What does the data say about wage setting and job search?
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Abstract

Do current wages depend on past labour market conditions? Wage posting models suggest so, Nash bargaining models do not, Bertrand competition between employers is somewhere in between. Recent evidence suggests Nash bargaining to be an adequate description of the data. We re-evaluate this evidence, using a more precise analytical framework and more efficient estimation techniques. Our evidence is easily squared with wage posting, but not with Nash bargaining, while Bertrand competition is clearly rejected. In wage posting models firms can pay hiring and retention premiums. Our evidence is consistent retention premiums, but it is more difficult to square with hiring premiums. Finally, our methodology allows two important side conclusions. First, we find strong support for the general class of models with on-the-job search. Second, the estimation results suggest the productivity distribution to have an fat tailed upper support, which is hard to square with pure sorting models.

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1 Introduction

Do past labour market conditions have an effect on wages paid in ongoing employment relations? Previous empirical evidence suggest so. Mark Bils (1985) reported that wages of new hires are more sensitive to labour market tightness than wages in ongoing jobs. Paul Beaudry and John DiNardo (1991) found that the current wage is related to the lowest the unemployment rate since the date of hiring. This type of findings is consistent with a variety of search models with On-the-Job-Search (OJS). The wage posting models of Kenneth Burdett and Dale Mortensen (1989), Christian Bontemps, Jean-Marc Robin and Gerard Van den Berg (1999), and Pieter Gautier, Coen Teulings, and Aico Van Vuuren (2010, GT&V hereafter) predict that the wage rate for a job is set at the date of hiring. The wage might subsequently be raised when the risk of quitting increases due to a tightening of the labour market, consistent with the evidence by Beaudry and DiNardo and with the general idea that wages are asymmetrically affected by labour market conditions, going up after tightening, but not going down after relaxation. In Fabien Postel-Vinay and Jean-Marc Robin (2002, PV&R hereafter) wages increase during the duration of a job spell by Bertrand competition between the current employer and an incoming outside offer.

However, Marcus Hagedorn and Iourii Manovskii (2013, H&M hereafter) offered a reinterpretation of this evidence. Job search models predict that workers gradually select into better matches. Hence, match quality increases over the career. The speed of this process varies with the tightness of the labour market. In a tight labour market, the job offer arrival rate is high and hence the selection process proceeds quickly, and the reverse when the labour market is weak. Hence, so H&M argue, the effect of past unemployment could well be due to the selection to better match qualities. In fact, expected match quality is a function of the accumulated job offers since the last time that a worker has been involuntary fired from his job. After firing, the worker has to start from the first rung of the ladder again. H&M use labour market tightness as an indicator for the job offer arrival rate. We refer to the sum of labour market tightness over an interval of time as the labour market history, which is a proxy for the expected number of job offers received during that interval and hence for the quality of the current job. H&M show that wages are indeed strongly correlated to the labour

market history, but not to the minimum unemployment rate since the date of hiring, as reported by Beaudry and DiNardo (1991) and as predicted by wage posting models, nor to the labour market history up to the moment of observation, as predicted by the Bertrand competition model of PV&R. H&M therefore conclude that the effect of past labour market conditions on current wages is due to their effect on match quality and not through their effect on wage setting. They conclude that period by period Nash bargaining over the surplus is an adequate description of wage setting.

In this paper, we argue that though the methodology proposed by H&M is interesting, their empirical conclusions do not survive further scrutiny. H&M use a first order Taylor expansion of the expected wage in terms of the log labour market history, at the start of the current job, during the current completed job spell, and from the start of the current job till the moment of observation. They show theoretically that the first order effects are positive under Nash bargaining due to the selection of better offers. They proceed to show that the log labour market history at job start and during the current job spell have a significant effect on log wages, but that neither the minimum unemployment during the spell nor the history in the current job up till the moment of observation have a significant effect on wages (as predicted by wage posting and the PV&R model respectively). From this they conclude that the data support Nash bargaining as the appropriate model of wage setting. Our analysis questions this conclusion. First, improving the empirical specification of their theoretical model allows for testing more detailed predictions. Second, H&M's conclusions are based on regressions on the level of log wages. We show that this estimation procedure is highly inefficient for discriminating between Nash bargaining and wage posting. First differencing the data and the elimination of data on job movers yields a more efficient procedure. This procedure finds empirical support for wage posting rather than Nash bargaining. In the wage posting model firms pay higher wages both to hire workers more easily and to retain them for longer. Our results support retention premiums, but are less easily squared with hiring premiums. This is important as GT&V show that there is excess vacancy creation in wage posting models where firms only pay retention premiums. The results support the idea stressed by Truman Bewley (1999) that firms prefer not to revise down wages. Our results clearly reject the PV&R model. The PV&R model predicts that the starting wage for workers finding a job directly from unemployment would depend

negatively on the match quality of the first job due to the higher option value of getting new offers and therefore being able to renegotiate the wage. The empirical evidence runs counter to this prediction. Since the labour market history up to the current point in time is a good proxy for the number of offers, the PV&R model predicts it to be an important determinant of wages. The data does not support this prediction.

The theoretical results we present are a more rigorous formulation of the results of H&M. We derive closed form approximations of the solutions to the steady state for the three models of wage setting and we analyze how wages respond to fluctuations in labour market conditions in these models. This analysis yields some additional testable implications, which help discriminating between the three models. In particular, wages in high quality jobs/matches turn out to be more sensitive to labour market conditions under Nash bargaining and wage posting models. Low quality matches have a small surplus and therefore little incentive to pay more for retention when the labour market is tight. In Beaudry and DiNardo (1991) and H&M unemployment and the minimum unemployment are included but not their interactions. We find that labour market conditions have the largest impact for high quality matches, whether it is measured by the current unemployment rate, as predicted by Nash bargaining, or by the minimum unemployment rate, as predicted by wage posting.

If wages were set via Nash bargaining then controling for match quality the minimum unemployment rate should have no independent effect on wages. Next consider the case of wage posting. Firms commit to a wage, hence wages only change if it is mutually beneficial. If firms pay retention premiums only then wages should depend on the minimum unemployment rate since the date of hiring, since firms increase their wage when the risk of a worker quitting goes up due to a tightening of the labour market. To the contrary, if firms pay both retention and hiring premiums then the unemployment rate at the date of hiring should be the best predictor of wages, since firms have no incentive to increase their wages after the worker has accepted the offer, even when labour market conditions tighten later on. We find that when controlling for match quality the minimum unemployment has an significant effect on wages. Furthermore we find that the the minimum unemployment rate is a better predictor of wages compared to the unemployment rate at the start of the job. The model that fits the data best is therefore wage posting with retention premiums only.

Our derivation of the distribution of match quality conditional on labour market history is based on a more convenient mathematical methodology than used by H&M. Our methodology is akin to the standard framework of job offer arrival rates. It has the advantage that it allows an exact characterization of the distribution of match quality. By the limit theorem for extreme value distributions, the precise functional form of the offer distribution is irrelevant for the functional form of the limit distribution after some minimum amount of labour market history. Only the fatness of the right tail matters, see Raouf Jaibi and Thijs ten Raa (1998). For a fat tail (e.g. the Gumbel distribution), wages increase logarithmically with labour market history. For a thin tail (e.g. the uniform and the normal distribution), wages converge to a fixed maximum. Our evidence suggests that the offer distribution has a fat tail. This runs counter to models with assortative matching with an interior upper bound of the matching set, like Robert Shimer and Lones Smith (2000) and GT&V, because in these models the upper support of the offer distribution is finite, implying a thin right tail. Hence, sorting cannot be the only source of variation in the offer distribution.

The precise structure of wage setting has important implications for the response of the economy to the business cycle. If wage setting is described by Nash bargaining or Bertrand competition, then job mobility is always efficient. Under wage posting, job mobility is inefficient in the downturn, since wages are set too high for jobs started in the upturn. Hence, workers will not always switch to better quality matches in the downturn. Both for wage posting and the PV&R model, there might be excess separation if workers and firms are unable to renegotiate an inefficient contract after a decline in labour productivity. Under Nash bargaining vacancy creation is efficient if the bargaining power of the workers satisfies the Hosios conditon whereas under wage posting vacancy creation is efficient if firms pay both hiring and retention premiums, see GT&V.

The structure of this paper is as follows. Section 2 develops the main theoretical concepts and derives the relation between wages and accumulated labour market experience for various models of wage setting. Section 3 then presents our empirical results. Section 4 concludes.

2 The theoretical argument

2.1 The distribution of job quality

The theoretical arguments are based on a search model with OJS. During their labour market career, workers receive job offers. We make the critical assumption that the wage bargaining process is efficient: workers accept any job offer that has a better quality than their current job. Most models of job search have this property. As workers labour market history accumulates, they receive ever more offers. Since the quality of their current job is the maximum of all offers received till so far, expected match quality is increasing in the accumulated labour market history. This selection process of ever better matches continues till the worker is laid off from his current job. Then, the worker becomes unemployed and the selection process starts all over again. Following Kenneth Wolpin (1992) and H&M, we refer to an interval from the start of an unemployment spell up till the moment that a worker gets fired as an *employment cycle*. The interesting new idea of H&M is that we can distinguish between the effect of this selection process of ever better matches and the effect of learning by using data on labour market tightness. While learning proceeds proportional to calendar time, the labour market history accumulates proportional to what we refer to as *labour market time*. In a tight labour market, the clock of labour market time runs faster than that of calendar time, speeding up the selection process. The speed at which labour market time runs will be proxied by labour marktet tightness.

Let θ_t denote labour market tightness at time t, and let $\lambda_t \equiv \lambda(\theta_t), \lambda'(\cdot) \geq 0$ and $\delta_t \equiv \delta(\theta_t), \delta'(\cdot) \leq 0$, be the job offer arrival rate and the job destruction rate respectively for employed workers. It is useful to define:

$$\begin{aligned} \Lambda_t &\equiv \int_0^t \lambda_r dr, \\ \Delta_t &\equiv \int_0^t \delta_r dr. \end{aligned}$$

We shall refer to Λ_t as labour market time. We normalize our measure of calendar time such that it takes the value 0 at the start of the first job of the current employment cycle. Hence, as long as a worker has not experienced a lay off, t is equal to labour market experience as usually defined. Finally, let $\underline{x}$ be a random variable measuring the quality

of a job offer. At this point we do not specify the precise metric of job quality. Hence, it can be scaled such that it is distributed uniformly between zero and one without loss of generality.

The distribution of match quality, $\underline{x}$, at time t for employed workers who have had k jobs prior satisfies:

$$\Pr\left(\underline{x} < x | t, k\right) = x^k \exp\left[-\Lambda_t(1-x)\right]. \tag{1}$$

Our notation distinguishes between underlined stochasts and their realisations only when required to avoid confusion. The first factor x^k is the probability that the all of the k jobs had a lower match quality than x. The second factor $\exp\left[-\Lambda_t(1-x)\right]$ is the probability that no offer greater than x has arrived since the start of the first job in this labour employment cycle.

Let a be the time at which the current job started and let b be the time at which this job will end. When using panel data, information on b is available for all jobs that do not last beyond the termination date of the panel. The density of b conditional on the quality of the job x and its starting date a satisfies:

$$\Pr(b|x,a) = \left[\delta_b + \lambda_b\left(1-x\right)\right] \exp\left[-\left(\Lambda_b - \Lambda_a\right)\left(1-x\right) - \left(\Delta_b - \Delta_a\right)\right]. \tag{2}$$

Using equation (1) for $t = a$ for the distribution of the quality in the previous job (which must be less than x), the joint probability of the job quality x in the current job and its termination time b conditional on its starting time a and having had k jobs before a can therefore be written as:

$$\begin{aligned} \Pr(x,b|a,k) &= C_1 \Pr\left(\underline{x} < x | a, k\right) \Pr\left(x | \text{offer at } a\right) \Pr(b|x,a) \\ &= \tilde{C}_1 x^k \left[\delta_b + \lambda_b(1-x)\right] \exp\left[-\Lambda_b(1-x) - \Delta_b\right] = \Pr(x,b|k). \end{aligned} \tag{3}$$

Throughout this section, the C_i's are appropriate constants of integration. The density of x at time a drops out in the second line, since x is uniformly distributed. Equation (3) implies that the jonint distribution of match quality x and the termination time b of the current job does not on its starting time a. The intuition for this result is that conditional on the moment of termination b, the arrival time of the maximum offer is uniformly distributed on the time interval $[0, b]$. The number of jobs is informative, since it imposes a lower bound on the number of job offers a worker has received since

the start of the current labour market run. It is only a lower bound because it is likely that there have been more, which remained unobserved because they have been rejected. But the precise timing of the previous jobs is not informative.

The probability of x conditional on a and b is obtained by dividing by the above probability by the marginal probability of b:[1]

$$\begin{aligned}\Pr(x|b,k) &= \frac{\Pr(x,b|k)}{\int \Pr(z,b|k)\,dz} \\ &= C_2 x^k \left[\delta_b + \lambda_b(1-x)\right] \exp\left[-\Lambda_b(1-x)\right] \\ &\cong \tilde{C}_2 \exp\left[-\left(\Lambda_b + k - \frac{\lambda_b}{\delta_b}\right)(1-x)\right] \end{aligned} \tag{4}$$

The distribution of match quality in the first job corresponds to the case of $a = 0$ and $k = 0$. For $\Lambda_b >> k$, the value of k contains only little information, because the actual number of job offers is likely to exceed by far the number job changes. When we do not condition on the number of prior jobs, the distribution at b is

$$\begin{aligned}\Pr\left(\underline{x} < x|b\right) &= x \exp\left[-\Lambda_b(1-x)\right], \\ \Pr\left(x|b\right) &= \left(\Lambda_b x + 1\right) \exp\left[-\Lambda_b(1-x)\right]. \end{aligned} \tag{5}$$

The second equation follows from differentiating the first. By weighting the density of match quality at b by the outflow rate $\delta_b + \lambda_b(1-x)$, we obtain the density of match quality x in jobs that terminate at date b:

$$\Pr(x|b) = \frac{\left[\delta_b + \lambda_b(1-x)\right]\left(\Lambda_b x + 1\right) \exp\left[-\Lambda_b(1-x)\right]}{\delta_b + \lambda_b\left[\Lambda_b^{-1} - \Lambda_b^{-2}\left(1 - \exp\left[-\Lambda_b\right]\right)\right]}.$$

We can further condition x on whether the job was terminated by either a lay off (occurring at rate δ_b) or a quit (ocurring at rate $\lambda_b\left(1-x\right)$). In the former case all matches are weighed equally since we assumed the lay off rate to be independent of job quality. In the latter case matches are weighed by the moving probability of moving to a better match, $1-x$. Hence, low quality matches are weighed more heavily. In the former case, the expected match quality satisfies:

$$\begin{aligned}\mathrm{E}(x|b, \text{lay off}) &= \int_0^1 x\left(\Lambda_b x + 1\right) \exp[-\Lambda_b(1-x)]dx, \\ &= 1 - \Lambda_b^{-1} + \Lambda_b^{-2}\left(1 - e^{-\Lambda_b}\right) \cong 1 - \Lambda_b^{-1}, \end{aligned} \tag{6}$$

[1]Where we use: $x = e^{\ln x} \cong e^{-(1-x)}$ and $\ln\left[\delta_b + \lambda_b(1-x)\right] \cong \ln \delta_b + \frac{\lambda_b}{\delta_b}(1-x)$.

while in the latter case it satisfies:

$$\begin{aligned} \mathrm{E}(x|b,\text{quit}) &= \int_0^1 \frac{\Lambda_b}{1-\Lambda_b^{-1}\left(1-e^{-\Lambda_b}\right)}(1-x)x\left(\Lambda_b x+1\right)\exp[-\Lambda_b(1-x)]dx \quad (7) \\ &= 1-2\frac{\Lambda_b+\Lambda_b e^{-\Lambda_b}+2e^{-\Lambda_b}-2}{\Lambda_b+e^{-\Lambda_b}-1}\Lambda_b^{-1} \cong 1-2\Lambda_b^{-1}. \end{aligned}$$

The expected match quality is increasing faster with b for jobs ending with a quit than with a lay off. Hence, Λ_b^{-1} is twice as informative on x when the job ends with a quit than when it ends with a layoff. When Λ_b goes to infinity, the expected match quality converges to the upper support of the offer distribution in both cases.

Alternatively we can use the approximation $e^{-\Lambda_b(1-x)} \cong x^{\Lambda_b}$, which applies for large values of Λ_b since then $x \to 1$. The expectation of x conditional on the termination with a lay off is then equal to the first order statistic (the maximum) from $\Lambda_b + 1$ draws:

$$\mathrm{E}(x|b,\text{lay off}) \cong \mathrm{E}(\underline{x}_{\Lambda_b+1|\Lambda_b+1}) = 1-(\Lambda_b+2)^{-1} \cong 1-\Lambda_b^{-1}.$$

where $\underline{x}_{j|N}$ refers to the j^{th} order statistic from N draws. Λ_b measures the expected number of draws while employed, the term 1 measures the draw obtained when unemployed, which led the worker into employment at time 0. The expectation of x conditional on termination with a quit is equal to the second order statistic (the second highest value) from $\Lambda_b + 2$ draws:

$$\mathrm{E}(x|b,\text{quit}) \cong \mathrm{E}(\underline{x}_{\Lambda_b+1|\Lambda_b+2}) = 1-2(\Lambda_b+3)^{-1} \cong 1-2\Lambda_b^{-1}.$$

The additional draw compared to the expression for the termination by a lay off is draw at time b that leads to the transition to a new job; $\underline{x}$ for the new job must be greater than x, hence x is the second highest draw.

2.1.1 The shape of the offer distribution

Till sofar, we have not given an interpretation of the match quality indicator x. For the analysis of wages setting, we have to take a stance on the interpretation of x. It is convenient to interpret x as match productivity. The value of leisure is assumed to be equal zero. Since x converges quickly to the upper support of the offer distribution, $1-x \cong -\ln x$. As long as wages do not deviate too much from productivity (as they won't for sufficiently small search frictions), this implies that $w \cong 1+\ln w$. These

relations will be used in our empirical analysis. For the sake of convenience and in line with much of the search literature, we consider the limiting case of the discount rate going to zero.

Till sofar we have assumed x to be uniformly distributed. This assumption is appropriate when the distribution is sufficiently thin tailed and results in a mean productivity that increases at the order $-\Lambda_b^{-1}$ and converges to a finite maximum. Instead, assume the distribution to be fat tailed, e.g. exponential or Gumbel, $\Pr(\underline{x} < x) = \exp\left(-e^{-(x-\mu)/\sigma}\right)$ with mean $\mu + \sigma\gamma$ and variance $\pi^2\sigma^2/6$, where γ and π are well known constants. In that case, the max after Λ periods satisfies:[2]

$$\begin{aligned} \Pr(\underline{x}_1 < x|\Lambda) &= F(x)e^{-\Lambda[1-F(x)]} \cong F(x)^{\Lambda+1} \\ &= \exp\left(-e^{-(x-\mu)/\sigma}\right)^{\Lambda+1} = \exp\left(-e^{-(x-\mu-\sigma\ln(\Lambda+1))/\sigma}\right), \\ \mathrm{E}[x|\Lambda] &\cong \mu + \sigma\left(\ln\Lambda + \Lambda^{-1}\right) + \sigma\gamma. \end{aligned} \tag{8}$$

The conditional mean increases with $\ln\Lambda$ instead of Λ^{-1} and hence does not converge to a finite maximum, while the variance does not depend on $\ln\Lambda$. This difference in the long run properties for wages allows testing the shape of the offer distribution, by checking whether wages converge to a constant or keep increasing, though at an ever slower rate. Similar to equation (6) and (7) we can calculate the expected productivity conditional on the termination date b. We use the feature that the expectation of x is the first order statistic after $\Lambda_b + 1$ draws when the job ends with a lay off, while it is equal to the second order statistic after $\Lambda_b + 2$ draws when the job ends with a quit. This implies:

$$\begin{aligned} \mathrm{E}[x|b, \text{lay off}] &\cong \mathrm{E}(\underline{x}_{\Lambda_b+1|\Lambda_b+1}) \cong \mu + \sigma\gamma + \sigma\left(\ln\Lambda_b + \Lambda_b^{-1}\right), \\ \mathrm{E}[x|b, \text{quit}] &\cong \mathrm{E}(\underline{x}_{\Lambda_b+1|\Lambda_b+2}) \cong \mu + \sigma\gamma + \sigma\left(\ln\Lambda_b - 1 + \Lambda_b^{-1}\right), \end{aligned} \tag{9}$$

The expected match quality is increasing slower with b for jobs ending with a quit than with a lay off, but the difference unlike that for the case of a uniform productivity distribution is negligible, see equation (6) and (7). If the true distribution is a mixture of a finite support and a fat right tail distribution we would therefore expect that the finite support component dominates.

[2] We use the approximation $e^{-\Lambda[1-F(x)]} \cong F(x)^{\Lambda}$. For the case of an exponential distribution it is easy to see that $\Pr(\bar{x} < x + \frac{1}{\gamma}\ln\Lambda) \cong (1 - e^{-\gamma x - \ln\Lambda})^{\Lambda} = (1 - e^{-\gamma x}\Lambda^{-1})^{\Lambda} \cong \exp[e^{-\gamma x}]$.

The variance of match productivity of employed workers can be calculated as the sum of the expected productivity conditional on the labour market history Λ_b (the between group variance) and the variance of productivity conditional on the labour market history (the within group variance):

$$\mathrm{Var}(x) = \mathrm{Var}(\mathrm{E}\left[x|\ln\Lambda_b\right]) + \mathrm{Var}(x|\ln\Lambda_b) = \left[\mathrm{Var}\left(\ln\Lambda_b\right) + \frac{\pi^2}{6}\right]\sigma^2. \tag{10}$$

2.2 Alternative models of wage setting

What would be the implications of alternative wage setting mechanisms for the relation between log wages and labour market time? We consider three wage setting mechanisms. The critical issue is the variation in wage setting over the business cycle. First, we analyze the steady state version of these models under the assumption that the job offer arrival rate and the job destruction rate are constant ($\lambda_t = \lambda$ and $\delta_t = \delta$). Next, we consider how an unexpected change in λ_t and δ_t would affect wages in new and ongoing jobs. In wage posting models and the PV&R model, these responses are asymmetric: wages can go up in the upturn, but cannot be lowered in downturn. Finally, we provide a qualitative discussion of the effect of workers' and firms' anticipations of future changes in labour market conditions.

2.2.1 Nash Bargaining

As a benchmark, we consider Nash bargaining on the current surplus. For convenience, we assume the threat point of the firm to be zero. Hence, Nash bargaining implies:

$$v(x) - v\left(0\right) = \beta s(x),$$

where $v\left(x\right)$ and $s\left(x\right)$ are the returns for a unit interval of time to the asset values of employment and the total surplus respectively; by construction, $v\left(0\right)$ is the return to the asset value of unemployment. The Bellman equations for a worker and the total

surplus read:

$$\begin{aligned} v(x) &= \frac{w(x)+\kappa\int_x^1 v(z)dz}{1+\kappa(1-x)} = \frac{w(x)+\beta\kappa\int_x^1 s(z)dz - v(0)}{1+\kappa(1-x)} + v(0), \\ s(x) &= \frac{x-\beta\kappa\int_0^x s(z)dz}{1+\kappa(1-x)} = \frac{\beta^{-1}w(x)-\kappa\int_0^x s(z)dz}{1+\kappa(1-x)}, \\ w(x) &= \beta x + \beta(1-\beta)\kappa\int_0^x s(z)dz. \end{aligned} \tag{11}$$

where $\kappa \equiv \lambda/\delta$. The steady state flow condition provides a simple rule of thumb for the value of κ. Let ψ be the efficiency of on-the-job search relative to job search when unemployed. Hence, the job offer arrival rate when unemployed is λ/ψ. The steady state condition for the equality of in- and outflow in and out unemployment yields:

$$u = \frac{1}{1+\psi^{-1}\kappa} \Rightarrow \kappa \cong \psi u^{-1}, \tag{12}$$

where u is unemployment rate. Estimates of ψ are around 0.5, see Pieter Gautier and Coen Teulings (Forthcoming). Hence, $u \cong 5\%$ implies $\kappa \cong 10$. The solution for $s(x)$ and $w(x)$ reads:

$$\begin{aligned} s(x) &= \frac{\kappa^{-1}}{1-\beta}\left[\left(\frac{\kappa^{-1}+1-x}{\kappa^{-1}+1}\right)^{\beta-1} - 1\right], \\ w(x) &= \left(1+\kappa^{-1}\right)\left[1-\left(\frac{\kappa^{-1}+1-x}{\kappa^{-1}+1}\right)^{\beta}\right] \\ &\cong 1-\kappa^{-\beta}\left[1+\beta\kappa(1-x)\right] = 1-\psi^{-\beta}u^{\beta}\left[1+\beta\psi u^{-1}(1-x)\right]. \end{aligned} \tag{13}$$

The final two steps follow from a Taylor expansions around the uppersupport of the offer distribution $x = 1$ and substituting κ for equation (12). Consider an unexpected change in the unemployment rate. Since firms and workers bargain continuously the wages will be determined by the equation (13). The Nash bargaining model predicts therefore that the wage rate in the best matches is most sensitive to fluctuations in the level of the unemployment rate:

$$\frac{w_{x\tilde{u}}}{w_x} \cong -\frac{1-\beta}{100u} \cong -0.1, \tag{14}$$

where the subscripts denote the partial derivatives of w, where $100u \cong 5$ is the unemployment rate in per cent, and where we pick $\beta \cong 0.5$ in the final expression.

Using the approximate density of x, see equation (6), the expectation of w conditional on the labour market history during the current employment cycle reads:

$$\mathrm{E}\left[w|b\right] = \int_0^1 \Pr(x|b) w\left(x\right) dx \cong 1 - \psi^{-\beta} u^{\beta} \left(1 + \beta\psi u^{-1} \Lambda_b^{-1}\right). \tag{15}$$

Equation (15) shows that the wage depends on the labour market experience up till the end of the current job Λ_b. As we have discussed in the previous section, this statistic is a good proxy for the highest offer. When Λ_b tends to infinity, a further increase in labour market experience does not lead to a higher expected wages, since the match quality reaches the uppersupport of the offer distribution. For high Λ_b , the wage converges to $1-\psi^{-\beta}u^{\beta}$, which converges to the upper support of the offer distribution for low labour market frictions, the unemployment being close to zero $u \to 0$.

What happens when workers and firms form expectations on future variation in u? Consider the case where u switches back and forth between u^- and u^+. The wage is still given by equation 11 but the surplus is different. The fact that agents expect the labour market conditions to change dampens the change in surplus coming from a change in u_t and the wage change is therefore also dampened.

In the case that the offer distribution has a fat right tail, the regression is similar: instead of the inverse of the labour market history, its log used, see Appendix B.

2.2.2 Wage posting

Next, we consider the implications of wage posting. There are two versions of this model. In the one version, firms can commit to pay a wage prior to contacting the worker. A higher wage offer serves to increase the hiring rate and reduce the quit rate. In the other version, firms cannot explicitly commit to future wage payment, but nevertheless have an incentive to pay fixed wage as to avoid the worker quitting. Reneging on this implicit commitment today will undermine the credibility of this type of commitment in the future, see GV&T for details. For the analysis of non-steady state wage setting, the latter version is more easy to analyse since wage setting depends only on the offer distribution, not on the wage distribution among employed workers. The former is independent of past wage setting, the latter not. As is well known, the derivative of

wage offer function in a wage posting model satisfies:

$$w'(x) = \kappa \frac{x - w(x)}{1 + \kappa(1 - x)},$$

assuming that the employers pay retention premiums only.[3] Using $w(0) = 0$ as an initial condition, the solution to this differential equation reads:

$$\begin{aligned} w(x) &= x + \left(\kappa^{-1} + 1 - x\right) \ln\left(\frac{\kappa^{-1} + 1 - x}{\kappa^{-1} + 1}\right) \\ &\cong 1 + \kappa^{-1} \ln\left(\frac{1}{\kappa + 1}\right) - (1 - x) \ln \kappa \cong 1 - \ln\left(\frac{\psi}{u}\right)\left[\frac{u}{\psi} + (1 - x)\right]. \end{aligned} \tag{16}$$

The final two steps follow again from a Taylor expansion around the uppersupport of x and applying equation (12).

Similar to the case of Nash bargaining, wages are most sensitive to labour market conditions for the best matches:

$$\frac{w_{x\tilde{u}}}{w_x} \cong \frac{1}{100u \ln(\psi^{-1}u)} \cong -0.1. \tag{17}$$

The ratio of the cross derivative to the derivative is the approximately equal for wage posting and Nash bargaining. Hence, this ratio does not provide much mileage to distinguish between both models. The expectation of w conditional on b reads:

$$\mathrm{E}[w|b] \cong 1 - \ln\left(\frac{\psi}{u}\right)\left[\frac{u}{\psi} + \Lambda_b^{-1}\right] \tag{18}$$

In this model where firms do not pay hiring premiums, the effect of fluctuations in labour market tightness is simple. First, consider an unexpected permanent change in u. Since firms care only about outside offers and since the offer distribution is constant, the only effect on wage setting for new jobs is the direct effect of the change in u. Like in the case of Nash bargaining, wage posting models predict that the wage rate in the best matches is most sensitive to fluctuations in labour market tightness. For ongoing jobs, firms cannot reduce wages since they have committed not to do so.[4] However, firms can increase wages when the labour market tightens. They will find it profitable

[3] The right hand side has to be multiplied by a factor two for the case that employers pay both hiring and retention premiums, see GT&V for a discussion.

[4] This is a particular interpretation of the version of wage posting where firms pay retention premiums only. Alternatively, firms could commit to paying a particular wage until the moment they

to do so. In fact, they will do so in the current model, just applying equation (16). This leads to a asymmetry in response of wages to the labour market conditions, similar to the findings by Beaudry and DiNardo (1991).

Note the difference to the case when firms pay hiring premiums. In that case, firms will only increase their wage offer when the labour market tightens substantially, since the worker has already accepted the job the firms will only have an incentive to increase pay for the purpose of retention. Hence, wages will not be changed unless the optimal wage offer paying a retention premium only is higher than the old wage paying both a retention and hiring premium. With hiring premiums, the world would therefore look more like Bils (1985) than like Beaudry and DiNardo (1991). In particular, the unemployment rate at the start of the job would be a good predictor of wages.

Under wage posting the efficiency of mobility decisions is not guaranteed. When the unemployment rate is low, firms pay higher wages. When unemployment increases later on, firms can therefore not respond by reducing the wages of their incumbent workers. The wage paid to new hires is not bound by this commitment. Hence, firms offer lower wages to them. Hence, when the productivity of an outside offer is only marginally better than in the current job, the worker does not move since the new job is paying a lower wage. The change in the minimum unemployment rate for job changers is therefore positively correlated related to the change in the match quality. Mobility will therefore also be lower after an increase in the unemployment rate. This inefficiency temporarily distorts the relation between labour market time Λ_b and match productivity x: after an increase in the unemployment rate, the standard effect of Λ_b^{-1} on x will overestimate the true effect. Since the minimum unemployment rate is posively related to this bias, standard OLS applied to equation (18) yields an upward bias in the estimated coefficient for the minimum unemployment rate, biassing it towards zero.

Again, when workers and firms form expectations on the future state of the labour market, these expectations dampen the direct effect of actual changes in labour market conditions: when firms expect a loosening of market conditions, they offer lower wages

announce to change it, which they would do only when the labour market tightness goes down. This version of the model would both support the payment of credible retention premiums and allow firms to reduce the wages when the labour market tightness decreases. The version of the model where firms cannot reduce wages turns out to be consistent with the data.

as to avoid that they pay their workers too much after this loosening. In the case that the offer distribution has a fat right tail, the regression is again similar only the log of the labour market history is used instead of the inverse, see Appendix B.

2.2.3 Bertrand competition

Finally, consider the PV&R where wages are set by Bertrand competition between the current and an incoming employer. Let $v(\chi, x)$ and $w(\chi, x)$ be return on the asset value and the wage rate respectively, both as a function of the highest offer x and the one but highest offer χ. Under Bertrand competition, the return to the asset value of unemployment will be equal to the value of leisure. In that case, the Bellman equation for $v(\chi, x)$ reads:

$$v(\chi, x) = w(\chi, x) + \kappa \int_{\chi}^{x} v(z) - v(\chi)\, dz + \kappa \int_{x}^{1} v(x) - v(\chi)\, dz,$$

The first term on the right hand side is the wage, the second term is the option value of receiving an outside offer that forces your employer to raise its wage offer, while the final term is the option value of getting an offer of a more productive firm. Due to Bertrand competition, the return on the asset value is fully determined by the one but highest job offer χ. Hence: $v(\chi, x) = v(\chi)$. Consider the wage rate in case the highest and the one but highest job offer are equal $\chi = x$. In that case, both competing firms will bid the wage up to the productivity level: $w(\chi, \chi) = \chi$. It is easy to see that in that case both option values vanish. Hence: $v(\chi) = \chi$. Substituting these results in the Bellman equation yields:

$$\begin{aligned} \chi &= w(\chi, x) + \kappa \int_{\chi}^{x} (z - \chi)\, dz + \kappa (x - \chi)(1 - x) \\ &= w(\chi, x) + \frac{1}{2}\kappa (x - \chi)(2 - x - \chi), \\ w(\chi, x) &= \chi - \frac{1}{2}\kappa (x - \chi)(2 - x - \chi). \end{aligned} \tag{19}$$

We use the density of x and χ conditional on b and t to calculate the expectation of

w conditional on the labour market history. This yields:

$$\begin{aligned} \mathrm{E}[w|b,t] &= \int_0^1 \Pr(x|b) \int_0^x \Pr(\chi|t,x) w(\chi,x) d\chi dx \\ &= \int_0^1 \Pr(x|b) \left[w(x,x) - \int_0^x \Pr\left(\underline{\chi} < \chi | t,x\right) w_\chi(\chi,x) d\chi \right] dx \\ &\cong 1 - \Lambda_b^{-1} - \Lambda_t^{-1} - \kappa \Lambda_b^{-1} \Lambda_t^{-1} + (1-\kappa)\left[\Lambda_b^{-2} + \Lambda_t^{-2}\right] \end{aligned} \tag{20}$$

The second line follows from integration by parts, while the third line applies equation (6), see Appendix A for details. This is a convenient formula as it shows the log wage to depend on the labour market experience up till the end of the current job Λ_b and the current labour market time Λ_t only. Roughly speaking, the former approximates the highest job offer till sofar x, while the second approximates the one but highest offer χ. Labour market experience at the end of the current job is a good proxy for the highest offer, because it is informative of the likelihood of getting an even better offer. The third line shows that the relation of w to Λ_b and Λ_t is well approximated by entering just two terms, the inverses of both labour market histories. When Λ_b and Λ_t tend to infinity, log wages converges to the maximum log productivity in the job offer distribution, $x = 1$.

Interestingly, the parameter κ and hence u only enters via the second order effects. This has consequences for the analysis of non-steady state behavior, since changes in u_t affect wages only via the option value which does not change much if the second highest offer is close to the upper support of the distribution.

3 Empirical analysis

3.1 Data

We use the data as constructed by H&M from the NLSY79. The cross-sectional sample is used over the years from 1979 to 2004. Jobs with fewer than 15 hours a week or with a shorter duration than 4 weeks as well as observations with an hourly wage lower or equal to \$0.1 or higher or equal to \$1,000 are dropped. Following H&M, wages are deflated by CPI. Observations are the times when a worker changes job or is interviewed. The variable Λ is constructed by summing the market tightness (vacancies over the unemployment rate) over an interval of time. Market tightness is constructed using

Table 1: Summary Table

Variables	Mean	SD	N
w	0.85	0.65	42741
u_t	6.49	1.43	42741
u_{min}	6.05	1.42	42741
$\ln \Lambda_b$	7.72	1.37	42741
$\ln \Lambda_t$	7.1	1.47	42741
$\ln \Lambda_T$	8.77	0.93	42741
$\ln \Lambda_{0a}$	7.1	1.14	15437
$\ln \Lambda_{ab}$	7.47	1.4	42741
$\ln \Lambda_{at}$	6.75	1.43	42741

the national seasonally adjusted versions of the unemployment rate and help-wanted advertising index constructed by the Bureau of Labor Statistics and the Conference Board respectably. Employment cycles are defined as the time elapsed since the last unemployment spell. To avoid classifying job movers as unemployed between the jobs H&M define unemployment as either a spell lasting longer than 8 weeks or they use the stated reason for leaving the job. For further details on the data, we refer to their paper.

3.2 Estimation

All our regressions include the same set of standard controls as used by H&M, including a full set of annual dummies for the incomplete job tenure and experience and quadratics in years of education and time, as well as dummies for industry, region, race, union status, marriage, and Standard Metropolitan Statistical Area (SMSA). Like H&M, we use the custom weighting of observations provided by the NLSY. We use the unemployment rate as a proxy for the labour market tension in order for the results to be comparable with the existing literature. Table 1 provides the summary statistics for the relevant variables. In the regressions we use deviations from the means of the variables to make the interactions terms easier to interpret.

3.2.1 Test of on-the-job search

Table 2 reports a set of regressions which guide our subsequent analysis. All reported standard errors are robust. Column 1 reports a regression that apart from the standard controls and the current labour market state u_t (which is included in all regressions), includes only the logarithm and the inverse of total labour market experience till the end of the current job, $\ln \Lambda_b$ and Λ_b^{-1}. As discussed in section 2 these are the most informative statistics for the current job quality if the right tail of the distribution is either fat or thin respectively. Column 1 provides strong support to the basic idea of search model with OJS, where workers move to ever better jobs/matches during the course of an employment run. Here, the distinction between calendar and labour market time provides great value added. Where learning proceeds in calendar time, job hopping proceeds in labour market time. The fact that we find a strongly significant effect of labour market time over and above calendar time (as captured by the full set of experience and tenure dummies) shows that job hopping is at least part of the story. Column 3 shows that the inverse comes in with the wrong sign from as predicted by a model with finite upper support whereas the sign matches that predicted by a fat right tail, see equation 9. The logarithm is highly significant which gives further evidence for a fat right tail. In Column 4 we include higher order terms. We therefore conclude that the logarithm is the appropriate form. This conclusion is theoretically important as it rules out particular type of matching models. In pure sorting models with assortative matching where matching sets are characterized by interior upper and lower bounds, like Shimer and Smith (2000) and GT&V, the uppersupport of the offer distribution is finite. The estimation results in column 1 are inconsistent with this prediction. Though there is empirical support for this type of sorting, see Gautier and Teulings (Forthcoming), it cannot be the full story. In all subsequent regressions, we use the logarithmic transformation.

It could be that the estimated effects from $\ln \Lambda_b$ is not due to OJS but instead because of the total labour market experience the worker experienced which is closely correlated with $\ln \Lambda_b$. To test for this we include the total labour market experience Λ_T which is the current labour market history Λ_b plus the labour market history from the worker's previous employment cycles. We also add the log of the number of jobs. In

Table 2: Wage regression on inverse and logs

Variables	Column 1	Column 2	Column 3	Column 4
u_t	-0.017837***	-0.020537***	-0.022334***	-0.022121***
	(2.4772e-03)	(2.5860e-03)	(2.5772e-03)	(2.5777e-03)
$\ln \Lambda_b$	0.086859***		0.11057***	0.088837***
	(3.5522e-03)		(5.5272e-03)	(0.015717)
Λ_b^{-1}		-33.7173***	21.7943***	-11.4691
		(2.62802)	(3.96002)	(22.4126)
Λ_b^{-2}				1,410.56
				(1,419.11)
$\ln \Lambda_b^2$				0.014034*
				(7.4852e-03)
Observations	42,741	42,741	42,741	42,741
R-squared	0.56720	0.56090	0.56778	0.56784

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3: Wage regression on match quality controlling for total labourmarket experience

Variables	Column 1: Full Sample	Column 2: 1-3	Column 3: 4-6	Column 4: >6
u_t	-0.018363***	-0.012258***	-0.023563***	-0.030044***
	(2.4864e-03)	(3.7402e-03)	(4.7199e-03)	(5.8781e-03)
$\ln \Lambda_b$	0.067328***	0.050569***	0.073685***	0.037631***
	(5.7871e-03)	(0.012334)	(0.013852)	(0.011092)
$\ln \Lambda_T$	0.064033***	0.053942***	0.057663**	0.14085***
	(0.011272)	(0.017672)	(0.027465)	(0.041948)
$\ln k$	7.9798e-03	0.038290**	-5.7263e-03	0.022972
	(8.4737e-03)	(0.016826)	(0.017430)	(0.016529)
Observations	42,741	13,920	14,216	14,605
R-squared	0.56768	0.72428	0.67029	0.55870

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3 we see that the labour market history is still highly significant.[5] Our theoretical analysis assumes we correctly classify the unemployment spells. If this is not the case we are likely to estimate a positive coefficient on Λ_T. In Column 2-4 the predictions of models with OJS is put to a further test. The effect of $\ln \Lambda_b$ should be the same for all employment runs. Column 2 runs the regression for the first three employment runs of each respondent only, column 3 runs it for employment run 4-6 and column 4 for the subsequent employment runs. Though the coefficient for $\ln \Lambda_b$ is somewhat lower for later employment runs, its coefficient remains of the same order of magnitude and highly significant, even though $\ln \Lambda_T$ is include in the specification. Hence, labour market time since the last layoff really matters for wages. This is a strong confirmation of the basis search model. The number of job changes k should affect wages positively. The results for this variable are mixed, however.

[5]The results are similar when employment spells with low hours or short tenures are not counted.

Table 4: Test using reason for separating

Variables	Column 1: Full Sample (U)	Column 2: 1st Job (U)	Column 3: Full Sample (E)	Column 4: 1st Job (E)
u_t	-0.013978***	-0.013249***	-0.021562***	-0.027910***
	(3.4100e-03)	(3.9638e-03)	(4.6764e-03)	(6.2146e-03)
$\ln \Lambda_b$	0.074620***	0.077900***	0.087607***	0.080222***
	(5.4627e-03)	(7.2677e-03)	(8.0274e-03)	(0.013595)
Observations	20,187	14,927	12,153	7,058
R-squared	0.57680	0.58806	0.66502	0.73145

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

As discussed in Section 2.1, see equation (6) and (7), the expected productivity of match depends conditional on the labour market history Λ_b depends on whether it ends by a quit to another job (E) or by a layoff into unemployment (U): productivity should be related more strongly to productivity when the job ends by a quit. As a further test we run the regressions of wages on labour market history for both groups separately. In Table 4 column 1 reports results for workers subsequently moving into unemployment and in column 2 we further restrict the analysis to the first jobs within an employment cycle. Column 3 and 4 repeats this analysis for workers moving to another job. The theory suggest that the increase in wages with $\ln \Lambda_b$ is faster for workers moving to another job and for the first job within a cycle. The data supports the first suggestion whereas the results are mixed for the other. The results are similar for the different samples suggesting that we do not introduce a large error by using the full sample, as we do in what follow for the analysis.

If the wage function is linear in productivity with coefficient α the coefficient estimated in our regressions above is $\alpha\sigma$. Multiplying the expression for the variance of match quality, Equation 10, with α^2 gives the total contribution of the selection process to wage dispersion. We estimate the coefficient $\alpha\sigma$ to be about 0.09. This contribution

is made up of two components, the between labour market histories variance and the within history variance:

$$\begin{aligned} \text{Var}(w) &= \left[\text{Var}(\ln \Lambda_b) + \frac{\pi^2}{6}\right] \left(\frac{\partial w}{\partial \ln \Lambda_b}\right)^2 \\ &\cong \left(1.4^2 + \frac{\pi^2}{6}\right) \times 0.09^2 \cong 0.03. \end{aligned}$$

We use Table 1 for the standard deviation of $\ln \Lambda_b$ and regression coefficients from Table 2 column 1 for $\partial w / \partial \ln \Lambda_b$. This calculation suggests that search frictions account for less than 10 % of the total variance of wages (which is about 0.4). Most studies report a higher contribution of difference in match quality to the total variance. Gautier and Teulings (Forthcoming) distinguish carefully between the effect on productivity and on wages, the former being smaller than the latter in a wage posting model. The estimate reported here is between their estimate for wage (2.5%) and their estimate for productivity (6%-8%).

3.2.2 Test of different wage setting models

Table 5 adds the current labour market experience $\ln \Lambda_t$ to $\ln \Lambda_b$, capturing the PV&R model (equation 20) and also the higher order terms. Column 2 enters second order terms in $\ln \Lambda_b$ and $\ln \Lambda_t$. The data do not support the implications from PV&R as the first and second order effect of $\ln \Lambda_t$ are insignificant and only the interaction comes in significantly. In Column 4 the interaction of the cross term $\ln \Lambda_t \times \ln \Lambda_b$ with the current unemployment rate is added, see equation (20). Models with Bertrand competition predict that the current labour market experience $\ln \Lambda_t$ is an important positive determinant of wages. In all our regressions we find a positive and significant coefficient on labour market history $\ln \Lambda_b$ but an insignificant coefficient with the wrong sign on labour market experience $\ln \Lambda_t$, suggesting that OJS but not Bertrand competition is supported by the data.

Table 6 provides some further evidence regarding the PV&R model. The model predicts that the starting wage in the first job of an employment cycle depends negatively on the quality of the job, since a better job provides a greater potential for future upward bargaining, see equation (19). Hence, the employer can afford to pay a lower wage.

Table 5: Wage regression test for PV&R

Variables	Column 1	Column 2	Column 3	Column 4
u_t	-0.018378***	-0.021258***	-0.020347***	-0.018191***
	(4.5311e-03)	(4.6903e-03)	(4.5955e-03)	(4.6459e-03)
$\ln \Lambda_b$	0.089808***	0.092365***	0.094961***	0.093610***
	(5.5867e-03)	(5.5152e-03)	(5.5910e-03)	(5.5447e-03)
$\ln \Lambda_t$	-4.5976e-03	6.8221e-03	-0.011130	-0.012324
	(5.1203e-03)	(5.2932e-03)	(8.0086e-03)	(8.0064e-03)
$\ln \Lambda_b^2$		0.011170***	7.2050e-03**	7.1463e-03**
		(2.8728e-03)	(2.9335e-03)	(2.9282e-03)
$\ln \Lambda_t^2$		2.7633e-03	-3.1721e-04	-1.6913e-03
		(2.1169e-03)	(2.4606e-03)	(2.6810e-03)
$\ln \Lambda_b \ln \Lambda_t$			2.7709e-06***	2.5773e-06***
			(7.6088e-07)	(7.2092e-07)
$u_t \ln \Lambda_b \ln \Lambda_t$				-6.7661e-07**
				(3.0197e-07)
Observations	42,741	42,741	42,741	42,741
R-squared	0.56722	0.56783	0.56800	0.56810

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 6: Wage regressions for the first job

Variables	Column 1: 8w	Column 2: 26w	Column 3: 52w
u_t	-0.016617	-0.013450**	-0.013215***
	(0.010989)	(5.2390e-03)	(4.4547e-03)
$\ln \Lambda_b$	0.067009***	0.074023***	0.080274***
	(0.013520)	(6.7266e-03)	(6.2589e-03)
$\ln \Lambda_t$	-0.021516	1.9432e-03	1.5211e-03
	(0.025194)	(9.0063e-03)	(7.1727e-03)
Observations	5,329	12,582	16,023
R-squared	0.61385	0.50821	0.49850

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Column 1, 2 and 3 provides a clean test this prediction using the first observations from the first 8, 26 and 52 weeks in the first job respectively. $\ln \Lambda_b$ is a good proxy for the quality of the match, which the theory predicts should affect wages negatively. The data reject this implication: the effect of $\ln \Lambda_b$ is significant but has the wrong sign. Hence, the regression results clearly reject the PV&R model.

Table 7 includes the cross term of the current unemployment rate and the proxy for match quality, that is, $u_t \ln \Lambda_b$, capturing Nash bargaining, see equation (15). The results provide support for Nash bargaining. The cross term is positive with an implied ratio of -0.07 which is close to the ratio predicted by the theory, see equation (17). Column 2 includes the minimum unemployment rate since the start of the current job u_{min} (where $u_{min} = \min_{s \in [a,t]} (u_s)$) and a cross-term $u_{min} \ln \Lambda_b$ instead of the cross term with the current unemployment rate, capturing wage posting, see equation (18) and as in Beaudry and DiNardo (1991). The regression results provide support for wage posting. The crossterm has the appropriate sign, showing that wages in better jobs are more sensitive to cyclical fluctuations. The ratio is -0.084 which again is close to what

Table 7: Wage regression test for wage posting

Variables	Column 1	Column 2	Column 3	Column 4
$\ln \Lambda_b$	0.089967***	0.091309***	0.087488***	0.099555***
	(3.7509e-03)	(3.9075e-03)	(3.6347e-03)	(4.4258e-03)
u_t	-0.020247***	-0.019705***	-0.020611***	-0.019558***
	(2.4788e-03)	(3.6436e-03)	(2.8931e-03)	(4.5350e-03)
$u_t \ln \Lambda_b$	-6.2593e-03***			-2.5587e-03
	(1.4603e-03)			(3.2232e-03)
u_{min}		-4.3302e-03		0.013789*
		(4.6692e-03)		(8.1624e-03)
$u_{min} \ln \Lambda_b$		-7.6305e-03***		-0.022808***
		(1.7552e-03)		(5.3004e-03)
u_a			-1.5361e-04	-0.015132***
			(2.9247e-03)	(4.8978e-03)
$\ln \Lambda_b u_a$			-3.7538e-03**	0.018893***
			(1.7269e-03)	(4.2470e-03)
Observations	42,741	42,741	42,741	42,741
R-squared	0.56747	0.56750	0.56728	0.56781

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

is predicted by the theory. Column 3 includes the unemployment rate at the start of the job instead of the minimum unemployment rate, that is, u_a and $u_a \ln \Lambda_b$, as suggested by wage posting with hiring and retention premiums. Then, column 4 includes both the minimum unemployment rate and the unemployment rate at the start of the jobs and their cross terms and the cross term for Nash bargaining. Only the coefficient consistent with wage posting remains significant. The coefficient on the crossterm $u_{min} \ln \Lambda_b$ has the expected sign whereas the coefficient on the minimum unemployment rate has the wrong sign. CONCLUSION

As we have shown in Section 2, see equation (14) and (17), the ratio of the cross and the level term do not provide much scope for discriminating between Nash bargaining and wage posting, because both models predict largely similar ratios. Again, the regression results provide support for both models, as the estimated ratios fit the predicted signs and magnitudes well.

As discussed in Section 2, the change in the minimum unemployment rate for job changes is negatively correlated with the change in match quality. This leads to a downward bias for the coefficient on u_{min}. We therefore look at wage changes within a job.

Wage posting models with hiring premiums further suggest that wages should be responsive to the minimum unemployment rate only after there minimum unemployment rate has changed substantial from the unemployment rate at the data of hiring, u_a as discussed in Section 2. We test this prediction using an interaction with an indicator variable for when the difference between the minimum unemployment rate and the unemployment rate at the start of the job is larger than 1%.

Table 8 reports the estimation results for this procedure. Column 1 enters u_{min} and $u_{min} \ln \Lambda_b$. Both variables are insignificant with the right sign. The coefficient on $u_{min} \ln \Lambda_b$ is similar to the coefficient from the regressions in levels. The ratio using the first difference regression is -0.09 and therefore very similar to that predicted by the theory. The coefficient on u_{min} is larger than the corresponding coefficient from the regressions in levels, suggesting that there is in fact a downward bias. Column 2 enters $u_t \ln \Lambda_b$. It is insignificant but u_{min} remains significant. Column 3 the the indicator for when the minimum unemployment rate is more than 1% smaller than the unemployment rate at the start of the job. variables interacted with u_{min} and $u_{min} \ln \Lambda_b$. Wage posting models with hiring premiums suggest that the coefficients should be negative as wages only respond after substantial changes in the unemployment rate for the data of hiring. Both coefficients are in fact positive suggesting that models with only retention premiums better capture the wage dynamics..

Table 8: Wage posting test in first differences

Variables	Column 1	Column 2	Column 3
Δu_t	-0.015839***	-0.012026**	-0.014530***
	(4.3541e-03)	(5.2575e-03)	(4.4006e-03)
Δu_{min}	-0.025985***	-0.030931***	-0.057518***
	(8.9380e-03)	(9.7296e-03)	(0.019465)
$\Delta u_{min} \ln \Lambda_b$	-8.2838e-03	-2.4577e-03	-0.027545*
	(5.7608e-03)	(7.2071e-03)	(0.015096)
$\Delta u_{min} I(u_a - u_{min} > 1\%)$			0.031719*
			(0.018812)
$\Delta u_{min} \ln \Lambda_b I(u_a - u_{min} > 1\%)$			0.021379
			(0.016346)
$\Delta u_t \ln \Lambda_b$		-5.2053e-03	
		(3.9421e-03)	
Observations	23,775	23,775	23,775
Number of idd	2,619	2,619	2,619

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

3.3 Comparison with H&M

H&M do not find evidence that labour market history up to the moment of observation matters as in PV&R nor that the unemployment rate at the date of hiring or the minimum unemployment rate over the job spell matters. Rather than using $\ln \Lambda_b$ and $\ln \Lambda_t$ as suggested by the theory, they use $\ln \Lambda_a$ and $\ln \Lambda_{ab} \equiv \ln \left(\int_a^b \lambda_s ds \right)$ as a proxy for match quality. They further include $\ln \Lambda_{at}$ instead of $\ln \Lambda_t$ as a proxy for the number of offers in order to test for PV&R. This approach is problematic as logs are not additive: while $\Lambda_b = \Lambda_a + \Lambda_{ab}$, this equation does not hold in logs. Hence, one could expect that second order terms to come in significant to correct for the misspecification of the functional form. Hence, this test is not robust to the inclusion of higher order terms. In the table below we have added second order terms to their regressions. The inclusion of these order effect makes the first order effect of $\ln \Lambda_{at}$ significant.[6]

Secondly H&M include u_{min} and u_a but not the interactions with $\ln \Lambda_b$. This makes the test weaker as the model and empirical analysis suggests that wages in good matches are most sensitive to cyclical fluctuations. The analysis is further done in levels which gives a biased coefficient on u_{min} according to the wage posting models by the reasoning in the previous section.

4 Conclusion

An innovation of H&M is that in models with OJS the distinction between calendar time and labour market time allows empirically the discrimination between the effect of learning and job hopping on the increase of wages over the career. In the paper we extend this framework and get an exact analytical relation between labour market history and productivity. We provide empirical support for OJS using data for different employment runs. Also, we show how the shape of the offer distribution of match quality, in particular the fatness of the right tail, matters for the evolution of expected wages over time. We provide a test for the fatness of the right tail of the match quality

[6] A further problem with HM estimation is that they assign individuals who do not have a previous job and therefore Λ_a is zero are assigned the individual average of $\ln \Lambda_a$ over all observations with positive Λ_a corresponding to that individual.

Table 9: H&M test for PV&R with higher orders

	(1)	(2)	(3)
Variables	Column 1	Column 2	Column 3
u_t	-0.017822***	-0.014875***	-0.016273***
	(2.6851e-03)	(2.7362e-03)	(2.7695e-03)
$\ln \Lambda_{at}$	4.4167e-03	-0.15888***	-0.10097***
	(5.1152e-03)	(0.032358)	(0.034200)
$\ln \Lambda_a$	0.028450***	0.028511***	0.031189***
	(5.4122e-03)	(5.4164e-03)	(5.5447e-03)
$\ln \Lambda_{ab}$	0.072627***	0.070785***	-0.051602*
	(3.6412e-03)	(3.6533e-03)	(0.030765)
$\ln \Lambda_{at}^2$		0.014656***	0.010246***
		(2.8531e-03)	(2.9734e-03)
$\ln \Lambda_{0a}^2$			0.013425***
			(3.8616e-03)
$\ln \Lambda_{ab}^2$			8.5064e-03***
			(2.0811e-03)
Observations	42,741	42,741	42,741
R-squared	0.566	0.566	0.566

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

distribution. The data supports a model with a fat right tail. Our results therefore reject models of match quality where the upper support of the offer distribution is finite. This implies that pure sorting models with an interior upper and lower bound of the matching set cannot fully explain the data. At least some forces driving the shape of the offer distribution must yield an unbounded upper support.

In addition, the paper reconsidered H&M conclusion that Nash bargaining provides an adequate description of wage setting. First, expressions for the effect of labour market time on job quality yields a precise characterization of the implications of various models of wage setting for wages. The models give stronger empirical predictions than those used in H&M. In particular, the theoretical model of wage posting and Nash bargaining suggests that the wages of high quality matches are the most cyclical. The data supports this conclusion.

The results reject the models with Bertrand competition between firms. The data clearly reject the implication that wages in the first job are decreasing in the quality of that job as is implied in Bertrand competition since the option value of a job is increasing in match quality. Furthermore the model with Bertrand competition suggest the the current labour market time should be an important determinant of wages since it is a proxy for the number of offers received till sofar. Again, the data does not support this conclusion.

The empirical results reject Nash bargaining since the minimum unemployment rate is an important determinant of wages. The data instead support wage posting models. The fact that only wage posting model are consistent with the data has important implications for macro economics, as it leads to inefficiently low job mobility in the downturn of the business cycle. Under Nash bargaining or PV&R wage setting, workers will always move between jobs when a more productive job comes along. Under wage posting that is not necessarily true. In the cross section, wages are increasing in the quality of the job, so that workers will always move when a more productive job offer comes along. However, in the downturn of the business cycle, this is not necessarily true. Firms are likely to post higher wages in the upturn compared to those in the down turn. This inefficiency results in too few job to job changes in the downturn. Furthermore, in the wage posting model where firms pay retention premiums but no hiring premiums, there is excess vacancy creation, see GT&V. Our evidence suggests this is to be the case.

These aspect of job mobility over the cycle deserve further empirical research.

A Appendix Expected wage in PV&R

$$\begin{aligned}
\mathrm{E}[w|b,h,t] &= \int_0^1 \Pr(x|b)\left[x - \int_0^x \frac{\chi}{x}\left[1+\kappa(1-\chi)\right]\exp\left[-\Lambda_t\left(x-\chi\right)\right]d\chi\right]dx \\
&\cong \int_0^1 \Pr(x|b)\left[x - \frac{1+\kappa(1-x)}{\Lambda_t} + \frac{1+\kappa(1-2x)}{x\Lambda_t^2}\right]dx \\
&\cong 1 - \Lambda_b^{-1} - \Lambda_t^{-1} - \kappa\Lambda_b^{-1}\Lambda_t^{-1} + (1-\kappa)\Lambda_b^{-2} + (1-\kappa)\Lambda_t^{-2}
\end{aligned}$$

In the second and the third line, we use:

$$\begin{aligned}
\int_0^x \frac{\chi}{x}\left[1+\kappa(1-\chi)\right]\exp\left[-\Lambda_t\left(x-\chi\right)\right]d\chi &\cong \frac{1+\kappa(1-x)}{\Lambda_t} - \frac{1+\kappa(1-2x)}{x\Lambda_t^2} \\
\int_0^1 \Pr(x|b)xdx &= 1 - \Lambda_b^{-1} + (1-\kappa)\Lambda_b^{-2}.
\end{aligned}$$

In the first step, we ignore terms of order $\exp\left(-\Lambda_t x\right)$ which will small for higher Λ_t and hence x close to unity. In the second line, we observe that the second term will be small relative to the first for $x\Lambda_t \gg 1$. In the third line, we use a similar approximation.

B Wages for a fat tailed productivity distribution

From the central limit theorem for the maximum of a set of i.i.d. random variables we know that if the productivity distribution is fat tailed then mean productivity of the highest draw increases with the log of the labour market history whereas if the distribution is thin tailed the maximum converges to the a fixed maximum. In the the theoretical section we derived the regressions under the assumption of a uniform distribution (thin tailed). Consider the case of an arbitrairy density and distribution function, $f\left(x\right), F\left(x\right)$:

$$\begin{aligned}
s(x) &= \frac{x - \beta\kappa\int_0^x s(z)f(z)dz}{1+\kappa\left[1-F(x)\right]} \\
w\left(x\right) &\cong \beta x + \beta(1-\beta)\kappa\int_0^1 s(z)f(z)dz + \beta(1-\beta)\kappa\frac{1-\beta\kappa\int_0^1 s(z)f(z)dz}{1+\kappa\left[1-F(1)\right]}(1-x)
\end{aligned}$$

where we apply a Taylor expansion around some arbitrary point, $x = 1$. This yields the regression

$$E(w|\Lambda_b) = \alpha_0 + \alpha_1 u + \alpha_2 \ln\Lambda_b + \alpha_3 u\ln\Lambda_b$$

Table 2: Wage regression on inverse and logs

Variables	Column 1	Column 2	Column 3	Column 4
u_t	-0.019060***	-0.022154***	-0.024106***	-0.023867***
	(2.5257e-03)	(2.6448e-03)	(2.6307e-03)	(2.6314e-03)
$\ln \Lambda_b$	0.090432***		0.11687***	0.097604***
	(3.5779e-03)		(5.5609e-03)	(0.015908)
Λ_b^{-1}		-34.5442***	24.3580***	-5.45971
		(2.65744)	(4.00418)	(22.6586)
Λ_b^{-2}				850.074
				(1,437.86)
$\ln \Lambda_b^2$				0.014851**
				(7.5447e-03)
Observations	42,741	42,741	42,741	42,741
R-squared	0.55326	0.54622	0.55399	0.55408

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For the case of wage posting we use

$$\begin{aligned} w(x) &= x - (1 + \kappa [1 - F(x)]) \int_0^x \frac{1}{1 + \kappa [1 - F(z)]} dz \\ &\cong x - (1 + \kappa [1 - F(1)]) \int_0^1 \frac{1}{1 + \kappa [1 - F(z)]} dz + \kappa f(1) \int_0^1 \frac{1}{1 + \kappa [1 - F(z)]} dz (1 - x) \end{aligned}$$

This gives the regression

$$\mathrm{E}(w|\Lambda_b) = \alpha_0 + \alpha_1 u + \alpha_2 \ln \Lambda_b + \alpha_3 u \ln \Lambda_b$$

Table 3: Wage regression on match quality controlling for total labourmarket experience

Variables	Column 1: Full Sample	Column 2: 1-3	Column 3: 4-6	Column 4: >6
u_t	-0.019972***	-0.013461***	-0.024941***	-0.033446***
	(2.5373e-03)	(3.8384e-03)	(4.7431e-03)	(5.9900e-03)
$\ln \Lambda_b$	0.063761***	0.049517***	0.072523***	0.032496***
	(5.8964e-03)	(0.012835)	(0.014127)	(0.011295)
$\ln \Lambda_T$	0.082230***	0.067456***	0.063797**	0.16251***
	(0.011401)	(0.017923)	(0.027568)	(0.041778)
$\ln k$	0.016846*	0.049935***	2.1322e-03	0.033751**
	(8.6453e-03)	(0.017413)	(0.017689)	(0.016883)
Observations	42,741	13,920	14,216	14,605
R-squared	0.55404	0.71586	0.66223	0.54641

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4: Test using reason for separating

Variables	Column 1: Full Sample (U)	Column 2: 1st Job (U)	Column 3: Full Sample (E)	Column 4: 1st Job (E)
u_t	-0.016562***	-0.015749***	-0.023455***	-0.029069***
	(3.5035e-03)	(4.0927e-03)	(4.7295e-03)	(6.2343e-03)
$\ln \Lambda_b$	0.077219***	0.079223***	0.089041***	0.080841***
	(5.5891e-03)	(7.4972e-03)	(8.0503e-03)	(0.013650)
Observations	20,187	14,927	12,153	7,058
R-squared	0.55405	0.56461	0.65785	0.72666

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Wage regression test for PV&R

Variables	Column 1	Column 2	Column 3	Column 4
u_t	-0.019884***	-0.023247***	-0.022414***	-0.020299***
	(4.2630e-03)	(4.4156e-03)	(4.3220e-03)	(4.3918e-03)
$\ln \Lambda_b$	0.094888***	0.098578***	0.10092***	0.099597***
	(5.5701e-03)	(5.5132e-03)	(5.5826e-03)	(5.4774e-03)
$\ln \Lambda_t$	-6.9784e-03	4.2762e-03	-0.012112	-0.013276*
	(5.0357e-03)	(5.0492e-03)	(7.8443e-03)	(7.8380e-03)
$\ln \Lambda_b^2$		0.014073***	0.010445***	0.010391***
		(2.8560e-03)	(2.8607e-03)	(2.8557e-03)
$\ln \Lambda_t^2$		1.4767e-03	-1.3303e-03	-2.6769e-03
		(2.1113e-03)	(2.4336e-03)	(2.6568e-03)
$\ln \Lambda_b \ln \Lambda_t$			2.5301e-06***	2.3387e-06***
			(7.4256e-07)	(7.0568e-07)
$\ln \Lambda_b \ln \Lambda_t u_t$				-6.6327e-07**
				(3.0284e-07)
Observations	42,741	42,741	42,741	42,741
R-squared	0.55329	0.55407	0.55421	0.55431

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 6: Wage regressions for the first job

Variables	Column 1: 8w	Column 2: 26w	Column 3: 52w
u_t	-0.016818	-0.016011***	-0.016061***
	(0.011267)	(5.3792e-03)	(4.5481e-03)
$\ln \Lambda_b$	0.069624***	0.074290***	0.080382***
	(0.013949)	(6.8893e-03)	(6.3527e-03)
$\ln \Lambda_t$	-0.014718	2.1469e-03	-3.3200e-04
	(0.025835)	(9.2271e-03)	(7.2974e-03)
Observations	5,329	12,582	16,023
R-squared	0.58883	0.47982	0.47196

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

C Results excluding industry and union control

Table 7: Wage regression test for wage posting

Variables	Column 1	Column 2	Column 3	Column 4
u_t	-0.021223***	-0.022184***	-0.022439***	-0.024251***
	(2.5223e-03)	(3.6931e-03)	(2.9371e-03)	(4.6629e-03)
u_{min}		-2.0412e-03		0.016669**
		(4.7281e-03)		(8.3174e-03)
$\ln \Lambda_b$	0.093227***	0.094968***	0.091243***	0.10297***
	(3.7812e-03)	(3.9416e-03)	(3.6630e-03)	(4.4732e-03)
$\ln \Lambda_b u_{min}$		-7.2525e-03***		-0.023840***
		(1.7781e-03)		(5.4045e-03)
$\ln \Lambda_b u_t$	-5.6085e-03***			1.1328e-03
	(1.4818e-03)			(3.3053e-03)
u_a			7.6872e-04	-0.013949***
			(2.9564e-03)	(4.9862e-03)
$\ln \Lambda_b u_a$			-3.9216e-03**	0.016577***
			(1.7463e-03)	(4.3026e-03)
Observations	42,741	42,741	42,741	42,741
R-squared	0.55347	0.55353	0.55335	0.55376

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 9: H&M test for PV&R with higher orders

	(1)	(2)	(3)
Variables	Column 1	Column 2	Column 3
u_t	-0.019620***	-0.016759***	-0.018430***
	(2.7454e-03)	(2.7977e-03)	(2.8306e-03)
$\ln \Lambda_{at}$	1.1349e-03	-0.15712***	-0.086617**
	(5.2000e-03)	(0.032860)	(0.034766)
$\ln \Lambda_a$	0.027639***	0.027690***	0.030325***
	(5.4994e-03)	(5.5030e-03)	(5.6448e-03)
$\ln \Lambda_{ab}$	0.077416***	0.075637***	-0.072521**
	(3.6573e-03)	(3.6683e-03)	(0.031205)
$\ln \Lambda_{at}^2$		0.014205***	8.8492e-03***
		(2.8973e-03)	(3.0212e-03)
$\ln \Lambda_{0a}^2$			0.013046***
			(3.9877e-03)
$\ln \Lambda_{ab}^2$			0.010274***
			(2.1087e-03)
Observations	42,741	42,741	42,741
R-squared	0.55146	0.55184	0.55235

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

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