Payments, Credit and Asset Prices*

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Abstract

This paper studies a monetary economy with two layers of transactions. In enduser transactions, households and institutional investors pay for goods and securities with deposits issued by banks. Endusers' payment instructions generate interbank transactions where banks pay with reserves or interbank credit. The model links the payments system and securities markets so that beliefs about asset payoffs matter for the money supply and the price level, and monetary policy matters for real asset values.

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1 Introduction

This paper studies the joint determination of payments, credit and asset prices. The starting point is that, in modern economies, transactions occur in two layers. In the *enduser layer*, nonbanks – for example, households, firms and institutional investors – trade goods and assets and pay for them with deposits or credit supplied by intermediaries. Intermediaries offering payments services to endusers include not only banks but also money market mutual funds as well as clearinghouses that offer short term (e.g. intraday) credit to institutional investors to pay for asset transactions.

Transactions in the enduser layer generate payment instructions to banks and thereby interbank transactions in the *bank layer*. Perhaps the most obvious example is direct payment out of a bank deposit account through a cheque or wire transfer: payments between customers of different banks generate interbank transfers of funds. Similarly, payment out of a money market fund is processed via the fund's account at its bank. Finally, settlement of clearinghouse credit positions also typically occurs via institutional investors' bank accounts. The resulting interbank transactions are paid either with reserves through a gross settlement system provided by the central bank or through various forms of interbank credit.

This paper formulates a stylized model of an economy with both layers of transactions. The endusers are households and institutional investors who must pay for some goods and assets with deposits supplied by a competitive banking sector. There is no currency, but banks use reserves as well as interbank credit to handle payment instructions due to enduser transactions. Banks as well as institutional investors face costs of leverage that lead to nontrivial capital structure decisions. Banks invest in short term credit but also compete with other investors in securities markets.

We use the model to study the effect of monetary policy on asset prices as well as the effect of beliefs about asset payoffs on credit, deposits and the price level. In particular, we model an increase in uncertainty perceived about future cash flows as a shift towards more pessimistic beliefs, motivated by models of ambiguity averse preferences. The model describes links between the payments system and asset markets that make shocks travel both directions. On the one hand, an increase in uncertainty that lowers securities prices through an increase in uncertainty premia makes it more costly for banks to create deposits and generates deflation. On the other hand, monetary policy can lower real interest rates and further boost asset prices by making it cheaper for investors to trade or build leveraged positions.

The model assumes that households have linear utility, and that banks and other financial firms maximize shareholder value and operate under constant returns to scale. Markets are competitive and all prices are perfectly flexible. There are however two sources of friction in financial markets. First, deposits and reserves are more liquid than other assets; formally, they relax cash-in-advance constraints in the enduser and bank layer, respectively. Second, firms issuing debt incur a cost that depends on the value of firm assets. Both frictions matter for portfolio choice, inside money creation and pricing. Since banks and other firms are free to adjust equity, the role of frictions is not to amplify shocks that temporarily lower financial institutions' capital. Instead, we study steady states in which changes in expectations and the

monetary policy regime have permanent effects.[1]

The model can be interpreted as describing the subset of worldwide transactions in a currency, rather than the closed economy of a country. The former interpretation is appropriate for economies like the United States that have banking systems and financial markets tightly integrated with those of other countries. We thus think of households in our models as agents who pay for goods out of dollar deposit accounts, while institutional investors may include foreign firms who obtain credit or payments from banks in terms of dollars. With this perspective in mind, the model can be used to think about how events in worldwide asset markets may affect nominal prices in the US.

A useful benchmark for comparison is the monetary asset pricing model of Lucas (1980). In that model, a given amount of goods market transactions occurs every period and is paid for with currency. Money is not needed in asset markets. With positive nominal rates, the nominal price level follows from the quantity equation: if more currency is available to pay for the same amount of transactions, then the price level rises. To first order, returns on nominal assets are given by the Fisher equation: investors receive real returns plus compensation for expected inflation. Real asset returns follow from the representative agent's marginal rate of substitution as in the purely real model of Lucas (1978). As the real values of money and other assets are determined separately, changes in expected asset payoffs do not change the nominal price level, and monetary policy does not affect real asset prices.

For the baseline version of our model, we follow Lucas in positing a representative household, a fixed amount of goods market transactions, and no need for money in asset markets. However, transactions are paid for with deposits issued by banks. Since banks face leverage costs, the equilibrium quantity of deposits depends on the value of assets that banks can purchase to back deposits. This creates a link between real asset values and the nominal price level. For example, if there is an increase in expected asset payoffs, banks compete to supply more deposits. As more deposits become available to pay for the same amount of transactions, the price level rises.

In the baseline version of the model, nominal returns still satisfy the Fisher equation. Real returns, however, do not follow simply from the representative agent's marginal rate of substitution. This is because banks receive collateral benefits from assets they invest in to back deposits and thus bid up the prices of those assets. In particular, the real interest rate on short term credit is so low in equilibrium that the representative household chooses not to lend short term. In the baseline model, the short term credit market primarily allows banks to absorb liquidity shocks. Since reserves are more liquid than credit, there will typically be a spread between the interest rates on short credit and reserves. However, if asset values fall enough, collateral becomes so scarce that the short interest rate falls to equal the interest rate on reserves: the economy enters a liquidity trap.

The fact that prices of assets held by banks (as well as institutional investors who borrow from banks) incorporate both collateral and liquidity benefits implies that monetary policy can permanently affect real asset prices and portfolios. Outside the liquidity trap, the central bank can change the asset mix towards more or less liquid bank assets. As long as reserves are more liquid than short credit, conventional open market purchases lower the need for interbank credit

[1]This approach also makes the model very tractable – despite the presence of heterogeneous agents it lends itself to simple graphical analysis.

and the real interest rate. In the liquidity trap, the asset mix does not matter: asset purchases have real effects only if they change the overall value of bank assets, for example because the central bank buys at prices that are higher than what others would pay.

In the baseline model, the quantity of real deposits and securities held by banks is held fixed. In two extensions, we introduce institutional investors whose demand for loans or deposits responds to changes in interest rates. We first consider *carry traders* who hold real assets and borrow against those assets using short term credit supplied by banks. Carry traders have no demand for deposits, but supply collateral in the form of short term loans to banks. One example is broker-dealers who finance securities holdings via repurchase agreements.

The key new feature in an economy with carry traders is that the price level now depends on carry traders' demand for loans. For example, a decrease in uncertainty perceived by carry traders increases the quantity of collateral, the supply of deposits and the price level. An asset price boom can thus be accompanied by inflation even if the supply of reserves as well as the amount of goods transacted remains constant and banks hold no uncertain securities themselves. Moreover, monetary policy that lowers the real short term interest rate lowers carry traders' borrowing costs and boosts the aggregate market by allowing more leverage.

The second extension introduces *active traders* who hold not only securities but also deposits, since they must occasionally rebalance their portfolio using cash payments. One example is asset management firms who sometimes want to exploit opportunities quickly before they can sell their current portfolio. Active traders' portfolio choice responds to the deposit interest rate offered by banks: if the real return on deposits is higher, active traders hold more deposits and the value of their transactions is higher. The strength of their response depends importantly on how much netting takes place among active traders though intraday credit systems.

The key new feature in an economy with active traders is that the price level now depends on active traders' demand for deposits. For example, a decrease in uncertainty perceived by active traders increases their demand for deposits. As more of deposits provided by banks are used in asset market transactions, less deposits are used in goods market transactions and the price level declines. During an asset price boom, we may thus see low inflation even if the supply of reserves increases. Moreover, monetary policy that lowers the real short term interest rate lowers active traders' trading costs and further boosts the aggregate market.

The end of the recent asset price boom in 2007 saw jointly a drop in the values of most claims on businesses and housing, a decrease in short term interest rates to essentially coincide with the interest rate on reserves, and an unprecedented increase in the level of reserves in the US banking system. At the same time, there was little movement in the overall price level. Our model offers an interpretation of this episode based on two exogenous forces: a shift in expectations towards a more pessimistic assessment of future asset payoffs (or, equivalently in our setting, an increase in uncertainty that leads investors to price assets as if they were more pessimistic) as well as an aggressive monetary policy move to swap reserves for both public and private bonds held by banks and institutional investors.

According to our model, a sufficiently large increase in uncertainty or pessimism pushes the economy to the liquidity trap, regardless of monetary policy. As collateral drops, bank leverage increases and interest rates decline. By itself, the pessimism would have led to deflation. However, expansive monetary policy counteracted the associated deflation.

Related literature to be written

2 Motivating facts on payments and credit

In this section we motivate the modeling exercise by providing a snapshot of the US payment system together with some recent time series facts. A key takeaway from the description of the payment system is that the quantity of interbank payments as well as asset transactions is large, and that netting arrangements between banks as well as securities market participants play an important role in the payment system. These facts guide us to include a bank layer as well as intraday credit directly in our model.

There are two main takeaways from the time series facts. First, the timing over the recent boom-bust cycle was different for nonfinancial transactions on the one hand and financial transactions as well as interbank transactions on the other. Indeed, nonfinancial transactions peaked in 2007, whereas financial and interbank transactions were still high through 2008. This fact suggests that thinking about financial transactions is important for understanding the joint dynamics of payments credit and nominal prices.

The second takeaway is a negative relationship between bank leverage and overnight lending rates across the key episodes. Before the crisis, bank leverage was low while overnight lending rates were high before the crisis, while we observe the opposite after the crisis. This change in bank portfolios went along with the well-documented increase in reserve holdings as well as the decline in interbank credit. Those facts guide our modeling of banks' liquidity management and capital structure tradeoffs.

A snapshot of the US payment system

We report numbers for the year 2011.[2] For comparison, 2011 GDP is 15 Trillion Dollars, and outstanding cash is \$1Trn. Nonfinancial payments – that is, payments by nonbanks excluding transactions in securities markets – are \$71 Trillion. These payments include payments for goods and services by households and nonfinancial firms. They are several times larger than GDP, as one would expect given that there are multiple stages of production and commerce before goods reach the consumer. Moreover trade in physical capital including real estate also is contained in this category. Payments are made using different types of payment instruction: the most important ones are \$31Trn in ACH transfers, \$26Trn in cheque payments, and \$4 Trn in card payments.

While some payments are netted at the bank level (about \$16trn of the above payments are within-bank transfers such as "on us" cheques), enduser payment instructions give rise to a large number of payments among banks. In the United States, there are two primary interbank payment systems. Fedwire, run by the Federal Reserve System, is a real time gross settlement system in which banks send reserves to each other. In 2011, Fedwire handled \$663 Trn of interbank payments. About 40% of these transactions involve a foreign bank counterparty, reflecting the international importance of the dollar.

[2] We combine data form the BIS Payments Statistics, the Payments Risk Committee sponsored by the Federal Reserve Bank of New York, the Federal Reserve Board's Flow of Funds Accounts, as well as publications of individual clearinghouse companies.

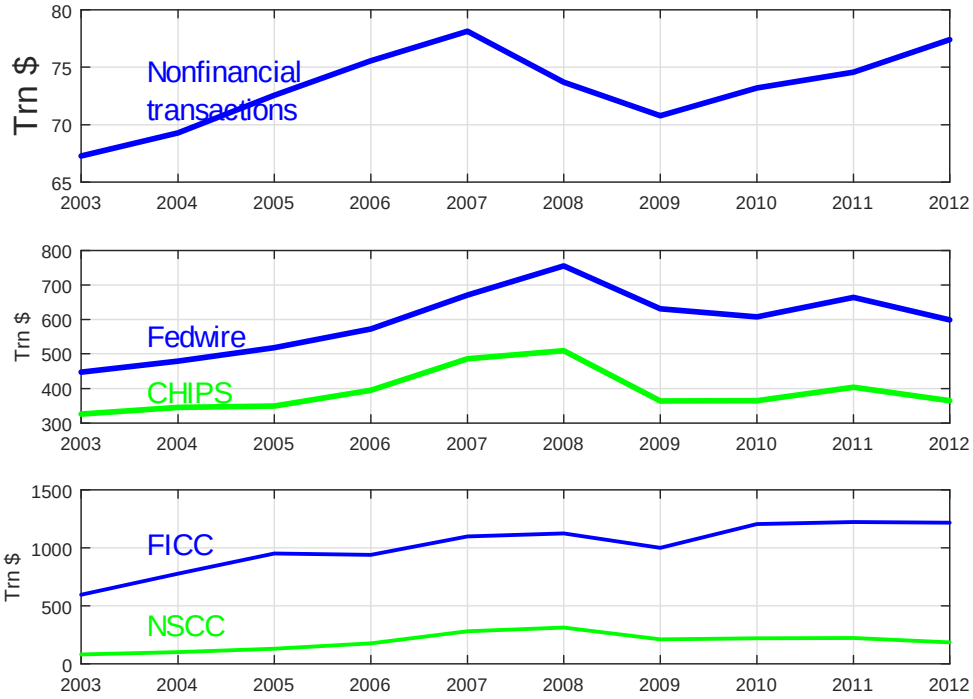


Figure 1: Nonfinancial transactions, interbank payments and securities clearing, 2003-now.

About 50 large banks participate in CHIPS, a large value transfer system that applies a netting algorithm to reduce the effective amount of transactions that need to be made among large banks. CHIPS members make a deposit into the system in the morning of each business day. As transactions are accumulate during the day, their intraday credit position varies and the deposit may be adjusted. The netting algorithm determines the effective payments that must be made – mostly at the end of the day but on busy days also during the day – and those payments are then executed over Fedwire. CHIPS cleared $440Trn worth of transactions in 2011, but the effective payment was only a small fraction of this amount.

Payment for securities involve intraday credit systems that are similar to CHIPS in that one of their key functions is netting of payment flows.[3] Members of these systems include not only banks but also institutional investors. Different systems serve different markets or sets of markets. For example, the National Securities Clearing Corporation (NSCC) handles trades on stock exchanges as well as over-the-counter trades in stocks, municipal and corporate bonds as well as mutual fund shares. NSCC cleared $221 Trn worth of such trades in 2011, which led to effective payments after netting of about $14 Trn. The Continuous Linked Settlement (CLS) group is a clearinghouse for foreign exchange spot and swap transactions. In 2011, it handled $1,440 Trn in payments, resulting in only $3 Trn worth of payments after netting.

The largest transactions numbers come from fixed income markets. The Fixed Income Clearing Corporation (FICC) clears Treasuries and agency security trades; some interbank trades are also handled by the Fedwire Securities service. In 2011, FICC cleared $1,126 of securities transactions. This number is high partially because every repurchase agreement involves two

[3] Another function in some cases is the management of counterparty risk: clearinghouses insert themselves in between parties as a buyer to every seller and a seller to every buyer.

separate security transactions (that is, the lender wires payment for a purchase to the borrower and the borrower wires payments back to the lender at maturity) It is difficult to determine the full amount of netting that occurs for fixed income trades, since FICC payments are settled on the books of two clearing banks – JP Morgan and Bank of New York Mellon – and one would have to know how much netting occurs on each bank's books.

Time series observations

The top panel of Figure 1 shows the nominal value of US nonbank payments between 2003 and 2012. Transactions grow at a roughly constant trend – similar to the trend in nominal GDP – except for the recession years 2007-9. In contrast, both interbank payments in the middle panel as well as securities clearing in the bottom panel are up in the boom, peak in 2008 (after the series of nonbank payments) and are then flat.

We have quarterly data on bank portfolios, deposits and credit from the Flow of Funds Accounts. We consolidate banks and money market mutual funds to obtain our modern concept of deposits, which can be used for payments. There was an expansion of deposits 2007-2008, mostly driven by an increase in institutional deposits. Figure 2 shows bank leverage in the upper panel and the Federal Funds rate in the lower panel. The main pattern here is that leverage is low at the peak of the boom in 2006, while the Federal Funds rate is high. After 2008, the economy is in the zero-lower bound with high bank leverage and a Federal Funds rate near zero.

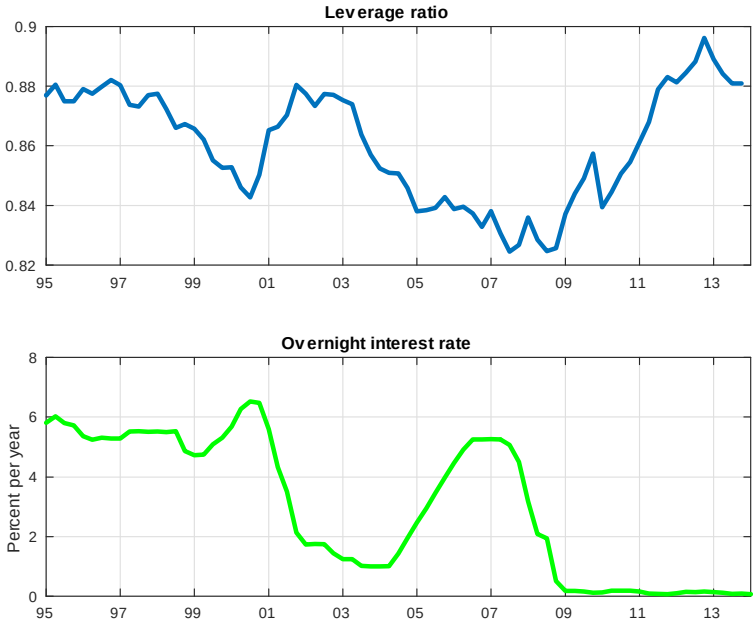


Figure 2: Aggregate bank leverage ratio and Federal Funds rate.

3 Model

Time is discrete, there is one good and there are no aggregate shocks. A representative household consumes an endowment of goods as well as fruit from trees. The total amount of goods

available for consumption is constant. The representative household also owns competitive financial firms. In this section, the only financial firms are banks who issue deposits; below we also introduce different types of asset management companies. All financial firms issue equity and participate in tree and credit markets along with the household itself.

Layers and timing

The model describes transactions and asset positions in both the enduser layer and the bank layer. In the enduser layer, households and nonbank financial firms trade goods and assets. Some of their transactions must be made with deposits supplied by banks. In the bank layer, banks trade securities and also borrow from and lend to each other. Since banks offer deposits to endusers, they may have to make payments to other banks.

To capture why some assets have liquidity benefits, we split every period into two subperiods. In the first subperiod, endusers buy goods or assets subject to deposit-in-advance constraints. This gives rise to a liquidity benefit for deposits in the enduser layer. At the same time, banks face deposit withdrawal shocks that require payments to other banks via reserves or credit. This gives rise to a liquidity benefit for reserves in the bank layer. In the second subperiod, banks adjust their portfolios and capital structure. Some other transactions – for example the payment of dividends and fiscal transfers – also take place in the second subperiod.

The model describes two types of frictions: in addition to the fact that some assets cannot be accessed in the first subperiod, financial firms incur leverage costs when issuing debt. However, both frictions are introduced such that financial firms' problems exhibit constant returns to scale. Since moreover financial firms can be recapitalized at no cost in the second subperiod of every period, a firm's history does not constrain its future portfolio and capital structure decisions. As in Lagos and Wright (2005), the distribution of heterogeneous agents' (here firms') histories thus plays essentially no role in the model.

Assets

There are five different asset classes. Reserves serve as numeraire; they are issued by the government, pay a nominal interest rate i_R and are held only by banks. Intraday credit does not pay interest but allows banks (and later asset management companies) to transfer resources between subperiods at the same date. Overnight credit pays an interest rate i, it is less liquid than reserves as funds lent out at date t cannot be used in the first subperiod at date $t+1$.

Trees are infinitely lived assets that each pay an exogenous quantity of goods x every period. Trees pay the same fruit x but may differ according to who can invest in them. Individual trees are indexed by types $j \in [0,1]$. The nominal price per unit of dividend for type j trees is denoted Q_j, so the nominal value of type j trees is $Q_j x$ and the nominal value of their fruit is Px, where P is the nominal price level.

Since we focus on steady states throughout, we write agents' constraints without time subscripts. Our convention for asset positions is that, in the constraint for any given date, the asset position chosen at that date is denoted by a prime: for example, households enter a period with deposits D chosen the previous period and exit with a new deposit position D'.

3.1 Households

Households have linear utility, discount the future at rate δ and receive an endowment Ω each period. Households enter the period with deposits D and buy consumption C at the nominal price P. They face a deposit-in-advance constraint

$$PC \leq \bar{v}D,$$

where $\overline{v}$ is a fixed parameter that determines velocity in the enduser layer..

Households can carry any unspent deposits over to the second subperiod. Combining budget constraints for the first and second subperiod, we write the overall budget constraint as

$$\begin{aligned} PC - P\Omega &= D\left(1+i_D\right) - D' + x\int_0^1 \left(\left(Q_j + P\right)\theta_j - Q_j\theta'_j\right) dj \\ &\quad + \text{dividends} + \text{fees} + \text{government transfers} \end{aligned} \tag{1}$$

Goods purchases in excess of the value of the endowment Ω must be financed either through changes in household asset positions in deposits or trees, or through exogenous income from dividends, fees or government transfers, described in more detail below.

We assume that interest on deposits is paid regardless of whether deposits are spent or not, but that it accrues only in the second subperiod. This assumption is convenient because it implies that the deposit rate does not affect velocity. Households are allowed to invest in all trees $j \in [0,1]$, but they cannot sell trees short, that is, we impose $\theta_j \geq 0$.

We think of the endowment as labor income (payoffs from human capital), while trees represent other long lived assets such as equity in nonfinancial firms or claims to housing services. Both trees and human capital are less liquid than deposits in the sense that they cannot be used to pay for consumption in the first subperiod. The difference between them is that human capital must be held by households, whereas trees can also be held by financial firms or the government, and their ownership affects the production of liquid deposits. A key equilibrium outcome is where in the economy trees are held.

Household choices

We focus on equilibria in which all nominal prices grow at the constant inflation rate π and the real rate on deposits $i_D - \pi$ is smaller than the discount rate δ. Households' optimal choice is then to hold as few deposits as necessary, that is, the deposit-in-advance constraint binds. Moreover, the household first-order condition for type j trees is

$$\frac{Q_j}{P} \geq e^{-\delta}\left(\frac{Q_j}{P} + 1\right),$$

where we have used that nominal tree and goods prices grow at the same rate, so Q_j/P is constant. The condition holds with equality if the household has a positive position in type j trees.

We think of our model period as a short period such as a day, and we do not want to study hyperinflation periods, so that all rates are small decimal numbers. We thus simplify formulas throughout by approximating $e^x = 1 + x$ for any small rate x, and setting products of rates

such as $\delta\pi$ to zero. For any tree the household invests in, the Euler equation then equates the real return on tree j, $r_j = \log(Q_j + P)/Q_j - \pi$ to the discount rate δ and determines the price payoff ratio as $Q_j/P = 1/\delta$. So far, tree returns only compensate for time discounting. Below we will introduce households' subjective beliefs or perception of uncertainty about trees, and returns will also compensate for that.

3.2 Banks

The household owns many competitive banks. We describe the problem of the typical bank i which maximizes shareholder value

$$\sum_t \exp(-\delta t)\, y_i^b(t).$$

Here bank dividends $y_i^b(t)$ are discounted at the household discount rate δ, as are payoffs from trees owned by households. Dividends can be positive or negative; the latter corresponds to recapitalization of the bank.

Liquidity management

The typical bank i enters the first subperiod with deposits D_i and reserves M_i. We want to capture the fact that enduser payments out of deposit accounts may lead to payments between banks, for example because the payments goes to an account holder at a different bank. We thus assume that the typical bank receives a withdrawal shock: an amount $\phi_i \bar{v} D_i$ must be sent to other banks, where ϕ_i is iid across banks with mean zero and cdf G. We also assume that ϕ_i is never so large that all deposits are withdrawn: the cdf G is increasing only up to a bound $\bar{\phi}$ with $\bar{\phi}\bar{v} < 1$.

In the cross section, some banks receive shocks $\phi_i > 0$ and must make payments, while other banks receive shocks $\phi_i < 0$ and thus receive payments. The scale of the required interbank payments depends on velocity on the enduser layer. In particular, if households almost never withdraw deposits, $\bar{v}$ is close to zero and few interbank payments are needed. The distribution of ϕ_i depends on the structure of the banking system as well as the pattern of payment flows among endusers.[4]

Banks have access to intraday credit I_i, to be paid back without interest in the second subperiod of the current period, or overnight credit $F_i' \geq 0$, to be paid back with interest in the second subperiod of the next period. Bank i's budget constraint in the first subperiod is

$$\phi_i \bar{v} D_i = M_i + I_i + F_i'.$$

Banks who must make an interbank payment ($\phi_i > 0$) can do so by spending predetermined reserves M_i or by borrowing intraday or overnight. Banks who receive interbank payments ($\phi_i < 0$) can carry funds forward to the second subperiod as a negative intraday credit position $I_i < 0$.

[4] We are assuming here that the likelihood of withdrawal is the same across banks, regardless of bank size. Since the size distribution of banks is not determinate in equilibrium below, little is lost in thinking about equally sized banks. Alternatively, one may think about large banks consisting of small branches that cannot manage liquidity jointly but instead each must deal with their own withdrawal shocks.

The intraday credit position satisfies a *liquidity constraint:* intraday credit cannot exceed a fraction γ of the bank's liquid funds

$$I_i \leq \gamma \left(M_i + F_i' \right), \tag{2}$$

which comprise both reserves and overnight borrowing. Both reserves and borrowed funds can thus be used as a downpayment in the intraday credit system. The parameter γ captures the netting efficiency of the interbank system. If γ is large, then banks are able to obtain a lot of intraday credit with few liquid funds.

If the real interest rate on overnight credit is positive, then it is always better to take out as little overnight credit as necessary. Banks thus optimally choose a threshold rule: do not borrow overnight unless ϕ_i is so large that the withdrawal $\phi_i \bar{v} D_i$ exhausts both reserves and the intraday credit limit available by just paying down reserves. The threshold shock is

$$\phi_i^* = \frac{(1+\gamma) M_i}{\bar{v} D_i}. \tag{3}$$

For shocks $\phi_i > \phi_i^*$, bank i borrows overnight

$$F_i' = \phi_i \frac{\bar{v} D_i}{1+\gamma} - M_i. \tag{4}$$

If there were no withdrawal shocks at all ($\phi_i = 0$ for sure), then there would be no liquidity benefit from reserves and no overnight borrowing by banks. In order to be held in equilibrium, reserves must then earn the same interest rate as other assets held by banks. As we will show below, this "liquidity trap" situation also occurs endogenously in the model if the real quantity of reserves is sufficiently large relative to the upper bound on the support of ϕ_i.

Portfolio and capital structure choice

In the second subperiod, banks pay back their intrabank credit positions. Moreover, they adjust their portfolio and capital structure subject to leverage costs. They invest in reserves, overnight credit and trees while trading off returns, collateral values and liquidity benefits. They issue deposits and adjust equity capital, either through positive dividend payout or negative recapitalizations y_i^b. Capital structure choices trade off returns, leverage costs and liquidity costs.

Substituting out the intraday credit position, we can write a single budget constraint for bank i, analogously to the household budget constraint (1):

$$\begin{aligned} P y_i^b &= M_i (1+i_R) - M_i' - D_i (1+i_D) + D_i' + (B_i - F_i)(1+i) - (B_i' - F_i') \\ &\quad + x \int_0^\beta \left((Q_j + P) \theta_{i,j} - Q_j \theta_{i,j}' \right) dj - c\left(L_i / K_i\right) L_i, \end{aligned} \tag{5}$$

Net payout to shareholder payout must be financed through changes in the bank's positions in reserves, deposits, overnight credit (where $B_i \geq 0$ is overnight lending) or trees. Banks also pay leverage costs $c(\cdot) L_i$, discussed in detail below.

Interest on reserves i_R accrues in the second subperiod to the bank that held the reserves overnight, regardless of whether those reserves were used by the bank to make a payment in the

first subperiod. Similarly, deposit interest i_D is paid by the bank that issued the deposits in the previous period, regardless of whether the deposits were used by endusers to make a payment in the first subperiod. Both conventions could be changed without changing the main points of the analysis, but at the cost of more cluttered notation.

Banks have access to trees $j \in [0, \beta]$, a subset of all the trees available to households. One way to think about this restriction is as the result of unmodelled contracting frictions: we could interpret the fruit from trees x as housing services, so the trees represent all claims on housing, which consist of mortgage bonds, as well as – perhaps due to a commitment problem – housing equity. Banks and households both participate in the market for mortgage bonds, whereas only households own housing equity. Alternatively the restriction could be due to regulation, as banks cannot own stocks in some countries.

Leverage costs

The last term in equation (5) represents leverage costs. Without these costs, the Modigliani-Miller theorem holds and bank capital structure is indeterminate. We assume instead that the function c is smooth, nonnegative, strictly increasing and concave in bank leverage

$$\ell_i := \frac{L_i}{K_i} = \frac{\rho_D D_i + F_i}{M_i + B_i + x \int_0^\beta Q_j \theta_{i,j} dj}.$$

Here the numerator is weighted debt L_i and the denominator is the value of bank assets K_i. If $\rho_D = 1$ then ℓ_i is simply the debt/asset ratio and $K_i - L_i$ is book equity (assuming that all assets and liabilities are stated at market value).

The basic idea behind the leverage cost function is that it takes effort to convince depositors and other debtholders that the bank will repay debt. The effort required per unit of debt is larger if more debt has been issued and smaller the more collateral is available in the form of bank assets. The weight on deposits $\rho_D \in [0, 1]$ allows for the possibility that depositors require less effort to be convinced, perhaps because they are insured and the regulator makes some additional effort. Its only role in the model is to make deposits a cheaper source of financing than overnight borrowing.

We assume that the leverage cost $c(\ell) L$ is a fee that is paid by banks to households and thus shows up on the right-hand side of the household budget constraint (1). One interpretation is that households inelastically supply specialized labor. For welfare analysis, it could be interesting to make explicit the utility cost of labor or alternatively interpret c not as effort but as a deadweight cost, for example a bankruptcy cost. The distinction does not matter for the positive analysis we undertake in this paper.

The effect of leverage costs in our model is to generate a determinate optimal leverage ratio. As we will see below, the equilibrium interest rates on deposits and overnight credit are below the rate of return on equity (that is, the discount rate). As a result, debt is a strictly cheaper source of financing at low levels of leverage. The bank will choose leverage to equate the marginal costs of equity and debt. In effect, the marginal cost of debt is a smoothed-out version of what it would be in a model with a collateral constraint: if we assumed e.g. $L \leq \alpha K$ for some number α, the extra cost of debt would be zero up to the constraint and infinite thereafter.

To derive bank first-order conditions, it is helpful to define the marginal cost of leverage as

the derivative of the leverage cost $c(L/K)L$ with respect to debt L, and the marginal benefit of collateral as the derivative with respect to K:

$$\begin{aligned}\lambda(\ell) &= c(\ell) + c'(\ell)\ell, \\ \kappa(\ell) &= c'(\ell)\ell^2.\end{aligned}$$

An extra unit invested in assets that contribute to collateral K earns not only a pecuniary return, but also the collateral benefit $\kappa(\ell)$. Similarly, a unit borrowed entails not only an interest cost but also the extra leverage cost $\lambda(\ell)$. Concavity implies that both λ and κ are increasing in leverage since $2c'(\ell) + c''(\ell)\ell > 0$. We further assume that the cost function c slopes up sufficiently fast that banks always choose $\ell < 1$. In this case, we also have that for a given level of leverage, the marginal cost of leverage is higher than the marginal collateral benefit: $\lambda(\ell) > \kappa(\ell)$.

Bank first-order conditions for assets

The typical bank's first-order conditions describe the key trade-offs of portfolio and capital structure choice. Since shareholders are risk neutral, the expected marginal benefits or costs of all assets and liabilities are compared to the return on equity δ. Importantly, benefits and costs are not only due to returns but also to leverage and liquidity. If the portfolio of the bank is chosen optimally, the marginal benefit from any asset cannot be larger than δ: if not, then the bank would choose to invest more. Moreover, the marginal benefit is equal to δ if the bank optimally holds a positive position in the asset. If the marginal benefit is below δ then the bank does not invest: it is better to pay dividends instead.

Consider the first-order condition for overnight lending, which not only earns the real interest rate $i - \pi$, but also the collateral benefit $\kappa(\ell_i)$. The return on equity must be higher than the marginal benefit:

$$\delta \geq i - \pi + \kappa(\ell_i), \tag{6}$$

with equality if bank i lends overnight. In the latter case, low real interest rates imply that overnight credit is valuable collateral, so banks optimally choose higher leverage. Put differently, highly levered banks obtain a high benefit from overnight lending as collateral and thus require a lower return on credit.

The first-order condition for trees is similar. The real rate of return on tree j held by banks is $r_j = \log(Q_j + P)/Q_j - \pi$. Since trees also deliver collateral benefits, we must have

$$\delta \geq r_j + \kappa(\ell_i),$$

with equality for trees held by bank i. Since the return on trees held by bank i is lower than the discount rate δ, their steady state price-dividend ratio is higher than that of trees held by households: we have $Q_j/P = (\delta - \kappa(\ell_i))^{-1}$. In particular, if banks holding tree j are more levered, then those trees are more valuable collateral, their cash flows are discounted at a lower rate and their price is higher.

Reserves differ from overnight credit and trees in that they not only provide returns $i_R - \pi$ and collateral benefits, but also liquidity benefits – they can be accessed in the first subperiod. The liquidity benefit depends on the Lagrange multiplier on the intraday credit limit (2) in the

first subperiod, denoted μ_i. The first order condition for reserves is

$$\delta \geq i_R - \pi + \kappa(\ell_i) + (1+\gamma) e^{-\delta} PE[\mu_i'].$$

The liquidity benefit (the second term) is higher the higher the expected discounted Lagrange multiplier $E[\mu_i']$ and the more intraday credit can be obtained per dollar of reserves (higher γ).

Bank first-order conditions for liabilities

Consider now banks' choice to finance themselves with deposits or overnight credit. If the capital structure of the bank is chosen optimally, the marginal cost of any liability type cannot be lower than δ, the cost of issuing equity: if not, banks would choose to borrow more. Moreover, the marginal cost is equal to δ if the bank holds a positive position in that particular liability type. If the marginal cost is above δ then the bank does not issue the liability: it is better to issue equity instead.

The first-order condition for deposits says that the equity return must be smaller than the sum of the real deposit rate plus the marginal leverage and liquidity costs of deposits

$$\delta \leq i_D - \pi + \rho_D \lambda(\ell_i) + e^{-\delta} PE[\mu_i' \phi_i'], \tag{7}$$

with equality if the bank issues deposits. Leverage costs increase with overall leverage through $\lambda(\ell_i)$ and are also scaled by the parameter ρ_D which makes deposits cheaper than other borrowing. Liquidity costs arise in those states next period when positive withdrawal shocks $\phi_i' > 0$ coincide with a binding intraday credit limit $\mu_i' > 0$.

The first-order condition for overnight borrowing says that the equity return must be smaller than the sum of the real overnight rate plus the marginal leverage cost, *less* the liquidity benefit provided by overnight credit:

$$\delta \leq i - \pi + \lambda(\ell_i) - \mu_i(1+\gamma). \tag{8}$$

For banks that borrow overnight, the condition holds with equality. This can happen because for those banks the intraday credit limit binds and μ_i can be positive. In contrast, banks with sufficient reserves we have $\mu_i = 0$ and do not borrow.

Since the bank problem exhibits constant returns to scale, the first-order conditions only pin down the leverage ratio ℓ_i. Given optimal leverage, banks are indifferent between positions in all assets and liabilities such that the first-order conditions hold with equality. The equilibrium quantities for the different positions are then determined by the other side of the market in equilibrium.

3.3 Equilibrium

We treat the government as a single entity that comprises the central bank and the fiscal authority. The government fixes a path for the supply of reserves M, nominal overnight government debt $\overline{B}$ and the reserve rate i_R. It may also choose to purchase trees. For now, we restrict attention to stationary policies: reserves and nominal government debt grow at the same rate, so $\bar{B}/M$ is constant and the number of trees held by the government is also constant. The

government makes lump sum net transfers to households so that its budget constraint is always satisfied.

Equilibrium requires that markets clear at the optimal choices of banks and households. Goods market clearing requires that households consume the endowment and all fruit from trees. We denote the quantity of real transactions in the economy by $T = \bar{v}(\Omega + x)$. Tree market clearing requires that banks or household hold all trees. The overnight credit market clears if borrowing by the government as well as borrower banks is held by lender banks. Finally,.banks must hold all reserves.

We focus on steady state equilibria with constant rates of return and a binding deposit-in-advance constraint. Since the quantity of transactions is fixed, deposits and the price level must grow at the same rate. Moreover, the key ratios chosen by banks, leverage ℓ_i and the liquidity ratio ϕ_i^*, must be constant over time. The government thus controls the steady state inflation rate through its choice of the growth rate of reserves. The nominal price level, however, is endogenous and depends on how many deposits banks produce for a given level of reserves.

Asset market participation and aggregation

The bank first-order condition (7) implies that, in equilibrium, the real rate on deposits must be below the discount rate. Indeed, deposits entail leverage costs, and banks would not choose to fund themselves with deposits unless they were cheaper than equity. Without deposits, however, the goods market could not clear. It then also follows from equation (7) that zero leverage cannot be optimal: if $i_D - \pi < \delta$, then at least the first dollar of deposits is always cheaper than equity, so all banks issue some deposits.

Since banks are levered in equilibrium, households do not invest in any asset markets that banks can invest in. Indeed, for levered banks, all accessible assets bring collateral benefits over and above the return. As a result, at least one bank will bid up the price of any asset accessible to banks until its return is below the discount rate. In particular, the equilibrium real overnight rate is always below the discount rate and banks are the only overnight lenders.

The first order conditions further imply that all active banks choose the same optimal leverage.[5] The optimal leverage ratio can be read off the first order condition for any asset that is accessible to banks. For example, the equilibrium spread between equity and the real overnight credit rate is related to aggregate banking sector leverage ℓ by

$$\delta - (i - \pi) = \kappa(\ell). \tag{9}$$

Liquidity management and the liquidity trap

Given equal leverage, banks also face the same one-step-ahead conditional distribution of the multiplier on the intraday credit limit (8). From equation (3), the threshold shock beyond which the intraday credit limit binds and banks borrow overnight is $\phi^* = M(1+\gamma)/\bar{v}D$. Substituting into the first-order condition for reserves, this threshold must satisfy

$$i - i_R = (1 - G(\phi^*))(\lambda(\ell) + i - \pi - \delta). \tag{10}$$

[5]Indeed, suppose bank 1 has higher leverage than bank 2, and hence a higher collateral benefit $\kappa(l_i)$. In equilibrium, bank 2 does not invest in illiquid assets (trees or overnight credit); due to the lower collateral benefit it is willing to pay strictly less than bank 1 for those assets. To be active, bank 1 must hold some assets, so suppose it holds only reserves. However, the liquidity benefit it earns from reserves cannot be higher than that of the more leveraged bank 2, a contradiction.

The spread between overnight and reserve rate must reflect the expected liquidity benefit of holding reserves. A liquidity benefit obtains only when the withdrawal shock exceeds ϕ^*, that is, with probability $1 - G(\phi^*)$. In this case, a bank holding an extra dollar of reserves saves the excess cost of overnight lending relative to equity. For given ϕ^*, a more leveraged banking system faces a larger penalty of running out of reserves and hence a larger spread.

In equilibria with $i > i_R$, the spread (10) shows how banks choose the reserve-deposit ratio as a function of interest rates and leverage. The interest rate i plays a dual role here. On the one hand, it affects the opportunity cost of reserves. Indeed, holding fixed the liquidity benefit, the equation works much like a money demand equation: if the overnight rate is higher, then it is more costly to hold reserves and banks choose smaller ϕ^*, or fewer reserves per dollar of deposits. On the other hand, the interest rate affects the liquidity benefit itself: holding fixed leverage ℓ and the spread, a higher interest rate increases the liquidity benefit of reserves and leads banks to choose more reserves to avoid the higher penalty of running out.

We can now compare how banks handle payments instructions generated by the total quantity of enduser transactions $T = \bar{v}D$ using either reserves or overnight credit. The withdrawal threshold can be written as

$$\phi^* = (1 + \gamma) \frac{M}{PT}, \tag{11}$$

that is, it is proportional to the ratio of real reserves to transactions.

Given leverage ℓ and a threshold ϕ^* below the upper bound, say $\bar{\phi}$, we can use (4) to compute the ratio of outstanding interbank credit to transactions

$$\frac{F}{PT} = f(\phi^*) := \frac{1}{1+\gamma} \int_{\phi^*}^{\bar{\phi}} (\phi - \phi^*) dG(\phi).$$

The function f is decreasing in ϕ^*: if interest rates are such that banks hold a lot of reserves, then ϕ^* is high and banks rarely run out of reserves, so outstanding interbank credit is low. In this sense, reserves and overnight are substitutes in liquidity management.

When equilibrium interest rates on overnight credit and reserves are equal, the economy is a in a liquidity trap. In this case, the reserve-deposit ratio is so high that $G(\phi^*) = 1$. Banks can always obtain more intraday credit than what is needed to withstand even the largest possible withdrawal shock without any overnight borrowing. As a result, the interbank overnight credit market shuts down completely. Of course, banks may still lend overnight to the government. The distinction between reserves and short government is now blurred; an amount of reserves $\bar{\phi}\bar{v}D/(1+\gamma)$ still serves to buffer liquidity shocks, but reserves beyond this amount are equivalent to government debt.

The costs of deposits

Combining banks' first-order condition for deposits and reserves, the spread between deposits and reserves is

$$\begin{aligned} i_D - i_R &= \kappa(\ell) - \rho_D \lambda(\ell) \\ &\quad + \left(1 - G(\phi^*) + \frac{\bar{v}}{1+\gamma} \int_{\phi^*}^{\bar{\phi}} \phi dG(\phi)\right) (\lambda(\ell) - \kappa(\ell)). \end{aligned}$$

It has both a leverage and a liquidity component. In the liquidity trap, we have that $\phi^* > \bar{\phi}$ so the term in the second line vanishes and the spread only reflects the difference between the collateral benefit of reserves and the leverage cost of debt.

If liquidity does not matter, the marginal cost of deposits can still differ from that of reserves because deposits require bank leverage. If ρ_D and ℓ are large enough, the deposit rate can even be below the reserve rate: enduser payments in our model cannot be made with reserves but require leverage and competitive banks pass on their leverage costs to depositors.[6] If ρ_D is sufficiently small, leverage via deposits is cheaper than other borrowing and the deposit rate is above the reserves rate. Movement of the spread with leverage also depends on ρ_D. In what follows, we assume that ρ_D is large enough that the deposit spread is decreasing in leverage for given liquidity ratio ϕ^*; this is always true for example if $\rho_D = 1$.

Outside the liquidity trap, the spread reflects in addition the difference between the liquidity benefit of reserves and the liquidity cost of deposits. The presence of netting within the banking system implies that this difference is always positive; moreover the spread is decreasing in ϕ^* for given leverage. Intuitively, an extra dollar of reserves allows the bank to access $1+\gamma$ dollars of intraday credit, whereas an extra dollar of deposits generates at most $\bar{\phi}\bar{v} < 1$ withdrawals and hence need for intraday credit. As long as the shadow price of intraday credit is positive, banks require less compensation for holding reserves than for issuing deposits, so the spread is higher.

4 Equilibrium: banks and households only

Throughout all our analysis, we consider comparative statics of steady states. The time period in the model should be thought of as very short, such as a day. Moreover, the model has no transition dynamics. We are therefore comfortable using the model for thinking about he behavior of asset prices, payments and credit over a sequence of years, such as the recent boom bust cycle. In particular, we are interested in the effect of shocks to agents' belief about future asset payoffs (such as those on claims to housing) as well as monetary policy responses and study those effects as a sequence of comparative statics.

4.1 Graphical analysis

The predictions of the model can be characterized by reducing the system of equations characterizing equilibrium to only two equations in the key ratios, leverage ℓ and the liquidity ratio ϕ^*. Since the liquidity ratio maps 1-1 into the price level and the leverage ratio maps 1-1 into the real interest rate, we then proceed to a graphical analysis that shows how these two relative prices are determined.

The liquidity management curve

As a first connection between the ratios ℓ and ϕ^*, we substitute equation (9) into (10) to obtain

[6] The term "deposit rate" should be interpreted broadly here: it is the only cost endusers pay for payments serives in our model, since we do not explicitly model other costs such as account and transaction fees.

$$\delta - \kappa(\ell) - (i_R - \pi) = (1 - G(\phi^*))(\lambda(\ell) - \kappa(\ell)). \tag{12}$$

This equation represents pairs (ℓ, ϕ^*) such that banks optimally choose their liquidity ratio given leverage ℓ. We thus refer to it as the *liquidity management curve*.

The opportunity cost of reserves on the left hand side is decreasing in leverage. The liquidity benefit of reserves $\lambda - \kappa$ on the right hand side is positive and increasing in leverage. This is because the leverage cost c is increasing and leverage is smaller than one. In equilibrium, any dollar of interbank credit entails leverage cost for the borrowing bank but adds collateral benefits for the lending bank. Since the former effect is larger, having to borrow overnight implies a penalty for running out of reserves that increases with leverage.

Together with the fact that G is a cdf, we now have that the liquidity management curve (12) describes an upward sloping curve in the (ϕ^*, ℓ) plane. For comparative statics, it is more interesting to plot the curve in the $(1/P, i - \pi)$ plane, so we can read off implications for observable endogenous prices. By definition, ϕ^* is proportional to the value of money $1/P$. Moreover, the interest rate is decreasing in leverage from equation (9). We can thus equivalently represent equation (12) as a downward sloping curve in the $(1/P, i - \pi)$ plane: this is the blue curve in Figure 3.

Moving along the curve in price space, a decrease in the real interest rate encourages banks to hold more reserves. Higher demand for reserves thus hits the same supply, so that the real value of reserves increases, or the nominal price level declines. Put differently, a decrease in the real interest rate makes it cheaper for banks to hold reserves relative to deposits. Since the supply of reserves has not changed, banks instead reduce the supply of deposits; with constant velocity in the enduser layer, the price level declines.

The liquidity management curve has a flat section for high values of money, where the real overnight interest rate equals the real interest rate on reserves. This region marks the liquidity trap. Movements along the curve correspond to changes in the quantity of deposits and hence changes the price level. However, since the real value of reserves is so high relative to the real quantity of transactions that banks can pay any withdrawal shocks without overnight borrowing, the interest rate is not affected in this region.

The capital structure curve

Starting from the definition of the leverage ratio and substituting in for banks' equilibrium asset and liability positions, we obtain a second relationship between the leverage and liquidity ratios:

$$\ell = \frac{\rho_D D + F}{M + B + x\int_0^\beta Q_j \theta_j dj} = \frac{\rho_D T/\bar{v} + Tf(\phi^*))}{\frac{\beta x}{\delta - \kappa(\ell)} + \phi^* \frac{T}{1+\gamma}\left(1 + \frac{\bar{B}}{M}\right) + Tf(\phi^*)}. \tag{13}$$

The equation represents pairs (ℓ, ϕ^*) so that banks' leverage is consistent with the quantities of debt and collateral implied by their liquidity management policies. We refer to it as the capital structure curve.

Other things equal, an increase in banks' liquidity ratio ϕ^* increases bank collateral and reduces leverage. Moreover, higher ϕ^* reduces outstanding interbank credit $Tf(\phi^*)$. Since every dollar of interbank credit is both an asset and a liability to the banking sector, a reduction in interbank credit also reduces overall leverage. As moreover the real value of assets held by

banks is increasing in leverage, the capital structure curve is downward sloping in the (ϕ, ℓ) plane. Equivalently, we can represent it as an upward sloping schedule in the $(1/P, i - \pi)$ plane: this is the green line in Figure 3.

Moving along the curve in price space, a decrease in the real interest rate goes along with lower leverage. This is accomplished by an increase in the value of collateral as well as a reduction in interbank credit. For a given quantity of reserves, both require a decline in the price level. Put differently, an increase in interest rates drives banks to reduce leverage. This entails in particular a reduction in the quantity of deposits relative to given nominal reserves; with constant velocity in the enduser layer the price level declines.

Neutrality of money

Our model describes a steady state equilibrium with flexible prices. As a result, money is neutral: increasing the money supply by a fixed factor only increases the price level by the same factor, leaving the real interest rate as well as all other real variables unchanged. Indeed, the supply of reserves M enters equations (12) and (13) only because real reserves M/P are contained in the threshold ϕ^*, and in the ratio $\bar{B}/M$ which we have assumed to be constant throughout. In terms of the figure, an increase in M thus shifts both curves to the left to generate a proportionate increase in the price level for the same real interest rate.

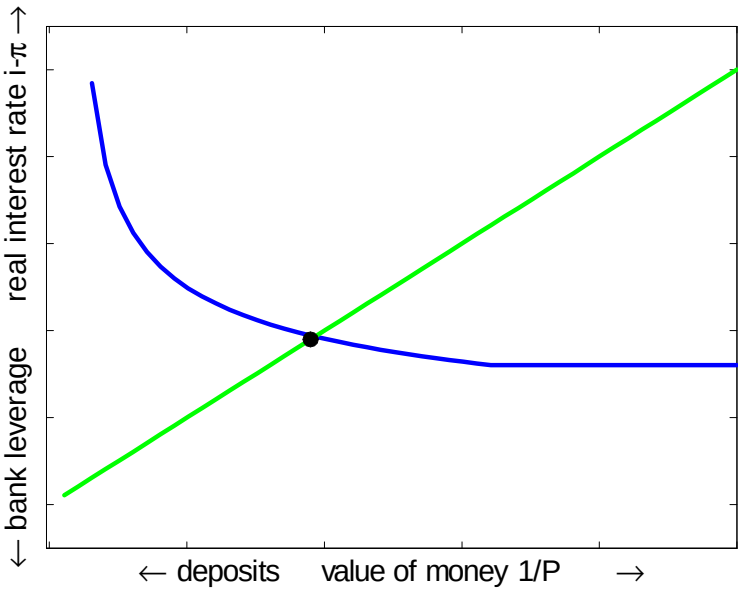


Figure 3: The green line is the liquidity management curve. The blue line is the capital structure curve.

4.2 Introducing uncertainty

We capture a change in uncertainty as a change in beliefs about the tree payoffs x. In particular, we assume that households behave *as if* tree payoffs fall at a rate s. At the same time, actual tree payoffs remain constant at x throughout. One way to think about these beliefs

is that households are simply pessimistic. Our preferred interpretation is ambiguity aversion: households contemplate a range of models for payoffs, and evaluate consumption plans using the worst case model. In either case, the key effect of pessimistic valuation is to generate premia on assets: an observer (such as an econometrician measuring the equity premium) will observe low prices relative to payoffs and hence high average returns.

To define equilibrium, we must take a stand not only on beliefs about exogenous variables, but also about endogenous variables such as the nominal price level and asset prices. We follow Ilut and Schneider (2014) who also capture the presence of uncertainty via low subjective mean beliefs about exogenous variables: beliefs about endogenous variables follow from agents' knowledge of the structure of the economy. In particular, agents know the policy rule of the government and that banks maximize shareholder value given the households' discount factor. Households' worst case beliefs thus also affect bank decisions.

Characterizing equilibrium is particularly simple if households expect the government to run a monetary policy that implements a constant inflation rate π. Households therefore perceive ambiguity only about x and hence the number of transactions T, but not about the evolution of the price level. They do perceive ambiguity about the path of reserves which differs from the actual constant growth path; the common denominator between the perceived and the actual policy is only that they keep inflation constant at π. This assumption serves to focus on changes in uncertainty in asset markets only.[7]

Working with mean beliefs allows us to capture the essential features of uncertainty with minimal changes to the model equations. The household first order condition relates the price households observe in the market in the current period with their expected future price. Since households behave as if they live – from next period onwards – in a steady state in which tree payoffs fall at the rate s, the first order condition is now

$$\frac{Q_j}{P} \geq e^{-\delta - s} \left(\frac{Q_j}{P} + 1 \right). \tag{14}$$

In other words, payoffs on trees held by households are now discounted at the rate $\delta + s$.

To see how ambiguity generates premia on assets, consider a tree j held by households so that (14) holds with equality. The equilibrium price of the tree is $(\delta + s)^{-1}$. An econometrician who observes prices and payoffs measures a return $\delta + s$ which is higher than the discount rate. If there was also a second "safe" tree held by households that earns exactly the discount rate, the econometrician would measure an equity premium on the uncertain tree. In terms of comparative statics, an increase in uncertainty captured by an increase in s leads to higher premia and lower prices.

The bank first order condition for trees accessible by banks is now

$$\delta = r_j - s + \kappa(\ell).$$

[7] An alternative equivalent approach assumes that there two goods - cash and credit goods - that are perfect substitutes in utility so they the relative price is one in equilibrium. The endowment is in cash goods that must be purchased with deposits, whereas trees pay off a credit good that does not require deposits. In this setup, we can assume that households perceive no ambiguity about monetary policy at all; they know that reserves grow at a constant rate.

The presence of uncertainty does not change the fact that all accessible trees are held within banks in equilibrium since trees still provide collateral benefits. Returns on asset held by banks are affected by two opposing forces: compensation for uncertainty born by shareholders tends to make returns higher, while the collateral benefit tends to make returns lower. The presence of uncertainty thus also puts a wedge between the return on trees held by banks and the return on safer overnight credit.

In terms of our graphical analysis, the only change is to the value of bank trees in the capital structure curve. In particular, the first term in the denominator on the right hand side of (13) should be replaced by

$$\frac{\beta x}{\delta + s - \kappa(\ell)},$$

the value of bank trees after discounting at the higher rate that includes the uncertainty premium. Holding fixed leverage, an increase in the uncertainty premium s lowers the value of bank trees for given leverage and the value of banks' collateral. For leverage to remain the same, the value of reserves must increase, that is, price level falls. It follows that the capital structure curve shifts to the right, as in Figure 4. The liquidity management curve is not affected by changes in s.

Increases in uncertainty and the liquidity trap

Starting from an equilibrium outside the liquidity trap, an increase in uncertainty lowers the real interest rate and the price level. This is because the drop in collateral values makes it more expensive for banks to create deposits. As a result, the supply of inside money declines and generates deflation. In the new equilibrium, bank leverage is higher, the reserve-deposit ratio is higher and the ratio of interbank credit to deposits is lower.

If the increase in uncertainty is large enough, the capital structure curve shifts so far to the right as to push the economy into the liquidity trap. At this point, further increases in uncertainty no longer change the overnight interest rate. The real quantity of reserves is now so high relative to the real quantity of transactions that all withdrawal shocks can be handled with reserves and intraday credit alone. The interbank overnight market shuts down entirely. Further increases in uncertainty do still lower the value of collateral, the nominal quantity of deposits and the price level.

4.3 Monetary and fiscal policy

An increase in uncertainty is an attractive candidate for a shock that could have occurred at the beginning of the recent financial crisis. It is consistent with an increase in asset premia, a drop in uncertain asset prices, a decline in the short term interest rate all the way to the reserve rate as well as an increase in bank leverage and an effective shutdown of interbank Federal Funds lending. However, we did not see a large deflation – after an initial small drop in late 2008 the price level remained quite stable over time.

According to our model, an obvious candidate for the absence of deflation is monetary policy. The simplest thought experiment says that if the central bank had simply given away a lot of reserves, then, by neutrality, the price level should have increased without further effects on the real variables listed above. *Simultaneous* increases in uncertainty about asset payoffs

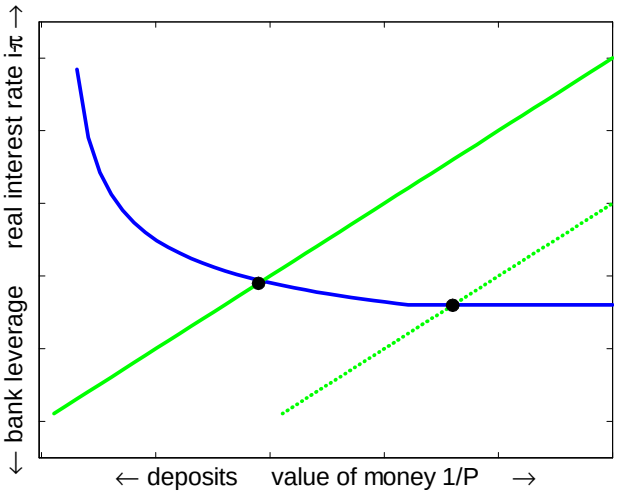


Figure 4: Increased uncertainty about tree payoffs

and the supply of reserves could thus explain why the economy moved into the liquidity trap while the price level remained stable.

Actual monetary policy did not consist of a give-away of reserves. Instead, policy involved various swaps of reserves for government bonds, government bonds for private assets (that is, trees) as well as reserves for trees. We now consider a sequence of comparative statics in which the government runs a policy that changes the asset mix in the economy.

Open market purchase of government bonds

Suppose the government uses reserves to buy bonds from banks. We start from an initial equilibrium outside the liquidity trap with reserves M and short government debt $\bar{B}$. We define a "conventional open market purchase" by new nominal quantities of reserves M^* and bonds $\overline{B}^*$. We assume that the total market value of government liabilities remains unchanged, but the share of reserves increases, that is, $M^* - M = \bar{B} - \bar{B}^* > 0$. Lump sum transfers are used to ensure the government budget constraint holds at all dates.

To see the mechanics, it is helpful to decompose the policy experiment into two components: a helicopter drop of reserves and a confiscation of bonds. The helicopter drop is neutral: both curves shift to the left to achieve a proportionally lower real value of money $1/P$, leaving the interest rate unchanged. The confiscation of bonds only affects the capital structure curve: in particular, it lowers the quantity of available collateral and thus shifts that curve to the right. Importantly, the shift does not go all the way back to the old capital structure curve, because the increase in reserves also decreases interbank credit. It follows that, outside the liquidity trap, an open market purchase of short bonds unambiguously lowers the real interest rate an increase the price level.

Intuitively, several effects are at work. The shift of the liquidity management curve represents a liquidity effect in the bank layer. In order for banks to absorb more reserves, the interest rate on overnight credit declines. An increase in reserves relative to other collateral allows banks to produce more deposits at lower liquidity cost. The resulting increase in deposits pushes up the price level. Banks also leverage more and demand collateral, further pushing

down the interest rate.

At the same time, the shift in the capital structure curve represents a counteracting effect on the interest rate and leverage, due to the presence of the interbank credit market. As the quantity of reserves increases, interbank borrowing becomes less important, thus "deleveraging" the banking sector. Lower leverage reduces the demand for collateral and increases the interest rate. It also allows banks to create more deposits so that the price level increases. Overall, the interbank credit effect on the interest rate is weaker than the liquidity effect, so that the interest rate falls.

If the initial equilibrium is in a liquidity trap, conventional open market purchases have no effect. The mechanics can be seen from the decomposition discussed above. The helicopter drop works as before. However, a confiscation of bonds now shifts the capital structure curve all the way back to its old location. This is because the policy only swaps two types of collateral and therefore has no effect on bank leverage. Conventional open market purchases can change the real interest rate only if it also affects leverage indirectly through the interbank market.

The government could affect interest rates in the liquidity trap through an open market sale of government bonds. Indeed, suppose the government does the reverse of the above policy and reduces the supply of reserves. Mechanically, the liquidity management curve moves to the right, also increasing the critical value of money at which the economy enters the liquidity trap. At the same time, the capital structure curve moves to the right if the reduction in reserves is large enough. For a large enough reduction in reserves, the new equilibrium is outside the liquidity trap at a higher interest rate and a lower price level.

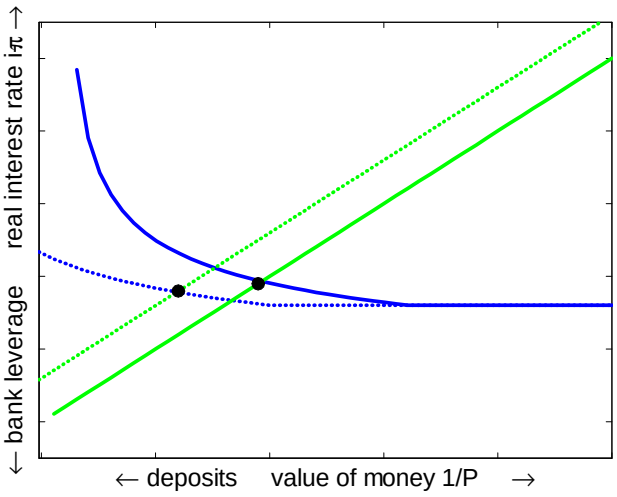


Figure 5: Open market operations buy bonds with reserves

Fiscal policy

To explore the government's ability to change the value of collateral, we consider an exchange of government bonds for trees. Since the quantity of reserves does not change, liquidity management is not directly affected. As a stark example, suppose the government takes all trees off banks' books in exchange for new government bonds $\bar{B}^* - \bar{B}$. The effect of the policy depends crucially on the price that the government pays for the trees. We first assume that the government sets the amount B^* so that interest rate payments on new bonds can always

be covered by tree payoffs, that is, $i\left(B^{*}-B\right)=P\beta x$. In other words, the government breaks even every period. From the perspective of households, however, the government has purchased trees at an "inflated" price $P\beta x/i$ that does not incorporate an uncertainty premium.

A purchase at inflated prices has the effect of increasing the overall value of collateral available to banks. From the perspective of households, "toxic" collateral is replaced by more valuable collateral. As a result, it makes sense to invest in banks and increase the creation of deposits. The economy exits the liquidity trap, as the real interest rate and the price level rise, as in the left hand panel of Figure 6.

Alternatively, suppose the government issues bonds to purchase trees at market price. We define a policy by a new quantity of reserves B^{*} such that $\bar{B}^{*}-\bar{B}=P\beta x/\left(\delta+s-\kappa\left(\ell\right)\right)$. The result is illustrated in the right hand panel in Figure 6: the capital structure curve does not shift but instead rotates upward through the old equilibrium. This is because the policy is designed to leave the overall value of collateral unchanged. The steeper slope is due to the fact that the government debt is nominal, so bank balance sheets are more sensitive to changes in the price level after the policy action.

An interesting feature of any government tree purchase, whether or not they change the value of collateral, is it leaves banks less exposed to the price of trees. While this may not have an any immediate effects, it does determine how the economy will react to future shocks. In particular, suppose that after the government has bought all bank trees, there is a reversal in beliefs towards lower uncertainty, that is, a reduction in the premium s. Prices of trees held by households now increase. However, the balance sheets of banks that have sold their trees do not improve along with tree prices. Instead bank leverage remains high and the economy remains in the liquidity trap. *Open market purchase of trees + interest on reserves*

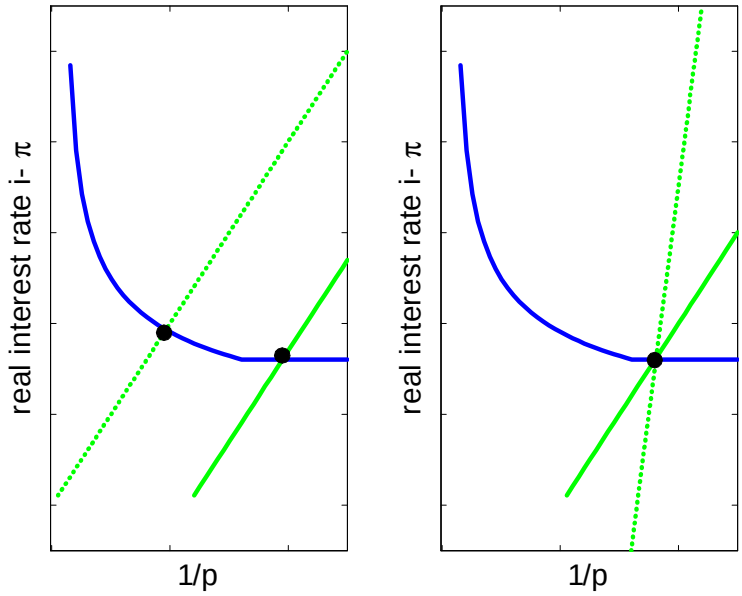


Figure 6: Unconventional monetary policy buys trees at inflated prices in the left panel and at market prices in the right panel

In general, monetary policy differs from fiscal policy in that it also affects liquidity man-

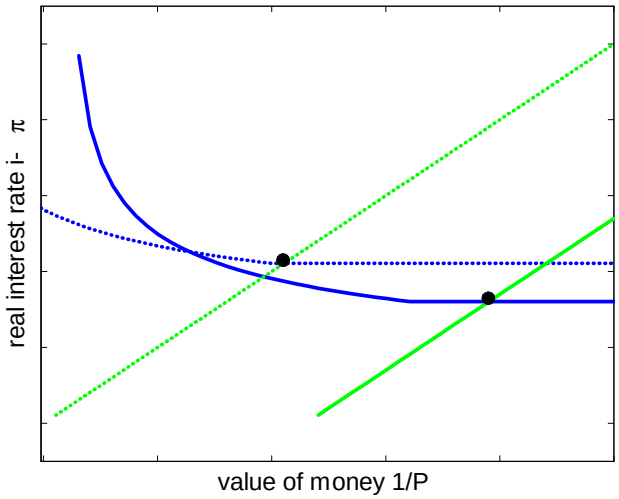


Figure 7: Open market purchase of trees and interest on reserves

agement. Indeed, an open market purchase of trees not only affects the value of collateral, but also banks liquidity ratio. In the liquidity trap, however, the effects of open market purchases are the same as the swaps of bonds for trees studied above. Indeed, consider an open market purchase of trees at market prices, defined by a new quantity of reserves that satisfies $M^* - M = P\beta x / (\delta + s - \kappa(\ell))$. The shift in the capital structure curve is the same as in the right hand panel of Figure 6. In addition, the increase in reserves induces a leftward shift in the liquidity management curve. This does not change the equilibrium however since the economy initially was in the liquidity trap.

Again similar to the case of fiscal policy, open market purchases of trees can increase the value of collateral if the government buys trees at inflated prices. As one example of such a policy, suppose that the government not only purchases trees but also increases the interest rate on reserves. In particular, the policy is set up such that interest on reserves can be paid out of tree payoffs and the government again breaks even in all periods after the first: the policy is defined by a new quantity of reserves M^* such that $i_R M^* = P\beta x$. Since the government again buys trees at high prices, the capital structure curve shifts to the left; in addition, the liquidity management curve shifts up (as interest on reserves increases) and to the left (as reserves also increase). The result, shown in Figure 7, is again an increase in the price level and the real interest rate away from the original liquidity trap equilibrium.

5 Institutional investors

So far in the paper, the only link between tree (that is, securities) markets and the payments system is that banks invest in trees. In this section, we extend the model to introduce two additional links: banks lend short overnight to institutional investors and institutional investors use deposits to trade assets. To clarify the effect of each link in isolation, we introduce two types of asset management firms: *carry traders* buy trees on margin, whereas *active traders* face a deposit-in-advance constraint for some asset purchases. Both types of firms otherwise

work like banks: they are competitive firms owned by households that have access to a subset of trees and maximize shareholder value.

5.1 Carry traders

Carry traders are competitive firms that issue equity, borrow overnight and invest in the subset of β^* trees $j \in [\beta, \beta + \beta^*]$, which is distinct from the subset accessible to banks. Like banks, carry traders face leverage costs, captured by an increase concave function c^* that could be different from the cost function c assumed for banks. The leverage ratio of carry trader i is defined as overnight credit F_i divided by the market value of the tree portfolio

$$\ell_i^* = \frac{F_i^*}{x \int_\beta^{\beta+\beta^*} Q_j \theta_{i,j}^* dj}.$$

Carry traders' marginal collateral benefit and marginal cost of leverage are denote κ^* and λ^*, respectively.

We assume further that carry traders are more optimistic about tree payoffs than households: they s and perceive less uncertainty $s^* < s$ about their payoffs. The idea here is that the firm employs specialized employees who households trust to make asset management decisions. As a result, the spread relevant for investment in carry trader trees *indirectly through investment by carry traders* carries the lower uncertainty premium s^*.

Optimal investment and borrowing

Carry traders' first order condition for overnight credit resembles that of banks in (8), except that overnight borrowing does not provide liquidity benefits: the return on equity must be smaller than the real overnight rate plus the marginal cost of leverage Since we already know that the real rate is lower than δ in equilibrium, it is always optimal for carry traders to borrow and we directly write the condition as an equality:

$$\delta = i - \pi + \lambda^* \left(\ell_i^* \right), \tag{15}$$

It follows that all carry traders choose the same leverage ratio and we drop subscripts from now on. Moreover, carry trader leverage is higher in equilibrium when interest rates are low.

Like banks, carry traders hold all trees accessible to them. This is due not only to the collateral benefit conveyed by trees, but also to carry traders' relative optimism. The first order condition for tree j is

$$\delta = r_j - s^* + \kappa^* \left(\ell^* \right).$$

When interest rates or uncertainty is low, carry traders apply a lower effective discount rate to trees, which results in higher tree prices.

The amount of carry trader borrowing in equilibrium is

$$F^* = \ell^* \frac{\beta^* x}{\delta + s^* - \kappa^* \left(\ell^* \right)}. \tag{16}$$

The fraction represents the market value of carry trader trees. Lower interest rates increase both leverage and the value of collateral and therefore increase borrowing. Moreover, an increase in

uncertainty perceived by carry traders (that is, an increase in s^*) lowers collateral values and borrowing.

Equilibrium with carry traders

Our graphical analysis of equilibrium remains qualitatively similar when carry traders are added to the model. The only change is that carry trader borrowing now enters on the asset side of the banking sector. We can thus add in the denominator of (13) a term $B^*(\ell)$ that expresses carry trader borrowing as a function of bank leverage. We obtain the function B^* by substituting for ℓ^* in (16) from (15) and then substituting for the interest rate from the bank first order condition from (9). The function B^* is increasing: if banks are more levered, the interest is lower and carry traders borrow more.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with carry traders. Suppose first that, starting from an equilibrium outside the liquidity trap, there is an increase in uncertainty perceived by carry traders only. As carry traders value trees less, they demand fewer loans from banks. This increases bank leverage and shifts the capital structure curve to the right. The liquidity management curve does not change. In the new equilibrium, bank leverage is higher and the interest rate is lower, as is the price level. The effect on prices is therefore the same as for an increase in uncertainty about bank trees. The additional prediction is that we should see a decline in funding of institutional investors via short term credit from payments intermediaries, such as a decline in repo extended by money market mutual funds to broker-dealers.

It is also interesting to reconsider the effect of monetary policy. Suppose policy engineers a change in the mix of bank assets or their value that lowers the real overnight interest rate. Carry trader borrowing increases and carry traders bid up the prices of the trees they invest in. As one segment of the tree market thus increases in value, the aggregate value of trees also rises: there is a tree market booms. Importantly, this is not a standard real interest rate effect: the discount rate of households, which is used the value trees held by households, is unchanged. The effect comes solely from the effect of monetary policy on the overnight rate and hence on carry traders' funding costs.

5.2 Active traders

Active traders are competitive firms that issue equity and invest in deposits as well as a subset of $\hat{\beta}$ trees $j \in [\beta^*, \beta^* + \hat{\beta}]$. There are many active traders and each is optimistic about one particular "favorite" tree: the trader perceives low uncertainty $\hat{s} < s$ about this tree. Every period, the identity of the favorite tree within the subset $[\beta^*, \beta^* + \hat{\beta}]$ changes with probability $\bar{v} \leq 1$ to some other tree in the subset Here $\bar{v}$ is chosen to be the same as velocity for deposits held by households – this assumption simplifies the analysis by leaving average enduser velocity unchanged.

To generate a need for deposits, we assume that active traders who change their tree position must place an order for trees in the first subperiod and pay with deposits or intraday credit. The first subperiod budget constraint is thus

$$z_i \int Q_j \hat{\theta}'_{i,j} \delta = I_i + \hat{D}_i$$

where z_i is a random variable that indicates whether or not the favorite tree of fund i has changed in the current period, I_i is the intraday credit position and $\hat{D}_i$ are deposits that the fund keeps at its bank.

Like banks, active traders faces a limit on intraday credit

$$I_i \leq \hat{\gamma}\hat{D}_i,$$

where $\hat{\gamma}$ is a parameter that governs netting in tree transactions. It is generally different from the parameter γ that governs netting among banks, since it captures netting by a clearing and settlement system for the securities that active traders invest in.

In the second subperiod, active traders settle their intraday credit positions and adjusts their portfolio and equity position. The portfolio choice is between trees and deposits. We focus on equilibria in which every active trader always holds only its favorite tree, as well as deposits. Since the real rate on deposits is below the discount rate, traders hold as few deposits as necessary in order to purchase the entire outstanding amount of their new favorite tree in case the identity of their favorite tree changes. It follows that the intraday credit limit binds in equilibrium, a form of "cash-in-the-market pricing".

Optimal investment and deposits

To make the marginal benefit of deposits equal to the discount rate, deposits must provide a liquidity benefit. The latter comes from traders' ability to invest in their favorite tree, which carries a return that compensates them for the opportunity cost of deposits. Indeed, the first order conditions for deposits and trees simplify to

$$\delta = i_D - \pi + (1 + \hat{\gamma}) (r_j - \hat{s} - \delta).$$

The equilibrium return r_j adjusts to provide an excess return $r_j - \hat{s} - \delta$ on the favorite trees. Since every trader perceives a lower spread $\hat{s}$ on his own favorite tree compared to the trees of other traders, this excess return can persist in equilibrium.

Equilibrium deposit holdings are proportional to the market value of active traders' favorite trees:

$$\hat{D} (1 + \hat{\gamma}) = \frac{\hat{\beta} x P}{\delta + \hat{s} + \frac{\delta - (i_D - \pi)}{1 + \hat{\gamma}}}.$$

The market value of trees, as well as deposit holdings, respond to the opportunity cost of trading: if the deposit rate is higher, then active traders earn lower returns on their trees, prices are higher and active traders hold more deposits. Moreover, an increase in uncertainty perceived by active traders (that is, an increase in $\hat{s}$) lowers asset values and the demand for deposits.

Equilibrium with active traders

Our graphical analysis of equilibrium remains qualitatively similar when active traders are added to the model. What changes is that transactions now include carry traders' asset purchases:

$$T = \Omega + x + \bar{v}\hat{D}/P.$$

Since we know that the equilibrium deposit rate is decreasing in banks' leverage and liquidity ratio, we can replace the exogenous parameter T in (13) by a function $T(\ell, \phi^*)$ that is decreasing in both arguments.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with active traders. Suppose first that, starting from an equilibrium outside the liquidity trap, there is an increase in uncertainty perceived by active traders only. As active traders value trees less, they demand fewer deposits. This lowers bank leverage and shifts the capital structure curve to the left. The liquidity management curve does not change. In the new equilibrium, bank leverage is lower and the interest rate is higher, as is the price level. In other words, active traders are a force that generate the opposite response to a change in uncertainty from banks and carry traders. Since in the typical economy all traders are present to some extent, we can conclude that it is important their relative strength. An additional prediction is that we should see a decline in deposits held by institutional investors.

We can also reconsider the effect of monetary policy. Suppose once more that policy lowers the real overnight interest rate. The opportunity cost of holding deposits falls and active traders demand more deposits. At the same time, they bid up the prices of the trees they invest in. Again a segment of the tree market increases in value and the aggregate value of trees also rises: there is a tree market boom. Again the effect is not due a change in the discount rate, but instead a change in the funding cost: here it affects carry traders' strategy which involves holding deposits to wait for trading opportunities.

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