K_t}{K_{t-1}}\right)\right)-\lambda_t\kappa q_t \tag{21}$$

and the laws of motion switch intercepts, so

$$\log A_t=a\left(s_t\right)+\rho_A\log A_{t-1}+\sigma_A\varepsilon_{A,t} \tag{22}$$

$$\log P_t=p\left(s_t\right)+\rho_P\log P_{t-1}+\sigma_P\varepsilon_{P,t} \tag{23}$$

3 Competitive Equilibrium

The competitive equilibrium is given by first-order conditions for the representative household-firm (5 equations)

$$\left(C_t-\frac{H_t^{\omega}}{\omega}\right)^{-\rho}=\mu_t$$

$$\left(1-\alpha-\eta\right)A_tK_{t-1}^{\eta}H_t^{\alpha}V_t^{-\alpha-\eta}=P_t\left(1+\phi r_t+\frac{\lambda_t}{\mu_t}\phi\left(1+r_t\right)\right)$$

$$\alpha A_tK_{t-1}^{\eta}H_t^{\alpha-1}V_t^{1-\alpha-\eta}=\phi W_t\left(r_t+\frac{\lambda_t}{\mu_t}\left(1+r_t\right)\right)+H_t^{\omega-1}$$

$$\mu_t = \lambda_t + \beta\left(s_t\right)\left(1+r_t\right)\mathbb{E}_t\mu_{t+1}$$

$$\mathbb{E}_t\mu_{t+1}\beta\left(s_t\right)\left(\begin{array}{c} 1-\delta+\left(\frac{\iota}{2}\left(\frac{k_{t+1}}{K_t}\right)^2-\frac{\iota}{2}\right) \\ +\eta A_{t+1}K_t^{\eta-1}H_{t+1}^{\alpha}V_{t+1}^{1-\eta-\alpha}\end{array}\right) = \mu_t\left(1-\iota+\iota\left(\frac{K_t}{K_{t-1}}\right)\right)-\lambda_t\kappa q_t$$

market price equations (2 equations)

$$q_t = 1+\iota\left(\frac{K_t-K_{t-1}}{K_{t-1}}\right)$$

$$W_t = H_t^{\omega-1}$$

budget constraints (2 equations)

$$C_t+I_t = A_tK_{t-1}^{\eta}H_t^{\alpha}V_t^{1-\alpha-\eta}-P_tV_t-\phi r_t\left(W_tH_t+P_tV_t\right)-\frac{1}{\left(1+r_t\right)}B_t+B_{t-1}$$

$$I_t = \delta K_{t-1}+\left(K_t-K_{t-1}\right)\left(1+\frac{\iota}{2}\left(\frac{K_t-K_{t-1}}{K_{t-1}}\right)\right)$$

the debt cushion and borrowing limit constraints (2 equations)

$$B_t^* = \frac{1}{\left(1+r_t\right)}B_t-\phi\left(1+r_t\right)\left(W_tH_t+P_tV_t\right)+\kappa q_tK_t$$

$$\varphi\left(s_t\right)B_t^* = \left(1-\varphi\left(s_t\right)\right)\lambda_t$$

and laws of motion (3 equations)

$$r_t = r^*+\left(\psi_r+\sigma_r\varepsilon_{r,t}\right)\left(e^{\overline{B}-B_t}-1\right)+\sigma_w\varepsilon_{w,t}$$

$$\log A_t = a\left(s_t\right)+\rho_A\log A_{t-1}+\sigma_A\varepsilon_{A,t}$$

$$\log P_t = p\left(s_t\right)+\rho_P\log P_{t-1}+\sigma_P\varepsilon_{P,t}$$

and an auxiliary equation connecting capital (1 equation)

$$k_t = K_t$$

3.1 Regime Switching Equilibrium

The 15 equilibrium conditions are written as

$$\mathbb{E}_tf\left(\mathbf{y}_{t+1},\mathbf{y}_t,\mathbf{x}_t,\mathbf{x}_{t-1},\chi\varepsilon_{t+1},\varepsilon_t,\theta_{t+1},\theta_t\right) = 0$$

where the variables are separated into the predetermined variables $\mathbf{x}_{t-1}$ and the nonpredermined variables $\mathbf{y}_t$. The variables are 4 predetermined variables

$$\mathbf{x}_{t-1} = \left[K_{t-1},B_{t-1},A_{t-1},P_{t-1}\right]$$

and 11 nonpredetermined variables

$$\mathbf{y}_t = [C_t, H_t, V_t, I_t, k_t, r_t, q_t, W_t, \mu_t, \lambda_t, B_t^*]$$

with 2 shocks

$$\varepsilon_t = [\varepsilon_{r,t}, \varepsilon_{w,t}, \varepsilon_{A,t}, \varepsilon_{P,t}]$$

and 4 switching variables

$$\theta_t = [\varphi(s_t), \beta(s_t), a(s_t), p(s_t)].$$

For solving the model, the functional forms are

$$\begin{aligned} \theta_{t+1} &= \bar{\theta} + \chi\hat{\theta}(s_{t+1}) \\ \theta_t &= \bar{\theta} + \chi\hat{\theta}(s_t) \end{aligned}$$

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$$

$$\begin{aligned} \mathbf{y}_t &= g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \\ \mathbf{y}_{t+1} &= g_{s_{t+1}}(\mathbf{x}_t, \chi\varepsilon_{t+1}, \chi) \end{aligned}$$

$$p_{s_t, s_{t+1}, t} = \pi_{s_t, s_{t+1}}(\mathbf{y}_t)$$

Using these in the equilibrium conditions and being more explicit about the expectation operator, given $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ and s_t, the

$$\begin{aligned} & F_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \\ \int \sum_{s'=0}^{1} \pi_{s_t, s'}(g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)) f \begin{pmatrix} g_{s_{t+1}}(h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \chi\varepsilon', \chi), \\ g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ \mathbf{x}_{t-1}, \chi\varepsilon', \varepsilon_t, \\ \bar{\theta} + \chi\hat{\theta}(s'), \bar{\theta} + \chi\hat{\theta}(s_t) \end{pmatrix} d\mu\varepsilon' &= 0 \end{aligned}$$

Stacking these conditions for each regime produces

$$\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \begin{bmatrix} F_{s_t=1}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \\ F_{s_t=2}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \end{bmatrix}$$

4 Steady State

The model has switching in parameters that would affect the steady state of the economy in a fixed parameter case. In other words, the switching parameters $\varphi(s_t)$, $\beta(s_t)$, $a(s_t)$, and $p(s_t)$ all affect the level of the economy directly, and will thus matter for steady state calculations. In steady state, the transition matrix satisfies

$$P_{ss} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(\gamma_{0,0}+\gamma_{0,1}B_{ss}^*)}{1+\exp(\gamma_{0,0}+\gamma_{0,1}B_{ss}^*)} & \frac{\exp(\gamma_{0,0}+\gamma_{0,1}B_{ss}^*)}{1+\exp(\gamma_{0,0}+\gamma_{0,1}B_{ss}^*)} \\ \frac{\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_{ss})}{1+\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_{ss})} & 1 - \frac{\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_{ss})}{1+\exp(\gamma_{1,0}+\gamma_{1,1}\lambda_{ss})} \end{bmatrix}$$

Let $\xi = [\xi_0, \xi_1]$ denote the ergodic vector of P_{ss}. Then define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1)$$
$$\bar{\beta} = \xi_0 \beta(0) + \xi_1 \beta(1)$$
$$\bar{a} = \xi_0 a(0) + \xi_1 a(1)$$
$$\bar{p} = \xi_0 p(0) + \xi_1 p(1)$$

The steady state of the economy depends on these ergodic means, and satisfies the following equations

$$\left(C_{ss} - \frac{H_{ss}^{\omega}}{\omega}\right)^{-\rho} = \mu_{ss}$$

$$(1-\alpha-\eta) A_{ss} K_{ss}^{\eta} H_{ss}^{\alpha} V_{ss}^{-\alpha-\eta} = P_{ss}\left(1 + \phi r_{ss} + \frac{\lambda_{ss}}{\mu_{ss}}\phi(1+r_{ss})\right)$$

$$\alpha A_{ss} K_{ss}^{\eta} H_{ss}^{\alpha-1} V_{ss}^{1-\alpha-\eta} = \phi W_{ss}\left(r_{ss} + \frac{\lambda_{ss}}{\mu_{ss}}(1+r_{ss})\right) + H_{ss}^{\omega-1}$$

$$\mu_{ss} = \lambda_{ss} + \bar{\beta}(1+r_{ss})\mu_{ss}$$

$$\mu_{ss}\bar{\beta}\left(\begin{array}{c} 1-\delta+\left(\frac{\iota}{2}\left(\frac{k_{ss}}{K_{ss}}\right)^2 - \frac{\iota}{2}\right) \\ +\eta A_{ss} K_{ss}^{\eta-1} H_{ss}^{\alpha} V_{ss}^{1-\eta-\alpha} \end{array}\right) = \mu_{ss}\left(1 - \iota + \iota\left(\frac{K_{ss}}{K_{ss}}\right)\right) - \lambda_{ss}\kappa q_{ss}$$

$$q_{ss} = 1 + \iota\left(\frac{K_{ss} - K_{ss}}{K_{ss}}\right)$$

$$W_{ss} = H_{ss}^{\omega-1}$$

$$C_{ss} + I_{ss} = A_{ss} K_{ss}^{\eta} H_{ss}^{\alpha} V_{ss}^{1-\alpha-\eta} - P_{ss} V_{ss} - \phi r_{ss}(W_{ss} H_{ss} + P_{ss} V_{ss}) - \frac{1}{(1+r_{ss})} B_{ss} + B_{ss}$$

$$I_{ss} = \delta K_{ss} + (K_{ss} - K_{ss})\left(1 + \frac{\iota}{2}\left(\frac{K_{ss} - K_{ss}}{K_{ss}}\right)\right)$$

$$B_{ss}^{*} = \frac{1}{(1+r_{ss})} B_{ss} - \phi(1+r_{ss})(W_{ss} H_{ss} + P_{ss} V_{ss}) + \kappa q_{ss} K_{ss}$$

$$\bar{\varphi} B_{ss}^{*} = (1-\bar{\varphi})\lambda_{ss}$$

$$r_{ss} = r^{*} + \psi_r\left(e^{\overline{B} - B_{ss}} - 1\right)$$

$$\log A_{ss} = \bar{a} + \rho_A \log A_{ss}$$

$$\log P_{ss} = \bar{p} + \rho_P \log P_{ss}$$

$$k_{ss} = K_{ss}$$

We can partially solve some of these directly

$$A_{ss} = \exp \frac{\bar{a}}{1 - \rho_A}$$

$$P_{ss} = \exp \frac{\bar{p}}{1-\rho_P}$$
$$q_{ss} = 1$$

Suppose know r_{ss}

$$\Omega_v = \frac{P_{ss}\left(1+\phi r_{ss}+\left(1-\bar{\beta}\left(1+r_{ss}\right)\right)\phi\left(1+r_{ss}\right)\right)}{(1-\alpha-\eta)}$$

$$\Omega_w = \left(\frac{\phi\left(r_{ss}+\left(1-\bar{\beta}\left(1+r_{ss}\right)\right)\left(1+r_{ss}\right)\right)+1}{\alpha}\right)$$

$$\Omega_k = \frac{\frac{1-\left(1-\bar{\beta}(1+r_{ss})\right)\kappa}{\bar{\beta}}-1+\delta}{\eta}$$

$$H_{ss} = \left(\frac{A_{ss}}{\Omega_k^{\eta}\Omega_w^{\alpha}\Omega_v^{1-\alpha-\eta}}\right)^{\frac{1}{\alpha(\omega-1)}}$$

$$V_{ss} = \frac{\Omega_w}{\Omega_v}H_{ss}^{\omega}$$

$$Y_{ss} = \Omega_w H_{ss}^{\omega}$$

$$K_{ss} = \frac{\Omega_w}{\Omega_k}H_{ss}^{\omega}$$

$$W_{ss} = H_{ss}^{\omega-1}$$

$$I_{ss} = \delta K_{ss}$$

$$k_{ss} = K_{ss}$$

$$B_{ss} = \bar{B} - \log\left(1+\frac{r_{ss}-r^*}{\psi_r}\right)$$

$$C_{ss} = Y_{ss} - \left(1+\phi r_{ss}\right)P_{ss}V_{ss} - \phi r_{ss}W_{ss}H_{ss} - \delta K_{ss} + \left(\frac{r_{ss}}{1+r_{ss}}\right)B_{ss}$$

$$\mu_{ss} = \left(C_{ss} - \frac{H_{ss}^{\omega}}{\omega}\right)^{-\rho}$$

$$\lambda_{ss} = \left(1-\bar{\beta}\left(1+r_{ss}\right)\right)\mu_{ss}$$

$$B_{ss}^* = \frac{1}{(1+r_{ss})}B_{ss} - \phi\left(1+r_{ss}\right)\left(W_{ss}H_{ss}+P_{ss}V_{ss}\right)+\kappa K_{ss}$$

And then r_{ss} solves

$$\bar{\varphi}B_{ss}^* - \left(1-\bar{\varphi}\right)\lambda_{ss} = 0$$

5 Calibration

We start the calibration by taking these parameters from Mendoza (2010).

Parameter	Value
Risk Aversion	$\rho = 2$
Labor Share	$\alpha = 0.592$
Wage Elasticity of Labor Supply	$\omega = 0.306$
Capital Depreciation (8.8% Ann)	$\delta = 0.0228$
Capital Adj Cost	$\iota = 2.75$
Collateral Constraint Parameter	$\kappa = 2$
Working Capital Parameter	$\phi = 0.2579$
Persistence of Processes	$\rho_A = \rho_P = 0.9$
Shock Standard Deviations	$\sigma_r = \sigma_w = \sigma_a = \sigma_p = 0.01$

The transition probabilities depend on steady state variables, and the steady state variables depends on the ergodic probabilities, which in turn depend on the steady state probabilities. To break this circle, it is possible to set the steady state transition probabilites directly. So setting $p_{00,ss} = 0.975$ and $p_{11,ss} = 0.5$ implies

$$\begin{aligned} \gamma_{0,0} &= -3.66356 + \gamma_{0,1} B^*_{ss} \\ \gamma_{1,0} &= -10\lambda_{ss} \end{aligned}$$

Setting $\gamma_{0,1} = \gamma_{1,1} = 10$ produces equal sensitivity to the cutoff values. These transition matrices lead to a vector of ergodic probabilities of $\xi = [0.952381, 0.047619]$.

First consider the steady state of the model in the case when only the non-binding regime occurs. Mendoza targets a discount factor in steady state where the constraint does not bind, so we set $\beta(0) = 0.97959$. The steady state interest rate in this case is $r_{ss} = r^* = \frac{1}{\beta(0)} - 1 = 0.0208352$, and the debt level is $B_{ss} = \bar{B}$. If we target a debt-to-output ratio equalling 1 in this case, then $\bar{B} = -2.06663$. We normalize $a(0) = p(0) = 1$.

Now consider the steady state of hte model in the case when only the binding regime occurs. Need some more guidance here on which moments and targets to hit. Note that, given r^* and $\bar{B}$, the steady state restriction

$$r_{ss} = r^* + \psi_r \left(e^{\overline{B} - B_{ss}} - 1 \right)$$

implies the steady state interest rate is increasing in debt when $\psi_r > 0$. So if the interest rate is higher in the binding regime case, so is the debt level. We set $\psi_r = 0.02$, which is about as high as possible while keeping non-negativity of the interest rate over the range $B \in [-4, 4]$. We assume that TFP decreases and import prices increase when the constraint is binding so $a(1) = -0.0025$, $p(1) = 0.0025$, and the interest rate would tend to increase even without debt dynamics, so $\beta(1) = 0.95959$

The following summarizes the rest of the parameterization

Parameter	Non-Binding	Binding
TFP	$a(0) = 0$	$a(1) = -0.0025$
Import Prices	$p(0) = 0$	$p(1) = 0.0025$
Discount Factor	$\beta(0) = 0.97959$	$\beta(1) = 0.95959$
Interest Rate Intercept	$r^* = 0.0208352$	
Interest Rate Elasticity	$\psi_r = 0.02$	
Neutral Debt Level	$\bar{B} = -2.06663$	
Probability Constant	$\gamma_{0,0} = -2.75088$	$\gamma_{1,0} = 0.91268$
Probability Reaction	$\gamma_{0,1} = 10$	$\gamma_{1,1} = 10$

The following table shows the steady state values for the variables in the case that each regime occurs in isolation. Note these imply some behavior that needs addressing. Beyond the fact that the differences between some variables is large, note that $B^*_{ss} = -1.74818$ in the non-binding only case suggests the constraint should bind in steady state (maybe this is a definitional issue)?

Variable	Non-Binding Only	Binding Only	Difference	Switching
K_{ss}	2.04098	1.42318	-30.27%	1.96686
B_{ss}	-2.06663	-0.18407	-91.09%	-2.25765
P_{ss}	1	1.02532	2.53%	1.00119
A_{ss}	1	0.97531	-2.47%	0.99881
C_{ss}	0.427791	0.385739	-9.83%	0.406643
H_{ss}	0.631022	0.551805	-12.55%	0.623439
V_{ss}	0.0736478	0.0560734	-23.86%	0.0719368
I_{ss}	0.179606	0.125239	-30.27%	0.173084
k_{ss}	2.04098	1.42318	-30.27%	1.96686
r_{ss}	0.0208352	0.00387925	0.017	0.025045
q_{ss}	1	1	0%	1
W_{ss}	0.677388	0.604714	-10.73%	0.670495
μ_{ss}	25.9677	23.8005	-8.35%	30.7956
λ_{ss}	0	2.39452	–	-0.0969333
B^*_{ss}	-1.74818	0	-100%	-1.93867
Y_{ss}	2.06663	1.76751	-14.47%	2.03415

6 Perturbation

For perturbation, we take the stacked equilibrium conditions $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, and differentiate with respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$. In general regime-switching models, the first-order derivative with respect to $\mathbf{x}_{t-1}$ produces a complicated polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0.$$

Often this system needs to be solved via Grobner bases, which finds all possible solutions in order to check them for stability. In our case, all the regime switching parameters show

up in the steady state, and we write $\theta_t = \bar{\theta} + \chi\hat{\theta}(s_t)$ so the steady state can be solved. This is the *Partition Principle* of Foerster, Rubio-Ramirez, Waggoner, and Zha (2015). Given these parameters, the regime switching in $\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0)$ disappears and simplifies to the standard no-switching case that can be solved via a generalized eigenvalue procedure.

After solving the eigenvalue problem, the other systems to solve are

$$\begin{aligned} \mathbb{F}_{\varepsilon}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= 0 \\ \mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= 0 \end{aligned}$$

and second order systems of the form (can apply equality of cross-partials)

$$\mathbb{F}_{\mathbf{i},\mathbf{j}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0,\ \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \varepsilon, \chi\}.$$

Recall the decision rules have the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$$

and so the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_t + H^{(1)}_{s_t} S_t + \frac{1}{2} H^{(2)}_{s_t}(S_t \otimes S_t)$$

$$\mathbf{y}_t \approx \mathbf{y}_t + G^{(1)}_{s_t} S_t + \frac{1}{2} G^{(2)}_{s_t}(S_t \otimes S_t)$$

where $S_t = \begin{bmatrix} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{bmatrix}'$.

7 Estimation of the Endogenous Regime-Switching Model

The advantage of writing the occasionally binding constraint model as an endogenous regime switching model is that we can take the model to the data using likelihood based estimation methods. Our estimation procedure is a Bayesian approach, based on a second-order solution of the model.

7.1 Priors for Financial Crises

A problem that we face with financial crisis models, such as the Mendoza (2010) sudden stop model, is that for any given country the number of periods spent in the constrained state is relatively small, and the number of crisis events is small as well. While this fact is good for the countries in question, it is a problem for the econometrician trying to estimate a model of these events. This contrasts with the existing regime switching literature where you typically switch between long lived regimes. For example, both regimes in hawk/dove monetary policy models typically last a decade or more. In the case of financial crises, with few observations the likelihood function may not be very informative for all of the

parameters in the constrained state. To address this problem we build a prior that incorporates information from a panel of countries that have experienced sudden stops. This dramatically increases the information we have on sudden stop episodes. We then combine this prior with the likelihood function from the endogenous regime switching model to form the posterior for a single country.

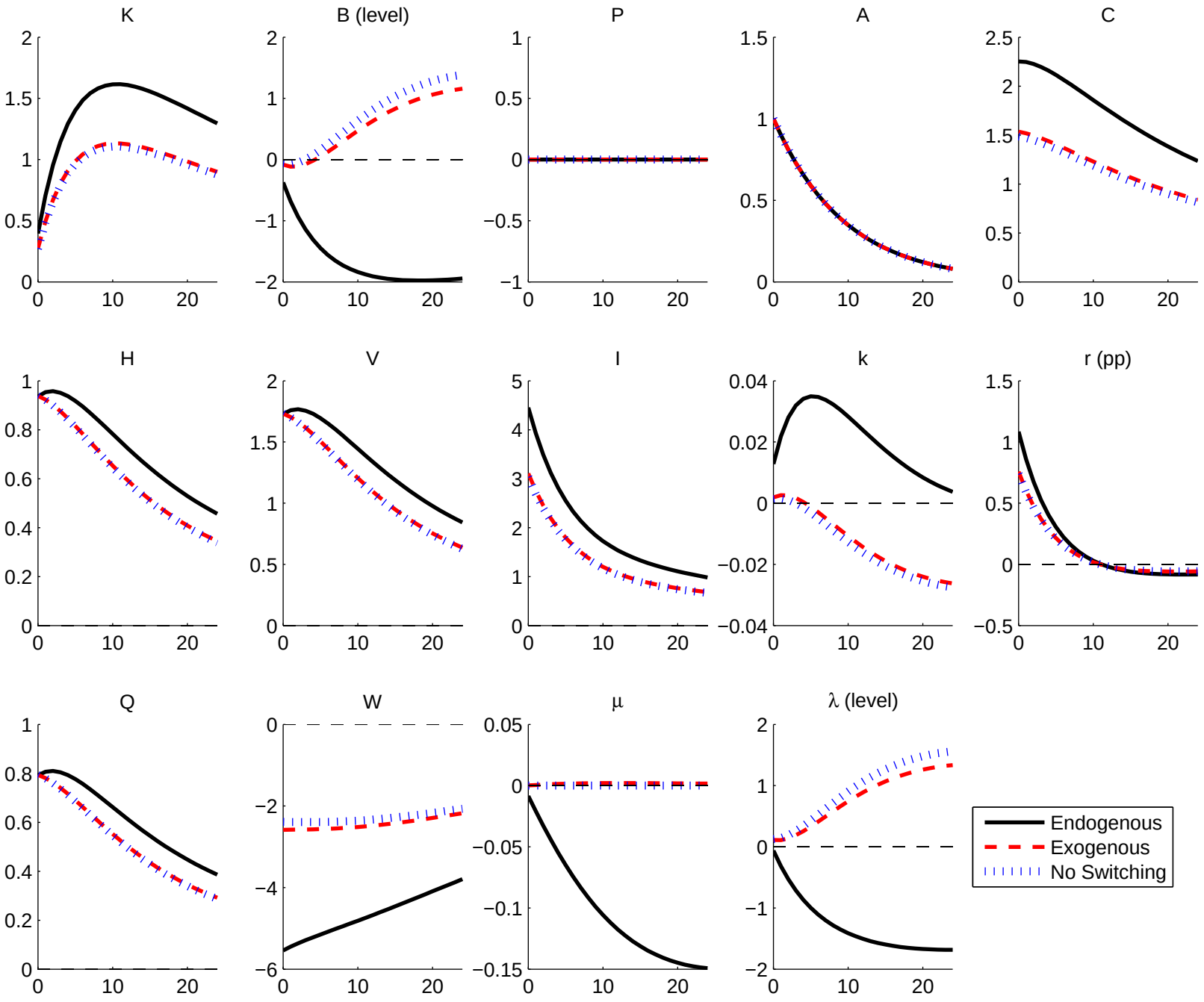
We build our prior with a preliminary estimation of the model based on a GMM limited-information approach. Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2014) provide analytical solutions for the moments of a model that has been computed to the second (or third) order. Given that computing the solution to the model is rapid with our perturbation approach, the availability of analytic moments means that is is computational feasible to quickly build a prior for the model. We choose moments to exploit the information from a large panel of countries to form our prior for the estimation of our target country. These moments include, means, variances, autocorrelations, unconditional probabilities of a crisis, and potentially higher order moments. Importantly, we calculate these moments by state, so we can develop prior information for parameters specific to each state.

The prior is formed by minimizing the distance between the model-implied moments and the empirical counterparts from the data. To build the prior we start by denoting by $\hat{\Psi}$ a vector of empirical moments from the data. Likewise, denote by $\Psi(\zeta)$ the same vector of moments implied by the MSDSGE model, where ζ contains all the structural parameters to be estimated, including state specific parameters. Our estimate of ζ will be the parameters that minimizes the distance between $\hat{\Psi} - \Psi(\zeta)$, weighted with the usual GMM weighting matrix. The properties and specification of this estimator is by now well known. There is also a literature that concerns itself with the finite sample properties of this type of estimate. Here we are unconcerned if the asymptotics hold (though they should since these are simple moments we calculate.) We are primarily interest in bringing in information outside of the likelihood function. Wwe can increase or decrease the variance of this prior, depending on how influential we want these moments to be. For example, we expect to want to use a stronger prior for the crisis period since there are few observations time series observations for the likelihood function for a single country. While the procedure here is to develop our prior, the results themselves may be interesting, as they represent an estimate of the model that can be thought of as a 'typical' emerging market economy.

8 Example Solution

We have solved the model to a second order approximation. In the Figure at the end of this draft we plot the IRF to a TFP shock for the case of the binding regime (we have IRF to all shocks in both regimes, but focus on this one as an example). To illustrate how endogenous switcing differs from exogenous switching we also plot IRF for exogenous swicthing and no-swicthing model. After the positive TFP shock we can see than the endogenous switching model has much stronger responses. We interpret this as a result of the fact that agents know that the likelihood of a bidning constraint has just decreased as income increases. This leads to an increased willingness to take on debt and a greater increase in capital accumulation. Note that with exogenous switching debt decreases, while it in increases with endogneous switching. We interpret this as indicating that the endogenous swicthing

model was very different behvaioral implications for exogenous swicthing.


K
B (level)
P
A
C
H
V
I
k
r (pp)
Q
W
μ
λ (level)
Endogenous
Exogenous
No Switching