Parameter Bias in an Estimated DSGE Model: Does Nonlinearity Matter?

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Abstract

How can parameter estimates be biased in a dynamic stochastic general equilibrium (DSGE) model that omits nonlinearity in the economy? To answer this question, we simulate data from a fully nonlinear New Keynesian model with the zero lower bound (ZLB) constraint and estimate a linearized version of the model. Monte Carlo experiments show that significant biases are detected in the estimates of monetary policy parameters and the steady-state inflation and real interest rates. These biases arise mainly from neglecting the ZLB constraint rather than linearizing the equilibrium conditions. The estimated impulse responses to a monetary policy shock are significantly different from the true responses. However, regarding the responses to shocks to the discount factor and productivity, the biased parameters contribute to making the differences small.

Keywords: Nonlinearity, Zero lower bound, DSGE model, Parameter bias, Bayesian estimation

JEL Classification: C32, E30, E52

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1 Introduction

Following the development of Bayesian estimation and evaluation techniques, many economists have estimated dynamic stochastic general equilibrium (DSGE) models using macroeconomic time series. In particular, estimated New Keynesian models, which feature nominal rigidities and monetary policy rules, have been extensively used by policy institutions such as central banks. Most of the estimated DSGE models are linearized around the steady state because a linear state-space representation along with the assumption of normality of exogenous shocks enables us to efficiently evaluate likelihood using the Kalman filter. However, Fernández-Villaverde and Rubio-Ramírez (2005) and Fernández-Villaverde, Rubio-Ramírez, and Santos (2006) demonstrate that parameter estimates, the level of likelihood, and the moments based on a linearized model can be significantly different from those based on its nonlinear counterpart. Moreover, in the context of New Keynesian models, Basu and Bundick (2012), Braun, Körber, and Waki (2012), Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015), Gavin, Keen, Richter, and Throckmorton (2015), Gust, López-Salido, and Smith (2012), Nakata (2013a, 2013b), and Ngo (2014) emphasize the importance of considering nonlinearity in assessing the quantitative implications of the models when the zero lower bound (ZLB) constraint on the nominal interest rate is taken into account.

Despite the existence of these studies, attempts to estimated DSGE models in a fully nonlinear and stochastic setting are still limited. A few exceptions are Fernández-Villaverde and Rubio-Ramírez (2005), who estimate a neoclassical growth model in a nonlinear form,[1] and Gust, López-Salido, and Smith (2012), who estimate a fully nonlinear New Keynesian DSGE model in which the ZLB constraint is occasionally binding. The reason for scarcity of such attempts is probably due to high computational costs for the estimation of nonlinear models. To evaluate the likelihood function in a nonlinear setting, one need to rely on a nonlinear solution method and a particle filter, both of which require iterative procedures, and their computational expense grows rapidly with an increase in the dimensionality of problems.

In this paper, we examine how and to what extent the parameter estimates can be biased in an estimated DSGE model when nonlinearity existing in the economy is omitted in the estimation process. If significant biases were detected, it would urge researchers to be equipped with nonlinear estimation techniques at any computational costs. Otherwise, it would at least assure practitioners that the common practice of estimating linearized models could lead to reliable estimates. To this

[1] Fernández-Villaverde and Rubio-Ramírez (2007) estimate a real business cycle model with investment-specific technological change, preference shocks, and stochastic volatility, using a second-order perturbation method.

end, we solve a fully nonlinear New Keynesian model that incorporates the ZLB constraint using a projection method and simulate artificial time series from the nonlinear model. The parameters calibrated in this data generating process (DGP) are regarded as true values. Then, using the simulated data, a Monte Carlo experiment is conducted, in which a log-linearized version of the model is estimated without imposing the ZLB. In the estimation, we employ Bayesian methods, which are now extensively used to estimate DSGE models. We set the prior means to the true parameter values and assess the parameter biases from neglecting the nonlinearity by comparing the posterior means and credible intervals with the true values. Moreover, in order to investigate to what extent the dynamic properties of the estimated model can differ from those of the true nonlinear model, we compare the estimated impulse response functions with the true ones.

The analysis in this paper is an extension of Hirose and Inoue (2015), who conduct a similar experiment using the data simulated from a New Keynesian model in which nonlinearity is considered only in a monetary policy rule to incorporate the ZLB constraint but the remaining equilibrium conditions are linearized.[2] While Hirose and Inoue (2015) point to parameter bias only resulting from omitting the ZLB, the present paper is able to investigate the bias arising from missing nonlinearities regarding both the ZLB and the other equilibrium conditions.

Our main results are summarized as follows. In the baseline experiment, while the estimates of the structural parameters related to preferences and nominal rigidities are not biased, significant biases are detected in the estimates of monetary policy parameters and the steady-state inflation and real interest rates. We show that these biases arise for the most part from missing the ZLB constraint rather than linearizing the equilibrium conditions. With the true parameter values, impulse response functions of the linearized model are substantially different from those of the nonlinear model. Because of the biased parameters, the differences between the estimated and true responses become larger regarding a monetary policy shock. We find, however, that the biased parameters can make the differences small for shocks to the discount factor and productivity. We also demonstrate that the biases in the parameter estimates become larger if the true parameter values in the DGP are altered so that the frequency of binding at the ZLB or the average duration of ZLB spells increases. Nevertheless, the increased biases in the parameter estimates do not necessarily amount to larger differences between the estimated and true impulse responses.

[2]Hirose and Inoue (2015) simulate the model using the solution algorithm of Erceg and Lindé (2014) and Bodenstein, Guerrieri, and Gust (2013). In their algorithm, if the model-implied nominal interest rate falls below zero, a sequence of contractionary monetary policy shocks is added in both the current and anticipated periods so that the contemporaneous and expected interest rates at the lower bound are zero.

A technical contribution of this paper is that the DGP used in our experiment has, to the best of our knowledge, the richest dynamic structure of all the New Keynesian models with the ZLB constraint solved in a fully nonlinear and stochastic setting.[3] To make our test economy realistic enough, we incorporate habit persistence in consumption preferences, price indexation of intermediate-good firms, and monetary policy smoothing into a standard New Keynesian model. In solving the model, we apply the efficient algorithm for the Smolyak-based projection methods using the unidimensional disjoint sets of grid points developed by Judd, Maliar, Maliar, and Valero (2014), to alleviate a computational burden because of the increased number of state variables.

The remainder of the paper proceeds as follows. Section 2 describes the model used in our analysis and a nonlinear solution method. Section 3 explains our Monte Carlo experiments and presents our results. Section 4 conducts a robustness analysis with alternative parameter settings. Section 5 is the conclusion.

2 The Model and Solution Method

This section describes the DGP used in our analysis. The DGP is characterized by a fully nonlinear New Keynesian DSGE model with the ZLB constraint on the nominal interest rate. The nonlinear model is solved using a Smolyak algorithm in the context of a projection method.

2.1 The Model

In the model economy, there are households, perfectly competitive final-good firms, monopolistically competitive intermediate-good firms that face price stickiness, and a central bank. For empirical validity, the model features habit persistence in consumption preferences, price indexation to recent past and steady-state inflation, and monetary policy smoothing.

Each household $h \in [0,1]$ consumes final goods $C_{h,t}$, supplies labor $L_{h,t}$, and purchases one-period riskless bonds $B_{h,t}$ so as to maximize the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=1}^{t} d_k \right)^{-1} \left[\frac{(C_{h,t} - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - L_{h,t} \right]$$

[3] While most of the existing studies that solve nonlinear New Keynesian models with the ZLB consider the models with the simplest lag structure and a limited number of shocks, Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) and Maliar and Maliar (2015) solve richer dynamic models with monetary policy smoothing and a variety of structural shocks.

subject to the budget constraint

$$P_t C_{h,t} + B_{h,t} = P_t W_t L_{h,t} + R_{t-1} B_{h,t-1} + T_{h,t},$$

where $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\gamma \in [0,1]$ is the degree of external habit persistence in consumption preferences, P_t is the price of final goods, W_t is the real wage, R_t is the gross nominal interest rate, and $T_{h,t}$ is the sum of a lump-sum public transfer and profits received from firms. Following Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011), a shock to the discount factor d_t affects the weight of the utility in period $t+1$ relative to the one in period t. The log of the discount factor shock follows an AR(1) process:

$$\log d_t = \rho_d \log d_{t-1} + \varepsilon_{d,t}, \tag{1}$$

where $\rho_d \in [0,1)$ is an autoregressive coefficient and $\varepsilon_{d,t}$ is a normally distributed innovation with mean zero and standard deviation σ_d. The first-order conditions for optimal decisions on consumption, labor supply, and bond-holding are identical among households and therefore become

$$\Lambda_t = (C_t - \gamma C_{t-1})^{-\sigma}, \tag{2}$$

$$W_t = \frac{1}{\Lambda_t}, \tag{3}$$

$$\Lambda_t = \frac{\beta}{d_t} \mathbb{E}_t \Lambda_{t+1} \frac{R_t^n}{\Pi_{t+1}}, \tag{4}$$

where Λ_t is the marginal utility of consumption and $\Pi_t = P_t / P_{t-1}$ denotes gross inflation.

The representative final-good firm produces output Y_t under perfect competition by choosing a combination of intermediate inputs $\{Y_{f,t}\}$ so as to maximize profit $P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} df$ subject to a CES production technology $Y_t = \left(\int_0^1 Y_{f,t}^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}$, where $P_{f,t}$ is the price of intermediate good f and $\theta > 1$ denotes the elasticity of substitution. The first-order condition for profit maximization yields the final-good firm's demand for intermediate good f, $Y_{f,t} = (P_{f,t}/P_t)^{-\theta} Y_t$, while perfect competition in the final-good market leads to $P_t = \left(\int_0^1 P_{f,t}^{1-\theta} df \right)^{\frac{1}{1-\theta}}$.

Each intermediate-good firm f produces one kind of differentiated goods $Y_{f,t}$ under monopolistic competition by choosing a cost-minimizing labor input L_t given the real wage W_t subject to the production function

$$Y_{f,t} = A_t L_{f,t},$$

where A_t represents total factor productivity. The log of the productivity level follows an AR(1) process:

$$\log A_t = \rho_a \log A_{t-1} + \varepsilon_{a,t}, \tag{5}$$

where $\rho_a \in [0,1)$ is an autoregressive coefficient and $\varepsilon_{a,t}$ is a normally distributed innovation with mean zero and standard deviation σ_a. The first-order condition for cost minimization shows that real marginal cost MC_t is identical across intermediate-good firms and is given by

$$MC_t = \frac{W_t}{A_t}. \tag{6}$$

In the face of the final-good firm's demand and marginal cost, the intermediate-good firms set prices of their products on a staggered basis as in Calvo (1983). In each period, a fraction $1-\xi \in (0,1)$ of intermediate-good firms re-optimize their prices while the remaining fraction ξ indexes prices to a weighted average of past inflation Π_{t-1} and steady-state inflation Π. The firms that re-optimize their prices in the current period then maximize expected profit

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota} \Pi^{1-\iota} \right) - MC_{t+j} \right] Y_{f,t+j}$$

subject to the final-good firm's demand

$$Y_{f,t+j} = \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota} \Pi^{1-\iota} \right) \right]^{-\theta} Y_{t+j},$$

where $\iota \in [0,1)$ denotes the weight of price indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized price P_t^o is given by

$$\frac{P_t^o}{P_t} = \frac{\theta^p}{\theta^p - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\left(\prod_{k=1}^{j} \left[\left(\frac{\Pi_{t+k-1}}{\Pi} \right)^{\iota} \frac{\Pi}{\Pi_{t+k}} \right] \right)^{-\theta} MC_{t+j} Y_{t+j} \right]}{\mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\left(\prod_{k=1}^{j} \left[\left(\frac{\Pi_{t+k-1}}{\Pi} \right)^{\iota} \frac{\Pi}{\Pi_{t+k}} \right] \right)^{1-\theta} Y_{t+j} \right]}$$

Let $S_t = \theta^p \mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\left(\prod_{k=1}^{j} \left[\left(\frac{\Pi_{t+k-1}}{\Pi} \right)^{\iota} \frac{\Pi}{\Pi_{t+k}} \right] \right)^{-\theta} MC_{t+j} Y_{t+j} \right]$ and $F_t = (\theta^p - 1) \mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\left(\prod_{k=1}^{j} \left[\left(\frac{\Pi_{t+k-1}}{\Pi} \right)^{\iota} \frac{\Pi}{\Pi_{t+k}} \right] \right)^{1-\theta} Y_{t+j} \right]$, so that $P_t^o / P_t = S_t / F_t$. Then, S_t and F_t can be written in the following recursive forms:

$$S_t = \theta MC_t Y_t + \xi \frac{\beta}{d_t} \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\left(\frac{\Pi_{t+1}}{\Pi} \right) \left(\frac{\Pi_t}{\Pi} \right)^{-\iota} \right]^{\theta} S_{t+1}, \tag{7}$$

$$F_t = (\theta - 1) Y_t + \xi \frac{\beta}{d_t} \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\left(\frac{\Pi_{t+1}}{\Pi} \right) \left(\frac{\Pi_t}{\Pi} \right)^{-\iota} \right]^{\theta-1} F_{t+1}. \tag{8}$$

Moreover, the final-good's price $P_t = \left(\int_0^1 P_{f,t}^{1-\theta} df \right)^{\frac{1}{1-\theta}}$ can be rewritten as

$$1 = (1-\xi) \left(\frac{S_t}{F_t} \right)^{1-\theta} + \xi \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1} \left(\frac{\Pi_{t-1}}{\Pi} \right)^{-\iota(\theta-1)}. \tag{9}$$

The final-good market clearing condition is

$$Y_t = C_t, \tag{10}$$

while the labor market clearing condition leads to

$$\frac{Y_t \Delta_t}{A_t} = \int_0^1 L_{f,t} df = L_t, \tag{11}$$

where $\Delta_t = \int_0^1 (P_{f,t}/P_t)^{-\theta} df$ represents price dispersion across the intermediate-good firms. Under the present pricing rule, the price dispersion evolve according to

$$\Delta_t = (1-\xi)\left(\frac{S_t}{F_t}\right)^{-\theta} + \xi\left(\frac{\Pi_t}{\Pi}\right)^{\theta}\left(\frac{\Pi_{t-1}}{\Pi}\right)^{-\iota\theta}\Delta_{t-1}. \tag{12}$$

A monetary policy rule is specified as

$$R_t = \max\left[R_t^*, 1\right], \tag{13}$$

where

$$R_t^* = \left(R_{t-1}^*\right)^{\phi_r}\left[R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\phi_r}\exp(\varepsilon_{r,t}). \tag{14}$$

R_t^* denotes the hypothetical interest rate that the central bank would set according to a Taylor (1993) type monetary policy rule where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, R is the steady-state gross nominal interest rate, and $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are the degree of interest rate policy response to inflation and output respectively. $\varepsilon_{r,t}$ is a monetary policy shock, which is normally distributed with mean zero and standard deviation σ_r. $R_t = R_t^*$ if $R_t^* \geq 1$ or the ZLB is not imposed. In the case of $R_t^* < 1$, $R_t = 1$ because the gross nominal interest rate cannot be lower than one.

An equilibrium is given by the sequence $\{Y_t, C_t, W_t, L_t, \Lambda_t, MC_t, S_t, F_t, \Pi_t, \Delta_t, R_t, R_t^*, d_t, A_t\}_{t=0}^{\infty}$ satisfying the equilibrium conditions (1)–(14).

2.2 Parameter Setting

The model is parameterized according to standard choices in the literature in order to make our test economy as representative as possible.

The inverse of intertemporal elasticity of substitution σ is set to be 1.5. The habit persistence and price indexation parameters (γ, ι) are both set at 0.5. These three values follow from the prior means used in Smets and Wouters (2007). As is often calibrated in the New Keynesian literature,

we set the Calvo parameter $\xi = 0.75$, which implies that the average duration of prices is four quarters.

The monetary policy parameters ($\phi_\pi = 2.0$, $\phi_y = 0.5$) that represent the degrees of interest rate responses to inflation and output are larger than the coefficients in the original Taylor (1993) rule, reflecting the recent literature in which the estimation of New Keynesian DSGE models leads to interest rate responses to inflation larger than 1.5.[4] Moreover, the larger degree of the policy responses are required to ensure the convergence of the nonlinear solution for broader sets of model parameters. The policy-smoothing parameter ϕ_r is set to be 0.5.

The steady-state gross inflation rate Π is set at 1.005, so that the central bank's target inflation rate is two percent annually. The inverse of the subjective discount factor β, which is equivalent to the steady-state gross real interest rate, is set at 1.0025, following the prior mean in Smets and Wouters (2007). The resulting steady-state nominal interest rate is annually 3 percent. This is almost the same as the average of the three-month Treasury bill rate in the post-1990 US sample.

The AR(1) coefficients for the shocks to the discount factor and productivity (ρ_d, ρ_a) are both set at 0.7. The standard deviations of these shocks (σ_d, σ_a) are both 0.003 while a relatively small value is assigned to that of the monetary policy shock ($\sigma_r = 0.001$). These parameter values are determined so that the nominal interest rate is bounded at zero for substantial periods in a simulated sample.

2.3 Solution Method

Our model has four endogenous state variables (lagged output Y_{t-1}, the inflation rate Π_{t-1}, the notional interest rate R^*_{t-1}, price dispersion Δ_{t-1}) and three exogenous shocks (the discount factor shock d_t, the productivity shock A_t, and the monetary policy shock ε_{rt}). We define the vector of endogenous and exogenous state variables:

$$\mathbb{S}_{t-1} = (Y_{t-1}, \Pi_{t-1}, R^*_{t-1}, \Delta_{t-1}),$$
$$\tau_t = (d_t, A_t, \varepsilon_{rt}).$$

Also, we define $\mathbb{S}_t = h(\mathbb{S}_{t-1}, \tau_t)$ as the policy functions satisfying the equilibrium conditions.

The nonlinear model is solved using a Smolyak algorithm in the context of a projection method. Below is a brief description of the computational procedure, and the details are explained in the

[4]See, for instance, Lubik and Schorfheide (2004), Smets and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010).

Appendix. To solve for the policy functions on each grid point $(\mathbb{S}_{t-1}, \tau_t)$, first we approximate each AR(1) process of the exogenous shocks τ_t by Markov chain. For each individual shock process, we use the method proposed by Rouwenhorst (1995) to determine the grid points and transition matrix for Markov chain. We used 7 discrete values for the discount factor and productivity shocks and 5 values for the monetary policy shock so that the total number of grid points is $N_\tau = 245$.

Given the exogenous shock processes, we obtain the values of the policy functions $h(\mathbb{S}_{t-1}, \tau_t)$ by a Smolyak algorithm as in Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015).[5] While Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) use a standard Smolyak method laid out in Malin, Krueger, and Kubler (2011), we apply a more efficient algorithm developed by Judd, Maliar, Maliar, and Valero (2014). Specifically, we compute the unidimensional disjoint sets of grid points of the endogenous variables $\mathbb{S}_{t-1}$ and projection functions onto the grid points. Then we evaluate each function when we calculate the future values of the function, which may be off the grid points. The grid points by the Smolyak algorithm are *sparse*; therefore the algorithm is more likely free from the curse of dimensionality.[6] We use the level of approximation $\mu = 2$ so that the total number of grid points is $N = 41$. This number is much smaller compared to the one obtained by a product of the maximum number of grid points in each dimension, $\tilde{N} = 5^4 = 625$.

As in Gust, López-Salido, and Smith (2012), we do not approximate $h(\mathbb{S}_t, \tau_{t+1})$ directly, as the ZLB in the Taylor rule generates kinks in the policy functions. Instead, we introduce the indicator function of the interest-rate regime and changes of variables so that the policy functions are smoother and easier to be obtained. In particular, we do not rely on any numerical optimization routines, which makes computation more reliable in finding an equilibrium.

3 Monte Carlo Experiments

In this section, we conduct Monte Carlo experiments to examine how the parameter estimates can be biased if the nonlinearity is omitted in the estimation process. Then, we analyze the effects of these parameter biases on the model properties by comparing the estimated impulse responses with those based on the true model.

[5] Gust, López-Salido, and Smith (2012) use a standard Chebyshev algorithm, as they have three endogenous state variables by using the Rotemberg pricing instead of the Calvo pricing for the price stickiness.

[6] We also find that the Smolyak algorithm is more accurate in terms of the Euler equation errors compared to the Chebyshev algorithm.

3.1 Design

Our Monte Carlo experiments proceed as follows. First, we generate an artificial time series of output (deviation from the steady state), inflation, and the nominal interest rate from the DGP as described in the previous section, given the parameter values presented in Section 2.2. Thus, the simulated series can be regarded as those that reflect the nonlinearity of the model economy including the ZLB constraint on the nominal interest rate. Each simulated sample consists of 200 observations,[7] corresponding to quarterly observations over a period of 50 years. This sample size is chosen because it is comparable to the size with which many researchers estimate DSGE models in practice.

Next, using the simulated data, we estimate a linearized version of the model. Log-linearizing the equilibrium conditions (1)–(14) around the steady state and rearranging the resulting equations yields

$$\tilde{Y}_t = \frac{1}{1+\gamma}\mathbb{E}_t\tilde{Y}_{t+1} + \frac{\gamma}{1+\gamma}\tilde{Y}_{t-1} - \frac{1-\gamma}{\sigma\,(1+\gamma)}\left(\tilde{R}_t - \mathbb{E}_t\tilde{\Pi}_{t+1} - \tilde{d}_t\right), \tag{15}$$

$$\tilde{\Pi}_t = \frac{\beta}{1+\beta\iota}\mathbb{E}_t\tilde{\Pi}_{t+1} + \frac{\iota}{1+\beta\iota}\tilde{\Pi}_{t-1} + \frac{(1-\xi)\,(1-\xi\beta)}{\xi\,(1+\beta\iota)}\left[\frac{\sigma}{1-\gamma}\tilde{Y}_t - \frac{\sigma\gamma}{1-\gamma}\tilde{Y}_{t-1} - \tilde{A}_t\right], \tag{16}$$

$$\tilde{R}_t = \phi_r\tilde{R}_{t-1} + (1-\phi_r)\left(\phi_\pi\tilde{\Pi}_t + \phi_y\tilde{Y}_t\right) + \varepsilon_{r,t}, \tag{17}$$

where the variables with $\tilde{}$ represent percentage deviations from their steady-state values. From the simulated data, the percentage deviations of output from the steady state $100\log(Y_t/Y)$, the inflation rate $100\log\Pi_t$, and the nominal interest rate $100\log R_t$ are assumed to be observable. Then, the observation equations are

$$\begin{bmatrix} 100\log(Y_t/Y) \\ 100\log\Pi_t \\ 100\log R_t \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{\pi} \\ r\bar{r} + \bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{Y}_t \\ \tilde{\Pi}_t \\ \tilde{R}_t \end{bmatrix}, \tag{18}$$

where $\bar{\pi} = 100\log\pi$ and $r\bar{r} = 100\log(1/\beta)$ are respectively the steady-state inflation rate and the real interest rate in percentage terms. In the subsequent analysis, $\bar{\pi}$ and $r\bar{r}$ are considered as parameters to be estimated, instead of Π and β.

In the estimation, we employ Bayesian methods. The prior distributions of parameters are presented in the second to fourth columns of Table 1. Each prior mean is set to the corresponding true parameter value used in the DGP. Gamma distributions with standard deviation 0.2 are

[7] The model is simulated for 250 periods, and the first 50 observations are eliminated.

chosen for the inverse of intertemporal elasticity of substitution σ, the monetary policy reaction parameters (ϕ_π, ϕ_y) and the steady-state inflation and real interest rates ($\bar{\pi}$, $\bar{rr}$). Beta distributions with standard deviation 0.2 are used for habit persistence γ, the Calvo parameter ξ, price indexation ι, the policy-smoothing parameter ϕ_r, and the autoregressive coefficients (ρ_d, ρ_a). For the standard deviations of the shocks (σ_d, σ_a, σ_r), inverse-gamma distributions with standard deviation 2 are assigned. To obtain the posterior distributions, we generate 200,000 draws using the random-walk Metropolis–Hastings algorithm and discard the first 25 percent of these draws.[8]

These steps are replicated for 200 times, and the posterior means and Bayesian credible intervals for the parameters are averaged over the replications. Then, we can evaluate the parameter bias arising from missing the nonlinearity by examining how the resulting posterior means and credible intervals differ from the true parameter values.

According to the simulated sample of 40,000 periods (200 Monte Carlo replications with a sample size of 200 observations) using the baseline parameter setting, the economy is at the ZLB for 9.7 percent of quarters, and the average duration of ZLB spells is 2.8 quarters. These statistics indicate that our model economy is more frequently constrained by the ZLB than the simulation results in the previous studies that employ nonlinear New Keynesian models similar to ours. Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012) simulate a model calibrated for the US economy and show that the economy spends 5.5 percent of quarters at the ZLB and that the average duration at the ZLB is 2.1 quarters. Gust, López-Salido, and Smith (2012) estimate a model in a fully nonlinear setting, and the simulation of their estimated model demonstrates that the economy is at the ZLB for 3.1 percent of quarters and that the average duration of the ZLB spells is about 3 quarters.

3.2 Results

3.2.1 Parameter estimates

The fifth to sixth columns in Table 1 (Baseline) present the averages of the posterior means and the 90 percent credible intervals for the estimated parameters in the baseline experiment.

The estimates of the structural parameters (σ, γ, ξ, ι), which are related to preferences and nominal rigidities, are not biased because the posterior mean estimates are very close to the true parameter values and the 90 percent credible intervals contain the true values. The policy-smoothing

[8]The scale factor for the jumping distribution in the Metropolis–Hastings algorithm is adjusted so that the acceptance rate of candidate draws is approximately 25 percent.

parameter ϕ_r, the autoregressive coefficients (ρ_d, ρ_a), and the standard deviations of the shocks (σ_d, σ_a, σ_r) are also not much affected by omitting the nonlinearity in the estimation.

However, we detect significant biases in the estimates of parameters that characterize the degree of a monetary policy response to inflation ϕ_π and the steady-state inflation and real interest rates ($\bar{\pi}$, $\bar{r}\bar{r}$), in the sense that the mean estimates for these parameters differ from the corresponding true values and that the credible intervals do not include the true values. The estimated monetary policy response to output ϕ_y is also substantially biased although its credible interval contains the true value.

For the biased estimates of the monetary policy reaction parameters (ϕ_π, ϕ_y), the same explanation as in Hirose and Inoue (2015) applies. In the DGP, the monetary policy reaction function has a kink where the ZLB constraint becomes binding, that is, the reaction function has positive slopes with respect to inflation and output if the unconstrained nominal interest rate is positive, but the slopes become flat if it turns negative. However, when such a kink is omitted in the estimation, as is the case in our experiment, the estimated slopes are approximated to lie between the positive and flat slopes, and thus these monetary policy parameters can be underestimated.

The downward biases detected in the estimates of the steady-state inflation and real interest rates ($\bar{\pi}$, $\bar{r}\bar{r}$) are in stark contrast to the result in Hirose and Inoue (2015), who find upward bias in the steady-state real interest rate.[9] In the present analysis, the DGP is characterized by the fully nonlinear model. In such an environment, the stochastic steady state can be substantially different from the deterministic steady state. In particular, once the ZLB constraint is taken into account, the presence of uncertainty can reduce inflation and output by a substantial amount, as emphasized by Nakata (2013a), and hence their levels in the stochastic steady state become lower than those in the deterministic one. In reality, the mean of the inflation rate simulated from the DGP, which should be equivalent to the stochastic steady-state value, is 0.39 percent while the corresponding deterministic value is calibrated at 0.5. Because the latter is treated as the true

[9]Hirose and Inoue (2015) explain the reason for the upward bias in the steady-state real interest rate as follows. The presence of the ZLB forces the nominal interest rate to be equal or greater than zero. However, if the model estimation fails to consider the ZLB, the nominal interest rate can be negative, and the mean of the model-implied nominal interest rate declines. Then, the estimate of the steady-state nominal interest rate must rise to adjust the difference between the mean of the model-implied nominal interest rate and that of the corresponding series simulated from the DGP with the ZLB, given the model is linearized around the steady state. In their experiment, such an adjustment has mostly emerged as a change in the estimate of the steady-state real interest rate, rather than the steady-state inflation rate.

value, the discrepancy between the two steady states is adjusted by the lower estimate of $\bar{\pi}$ in our experiment. The nominal interest rate in the stochastic steady state can be either larger or smaller than its deterministic steady-state value. It will be larger if the direct effect of the ZLB constraint is strong. It will be smaller if the feed-back effect of reduced inflation and output on the nominal interest rate through the Taylor-type monetary policy rule is dominant. In the present experiment, the mean of the nominal interest rate simulated from the DGP is 0.57 percent, which is much lower than the deterministic steady-state value of 0.75 percent. Consequently, the steady-state real interest rate $\bar{r}\bar{r}$ is underestimated while the downward bias in the estimate of $\bar{\pi}$ partially contribute to filling the gap between the two steady states.

These sources of the parameter biases could potentially affect the estimates of other structural parameters that characterize the dynamic IS equation (15) and the New Keynesian Phillips curve (16) because we employ the system-based estimation approach. However, our experiment indicates that such an effect is quite limited.

3.2.2 Impulse responses

From a practical perspective, it is important to investigate to what extent the dynamic properties of the estimated model that omits nonlinearity can differ from those of the true nonlinear model. To examine this issue, we compare the impulse response functions estimated without the nonlinearity with those computed using the true model. Figure 1 depicts the impulse responses of output, inflation, and the nominal interest rate to one standard deviation shocks to the (1) discount factor, (2) productivity, and (3) monetary policy. The responses are expressed in terms of the percentage deviation from the steady state in order to exclude the effects of the biased estimates of the steady-state parameters and to focus on the changes in the propagation of shocks. In each panel, the solid thick (red) line represents the true response, and the solid thin (blue) line and dashed (blue) lines are respectively the posterior mean and 90 percent credible interval of the estimated response given the biased parameters. For comparison, each panel contains the response of the linear model with the true parameters, shown by (black) dots.

Sizeable differences are found between the responses of the linear model with the true parameters and those of the true model. These differences arise from the fact that policy functions of the nonlinear model with the ZLB constraint are substantially different from those of the linear model without the constraint. In contrast, the mean estimates of the responses become closer to the true responses regarding the shocks to the discount factor and productivity, implying that the biased

parameters contribute to partially replicating the true responses to these shocks. For the responses to the monetary policy shock, however, the estimated responses deviate more from the true ones than the responses of the linear model with the true parameters.

In terms of 90 percent credible intervals, the estimated responses of the interest rate to the productivity shock and those of output and inflation to the monetary policy shock are different from the true responses in that the true responses are outside the credible intervals. Therefore, the dynamic properties of the estimated model missing the nonlinearity are no longer the same as those of the true model.

3.2.3 Which nonlinearity matters?

The results so far suggest that omitting nonlinearity in estimation leads to substantial biases in parameter estimates and impulse response functions. Which missing nonlinearity causes such biases, the ZLB constraint or the nonlinear equilibrium conditions? To answer this question, we conduct two additional Monte Carlo experiments. First, the DGP in the baseline experiment is replaced with the one where the ZLB constraint is not imposed but the other nonlinearities remain unchanged. Second, we consider the DGP where the ZLB constraint is imposed but all the equilibrium conditions are linearized. The latter experiment is comparable to the analysis in Hirose and Inoue (2015) in the sense that the DGP is quasi-linear, although the solution method is different from each other.

The results in the first additional experiment are presented in the seventh to eighth columns of Table 1 (No ZLB in DGP). The parameter estimates are unbiased in that all the mean estimates are almost the same as the true values. This finding implies that omitting nonlinearity in estimation would not affect parameter estimates if the ZLB were not an issue. To understand this result, Figure 2 shows the impulse responses in the present experiment. In the same way as the baseline experiment, each panel compares the true response, the posterior mean and 90 percent credible interval for the estimated response, and the response of the linear model with true parameters. Throughout the figure, the true responses exhibit almost the same paths as those of the linear model with the true parameters, indicating that the policy functions of the linear model can well approximate those of the nonlinear model in the absence of the ZLB.

The biased parameters in the baseline experiment are affected by ignoring both the ZLB constraint and the nonlinear equilibrium conditions. The second additional experiment is to quantify the effect of omitting solely the ZLB constraint on the parameter estimates. The results are shown in the last two columns of Table 1 (Quasi-linear DGP). As in the baseline experiment, non-negligible

biases are found in the estimates of the monetary policy reaction parameters (ϕ_π, ϕ_y) and the steady-state inflation and real interest rates ($\bar{\pi}$, $\bar{rr}$), although the biases are smaller than those in the baseline.

Therefore, from these two experiments, the biases detected in the baseline experiment are mainly due to ignoring the ZLB constraint rather than linearizing the equilibrium conditions.

4 Robustness Analysis

According to the DGP in our baseline experiment, the nominal interest rate is bounded at zero for 9.7 percent of quarters, and the average duration of ZLB spells is 2.8 quarters. As the probability of hitting the ZLB increases in the simulated sample, parameter biases due to excluding the nonlinearity in the estimation might become large. To analyze such a possibility, we change the parameters assigned to the DGP so that the model economy is more frequently constrained by the ZLB. We then conduct the same Monte Carlo exercises as the baseline experiment using the data simulated by the DGP with the alternative parameter settings. Specifically, we consider two cases. One is where the true steady-state real interest rate $\bar{rr}$ falls by 0.03 (0.12 percent annually) so that $\bar{rr} = 0.22$. In this case, the probability of being stuck at the ZLB is 14.8 percent of the simulated sample. The other is where the standard deviation of the discount factor shock $100\sigma_d$ increases from 0.3 to 0.33. In the latter case, the number of zero interest rate periods increases because large negative shocks are more likely to depress the economy and to lower the nominal interest rate, and the model economy is at the ZLB for 14.2 percent of quarters. In both cases, the average duration of ZLB spells is 3.1 quarters, which is longer than that in the baseline experiment.[10]

Table 2 presents the averages of the posterior means and the 90 percent credible intervals for the parameters in the two alternative experiment: the case of low $\bar{rr}$ and the case of large σ_d. In both cases, the biases in the estimates of the monetary policy reaction parameters (ϕ_π, ϕ_y) and the steady-state inflation and real interest rates ($\bar{\pi}$, $\bar{rr}$), which are found in the baseline experiment (Table 1), become large. In particular, the credible interval for ϕ_y no longer includes the true value. Moreover, a significant bias is detected in the estimates of the standard deviation of the monetary policy shock σ_r. The reason for the upward bias in σ_r is straightforward. In the simulated data, the nominal interest rate never falls below zero because the DGP takes account of the ZLB constraint. In the estimation process, however, the model-implied nominal interest rate can be negative, and

[10]The solutions did not converge if we parameterized the DGP so that the probability of being at the ZLB or the duration of ZLB spells increases more.

such a discrepancy must be captured by the monetary policy shock ε_t^r, which results in the increase in σ_r. The estimates of the other parameters are almost unbiased, as in the baseline experiment, and hence are robust to the increased probabilities and durations of the ZLB in the DGP.

Figure 3 and 4 show the impulse responses of output, inflation, and the nominal interest rate to each one standard deviation shock in terms of the percentage deviation from the steady state in the case of low $\bar{r}\bar{r}$ and the case of large σ_d, respectively. As in the preceding section, each panel draws the true response, the posterior mean and 90 percent credible interval for the estimated response, and the response of the linear model with true parameters. Compared with the baseline experiment (Figure 1), the differences between the responses of the linear model with true parameters and those of the true model become large throughout the figures. As for the differences between the estimated and true responses, the widening differences are found in the responses to the monetary policy shock because of the upward bias in the estimates of its standard deviation σ_r. However, changes in the differences are limited regarding the responses to the discount factor shock and the productivity shock. Thus, the increased biases in parameter estimates do not necessarily make the estimated responses deviate further from the true ones.

5 Concluding Remarks

This paper has investigated parameter bias in an estimated DSGE model that neglects nonlinearity intrinsic to the economy. According to the results in our Monte Carlo experiments, significant biases have been detected in the parameter estimates associated with the monetary policy rule and the steady-state value of the inflation and real interest rate. These biases are due to ignoring the nonlinearity arising from the ZLB rather than linearizing the equilibrium conditions. We demonstrate that the estimated impulse response functions can be substantially different from the true ones although the biased parameters partially contribute to making the differences small. These findings caution researchers against the common practice of estimating linearized DSGE models without considering nonlinearity.

Our finding about the source of parameter bias indicates that omitting nonlinearity in estimation would not affect parameter estimates if the ZLB were not an issue. However, it might not be the case if the DGP is characterized by a more highly nonlinear model than considered in this paper, such as the one with recursive preferences, state-dependent pricing, or increased uncertainty. Investigating such a possibility is left for future research.

Appendix

To solve for the policy functions on each grid point $(\mathbb{S}_{t-1}, \tau_t)$, we adapt the ideas developed by Christiano and Fisher (2000) and Gust, López-Salido, and Smith (2012). We consider the regime-specific policy functions $h_i(\mathbb{S}_{t-1}, \tau_t)$ for $i = \{1, 2\}$, whose index is associated with the interest-rate regime. Specifically, the policy functions are defined as weighted averages of the regime-specific functions

$$
\begin{aligned}
Y_t &= h_Y(\mathbb{S}_{t-1}, \tau_t) = h_{Y,1}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + h_{Y,2}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}}, \\
\Pi_t &= h_\Pi(\mathbb{S}_{t-1}, \tau_t) = h_{\Pi,1}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + h_{\Pi,2}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}}, \\
\Delta_t &= h_\Delta(\mathbb{S}_{t-1}, \tau_t) = h_{\Delta,1}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + h_{\Delta,2}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}}, \\
R_t^* &= h_R(\mathbb{S}_{t-1}, \tau_t) = h_{R,1}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + h_{R,2}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}},
\end{aligned}
$$

where

$$
\begin{aligned}
h_{R,1}(\mathbb{S}_{t-1}, \tau_t) &= (R_{t-1}^*)^{\phi_r} \left[R \left(\frac{h_{\Pi,1}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right)^{\phi_\pi} \left(\frac{h_{Y,1}(\mathbb{S}_{t-1}, \tau_t)}{Y} \right)^{\phi_y} \right]^{1-\phi_r} \exp(\varepsilon_{r,t}), \\
h_{R,2}(\mathbb{S}_{t-1}, \tau_t) &= 1,
\end{aligned}
$$

and $1_{\{D\}}$ is the indicator function that is one if the condition D is true and zero otherwise. The hypothetical interest rate, R_t^*, is either above or below the lower bound of $\bar{R} = 1$. The indexed policy functions do not depend on the current indicator function.

Given the processes of τ_t, the regime-specific functions in each regime $i = 1, 2$ satisfy the following equilibrium conditions:

$$
\begin{aligned}
\frac{1}{\mathcal{W}_i(\mathbb{S}_{t-1}, \tau_t)} &= \beta h_{R,i}(\mathbb{S}_{t-1}, \tau_t) \mathbb{E}_t \left\{ \frac{1}{\mathcal{W}(\mathbb{S}_t, \tau_{t+1}) h_\Pi(\mathbb{S}_t, \tau_{t+1})} \right\}, \\
S_i(\mathbb{S}_{t-1}, \tau_t) &= \theta \mathcal{W}_i(\mathbb{S}_{t-1}, \tau_t) A_t^{-1} \\
&\quad + d_t^{-1} \beta \xi \mathbb{E}_t \left\{ \left[\left(\frac{h_\Pi(\mathbb{S}_t, \tau_{t+1})}{\Pi} \right) \left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right)^{-\iota} \right]^{\theta} \frac{\mathcal{W}_i(\mathbb{S}_{t-1}, \tau_t)}{\mathcal{W}(\mathbb{S}_t, \tau_{t-1})} \frac{h_Y(\mathbb{S}_t, \tau_{t+1})}{h_{Y,i}(\mathbb{S}_{t-1}, \tau_t)} S(\mathbb{S}_t, \tau_{t+1}) \right\}, \\
F_i(\mathbb{S}_{t-1}, \tau_t) &= (\theta - 1) \\
&\quad + d_t^{-1} \beta \xi \mathbb{E}_t \left\{ \left[\left(\frac{h_\Pi(\mathbb{S}_t, \tau_{t+1})}{\Pi} \right) \left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right)^{-\iota} \right]^{\theta-1} \frac{\mathcal{W}_i(\mathbb{S}_{t-1}, \tau_t)}{\mathcal{W}(\mathbb{S}_t, \tau_{t-1})} \frac{h_Y(\mathbb{S}_t, \tau_{t+1})}{h_{Y,i}(\mathbb{S}_{t-1}, \tau_t)} F(\mathbb{S}_t, \tau_{t+1}) \right\}, \\
1 &= (1 - \xi) \left(\frac{S_i(\mathbb{S}_{t-1}, \tau_t)}{F_i(\mathbb{S}_{t-1}, \tau_t)} \right)^{1-\theta} + \xi \left[\left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right) \left(\frac{\Pi_{t-1}}{\Pi} \right)^{-\iota} \right]^{\theta-1}, \\
h_{\Delta,i}(\mathbb{S}_{t-1}, \tau_t) &= (1 - \xi) \left(\frac{S_i(\mathbb{S}_{t-1}, \tau_t)}{F_i(\mathbb{S}_{t-1}, \tau_t)} \right)^{-\theta} + \xi \left[\left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right) \left(\frac{\Pi_{t-1}}{\Pi} \right)^{-\iota} \right]^{\theta} \Delta_{t-1},
\end{aligned}
$$

where

$$\mathcal{W}_i(\mathbb{S}_{t-1},\tau_t) \equiv (h_{Y,i}(\mathbb{S}_{t-1},\tau_t) - \gamma Y_{t-1})^{\sigma},$$

and $h_{\Pi}(\mathbb{S}_t,\tau_{t+1})$, $h_Y(\mathbb{S}_t,\tau_{t+1})$, $S(\mathbb{S}_t,\tau_{t+1})$, $F(\mathbb{S}_t,\tau_{t+1})$ and $\mathcal{W}(\mathbb{S}_t,\tau_{t+1})$ are weighted averages of the regime-specific functions in the next period. Note that the regime-specific functions depend on the expectation of binding constraint in the next period. They are also affected by binding constraint in the current period through the values of the policy functions, $\mathbb{S}_t = h_i(\mathbb{S}_{t-1},\tau_t)$. For example, in the first equation, the future possibility of binding constraint enters through $h_{\Pi}(h_i(\mathbb{S}_{t-1},\tau_t),\tau_{t+1})$ or $\mathcal{W}(h_i(\mathbb{S}_{t-1},\tau_t),\tau_{t+1})$.

Furthermore, as in Gust, López-Salido, and Smith (2012), we apply the following changes of variables

$$\mathcal{V}_i(\mathbb{S}_{t-1},\tau_t) = \mathbb{E}_t\left\{\left[\left(\frac{h_{\Pi}(\mathbb{S}_t,\tau_{t+1})}{\Pi}\right)\left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1},\tau_t)}{\Pi}\right)^{-\iota}\right]^{\theta}\frac{h_Y(\mathbb{S}_t,\tau_{t+1})S(\mathbb{S}_t,\tau_{t+1})}{\mathcal{W}(\mathbb{S}_t,\tau_{t-1})h_{Y,i}(\mathbb{S}_{t-1},\tau_t)}\right\},$$

$$\mathcal{K}_i(\mathbb{S}_{t-1},\tau_t) = \mathbb{E}_t\left\{\left[\left(\frac{h_{\Pi}(\mathbb{S}_t,\tau_{t+1})}{\Pi}\right)\left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1},\tau_t)}{\Pi}\right)^{-\iota}\right]^{\theta-1}\frac{h_Y(\mathbb{S}_t,\tau_{t+1})F(\mathbb{S}_t,\tau_{t+1})}{\mathcal{W}(\mathbb{S}_t,\tau_{t-1})h_{Y,i}(\mathbb{S}_{t-1},\tau_t)}\right\},$$

so that the second and third equations become

$$S_i(\mathbb{S}_{t-1},\tau_t) = \theta\mathcal{W}_i(\mathbb{S}_{t-1},\tau_t)A_t^{-1} + d_t^{-1}\beta\xi\mathcal{W}_i(\mathbb{S}_{t-1},\tau_t)\mathcal{V}_i(\mathbb{S}_{t-1},\tau_t), \tag{A.1}$$

$$F_i(\mathbb{S}_{t-1},\tau_t) = (\theta-1) + d_t^{-1}\beta\xi\mathcal{W}_i(\mathbb{S}_{t-1},\tau_t)\mathcal{K}_i(\mathbb{S}_{t-1},\tau_t). \tag{A.2}$$

The policy function iteration algorithm is used with initial values of $\mathcal{W}_i^{(0)}(\mathbb{S}_{t-1},\tau_t)$, $\mathcal{V}_i^{(0)}(\mathbb{S}_{t-1},\tau_t)$ and $\mathcal{K}_i^{(0)}(\mathbb{S}_{t-1},\tau_t)$ on the grid points $(\mathbb{S}_{t-1},\tau_t)$. The values of $S_i(\mathbb{S}_{t-1},\tau_t)$ and $F_i(\mathbb{S}_{t-1},\tau_t)$ are obtained from the values of $\mathcal{W}_i^{(j-1)}(\mathbb{S}_{t-1},\tau_t)$, $\mathcal{V}_i^{(j-1)}(\mathbb{S}_{t-1},\tau_t)$ and $K_i^{(j-1)}(\mathbb{S}_{t-1},\tau_t)$ on the grid points in each iteration $j = 1, 2, ...$[11] Therefore, we can solve for the value of the policy functions $h_i(\mathbb{S}_{t-1},\tau_t)$ without relying on any numerical optimization routines. The policy functions of $h_{\Pi,i}(\mathbb{S}_{t-1},\tau_t)$ and $h_{Y,i}(\mathbb{S}_{t-1},\tau_t)$ are given by

$$h_{\Pi,i}(\mathbb{S}_{t-1},\tau_t) = \Pi\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\iota}\left(\xi^{-1}\left[1-(1-\xi)\left(\frac{S_i(\mathbb{S}_{t-1},\tau_t)}{F_i(\mathbb{S}_{t-1},\tau_t)}\right)^{1-\theta}\right]\right)^{\frac{1}{\theta-1}}, \tag{A.3}$$

$$h_{Y,i}(\mathbb{S}_{t-1},\tau_t) = \mathcal{W}_i(\mathbb{S}_{t-1},\tau_t)^{1/\sigma} + \gamma Y_{t-1}. \tag{A.4}$$

Equations (A.1)-(A.4) imply that there is a one-to-one mapping from $\mathcal{W}_i(\mathbb{S}_{t-1},\tau_t)$, $\mathcal{V}_i(\mathbb{S}_{t-1},\tau_t)$ and $\mathcal{K}_i(\mathbb{S}_{t-1},\tau_t)$ to $h_{\Pi,i}(\mathbb{S}_{t-1},\tau_t)$ and $h_{Y,i}(\mathbb{S}_{t-1},\tau_t)$. Then the values of $\mathcal{W}_i^{(j)}(\mathbb{S}_{t-1},\tau_t)$, $\mathcal{V}_i^{(j)}(\mathbb{S}_{t-1},\tau_t)$ and

[11] For simplicity of notation, we omit the index j from the functions except for $\mathcal{W}_i^{(j)}(\mathbb{S}_{t-1},\tau_t)$, $\mathcal{V}_i^{(j)}(\mathbb{S}_{t-1},\tau_t)$ and $\mathcal{K}_i^{(j)}(\mathbb{S}_{t-1},\tau_t)$.

$\mathcal{K}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t)$ are updated as

$$\mathcal{W}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t) = \left[\beta h_{R,i}(\mathbb{S}_{t-1}, \tau_t) \sum_{\tau_{t+1}} \left\{ \frac{\Phi(\tau_{t+1}|\tau_t)}{\hat{\mathcal{W}}(\mathbb{S}_t; \tau_{t+1}) \hat{h}_\Pi(\mathbb{S}_t; \tau_{t+1})} \right\} \right]^{-1},$$

$$\mathcal{V}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t) = \sum_{\tau_{t+1}} \left\{ \left[\left(\frac{\hat{h}_\Pi(\mathbb{S}_t; \tau_{t+1})}{\Pi} \right) \left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right)^{-\iota} \right]^{\theta} \frac{\hat{h}_Y(\mathbb{S}_t; \tau_{t+1})}{h_{Y,i}(\mathbb{S}_{t-1}, \tau_t)} \right.$$
$$\left. \times \left(\theta A_{t+1}^{-1} + d_{t+1}^{-1} \beta \xi \hat{\mathcal{V}}(\mathbb{S}_t; \tau_{t+1}) \right) \Phi(\tau_{t+1}|\tau_t) \right\},$$

$$\mathcal{K}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t) = \sum_{\tau_{t+1}} \left\{ \left[\left(\frac{\hat{h}_\Pi(\mathbb{S}_t; \tau_{t+1})}{\Pi} \right) \left(\frac{h_{\Pi,i}(\mathbb{S}_{t-1}, \tau_t)}{\Pi} \right)^{-\iota} \right]^{\theta-1} \frac{\hat{h}_Y(\mathbb{S}_t; \tau_{t+1})}{h_{Y,i}(\mathbb{S}_{t-1}, \tau_t)} \right.$$
$$\left. \times \left((\theta - 1)/\hat{\mathcal{W}}(\mathbb{S}_t; \tau_{t+1}) + d_{t+1}^{-1} \beta \xi \hat{\mathcal{K}}(\mathbb{S}_t; \tau_{t+1}) \right) \Phi(\tau_{t+1}|\tau_t) \right\},$$

where $\Phi(\tau_{t+1}|\tau_t)$ is a transition probability from τ_t to τ_{t+1}. We approximate the functions $\hat{\mathcal{W}}(\mathbb{S}_{t-1}; \tau_t) \approx \mathcal{W}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$, $\hat{\mathcal{V}}(\mathbb{S}_{t-1}; \tau_t) \approx \mathcal{V}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$ and $\hat{\mathcal{K}}(\mathbb{S}_{t-1}; \tau_t) \approx \mathcal{K}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$ for any values in $\mathbb{S} \subset \mathbb{R}^4$ for given τ_t by a projection method. The values of the functions on the grid points $(\mathbb{S}_{t-1}, \tau_t)$ are given by

$$\mathcal{W}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) = \mathcal{W}_1^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + \mathcal{W}_2^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}},$$
$$\mathcal{V}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) = \mathcal{V}_1^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + \mathcal{V}_2^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}},$$
$$\mathcal{K}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) = \mathcal{K}_1^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) > 1\}} + \mathcal{K}_2^{(j-1)}(\mathbb{S}_{t-1}, \tau_t) 1_{\{h_{R,1}(\mathbb{S}_{t-1}, \tau_t) \leq 1\}}.$$

Then we evaluate these approximated functions at $(\mathbb{S}_t; \tau_{t+1})$. The estimated values of $\hat{\mathcal{W}}(\mathbb{S}_t; \tau_{t+1})$, $\hat{\mathcal{V}}(\mathbb{S}_t; \tau_{t+1})$ and $\hat{\mathcal{K}}(\mathbb{S}_t; \tau_{t+1})$ are used to obtain the values of $\hat{h}_\Pi(\mathbb{S}_t; \tau_{t+1})$ and $\hat{h}_Y(\mathbb{S}_t; \tau_{t+1})$.

The algorithm is summarized as follows.

1. $\mathcal{W}_i^{(0)}(\mathbb{S}_{t-1}, \tau_t)$, $\mathcal{V}_i^{(0)}(\mathbb{S}_{t-1}, \tau_t)$ and $\mathcal{K}_i^{(0)}(\mathbb{S}_{t-1}, \tau_t)$ are given on each grid point $(\mathbb{S}_{t-1}, \tau_t)$ as initial values.

2. $h_i(\mathbb{S}_{t-1}, \tau_t)$ is computed for each regime $i = 1, 2$, as the values of $\mathcal{W}_i^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$, $\mathcal{V}_i^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$ and $\mathcal{K}_i^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)$ are given on each grid point.

3. The estimated values $\hat{\mathcal{W}}(\mathbb{S}_t; \tau_{t+1})$, $\hat{\mathcal{V}}(\mathbb{S}_t; \tau_{t+1})$ and $\hat{\mathcal{K}}(\mathbb{S}_t; \tau_{t+1})$ in the next period are obtained by a projection method.

4. $\hat{h}(h_i(\mathbb{S}_{t-1}, \tau_t); \tau_{t+1})$ is computed for each regime $i = 1, 2$ on each grid point $(\mathbb{S}_{t-1}, \tau_t, \tau_{t+1})$.

5. $\mathcal{W}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t)$, $\mathcal{V}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t)$ and $\mathcal{K}_i^{(j)}(\mathbb{S}_{t-1}, \tau_t)$ are updated on each grid point $(\mathbb{S}_{t-1}, \tau_t)$ by taking expectations over the grid points τ_{t+1}.

6. Iterate the procedure 2–5 until the algorithm converges as $\left\|\mathcal{W}^{(j)}(\mathbb{S}_{t-1}, \tau_t) - \mathcal{W}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)\right\| < \epsilon$, $\left\|\mathcal{V}^{(j)}(\mathbb{S}_{t-1}, \tau_t) - \mathcal{V}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)\right\| < \epsilon$ and $\left\|\mathcal{K}^{(j)}(\mathbb{S}_{t-1}, \tau_t) - \mathcal{K}^{(j-1)}(\mathbb{S}_{t-1}, \tau_t)\right\| < \epsilon$ hold.

References

Basu, Susanto, and Brent Bundick, 2012. "Uncertainty Shocks in a Model of Effective Demand," NBER Working Paper 18420, National Bureau of Economic Research.

Bodenstein, Martin, Luca Guerrieri, and Christopher J. Gust, 2013. "Oil Shocks and the Zero Bound on Nominal Interest Rates." *Journal of International Money and Finance*, 32, 941–967.

Braun, R. Anton, Lena M. Körber, and Yuichiro Waki, 2012. "Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate is Zero." Working Paper 2012-05a, Federal Reserve Bank of Atlanta.

Calvo, Guillermo A., 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3), 383–398.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, 2011. "When Is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119(1), 78–121.

Eggertsson, Gauti B., and Michael Woodford, 2003. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1), 139–235.

Erceg, Christopher J., and Jesper Lindé, 2014. "Is There a Fiscal Free Lunch in a Liquidity Trap." *Journal of European Economic Association*, 12(1), 73–107.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramírez, 2005. "Estimating Dynamic Equilibrium Economies: Linear Versus Nonlinear Likelihood." *Journal of Applied Econometrics*, 20(7), 891–910.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramírez, 2007. "Estimating Macroeconomic Models: A Likelihood Approach." *Review of Economic Studies*, 74(4), 1059–1087.

Fernández-Villaverde, Jesús, Juan F. Rubio-Ramírez, and Manuel S. Santos, 2006. "Convergence Properties of the Likelihood of Computed Dynamic Models." *Econometrica*, 74(1), 93–119.

Fernández-Villaverde, Jesús, Grey Gordon, Pablo A. Guerrón-Quintana, and Juan Rubio-Ramírez, 2012. "Nonlinear Adventures at the Zero Lower Bound." *Journal of Economic Dynamics and Control*, 57, 182–204.

Gavin, William T., Benjamin D. Keen, Alexander Richter, and Nathaniel Throckmorton, 2015. "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics." *Journal of Economic Dynamics and Control*, 55, 14–38.

Gust, Christopher, David Lopez-Salido, and Matthew E. Smith, 2012. "The Empirical Implications of the Interest-Rate Lower Bound." Finance and Economics Discussion Series 2012-83, Board of Governors of the Federal Reserve System.

Hirose, Yasuo, and Atsushi Inoue, 2015. "The Zero Lower Bound and Parameter Bias in an Estimated DSGE Model." *Journal of Applied Econometrics*, forthcoming.

Judd, Kenneth L., Lilia Maliar, Serguei Maliar, and Rafael Valero, 2015. "Smolyak Method for Solving Dynamic Economic Models: Lagrange Interpolation, Anisotropic Grid and Adaptive Domain." *Journal of Economic Dynamics and Control*, 44, 92-123.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, 2010. "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57, 132–145.

Malin, Benjamin A., Dirk, Krueger, and Felix Kubler, 2011. "Solving the Multi-Country Real Business Cycle Model using a Smolyak-Collocation Method." *Journal of Economic Dynamics and Control*, 35(2), 229–239.

Lubik, Thomas A., and Frank Schorfheide, 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1), 190–217.

Maliar, Lilia, and Serguei Maliar, 2015. "Merging Simulation and Projection Aproaches to Solve High-Dimensional Problems with an Application to a New Keynesian Model", *Quantitative Economics*, 6, 1–47.

Nakata, Taisuke, 2013a. "Uncertainty at the Zero Lower Bound." Finance and Economics Discussion Series 2013-09, Board of Governors of the Federal Reserve System.

Nakata, Taisuke, 2013b. "Optimal Fiscal and Monetary Policy with Occasionally Binding Zero Bound Constraints." Finance and Economics Discussion Series 2013-40, Board of Governors of the Federal Reserve System.

Ngo, Phuong V., 2014. "Optimal Discretionary Monetary Policy in a Micro-Founded Model with a Zero Lower Bound on Nominal Interest Rate." *Journal of Economic Dynamics and Control*, 45, 44–65.

Rouwenhorst, K. Geert, 1995. "Asset Pricing Implications of Equilibrium Business Cycle Models." In: Cooley, Thomas F. (ed.), *Frontiers of Business Cycle Research*, 294–330. Princeton University Press, Princeton, NJ.

Smets, Frank, and Rafael Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3), 586–606.

Taylor, John B., 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

Table 1: Prior and Posterior distributions of parameters

	Prior			Posterior					
				Baseline		No ZLB in DGP		Quasi-linear DGP	
Parameter	Distribution	Mean	S.d.	Mean	90% interval	Mean	90% interval	Mean	90% interval
σ	Gamma	1.500	0.200	1.508	[1.192, 1.820]	1.471	[1.160, 1.776]	1.492	[1.178, 1.801]
γ	Beta	0.500	0.200	0.510	[0.412, 0.609]	0.487	[0.385, 0.590]	0.500	[0.401, 0.600]
ξ	Beta	0.750	0.200	0.734	[0.693, 0.774]	0.743	[0.703, 0.783]	0.738	[0.697, 0.778]
ι	Beta	0.500	0.200	0.548	[0.363, 0.733]	0.498	[0.311, 0.683]	0.524	[0.340, 0.707]
ϕ_π	Gamma	2.000	0.200	1.731	[1.574, 1.887]	2.001	[1.815, 2.187]	1.838	[1.680, 1.995]
ϕ_y	Gamma	0.500	0.200	0.416	[0.307, 0.525]	0.495	[0.384, 0.605]	0.442	[0.334, 0.548]
ϕ_r	Beta	0.500	0.200	0.498	[0.453, 0.544]	0.495	[0.447, 0.543]	0.491	[0.447, 0.535]
$\bar{\pi}$	Gamma	0.500	0.200	0.384	[0.333, 0.434]	0.498	[0.459, 0.537]	0.432	[0.383, 0.481]
$\bar{r}$	Gamma	0.250	0.200	0.175	[0.130, 0.220]	0.248	[0.203, 0.293]	0.203	[0.156, 0.251]
ρ_d	Beta	0.700	0.200	0.723	[0.645, 0.802]	0.698	[0.614, 0.782]	0.732	[0.655, 0.810]
ρ_a	Beta	0.700	0.200	0.683	[0.557, 0.809]	0.707	[0.586, 0.829]	0.693	[0.570, 0.817]
$100\sigma_d$	Inv. Gamma	0.300	2.000	0.337	[0.247, 0.424]	0.296	[0.219, 0.371]	0.316	[0.235, 0.396]
$100\sigma_a$	Inv. Gamma	0.300	2.000	0.334	[0.213, 0.451]	0.293	[0.187, 0.397]	0.323	[0.206, 0.435]
$100\sigma_r$	Inv. Gamma	0.100	2.000	0.108	[0.099, 0.117]	0.100	[0.091, 0.108]	0.103	[0.094, 0.112]

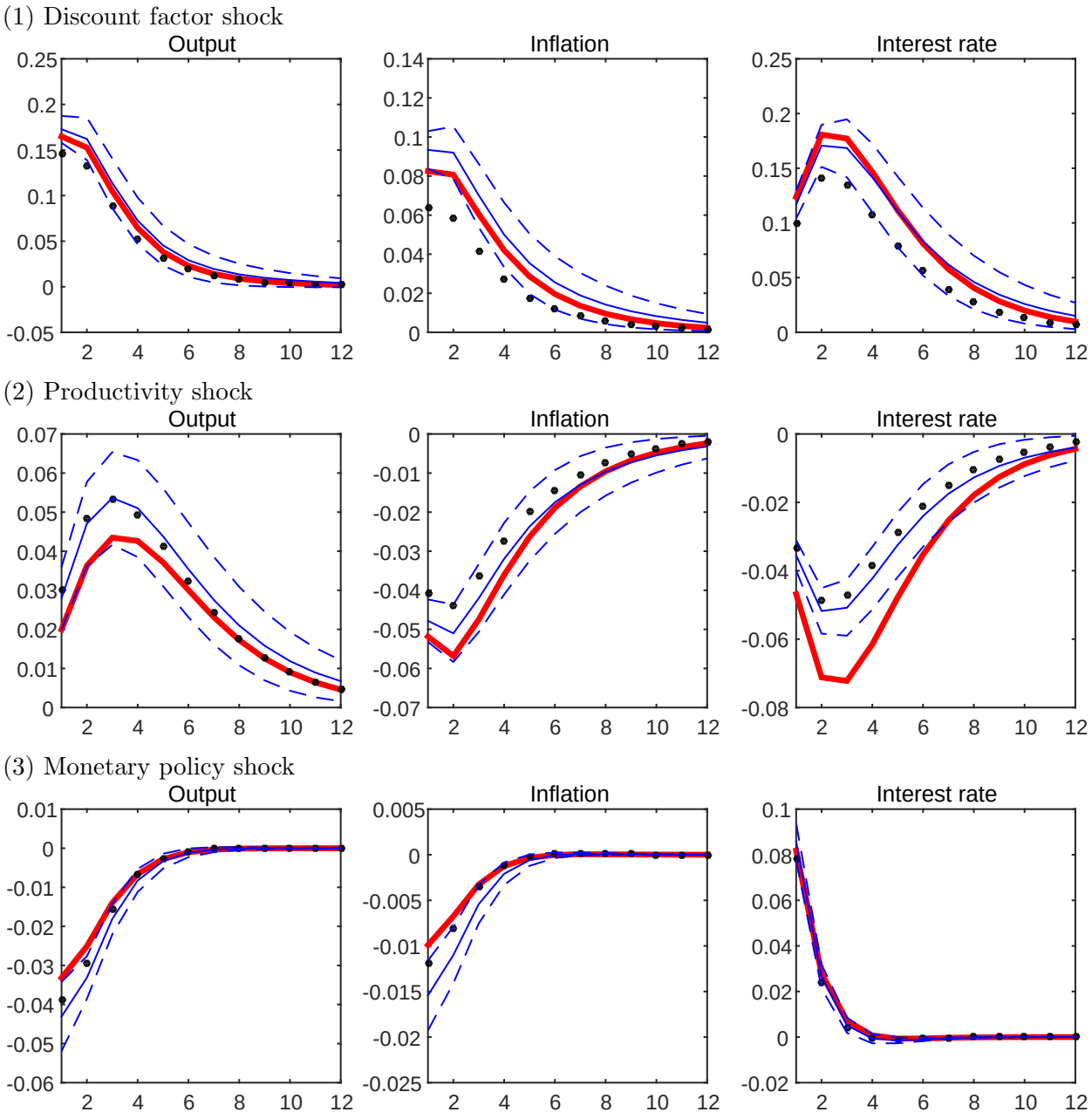
Notes: Means and 90% intervals are the averages of posterior means and 90% credible intervals based on 200 Monte Carlo replications using a sample size of 200 observations. In each of the Monte Carlo replications, 200,000 MCMC draws are generated and the first 25 percent of the draws are discarded. S.d. denotes standard deviation.

Table 2: Posterior distributions of parameters in alternative experiments

		Case of low $\bar{rr}$		Case of large σ_d	
Parameter	True value	Mean	90% interval	Mean	90% interval
σ	1.500	1.530	[1.212, 1.844]	1.528	[1.209, 1.842]
γ	0.500	0.521	[0.424, 0.619]	0.521	[0.425, 0.619]
ξ	0.750	0.733	[0.693, 0.773]	0.733	[0.694, 0.773]
ι	0.500	0.570	[0.388, 0.754]	0.573	[0.390, 0.755]
ϕ_π	2.000	1.614	[1.456, 1.771]	1.619	[1.459, 1.778]
ϕ_y	0.500	0.376	[0.263, 0.487]	0.376	[0.265, 0.486]
ϕ_r	0.500	0.514	[0.466, 0.562]	0.515	[0.468, 0.561]
$\bar{\pi}$	0.500	0.347	[0.294, 0.400]	0.352	[0.299, 0.405]
$\bar{rr}$	0.22/0.25	0.138	[0.094, 0.182]	0.175	[0.130, 0.219]
ρ_d	0.700	0.721	[0.644, 0.800]	0.716	[0.638, 0.794]
ρ_a	0.700	0.672	[0.544, 0.800]	0.670	[0.543, 0.799]
$100\sigma_d$	0.30/0.33	0.347	[0.251, 0.440]	0.382	[0.277, 0.484]
$100\sigma_a$	0.300	0.355	[0.225, 0.481]	0.356	[0.226, 0.481]
$100\sigma_r$	0.100	0.115	[0.105, 0.124]	0.118	[0.108, 0.128]

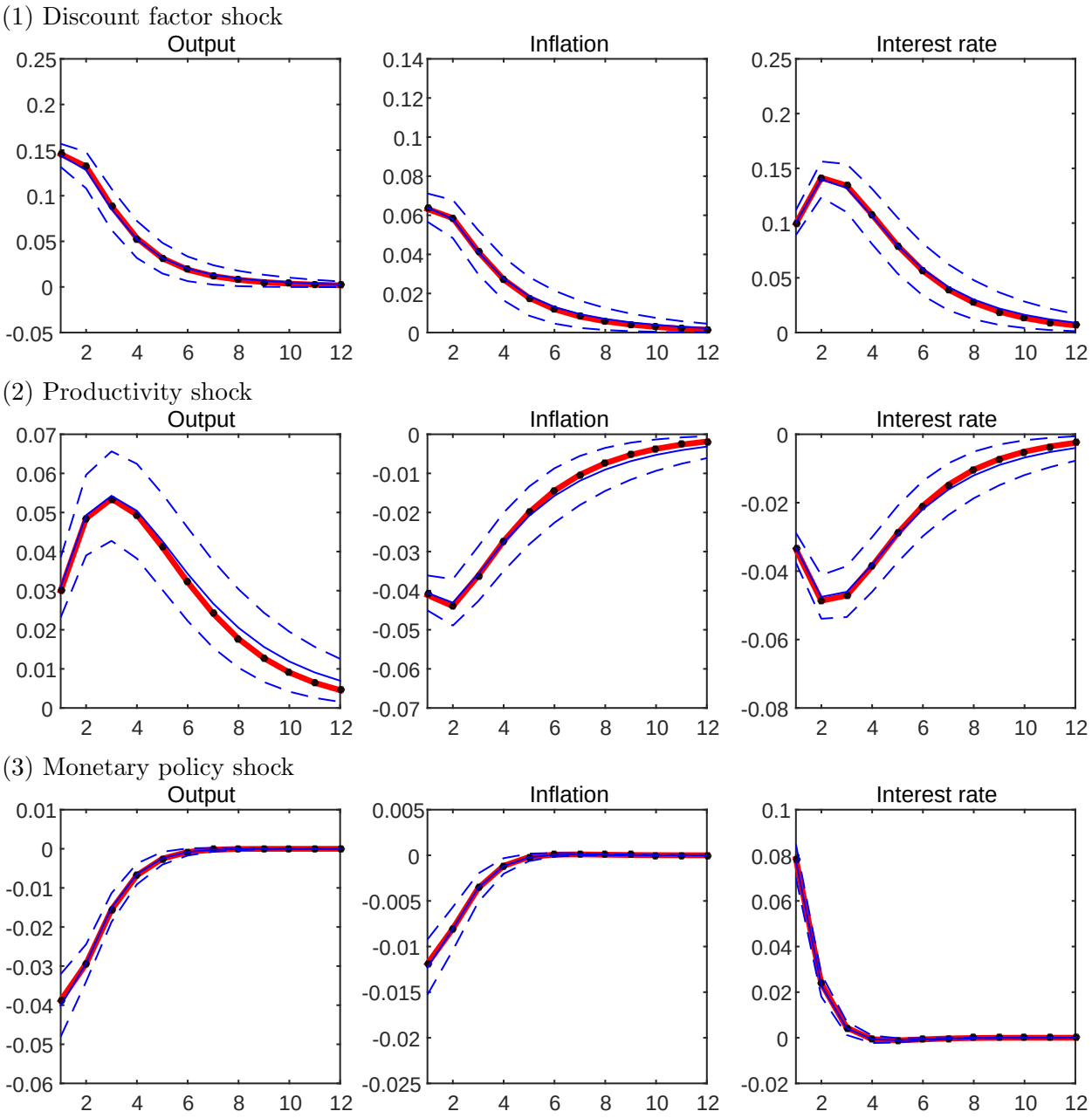
Notes: In the case of low $\bar{rr}$, the true value of $\bar{rr}$ is 0.22. In the case of large σ_d, the true value of $100\sigma_d$ is 0.33. Means and 90% intervals are the averages of posterior means and 90% credible intervals based on 200 Monte Carlo replications using a sample size of 200 observations. In each of the Monte Carlo replications, 200,000 MCMC draws are generated and the first 25 percent of the draws are discarded.

Figure 1: Impulse responses in the baseline experiment



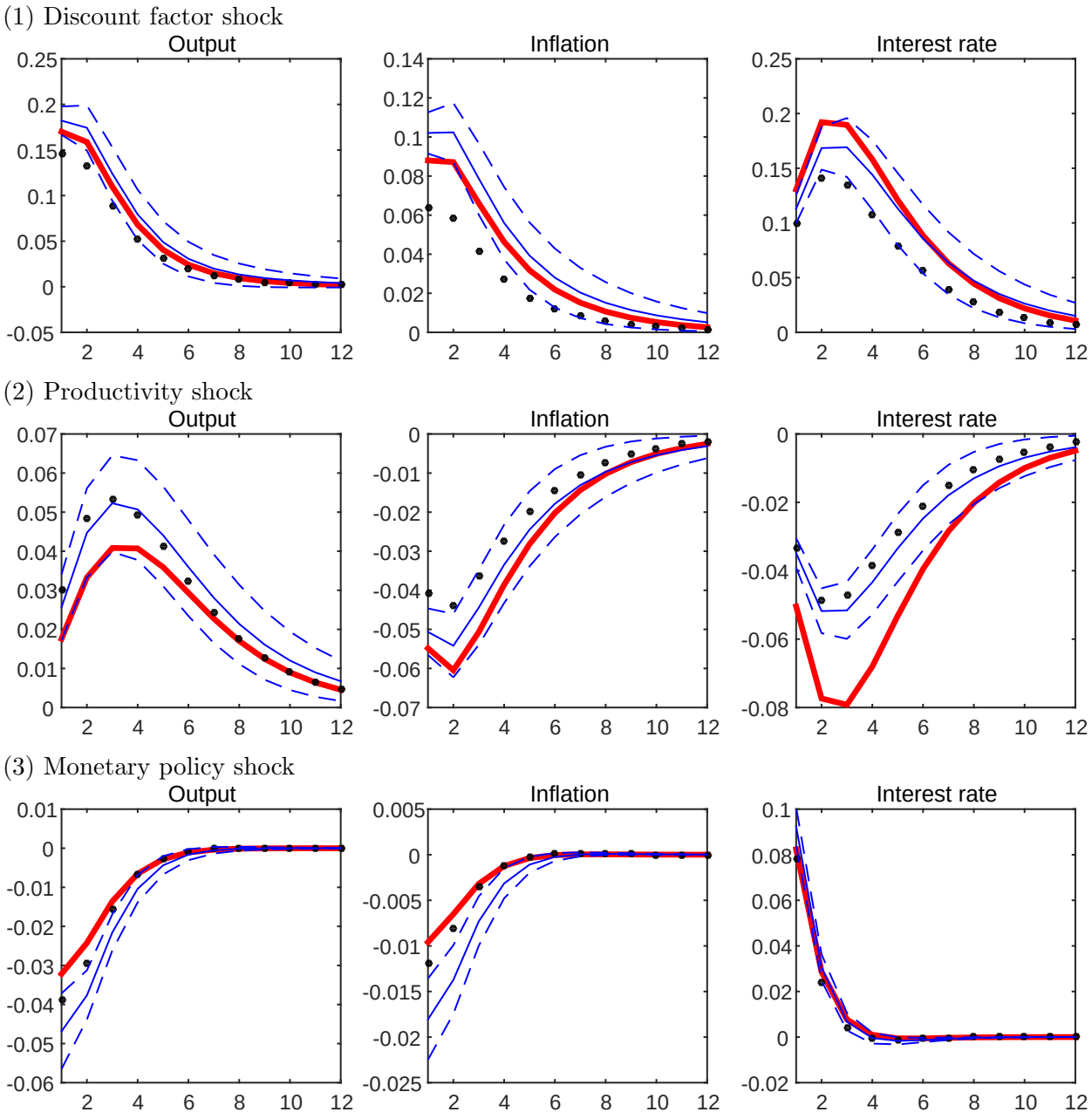
Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks to the discount factor, productivity, and monetary policy. The solid thick lines represent the true responses, the dots are the responses of the linearized model with true parameters, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are taken over 200 Monte Carlo replications.

Figure 2: Impulse responses in the case of no ZLB in DGP



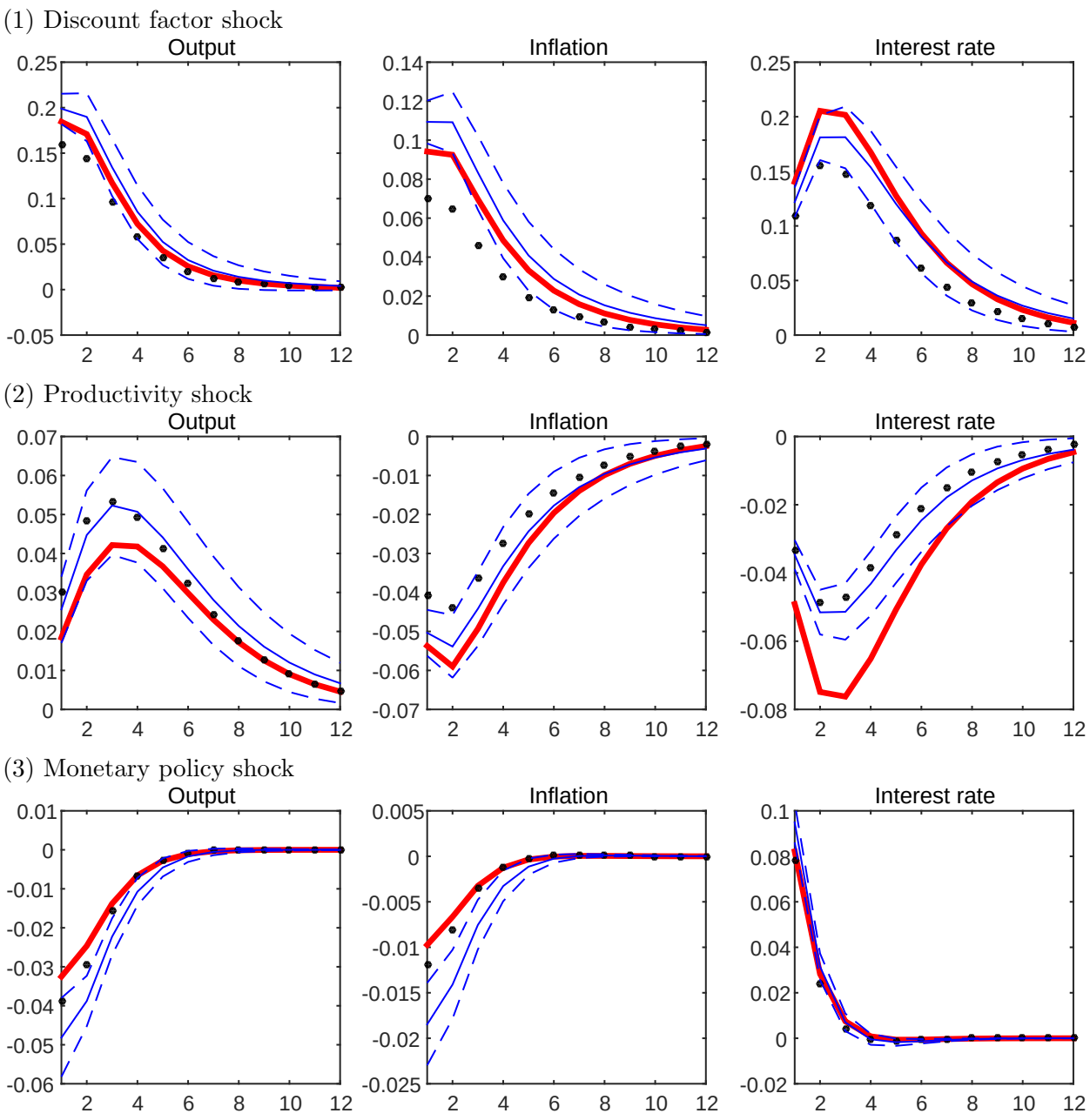
Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks to the discount factor, productivity, and monetary policy. The solid thick lines represent the true responses, the dots are the responses of the linearized model with true parameters, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are taken over 200 Monte Carlo replications.

Figure 3: Impulse responses in the case of low $\bar{rr}$



Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks to the discount factor, productivity, and monetary policy. The solid thick lines represent the true responses, the dots are the responses of the linearized model with true parameters, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are taken over 200 Monte Carlo replications.

Figure 4: Impulse responses in the case of large σ_d



Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks to the discount factor, productivity, and monetary policy. The solid thick lines represent the true responses, the dots are the responses of the linearized model with true parameters, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are taken over 200 Monte Carlo replications.