Does the New Keynesian Model Have a Uniqueness Problem?

Lawrence J. Christiano Martin Eichenbaum Benjamin K. Johannsen

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Introduction

- New Keynesian (NK) model has played an important role in thinking about the causes of the Great Recession, as well as possible remedies.

  - Eggertsson and Woodford: collision between reduced spending and zero lower bound as cause of Great Recession.
  - Forward guidance.
  - Government spending multiplier.

- It has been discovered that the NK model has multiple rational expectations equilibria.

  - Benhabib, Schmitt-Grohe, Uribe (2000), Mertens and Ravn (2014), Braun, Körber, Yuichiro Waki (2014).
  - The different equilibria have very different implications for the Great Recession and for policy.

What We Do

- We study multiplicity properties of NK model.
  - Interested in ZLB and non-ZLB.
  - For today, we report results for ZLB.
- Describe the multiplicity problem in ZLB.
- Study usefulness of learnability as an equilibrium selection device.

Findings for Linearized Equilibrium Conditions in ZLB

- Analysis based on linearized equilibrium conditions
  - equilibrium unique and gov't spending multiplier big.
- Size of multiplier and drop in GDP in ZLB:
  - Bigger the more flexible are prices and the longer the expected duration of ZLB.
- Linearization appealing because results are analytic.
  - But, linearization may be misleading.

Findings Based on Actual Equilibrium Conditions in ZLB

- Two equilibria: **not-so-bad** and **really-bad**
- Not so bad: resembles equilibrium identified by linearization.
  - Government spending multipler big
  - Size of multiplier and drop in GDP in ZLB:
    - Bigger the more flexible are prices and the longer the expected duration of ZLB.
- Really bad:
  - *Huge* output drop.
  - Properties of equilibrium reversed.
  - Size of multiplier and drop in GDP in ZLB:
    - *Smaller* the more flexible are prices and the longer the expected duration of ZLB.

Learning

- Not-so-bad equilibrium: stable under learning.
- Really bad equilibrium: not stable under learning.

Outline

- Properties of Rational Expectation Equilibrium.
- Simple example to illustrate learning as an equilibrium selection device.
  - Laffer curve.
- Learning in the New Keynesian model.

Model

- Standard NK model
  - Representative household,
  - Monopolistically competitive firms face price-adjustment costs (Rotemberg, 1982),
  - Government.

- Results based on non-linear analysis of Calvo-pricing model.
  - Very similar conclusions.

Model

- A representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right]$$

  subject to

$$P_t C_t + B_t \leq \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t,$$

- Aggregate output, Υ_t, is produced by representative, competitive final good producer using intermediate goods, Υ_{jt}, $j \in [0,1]$.

$$\Upsilon_t = \left(\int_0^1 \Upsilon_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon \geq 1,$$

Model

- The monopolist that produces the j^{th} good has the following objective:

$$E_t \sum_{l=0}^{\infty} \beta^l v_{t+l} [(1+\nu) P_{j,t+l} Y_{j,t+l} - \overbrace{s_{t+l} P_{t+l} Y_{j,t+l}}^{\text{labor costs of production}}$$

$$- \overbrace{\frac{\phi}{2} \left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1 \right)^2 (C_{t+l} + G_{t+l})}^{\substack{\text{cost (in terms of final goods) of adjusting prices} \\ \text{related to aggregate level of output}}} \times P_{t+l}],$$

- v_t: state and date-contingent value assigned to payments sent to households.

- ν: subsidy to firms to address distortions due to monopoly power.

Equations Defining a RE Equilibrium

$$
\begin{aligned}
R_t &= \max\left\{1, \frac{1}{\beta} + \alpha(\pi_t - 1)\right\} \\
\frac{1}{R_t} &= \frac{1}{1+r} E_t \frac{C_t}{C_{t+1}\pi_{t+1}} \text{ where } \beta = 1/(1+r) \\
(\pi_t - 1)\pi_t &= \frac{1}{\phi}\epsilon(s_t - 1)\frac{Y_t}{C_t + G_t} \\
&\quad + \frac{1}{1+r} E_t(\pi_{t+1} - 1)\pi_{t+1}\frac{C_{t+1} + G_{t+1}}{C_{t+1}}\frac{C_t}{C_t + G_t} \\
Y_t &= h_t = C_t + G_t + \frac{\phi}{2}(\pi_t - 1)^2(C_t + G_t)
\end{aligned}
$$

The ZLB

- As in Eggertsson and Woodford (2003), we assume:
  - $r = r^{\ell} \leq r^{h}$ at time zero
  - r jumps to $r^{h} > 0$ with probability $1 - p$
  - r^{h} is an absorbing state.

- We assume that agents expect equilibrium C and π return to zero-inflation SS when $r = r^{h}$.

- As in EW, focus on equilibria in which

$$C_t = C^{\ell}, \ \pi_t = \pi^{\ell},$$

  for all t while ZLB lasts.

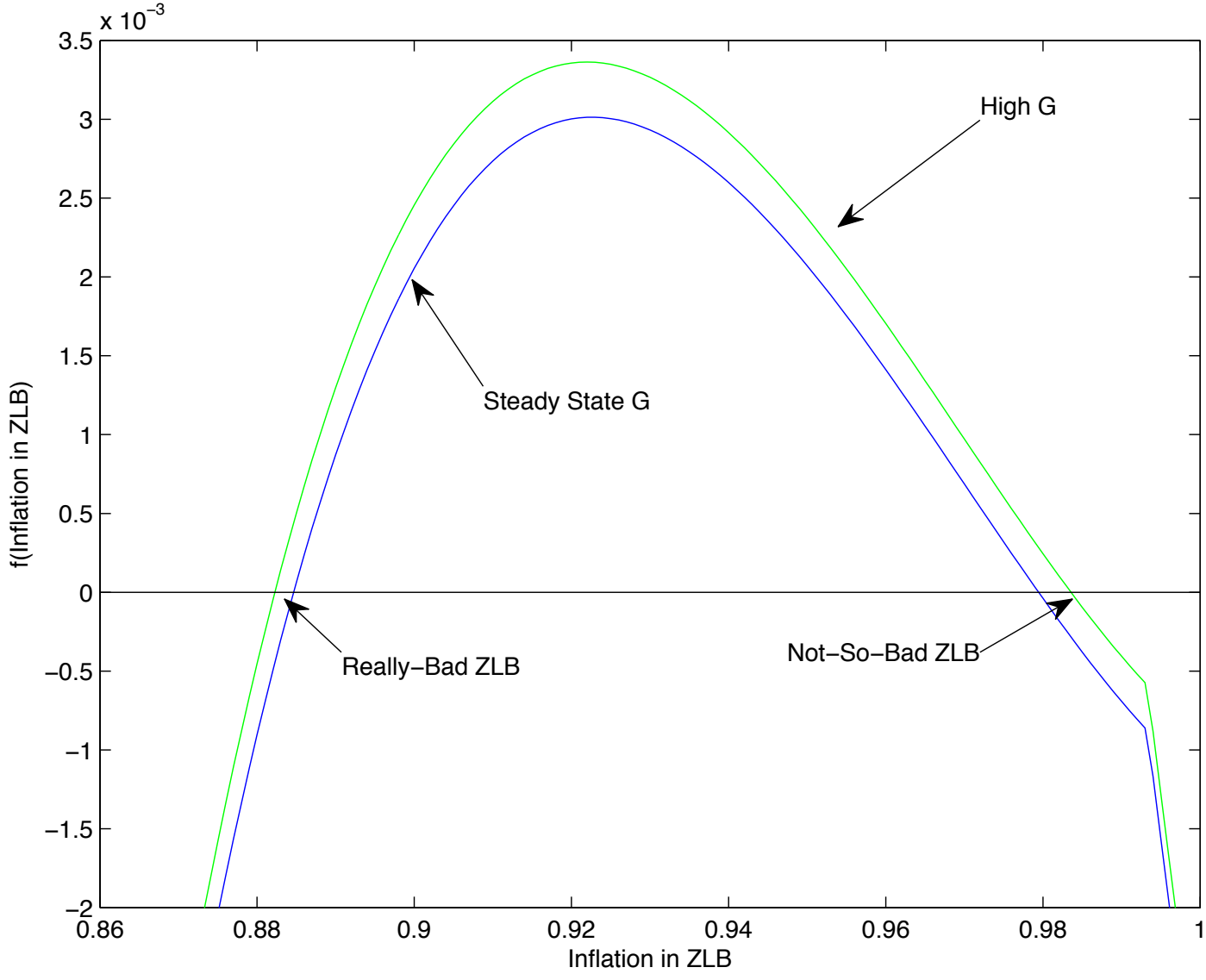
The ZLB

- Phillips curve, resource constraint and intertemporal Euler equation collapse into one equation in one unknown, π^{ℓ} :

$$f(\pi^{\ell}) = 0$$

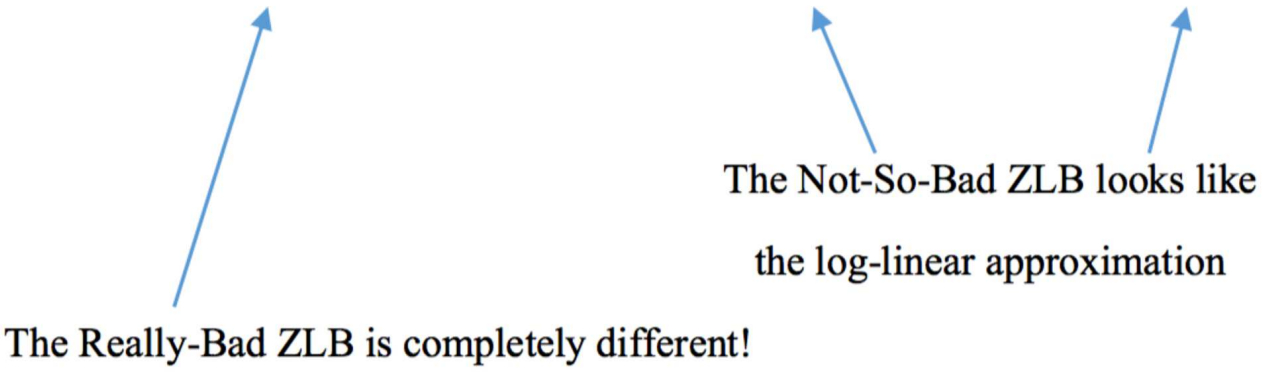
- Function, f, has inverted U shape on set of potential equilibrium values of π^{ℓ}.
  - Implication: generically, either there is no equilibrium, or two.

Inflation at the ZLB

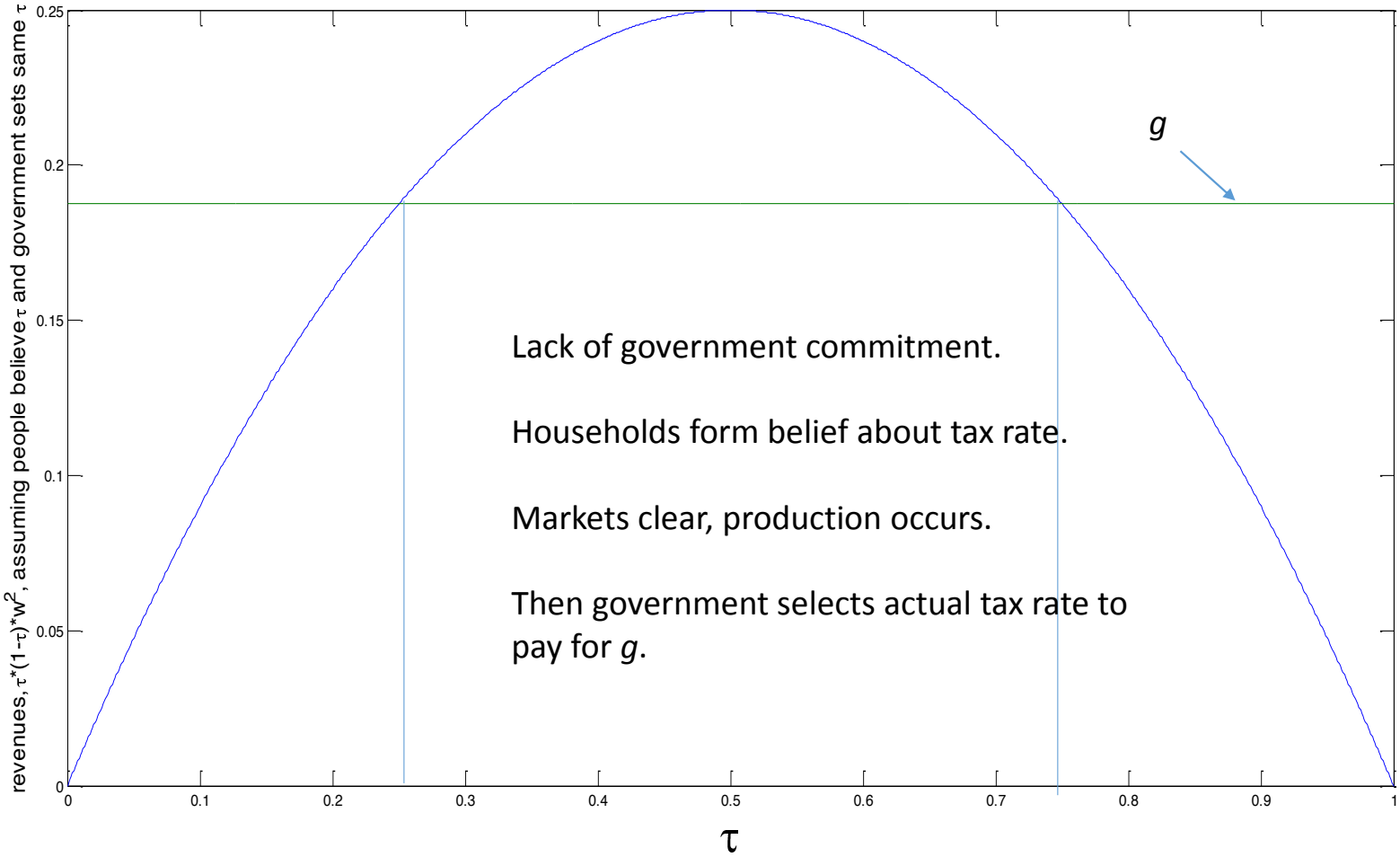


Numerical Results for Two Equilibria

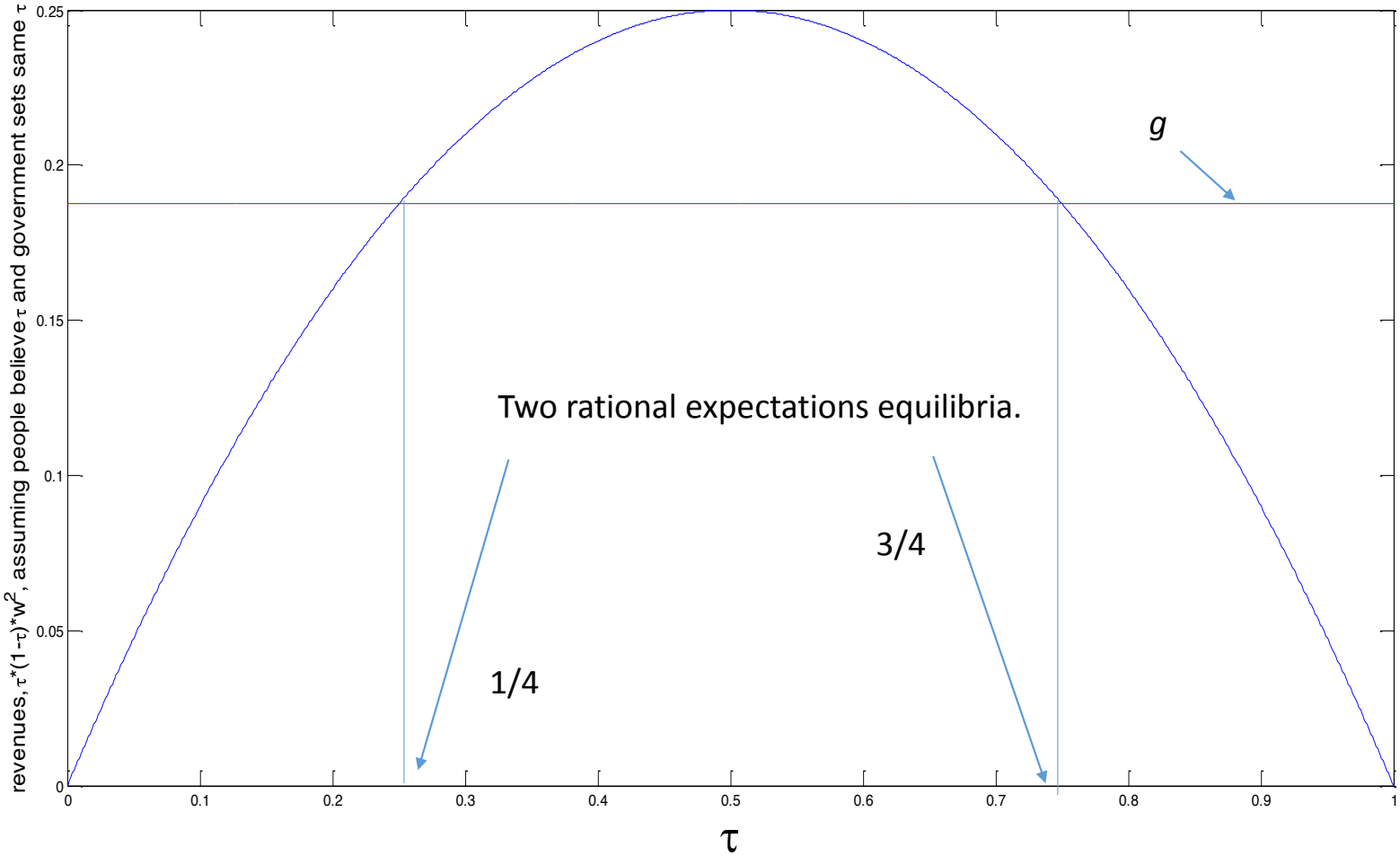
	Really-Bad ZLB	Not-So-Bad ZLB	Log-Linear
Multiplier	0.26	2.36	2.77
% Drop in GDP	37.55	5.38	5.99
Drop in Inflation Rate	11.77	1.64	1.90



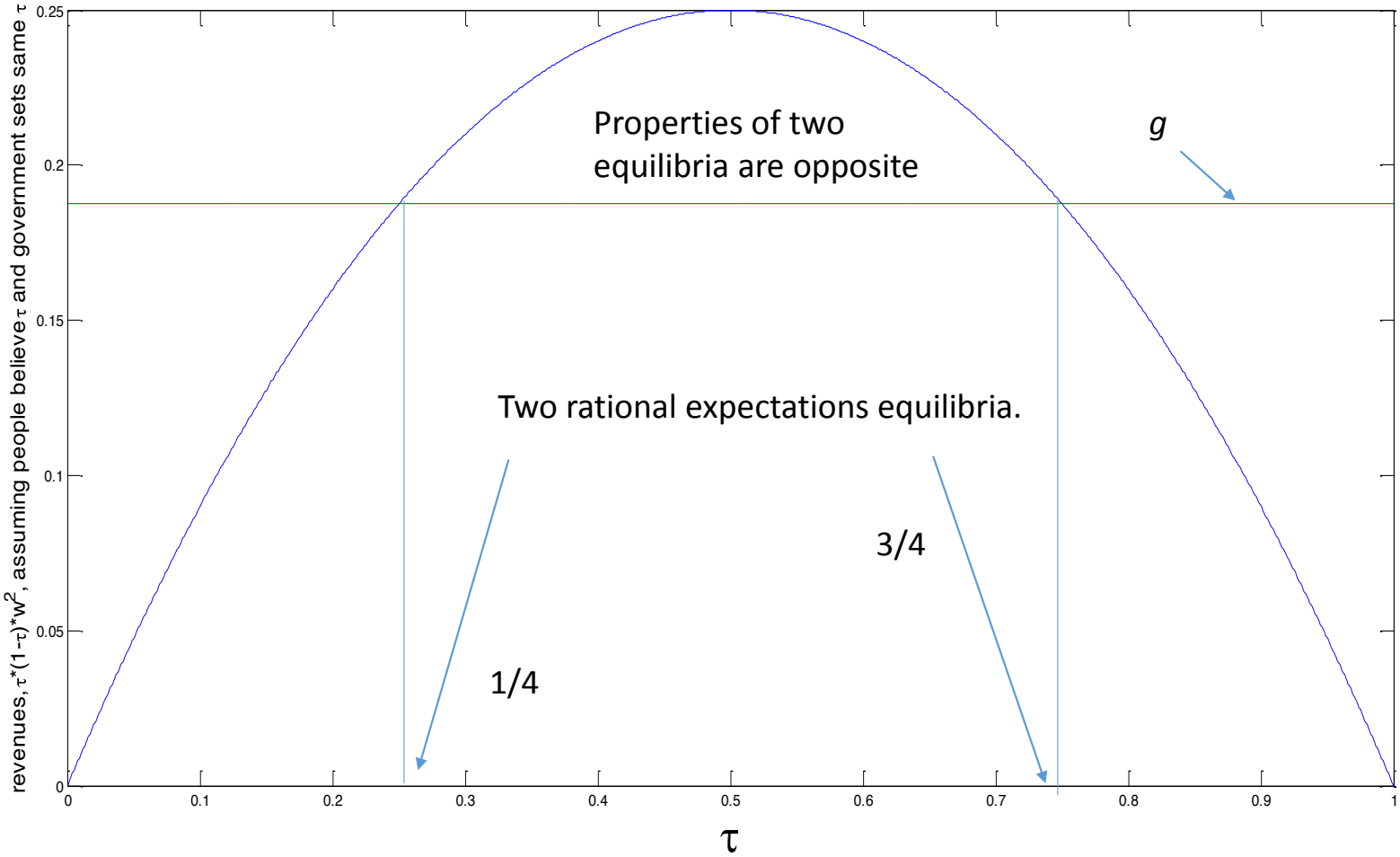
Laffer Curve Model



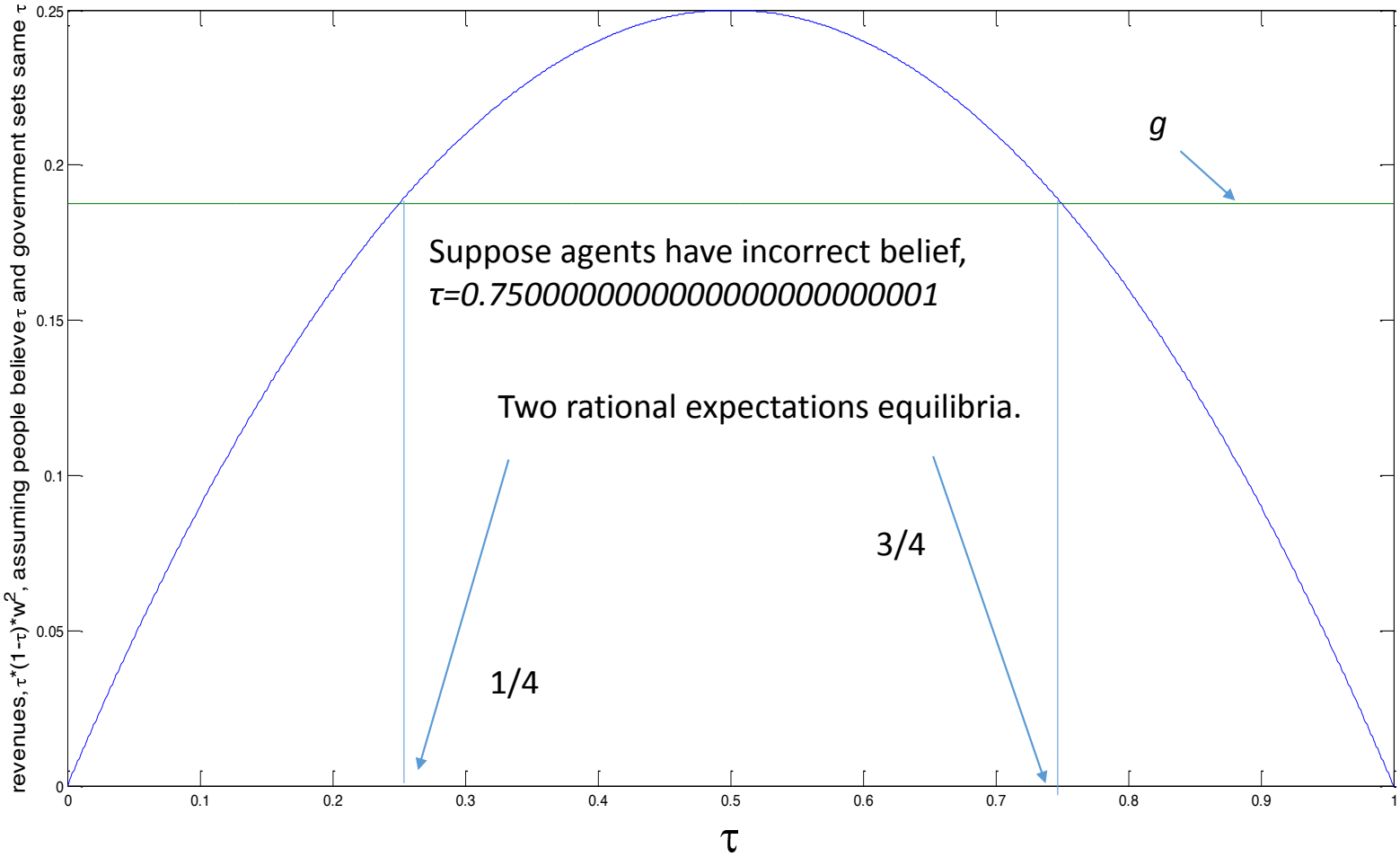
Laffer Curve Model



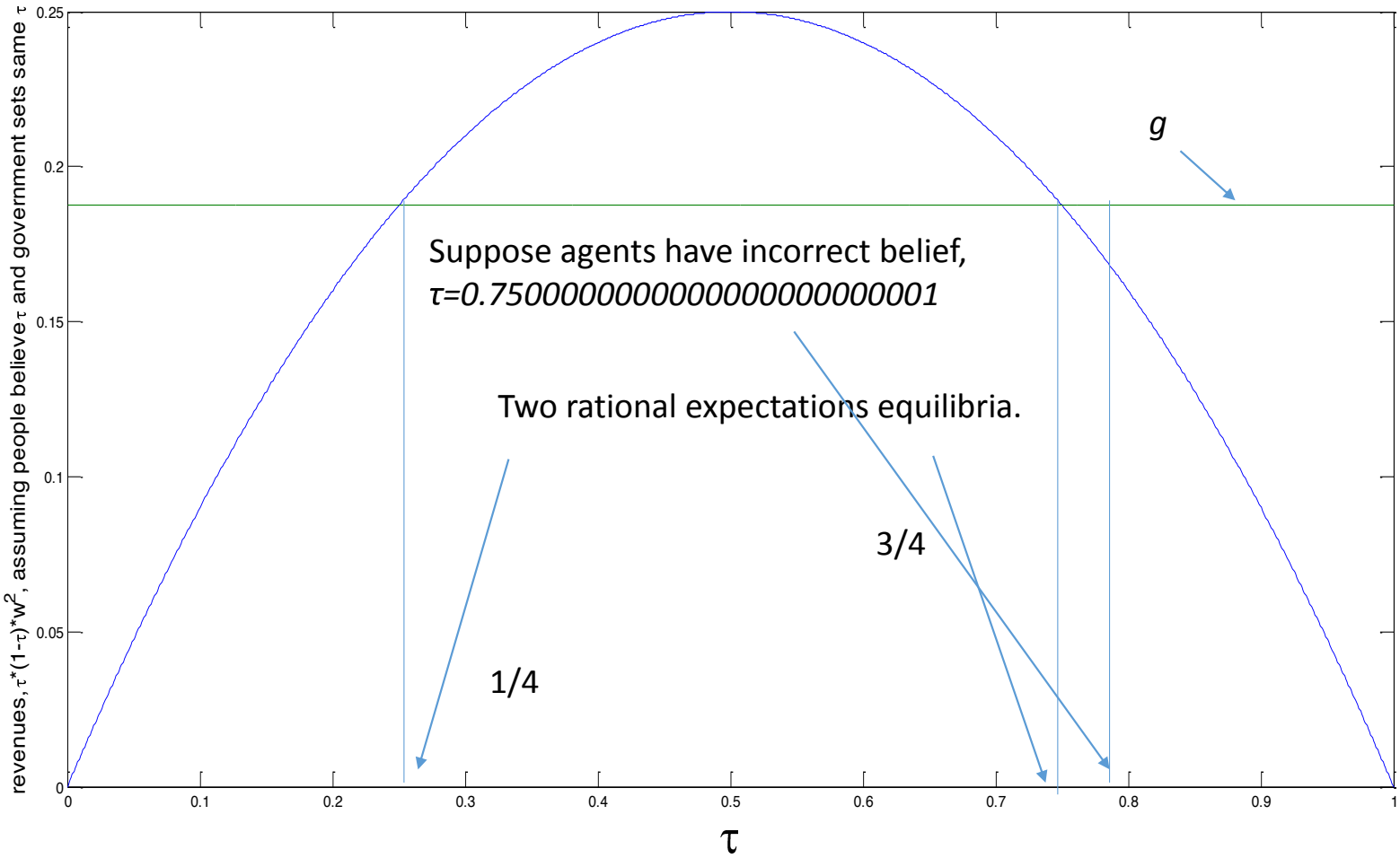
Laffer Curve Model



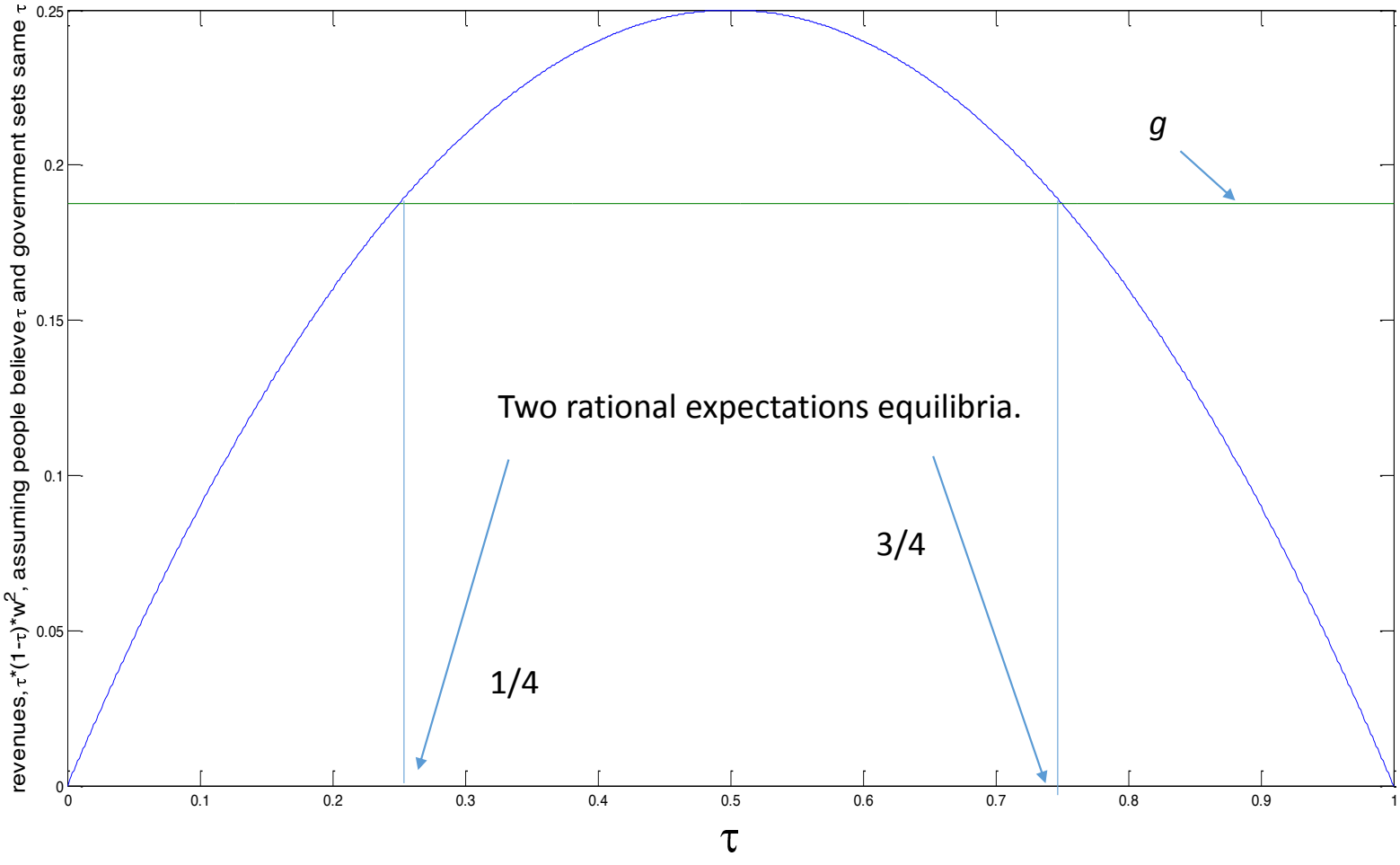
Laffer Curve Model



Laffer Curve Model



Laffer Curve Model



Learning in the New Keynesian Model

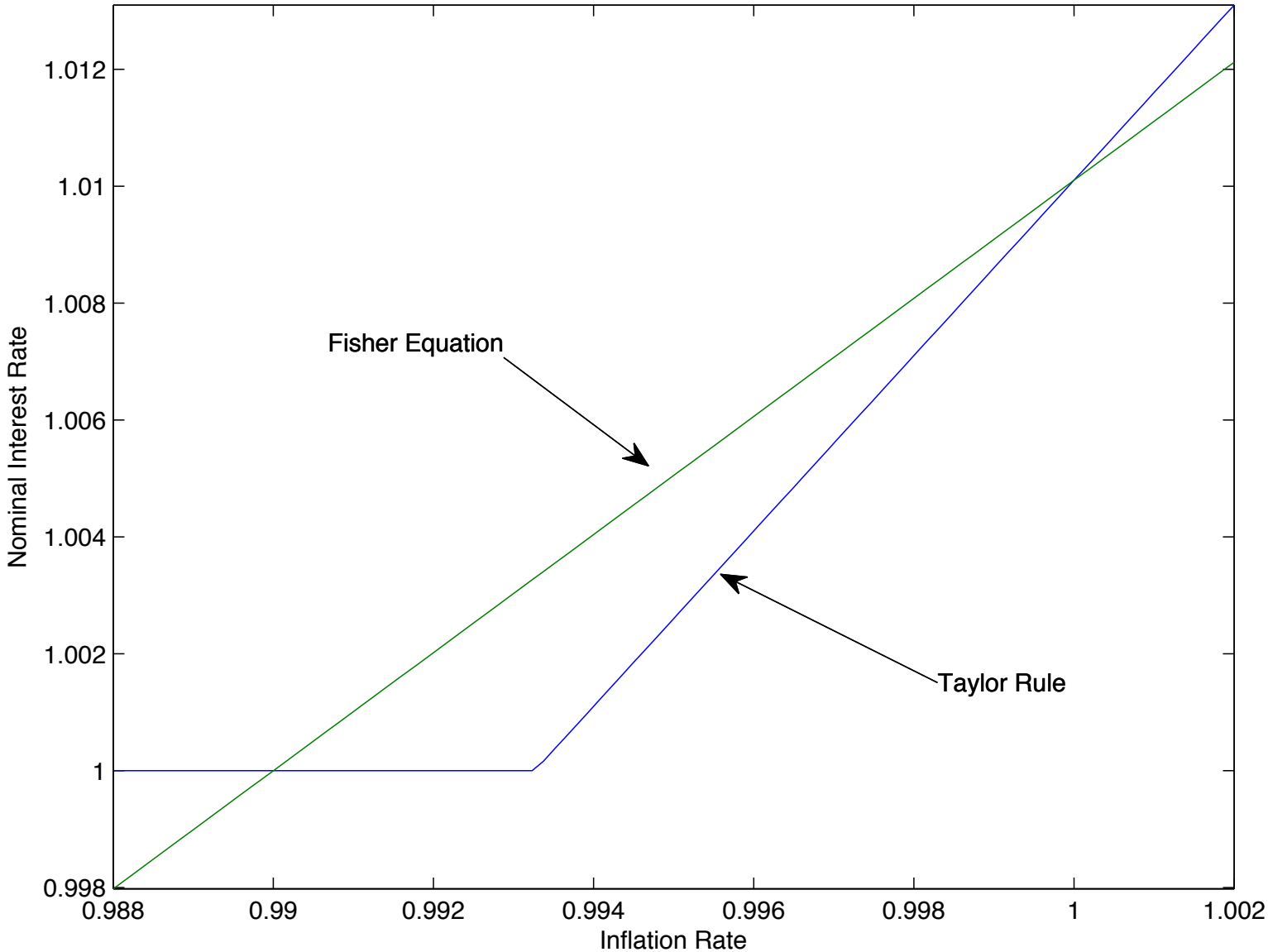
- Firms need to have expectations about variables beyond their control.
- Current period variables.
  - Firms do not observe actual aggregate price index at the time they choose their price.
    - 'Our learning' versus 'Evans-Honkapohja' learning.
  - Current aggregate output, consumption and price (inflation):

$$\begin{aligned} x_t^e &= \omega x_{t-1} + (1-\omega)\, x_{t-1}^e \\ & \quad \omega \text{ ~ gain parameter} \end{aligned}$$

- Future Variables while in ZLB:

$$x_{t+1}^e = x_t^e.$$

Future Variables out of ZLB



Learning in the New Keynesian Model

- Problem:

$$E_t \sum_{l=0}^{\infty} \beta^l v_{t+l} [(1+\nu) P_{j,t+l} Y_{j,t+l} - s_{t+l} P_{t+l} Y_{j,t+l}$$
$$-\frac{\phi}{2} \left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1 \right)^2 (C_{t+l} + G_{t+l}) \times P_{t+l}],$$

- First order condition:

$$(1+\nu) \frac{P_{j,t}}{P_t^e} = \frac{\varepsilon}{\varepsilon - 1} \chi h_t^e C_t^e +$$
$$\phi \frac{1}{\varepsilon - 1} \left(\frac{P_{j,t}}{P_t^e} \right)^{\varepsilon} \frac{C_t^e}{Y_t^e} [- \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{P_{j,t}}{P_{j,t-1}} \frac{(C_t^e + \psi G_t^e)}{C_t^e}$$
$$+ \frac{p}{1+r^l} \left(\left(\frac{P_{j,t+1}}{P_{j,t}} \right)^e - 1 \right) \left(\frac{P_{j,t+1}}{P_{j,t}} \right)^e \left(\frac{C_{t+1}^e + \psi G_{t+1}}{C_{t+1}^e} \right)]$$

Equilibrium Conditions, Learning

- Phillips curve

$$0 = \left[(1+\nu)(1-\varepsilon) \left(\frac{\pi_t^l}{\pi_t^e} \right)^{-1-\varepsilon} + \varepsilon \chi h_t^e C_t^e \left(\frac{\pi_t^l}{\pi_t^e} \right)^{-\varepsilon-2} \right] \frac{Y_t^e}{C_t^e}$$
$$-\phi \left(\pi_t^l - 1 \right) \pi_t^e \frac{(C_t^e + G_t^e)}{C_t^e} + \frac{p\phi\pi_t^e}{1+r^l} \left(\pi_t^l - 1 \right) \left(\frac{C_{t+1}^e + G_{t+1}}{C_{t+1}^e} \right)$$

- Household intertemporal Euler equation:

$$1 = \frac{1}{1+r^l} \left[p \frac{C_t^l}{\left(C_{t+1}^l\right)^e \left(\pi_{t+1}^l\right)^e} + (1-p) \frac{C_t^l}{C^h} \right]$$

- Resource constraint

$$h_t^l = C_t^l + G_t^l + \frac{\phi}{2} \left(\pi_t^l - 1 \right)^2$$

Compact Representation of Equilibrium Conditions and Learning

$$z_t = \begin{pmatrix} C_t^l \\ h_t^l \\ \pi_t^l \end{pmatrix}, \; z_t^e = \begin{pmatrix} \left(C_t^l\right)^e \\ \left(h_t^l\right)^e \\ \left(\pi_t^l\right)^e \end{pmatrix}$$

$$\begin{pmatrix} z_t \\ z_t^e \end{pmatrix} = f \begin{pmatrix} z_{t-1} \\ z_{t-1}^e \end{pmatrix}.$$

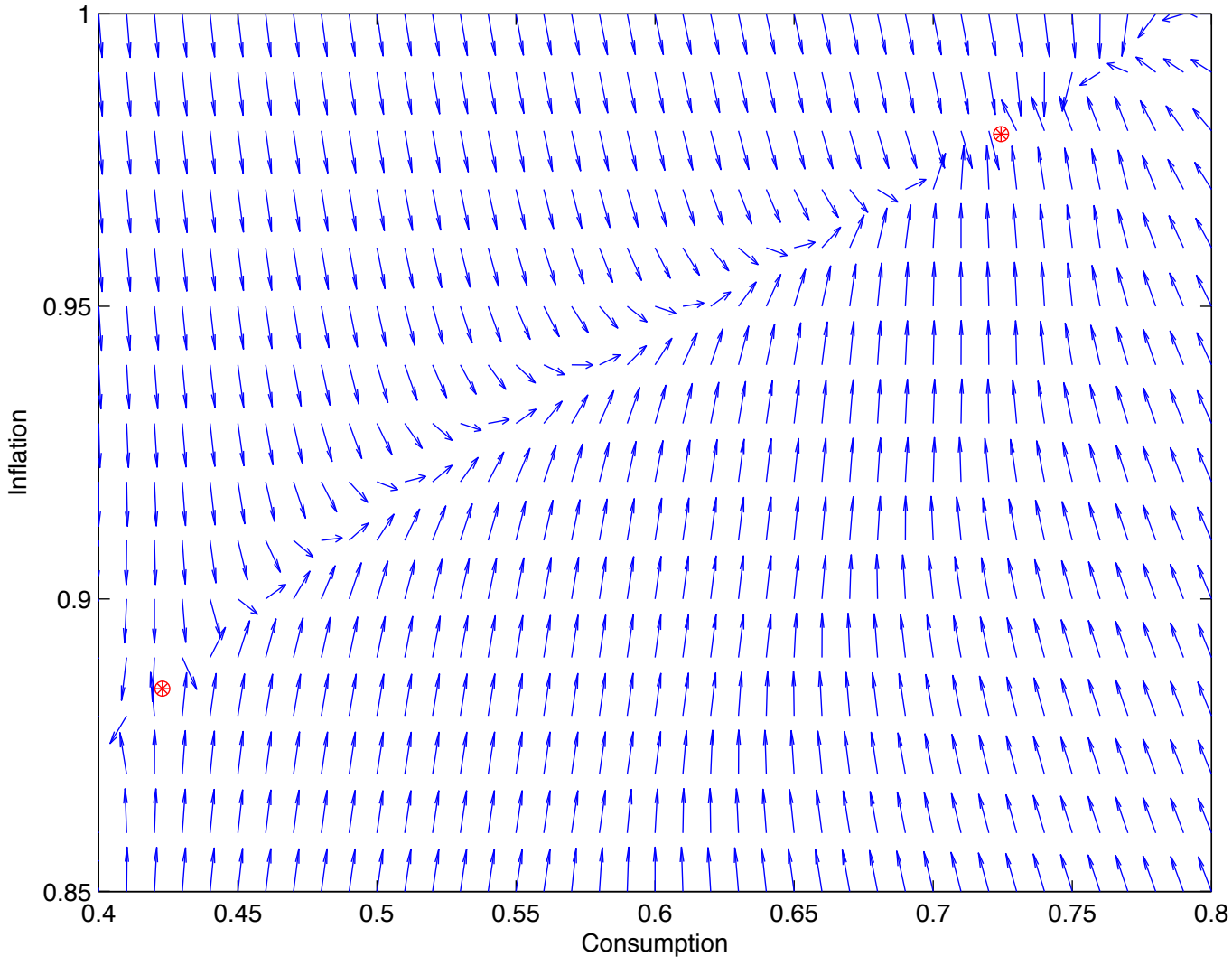
Parameterizing the model

$$
\begin{aligned}
G &= 0.20,\ \beta = 0.99,\ \frac{\varepsilon - 1}{\phi} = 0.02,\ \phi = 100, \\
\chi &= 1.25,\ \omega = 0.75,\ \Upsilon = h = 1.
\end{aligned}
$$

Stability and the ZLB

- If we start near the not-so-bad ZLB, we *converge* back to it.
  - There exist expectational points far from not-so-bad ZLB from which we diverge to negative consumption.
- Not-so-bad ZLB is stable.
- If we start near the really-bad ZLB, we either converge to the not-so-bad ZLB (generic case) or we diverge to negative consumption.
- Conclude: really-bad ZLB is not stable, not-so-bad ZLB is stable.

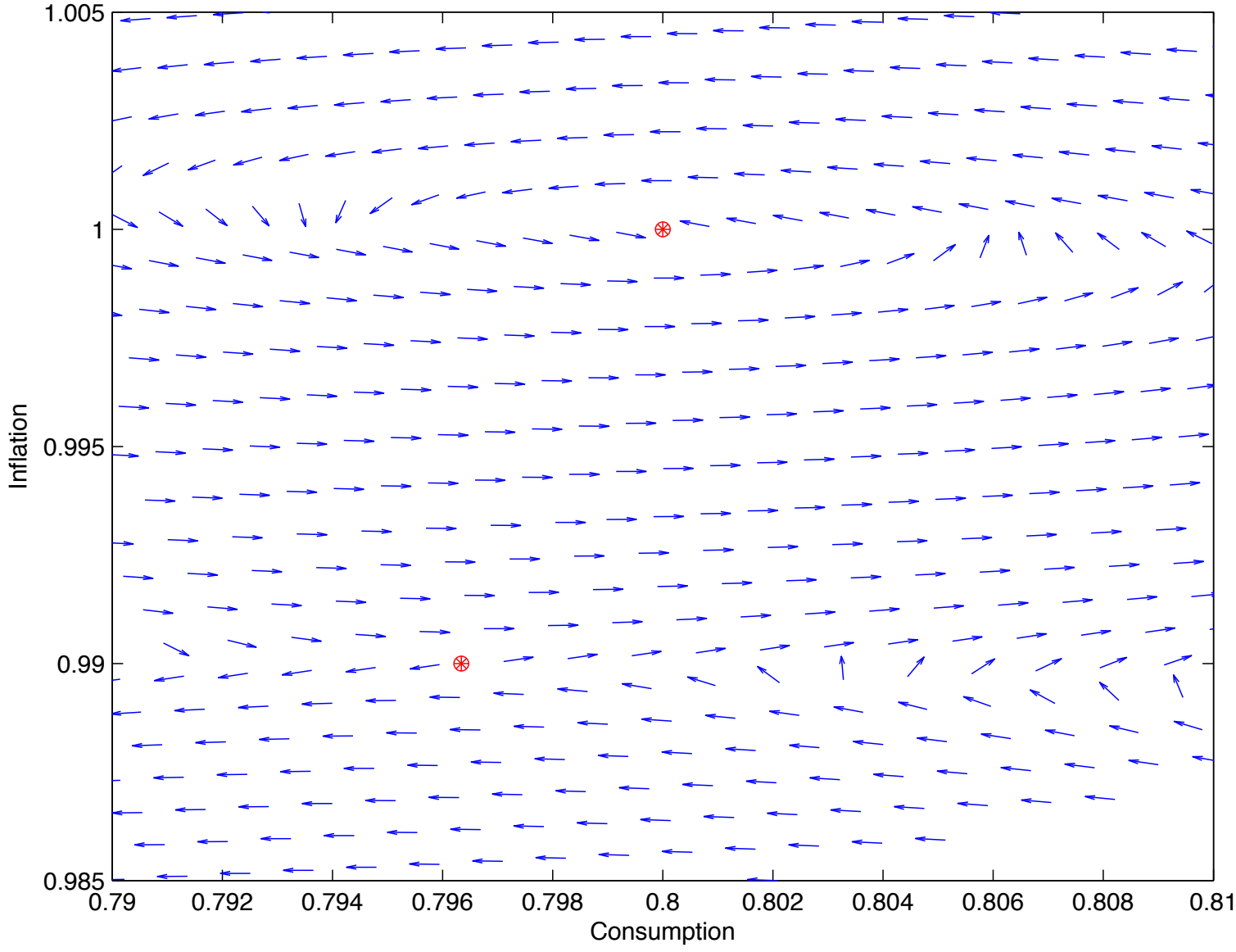
Alternative initial expectations



Benhabib, Schmitt-Grohe and Uribe

- Previous computations assume people expect to go to zero inflation steady state.
- What if they are slightly wrong about the equilibrium quantities in the steady state?
- Next, show:
  - zero inflation steady state stable under learning,
  - BSGU zero interest rate steady state not stable under learning.
  - See also Evans, Guse and Honkapohja (EER, 2008) and Benhabib, Evans and Honkapohja (2014).

Stability analysis



Robustness

- None of our results regarding stability are affected by
  - whether we do E-H or our learning,
  - which values of $\omega > 0$ that we use.

Learning Dynamics

- We have established that learning allows one to select a rational expectation equilibrium for ZLB analysis.
  - So, if you're into rational expectations, we're done.

- But, in the Great Recession, learning dynamics may be more interesting than rational expectations dynamics.

- Next we turn to government spending multipliers and convergence to rational expectations in learning equilibria.
  - Message: properties of learning equilibria sensitive to details.

The Government Spending Multiplier in the ZLB

- Multiplier in not-so-bad equilibrium is large, even under learning.
- Multiplier in really-bad equilibrium under learning is also large.
  - We will explain why M-R obtain a different result.
- As we saw, economy converges to not-so-bad ZLB.
- Fiscal multiplier converges to the large multiplier associated with the not-so-bad ZLB.

Two Experiments

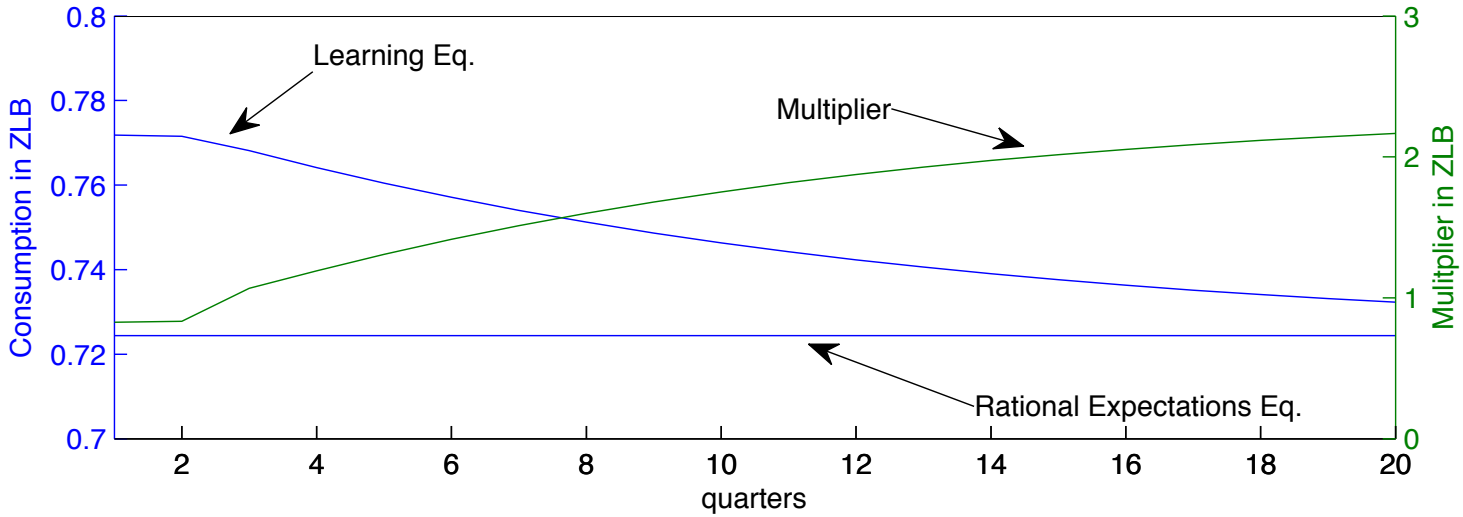
- Simulate:

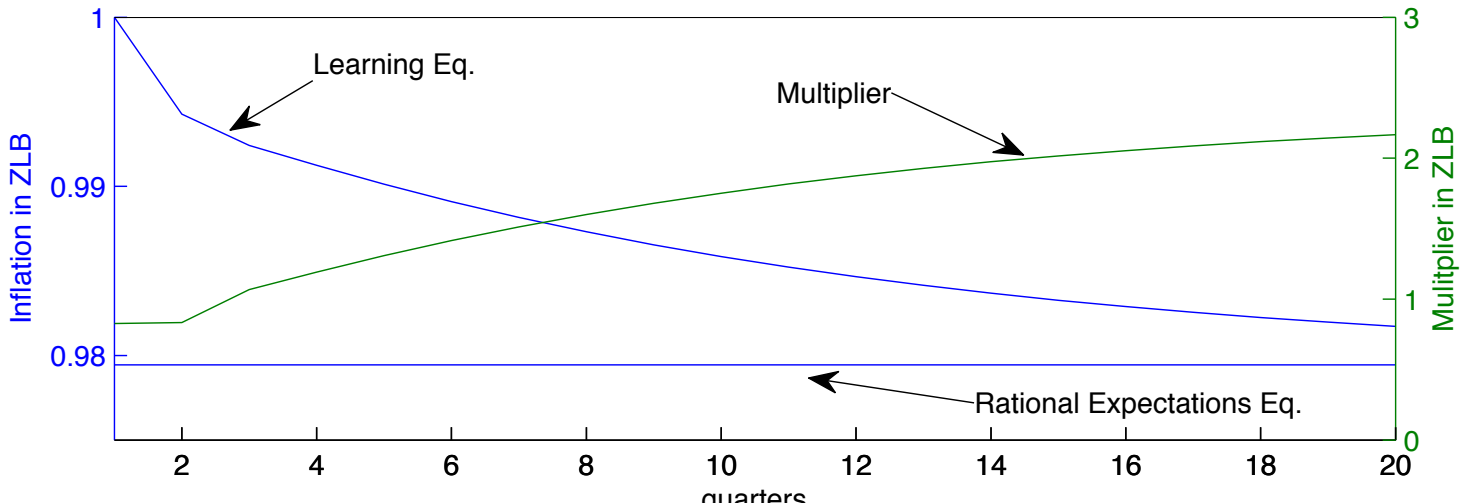
$$\begin{pmatrix} z_t \\ z_t^e \end{pmatrix} = f \begin{pmatrix} z_{t-1} \\ z_{t-1}^e \end{pmatrix}.$$

- Experiment #1: z_{t-1}, z_{t-1}^e correspond to old steady state.

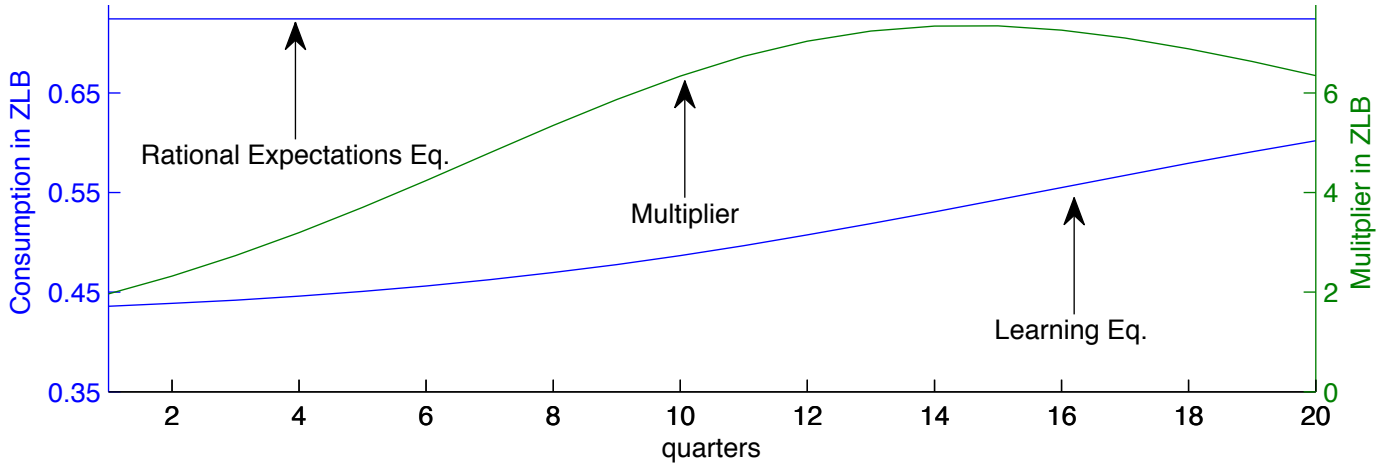
- Experiment #2: z_{t-1}, z_{t-1}^e near really bad ZLB.

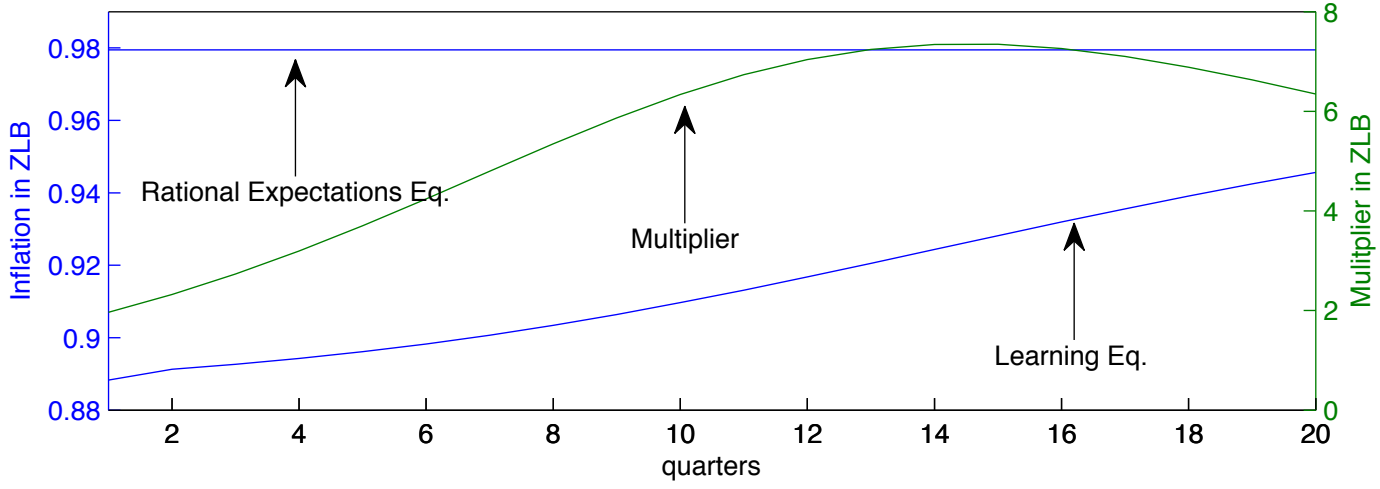
Experiment #1: Initial Conditions Equal to Old Steady State





Experiment #2: Initial Conditions Near Really-Bad ZLB





Comparison with MR

- MR get very small multipliers when they start near to the really-bad ZLB.
  - We get big multipliers.

- Reason for difference
  - MR force inflation expectations in really-bad equilibrium to respond to government spending in the way they respond in the corresponding rational expectations equilibrium.
    - In rational expectations equilibrium, government spending drives inflation down.
    - Makes real interest rate high and reduces spending.
  - We follow our learning rule.
    - In our simulations, inflation rises a little with increase in G.
    - Makes real interest rate low and increases spending.

Conclusion

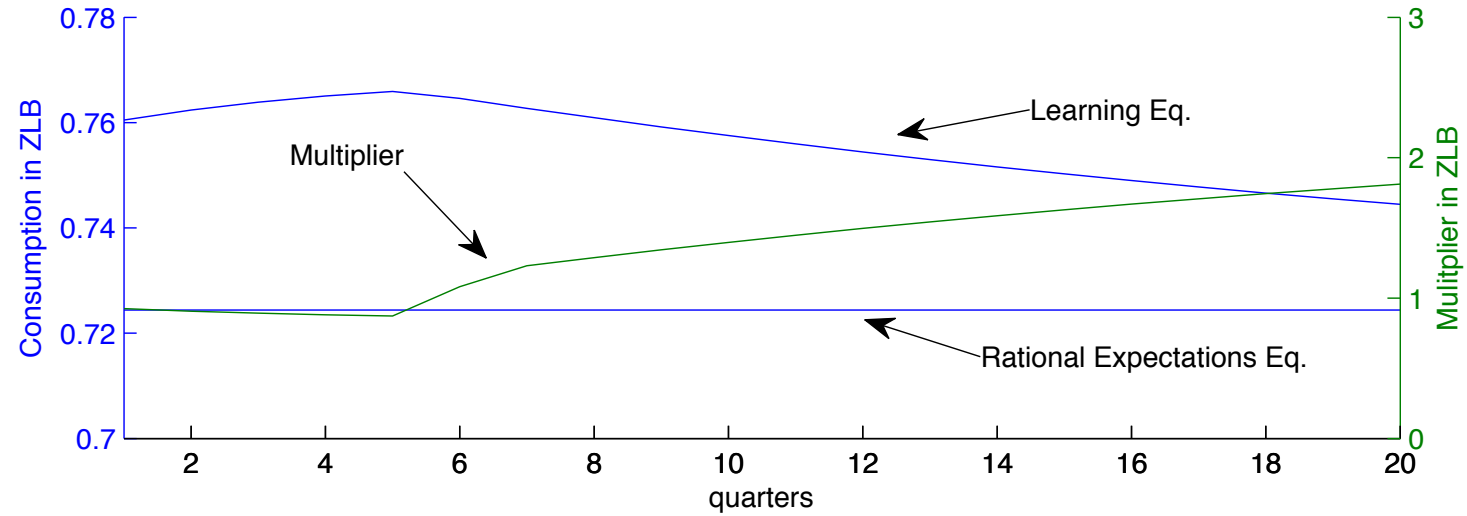
- If we require that RE equilibria are robust to small deviations from RE, then NK model does not have an interesting uniqueness problem in the ZLB.

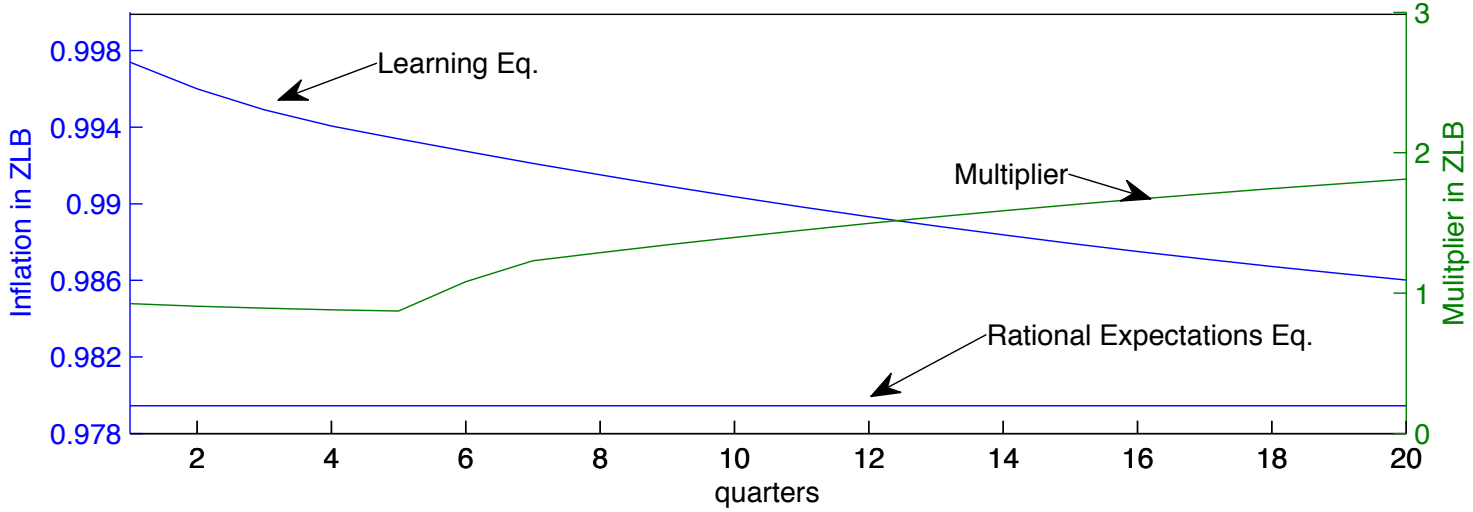
- The qualitative conclusions from analysis based on linear approximations correspond closely to those of the learnable RE equilibrium.

- The quantitiative conclusions based on linear approximations must be handled with care.
  - When expected duration of ZLB is long and prices relatively flexible, linearization has some 'crazy' implications, like enormous multipliers (> 400).
  - These implications represent approximation error.

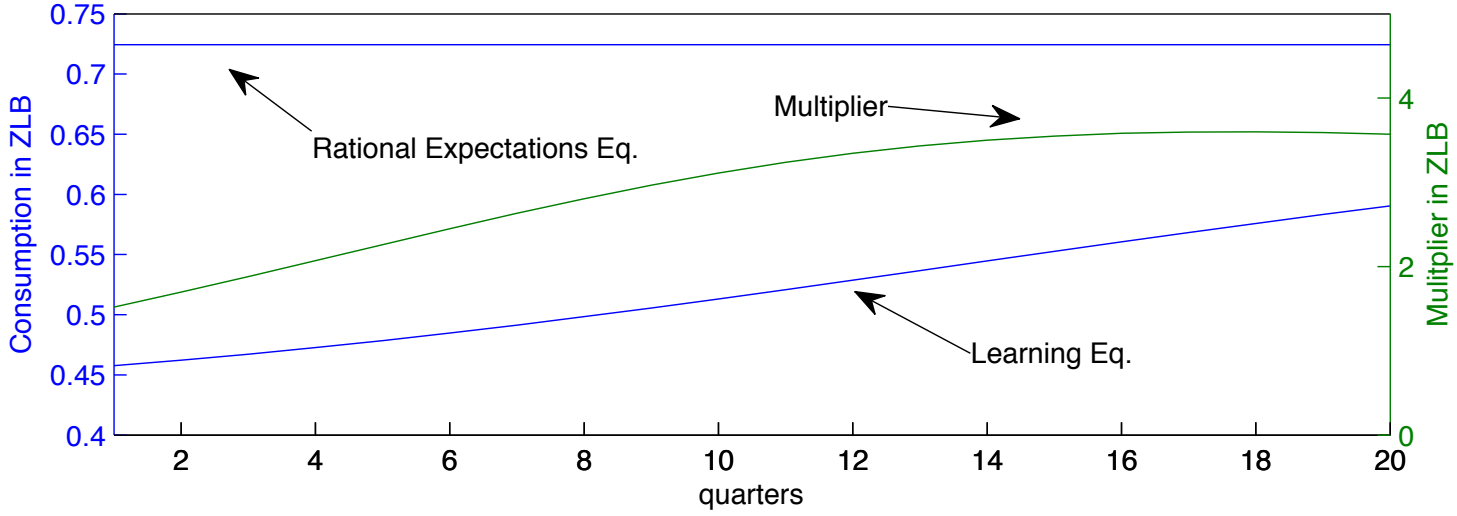
- Our analysis complements all the other evidence we have which suggests that how agents actually form beliefs matters.

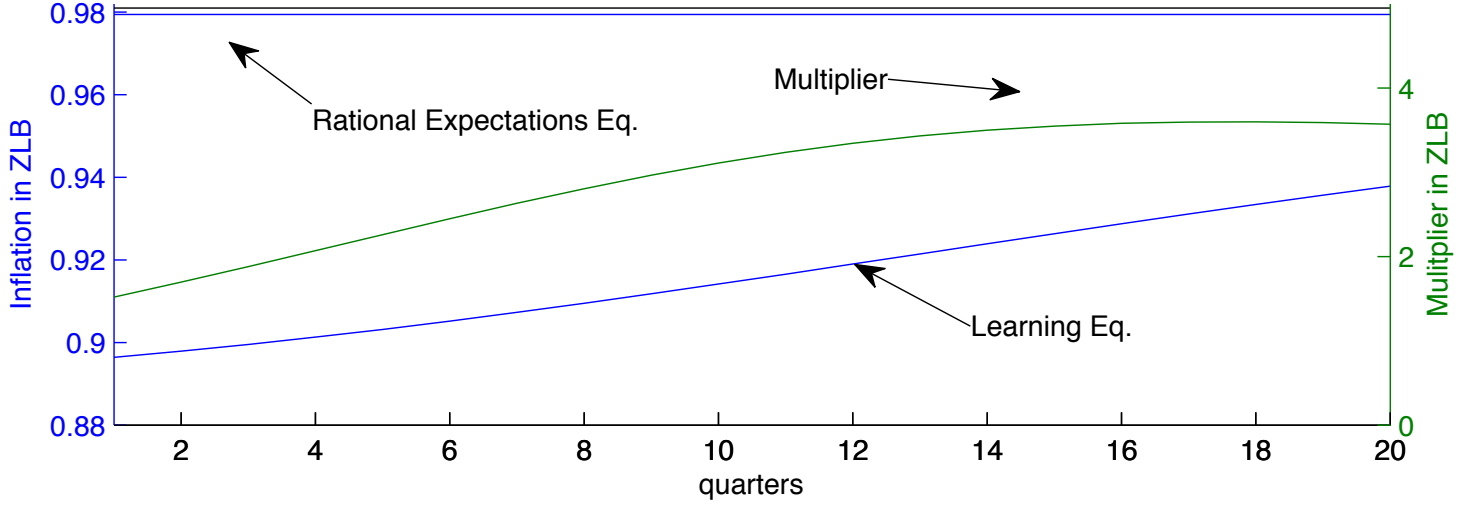
EH Learning, agents expect zero-inflation SS





EH Learning, agents expect really-bad ZLB

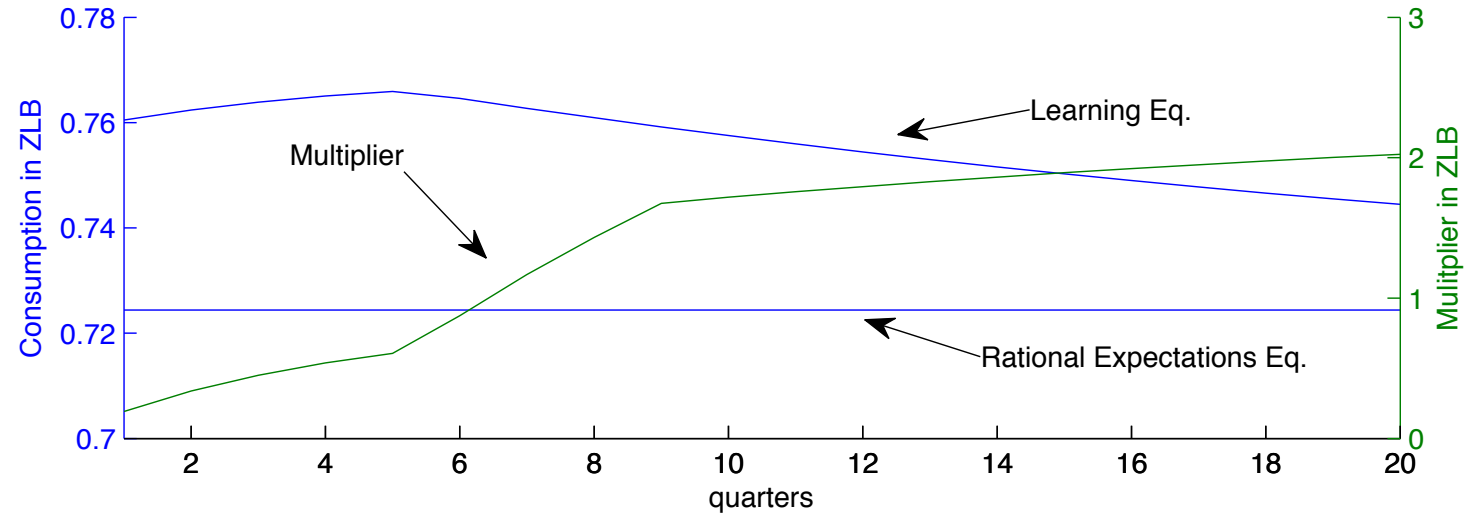


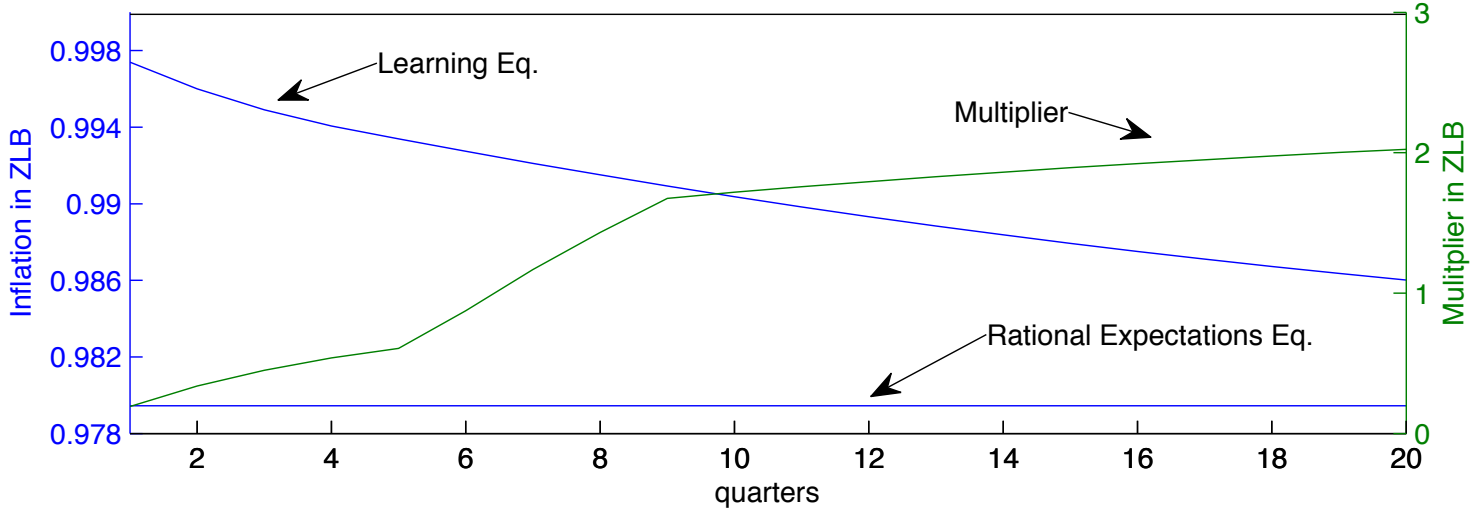


Subtly different experiments

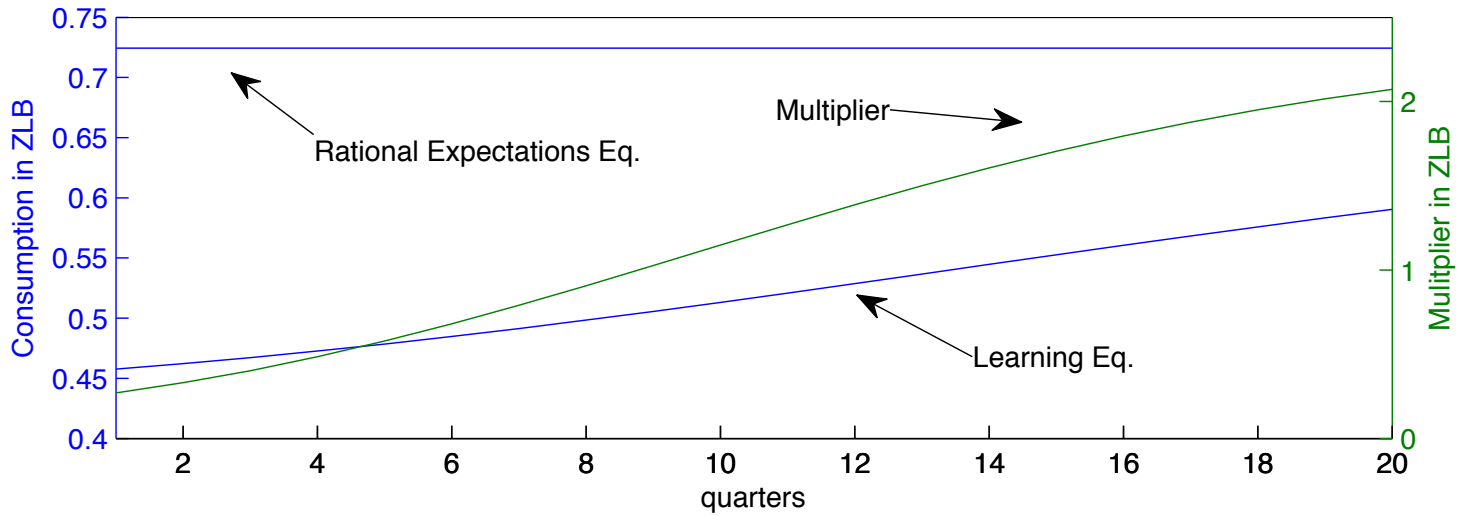
- After discount rate shock, agents think equilibrium will either be near zero-inflation SS or the really-bad ZLB.
- When we increase G we don't change those expectations.
- MR change expectations by amount that RE equilibrium changes when there's an increase in G.
- Turns out this modification accounts for the difference in our results.

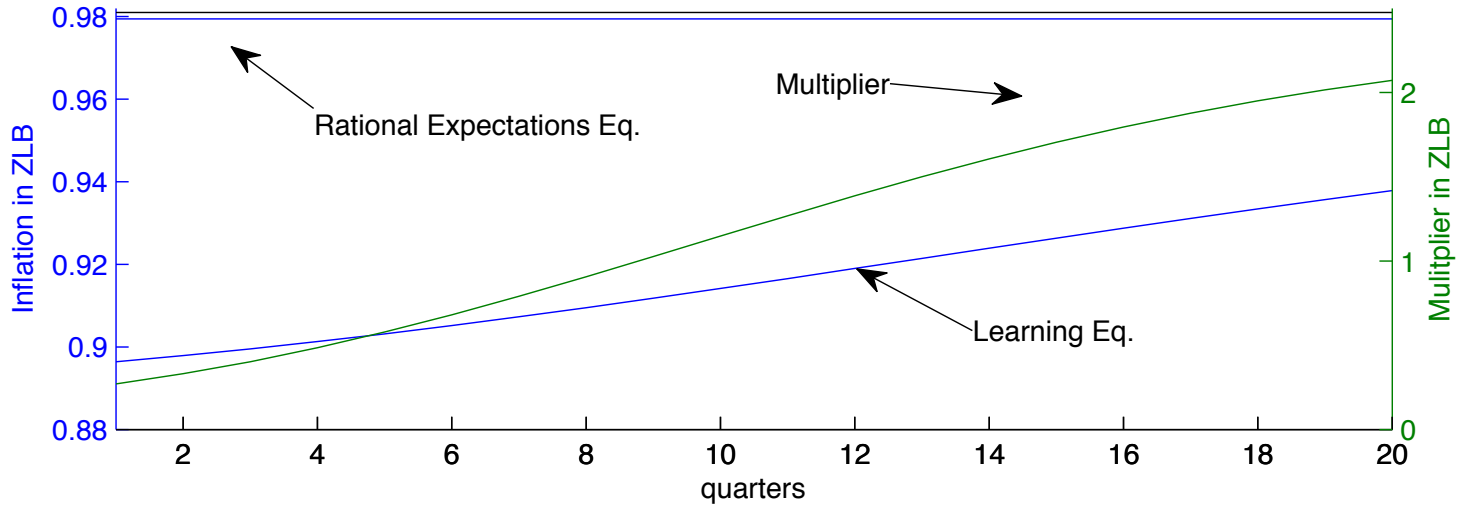
MR experiment, agents expect to be in zero-inflation SS





MR experiment, agents expect to be in really-bad ZLB

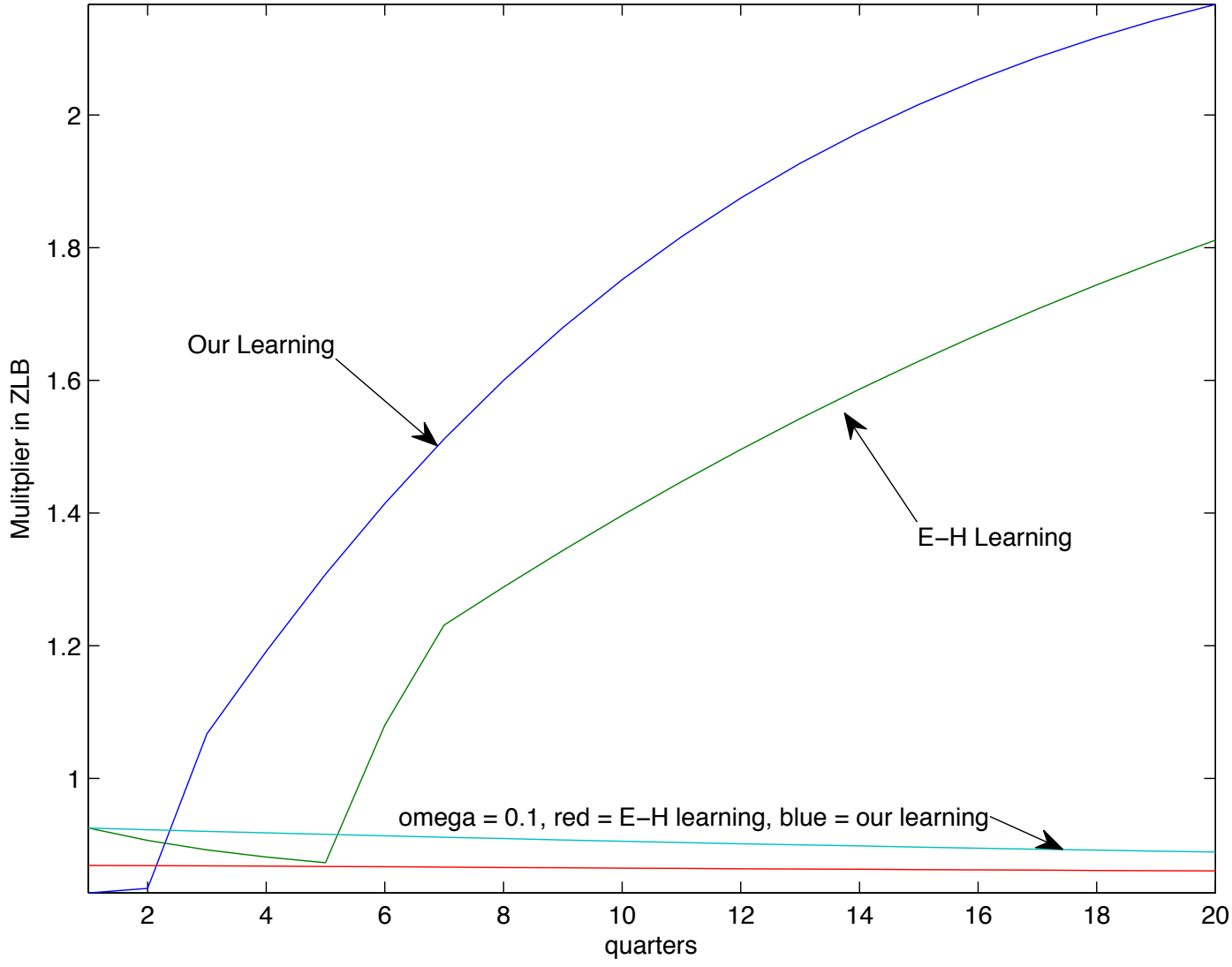




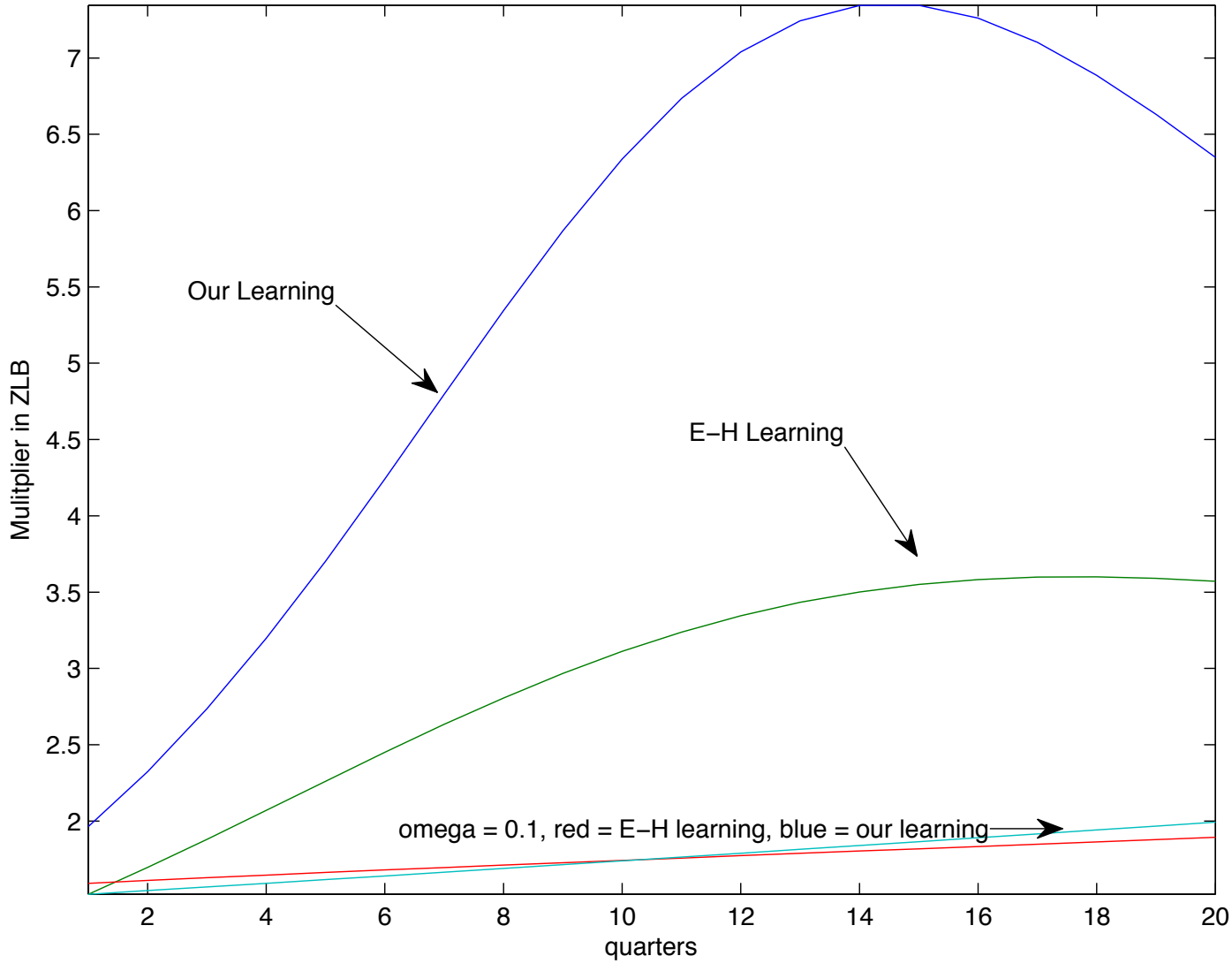
Speeds of Convergence

- Depend critically on ω.
- Recall literature typically uses very low ω when studying stability of SS.
- *Heuristically*, we think ω should be bigger if agents think they are in a novel situation like the ZLB.
- Setting ω to a small number like 0.1 (e.g. MR) makes a huge difference.
- Rates of convergence/divergence fall dramatically.

Speeds of Convergence, impact of omega



Speeds of Convergence, impact of omega



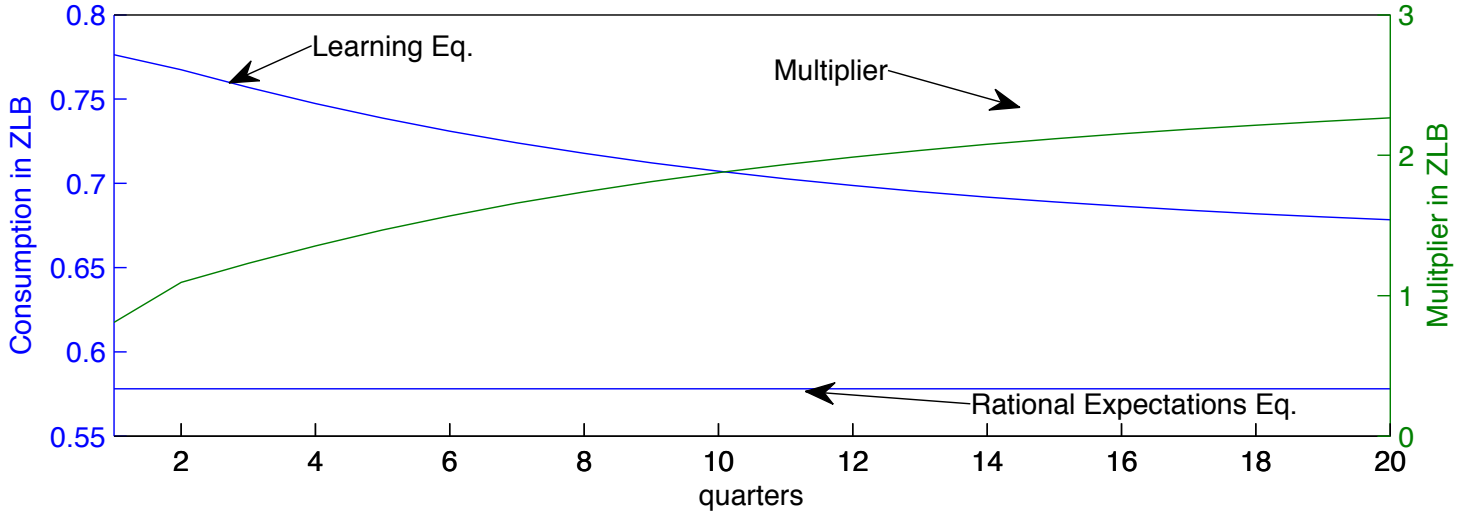
Multipliers and Learning

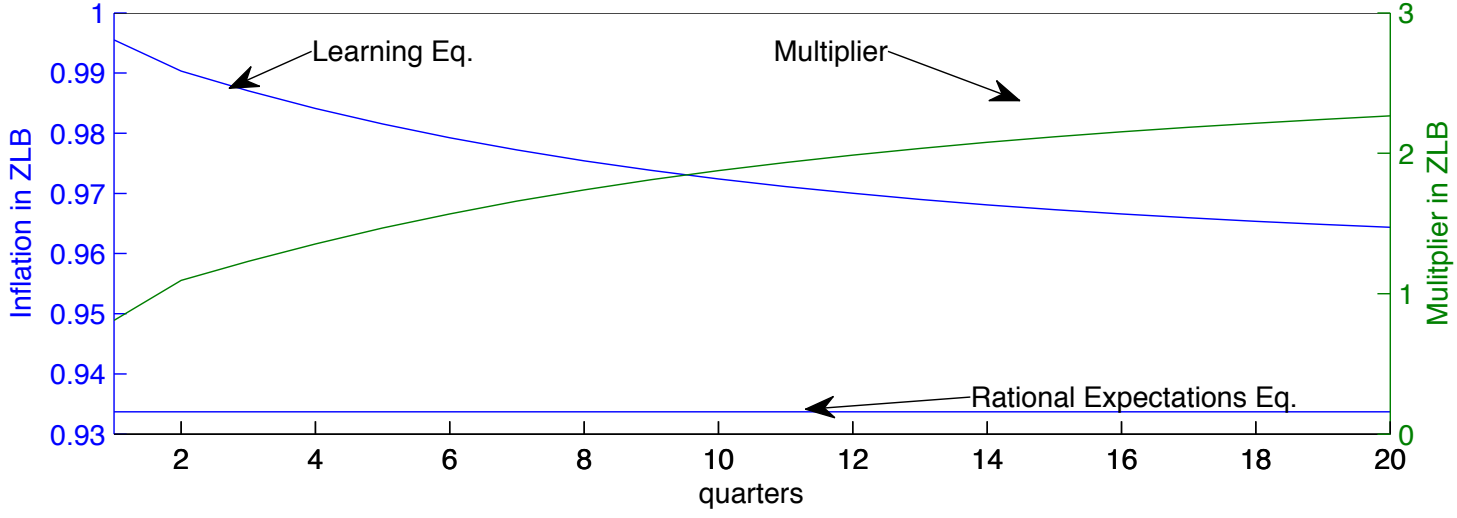
- Details of expectations formations matter a lot for detailed predictions of model.
- Speeds of convergence depend on EH learning versus our setup, as well as value of ω.
- We just use learning as an input into equilibrium selection.
- Regardless of our specification, we obtain a unique stable RE equilibrium.

Completing the uniqueness argument

- Until now we assumed that agents assume that once β goes back to its normal value, the economy goes back to zero-inflation SS.
- What if agents assume post-ZLB, economy will be somewhere other than the stable SS?
- These alternative assumptions don't affect our stability results.
- Economy always converges to zero-inflation SS diverge to negative consumption levels.

Alternative scenario post-ZLB





Linear Approximations

- Eggertsson and Woodford (2003) and Christiano, et. al (2011) study linear approximation to equilibrium conditions at the ZLB.

- For perturbations of reasonable size, conclusions arrived at in ZLB analyses using linear approximations are robust.

Linear Approximations and Cochrane's Analysis

- Cochrane (2011) linearizes NK model around zero-deflation SS.
  - Monetary policy satisfies Taylor principle.
- He concludes there are some equilibria where inflation wanders off to infinity but all real quantities are non-negative.
- Ongoing work:
  - Cochrane observations don't hold in non-linear versions of model or if we linearize anywhere (epsilon away) from SS.
  - Consumption goes to zero.
  - Costs of changing prices eat all of output.

Learning

- RE models typically assume that agents have an extraordinary amount of information about structure of economy.
  - These assumptions are *obviously* wrong.

- One interpretation: assumptions not literally satisfied.

  "...the model assumes that agents know a great deal about the structure of the economy, and perfom some non-routine computations. It is in order to ask, then: will an economy with agents armed with 'sensible' rules-of-thumb, revising these rules from time to time so as to claim observed rents, tend as time passes to [converge to the rational expectations equilibrium]?" Lucas (1978)

- Evans (1983, 1985), Marcet and Sargent (1989), Evans and Honkapohja (1995, 2001), McCallum (2002).