The Long and the Short of It:
Sovereign Debt Crises and Debt Maturity

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Abstract

We present a simple model of sovereign debt crises in which a country chooses its optimal mix of short and long-term debt contracts subject to standard contracting frictions: the country cannot commit to repay its debts nor to a specific path of future debt issues, and contracts cannot be made state contingent nor renegotiated. We show that in order to satisfy incentive compatibility the country must issue short-term debt, which exposes it to roll-over crises and inefficient repayments. We examine two policies – restructuring and reprofiling – and show that both improve ex ante welfare if structured correctly. Key to the welfare results is the country's ability to choose its debt structure so as to neutralize any negative effect resulting from the redistribution of payments across creditors in times of crises.

Keywords: sovereign debt, dilution, optimal maturity, restructuring, reprofiling, IMF.

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1 Introduction

Sovereign debt is once again at the center stage of the academic and policy debate. This attention has been largely fueled by the growth of public debt within the Eurozone, which has raised concerns regarding the ability of the current debt-restructuring framework to deal efficiently with large-scale debt crises. These concerns were vindicated by the experience of Greece, where the debt restructuring of 2012 is widely perceived to have come inefficiently late, after years of low growth and increasing official indebtedness.

It is against this backdrop that various reforms of the international lending framework are being discussed. The International Monetary Fund in particular is considering modifying its lending framework to allow greater flexibility in how it deals with debt crises. The centerpiece of the proposed reform is to increase the flexibility of the institution to deal with debt crises through debt "reprofiling". Intuitively, the idea is that when a country is faced with a debt crisis, it may be helpful to extend the maturities of its private debts and postpone payments until greater certainty is obtained regarding the country's prospects and the sustainability of its debt burden. If the country recovers, it can pay its debt without engaging in a full-blown restructuring; if not, restructuring is needed. This proposal has generated a lively debate. Adherents stress its ex post benefits, i.e., it will help countries deal more efficiently with debt crises when these happen. Opponents emphasize instead its ex ante costs, i.e., it will make short-term debt more similar to long-term debt, thereby raising the cost of funding, and ultimately lead to more frequent crises or prevent some countries from borrowing altogether.[1] Assessing the merits of these views requires an analytical framework to answer the following questions: What are the costs and benefits of reprofiling? When is reprofiling likely to be used relative to restructuring? What are its effects on short- and long- term interest rates? How does it affect the likelihood and cost of debt crises? The objective of this paper is to provide a simple framework in which to analyze these and related questions regarding sovereign debt reforms.

We develop a model that incorporates several features that are important to an analysis of these issues. First, borrowing is subject to standard contracting frictions: debt contracts are non-contingent, they cannot be renegotiated ex post, and the country is unable to commit to a path of future debt issues or repayment. Second, the timing of payments can matter. When there is uncertainty regarding future outcomes, the refusal by short-term creditors to roll-over their debt can result in inefficient payments that negatively impact the country's ability to generate future output. Lastly, and key as we will show to understanding the welfare consequences of reforms, the country's debt maturity structure is endogenous.

The model features a small-open economy that borrows resources from the international financial market in order to finance an ex-ante profitable investment opportunity. The project takes time to mature: early payments divert funding from the project and decrease its associated expected output. The country is subject to shocks which affect its productivity and thus its ability and willingness to repay its debt. In this

[1] For an expostision of these arguments, see "Creditors likely losers from IMF Rethink" (Financial Times, 6/4/2013), "IMF sovereign debt plans in jeopardy" (Financial Times, 1/26/2014), and "Finance: In search of a better bailout" (Financial Times, 1/26/2014).

environment, short-term debt is costly because creditors may be unwilling to roll it over in the face of bad economic prospects, triggering a debt crisis. If the country could commit to a path of future debt issues, these risks could be avoided by issuing long-term debt. Absent this ability to commit though, long-term debt is also problematic. The reason is that, once issued, the country will be tempted to dilute its long-term debt by issuing more debt in the future. In this regard, as in much of the recent sovereign debt literature that incorporates debt maturity, short-term debt plays a "disciplinary" role by reducing the incentives to dilute. This gives rise to a set of incentive compatible maturity structures that keep dilution in check.

Key to choosing among these possible maturity structures is how they affect both the likelihood and the cost of debt crises. In equilibrium, the country's choice of maturity structure optimally trades off the costs and benefits of short-term debt. We find that when a country faces ex ante "favorable" circumstances in which long-term debt is endogenously relatively cheap, there is no trade-off: the country chooses the longest maturity structure that is possible among those that are incentive compatible, minimizing both the likelihood and the cost of debt crises. When a country faces instead ex ante "unfavorable" circumstances in which long-term debt is endogenously relatively expensive, a tension ensues: increasing the share of long-term debt reduces the costs of crises but, since long-term debt is expensive, it may also raise their likelihood. In this case, the country may opt for a shorter maturity structure than dictated by incentive compatibility considerations alone.

We use the model to analyze the effects of potential reforms of the sovereign debt system. We assume that a country can request help from an international financial institution (e.g., the IMF) and that the latter coordinates creditors and country by choosing the appropriate remedy, be it debt restructuring, debt reprofiling, or non-intervention in the event of a debt crisis.[2] A debt restructuring is a proportional write-down of the value of the debt to a level that allows it to be rolled over at actuarially fair rates. Because restructuring is a credit event, it imposes the same penalty as default on the country. In addition to redistributing resources between creditors and the country, restructuring has an efficiency-enhancing effect since, by preventing inefficient payments during crises, it increases the total amount of available resources.

A debt reprofiling is considered a lighter version of restructuring. Instead of writing down the value of the debt immediately, payments due are postponed (i.e., debt maturity is lengthened). Naturally, reprofilings can only be successful if they impose a haircut or a *de facto* write-down on short-term creditors, i.e., if they are forced to roll over their debt at an interest rate that does not fully reflect the expected risk of default. Relative to restructurings, reprofilings have two effects. First, they redistribute resources from short-term creditors, who absorb the entire haircut, to long-term creditors. This implies that short-term debt becomes more expensive, for a given maturity structure. Second, reprofilings may entail a lower average haircut than restructurings, and thus presumably a lower penalty of default as well.

Both restructuring and reprofiling have the potential to be welfare improving because they eliminate the

[2] Given the large literature that already exists on the topic of solvency crises.(e.g., Cole and Kehoe 2000, Corsetti et al. 2006), we ignore self-fulfilling crises and the role for an IFI as a potential lender of last resort.

inefficiency associated with early payments to short-term creditors when these are unwilling to roll-over their debt. Both reforms also affect how resources are distributed among creditors and between creditors and the country. We show that a sufficient condition for these interventions to be welfare enhancing is that they *increase* the expected total value of payments to creditors during times of crises. This is a surprising result. After all, what guarantees that redistribution away from short-term creditors doesn't result in higher short-term interest rates and thus a greater frequency of debt crises? More subtly, even if all creditors are paid more in times of crises, these payments affect the relative probability with which long-term creditors are paid in full versus partially, which may then increase the probability of a crisis. A central contribution of our paper is to show that these concerns are rendered moot by the country's ability to choose its debt maturity structure. In particular, we prove a general result: any negative effect arising either from the redistribution of payments between creditors (short versus long), or from changes in the frequency with which long term debt is paid in part versus in full, can be neutralized by the country through an appropriate, incentive-compatible change of its debt maturity structure. Thus, our analysis allows us to conclude that the two interventions increase ex ante welfare and that, to the extent that reprofiling is a "light restructuring", it dominates outright restructuring.

This paper complements the literature on the optimal maturity structure of debt. Most theories build on the notion that, in a context of incomplete markets, debts of different maturities allow for greater hedging of risk.[3] More relevant for our purposes, the maturity structure of debt may also impact the repayment prospects and incentives faced by a borrower whose actions are unobservable or who is unable to commit to a future path of actions. In a model of corporate debt that shares several features with ours, for instance, Brunnermeier and Ohemke (2013) show how the inability of creditors to observe a borrower's total debt obligations leads to an excessively short maturity structure of debt. In the banking literature, Diamond and Rajan (2001) have shown how short-term debt may discipline borrowers and induce them to undertake certain actions that are desirable *ex ante* but which would not otherwise be incentive compatible *ex post*.[4]

This "disciplinary" view of short-term debt has been recently extended to the context of sovereign debt, where the critical additional assumption is the inability of a government to commit to repay its debts.[5] In this context, as shown by Jeanne (2009), short-term debt can provide incentives even when the government can commit to a future path of debt issues and hence is immune to the temptation to dilute. Instead, the

[3] In a closed-economy model with full commitment, for instance, Angeletos (2002) and Buera and Nicolini (2004) have shown that, even if the government has access only to non-contingent bonds, it can structure an optimal portfolio of maturities to replicate the hedging properties of contingent bonds.

[4] In a related vein, Debortoli et al. (2014) have recently shown that a government's ability to commit to repayment but not to a path of debt issuance limits its ability to use the maturity structure for hedging purposes.

[5] Of course, there are other motives for why a sovereign may wish to issue short-term debt. Broner, Lorenzoni and Schmukler (2008), for instance, document that the relative cost of long-term borrowing is high for emerging markets and show that this is consistent with the existence of risk-averse creditors. The latter favor short-term debt, thereby making it relatively cheaper and more attractive for governments to borrow in this fashion. See Niepelt (2014) for relevant theoretical results in an infinite horizon model with no government commitment.

role of short-term debt is to expose the government to costly roll-over crises in the absence of pro-market reforms, rendering the latter incentive compatible. Bolton and Jeanne (2009), on the other hand, assume that the government cannot commit to a future path of debt issues which leads, as in our model, to the threat of dilution. They use their model to study optimal mix of government debt that is differentially difficult to renegotiate (either fully renegotiable or non-renegotiable). They show that, although the first-best would be to issue only fully renegotiable debt and thus obtain state-contingency, the possibility of dilution forces the government to issue an inefficiently large amount of non-renegotiable debt. Our paper complements this literature by developing a simple model of sovereign debt maturity and by using it to assess the welfare implications of reforms to the international financial system.[6]

The quantitative literature in this area has also explored the role of dilution in determining the structure of debt maturity.[7] Hatchondo et al. (2011) find that the costs imposed by dilution are quantitatively important, in the sense that eliminating the government's ability to dilute existing debt (for instance, though seniority clauses) has a substantial negative effect both on the frequency of defaults and on average spreads. In a related vein, Chatterjee and Eyigungor (2012) find that – absent the possibility of self-fulfilling crises – the costs associated to dilution are high enough to discourage the use of long-term debt altogether and Arellano and Ramanarayanan (2012) show that a calibrated model of endogenous debt maturity can account for the observed relationship between maturity length and the term structure of interest rate spreads in emerging markets. This literature has not studied, however, the welfare consequences of different reforms.

Relative to the literature, our paper contributes three important insights to our understanding of the role of short-term debt. First, it shows that even if short-term debt is issued only for "disciplinary" reasons, countries may find it beneficial to issue more than what would be strictly required for these purposes. The reason is that, once short-term debt is issued, the incidence of crises rises and so does the cost of long-term debt. This may make long-term debt relatively expensive and thus lead the country to substitute away from it in order to lower the overall costs of debt and default. Second, our paper shows that the welfare consequences of reforms to existing debt-crisis management practices depend critically on how they would affect total expected payments to creditors in these crises episodes. Although these reforms are likely to have important redistributional effects between short- and long-term creditors and on the probability of a crisis, we show that these effects can be addressed by appropriately changing the maturity structure of debt. Lastly, the analysis shows that short-term debt may play a novel disciplining role by creating a greater incentive to deal with a crisis via an international financial institution's (IFI) reprofiling of sovereign debt rather than via the dilution of long-term debt. This may lead countries to issue more short-term debt even as it becomes more expensive.

[6] Also related is Aguiar and Amador (2014) which explores the disciplinary role of short-term debt in providing incentives to a government to repay an inherited stock of short and long-term debt.

[7] To preserve recursivity in an infinite horizon environment the quantitative literature has also made simplifying assumptions (e.g., constant-coupon bonds that mature probabilistically (Chatterjee and Eyigungor (2012), or perpetuities with coupon payments that decay at a geometric rate (Hatchondo and Martinez (2009)).

The paper proceeds as follows. Section 2 develops the basic model. Sections 3 and 4 characterize the properties of equilibrium. Section 5 introduces the IFI and explores the implications of debt restructurings and reprofilings. Section 6 concludes with a discussion of robustness and extensions.

2 The Model

In this section we present a model with which to analyze the basic trade-offs a country faces in choosing its debt maturity. In particular, we examine the implications of the maturity structure for the expected frequency and severity of debt crises. We make several simplifying assumptions (e.g., a finite horizon and two states of nature) that allow us to obtain analytical solutions and to better understand the mechanisms underlying the main results. They key tradeoff between short and long-term debt – the former increasing the likelihood of costly crises whereas the latter increasing the scope for dilution – and the analysis of the factors that influence this tradeoff should carry through to more complicated environments, as discussed in section 6.

Consider therefore a model in which time is discrete and there are three periods: $t = 0, 1, 2$. There are two types of agents (in the next section we introduce a third – an International Financial Institution or IFI): a risk-neutral country that is small in world capital markets and consumes only in period 2, and perfectly competitive risk-neutral creditors. Below we specify the timing in which decisions are made and introduce the main assumptions.

In period 0, the country receives an indivisible productive investment opportunity of size 1, which matures in period 2. This project can only be financed, for simplicity, by borrowing. The latter is subject to three contracting frictions: debt contracts are non-contingent, they cannot be renegotiated ex post, and the country cannot commit to a path of future debt issues.[8]

The project can be interrupted by the country before it matures. In particular, the country can disinvest a portion $\delta \in [0, 1]$ of its project in period 1, as will be discussed in greater detail below. Thus, at the beginning of period 2, the amount of capital remaining in the project is $k = 1 - \delta$. The return to the project in period 2 is stochastic. Specifically, period 2 output y is given by

$$y(\theta; k) = \theta k,\ \theta \in \{\theta_L, \theta_H\} \tag{1}$$

where $\theta_H > \theta_L$. Henceforth we simplify the algebra by setting $\theta_L = 0$.

Information about the project is as follows: in period 0, before accessing capital markets, the initial probability that $\theta = \theta_H$ is given by p_0. At the beginning of period 1, a signal regarding the productivity of the project is received and the updated probability of $\theta = \theta_H$ becomes $p \sim G(p)$, where $p \in [0, 1]$ and $p_0 = \int_0^1 p dG(p)$. At the beginning of period 2, $\theta \in \{\theta_L, \theta_H\}$ is realized.

[8] For a broad discussion of the role of these assumptions in the sovereign debt literature, see Aguiar and Amador (2013). See Yue (2010) for a model of sovereign debt renegotiation.

Financing the project requires the country to borrow a unit of capital from the international capital market in period 0. The country can offer its preferred mix of short and long-term contracts, but all contracts must be non-contingent. Each contract thus specifies a gross rate of return R_{0t}, where t denotes the period in which repayment is due, i.e., R_{01} is the gross return due to a unit of short-term (ST) debt in period 1 and R_{02} is the equivalent for long-term (LT) debt due in period 2. The gross safe rate of return in the international capital market is given by $R > 1$ per period. This is also the rate at which the country can save.

In period 1, after p has been realized, the payment of short-term debt becomes due. At this point, in order not to default, the country must either repay its ST creditors or roll-over their debt at an endogenous gross rate R_{12}. Given a competitive market for credit, rolling over ST debt is equivalent to obtaining new loans. Failure to entirely repay or roll-over its short-term debt means that the country is in default, whereupon creditors are able to insist upon repayment to the extent possible. We assume that repayment in period 1 is more costly than in period 2. This can be thought of as arising from the fact that the project takes time to mature and taxation is therefore costly, especially when future prospects look bad. For example, the country may need to engage in austerity measures that reduce spending in complementary areas (e.g. cuts in social spending and infrastructure projects), or it may need to engage in excessive taxation that distorts economic activity and reduces potential output. In particular, we assume that diverting δ units of capital from the project in period 1, given a signal p, yields $F(\delta, p)$ units of output in period 1 where

$$F(\delta, p) < \delta \cdot \frac{p\alpha\theta_H}{R} \text{ for } p \in (0,1] \text{ with } F(\delta, 0) = 0,\ F_1 > 0,\ F_2 \geq 0 \tag{2}$$

and where $\alpha \in (0,1)$, as we shall see, is a constant that denotes the maximum share of period-2 output that can be effectively transferred to creditors. Intuitively, equation (2) implies that disinvestment is inefficient in the sense that it reduces the present value of total output that can be used to pay creditors.[9]

In order for the country to be able to borrow in equilibrium, it must be willing and able to repay its debt in some states of nature. There are three features of our environment that provide incentives for repayment. First, as discussed previously, default in period 1 effectively allows creditors to obtain some repayment by forcing the country to divert funds from the investment project. Second, upon default in period 2, it is assumed that creditors are effectively able to seize a proportion $\alpha \in (0,1)$ of output $y(\theta, k)$. To simplify matters, we assume that α also denotes the maximum proportion that the country is able to pay its creditors from period-2 output (e.g., there is a limit to the country's ability to raise taxes on output). We refer to this quantity as "pledgeable output." In addition to using the latter, the country can pay creditors via its savings which, we assume, cannot be seized by creditors.[10] Third, default in any period subjects the

[9] The assumption of inefficient disinvestment is adopted for simplicity (e.g. Jeane (2009)). Alternatively, one could assume that funds are raised via distortionary taxation that entails convex costs. This creates a desire to smooth debt payments over time, raising the cost of making high payments in period 1.

[10] By assuming that savings cannot be seized, we provide the country with the maximum incentives to divert resources from creditors by disinvesting before period 2 and accumulating savings. As we show below, however, this option is never used in equilibrium.

country to a default penalty in period 2 that is proportional to productivity: we denote this penalty by $\pi\theta$ with $0 < \pi < \alpha$. This penalty can be thought of as encapsulating any future losses that a country faces after a default.[11] Finally, note that debt cannot be renegotiated ex post: upon default, creditors obtain funds according to whether default is early (period 1) or late (period 2) and the country suffers the default penalty $\pi\theta$.[12]

Note that when $\theta = \theta_L = 0$, the country is not able to make any repayments in period 2 and must necessarily default. Thus, borrowing is only possible if the project is sufficiently productive to repay creditors an actuarially fair return when $\theta = \theta_H$. Given that the project lasts for two periods and that the ex ante probability that $\theta = \theta_H$ is p_0, a sufficient condition for this to be true is

$$p_0 \alpha \theta_H > R^2, \tag{3}$$

which we henceforth assume.

The timeline of decisions and realizations is given in Figure 1. In brief, in period 0 the country decides its optimal mix of short and long-term debt, respectively denoted by γ and $1-\gamma$, and offers contracts specifying R_{0t}, $t = 1, 2$. In period 1, after information about the true state of nature is revealed via p, the country decides how much short-term debt to repay, how much to disinvest and save, and the quantity of short-term debt to issue at a contractual interest rate of R_{12}. In period 2, the state of nature θ is revealed, payments are made, a fraction α of output is seized if the country is in default, and consumption and any default penalty takes place.

Before turning to the solution of the model, we specify the rules that govern creditor repayment. Suppose that in period 1 the country is unwilling or unable to roll-over its debt. This forces it to disinvest its project in order to repay its short-term creditors. If these funds suffice, the country can continue to period 2 without declaring default in period 1. If these funds do not suffice, then only ST creditors are repaid to the extent possible and default is declared.[13] In particular, if the entire project were disinvested (leaving the country with no period-2 output), creditors would obtain:

$$V_1\left(R_{01}, R_{02}; p\right) = \begin{cases} \min\left(R_{01}, \frac{F(1,p)}{\gamma}\right) & \text{for ST debt} \\ \min\left(\frac{R_{02}}{R}, z\right) & \text{for LT debt} \end{cases}, \tag{4}$$

[11] This is a common assumption in the literature. The costs typically emphasized encompass: loss of reputation that reduces trade in goods or assets between the defaulting country and the rest of the world, economic sanctions imposed on the defaulting country by the rest of the world, costs associated to the economic content of defaults, or costs related to domestic holdings of public debt. For a discussion of the theoretical undeprinnings and the empirical evidence, see Borensztein and Panizza (2009) and Sandleris (2012).

[12] In particular, creditors are unable to negotiate as a group, forgive debt, or postpone payments, giving rise to a potential role for outside parties (the International Financial Institution" or IFI).

[13] We have in mind that when the ST debt refuses to roll-over, the country services debt sequentially with default declared only once there is no (liquidated) output left over. At that point some ST debt will still be unpaid as will all the LT debt. Thus when default is actually declared, there are no resources available for repayment of LT debt. See section 6 for a discussion of an alternative assumption in which all debt is paid pro rata whenever there is default. None of the welfare conclusions are affected.

per unit of debt, where $z = \max\left(0, \frac{F(1,p)-\gamma R_{01}}{(1-\gamma)}\right)$.[14]

If instead the country fails to meet its debt obligations in period 2, then default is declared and all creditors are paid pro rata obtaining, for $\theta = \theta_H$, repayment, per unit of debt, of:

$$V_2\left(R_{12,}R_{02};k,D_2\right) = \begin{cases} R_{12}\cdot\min(1,\dfrac{\alpha\theta_H k}{D_2}) \text{ for ST debt} \\ R_{02}\cdot\min(1,\dfrac{\alpha\theta_H k}{D_2}) \text{ for LT debt} \end{cases} \tag{5}$$

and 0 if $\theta = \theta_L$. Here $D_2 = (1-\gamma)R_{02} + D_{12}R_{12}$ where, abusing notation slightly, D_{12} denotes ST debt obligations incurred in period 1 at rate R_{12}.

3 Feasible Strategies

In this section we describe the feasible strategy set; we solve for the equilibrium in the next section. Let us begin with period 2. If the country has already defaulted in period 1, the only action available to it is to consume any available income. If the country has not defaulted in period 1, it can now choose whether to repay its debt obligation if feasible (i.e., if $D_2 \leq \alpha\theta k + R\cdot s$ where s denotes its period 1 savings) or default. If full repayment is not feasible, it will necessarily default.

Period 1 is when most of the action takes place. In this period, the country can always choose to default. It may also be able to avoid default in that period by meeting its ST debt obligations, either by repaying this debt or by rolling it over. To do this, the country must compensate ST creditors for the risk associated with the realization of p (the updated period-1 belief that $\theta = \theta_H$). Hence, debt issued in period 1 must obtain an ex post return of $\dfrac{R}{p}$ when $\theta = \theta_H$.[15] We characterize below the parameters for which each of these strategies is feasible in period 1.

The country's debt structure id determined in period 0: we denote it by $(\gamma.r)$, where γ denotes the proportion of debt that is short term and $r = (R_{01,}R_{02})$ is the vector of promised returns in period 0. Given (γ, r), though, we can divide the period-1 feasible strategy space into three basic categories as a function of p: roll-over with full repayment in period 2, roll-over with full dilution and late default, and disinvestment and early default.

3.1 Roll-over with Full Repayment in Period 2 ($p \geq \underline{p}$)

If the country plans to repay its debts should $\theta = \theta_H$ in period 2, it needs to roll over its ST debt at the rate $\frac{R}{p}$. For a low-enough realization of p, however, this strategy is not feasible because the country's debt obligations exceed its pledgeable output of $\alpha\theta_H$. For a given debt structure (γ, r), we define $\underline{p}(\gamma, r)$ as the lowest realization of p such that it is *feasible* for the country to roll-over its ST debt and repay all debts in the high state in period 2 assuming no disinvestment in period 1 ($\delta = 0$). Formally, $\underline{p}(\gamma, r)$ satisfies:

[14] We have assumed that R_{02} is adjusted to $\frac{R_{02}}{R}$ in order to reflect that it is being repaid early.

[15] The nominal gross rate it offers, R_{12}, may differ from $\frac{R}{p}$, however, as the payment creditors receive depend on the country's strategy as will be shown.

$$\alpha\theta_H - (1-\gamma)\, R_{02} - \gamma R_{01}\frac{R}{p} = 0. \tag{6}$$

Thus, as long as $p \geq \underline{p}\,(\gamma, r)$ it is feasible for the country to avoid default if $\theta = \theta_H$ in period 2.

3.2 Roll-over with Full Dilution and Late Default ($p \geq \widehat{p}$)

If $p < \underline{p}\,(\gamma, r)$, resources are insufficient to repay the debt fully in period 2 and ST debt cannot be rolled over at $\frac{R}{p}$. The country need not default immediately in period 1, however. An alternative may be to roll over the ST debt by fully diluting LT debt, thereby postponing default until period 2. When is this strategy feasible?

Define $\widehat{p}\,(\gamma; R_{01})$ as the minimum value of p that is compatible with the roll over of ST debt given R_{01}. In order to derive $\widehat{p}$, note that the country can offer its ST creditors at most $R_{12} = \infty$. Of course, this implies that the country will necessarily default in period 2 independently of the realization of θ. Upon default, though, all pledgeable output will be used to repay ST debt and the latter will obtain a unit return of $\frac{\alpha\theta_H}{D_{12}}$. LT debt will in this case be completely diluted and obtain a return of zero (see equation (5)).[16]

The strategy above enables the country to issue a maximum of $D_{12} = \frac{p\alpha\theta_H}{R}$ units of new (or rolled-over) debt in period 1, since it is in essence selling the discounted expected value of its entire future payable output, including any funds that would have been used to repay LT creditors. Note that this strategy is feasible only if the funds it generates suffice to pay ST creditors, i.e., if and only if $\frac{p\alpha\theta_H}{R} \geq \gamma R_{01}$. This implies that, given γ and R_{01}, the lowest realization of p that is compatible with this strategy is given by

$$\widehat{p}\,(\gamma; R_{01}) = \frac{\gamma R_{01} R}{\alpha\theta_H} \tag{7}$$

Whenever $p > \widehat{p}\,(\gamma; R_{01})$ this strategy generates funds in excess of the amount due to ST debt, which can then be saved at the international rate R and used by the country to consume in period 2.[17] This, as we will see, creates incentives for debt dilution in equilibrium.

3.3 Early Default ($p \geq 0$)

Lastly, partially or fully diverting funds from the project in period 1 (i.e., setting $\delta > 0$) is always feasible. Furthermore, it is the only feasible strategy if $p < \widehat{p}\,(\gamma; R_{01})$. For such realizations of p, ST debt cannot be rolled over even with full dilution of LT debt, leading ST creditors to demand repayment in period 1

[16] In this setup there is only one state of nature in which a country may be able to repay and hence partial dilution is never optimal. In an economy with various (or a continuum of) possible values of θ, the country would choose the set of states in which to dilute LT debt as well as the extent of this dilution by setting R_{12} accordingly.

[17] We have assumed that all consumption is in period 2, implying that any funds gained from dilution would be saved (away from the reach of creditors) and consumed that period. At the expense of some additional notation (e.g. introducing a discount factor), an alternative formulation would endow the country with linear preferences over consumption in each of the three periods. In this case, savings would play no role as any gains from dilution could be immediately consumed by the country in period 1.

thereby forcing early default. By definition of $\widehat{p}(\gamma; R_{01})$, the promised payments to ST debt exceed the appropriately discounted expected value of pledgeable output. Although it follows from equation (2) that the output available from divestment, $F(1,p)$, is less than what would be obtained via full dilution, the atomistic nature of creditors implies that the latter cannot agree to collectively roll-over their debt. Each owner of ST debt would find it a dominant strategy to holdout and insist upon early repayment. Note that the same conclusion holds for any amount of disinvestment. Hence, whenever $p < \widehat{p}(\gamma; R_{01})$, the project will be fully disinvested and the country will default early. To economize on notation, hereafter we refer to $F(1,p)$ as $f(p)$.

This concludes our description of feasible strategies and we next turn to solving for the equilibrium.

4 Equilibrium Strategies

In this section we describe the equilibrium strategy of the country. As we show, depending on (γ, r) and p, the optimal strategy in period 1 will take one of three forms: if $p < \widehat{p}(\gamma, r)$, the country fully disinvests in period 1 and defaults; if $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$, the country completely dilutes the LT debt and then defaults in period 2; finally, if $p \geq \underline{p}(\gamma, r)$, the country rolls over its ST debt at rate $R_{12} = R/p$ and repays the entire debt in period 2 if $\theta = \theta_H$. In period 0, the country chooses (γ, r) taking these strategies into account.

We now derive the equilibrium strategies by starting in period 2 and working our way backwards.

4.1 Final Period ($t = 2$)

The equilibrium strategy in period 2 is straightforward. Given the state θ, capital $k \leq 1$, and its debt repayments due that period, D_2, if the country repays its debt (assuming feasibility, i.e., if $\alpha\theta k + R \cdot s - D_2 \geq 0$) its payoff is:

$$W_{nd}(\theta; k, s, D_2) = \theta k + R \cdot s - D_2 \tag{8}$$

If it does not repay its debt, either because it is infeasible or because it chooses not to, its payoff is

$$W_d(\theta; k, s) = (1 - \alpha)\theta k + R \cdot s - \pi\theta \tag{9}$$

Hence, if $\alpha\theta k + R \cdot s - D_2 \geq 0$, the country will choose to repay its debt iff $\alpha\theta k + \pi\theta \geq D_2$. That is, a country will only repay if the cost of not doing so (having a portion α of output seized and suffering the default penalty) exceeds its debt obligations, assuming that repayment is feasible.

4.2 Intermediate Period ($t = 1$)

Given p and (γ, r), we start by noting that if the country plans to default in period 2 it will always choose to fully dilute its LT debt if feasible. To see this, note that dilution allows the country to consume the expected value of output next period net of payments to ST creditors. Thus, its expected welfare from

dilution if this strategy is feasible is,

$$W_{dd}(p;\gamma,r) = p\theta_H - \gamma R_{01} R - p\pi\theta_H, \tag{10}$$

whereas defaulting in period 2 without full dilution yields at most $(1-\alpha)\, p\theta_H - \gamma R_{01} R - p\pi\theta_H$. A comparison of these terms shows that, when feasible, dilution allows the country to increase its consumption by $\alpha p\theta_H$. The reason is that, by fully diluting LT debt, the country is essentially selling the expected value of seizable output to new ST creditors. When feasible, default with dilution therefore dominates default without dilution.

Suppose instead that repayment is feasible, i.e., $p \geq \underline{p}$, so that the country does not have to default in period 2. Does dilution (and default) dominate repayment? In this case, the country's expected welfare from repayment is:

$$W_{pay}(p;\gamma,r) = p\theta_H - p(1-\gamma) R_{02} - \gamma R_{01} R, \tag{11}$$

so that repayment is weakly preferred to dilution and default if and only if:

$$\pi\theta_H \geq (1-\gamma) R_{02}. \tag{12}$$

Equation (12) is intuitive: it says that repayment is better than dilution if and only if the punishment from default is greater than the gain obtained from reselling the funds that would be used to repay LT debt. Note that this incentive compatibility constraint is independent of p; the realization of p, however, determines whether repayment or dilution is feasible.

Lastly, let us consider the strategy or outcome of diverting all funds from the project. Recall first that, if $p < \widehat{p}(\gamma, r)$, the country's only feasible strategy in period 1 is to default without rolling over its ST debt. Hence, for such values of p the country's expected welfare is necessarily given by:

$$W_{\text{div}}(p;\gamma,r) = -p\pi\theta_H \tag{13}$$

Will the country want to divert funds (partially or fully) for other realizations of p? Note that the only reason to do so would be either to increase its consumption in period 2 by shielding these funds from creditors or to allow it to increase its payments to creditors. The latter possibility is not feasible, however, given the inefficiency inherent in not allowing the project to mature. Disinvesting from the project in order to save the proceeds for consumption in period 2, on the other hand, is dominated by dilution of LT debt.[18] The intuition for this is clear: the country can shield the expected value of seizable output by selling it to ST creditors in period 1 without needing to engage in inefficient diversion of funds. From this we can conclude that funds will be diverted from the project only when unavoidable, i.e., when dilution is no longer a feasible strategy.

[18] To see this, compare the payoff from dilution and default, W_{dd} in equation (10), with the expected payoff from diverting a fraction δ of the capital in the project, saving the output associated with it ($F(\delta, p)$), and shielding it from creditors: $(1-\alpha)\, p\theta_H (1-\delta) + RF(\delta, p) - \gamma R_{01} R - p\pi\theta_H$. A quick comparison reveals that dilution and default dominates disinvestment; its expected payoff is greater by $p\theta_H (1-\alpha)\,\delta + \alpha p\theta_H - RF(\delta, p) > 0$.

Summarizing the country's optimal strategy in period 1: if $p < \widehat{p}(\gamma, r)$, the country is forced to disinvest its project in period 1 and defaults in that period. If $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$, the country dilutes completely its LT debt (by offering $R_{12} = \infty$ and obtaining the maximum amount of new ST debt compatible with this) and then defaults in period 2. If $p \geq \underline{p}(\gamma, r)$ and the IC constraint in equation (12) is satisfied, the country rolls over its short term debt at rate $R_{12} = R/p$ and (if $\theta = \theta_H$) repays the entire debt in period 2. Finally, if $p \geq \underline{p}(\gamma, r)$ and the IC constraint is not satisfied, the country follows the same strategy as for $\underline{p}(\gamma, r) > p \geq \widehat{p}(\gamma, r)$ – full dilution and default in period 2.

Figures 2 and 3 show the feasible and optimal strategies, respectively, for the country for all p given (γ, r), where DL is default and disinvest in period 1, DD_2 is full dilution and default in period 2, and ND is no default (full repayment). The range for which ND is optimal is shown assuming that the IC constraint is met.

4.3 Initial Period ($t = 0$)

We can now determine a country's optimal choice of debt structure (γ, r). We assume that the country can set both the proportions of debt and the nominal rates of return, thus ignoring the multiplicities generated solely by expectations.[19] This is equivalent to finding, for each γ, the lowest contract rates compatible with equilibrium behavior. The country's objective thus is to choose (γ, r) so as to maximize its period-0 expected welfare

$$\begin{aligned} W_0(\gamma, r) &= \int_0^1 W(p; \gamma, r)\, dG(p) \\ s.t.\ W_0(\gamma, r) &\geq 0 \end{aligned} \tag{14}$$

understanding how each choice influences its subsequent strategies.

The First Best

Before deriving the equilibrium choice of contracts, it is useful to characterize the first-best outcome. Note that since creditors must, in equilibrium, obtain an expected payoff of R per unit of debt per period, a country's welfare would be maximized by minimizing all instances of disinvestment and by avoiding all instances of default, with or without disinvestment. Both outcomes simply lower its ex ante expected welfare without reducing the expected value of the real transfers it must make to creditors.

Suppose that the country could credibly promise not to issue any short-term debt in period 1. In our environment, this would allow it to attain the first-best allocation. To see this, note that it could simply set $\gamma = 0$ and – assuming full repayment whenever $\theta = \theta_H$ – perfect competition in the capital market would

[19] That is, if creditors expect the country to default for a large range of realizations of p, they will demand high interests rate that may – in a self-fulfilling manner – validate the initial expectations of default. Calvo (1998) provides a classic example if this type of multiplicity. More recently, Lorenzoni and Werning (2014) have explored this feature in a dynamic setting. See also Cole and Kehoe (2008) for an alternative timing of repayment and borrowing decisions that gives rise to multiple equilibria.

imply $R_{02} = \frac{R^2}{p_0}$. Note that the expected productivity of the project (see equation (3)) would indeed allow for full repayment of LT debt at this rate, validating the assumption. Thus, the country's expected ex ante welfare would in this case be given by

$$W_0^{fb} = \int_0^1 p\theta_H dG(p) - R^2 = p_0\theta_H - R^2 \tag{15}$$

since disinvestment would always be avoided and default would occur only for $\theta = \theta_L = 0$.[20]

Can the country achieve the first best without the ability to commit to not issuing new debt in period 1? Only if the IC constraint is satisfied when $\gamma = 0$. Otherwise, no creditor would be willing to purchase LT debt because she would (correctly) expect this debt to be fully diluted in period 1. A necessary and sufficient condition for the IC to be violated for $\gamma = 0$ is

$$R^2 > p_0\pi\theta_H \tag{16}$$

We henceforth restrict attention to parameters that satisfy this restriction as it guarantees that the first-best solution is not feasible and thus any equilibrium must entail $\gamma > 0$. To understand the behavior of such equilibria, we turn next to deriving their associated contract rates.

Equilibrium Contract Rates

We start by deriving the equilibrium contractual rates as a function of γ. Note first that any equilibrium must satisfy the incentive compatibility constraint of equation (12), i.e.,

$$\gamma \geq 1 - \frac{\pi\theta_H}{R_{02}} \tag{17}$$

must hold.

To derive equilibrium interest rates we assume that the IC constraint holds and require rates such that creditors obtain an expected payoff of R per period and per unit of debt. We then restrict the country's equilibrium choice of debt structure to those values of γ that are incentive compatible: this set is clearly not empty since $\gamma = 1$ trivially satisfies incentive compatibility. Taking this into account, we can find – for each γ – the associated values of $\widehat{p}(\gamma)$ and $\underline{p}(\gamma)$ as well as the vector $r(\gamma) = (R_{01}(\gamma), R_{02}(\gamma))$. Recall that $\widehat{p}(\gamma)$ solves equation (7) for a given $R_{01}(\gamma)$, whereas $\underline{p}(\gamma)$ solves equation (6) for given $R_{01}(\gamma)$ and $R_{02}(\gamma)$. It is important to note that some maturity structures may not be feasible for the country. This is because, for a given γ', there may not exist a $\underline{p}(\gamma') < 1$ that is consistent with the zero-profit conditions and hence $\widehat{p}(\gamma')$ and $\underline{p}(\gamma')$ are not defined. We denote such cases by $\widehat{p}(\gamma') = \underline{p}(\gamma') = r(\gamma') = \varnothing$.

From a creditor's zero profit condition we can implicitly write $R_{02}(\gamma)$ as

$$R^2 = R_{02}(\gamma) \int_{\underline{p}(\gamma)}^1 pdG(p), \tag{18}$$

[20] Note that the result that this is a first-best relies on the absence of effective punishment when $\theta = \theta_L$ which occurs only as a result of assuming $\theta_L = 0$. More generally, this would be a constrained first best.

reflecting that LT debt is repaid only if $p \geq \underline{p}$, and we can implicitly write $R_{01}(\gamma)$ as

$$R = R_{01}(\gamma)\left[1 - G\left(\widehat{p}(\gamma)\right)\right] + \int_0^{\widehat{p}(\gamma)} \frac{f(p)}{\gamma} dG(p), \tag{19}$$

reflecting that ST debt is repaid in full whenever $p \geq \widehat{p}$ and is repaid from complete disinvestment otherwise. Thus, for any level of γ, the four equations ((7), (6), (18), and (19)) determine the four endogenous variables: $\widehat{p}(\gamma)$, $\underline{p}(\gamma)$, $R_{01}(\gamma)$, and $R_{02}(\gamma)$. The debt structures available to the country at $t = 0$ are restricted to those values of γ that satisfy feasibility, i.e., for which $\widehat{p}(\gamma), \underline{p}(\gamma)$, and $r(\gamma) \neq \varnothing$, and incentive compatibility, i.e., for which the IC constraint in equation (12) is satisfied given the associated $R_{02}(\gamma)$. If the set of debt maturity structures that satisfy the IC constraint and zero-profit for creditors is empty, the country is shut out of the market. Note that a sufficient condition for this not to occur is that it be possible to fund the project solely issuing ST debt.[21]

It follows that the country's ex ante welfare can be written as:

$$W_0(\gamma) = \int_{\widehat{p}(\gamma)}^{1} p\theta_H dG(p) + \int_0^{\widehat{p}(\gamma)} Rf(p)\, dG(p) - \int_0^{\underline{p}(\gamma)} p\pi\theta_H dG(p) - R^2. \tag{20}$$

Relative to the first-best in which only LT debt is offered, equation (20) shows that the country's use of ST debt generates two types of losses: first, it leads to default (and its associated costs) whenever $p < \underline{p}$; second, in addition to default, it leads to inefficient disinvestment whenever $p < \widehat{p}$. Note therefore that $\widehat{p}$ and $\underline{p}$ are sufficient statistics for the country's welfare; the country is best off with the lowest feasible values of both cutoff probabilities. In particular, the gross contractual rates r and the debt composition γ matter only because they affect these cutoff values of p. Thus, to understand the characteristics of the optimal γ we need to examine how $\widehat{p}$ and $\underline{p}$ vary with γ. We turn next to deriving these properties.

Properties of $\widehat{p}$ and $\underline{p}$

First note that $\widehat{p}$ and R_{01} can be solved for independently of $\underline{p}$ and R_{02}. Using the definition of $\widehat{p}$ in equation (7) and the break-even condition of creditors in equation (19), we can rewrite $\widehat{p}$ as

$$\widehat{p}(\gamma) = \frac{\gamma R}{\alpha\theta_H} \cdot \frac{R - \int_0^{\widehat{p}(\gamma)} \left(\frac{f(p)}{\gamma}\right) dG(p)}{1 - G(\widehat{p})}. \tag{21}$$

Figure 4 depicts both sides of equation (21) for a given γ. It is easy to see that there may be multiple values of $\widehat{p}$ consistent with equilibrium. Of these, the country will always choose the lowest value of $\widehat{p}(\gamma)$, and hence of $R_{01}(\gamma)$.[22] We refer to these equilibrium values as $R_{01}^*(\gamma)$ and $\widehat{p}^*(\gamma)$.

Let us consider first the comparative static properties of $\widehat{p}^*$ and R_{01}^* with respect to γ. Since the partial derivative of the right-hand side of equation (21) with respect to γ is positive, an increase in the proportion

[21] After a bit of algebra, this condition can be written implicitly as $R^2\left[1 - \int_0^{\widehat{p}(1)} \frac{f(p)}{R}\right] < \left[1 - G\left(\widehat{p}(1)\right)\right]\alpha\theta_H$ and thus is guaranteed if $\left[1 - \int_0^{\widehat{p}(1)} \frac{f(p)}{R}\right] < \left[1 - G\left(\widehat{p}(1)\right)\right]$.

[22] An infeasible maturity structure would give rise to curves that failed to intersect.

of ST debt increases $\widehat{p}^*$ and hence R_{01}^* as shown in figure 4. The intuition for this result is straightforward. An increase in γ means that there are more instances in which ST debt cannot be rolled over by diluting LT debt (since $p\alpha\theta_H$ must be shared among a greater number of ST creditors), and it must therefore lead to more instances of disinvestment, i.e., to an increase in $\widehat{p}^*$. This requires an increase in R_{01}^*, and thus a further increase in $\widehat{p}^*$. We note here that the expression in equation (21) may not be differentiable and that, for some values of γ, an increase in γ could lead to a discontinuous increase in $\widehat{p}^*$ and R_{01}^*. We will return to this issue later on.

We can now turn to the properties of $\underline{p}$ and R_{02}. Using its definition in equation (6), we can write $\underline{p}$ as a function of γ, $R_{01}^*(\gamma)$, and R_{02}:[23]

$$\underline{p}(R_{02};\gamma) = \frac{\gamma R_{01}^*(\gamma) R}{\alpha\theta_H - (1-\gamma) R_{02}} . \tag{22}$$

From the break-even condition of LT creditors in equation (18), moreover, we can obtain $R_{02}\left(\underline{p}\right)$. Figure 5 depicts both expressions for a given value of γ (and thus, of $R_{01}^*(\gamma)$). Both curves are upward sloping and may intersect more than once or not at all. Once again, the country will choose the intersection with the lowest value of $\underline{p}$ and hence of R_{02}. We denote these equilibrium values by $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$.

Interestingly, the comparative static properties of $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$ with respect to γ depend on whether $\alpha\theta_H$ is greater or smaller than R_{02}. For brevity, we sometimes refer to the former case as a "favorable" economy and to the latter as an "unfavorable" economy. These comparative statics are best seen graphically through Figure 5. In the figure, the $R_{02}\left(\underline{p}\right)$ curve is unaffected by changes in γ. The $\underline{p}(R_{02};\gamma)$ curve instead shifts in response to changes in γ, but the way in which it shifts depends on the size of $R_{02}(\gamma)$ relative to $\alpha\theta_H$. Formally, differentiation of equation (22) yields:

$$\frac{\partial \underline{p}(R_{02};\gamma)}{\partial\gamma} = \frac{R_{01}^* R(\alpha\theta_H - R_{02}) + \gamma R(\alpha\theta_H - (1-\gamma) R_{02}) \frac{dR_{01}^*}{d\gamma}}{(\alpha\theta_H - (1-\gamma) R_{02})^2} \tag{23}$$

Recalling that $\frac{dR_{01}^*}{d\gamma} > 0$, a sufficient condition for this expression to be positive is $R_{02} \leq \alpha\theta_H$. Thus, when γ increases, the $\underline{p}(R_{02};\gamma)$ curve shifts right if $R_{02} \leq \alpha\theta_H$; for R_{02} sufficiently large, however, it shifts left. This means, as shown in figure 6, that the ultimate effect of an increase in γ on $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$ depends on whether the initial equilibrium corresponded to a favorable economy or an unfavorable one. In the first case, which is depicted in panel A of the figure, $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$ increase with γ; although in the second case the effect is ambiguous, panel B depicts the case in which $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$ decrease with γ.

The intuition for the result above is clear. When the economy is favorable, $R_{02} < \alpha\theta_H < \frac{R_{01}R}{\underline{p}}$ and long term debt is relatively "cheap". Thus, issuing a slightly higher proportion of ST debt raises the total cost of borrowing and increases the range of p which lead to default. When the economy is unfavorable, however, $R_{02} > \alpha\theta_H > \frac{R_{01}R}{\underline{p}}$ and long term debt is relatively "expensive". In this case, small increases in the proportion of ST debt have an ambiguous effect on $\underline{p}$. A higher proportion of ST debt now increases

[23] Of course this equation is valid only if $\alpha\theta_H - (1-\gamma) R_{02}(\gamma) > 0$. If the latter didn't hold, the country would not be able issue borrow long term debt ($R_{02}(\gamma)$ would be infinite).

the amount of debt that is relatively cheap, decreasing $\underline{p}$, but it also raises the required rate of return on ST debt R_{01}^*, increasing $\underline{p}$. The final effect on $\underline{p}^*$ and R_{02}^* (which in equilibrium must move in the same direction) depends on the relative strength of these two effects. Note again that there is no guarantee of differentiability of $R_{02}^*(\gamma)$ and $\underline{p}^*(\gamma)$. First, for some values of γ, a change in γ could lead to a jump in $R_{01}^*(\gamma)$. Second, if the $R_{02}(\underline{p})$ locus (which depends on the distribution $G(p)$) is non-convex, small shifts in the $\underline{p}(R_{02};\gamma)$ curve can cause $\underline{p}^*(\gamma)$ and $R_{02}^*(\gamma)$ to jump.

The Equilibrium Choice of Contracts

We are now set to describe the equilibrium choice of the maturity structure γ. Let Γ_{IC} denote the set of γ that satisfy the incentive compatibility constraint of equation (17) for equilibrium interest rates (i.e. $\gamma \geq 1 - \frac{\pi\theta_H}{R_{02}^*(\gamma)}$) and let $\gamma_{IC} = \min\{\gamma | \gamma \in \Gamma_{IC}\}$ denote the minimum value γ in this set. The country's welfare maximizing maturity structure, γ^*, is the solution to

$$\begin{aligned} \underset{\gamma}{Max}\ W_0(\gamma) \quad &= \int_{\widehat{p}^*(\gamma)}^{1} p\theta_H dG(p) + \int_0^{\widehat{p}^*(\gamma)} Rf(p)\, dG(p) - \int_0^{\underline{p}^*(\gamma)} p\pi\theta_H dG(p) - R^2 \qquad (24) \\ &\quad s.t. \\ (IC) \quad &: \quad \gamma \in \Gamma_{IC}, \\ \text{(feasibility):} \quad &\gamma \text{ is feasible with } r^*(\gamma), \widehat{p}^*(\gamma), \underline{p}^*(\gamma) \text{ satisfying (6), (7), (18), and (19)} \end{aligned}$$

In what follows, we will assume that there is a non-empty set of values of γ that satisfies both IC and feasibility as discussed previously so that the country has access to capital markets.

If $R_{02}^*(\gamma_{IC}) \leq \alpha\theta_H$ so that the economy is "favorable" when $\gamma = \gamma_{IC}$, then $\gamma^* = \gamma_{IC}$. To see this note first that, in this case, γ_{IC} is clearly preferred to any $\gamma' \in \Gamma_{IC}$ in the favorable region, i.e., for which $R_{02}^*(\gamma') \leq \alpha\theta_H$. In this region both $\widehat{p}^*(\gamma)$ and $\underline{p}^*(\gamma)$ are increasing functions of γ and welfare is thus maximized by the lowest γ feasible. Second, γ_{IC} is also clearly preferred to any $\gamma' \in \Gamma_{IC}$ in the unfavorable region, i.e., for which $R_{02}^*(\gamma') > \alpha\theta_H$. Since $\gamma' > \gamma_{IC}$, it follows that $\widehat{p}^*(\gamma_{IC}) < \widehat{p}^*(\gamma')$ and thus $R_{01}^*(\gamma_{IC}) < R_{01}^*(\gamma')$; since γ_{IC} is in the favorable region whereas γ' is not, it follows that $R_{02}^*(\gamma_{IC}) < R_{02}^*(\gamma')$. Thus, $\underline{p}^*(\gamma_{IC}) < \underline{p}^*(\gamma')$, which – together with $\widehat{p}^*(\gamma_{IC}) < \widehat{p}^*(\gamma')$ – guarantees that welfare is maximized by setting $\gamma^* = \gamma_{IC}$.

If instead $R_{02}^*(\gamma_{IC}) > \alpha\theta_H$ (i.e., the economy is "unfavorable"), setting $\gamma^* = \gamma_{IC}$ may not be the welfare maximizing choice for the country: although $\frac{d\widehat{p}^*(\gamma)}{d\gamma} > 0$, $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ may now be negative. If so, there is a welfare trade-off generated by choosing any other $\gamma' \in \Gamma_{IC}$. On the one hand, more ST debt increases the range of p associated with early default and inefficient disinvestment because it reduces the feasibility of dealing with debt crises through dilution. On the other hand, more ST debt may decrease the range of p associated with late default by allowing the country to have a less costly mix of LT and ST debt. As a result, γ^* may be interior to the IC constraint in this case.

Summing up, if the economy is favorable at the minimum level of ST debt that is incentive compatible, the optimal maturity structure is clear. The country should issue as little ST debt as possible as the latter

only increases the likelihood of both early and late default. If, on the other hand, LT debt is expensive at this minimum level of incentive-compatible ST debt, the country faces a trade-off. While issuing more ST debt may expose it to more instances of the costlier early default, it may also allow it to repay its debt over a greater range of realizations of p, thereby decreasing the instances of late default. How this tradeoff is resolved depends on $G(p)$. Lastly, in order for the solutions above to be equilibrium choices, they must satisfy $W_0(\gamma^*) \geq 0$. If this participation constraint is not satisfied, the country will not undertake the project.

4.4 Discussion

This completes the characterization of the equilibrium maturity structure. The general message is that, when a country is unable to commit to a certain path of debt, the maturity structure shortens relative to the first best. This is costly because debt contracts are non-contingent and renegotiation is not possible (or efficient), so that ST debt exposes the country to costly crises and inefficient repayments whenever the economic outlook is bleak. We next use this model to evaluate the role of an international financial institution such as the IMF and to explore the merits of debt restructuring or reprofiling in the event of a crisis.

Before doing so, however, we can use the model to ask how a country's optimal maturity choice is affected by changes in the environment. Broadly speaking, we can classify these changes into two categories. Changes in the economic environment are those related to the country's economic outlook: this category includes changes in the international interest rate R and in the distribution over outcomes $G(p)$. Changes in the enforcement technology are those related to the ability of foreign creditors to extract repayment: this category includes changes in the cost of default π, the efficiency of making early payments F, and in the share of output that can be seized by creditors α.[24] It must be remembered throughout that continuity is not guaranteed, so that the endogenous variables could vary discontinuously in response to continuous changes in the environment.

We summarize here our main findings and relegate a more formal discussion to the appendix. As a general rule, the response of the optimal maturity structure to changes in the environment depends on whether the economy is "favorable" or "unfavorable". In the former case, as we have seen, the maturity structure of debt is restricted by the IC constraint and it thus suffices to know how changes in the economy affect the latter. For an unfavorable economy, the choice of debt maturity is instead the result of an optimal trade-off between the default costs associated with LT debt and the efficiency costs associated with ST debt. To the extent that both costs are affected by changes in the environment, the net effect on the choice of maturity structure is ambiguous.

We find that a deterioration in the country's economic environment (e.g., through a higher international

[24] This distinction between economic environment and enforcement technology is not stark, of course. The costs of default and the efficiency of liquidation/early repayments, for instance, are clearly related to the underlying economic environment.

interest rate R or a rightward shift in $G(p)$) leads both to higher spreads and, if the economy is favorable, to a shortening of the maturity structure. The same is true with respect to a deterioration in the country's enforcement environment (e.g., through a lower cost of default π, a lower efficiency of disinvestment $F(\delta, p)$, or a lower ability to pledge output α). For the case of a favorable economy, the model's predictions are therefore consistent with the evidence for emerging markets documented by Broner et al. (2008) and Arellano and Ramanarayanan (2012), who find that periods of high country spreads are accompanied by a shortening of the maturity structure of public debt. For an unfavorable economy, the ultimate effect of any change in the environment depends instead on how it affects the marginal cost of ST debt (in terms of disinvestment) relative to the marginal cost of LT debt (in terms of default). In general, as we show in the appendix, these changes have an ambiguous effect on the maturity structure of debt.

5 A Role for an International Financial Institution?

The model concludes that a country's inability to commit to a path of debt issuance forces it to issue a positive amount of ST debt. Because debt contracts are non-contingent and renegotiation is not possible, ST debt exposes the country to costly crises and inefficient repayments whenever there are signals that the economy may not perform well (i.e., whenever $p < \underline{p}(\gamma^*)$). We now use the framework to ask whether there is a useful role for an international financial institution (IFI).

We focus on two types of intervention by the IFI: debt restructuring and debt reprofiling.[25] The first policy has long been in place and has been used in a variety of situations. The second is a new policy which the IMF is currently considering and which has been the subject of much recent attention.[26] Of course, the IMF does not have the power to impose or mandate debt restructurings or reprofilings in practice. But the proposed reform rests on the notion that the IMF does have sufficient leverage to "coordinate" creditors into accepting a deal, either by conditioning its resources or via its assistance to the country in distress.

5.1 The Model with the IFI

Formally, we modify the model to include the IFI as follows. The IFI has a given tool-kit of policies. After observing p in period 1, the country decides whether to approach the IFI for help. The IFI then decides how (including whether) to intervene, given the set of policy tools at its disposal.

The IFI is assumed to maximize the equally weighted sum of creditor and sovereign welfare. Hence, it is not concerned with redistribution per se, neither between the country and its creditors nor within the group of creditors. Because preferences are linear, this amounts to maximizing expected output net of default

[25] Once again, we ignore self-fulfilling crises so that there is no room for the IFI to act as a lender of last resort, a role that is already well understood.

[26] See "Creditors likely losers from IMF Rethink" (Financial Times, 6/4/2013), "IMF sovereign debt plans in jeopardy" (Financial Times, 1/26/2014), , "Finance: In search of a better bailout" (Financial Times, 1/26/2014), Buchheit et al. "Guest post: The case for sovereign reprofiling the IMF way, part one" (Financial Times, 11/7/2014) and IMF (2014).

costs – a useful benchmark. We model the IFI as having the same commitment technology as the country: it cannot commit to a policy rule in advance. Similarly, the IFI does not have a superior punishment or seizure technology than the creditor community: it cannot seize more resources nor change the default penalty. The key contribution of the IFI, then, is that it can coordinate creditors into accepting a debt restructuring or reprofiling that may not be perceived as desirable from each creditor's perspective even if it is welfare improving for creditors as a whole.

In the welfare comparisons that follow, we do not impose any restrictions in the way in which payments are shared among creditors for $p < \widehat{p}$ in the no-IFI model. That is, we will not impose the restriction that LT creditors are excluded from payments for these instances. In this sense, our welfare results are fully general – they are independent of how payments were originally shared below $\widehat{p}$.

5.2 Debt Restructuring

A typical debt restructuring consists of a write-down of the value of promised future debt payments such that the probability of future default is lower. Achieving a restructuring agreement requires complicated negotiations among creditors, country, and the IFI. In this model we abstract from this procedure and instead restrict the IFI to restructurings that impose the same haircut on all creditors and collectively increase the expected value of repayments in times of crises, in a sense that will be made precise.[27] Given that debt can never be repaid when $\theta = \theta_L = 0$, restructuring implies a write-down of the value of the debt such that it is repayable when $\theta = \theta_H$. Debt restructuring is considered a credit event and thus we assume that it entails the same costs, $\theta\pi$, as a default.

Under what p realizations would the IFI restructure debt? Since it effectively maximizes expected output net of default costs, the IFI is strictly willing to undertake a restructuring whenever $p < \hat{p}$. By doing so, it prevents disinvestment and increases the resources available to distribute between the country and its creditors. For $p \in \left[\hat{p}, \underline{p}\right)$, on the other hand, debt restructurings redistribute resources without affecting efficiency so the IFI would choose not to intervene even if the country desired it.[28] The country, in turn, is always willing to call the IFI when $p < \hat{p}$, since it can only be made better off by preventing disinvestment. It thus follows that debt payments are restructured in equilibrium whenever $p < \hat{p}$.

Let (γ, r^s) be a country's debt structure in an environment with an IFI capable of restructuring (henceforth referred to as a restructuring regime). Given that restructuring writes down the debt to D^s so that it can be repaid when $\theta = \theta_H$, i.e., $D^s \leq \alpha\theta_H < \frac{\gamma R_{01} R}{p} + (1-\gamma)R_{02}$, and that all creditors face the same proportional haircut (i.e., repayment is *pro rata*), the share of the total expected payment pD^s received by

[27] Assuming the same haircut across creditors is without loss of generality as shown in Proposition 1.

[28] Of course, the IFI would not intervene when there was no crisis either, $p \geq \underline{p}$, as this imposes a default cost on the country and thus decreases total welfare.

LT creditors at a $p < \hat{p}(\gamma; r^s)$ can be written as:[29]

$$\beta^s(\gamma, p; D^s) \equiv \beta^s(\gamma R_{01}^s, (1-\gamma) R_{02}^s, p; D^s) = \frac{(1-\gamma)R_{02}^s(\gamma)}{\gamma R_{01}^s(\gamma) \cdot \dfrac{R}{p} + (1-\gamma)R_{02}^s(\gamma)}, \tag{25}$$

with the remainder allocated to ST creditors. Perfect competition among creditors requires that gross returns reflect the possibility of this intervention. Thus, equations (18) and (19) become:

$$R^2 = \int_0^{\widehat{p}^s(\gamma)} \frac{\beta^s(\gamma, p; D^s)}{1-\gamma} \cdot pD^s dG(p) + R_{02}^s \int_{\underline{p}^s(\gamma)}^1 pdG(p) \tag{26}$$

$$R = \int_0^{\widehat{p}^s(\gamma)} \frac{(1-\beta^s(\gamma, p; D^s))}{\gamma} \cdot pD^s dG(p) + R_{01}^s (1 - G(\widehat{p}^s(\gamma))) \tag{27}$$

and, together with equations (7) and (22), these expressions determine break-even interest rates and early versus late crises probabilities for a given γ. Note that, as we did for the non-IFI regime, we ignore the IC constraint in the construction of these rates and impose the constraint later.

Clearly, the system of equations (7), (22), (26) and (27), together with the incentive compatibility constraint of equation (17), may have multiple solutions for a given γ or no solution at all. Whenever there are multiple solutions, however, the country will pick the contract structure that yields the lowest value of $\underline{p}(\gamma)$. To understand why, note that the country's ex ante welfare in the presence of restructuring, $W_0^s(\gamma)$, is given by:

$$W_0^s(\gamma) = \int_0^1 p\theta_H dG(p) - \int_0^{\underline{p}^s(\gamma)} p\pi\theta_H dG(p) - R^2, \tag{28}$$

which, for any given (feasible) γ, is maximized by minimizing the probability of default $\underline{p}^s(\gamma)$. It is these equilibrium values that we denote by $r^s(\gamma) = (R_{01}^s(\gamma), R_{02}^s(\gamma))$, $\underline{p}^s(\gamma)$ and $\hat{p}^s(\gamma)$.

Let γ^s denote the optimal choice of γ under restructuring. Comparing the ex ante welfare in an environment with and without restructuring (using (20) and (28)), yields:

$$W_0^s(\gamma^s) - W_0(\gamma^*) = \int_0^{\widehat{p}^*(\gamma^*)} [p\theta_H - Rf(p)] dG(p) + \int_{\underline{p}^s(\gamma^s)}^{\underline{p}^*(\gamma^*)} p\pi\theta_H dG(p). \tag{29}$$

This equation has a natural interpretation. The first term, which is strictly positive, captures the benefit of restructuring obtained by avoiding inefficient disinvestment whenever $p < \widehat{p}(\gamma^*)$. The second term captures the effect of restructuring associated with the change in the expected likelihood of default. If debt restructuring raises the equilibrium likelihood of default, then $\underline{p}^s(\gamma^s) > \underline{p}^*(\gamma^*)$ and this term is negative; if instead restructuring lowers the equilibrium likelihood of default, then $\underline{p}^s(\gamma^s) < \underline{p}^*(\gamma^*)$ and this term is positive. Thus a sufficient condition for restructuring to increase the country's ex ante welfare is that it reduce the likelihood of default, i.e., that $\underline{p}^s(\gamma^s) < \underline{p}^*(\gamma^*)$.

[29] The superscript s on a variable is used to denote the variable's equilibrium value under a restructuring regime for a given γ.

Before examining the welfare consequences of restructuring, we need to introduce an important definition. We say that restructuring is *payment-enhancing* relative to the no-restructuring regime if, keeping fixed the states in which disinvestment occurs in the no-restructuring regime, the switch from disinvestment to restructuring would increase total expected payments to creditors in those states. Formally,

Definition 1 *A restructuring regime to be* payment enhancing *relative to a no-restructuring regime with debt structure* $(\gamma, r(\gamma))$ *if and only if*

$$\int_0^{\widehat{p}(\gamma)} pD^s dG(p) > \int_0^{\widehat{p}(\gamma)} Rf(p)\, dG(p), \tag{30}$$

where $\widehat{p}(\gamma)$ *denotes the equilibrium value of* $\widehat{p}$*, given a feasible* γ*, in an environment without restructuring (i.e., with no IFI).*

At first blush, it may seem that a payment-enhancing regime is trivially welfare improving. After all, if a country can increase payments to creditors when $p < \widehat{p}$, shouldn't this lower contractual rates and thus decrease the probability of default? This reasoning, however, is faulty. It ignores two important consequences of restructuring. First, it may *directly* affect the distribution of payments across creditors. For example, it might call for larger payments to LT creditors when $p < \widehat{p}$ that come in part at the expense of payments previously made to ST creditors. This would affect contractual rates and hence the probability of early and late default. Second, restructuring may *indirectly* affect payments to creditors by changing the frequency with which debt classes are paid in full or in part. If, for example, restructuring implies that ST creditors will be paid more when $p < \widehat{p}$, ceteris paribus this lowers R_{01} and thus lowers $\widehat{p}$ as well. But a lower $\widehat{p}$ may, in general, reduce the frequency with which LT debt is paid in part through disinvestment – recall that LT debt is completely diluted between $\widehat{p}$ and $\underline{p}$ – thereby affecting R_{02} as well.[30] This likewise implies ambiguous consequences for the frequencies of early and late default.

We next show that a payment-enhancing restructuring regime raises ex-ante welfare.[31] This result holds irrespective of the exact rules used to share payments across creditors in the original economy. In particular, the welfare result does not rely on only short-term debt being repaid in the event of an early crisis in the pre-existing regime. Instead, it is the country's ability to change γ in response to the new regime and the greater repayment capacity that are key to obtaining positive welfare results.

Proposition 1 *Consider an economy with no IFI intervention and with an incentive compatible maturity structure* $(\gamma_0, r(\gamma_0))$ *in which, at each* $p < \widehat{p}(\gamma_0) \equiv \widehat{p}_0$*, a fraction* $\beta^0(\gamma, p; D_0(p))$ *of total payments* $D_0(p)$ *is paid to LT creditors (with* $1 - \beta^0(\cdot)$ *paid to ST creditors). Consider a reform to a restructuring regime that is payment-enhancing relative to the original regime (as defined in equation (30)) and that distributes*

[30] In the baseline model this cannot occur as we have assumed that LT debt is paid zero below $\widehat{p}$. For more general ways in which repayment is shared below $\widehat{p}$, this concern is valid.

[31] Note that asking a restructuring regime to be payment enhancing is not particularly demanding. Once inefficient disinvestment is avoided, nothing prevents a restructuring from increasing total payments to creditors.

expected resources pD^s *for* $p < \widehat{p}$ *according to* $\beta^s(\gamma, p; D^s) \equiv \beta^s(\gamma R_{01}(\gamma), (1-\gamma)R_{02}, p; D^s)$ *as defined in equation (25). Then there exists an incentive-compatible* γ' *such that* $\underline{p}^s(\gamma') < \underline{p}(\gamma_0) \equiv \underline{p}_0$.

Proof. Here we sketch the main arguments of the proof; the full proof is in the appendix. We first show that, holding constant $\underline{p} = \underline{p}_0$ (in a sense that is made precise in the proof), it is always possible under the new restructuring regime to appropriately change the maturity structure to γ' such that expected contractual payments to ST creditors are the same as in the original (non-restructuring regime) economy. Given that restructuring is payment-enhancing, this is shown to imply that the expected value of total contractual payments at $\underline{p} = \underline{p}_0$ is lower than in the original economy, which in turn implies that expected contractual payments to LT creditors must have fallen. We then show that, with this new maturity structure, the new equilibrium contractual rates generate $\underline{p}^s(\gamma') < \underline{p}_0$ and satisfy the IC constraint. ■

The key intuition in the proof above is that there exists a maturity structure that keeps promised contractual payments to ST creditors unchanged. This ensures that the greater availability of funds allows the country to lower its contractual obligations to LT creditors and thus improves welfare in an incentive compatible fashion. The proof of the proposition is fairly intricate, however. The reason is that, for the reasons explained in section 4.3, it must deal with both the potential non-existence of equilibrium for any given γ and with the potential lack of continuity of equilibrium contract rates in γ. These obstacles are overcome by the construction of contractual rates that, while being zero-profit, are not required to be equilibrium objects because they are defined for exogenous cutoff levels $\left(\underline{p}^s(\gamma), \widehat{p}^s(\gamma)\right)$. These rates are continuous in γ, which permits us to establish the existence of the desired maturity and its properties. We can now use this proposition to establish the main result of this section

Corollary 1 *Consider a regime change from non-restructuring to restructuring. If the restructuring regime is payment-enhancing relative to the original, the regime change is welfare enhancing.*

Proof. As discussed previously, a country will always ask the IFI to intervene when $p < \hat{p}$ and the IFI will be willing to restructure under those p realizations. From equation (28), a sufficient condition for restructuring to be welfare enhancing is $\underline{p}^s(\gamma^s) < \underline{p}^*(\gamma^*)$ (i.e., once restructuring eliminates the production inefficiency associated with early default, the country need only worry about minimizing its overall probability of default). It follows immediately from the proposition above that there exists an incentive-compatible γ' such that $\underline{p}^s(\gamma') < \underline{p}^*(\gamma^*)$. The country's optimal γ^s may differ from γ' but, by revealed preference, γ^s must necessarily increase ex ante welfare. Since creditors are always paid R^2 in expectation, the regime change is welfare-enhancing. ■

Having shown that the country is made better off by a restructuring regime, we can now ask how the latter affects the country's optimal maturity structure.[32] Note first that, as shown in the proof of Proposition 1,

[32] We now return to the assumptions of the original model regarding payments below $\widehat{p}$ in the absence of an IFI as we cannot characterize the equilibrium maturity structure otherwise.

a restructuring regime loosens the IC constraint. Letting γ_{IC}^s denote the minimum incentive-compatible level of γ under restructuring, we have that $\gamma_{IC}^s < \gamma_{IC}$. If $R_{02}^s(\gamma_{IC}^s) < \alpha\theta_H$, so that the equilibrium under restructuring is favorable when $\gamma = \gamma_{IC}^s$, the country sets $\gamma^s = \gamma_{IC}^s$; since $\gamma_{IC}^s < \gamma_{IC} \leq \gamma^*$, restructuring lengthens the equilibrium maturity structure.

If instead $R_{02}^s(\gamma_{IC}^s) > \alpha\theta_H$, i.e., the equilibrium under the restructuring regime (and hence without restructuring as well) is unfavorable, the effect on γ is ambiguous. The reason is that the country may be better off with an interior choice of γ. For the sake of illustration, assume differentiability of $d\underline{p}^*(\gamma)$ and $\widehat{p}^*(\gamma)$ and note that, in the absence of restructuring, an interior choice of γ must satisfy

$$\frac{dW_0}{d\gamma} = -\pi\theta_H g(\underline{p}^*(\gamma))\frac{d\underline{p}^*(\gamma)}{d\gamma} - \left(\widehat{p}^*(\gamma)\theta_H - R\frac{f(p)}{\gamma}\right) g(\widehat{p}^*(\gamma))\frac{d\widehat{p}^*(\gamma)}{d\gamma} = 0, \tag{31}$$

whereas the equivalent expression under restructuring is given by

$$\frac{dW_0^s}{d\gamma} = -\pi\theta_H g(\underline{p}^S(\gamma)))\frac{d\underline{p}^s(\gamma)}{d\gamma} = 0. \tag{32}$$

A quick comparison of both expressions reveals that restructuring makes ST debt more attractive by eliminating the cost of disinvestment $\left(\widehat{p}^*(\gamma)\theta_H - R\frac{f(p)}{\gamma}\right)$. Furthermore, the behavior of $\underline{p}^s(\gamma)$ differs from $\underline{p}^*(\gamma)$. Like $\underline{p}^*(\gamma)$, it need not be increasing in γ when the economy is unfavorable but the mechanisms for this are now twofold. As before, increases in γ substitute away from relatively expensive LT debt. In addition, increases in γ raise $\widehat{p}$, increasing R_{01} but also – since this reduces the instances of full dilution – reducing R_{02}. The final effect of a marginally higher γ on $\underline{p}$ is ambiguous and thus so is the change in the optimal maturity structure.

5.3 Debt Reprofiling

The IMF is currently considering extending its lending framework to allow for the "reprofiling" of debt payments (IMF 2014a). This is a postponement of payments during crises or, equivalently, a suspension of payments accompanied by a lengthening of debt maturity. The objective of reprofiling is to allow the country to avoid a full default if the economy eventually recovers. We use the model to analyze the main effects of this policy proposal.

The tool kit available to the IFI is thus assumed to contain both restructuring and reprofiling. The latter consists of the ability to postpone all payments to creditors once p is revealed at $t = 1$ and the IFI is called in. For long-term creditors, this intervention has no direct effect. For short-term creditors, though, this intervention essentially implies that they must forcefully roll over their debt.

More formally, let $R_{12}^f(p)$ denote the rate at which ST debt is rolled over in the event of a reprofiling, where f stands for reprofiled. Whenever debt is reprofiled, short-term creditors obtain an expected return in period 2 of

$$p \cdot R_{01}^f \cdot R_{12}^f(p)$$

per unit of debt. It is immediate that, in order to fulfill its stated objective of allowing the country avoid a default, reprofiling must necessarily impose a "haircut" on ST creditors.[33] That is, in order for reprofiling to reduce the probability of default, it must roll-over ST creditors at a rate that is lower than what would be required by the market, i.e.,

$$R_{12}^f(p) < \frac{R}{p} \quad \text{for } p < \underline{p}\left(\gamma; r^f\right) \tag{33}$$

In particular, we assume that $R_{12}^f(p)$ is set so that total payments to creditors equal $D^f \leq \alpha\theta_H$ when $\theta = \theta_H$, i.e.,

$$R_{12}^f(p) = R_{12}^f = \max\left\{\frac{D^f - (1-\gamma)\cdot R_{02}^f}{\gamma \cdot R_{01}^f}, 0\right\} \tag{34}$$

Defined in this manner, for a given $p < \underline{p}$, expected total payments to creditors under a debt reprofiling equal $p \cdot D^f$, where a fraction

$$\beta^f(\gamma, p; D^f) \equiv \beta^f(\gamma R_{01}^f, (1-\gamma) R_{02}^f, p; D^f) = \max\left\{\frac{(1-\gamma)\cdot R_{02}^f}{D^f}, 1\right\}, \tag{35}$$

is paid to LT creditors, and the remaining share is paid to ST creditors.

Since reprofiling requires a haircut on creditors (albeit only on short-term creditors) it constitutes a credit event and hence is bound to impose a default penalty on the country. The IMF has argued, however, that since the haircut is borne only by a portion of a debt, the associated penalty is bound to be lower. Accordingly, we assume that debt reprofiling entails a loss of $\theta \cdot \pi^f$ of output at $t = 2$, with $\pi^f < \pi$.[34]

In order to determine how a reprofiling regime affects equilibrium, we must first determine the circumstances under which reprofiling will be used. Note that, conditional on being approached by the country, the IFI will choose to intervene and coordinate creditors and country to reprofile the debt whenever $p < \underline{p}^f(\gamma; r^f)$ since the latter has lower costs than either a debt restructuring or an outright default. Anticipating this, will the country choose to approach the IFI knowing that reprofiling will be the outcome when $p < \underline{p}^f(\gamma; r^f)$? By reprofiling its debt, the country is able to avoid the full costs of an outright default; it loses, however, its ability to dilute long-term debt. Hence, the country will only approach the IMF when the gains of reprofiling exceed those of dilution.

Formally, to determine the range of p for which reprofiling takes place requires comparing the country's payoff from reprofiling

$$p\theta_H - pD^f - p\pi^f\theta_H,$$

[33] As with restructuring, in an environment with many values of θ the choice of $R_{12}^f(p)$ would more generally imply a choice regarding the range of θ which would result in default in period 2.

[34] Clearly, if $\pi^f = \pi$, reprofiling boils down to a form of restructuring and the analysis of the previous section applies. But there are reasons for which, in practice, it may be easier to reprofile debt than to restructure it outright. Reprofiling requires an agreement on the rate at which a fraction of the debt is rolled over, whereas restructuring requires an agreement on the haircut imposed on the entire debt. IMF (2014a) provides suggestive evidence that, in practice, reprofilings do seem to be less costly than debt restructurings.

to its welfare in the absence of IFI intervention

$$p\theta_H - \gamma R_{01}^f R - p\pi\theta,$$

which yields that reprofiling is preferred by the country for $p \leq \frac{\gamma R_{01} R}{D^f - (\pi - \pi^f)\theta_H}$. Thus, reprofiling takes place in equilibrium whenever,

$$p < \widetilde{p}^f(\gamma; r^f) = \min\left\{\frac{\gamma R_{01}^f R}{D^f - (\pi - \pi^f)\theta_H}, \underline{p}^f(\gamma; r^f)\right\} = \min\left\{\kappa \cdot \hat{p}^f\left(\gamma; r^f\right), \underline{p}^f(\gamma; r^f)\right\}, \tag{36}$$

where

$$\kappa = \frac{\alpha\theta_H}{D^f - (\pi - \pi^f)\theta_H} \geq 1 \tag{37}$$

Perfect competition among creditors requires that gross returns now reflect the possibility of reprofiling. In particular, zero-profit contract rates are now given by:

$$R^2 = \int_0^{\widetilde{p}^f(\gamma)} \beta^f(\gamma, D^f) p D^f dG(p) + R_{02}^f \int_{\underline{p}^f(\gamma)}^1 p dG(p), \tag{38}$$

$$R = \frac{1}{R}\int_0^{\widetilde{p}(\gamma)} \left(1 - \beta^f(\gamma, D^f)\right) p D^f dG(p) + R_{01}^f \left(1 - G\left(\widetilde{p}^f(\gamma)\right)\right). \tag{39}$$

Together with equations (7), (22) and (36) and with the incentive compatibility constraint of equation (17), these expressions determine equilibrium interest rates and crisis and reprofiling probabilities for a given level of γ. As before, there may be values of γ that are not feasible for the country as they are unable to satisfy both the IC constraint and the creditors' break-even conditions. There may also be values of γ that admit multiple contractual rates and cutoff levels of p. Whenever the system of equations has multiple solutions for a given γ, we define an equilibrium under reprofiling – denoted by $r^f(\gamma) = \left(R_{01}^f(\gamma), R_{02}^f(\gamma)\right)$, $\underline{p}^f(\gamma)$, $\hat{p}^f(\gamma)$ and $\widetilde{p}^f(\gamma)$ – as the solution that attains the maximum level of ex ante welfare for that particular γ:

$$W_0^f(\gamma) = \int_0^1 p\theta_H dG(p) - \int_0^{\widetilde{p}^f(\gamma)} p\pi^f\theta_H dG(p) - \int_{\widetilde{p}^f(\gamma)}^{\underline{p}^f(\gamma)} p\pi\theta_H dG(p) - R^2. \tag{40}$$

This expression summarizes welfare under reprofiling. Like restructuring, reprofiling eliminates the need for inefficient payments (disinvestment). Thus, the country suffers the penalty associated with reprofiling when $p \leq \widetilde{p}^f(\gamma)$, and the full penalty from default when $p \in \left(\widetilde{p}^f(\gamma), \underline{p}^f(\gamma)\right)$.

Before turning to the welfare implications of reprofiling, it is useful to define a payment-enhancing regime in this environment. Let $\widetilde{p}^s(\gamma) \equiv \min\left\{\kappa \cdot \hat{p}^s(\gamma; r), \underline{p}^s(\gamma; r)\right\}$, i.e., even though there is no reprofiling in a restructuring regime, let us define $\widetilde{p}^s(\gamma)$ nonetheless. We say that reprofiling is payment-enhancing relative to a restructuring regime if total expected payments to creditors over the states $p < \widetilde{p}^s(\gamma)$ to creditors is greater. Formally:

Definition 2 *A reprofiling regime is said to be payment-enhancing relative to a restructuring regime with maturity structure* $(\gamma, r(\gamma))$ *iff* $D^f \geq D^s$ *and*

$$\int_0^{\widetilde{p}^s(\gamma)} pD^f dG(p) > \int_0^{\widehat{p}^s(\gamma)} pD^s(p)dG(p) + \int_{\widehat{p}^s(\gamma)}^{\widetilde{p}^s(\gamma)} \gamma R^s_{01}(\gamma)RdG(p), \tag{41}$$

The left-hand-side of equation (41) represents expected payments to creditors over all states with $p \leq \widetilde{p}^s(\gamma)$ assuming that they are paid D^f in each state. The right-hand side of the equation represents instead expected payments to creditors over all states with $p \leq \widetilde{p}^s(\gamma)$ under the restructuring regime , i.e., actual debt restructuring payments for $p \in [0, \widehat{p}^s(\gamma))$ and payment in full to ST creditors for $p \in [\widehat{p}^s(\gamma), \widetilde{p}^s(\gamma)]$. Thus, the inequality says that a reprofiling regime is payment enhancing if and only if, given the original maturity structure and interest rates, debt reprofiling raises expected payments relative to the restructuring regime. Intuitively, the condition requires reprofiling to reduce the "average haircut" on debt payments relative to the restructuring regime. The condition $D^f \geq D^s$ requires restructuring to impose a higher haircut on creditors than reprofiling when restricted only to those states in which restructuring would occur.[35]

We are now ready to present the main result of this section, namely, that a payment-enhancing reprofiling regime increases ex ante welfare. At first glance, this result may seem obvious: if reprofiling makes it possible to increase total payments to creditors during a crisis, shouldn't it follow that it would reduce interest rates and thus the likelihood of default? The answer to this question is negative though, for the same reason invoked for the case of restructuring, i.e., debt reprofiling, by changing payments, affects all endogenous variables. In particular, debt reprofiling may substantially reduce payments to ST creditors in the event of a crisis, thereby raising the cost of ST borrowing, as critics of this reform have warned. As the following proposition shows, however, the country can always undo the negative effects of these changes by appropriately adjusting the maturity structure of its debt.

Proposition 2 *Consider a restructuring regime with an incentive compatible maturity structure* $(\gamma_0, r^s(\gamma_0))$. *In this economy, total expected payments to creditors equal* pD^s *for* $p < \widehat{p}^s(\gamma_0) = \widehat{p}_0$, *a fraction* $\beta^s(\gamma_0, p; D^s)$ *of which is paid to LT creditors (with* $1 - \beta^s(\cdot)$ *paid to ST creditors). Consider a reform to a reprofiling regime* f *that is payment-enhancing relative to the original regime (as defined in equation (41)) and that distributes resources* D^f *for* $p < \widetilde{p}^f$ *according to* $\beta^f(\gamma, p; D(p)) \equiv \beta^f(\gamma R^f_{01}, (1-\gamma)R^f_{02}, p; D^f)$ *as defined in equation (35). Then there exists an incentive-compatible* γ' *such that* $\underline{p}^f(\gamma') < \underline{p}^s(\gamma_0) = \underline{p}_0$.

Proof. See appendix. ■

The key intuition behind the proof of Proposition 2 is similar to that given for the proof of Proposition 1: by finding a new maturity structure that keeps promised contractual payments to ST creditors unchanged, the greater availability of resources allowed by reprofiling makes it possible for the country to lower its

[35] Note that payment enhancing is a fairly natural property for a reprofiling regime to satisfy: for $p \in [0, \widehat{p}^s(\gamma))$, there is no reason why reprofiling should impose a higher haircut than restructuring; for $p \in [\widehat{p}^s(\gamma), \widetilde{p}^s(\gamma)]$, instead, total payments to creditors were limited not by expected output but rather by dilution, which is eliminated when debt is reprofiled.

contractual obligations to LT creditors and thus improve welfare in an incentive compatible fashion. As in the case of restructuring, though, the proof of the proposition is fairly intricate because of potential problems of non-existence of equilibrium for some values of γ and the lack of continuity of equilibrium contract rates in γ. To overcome these obstacles, the proof constructs contractual rates for exogenous cutoff levels $\left(\underline{p}^f(\gamma), \widetilde{p}^f(\gamma)\right)$: these rates satisfy creditors' zero-profit conditions but, since they are not required to be equilibrium rates, they are continuous in γ. This allows us to establish the existence of a maturity structure γ' and to study its properties. Proposition 2 leads directly to the main result of this section:

Corollary 2 *Consider a regime change from restructuring to reprofiling. If the reprofiling regime is payment-enhancing relative to the original, the regime change is welfare improving.*

Proof. As discussed previously, a country will ask the IFI to intervene if $p < \widetilde{p}$, and the IFI will be willing to reprofile for those realizations of p. Since the penalty suffered by the country from reprofiling is lower than from restructuring, it follows from equation (40) that a sufficient condition for reprofiling to increase ex ante welfare over restructuring is that it allow the country to reduce the probability of a crisis in equilibrium, i.e., that there exist a γ^f satisfying $\underline{p}^f(\gamma^f) < \underline{p}^s(\gamma^s)$. It follows immediately from the proposition above that if the reprofiling regime is payment enhancing there exists an incentive-compatible γ' such that $\underline{p}^f(\gamma') < \underline{p}^s(\gamma^s)$. The country's optimal γ^f may differ from γ' but, by revealed preference, it must necessarily increase its ex ante welfare. Since creditors are always paid R^2 in expectation, the regime change is welfare increasing. ■

We have now shown that the country will be made better off adopting a payment-enhancing reprofiling regime; not only does reprofiling entail a lower penalty than either restructuring or outright default, it also makes it possible for the country to reduce the overall probability of default. The effect of reprofiling on the maturity structure of debt, however, is ambiguous. To see this more formally, note from equation (40) that the optimal maturity structure under reprofiling must trade-off two effects: first, it seeks to minimize the probability of a default as captured by $\underline{p}^f(\gamma)$; second, conditional on defaulting, it seeks to maximize the likelihood of reprofiling, as captured by $\widetilde{p}^f(\gamma)$, thereby reducing the costs of default. Formally, assuming differentiability for the sake of illustration, whereas any interior choice of γ in the absence of reprofiling must satisfy equation (32), the equivalent expression under reprofiling is given by

$$\frac{dW_0^f}{d\gamma} = -\pi\theta_H g(\underline{p}^f(\gamma))\frac{d\underline{p}^f(\gamma)}{d\gamma} + \left(\pi - \pi^f\right)\theta_H g(\widetilde{p}^f(\gamma))\frac{d\widetilde{p}^f(\gamma)}{d\gamma} = 0. \tag{42}$$

Equation (42) shows why the effect of reprofiling on the optimal maturity of debt is ambiguous. First, and for the same reasons discussed for the economy under restructuring, $\underline{p}^f(\gamma)$ need no longer be an increasing function of γ. Second, reprofiling introduces a new additional ex ante benefit of raising γ. To see this, note from equation (36) that, at least when $\widetilde{p}^f(\gamma) = \kappa\widehat{p}^f$, $\widetilde{p}^f(\gamma)$ may be increasing in γ**.** If so, a shorter maturity structure reduces the gains from debt dilution at $t = 1$, which provides the country with additional incentives to approach the IFI and reprofile the debt. This is beneficial from an ex ante perspective because the cost of

reprofiling is lower than the cost of an outright default. Thus, reprofiling may actually shorten the optimal maturity structure of debt even though it relaxes the incentive compatibility constraint. A shorter maturity structure may be optimal even if it raises the likelihood of a crisis at the margin since it also increases the ex post incentive to deal with a crisis by approaching the IMF and reprofiling the debt. Thus, one cannot rule out the possibility that reprofiling raises the incidence of debt crisis while simultaneously reducing their cost. Of course, the country will only choose this option if it increases ex ante welfare and hence is Pareto improving.

6 Discussion and Concluding Remarks

This paper presented a model of endogenous debt maturity and used it to analyze the effects of two alternative policies for debt-crisis resolution: restructuring and reprofiling. Our main result is that these policies are guaranteed to be welfare enhancing if they raise total expected payments to creditors in times of crises. We show that any potentially negative effects of these policies can be eliminated by appropriately adjusting the maturity structure of debt. While our analysis focused on a country that is able to access international capital markets even in the absence of an IFI, note that another benefit from introducing these policies is that they may permit countries that would otherwise be shut out of international markets to access them. To conclude, we now discuss several ways in which the model may be extended or modified and how these changes may affect the main results.

First, we have assumed that the IFI has the ability to coordinate creditors into accepting a debt restructuring or reprofiling if approached by the country. Insofar as debt restructurings and reprofilings are payment-enhancing, this may seem natural since it is in the interest of creditors as a whole to accept these policies. Our model does not take into account the potential problems that may arise across different types of creditors, however, which may lead to holdouts and make it difficult to reach an agreement. This is an important concern that needs to be addressed by thinking about the interaction between renegotiation, the trading of debt in the secondary market, and the potential use of legal instruments such as collective action clauses.

Second, we have implicitly assumed that the country does not reaccess financial markets immediately after a debt restructuring or reprofiling. This is important because both types of operations entail a lengthening of the maturity structure: after either, the country is left only with debt due in period 2. Becuase of this, the country may face strong incentives to borrow in period 1 in order to dilute the debt that has just been restructured/reprofiled. This implies that, in reality, it may be critical to restrict a country's ability to access markets for some time after a restructuring or reprofiling event, at least until fundamentals improve significantly.

Third, our model was silent about a potential concern related to debt reprofiling: debt overhang. It is widely perceived that an advantage of restructuring relative to reprofiling is that the former deals with debt

problems "once and for all", whereas reprofiling may exacerbate debt overhang by postponing resolution. This possibility could be introduced into our model by assuming that, although reprofiling has a lower cost of default than reprofiling, it entails a loss of output due to debt overhang. This should not be a problem as long as the country and the IFI correctly internalize these costs. One might imagine, however, situations in which the IFI's preferences are skewed towards creditors so that it reprofiles debt even when restructuring would be socially optimal. In such cases, the country might prefer not to approach the IFI altogether and the option to reprofile may reduce welfare relative to a restructuring only policy.

Fourth, the model did not address the role of IFI lending in debt crises. Debt restructurings are typically accompanied by IMF programs that entail some degree of lending by the institution. What would change if this role was incorporated in our framework? Assuming that the IFI breaks even on its lending, the answer depends on the institution's ability to extract payments from the country relative to private creditors, i.e. on whether the IFI faces a different α than private creditors. If both have the same capacity to extract payments from the country, then any payments to the IFI during crises will crowd out payments to private creditors. If the IFI instead has a higher ability to extract payments from the country, its lending will trivially be useful: by pledging output to the IFI that it cannot pledge to private creditors, the country will be able to access additional funds in times of crises, thereby reducing the likelihood of default.

At a more general level, the insights offered by this paper should play an important role in the various debates regarding the reform of the international financial architecture. They highlight the fact that it is critical to endogenize the structure of debt in order to analyze the effects of any such reforms. In this paper, we have focused on the maturity structure of debt as the reforms have different effects on short versus long-term creditors. To examine the effects of other reforms under discussion, such as the strengthening of collective action clauses, it will be important to endogenize other dimensions of the debt structure, such as the type and jurisdiction of instruments being issued.[36] This constitutes an important avenue for future research.

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7 Appendix

7.1 Comparative statics

Consider first an increase in the international interest rate R. From equations (18) and (19), this has a direct impact on ST and LT interest rates $R_{01}^*(\gamma)$ and $R_{02}^*(\gamma)$ and therefore increases both $\underline{p}(\gamma)$ and $\widehat{p}(\gamma)$. The increase in the probabilities of early and late default in turn feed back into $R_{01}^*(\gamma)$ and $R_{02}^*(\gamma)$, raising the spreads that the country must pay on ST and LT debts. Finally, a higher $R_{02}^*(\gamma)$ tightens the IC constraint. In the favorable economy, this forces the country to shorten its maturity structure, i.e., to increase γ^*. For an unfavorable economy, the effect of an increase in R on γ is instead ambiguous. In this case, assuming differentiability for the sake of illustration, an interior solution satisfies:

$$\frac{dW_0}{d\gamma} = -\pi\theta_H g(\underline{p}^*(\gamma))\frac{d\underline{p}^*(\gamma)}{d\gamma} - \left(\widehat{p}^*(\gamma)\theta_H - R\frac{f(p)}{\gamma}\right) g(\widehat{p}^*(\gamma))\frac{d\widehat{p}^*(\gamma)}{d\gamma} = 0. \tag{43}$$

Although it is straightforward to show that $\dfrac{d\widehat{p}^*(\gamma)}{d\gamma}$ is positive with R, the sign of $\dfrac{d\underline{p}^*(\gamma)}{d\gamma}$ is ambiguous. Thus, without further parametric assumptions, it is impossible to state how γ^* responds to a higher R.

The same analysis carries over to a change in the distribution function $G(p)$. In particular, one can capture the effects of deteriorating economic prospects for the country by assuming that the distribution function changes to $H(p)$, where $H(p) \geq G(p)$ for all $p \in [0,1]$. From equations (19) and (18), this shift in the distribution of p has a direct impact on ST and LT interest rates $R_{01}^*(\gamma)$ and $R_{02}^*(\gamma)$, raising the spreads paid by the country as well as the probabilities of early and late default by increasing $\underline{p}^*(\gamma)$ and

$\widehat{p}^*(\gamma)$. For a favorable economy, the effect is to increase γ^*: once again, the rise in R^*_{02} tightens the incentive compatibility constraint and this forces the country to shorten its maturity structure. For the unfavorable economy, though, the probability has an ambiguous effect on γ as before. In particular, it depends on how the change affects $g(\underline{p}^*(\gamma))$, $g(\widehat{p}^*(\gamma))$, and, assuming differentiability, $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ and $\frac{d\widehat{p}^*(\gamma)}{d\gamma}$ in equation (43).

Turning next to changes in the enforcement environment of the country, consider an increase in the cost of default π. The direct effect is to relax the incentive compatibility constraint. For a favorable economy, this enables the country to lower γ, thereby reducing both $\widehat{p}^*(\gamma)$ and $\underline{p}^*(\gamma)$. Note that the welfare consequences of this change is ambiguous, though: although default becomes less likely in equilibrium, it is also more costly when it happens. This is, by the way, a standard feature of models with equilibrium default (see, e.g., Cole and Kehoe (2000)). For an unfavorable economy, if the equilibrium remains interior, an increase in π leads instead to an increase in γ. The reason is that, here, the probability of default $\underline{p}(\gamma)$ is decreasing in the share of ST debt: when default becomes more expensive relative to the efficiency cost of disinvestment, the country has an incentive to reduce its likelihood by shortening its maturity structure (see equation (43)). Globally, however, the ability to sustain a lower level of γ_{IC} may prove more attractive. Thus the comparative static implications remain ambiguous overall.

Consider next the consequences of a rise in $F(\delta, p)$ – i.e., an increase in the efficiency of payments upon early default. The direct effect of this is to raise increase payments to ST creditors and thus reduce $R^*_{01}(\gamma)$. This in turn leads to a decrease in both $\underline{p}^*(\gamma)$ and $\widehat{p}^*(\gamma)$, which means that $R^*_{02}(\gamma)$ must also fall. For a favorable economy, the IC constraint is relaxed and γ^* falls in consequence. But for an unfavorable economy the effect is once again ambiguous. On the one hand, if early default is less costly it becomes more attractive to increase γ at the margin in order to reduce the overall probability of default associated with $\underline{p}^*(\gamma)$. On the other hand, these changes affect $g(\underline{p}^*(\gamma))$, $g(\widehat{p}^*(\gamma))$, and, assuming differentiability, $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ and $\frac{d\widehat{p}^*(\gamma)}{d\gamma}$, rendering the net effect ambiguous even though the ultimate effect on welfare is unambiguously positive.

Lastly, consider the effect of an increase in α. By raising the share of output that can be pledged to creditors, this change reduces both $\underline{p}^*(\gamma)$ and $\widehat{p}^*(\gamma)$ and, from equations (18) and (19), it also reduces $R^*_{01}(\gamma)$ and $R^*_{02}(\gamma)$. For a favorable economy, γ falls because the IC constraint is relaxed. For an unfavorable economy, and assuming differentiability, the overall effect of this change depends on how the increase in α affects $\frac{d\widehat{p}^*(\gamma)}{d\gamma}$ relative to $\frac{d\underline{p}^*(\gamma)}{d\gamma}$ and hence is ambiguous though clearly welfare increasing.

7.2 Proof of Propositions

Proposition 1 *Consider an economy with no IFI intervention and with an incentive compatible maturity structure* $(\gamma_0, r(\gamma_0))$ *in which, at each* $p < \widehat{p}(\gamma_0) \equiv \widehat{p}_0$*, a fraction* $\beta^0(\gamma, p; D_0(p))$ *of total payments* $D_0(p)$ *is paid to LT creditors (with* $1 - \beta^0(\cdot)$ *paid to ST creditors). Consider a reform to a restructuring regime that is payment-enhancing relative to the original regime (as defined in equation (30)) and that distributes resources* D^s *for* $p < \widehat{p}$ *according to* $\beta^s(\gamma, p; D^s) \equiv \beta^s(\gamma R_{01}(\gamma), (1-\gamma)R_{02}, p; D^s)$ *as defined in equation* (25)*. Then there exists an incentive-compatible* γ' *such that* $\underline{p}^s(\gamma') < \underline{p}(\gamma_0) \equiv \underline{p}_0$.

Proof. The structure of the proof is as follows. We first show that, given $\underline{p} = \underline{p}_0$, it is always possible under the new regime to reduce expected payments to creditors by appropriately changing γ'. We then show that with this new maturity structure there exist contractual interest rates that satisfy both the break-even condition of creditors and the IC constraint and for which $\underline{p}^s(\gamma') < \underline{p}^s_0$.

To prove the lemma it is useful to construct a pair of interest rates, $R^s_{01}(\gamma; \underline{p}, \widehat{p})$ and $R^s_{02}(\gamma; \underline{p}, \widehat{p})$, that satisfy creditor break-even conditions under restructuring for *exogenously given* levels of $\underline{p}$ and $\widehat{p}$. We will be interested in these rates evaluated at $\underline{p} = \underline{p}_0$, $\widehat{p} = \widehat{p}_0$. These rates are constructed assuming that i. all creditors are paid in full for $p \geq \underline{p}_0$; ii. ST debt is paid in full, whereas LT is paid nothing, for $p \in \left[\widehat{p}_0, \underline{p}_0\right)$; and, iii. ST and LT creditors are paid a total of D^s, shared according to $\beta^s(\gamma, p; D^s)$ for $p < \widehat{p}_0$, where β^s is calculated using the contractual rates $R^s_{01}(\gamma; \underline{p}_0, \widehat{p}_0)$ and $R^s_{02}(\gamma; \underline{p}_0, \widehat{p}_0)$.[37] Note that these contractual rates differ from the equilibrium rates $R^s_{01}(\gamma)$ and $R^s_{02}(\gamma)$ (as defined in equations (26) and (27)), since they are neither required to be consistent with the endogenously generated cutoffs $\left(\underline{p}^s(\gamma), \widehat{p}^s(\gamma)\right)$ nor are the implied payments required to be feasible. Thus, unlike equilibrium rates, $R^s_{01}(\gamma; \underline{p}_0, \widehat{p}_0)$ and $R^s_{02}(\gamma; \underline{p}_0, \widehat{p}_0)$ are continuous functions of γ and are guaranteed to exist as feasibility is not a concern.

We will show that one of two possible scenarios holds when the restructuring economy is evaluated using the contractual rates constructed above: (i) there exists an incentive-compatible γ' such that $\widehat{p}\left(\gamma'; \underline{p}_0, \widehat{p}_0\right) = \widehat{p}_0$ which we then show implies $\underline{p}^s(\gamma') < \underline{p}_0$ or, (ii) if such a γ' does not exist, then setting $\gamma' = 1$ guarantees $\underline{p}^s(1) < \underline{p}_0$.

If there exists a γ' such that $\widehat{p}\left(\gamma'; \underline{p}_0, \widehat{p}_0\right) = \widehat{p}_0$, it follows that $\gamma' R^s_{01}\left(\gamma'; \underline{p}_0, \widehat{p}_0\right) = \gamma_0 R_{01}(\gamma_0)$. Thus, we have

$$\gamma' \cdot \frac{R - \frac{1}{R}\int_0^{\widehat{p}_0}(1 - \beta^s(\gamma', p; D^s))p\frac{D^s}{\gamma'}dG(p)}{1 - G(\widehat{p}_0)} = \gamma_0 \cdot R_{01}(\gamma_0) \tag{44}$$

yielding:

$$\gamma' = h(\gamma') = \gamma_0 \frac{R_{01}(\gamma_0)}{R}(1 - G(\widehat{p}_0) + \frac{1}{R^2}\int_0^{\widehat{p}_0}(1 - \beta^s(\gamma', p; D^s))pD^s dG(p), \tag{45}$$

where $h(\gamma')$ is a continuous function of γ'. Continuity of h follows from continuity of $R^s_{01}\left(\gamma; \underline{p}_0, \widehat{p}_0\right)$, $R^s_{02}\left(\gamma; \underline{p}_0, \widehat{p}_0\right)$, and $\beta^s(\cdot)$ in γ. Note that $\lim_{\gamma' \to 0} h(\gamma') = \gamma_0 \cdot \frac{R_{01}(\gamma_0)}{R}(1 - G(\widehat{p}(\gamma_0))$ and thus that $\lim_{\gamma' \to 0} \gamma' R^s_{01}\left(\gamma'; \underline{p}_0, \widehat{p}_0\right) = 0 < \gamma_0 R_{01}(\gamma_0)$. By continuity of h, either a fixed point $\gamma' = h(\gamma')$ exists or, if it does not, then $h(1)$ must be greater than 1. We now examine both cases.

Suppose first that a fixed point exists. We now want to show that $\underline{p}^s(\gamma') < \underline{p}(\gamma_0)$. To prove this, it is useful to define a variant of the contractual interest rates used above, namely $R^s_{01}(\gamma; \underline{p})$ and $R^s_{02}(\gamma; \underline{p})$. These rates are calculated analogously to the ones described above but in this case only $\underline{p}$ is exogenous. This introduces the additional wrinkle that, since $\widehat{p}\left(\gamma; \underline{p}\right)$ is now generated endogenously, it is possible for

[37] Note that since $\widehat{p}_0$ is not an equilibrium cutoff since it is imposed exogenously, it is possible for the share of D^s that would be paid to a type of creditor to exceed the contractual rate for some $p < \widehat{p}_0$. In that case, we assume that the excedent is returned to the country.

$\widehat{p}\left(\gamma;\underline{p}\right) \geq \underline{p}$, and thus the payment rules must be specified for this eventuality. Accordingly, we assume that if $\widehat{p}\left(\gamma;\underline{p}\right) \geq \underline{p}$, the payment rules are those given by $\underline{p}$, i.e., all creditors are paid in full above $\underline{p}$ and share D^s according to β^s below $\min\left(\underline{p},\widehat{p}\left(\gamma;\underline{p}\right)\right)$.

Let us define the expected value of the contractual expenditures generated by the rules above, for a given γ and at a given $p = \underline{p}$, as:

$$E^s\left(\gamma;\underline{p}\right) \equiv \gamma R^s_{01}(\gamma;\underline{p})R + (1-\gamma)\, R^s_{02}(\gamma;\underline{p})\underline{p} \tag{46}$$

We now note two critical features of $E^s\left(\gamma;\underline{p}\right)$. First, it is increasing in $\underline{p}$: a higher $\underline{p}$, ceteris paribus, increases $R^s_{02}(\gamma;\underline{p})$ as LT debt is paid in full in fewer states. Given the sharing rules β^s, this in turn decreases the share of D^s received by ST creditors below $\widehat{p}(\gamma;\underline{p})$, thus increasing $R^s_{01}(\gamma;\underline{p})$ as well.[38] Second, the expected value of the contractual expenditures generated by γ' using the contractual rates $R^s_{01}\left(\gamma';\underline{p}_0,\widehat{p}_0\right)$, $R^s_{02}\left(\gamma';\underline{p}_0,\widehat{p}_0\right)$ are given by $E^s\left(\gamma';\underline{p}_0\right)$ since the endogenously generated $\widehat{p}\left(\gamma';\underline{p}_0,\widehat{p}_0\right) = \widehat{p}_0$ and thus $R^s_{01}\left(\gamma';\underline{p}_0,\widehat{p}_0\right) = R^s_{01}\left(\gamma';\underline{p}_0\right)$ and $R^s_{02}\left(\gamma';\underline{p}_0,\widehat{p}_0\right) = R^s_{02}\left(\gamma';\underline{p}_0\right)$.[39]

We will now show that the expected value of the contractual expenditures in the equilibrium of the original (non-restructuring) economy $(\gamma_0, r(\gamma_0))$, evaluated at $\underline{p}_0$, are greater than those generated by the restructuring regime evaluated at $\underline{p}_0$. Note that the former expenditures must equal $\underline{p}_0\alpha\theta_H$ as $\underline{p}_0$ is the equilibrium cutoff for the original economy. Thus, our goal is to show $E^s\left(\gamma';\underline{p}_0\right) < \underline{p}_0\alpha\theta_H$. To see why this inequality holds, note that

$$E^s\left(\gamma';\underline{p}_0\right) = \gamma' R^s_{01}(\gamma';\underline{p}_0)R + (1-\gamma')\,\frac{R^2 - \int_0^{\widehat{p}_0}\beta^s(\gamma',p;D^s)\frac{D^s}{1-\gamma'}pdG(p)}{\int_{\underline{p}_0}^1 pdG(p)}\underline{p}_0 \tag{47}$$

whereas,

$$E^0\left(\gamma_0;\underline{p}_0\right) = \gamma_0 R_{01}(\gamma_0)R + (1-\gamma_0)\,\frac{R^2 - \int_0^{\widehat{p}_0}\beta^0\left(\gamma_0,p;D\left(p\right)\right)\frac{D(p)}{1-\gamma_0}pdG(p)}{\int_{\underline{p}_0}^1 pdG(p)}\underline{p}_0 = \underline{p}_0\alpha\theta_H. \tag{48}$$

Recalling $\gamma' R^s_{01}(\gamma';\underline{p}_0) = \gamma_0 R_{01}(\gamma_0)$, it follows that $E^s\left(\gamma';\underline{p}_0\right) < E^0\left(\gamma_0;\underline{p}_0\right)$ iff $(1-\gamma')R^s_{02}(\gamma';\underline{p}_0) < (1-\gamma_0)R_{02}(\gamma_0)$, which reduces to requiring

$$\gamma' - \gamma_0 > \frac{1}{R^2}\int_0^{\widehat{p}_0}\left[\beta^0\left(\gamma_0,p;D^0\left(p\right)\right)D\left(p\right) - \beta^s(\gamma',p;D^s)D^s)\right]pdG(p) \tag{49}$$

[38] These rates may not be uniquely defined in which case we simply choose the lowest at each $\underline{p}$ (the ones that minimize E^s).

[39] Note that the rates may now jump, causing E^s to be discontinuous. As will be clear from what follows below, the proof does not rely on continuity of E^s in $\underline{p}$, but on it being an increasing function.

From the definition of γ' in equation (45), after substituting R_{01}, we have

$$\begin{aligned}\gamma' - \gamma_0 &= \frac{1}{R^2}\int_0^{\widehat{p}_0}\left(D^s - D^0(p)\right)pdG(p) + \\ &\quad \frac{1}{R^2}\int_0^{\widehat{p}_0}\left[\beta^0\left(\gamma_0, p; D^0(p)\right)D^0(p) - \beta^s(\gamma', p; D^s)D^s\right]pdG(p) \\ &> \frac{1}{R^2}\int_0^{\widehat{p}_0}\left[\beta^0\left(\gamma_0, p; D^0(p)\right)D^0(p) - \beta^s(\gamma', p; D^s)D^s\right]pdG(p)\end{aligned}$$

where the strict inequality follows from the premise that the restructuring regime is payment enhancing relative to the original regime. Thus, the condition expressed in equation (49) is satisfied establishing $E^s\left(\gamma'; \underline{p}_0\right) < E^0\left(\gamma_0; \underline{p}_0\right)$.

We are now set to show $\underline{p}^s(\gamma') < \underline{p}(\gamma_0)$. Given that $E^s\left(\gamma'; \underline{p}\right)$ is increasing in $\underline{p}$, that $E^s\left(\gamma'; \underline{p}_0\right) < \underline{p}_0\alpha\theta_H$, and that $E^s(\gamma'; 0) > 0$, it follows that there exists a $\underline{p}^s \in \left(0, \underline{p}_0\right)$ for which $E^s\left(\gamma'; \underline{p}^s\right) = \underline{p}^s\alpha\theta_H$. At this point we have found an equilibrium for the restructuring regime, i.e., $\underline{p}^s = \underline{p}^s(\gamma')$ and $R^s_{02}(\gamma') = R^s_{02}(\gamma'; \underline{p}^s(\gamma'))$, $R^s_{01}(\gamma') = R^s_{01}(\gamma'; \underline{p}^s(\gamma'))$, i.e., the artificially constructed contractual rates are equilibrium rates at $\underline{p}^s$. Thus $\underline{p}^s(\gamma') < \underline{p}(\gamma_0)$. Moreover, this allocation is incentive compatible since $R_{02}(\gamma'; \underline{p})$ is increasing in $\underline{p}$, implying $(1-\gamma')R^s_{02}(\gamma') < (1-\gamma')R^s_{02}(\gamma'; \underline{p}_0)$ and thus that $(1-\gamma')R^s_{02}(\gamma'; \underline{p}_0) < (1-\gamma_0)R_{02}(\gamma_0) \leq \pi\theta_H$ where the last inequality follows from the premise that the original equilibrium satisfied the IC constraint.

Lastly, it remains to be shown that $\underline{p}^s < \underline{p}_0$ when a fixed point does not exist. In that case $h(1) > 1$ implying $R^s_{01}\left(1; \underline{p}_0, \widehat{p}_0\right) < \gamma_0 R_{01}(\gamma_0)$. Suppose that we now allow $\widehat{p}$ to adjust endogenously, keeping $\underline{p} = \underline{p}_0$. It follows that $\widehat{p}$ will decrease (since it must equal $\frac{R^s_{01}\left(1; \underline{p}_0\right)}{\alpha\theta_H}$ in equilibrium), allowing payments to ST debt to increase, and further decreasing the contractual rate. It thus follows that $E^s\left(1; \underline{p}_0\right) < E^0\left(\gamma_0; \underline{p}_0\right)$. One can now use the same logic as above to show that this implies $\underline{p}^s(\gamma') < \underline{p}_0$. Furthermore, as there is no LT debt, the IC constraint is trivially satisfied. This concludes the proof of the proposition. ■

Proposition 2. *Consider a restructuring economy with an incentive compatible maturity structure* $(\gamma_0, r^s(\gamma_0))$. *In this economy, total expected payments to creditors equal* pD^s *for* $p < \widehat{p}^s(\gamma_0) \equiv \widehat{p}_0$, *a fraction* $\beta^s(\gamma_0, p; D^s)$ *of which is obtained by LT creditors (with* $1-\beta^s(\cdot)$ *going to ST creditors). Consider a reform to a reprofiling regime* f *that is payment-enhancing relative to the original regime (as defined in equation* (41)*) and that distributes resources* D^f *for* $p < \widetilde{p}^f$ *according to* $\beta^f(\gamma, p; D(p)) \equiv \beta^f(\gamma R^f_{01}, (1-\gamma)R^f_{02}, p; D^f)$ *as defined in equation (35). Then there exists an incentive-compatible* γ' *such that* $\underline{p}^f(\gamma') < \underline{p}^s(\gamma_0) \equiv \underline{p}_0$.

Proof. The structure of the proof is as follows. We first show that, given $\underline{p} = \underline{p}_0$, the new regime always makes it possible to reduce expected payments to creditors by appropriately changing γ'. We then show that, under this new maturity structure, there exist interest rates that satisfy both the break-even condition of creditors and the incentive compatibility constraint and for which $\underline{p}^f(\gamma') < \underline{p}_0$.

To prove the lemma we construct a pair of partial equilibrium interest rates, $R^f_{01}(\gamma; \underline{p})$ and $R^f_{02}(\gamma; \underline{p})$

that satisfy creditor break-even conditions *given* a level of $\underline{p}$.[40] We use $\widehat{p}^f(\gamma;\underline{p})$ and $\widetilde{p}^f(\gamma;\underline{p})$ to denote the associated cutoff probabilities, i.e., $\widehat{p}^f(\gamma;\underline{p}) = \dfrac{\gamma R^f_{01}(\gamma;\underline{p})R}{\alpha\theta_H}$ and $\widetilde{p}^f(\gamma;\underline{p}) = \min\left\{\kappa\cdot\widehat{p}^f(\gamma;\underline{p}),\underline{p}\right\}$. These partial equilibrium rates are derived assuming that: i. all creditors are paid in full for $p \geq \underline{p}$; ii. ST debt is paid in full, whereas LT is paid nothing, for $p \in \left[\widetilde{p}^f(\gamma;\underline{p}),\underline{p}\right)$; and, iii. for $p < \widetilde{p}^f(\gamma;\underline{p})$, ST and LT creditors are paid either pD^f shared according to $\beta^f(\gamma,\underline{p};D^f) = \beta^f(\gamma R^f_{01}(\gamma;\underline{p}),(1-\gamma)\,R^f_{02}(\gamma;\underline{p}),p;D^f)$, or they are paid in full if $pD^f > \gamma R^f_{01}(\gamma;\underline{p}) + (1-\gamma)\,R^f_{02}(\gamma;\underline{p})p$. This last modification of the reprofiling rule is necessary because, since $R^f_{01}(\gamma;\underline{p})$ and $R^f_{02}\ (\gamma;\underline{p})$ are calculated for an exogenous $\underline{p}$, it is possible for the contractual promised payments to creditors to be lower than pD^f for some $p \leq \widetilde{p}^f(\gamma;\underline{p})$. We introduce this modification formally by assuming that, for $p < \widetilde{p}^f(\gamma;\underline{p})$, creditors are paid an expected value of $p\widetilde{D}^f(p) = \min\left\{pD^f, \gamma R^f_{01}(\gamma;\underline{p})R + (1-\gamma)\,R^f_{02}(\gamma;\underline{p})p\right\}$ shared according to $\beta^f(\gamma,\underline{p};\widetilde{D}^f(p;\underline{p}))$. As we will show, this artificial modification to the way payments are made does not bind in the feasible allocation that we ultimately find. Finally, there is an additional wrinkle as it is possible to obtain $\widehat{p}\left(\gamma;\underline{p}\right) \geq \underline{p}$, and we must thus specify the payment rules for this eventuality. We assume that if $\widehat{p} \geq \underline{p}$, the payment rules are those given by $\underline{p}$, i.e., all creditors are paid in full above $\underline{p}$ and share $\widetilde{D}^f(p)$ according to β^f below $\min\left(\underline{p},\widehat{p}\right)$.[41]

We start by asking whether there exists a γ' for which, given $\underline{p}_0$, contractual payments to ST creditors are the same under the reprofiling regime as they were under the restructuring regime. To do so, we construct an alternative version of the partial equilibrium interest rates mentioned above, denoted by $R^f_{01}(\gamma;\underline{p}_0,\widetilde{p}_0)$ and $R^f_{02}\ (\gamma;\underline{p}_0,\widetilde{p}_0)$, which are computed holding both $\underline{p}_0$ and $\widetilde{p}_0$ constant. Note that these rates, unlike the prior ones, are necessarily continuous in γ. Given these interest rates and the associated $\beta^f(\gamma',\underline{p}_0,\widetilde{p}_0;\widetilde{D}^f)$, we ask whether there exists a γ' that satisfies:

$$\gamma^{'}\cdot\frac{R-\dfrac{1}{R}\int\limits_0^{\widetilde{p}_0}(1-\beta^f(\gamma',\underline{p}_0,\widetilde{p}_0;\widetilde{D}^f(p)))\dfrac{p\widetilde{D}^f(p)}{\gamma'}dG(p)}{1-G(\widetilde{p}_0)} = \gamma_0 R^s_{01}(\gamma_0). \tag{50}$$

After a bit of algebra, equation (50) can be rewritten as

$$\gamma' = h(\gamma') = \gamma_0\frac{R^s_{01}\left(\gamma_0\right)}{R}(1-G(\widetilde{p}_0)) + \frac{1}{R^2}\int\limits_0^{\widetilde{p}_0}\left[(1-\beta^f(\gamma',\underline{p}_0,\widetilde{p}_0;\widetilde{D}^f(p;\underline{p})))p\widetilde{D}^f(p)\right]dG(p). \tag{51}$$

where $h(\gamma')$ is a continuous function of γ'. Continuity of h follows from continuity of $R^f_{01}\left(\gamma;\underline{p}_0,\widetilde{p}_0\right)$, $R^f_{02}\left(\gamma;\underline{p}_0,\widetilde{p}_0\right)$, and $\beta^f\left(\cdot\right)$ in γ. Note that $\lim\limits_{\gamma'\to 0}h(\gamma') = \gamma_0\cdot\dfrac{R^s_{01}\left(\gamma_0\right)}{R}(1-G(\widetilde{p}_0))$ and thus that $\lim\limits_{\gamma'\to 0}\gamma' R^f_{01}(\gamma';\underline{p}_0,\widetilde{p}_0) = 0 < \gamma R^s_{01}\left(\gamma_0\right)$. By continuity of h, either a fixed point of $\gamma' = h(\gamma')$ exists or, if it does not, then $h\left(1\right)$ is well-defined and greater than 1. We now examine both cases.

Suppose first that a fixed point $\gamma' = h(\gamma')$ exists. Then, contractual payments to ST creditors are the same as they were under restructuring, i.e., $\gamma' R^f_{01}(\gamma';\underline{p}_0,\widetilde{p}_0) = \gamma_0 R_{01}(\gamma_0)$ and it therefore follows that $\widehat{p}^f(\gamma;\underline{p}_0) = \widehat{p}_0$. This, in turn, guarantees that $\widetilde{p}^f(\gamma;\underline{p}_0) = \min\left\{\kappa\cdot\widehat{p}^f(\gamma,\underline{p}_0),\underline{p}_0\right\} = \min\left\{\kappa\cdot\widehat{p}_0,\underline{p}_0\right\} = \widetilde{p}_0$ and

[40] These rates may not be uniquely defined in which case we simply choose the lowest ones at each $\underline{p}$.

[41] The exact way in which payments are allocated when $\underline{p} < \widehat{p}$ – an impossible event in equilibrium but which must be specified here for an exogenous $\underline{p}$ – does not play a role in the equilibrium.

that $R^f_{01}(\gamma';\underline{p}_0,\widetilde{p}_0)=R^f_{01}(\gamma';\underline{p}_0)$ and $R^f_{02}(\gamma';\underline{p}_0,\widetilde{p}_0)=R^f_{02}(\gamma';\underline{p}_0)$, i.e., these interest rates are effectively the ones that arise for a given $\underline{p}_0$ if $\widetilde{p}^f(\gamma;\underline{p}_0)$ is determined endogenously.

Let us define $E^f\left(\gamma';\underline{p}\right)\equiv\gamma R^f_{01}(\gamma';\underline{p})R+(1-\gamma)\,R^f_{02}(\gamma';\underline{p})\underline{p}$ as the country's total expected contractual obligations under reprofiling, for a given debt maturity γ' and an exogenous level of $\underline{p}$.[42] We want to establish two features of $E^f\left(\gamma';\underline{p}\right)$. First, we want to show that expected contractual payments evaluated at $\underline{p}_0$ are lower under reprofiling than under restructuring, i.e.,

$$E^f\left(\gamma';\underline{p}_0\right)=\gamma' R^f_{01}(\gamma';\underline{p}_0)R+(1-\gamma')\,\frac{R^2-\int_0^{\widetilde{p}_0}\beta^f(\gamma',\underline{p}_0;\widetilde{D}^f(p))\dfrac{p\widetilde{D}^f(p)}{1-\gamma'}dG(p)}{\int_{\underline{p}_0}^1 pdG(p)}\underline{p}_0<E^s\left(\gamma';\underline{p}_0\right)\tag{52}$$

where

$$E^s\left(\gamma_0;\underline{p}_0\right)=\gamma_0 R^s_{01}(\gamma_0)R+(1-\gamma_0)\,\frac{R^2-\int_0^{\widehat{p}_0}\beta^s(\gamma_0,p;D^s)\dfrac{pD^s}{1-\gamma_0}dG(p)}{\int_{\underline{p}_0}^1 pdG(p)}\underline{p}_0.\tag{53}$$

Note that in the expression above, the limit of integration in the numerator of the second term is given by $\widehat{p}_0$ instead of by $\widetilde{p}_0$: this is because, under restructuring, LT creditors are paid zero for $p\in(\widehat{p}_0,\widetilde{p}_0]$. Since $\gamma' R^f_{01}(\gamma';\underline{p}_0)R=\gamma_0 R^s_{01}(\gamma_0)$, a necessary and sufficient condition for inequality (52) to hold is that $(1-\gamma')R^f_{02}(\gamma';\underline{p}_0)<(1-\gamma_0)R^s_{02}(\gamma_0)$, i.e.,

$$\gamma'-\gamma_0>\frac{1}{R^2}\left[\int_0^{\widehat{p}_0}\beta^s(\gamma_0,p;D^s)pD^s dG(p)-\int_0^{\widetilde{p}_0}\beta^f(\gamma',\underline{p}_0;\widetilde{D}^f(p))p\widetilde{D}^f(p)dG(p)\right].\tag{54}$$

To see whether this inequality holds, we can use the definition of γ' in equation (45) and, noting that we can write $R^s_{01}(\gamma_0)=\frac{R-R_{01}(G(\widetilde{p})-G(\widehat{p}))-\frac{1}{R}\int_0^{\widehat{p}_0}(1-\beta^s(\gamma_0,p;D^s))\frac{pD^s}{\gamma}dG(p)}{1-G(\widetilde{p}_0)}$, we obtain

$$\begin{aligned}\gamma'-\gamma_0 \;=\;& \frac{1}{R^2}\left[\int_0^{\widehat{p}_0}\beta^s(\gamma_0,p;D^s)pD^s dG(p)-\int_0^{\widetilde{p}_0}\beta^f(\gamma',\underline{p}_0;\widetilde{D}^f(p))p\widetilde{D}^f(p)dG(p)\right]\\ &+\frac{1}{R^2}\left[\int_0^{\widetilde{p}_0}p\widetilde{D}^f(p)dG(p)-\int_0^{\widehat{p}_0}pD^s dG(p)-\int_{\widehat{p}_0}^{\widetilde{p}_0}\gamma_0 R^s_{01}(\gamma_0)RdG\,(p)\right].\end{aligned}$$

Here there are two possibilities. If $\widetilde{D}^f(p)=D^f$ for all $p\le\widetilde{p}_0$, the assumption that reprofiling is payment enhancing guarantees that the last term in brackets is strictly positive, which in turn implies that inequality (54) is satisfied. The other possibility is that there exists $\mathring{p}<\widetilde{p}_0$ such that $\widetilde{D}^f(p)=D^f$ for all $p\le\mathring{p}$ and $\widetilde{D}^f(p)=\gamma R^f_{01}(\gamma';\underline{p})+(1-\gamma)\,R^f_{02}(\gamma';\underline{p})p$ for $p\in(\mathring{p},\widetilde{p}_0)$. In this case, though, for $p\in(\mathring{p},\widetilde{p}_0)$,

$$\gamma R^f_{01}(\gamma';\underline{p})R+(1-\gamma)\,R^f_{02}(\gamma';\underline{p})p<pD^f\le p\alpha\theta_H<\gamma_0 R^s_{01}(\gamma_0)R+(1-\gamma_0)\,R^s_{02}p\tag{55}$$

[42] Note that $R^f_{01}(\gamma';\underline{p})$ and $R^f_{02}(\gamma';\underline{p})$ may now jump when $\underline{p}$ changes, causing E^f to be discontinuous. As will be clear from what follows below, the proof does not rely on continuity of E^f in $\underline{p}$, but on it being an increasing function.

where the last inequality follows because $\widetilde{p}_0 \leq \underline{p}_0$. Since $\gamma R_{01}^f(\gamma';\underline{p}) = \gamma_0 R_{01}^s(\gamma_0)R$, though, expression (55) directly implies that $(1-\gamma)\,R_{02}^f(\gamma';\underline{p}) < (1-\gamma_0)\,R_{02}^s(\gamma_0)$ and thus that the inequality of equation (52) is satisfied.

Second, we want to show that $E^f\left(\gamma';\underline{p}\right) \equiv \gamma R_{01}^f(\gamma';\underline{p}) + (1-\gamma)\,R_{02}^f(\gamma';\underline{p})\underline{p}$ is weakly increasing in $\underline{p}$. Note first that, ceteris paribus, $R_{02}^f(\gamma';\underline{p})$ must weakly decrease as $\underline{p}$ falls: a decrease in $\underline{p}$ either expands the set of states in which LT creditors are paid in full vs. nothing (if $\widetilde{p}^f(\gamma';\underline{p}) < \underline{p}$), or in which they are paid in full vs. partially (if $\widetilde{p}^f(\gamma';\underline{p}) = \underline{p}$ and $D^f < (1-\gamma')R_{02}^f(\gamma';\underline{p})$); in the only other remaining option (i.e., $\widetilde{p}^f(\gamma';\underline{p}) = \underline{p}$ and $D^f \geq (1-\gamma')R_{02}^f(\gamma';\underline{p})$), a fall in $\underline{p}$ has no effect on $R_{02}^f(\gamma';\underline{p})$. Thus, a decrease in $\underline{p}$ weakly raises the share of total payments received by ST creditors below $\widetilde{p}^f(\gamma';\underline{p})$ so that, weakly, $R_{01}^f(\gamma;\underline{p})$ falls as well.

We are now set to show $\underline{p}^f(\gamma') < \underline{p}(\gamma_0)$. Given that $E^f\left(\gamma';\underline{p}\right)$ is increasing in $\underline{p}$, that $E^f\left(\gamma';\underline{p}_0\right) < \underline{p}_0\alpha\theta_H$, and that $E^f\left(\gamma';0\right) > 0$, it follows that there exists a $\underline{p}^f \in \left(0,\underline{p}_0\right)$ for which $E^f\left(\gamma';\underline{p}^f\right) = \underline{p}^f\alpha\theta_H$. At this point we have found a feasible allocation, i.e., $\underline{p}^f = \underline{p}^f\left(\gamma'\right)$ since the contractual rates satisfy creditors' break-even conditions given $\underline{p}^f$. At $\underline{p}^f(\gamma')$, moreover, the reprofiling rule being used is indeed the correct one, i.e., $\widetilde{D}^f(p) = D^f$ for all $p \leq \widetilde{p}^f(\gamma;\underline{p}^f(\gamma'))$. This follows from the fact that

$$\alpha\theta_H = \frac{\gamma R_{01}^f(\gamma;\underline{p}^f(\gamma'))R}{\underline{p}^f(\gamma')} + (1-\gamma)\,R_{02}^f(\gamma;\underline{p}^f(\gamma')),$$

and $D^f \leq \alpha\theta_H$ and $\widetilde{p}^f(\gamma;\underline{p}^f(\gamma')) \leq \underline{p}^f(\gamma')$. Finally, this feasible allocation is incentive compatible because $R_{02}^f(\gamma';\underline{p})$ is increasing in $\underline{p}$ and, as shown above, $(1-\gamma')R_{02}^f(\gamma';\underline{p}_0) < (1-\gamma_0)R_{02}^s(\gamma_0) \leq \pi\theta_H$ where the last inequality follows from the premise that the original equilibrium satisfied the IC constraint.

To complete the proof, we analyze the case for when the fixed point $\gamma' = h(\gamma')$ fails to exist. Then $h(1) > 1$, and setting $\gamma' = 1$ yields $R_{01}^f\left(1;\underline{p}_0,\widetilde{p}_0\right) < \gamma_0 R_{01}^s\left(\gamma_0\right)$. This inequality implies that $\widehat{p}^f(1;\underline{p}_0,\widetilde{p}_0) < \widehat{p}_0$, so that $\widetilde{p}$ must weakly fall once is allowed to adjust endogenously given $\underline{p}_0$, i.e., $\widetilde{p}^f(1;\underline{p}_0) \leq \widetilde{p}_0$. To see why this is, note that if $\widetilde{p}_0 = \kappa\widehat{p}_0$, then dropping the requirement that ST creditors be paid $\widetilde{D}^f$ between 0 and $\widetilde{p}_0$ allows payments to these creditors to weakly increase. This decreases R_{01} and thus $\widehat{p}$ as well until a new equilibrium is reached at a lower $\widehat{p}$ and $\widetilde{p}$, thus obtaining $R_{01}^f\left(1;\underline{p}_0\right) < R_{01}^f\left(1;\underline{p}_0,\widetilde{p}_0\right)$. If instead $\widetilde{p}_0 = \underline{p}_0$, then either $R_{01}^f\left(1;\underline{p}_0\right) = R_{01}^f\left(1;\underline{p}_0,\widetilde{p}_0\right)$ or, if $\frac{\kappa R_{01}^f\left(1;\underline{p}_0,\widetilde{p}_0\right)R}{\alpha\theta^H} < \underline{p}_0$, then once again dropping the requirement that ST creditors be paid $\widetilde{D}^f$ between 0 and $\widetilde{p}_0$ allows payments to these creditors to weakly increase, thus decreasing R_{01}^f further and obtaining $R_{01}^f\left(1;\underline{p}_0\right) < R_{01}^f\left(1;\underline{p}_0,\widetilde{p}_0\right)$. This establishes, in terms of our previous notation, that $E^f\left(1;\underline{p}_0\right) < E^s\left(\gamma_0;\underline{p}_0\right)$.

As before, we now want to show that $E^f\left(1;\underline{p}\right)$ is weakly increasing in $\underline{p}$. As we reduce $\underline{p}$ below $\underline{p}_0$, there are two possibilities. The first is that $\widetilde{p}^f(1;\underline{p}) = \kappa\widehat{p}^f(1;\underline{p})$, in which case $\widetilde{p}^f(1;\underline{p})$ does not move with $\underline{p}$ and $R_{01}^f(1;\underline{p})$ is therefore unaffected. The second is that $\widetilde{p}^f(1;\underline{p}) = \underline{p}$, in which case a decrease in $\underline{p}$ reduces $\widetilde{p}^f(1;\underline{p})$ and this either expands the set of states in which ST creditors are paid in full vs. partially (if $\underline{p}D^f \leq R_{01}^f(1;\underline{p})$) or it does not affect payments to ST creditors (if $\underline{p}D^f > R_{01}^f(1;\underline{p})$): in any case, a fall in $\underline{p}$ leads to a (weak) decline in $R_{01}^f(1;\underline{p})$. This establishes that $E^f\left(1;\underline{p}\right)$ is weakly increasing in $\underline{p}$. Together with $E^f\left(1;0\right) > 0$,

these two properties of $E^f\left(1;\underline{p}\right)$ imply that there exists a $\underline{p}^f \in \left(0,\underline{p}_0\right)$ for which $E^s\left(1;\underline{p}^f\right) = \underline{p}^s\alpha\theta_H$. At this point we have found a feasible equilibrium allocation, i.e., $\underline{p}^f = \underline{p}^f(1) = \tilde{p}^f(1) = \hat{p}^f(1)$ since the contractual rate satisfies the break-even condition of creditors given $\underline{p}^f$. Incentive compatibility follows trivially because the allocation entails only ST debt. ■

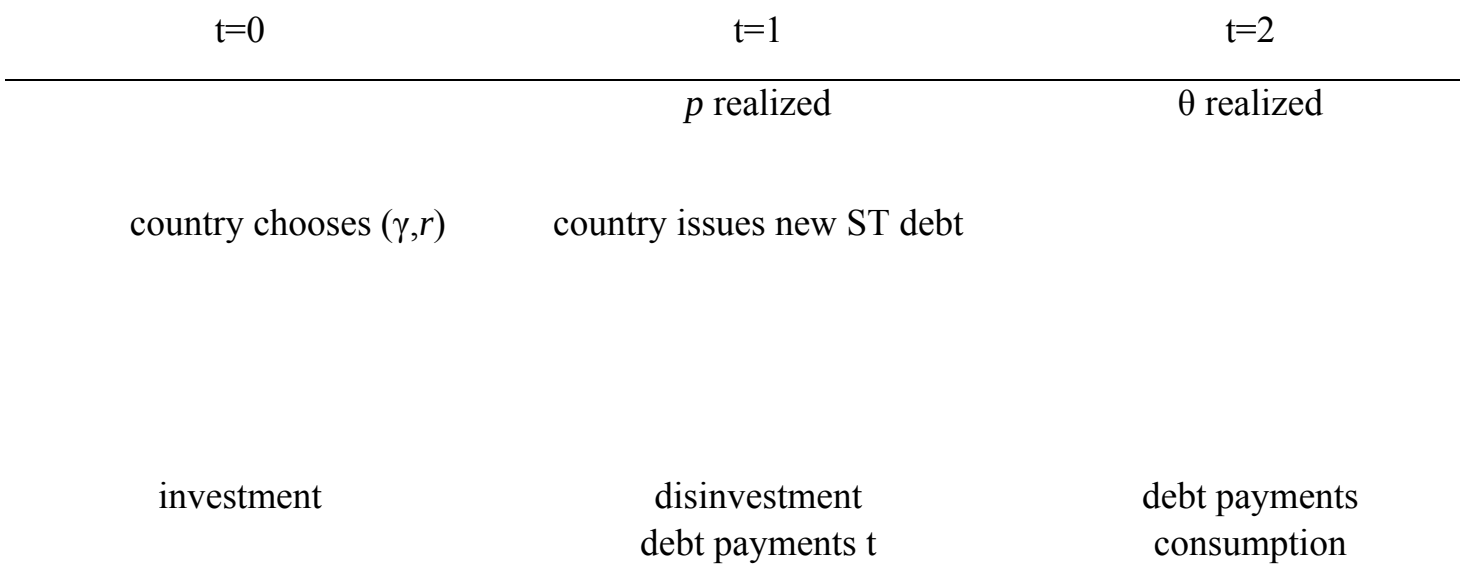

t=0
t=1
t=2
p realized
θ realized
country chooses (γ,r)
country issues new ST debt
investment
disinvestment
debt payments t
debt payments
consumption

Figure 1

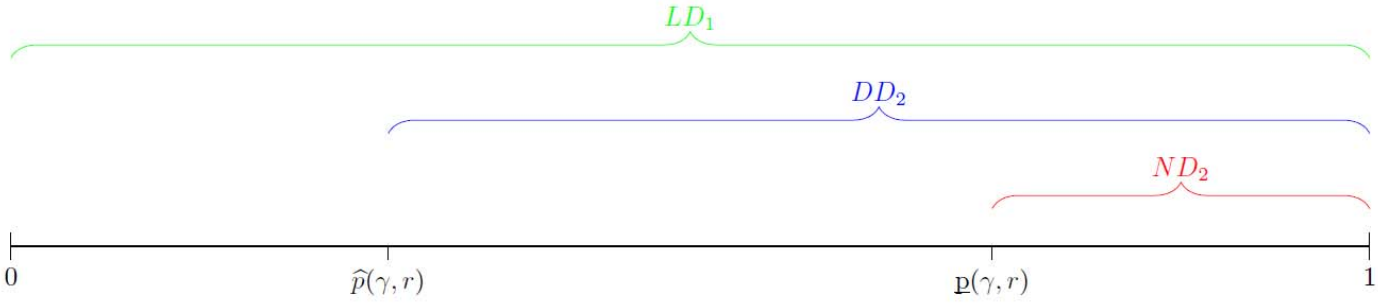


Figure 2: feasible strategies

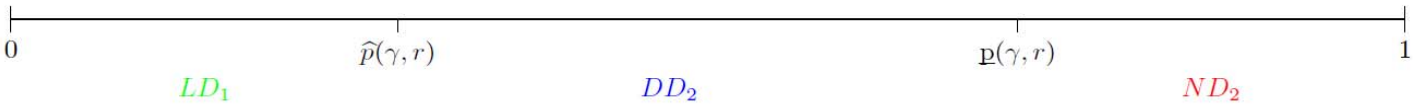


Figure 3: optimal strategies

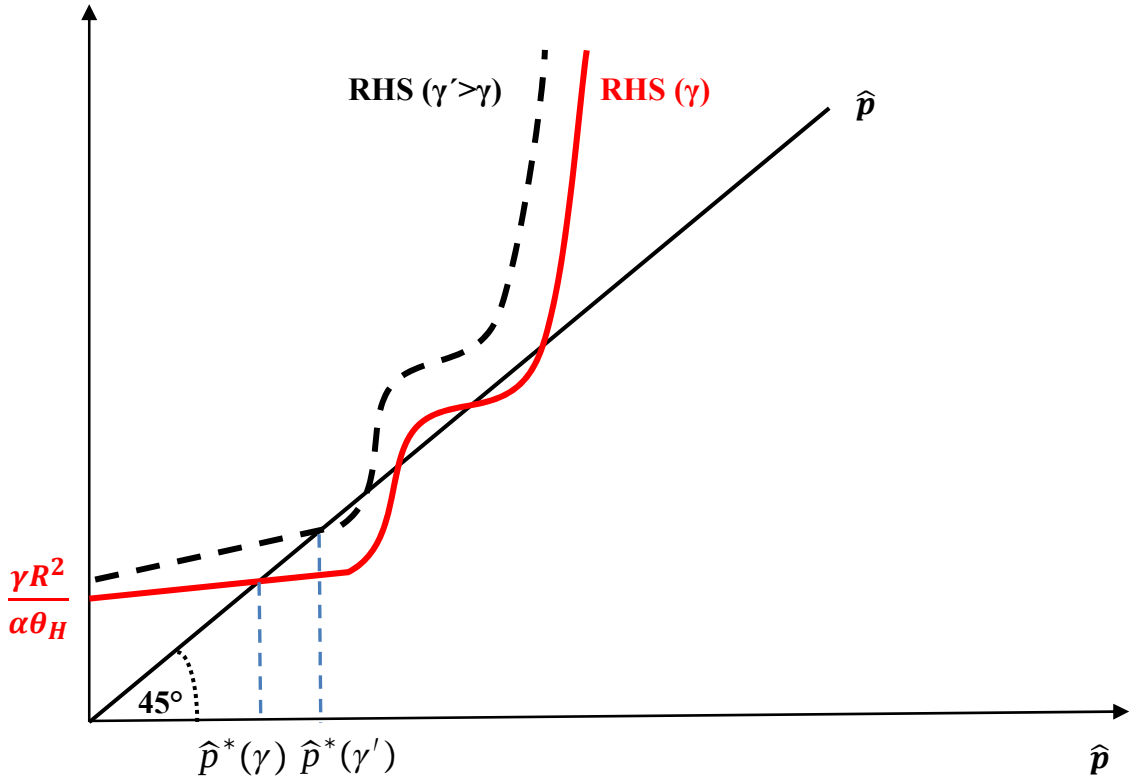


Figure 4

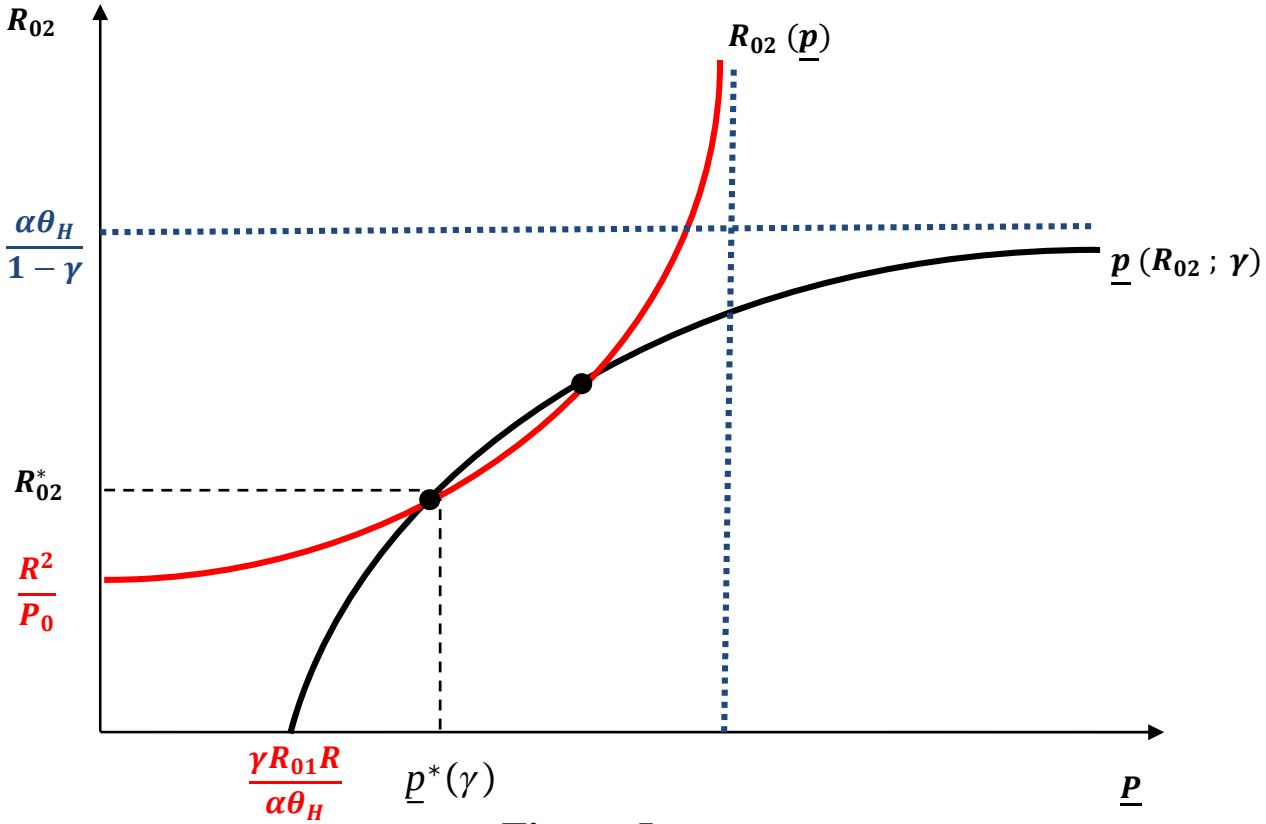


Figure 5

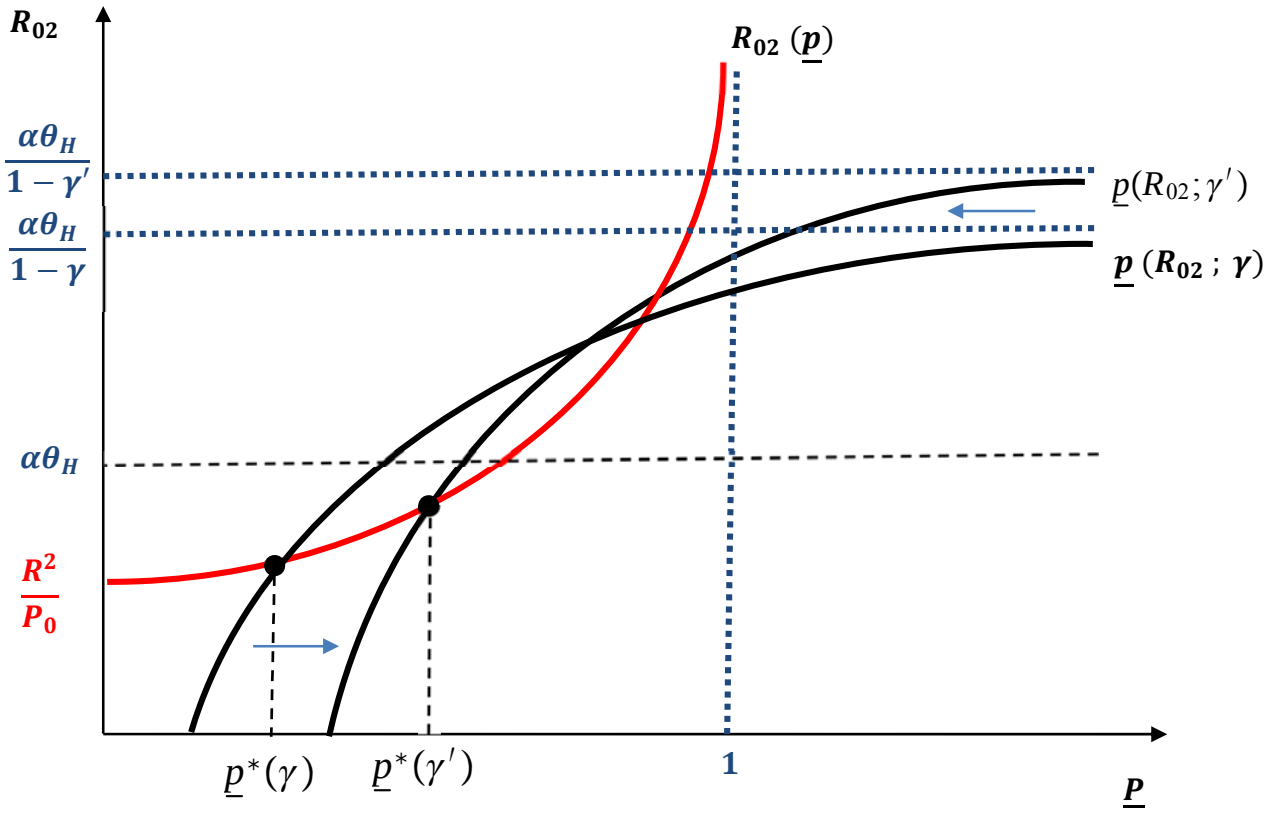


Figure 6A

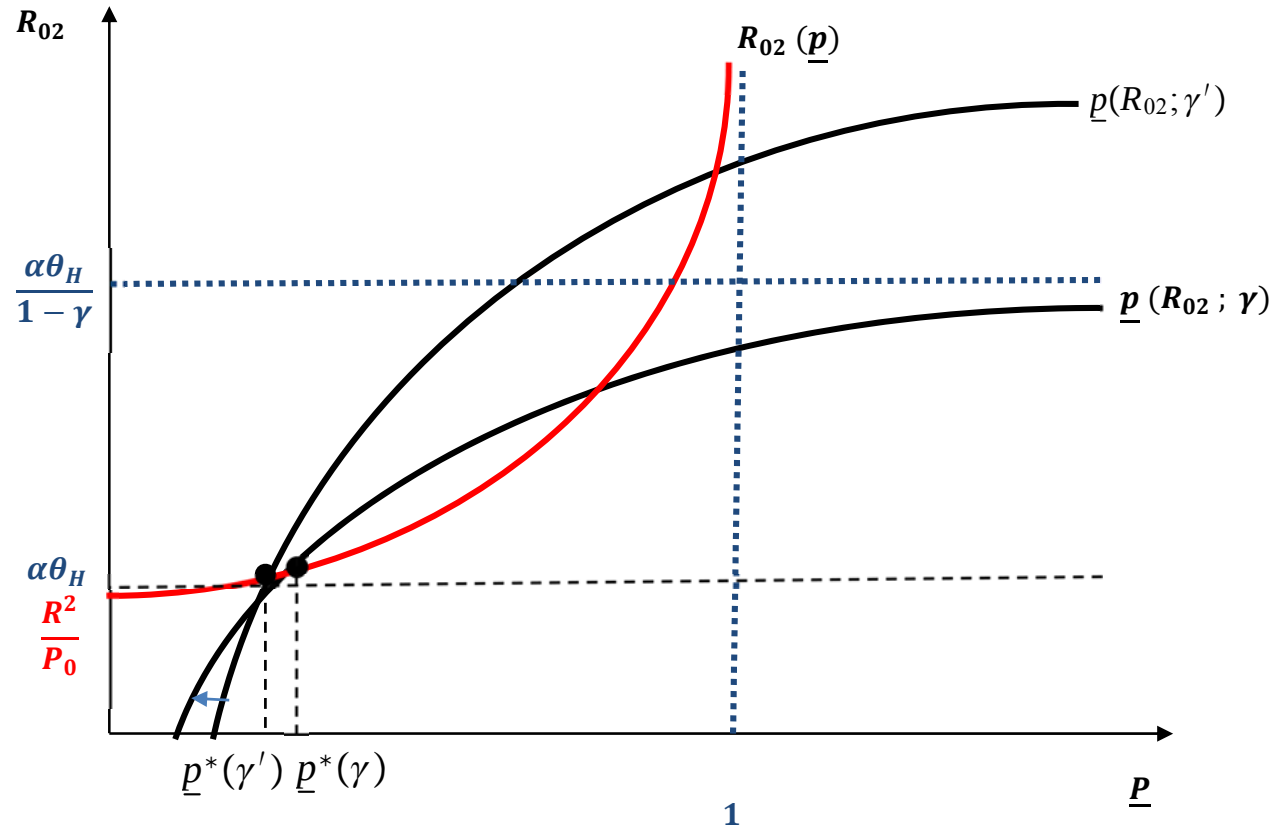


Figure 6B