Sovereign Default: The Role of Expectations*

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Abstract

We study a variation of the standard model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), and show that it is consistent with multiple equilibrium interest rates. Some of those equilibria correspond to the ones identified by Calvo (1988) where default is likely because rates are high, and rates are high because default is likely. The model is used to simulate equilibrium movements in sovereign bond spreads that resemble sovereign debt crisis. It is also

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used to discuss lending policies similar to the ones announced by the European Central Bank in 2012.

Key words: Sovereign default; interest rate spreads; multiple equilibria. JEL Codes: E44, F34.

1 Introduction

This paper is on the origins of sovereign debt crises. Are sovereign debt crises caused by bad fundamentals, alone, or, instead, do expectations play an independent role? The main point of the paper is that, indeed, both fundamentals and expectations can play important roles. High interest rates can be triggered by self confirming expectations, but those high rates are more likely when debt levels are relatively high. The model analyzed can help explain the large and abrupt increases in spreads during sovereign debt crises, particularly in countries that have accumulated large debt levels, as in the recent European experience. It can also justify the policy response by the European Central Bank, to be credited for the equally large and abrupt reduction in sovereign spreads.

The literature on sovereign debt crises is ambiguous on the role of expectations. In a model with rollover risk, Cole and Kehoe (2000) have established that sunspots can play a role, that is strengthened by bad fundamentals. Using a different mechanism, Calvo (1988) also shows that there are both low and high interest rate equilibria. The reason for the multiplicity in Calvo is that, while interest rates may be high because of high default probabilities, it is also the case that high interest rates induce high default probabilities. This gives rise to equilibria with high rates/likely default and low rates/unlikely default. In contrast with the results in those models, in the standard quantitative model of sovereign default as in Aguiar and Gopinath (2006) or Arellano (2008) there is a single low interest rate equilibrium.

In this paper, we take the model of Aguiar and Gopinath (2006) and

Arellano (2008), that build on Eaton and Gersovitz (1988) and make minor changes in the modelling choices concerning the timing of moves by debtors and creditors, and the actions that they may take. In so doing, we are able to produce expectation-driven movements in interest rates. The reason for those movements is the one identified by Calvo (1988), and more recently analyzed in related, independent work by Lorenzoni and Werning (2013). The change in the modelling choices is minor since direct evidence cannot be used to discriminate across them. On the other hand, there is ample indirect evidence provided by large and abrupt movements in spreads, apparently unrelated to fundamentals, during sovereign debt crises.

Our theoretical exploration of self-fulfilling equilibria in interest rate spreads is motivated by two particular episodes of sovereign debt crises. The first is the Argentine crisis of 1998-2002. Back in 1993, Argentina had regained access to international capital markets. The debt to GDP ratio, was roughly between 35% and 45% during the period, very low by international standards. The average yearly growth rate of GDP was around 5%. But the average country risk spread on dollar denominated bonds for the period 1993-1999, relative to the US bond, was 7%. Notice that a 7% spread on a 35% debt to GDP ratio amounts to almost 2.5% of GDP on extra interest payments per year.[1] Accumulated over the 1993-1999 period, this is 15% of GDP, almost half the debt to GDP ratio of Argentina in 1993. An obvious question arises: Had Argentina faced lower interest rates, would it have defaulted in 2002?

The second episode is the recent European sovereign debt crisis that started in 2010 and receded substantially since the policy announcements by the European Central Bank (ECB) in September 2012. The spreads on Italian and Spanish public debt, very close to zero since the introduction of the euro and until April 2009, were higher than 5% by the summer of 2012, when the ECB announced the program of Outright Monetary Transactions

[1] This calculation unrealistically assumes one period maturity bonds only. Its purpose is just to illustrate the point in a simple way.

(OMTs). They were considerably higher in Portugal, and specially in Ireland and Greece. With the announcement of the OMTs, according to which the central bank stands ready to purchase euro area sovereign debt in secondary markets, the spreads in most of those countries slid down to less than 2%, even though the ECB did not actually intervene. The potential self-fulfilling nature of the events leading to the high spreads of the summer 2012 was explicitly used by the president of the ECB to justify the policy.

The model in this paper is of a small open economy with a random endowment, very similar to the structure in Aguiar and Gopinath (2006) or Arellano (2008). A representative agent can borrow noncontingent and cannot commit to repay. There is a penalty for defaulting. The borrower faces atomistic foreign creditors, each of them with limited funds. They are risk neutral, so that the average return from lending to this economy taking into account the probability of default has to be equal to the risk-free international rate of interest. The timing and action assumptions are the following: In the beginning of the period, given the level of debt gross of interest and the realization of the endowment, the borrower decides whether to default. If there is default, the endowment is forever low. Otherwise, creditors move first and offer their limited funds at some interest rate. The borrower moves next and borrows from the low rate creditors up to some total optimal debt level. In equilibrium the creditors all charge the same rate, which is the one associated with the probability of default for the optimal level of debt chosen by the country. With these timing assumptions, there are multiple interest rate equilibria. High interest rates can generate high default rates which in turn justify high interest rates. In equilibria such as these, there is a sense in which interest rates can be "too high".

With this timing, when deciding how much to borrow, the borrower takes the interest rate as given. This does not mean that the borrower behaves as a small agent. Even if it takes current prices as given, it still takes into account the effects of its current choices on future prices. The borrower is

just not benefiting from a first mover advantage.[2]

The timing assumptions in Aguiar and Gopinath (2006) and Arellano (2008) are such that the borrower moves first, before the creditors. They also assume the borrower chooses the debt level at maturity including interest payments. Creditors move next and respond with a schedule that specifies a single interest rate for each level of debt gross of interest. By moving first and choosing the debt at maturity, gross of interest, the borrower is able to select a point in the schedule, therefore pinning down the interest rate. It follows that there is a single equilibrium. The first mover advantage allows the borrower to coordinate the creditors actions on the low interest rate equilibrium.

An alternative structure has the same sequence of moves, except that the borrower chooses current debt instead of debt at maturity. The interest rate schedule will then be a function of current debt, rather than debt at maturity. In this case, there will in general be multiple schedules.[3] Given current debt, if the interest rate is high, so is debt at maturity, and therefore the probability of default is also high. This is the spirit of the analysis in Calvo (1988).

Current debt in Calvo (1988) is exogenous, but debt at maturity is not since it depends on the endogeneously determined interest rate. If the borrower were to choose debt at maturity, given current debt, the interest rate would be pinned down, and, again, there would be a single equilibrium. Lorenzoni and Werning (2013) analyze a dynamic version of Calvo's model with exogenous public deficits, and argue against the possibility of the gov-

[2] A similar timing assumption in Bassetto (2005), also generates multiple Laffer curve equilibria. In Bassetto, if the government were to move first and pick the tax, there would be a single low tax equilibrium. Instead, if households move first and supply labor, there is also a high tax equilibrium. Bassetto convincingly argues that the assumption that the government is a large agent is unrelated to the timing of the moves.

[3] In Eaton and Gersovitz (1981), even if that is the assumption on the actions of the country, they dismiss the multiplicity by assumption (this is discussed in Section 2.3).

ernment choosing debt at maturity. For that, they build a game with an infinite number of subperiods, and assume that the government cannot commit not to reissue debt in those subperiods. As a result, the government is unable to select a point on the interest rate schedule.

As mentioned above, the reason for expectation-driven high interest rate equilibria in these models is different from the one in Cole and Kehoe (2000). Still, in that set up it is the timing of moves that is crucial to generate multiplicity. In Cole and Kehoe, there is multiplicity when the choice of how much debt to issue takes place before the decision to default. In that case, it may be individually optimal for the creditors not to roll over the debt, which amounts to charging arbitrarily high interest rates. This may induce default, confirming the high interest rates. In our model there is no rollover risk because the decision of default is at the beginning of the period. Still, a similar timing assumption to the one in Cole and Kehoe generates the multiplicity. As creditors move first, it can be individually optimal to ask for high rates. That will induce a high probability of default, confirming the high rates.

Most of the theoretical analysis in this paper is done in a two period version of the model where the intuitions are very clear. We discuss the relevance of the alternative timing and action assumptions. The model is first solved with our preferred timing in which the borrower behaves as a price taker. The solution can be derived very simply using a demand curve of debt by the borrower and a supply curve of funds by the creditors. In general there are multiple intersections of the demand and the supply curve. These are all potential equilibria, but some are more compelling than others.

For standard distributions of the endowment, the high rate equilibria have properties that make them fragile to reasonable refinements. Those high rates can be in parts of the supply curve in which the rates decrease with an increase in the level of debt. If that is the case, then the total gross service of the debt also decreases with an increase in the level of debt. For

those high rates, creditors also jointly benefit from lowering interest rates, because of their effect on probabilities of default. These are all features of the high rate equilibria in Calvo (1988). But as we show, multiplicity does not disappear even if those equilibria are refined away. To show this, we consider bimodal distributions for the endowment, with good and bad times. With those distributions, there are low and high rate equilibria, equally robust, for the same level of debt. The set of equilibria has the feature that for low levels of debt there is only one equilibrium. Interest rates are low and increase slowly with the level of debt. As debt becomes relatively high, then there are both low and high rate equilibria. For even higher levels of debt, there is a single high rate equilibrium, until eventually there is none. As we explain in detail in the paper, we associate these binomial distributions as reflecting the likelihood of relatively long periods of stagnation, as currently discussed in Europe, in a way that resembles the Markov-switching processes for output popularized by Hamilton (1989).[4]

In the region where the interest rates are unnecessarily high, policy can be effective in selecting a low rate equilibrium. A large lender can accomplish the missing coordination, by lending up to a maximum amount at a penalty rate. In equilibrium only private creditors would be lending. This may help understand the role of policies such as the OMTs introduced by the ECB, following the announcement by its president that it would do "whatever it takes" to avoid a sovereign debt crisis in the euro area.

The paper also includes a quantitative section with a dynamic model in which a sunspot variable is introduced that triggers coordination on high or low interest rates. To stay closer to the quantitative literature, and also for simplicity in the computations, we consider the standard timing in the literature that has the borrower moves first and faces an interest rate schedule. In order to have multiplicity, the schedules are in terms of debt net of interest. The model is shown to be consistent with a sovereign debt crisis unraveling,

[4]See also the evidence in Jones and Olken (2008) for an international perspective.

in particular when debt is relatively large - as it is the case in Europe now - and the probability of a relatively long period of stagnation is high.

As mentioned, this paper is closely related in its motivation to Lorenzoni and Werning (2013), but there are some very important differences.[5] First, Lorenzoni and Werning consider long maturity debt, and focus the analysis on equilibria with debt dilution, while we do not. In our set up, the multiplicity is closer to the one analyzed by Calvo (1988) - it arises with only short term debt. We emphasize the role of large debt levels and the plausibility of long periods of stagnation as possible drivers of the multiplicity. Second, they study a model where fiscal policy is exogenous. We instead characterize equilibria with optimal debt choices. In our model, the possibility of a long stagnation is key, so we view our results as illustrative of possibilities over low frequencies, and it seems natural to consider aggregate fiscal policy as endogenous in that case. Our choice has the advantage that we can discuss the role of the endogenous decision to borrow on the likelihood and characteristics of debt crisis. This "disciplinary effect of crisis" is discussed in our quantitative section. From the theoretical point of view, we only analyze a two-period model and focus on the importance of timing and action assumptions for multiple interest rate equilibria to arise. By exposing the importance of these assumptions we argue for the empirical relevance of that multiplicity. Along similar lines, Lorenzoni and Werning analyze games that also provide support for multiplicity.

2 A two period model

It is useful to analyze first the case of a simple two period model, where analytical results can be derived and some of the features of the model can be seen clearly. In particular it is easier to understand in the two period model what drives the multiplicity of spreads and default probabilities that

[5] We started working independently. Their first version precedes ours.

resembles the result in Calvo (1988).

We analyze a two-period, endowment economy populated by a representative agent that draws utility from consumption in each period and by a continuum of risk neutral foreign creditors. Each creditor has limited capacity, but there are enough of them so that there is no constraint on the aggregate credit capacity. The period utility function of the representative agent, U, is assumed to be strictly increasing, strictly concave and to satisfy standard Inada conditions. The endowment is assumed to be equal to 1 in the first period. That is the lower bound of the support of the distribution of the endowment in the second period. Indeed, uncertainty regarding future outcomes is described by a stochastic endowment $y \in [1, Y]$, with density $f(y)$ and corresponding cdf $F(y)$. The outstanding initial level of debt is assumed to be zero, and, in period one, the representative agent can borrow b in a noncontingent bond in international financial markets. The risk neutral gross international interest rate is R^*. In period two, after observing the realization of the shock, the borrower decides either to pay the debt gross of interest, Rb, or default. If there is default, consumption is equal to the lower bound of the endowment process, 1. Note that there may be contingencies under which the borrower chooses to default, the interest rate charged by foreign creditors, R, may differ from the risk free rate R^*.

The timing of moves is the following: In the first period each creditor $i \in [0, 1]$ offers the limited funds at gross interest rate R_i. The borrower moves next and picks the level of debt $b = \int_0^1 b_i di$, where b_i is how much is borrowed from each creditor. The borrower's best response is to borrow from the low interest rate lenders first. Since each creditor has limited funds, neither of them can individually affect the probability of default, so in order to make zero profits in equilibrium the interest rates they charge will have to be the same, $R_i = R$. Without loss of generality we focus on symmetric outcomes where if $R_i = R_j$, then $b_i = b_j$. Then, $b_i = b$ for all $i \in [0, 1]$, so $\int_0^1 b_i R_i di = a = Rb$.

In the second period, the borrower decides whether to default fully or to pay the debt in full. The borrower decides to default if and only if $U\left(y - \int_0^1 b_i R_i di\right) \leq U(1)$, or

$$y \leq 1 + \int_0^1 b_i R_i di.$$

In order for creditors to make zero profits in equilibrium the interest rates they charge will have to be the same, $R_i = R$. Assuming the country borrows the same amount from each creditor, default happens whenever

$$y \leq 1 + bR,$$

which defines a default threshold for output. The probability of default is then $F[1 + bR]$.

Since creditors are risk neutral, the expected return of lending to the borrower in this economy must be the same as R^*, so

$$R^* = R[1 - F(1 + bR)]. \tag{1}$$

This defines a locus of points (b, R) such that each point solves the problem of the creditors, which can be interpreted as a supply curve of funds. The mapping from debt levels to interest rates is a correspondence since, in general, for each b there are multiple Rs that satisfy equation (1). Multiple functions can be built with the points of the correspondence. We call those functions interest rate schedules.

The optimal choice of debt by the borrower is the one that maximizes utility

$$U(1 + b) + \beta \left[F(1 + bR)U(1) + \int_{1+bR}^{Y} U(y - bR) f(y) dy \right]. \tag{2}$$

subject also to an upper bound restriction on the maximum level of debt.

Absent this condition, the optimal choice would be to borrow an arbitrarily large amount and default with probability one. The supply of debt would be zero in equilibrium.

The marginal condition, for an interior solution, is

$$U'(1+b) = R\beta \int_{1+bR}^{Y} U'(y-bR) f(y)\, dy. \tag{3}$$

The optimal choice of debt for a given interest rate defines a locus of points (b, R) that can be interpreted as a demand curve for funds. The possible equilibria will be the points where the demand curve intersects the supply curve above described by (1).

An equilibrium in this economy can then be defined as:

Definition 1 An equilibrium is an interest rate $\widetilde{R}$ and a debt level $\widetilde{b}$ such that: given $\widetilde{R}$, $\widetilde{b}$ maximizes (2); and (ii) the arbitrage condition (1) is satisfied.

2.1 Multiple equilibria

As mentioned above, there are in general multiple equilibria in this model, low rate equilibria and high rate equilibria, that resemble the multiple equilibria in Calvo (1988).

The supply curve defined implicitly by (1) is analyzed now. For that purpose, it is useful to define the function for the expected return on the debt,

$$h(R; b) = R\left[1 - F(1+bR)\right],$$

that must be equal to the riskless rate, R^*. For $R = 0$, $h(0; b) = 0$. If the distribution of the endowment has a bounded support, for R high enough, if $1 + bR \geq Y$, then $h(R; b) = 0$. For standard distributions, the function $h(R; b)$ is concave, so that there are at most two solutions of $R^* = h(R; b)$.

In the case of the uniform distribution it is straightforward to obtain the solutions of $R^* = h(R;b)$, so that the supply curve can be described analytically. Let the distribution of the endowment process be the uniform, $f(y) = \frac{1}{Y-1}$, so that $F(y) = \frac{y-1}{Y-1}$. Then, from (1), the equilibrium interest rates must satisfy

$$R = \frac{1 \pm \left(1 - 4\frac{R^* b}{Y-1}\right)^{\frac{1}{2}}}{2\frac{b}{Y-1}},$$

provided $1 - 4\frac{R^* b}{Y-1} \geq 0$. The maximum level of debt consistent with an equilibrium with borrowing is given by $b^{\max} = \frac{Y-1}{4R^*}$. Below this value of debt, for each b, there are two possible levels of the interest rate.

In Figure 1, the curve $h(R;b)$ is depicted against R, where F is the cumulative normal.[6] An increase in b shifts the curve h, downwards, so that the solutions for b are closer to each other. The second derivative of $h(R;b)$ is negative when $2f(1+bR) \geq -f'(1+bR)bR$. The function $h(R;b)$ does not have to be everywhere concave. This depends on the cumulative distribution $F(1+bR)$.[7] We discuss below conditions for the non concavity of the function $h(R;b)$.

Figure 2 plots the solutions for R of equation (1) for each level of debt, also for the normal distribution.

The supply curve of Figure 2 has two monotonic schedules. For lower values of the interest rate, there is a flat schedule that is increasing in b. There is also a steeper decreasing schedule for higher values of the interest rate.

The equilibrium must also be on a demand curve for the borrower, obtained from the solution of the problem defined in (2). Figure 3 depicts the two curves, the supply and the demand curves.

The points on the decreasing schedule have particularly striking proper-

[6] The black vertical dotted lines are grid lines. We kept them in the plots throughout the paper to make the exposition clearer.

[7] In the appendix we further characterize conditions for concavity and study several commonly used distributions.

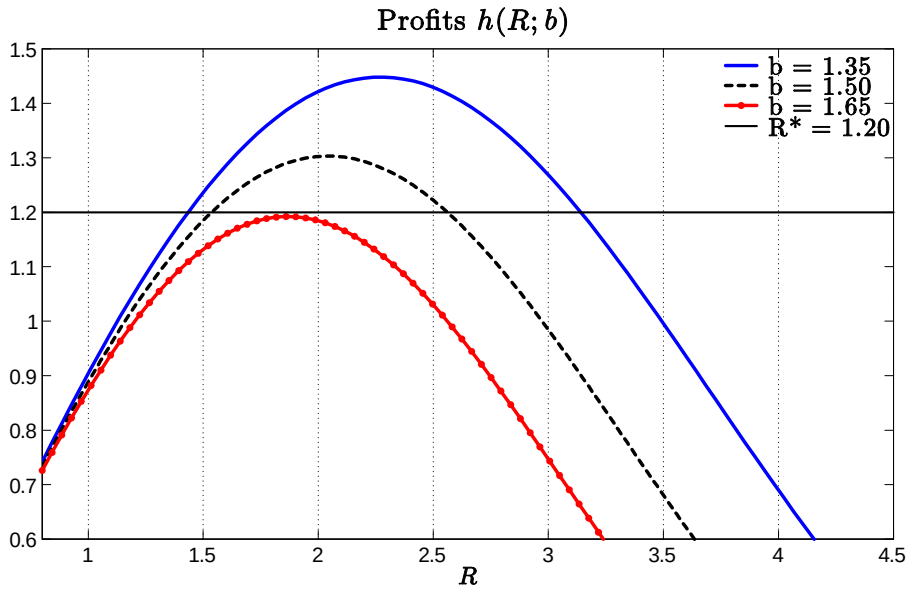


Figure 1: Expected return $h(R;b)$

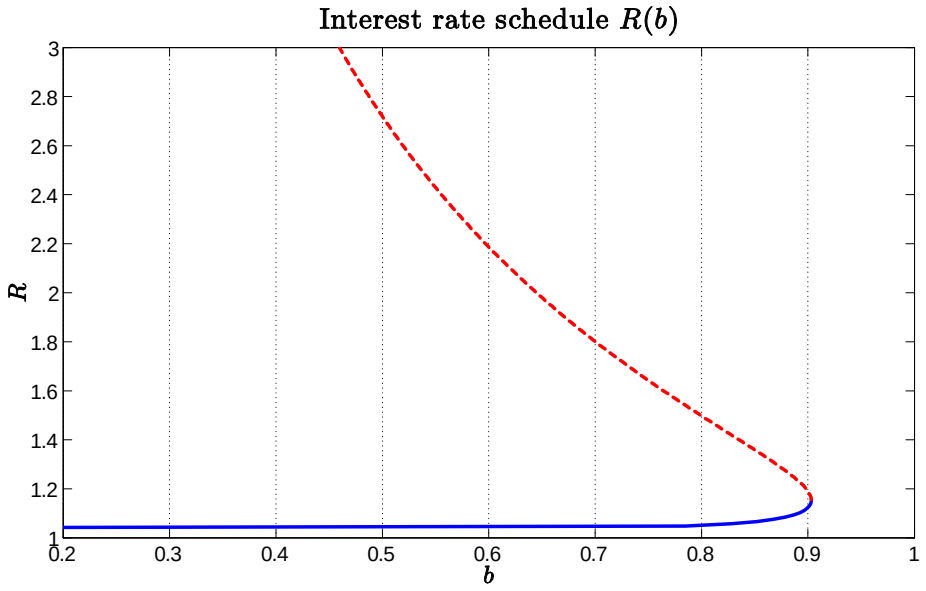


Figure 2: Interest rate schedules

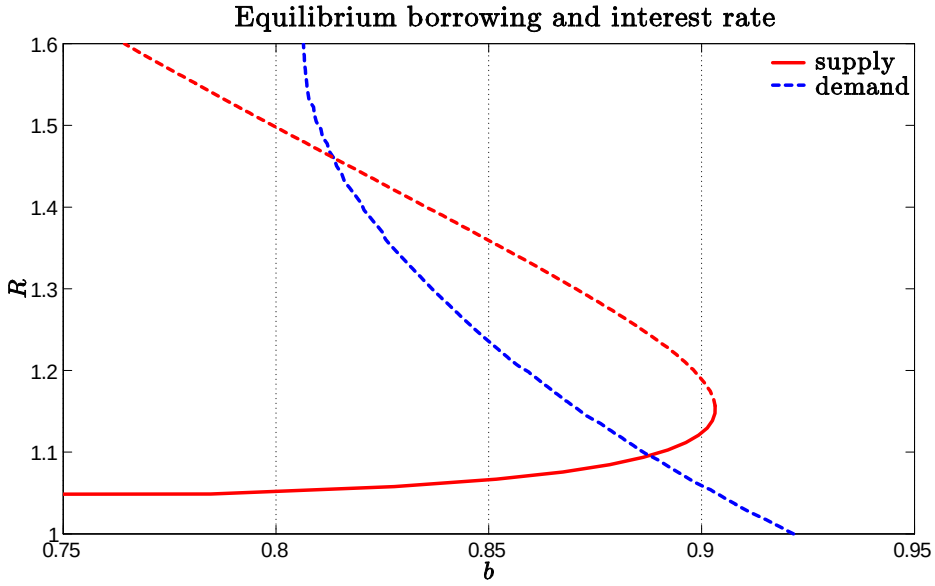


Figure 3: Supply and demand curves

ties. For those points in the supply curve, not only does the interest rate go down with the level of debt, b, but the gross service of the debt, Rb, also decreases with the level of debt, b. To see this, notice that from (1), R increases in the level of Rb. The points on the decreasing schedule are fragile as candidates for equilibria in the following sense. Consider a perturbation of a point $(\widehat{R}, \widehat{b})$ in that schedule, that consists of the same value for the interest rate, but a slightly lower value for debt $(\widehat{R}, \widehat{b} - \varepsilon)$. This point would lie below the schedule. At the point $(\widehat{R}, \widehat{b} - \varepsilon)$, the interest rate is the same as in $(\widehat{R}, \widehat{b})$, but the debt lower, so the probability of default is also lower. Thus, profits for the creditors are higher than at $(\widehat{R}, \widehat{b})$, where profits are zero. This means that a small reduction in the interest rate is beneficial for the borrower and for the lender, which suggest that these equilibria may not survive reasonable refinements. In Appendix 1 we perturb the extensive form game in the first period and offer a refinement that, indeed, rules out

equilibria on the decreasing schedules.[8] One could then hope that, subject to this refinement, the equilibrium would therefore be unique. As we now show, such hopes are not realized.

2.1.1 A distribution with good and bad times

Equation (1) may have more than two solutions for R, for a given b, depending on the distribution of the endowment process.[9] One case in which there can be multiple increasing schedules is when the distribution combines two normal distributions, a distribution for good times and a distribution for bad times.

Consider two independent random variables, y^1 and y^2, both normal with different mean, μ^1 and μ^2, respectively, and the same standard deviation, σ. Now, let the endowment in the second period, y, be equal to y^1 with probability p, and equal to y^2 with probability $1-p$.

If the two means, μ^1 and μ^2, are sufficiently apart, then (1) has four solutions, for some values of the debt, as Figure 4 shows. The correspondence between levels of debt and R, as solutions to the arbitrage equation above, is plotted in Figure 4, in which $p = 0.8$, $\mu^1 = 5.5$, $\mu^2 = 4.2$, $\sigma = 0.1$.[10] Clearly, there are debt levels for which there are only two solutions, so there is only one increasing schedule. But for intermediate levels of debt, the equation has four solutions and therefore multiple increasing schedules. This means that, even if one is restricted not to consider equilibria on decreasing schedules, the model may still exhibit multiplicity. Notice that the multiplicity on the

[8] There are two important assumptions, as we explain in detail in the appendix: First, there must exist a minimal degree of coordination, which, for some equilibria in the decreasing schedule, may be large. Second, the fist period auction must be anonymous, in the sense that ex-ante differences that arise because of the perturbation cannot be observed by the borrower.

[9] In appendix 5.1, sufficient conditions are provided on the density so there are only two solutions. Conditions under which more than two solutions are likely to arise are also described.

[10] The relatively high probability and the average severity of a disaster can be thought of as the relatively frequent, long periods of stagnation.

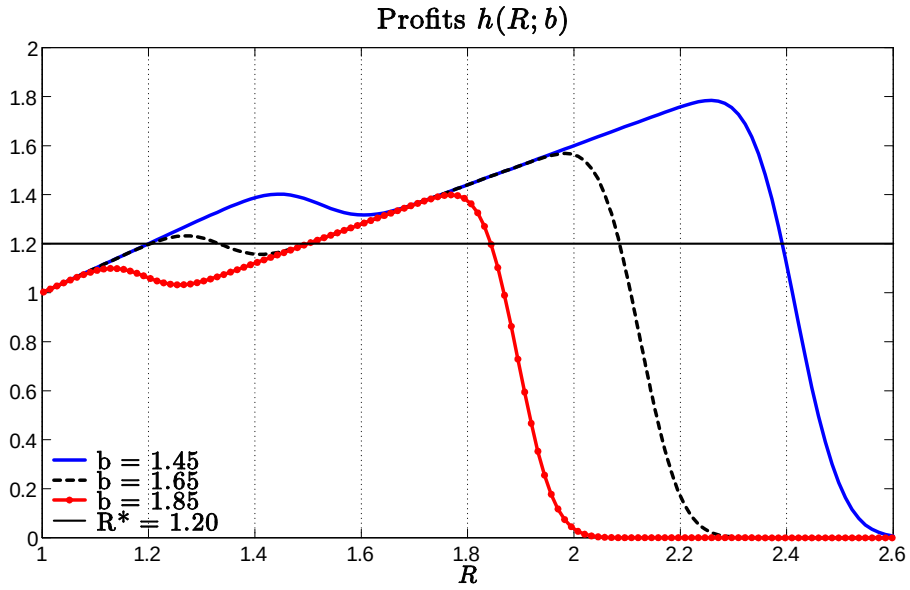


Figure 4: Expected return for the bimodal distribution $h\left(R;b\right)$

increasing schedules arises for relatively high levels of debt.

The supply curve for this case of the bimodal distribution is the red-dotted line depicted in Figure 5, below. The demand is the blue-filled line in the same figure. Notice that multiplicity only arises if the demand curve is high enough, so the resulting equilibrium level of debt is high. The demand is discontinuous in this case, since the maximum problem in (2) has two interior local maxima, due to the bimodal distribution. As the interest rate changes, the relative value of utility between the two local maxima changes. The details are in an appendix available upon request.[11]

If the debt level is relatively large, multiple equilibria are more likely to arise. This is the case with the bimodal distribution analyzed above, but it is particularly so when the value of the debt is close to the maximum

[11] Note that theoretically, it would be possible to have multiplicity with low levels of debt, due just to the discountinuity of the demand. This never happened in our simualtions, so we ignore it.

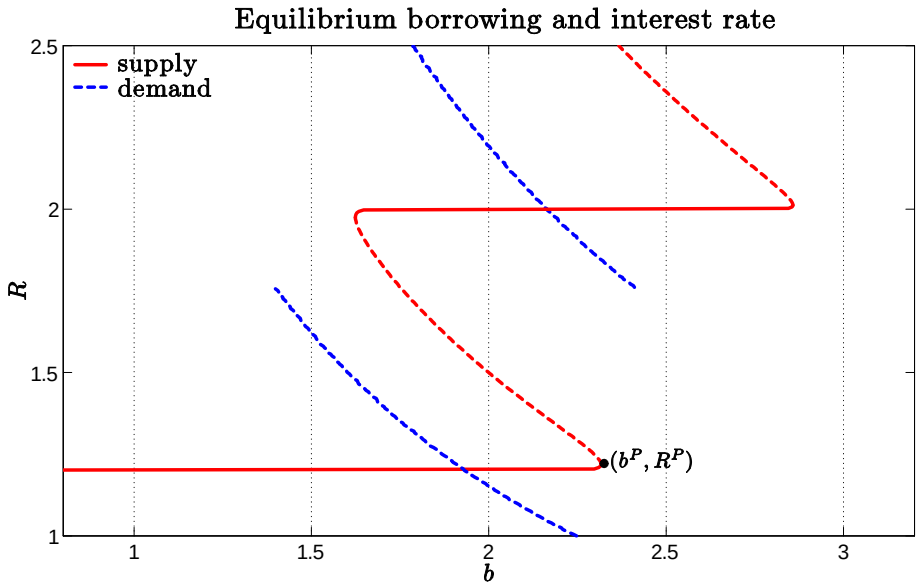


Figure 5: Supply and demand for the bimodal distribution

and any single mode distribution is perturbed by adding a non-monotonic transformation. The details are in Appendix 2.

2.2 Policy

To illustrate the effects of policy, the case of the bimodal distribution depicted in Figure 5 is considered. The extensions to other cases are straightforward.

Consider there is a new agent, a foreign creditor that can act as a large lender, with deep pockets.[12] This large lender can offer to lend to the country, at a policy rate R^P, any amount lower than or equal to a maximum level b^P. It follows that there cannot be an equilibrium with an interest rate larger than R^P.

Now, let us imagine that b^P and R^P are the debt level and interest rate corresponding to the maximum point of the low (solid line) increasing sched-

[12] If the borrower was a small agent, rather than a sovereign, any creditor could possibly play this role.

ule in Figure 5. In this case, the only equilibrium is the point corresponding to the intersection of demand and supply on the low interest rate, increasing schedule. In addition, the amount borrowed from the large lender is zero. The reason the equilibrium interest rate is lower than the one offered by the large lender is that at that interest rate R^P and for debt levels strictly below b^P, there would be profits.

Notice that the large lender cannot offer to lend any quantity at the penalty rate. Whatever is the rate, the level of lending offered has to be limited by the points on the supply curve, otherwise, the borrower may borrow a very high amount and then default.

2.3 Current debt versus debt at maturity

The borrower in the model analyzed above chooses current debt. Would it make a difference if the borrower were to choose debt at maturity, gross of interest? We now consider an alternative game in which the timing of the moves is as before, but now the borrower chooses the value of debt at maturity that we denote by a, rather than the amount borrowed, b. Are there still multiple equilibria in this set up? The answer is yes. With this timing of moves, there are multiple interest rate equilibria whether the government chooses the amount borrowed b, or the amount paid back a. This is a relevant question, because in the models of Calvo (1988) and Arellano (2008) it is that assumption, of whether the borrower chooses b or a, that is key to have uniqueness or multiplicity of equilibria, as will be discussed later.[13]

Again, here, the creditors move first and offer the limited funds at gross interest rate R_i, $i \in [0, 1]$. The borrower moves next and picks the level of debt at maturity $a = \int_0^1 a_i di$. As before, the rate charged by each creditor will have to be the same in equilibrium. In the second period, the borrower defaults if and only if $y \leq 1 + a$. Arbitrage in international capital markets

[13] The key for the different results is the timing assumption, as clarified in section 3.

implies that

$$R^* = R\left[1 - F\left(1 + a\right)\right]. \tag{4}$$

The locus of points (a, R) defined by (4), that we interpret as a supply curve of funds, is monotonically increasing (which is not the case for the supply curve in b and R defined in (1)).

The utility of the borrower is

$$U(1 + \frac{a}{R}) + \beta \left[F(1 + a)U(1) + \int_{1+a}^{Y} U(y - a)f(y)dy\right]. \tag{5}$$

where $\frac{1}{R}$ is the price of one unit of a as of the first period. The marginal condition is

$$U'(1 + \frac{a}{R}) = R\beta \int_{1+a}^{Y} U'(y - a)f\left(y\right)dy. \tag{6}$$

The locus of points (a, R) defined by the solution of this maximization problem can be interpreted as a demand curve for funds. There are again multiple intersection points of this demand curve with the supply curve. Provided the choice of a is interior, those points are the solutions of the system of two equations, (4) and (6), but those are the exact same two equations (1) and (3) that determine the equilibrium outcomes for R and b for $a = Rb$.

Figure 6 plots the supply curves for (b, R) and (a, R) defined in (1) and (4), respectively, for the normal distribution. It also plots the demand curves defined in (6) and (3), for the logarithmic utility function. With the timing assumed so far, whether the borrower chooses debt net or gross of interest is inessential.

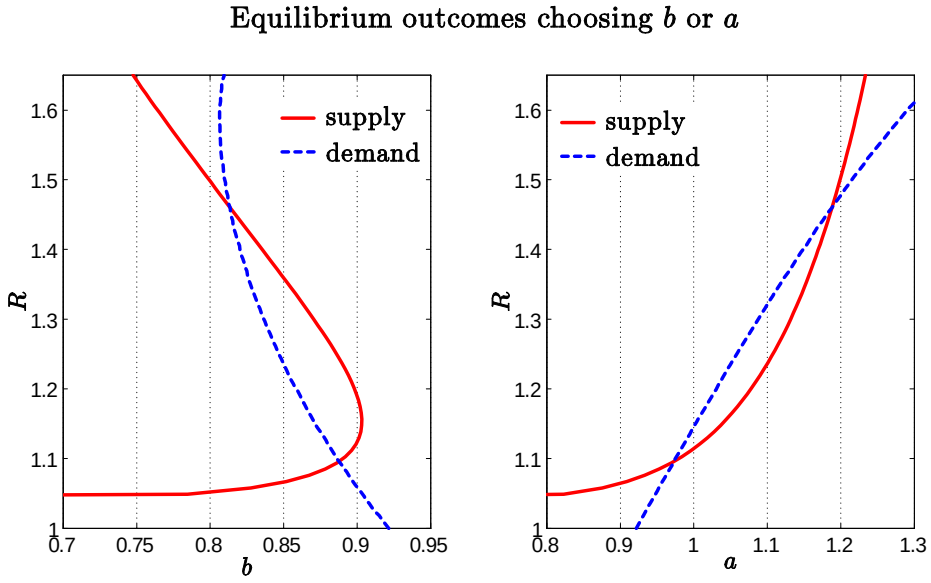


Figure 6: Choosing value of debt at maturity a or amount borrowed b

3 Timing of moves and multiplicity: Related literature

The timing of moves assumed above, with the creditors moving first, amounts to assuming that the borrower in this two period game takes the current price of debt as given.[14] The more common assumption in the literature is that the borrower moves first, choosing debt levels b or a, facing a schedule of interest rates as a function of those levels of debt, $R = R(b)$ or $R = \frac{1}{q(a)}$, depending on whether the choice is on b or a, respectively.

Suppose the schedule the borrower faces is $q(a)$ corresponding to the supply curve derived from (4) and depicted in the right hand panel of Figure 6. This is a monotonically increasing function. Since the borrower can choose a it is always going to choose in the low R/low a part of the schedule. The

[14] In the dynamic game the contemporaneous price is taken as given, but not so the future prices.

borrower is also going to take into account the monopoly power in choosing the level of a. These are the assumptions in Aguiar and Gopinath (2006) and Arellano (2008). The equilibrium is unique.

Suppose now that the borrower faces the full supply curve as depicted in Figure 2 with an increasing low rate schedule and a decreasing high rate schedule. Then by picking b the borrower is not able to select the equilibrium outcome.[15] There are multiple possible interest rates that make creditors equally happy. The way this can be formalized, as in Calvo (1988),[16] is with multiple interest rate functions $R(b)$, which can be the low rate increasing schedule or the high rate decreasing one. Any other combination of those two schedules is also possible. The borrower is offered one schedule of the interest rate as a function of the debt level b and chooses debt optimally given the schedule.

In summary, the assumption on the timing of moves is a key assumption to have multiple equilibria or a single equilibrium. If the creditors move first, there are multiple equilibrium interest rates and debt levels, and they are the same equilibria whether the borrower chooses current debt or debt at maturity. Instead, if the borrower moves first, and chooses debt at maturity, as in Aguiar and Gopinath (2006) and Arellano (2008), there is a single equilibrium. Choosing debt at maturity amounts to picking the probability of default, and therefore also the interest rate. Finally if the borrower moves first and chooses the current level of debt, given an interest rate schedule defined as a one-to-one mapping from b to R, then the equilibrium will depend on the schedule and there is a continuum of equilibrium schedules. This is the approach in Calvo (1988). It is also the approach that we will follow in the dynamic computations in the next section.

[15] Trivially, it is still possible to obtain uniqueness in the case where the borrower faces the supply curve in R and b defined by (1). If the borrower picks R, then it is able to select directly the low rate equilibrium. That is essentially what happens when the borrower faces the schedule $R(a)$ and picks a.

[16] In Calvo (1988) debt is exogenous.

Lorenzoni and Werning (2013) Lorenzoni and Werning (2013) use a dynamic, simplified version of Calvo (1988)'s model, in which the borrower is a government with exogenous deficits or surpluses. In a two-period version, there is an exogenous deficit in the first period $-s^h$, with $s^h > 0$. In the second period, the surplus is stochastic, $s \in \left[-s^h, S\right]$, with density $f(s)$ and corresponding cdf $F(s)$. In order to finance the deficit in the first period the government needs to borrow $b = s^h$. In the second period, it is possible to pay back the debt if $s \geq bR$, where R is the gross interest rate charged by foreign lenders.

The creditors are competitive; they must make zero profits. It follows that $R^* = R\left(1 - F\left(bR\right)\right)$. If we were to have written $q = \frac{1}{R}$, and $a = bR$, the condition would be $R^* = \frac{1}{q}\left(1 - F\left(a\right)\right)$. As before, it is possible to use these equations to obtain functions $R\left(b\right)$, using the first equation, and $q\left(a\right)$ using the second equation. These would be the two classes of schedules that were identified in the analysis above, when the government moves first. For the normal distribution, the schedules $R\left(b\right)$ and $q\left(a\right)$ will look like the supply curves in Figure 8. There are multiple equilibrium schedules $R\left(b\right)$. There's the good, increasing schedule and the bad, decreasing schedule, and there is a continuum of other schedules with points from any of those two schedules. The government that borrows $b = s^h$, may have to pay high or low $a = R\left(b\right)b$, depending on which schedule is being used, with the corresponding probabilities of default.

What if the schedule, instead, is $q\left(a\right)$? The schedule is unique, but there are multiple points in the schedule that finance b. The government that borrows $q\left(a\right)a = s^h$, can do it with low a and low $\frac{1}{q}$, or with high a and high $\frac{1}{q}$. If the government is able to pick a, then implicitly it is picking the interest rate. Lorenzoni and Werning (2013) use an interesting argument for the inability of the government to pick the debt level a. For that they write down a game in which they divide the period into an infinite number of sub-periods, and do not allow for commitment in reissuing debt within the

period. In that model the government takes the price as given. The intuition is similar to the durable good monopoly result. In our model, the large agent also takes the price as given due to the timing assumption.

Even if there are multiple equilibria, with high and low interest rates, the high interest rate equilibria that Lorenzoni and Werning focus on are of a different type. They assume that debt is long term and characterize high rate equilibria with debt dilution. Because we assume debt is only short term the model we analyze does not exhibit those equilibria.

Eaton-Gersovitz (1981) In the model in Eaton and Gersovitz (1981) the borrower moves first, so it is key whether the equilibrium schedule is in b or a. In our notation they consider a schedule for $R(b)$. To be more precise, they assume that $a = \overline{R}(b)$, where $\overline{R}(b) = R(b)\, b$. Their equation (8) can be written using our notation as $\left[1 - \lambda\left(\overline{R}(b)\right)\right] \overline{R}(b) = R^* b$, where λ is the probability of default that depends on the level of debt at maturity. This is equivalent to

$$\left[1 - \lambda\left(R(b)\, b\right)\right] R(b) = R^*,$$

which is analogous to equation (1) in our model. As seen above there are multiple schedules in this case.

For the case of the uniform or normal distributions, there is both an increasing and a decreasing schedule $R(b)$. In that case, $\overline{R}(b) = R(b)\, b$ first goes up with b, and then goes down. Eaton and Gersovitz dismiss the decreasing schedule by assuming that $R(b)\, b$ cannot go down when b goes up. This amounts to excluding decreasing schedules by assumption.[17]

[17] See proof of Theorem 3 in Eaton and Gersovitz (1981).

4 The infinite period model: Numerical exploration

In order to keep the analysis closer to the literature that has computed equilibria with sovereign debt crises in models without a role for sunspots, as in Aguiar and Gopinath (2006) and Arellano (2008), we consider the standard timing in which the borrower moves first. In order for there to be a role for sunspots, the borrower chooses the current debt, rather than debt at maturity.[18]

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The endowment y follows a Markov process with conditional distribution $F(y'|y)$. At the beginning of every period, after observing the endowment realization y, the borrower can decide whether to repay the debt or to default. Upon default, the borrower is permanently excluded from financial markets and the value of the endowment becomes $y^d \in \mathbb{R}_+$ forever.[19]

The period utility function, $U(c)$, is assumed to be strictly increasing, strictly concave and to satisfy standard Inada conditions. Thus,

$$V^{aut} = \frac{U(y^d)}{1-\beta}. \tag{7}$$

is the value of default.

We allow for a sunspot variable s that takes values in $S = \{1, 2, ..., N\}$ and follows a Markov distribution $p(s'|s)$. Upon not having defaulted in the past, every period the borrower chooses the current debt b' given an interest rate schedule that may depend on the realization of the sunspot variable s.

[18] The computations of the alternative timing, where the lenders move first so the borrower takes the price as given is harder, due to the discontinuty of the demand function.

[19] Note that the value of autarky is independent of the state previous to default. This substantially simplifies the analysis.

The case with two schedules We analyze the case with two possible schedules for the bimodal distribution studied above. The sunspot variable can take two possible realizations, $s = 1, 2$, which indexes the interest rate schedule $R(b', y, s)$ faced by the borrower. The value for the borrower that did not default is given by $V(\omega, y, s)$, satisfying

$$\begin{aligned} V(\omega, y, s) &= \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E}_{y', s'} \left[\max \left\{ V^{aut}, V(\omega', y', s;) \right\} | y, s \right] \right\} \quad (8) \\ & \quad \text{subject to} \\ c &\leq \omega + b' \\ \omega' &= y' - b' R(b', y, s) \\ b' &\leq \underline{b} \end{aligned}$$

Wealth ω is used as a state variable (instead of current debt) because it reduces the dimensionality of the state space.[20] The borrowing limit is important. Since the borrower receives a unit of consumption for every unit of debt issued, it could always postpone default by issuing more debt. This is ruled out by imposing a maximum amount of debt.

The interest rate schedule $R(b', y, s)$ is a function of the amount of debt because default probabilities depend on it, and the interest rate reflects the likelihood of default. It is also a function of current output because, since the endowment follows a Markov process, it contains information about future default probabilities.

Default follows a threshold $\underline{y}(b', y, s, s')$, such that the optimal rule is to pay the debt as long as $y' \geq \underline{y}(b', y, s, s')$ and default otherwise.[21] The threshold for default is the level of $y' = \underline{y}(b', y, s, s')$ that solves

$$V^{aut} = V(\omega', y', s') = V(y' - b' R(b', y, s), y', s'). \quad (9)$$

[20] If we were to keep current debt b as a state, we would also need to know the previous period interest rate that is a function of the debt level in the previous period.

[21] All equilibria have this property as long as $\frac{\partial V(\omega, y, s)}{\partial y} \geq 0$, which is the case with non-negative serial correlation of the endowment process.

Creditors offer their amount of funds, as long as the expected return is R^*. The arbitrage condition for the risk free creditors that pins down the schedule $R(b', y, s)$ is

$$R^* = R(b', y, s) \sum_{s'=1,2} p(s'|s) \left[1 - F\left(\underline{y}(b', y, s, s')|y\right)\right] \tag{10}$$

where $\underline{y}(b', y, s, s')$ is defined by (9).

Equilibrium An equilibrium is given by functions

$$V(\omega, y, s), c(\omega, y, s), b'(\omega, y, s), R(b', y, s), \underline{y}(b', y, s, s')$$

such that,

1. given $R(b', y, s)$, policies $c(\omega, y, s)$ and $b'(\omega, y, s)$ solve (8) and achieves $V(\omega, y, s)$.
2. given $V(\omega, y, s)$, the default threshold $\underline{y}(b', y, s, s')$ solves (9).
3. the schedule $R(b', y, s)$ satisfies condition (10).

4.1 Simulations

In this section, we compute equilibria to show that the model can replicate salient features of the recent European sovereign debt crises. We discuss to what extent the European spread data can be generated by a model of this type. That includes the effects of policy interventions that resemble the OMT program announced by the ECB in 2012.

Parameter values As discussed above, the key parameters to generate multiplicity are the ones that govern the stochastic process for the endowment, which must alternate from being relatively high to being relatively low. We interpret these regimes with the possibility of relatively long periods of

stagnation, alternating with periods of relatively high growth. Our interpretation is motivated by the ongoing debate regarding secular stagnation in Europe.[22] We construct a bimodal distribution made out of two normals. Specifically, we assume that, every period, with probability π, the endowment is drawn from $N(\mu_1, \sigma_1)$, while with probability $1-\pi$, the endowment is drawn from $N(\mu_2, \sigma)$, with $\mu_1 < \mu_2$. For simplicity, we assume both distributions have the same standard deviation. The relatively large differences between the means of the two distributions, required to exhibit multiplicity, are interpreted as the effect of different growth rates of output for a prolonged period of time. A period in the model is several years, between 8 and 10 years. This is the average maturity of debt for most of the European countries under discussion, so it is consistent with a single period maturity in the model. A growth rate differential between the high growth regime and the low growth regime of between 4% and 6% delivers an income gap between 35% to 50% in 10 years. These numbers are in line with the growth differentials Hamilton (1989) obtains (close to 1.5% per quarter) for the US, though his analysis is focused on quarterly frequencies.[23] Thus, we assume in our benchmark case, that $\mu_2 = 1.5\mu_1$. To obtain multiple interest rate schedules, it is necessary that the ratio $\frac{\mu_2 - \mu 1}{\sigma}$ be large enough. In the example, we set $\mu_1 = 4, \mu_2 = 6$ and $\sigma = 0.1$. For example, if we set $\mu_1 = 4.5$ or higher, or if we set $\sigma = 0.4$ or higher, there is only one increasing schedule.[24] In addition, we set the probability of drawing a value for the endowment from the low distribution to be 0.3. Computing the unconditional probabilities from Hamilton's estimates, one obtains a value of $0,27$.[25]

[22] The very different growth rates in Argentina in the three decades between 1970 and 1999 are another stricking example. Jones and Olken (2008) provide ample international evidence of structural breaks in growth rates.

[23] The evidence is for the US, but European evidence can only reinforce it further.

[24] Technically, for $\sigma = 0.1$, there is still mutiplicity even if $\mu_1 = 5.5$. However, the region of multiplicity is very small. An appendix with robustness exercises is available on the website of the working paper.

[25] It should be emphasized, though that Hamilton estimates are not consistent with a symmetric Markov matrix. They imply a longer than a decade duration for the good state

The model has a relatively low number of additional parameters. The first three are not controversial. First, we set the international interest rate $R^* = 1.20$, roughly consistent with a 2% yearly rate for a decade. We allow the discount factor in preferences to be higher, so the borrower has a reason to borrow, thus it is set $\beta = 0.7$, consistent with a yearly discount factor of 0.96. Preferences exhibit constant relative risk aversion with parameter $\gamma = 6$, so as to have a relatively strong preference for consumption smoothing.

There are two remaining parameters, the value for consumption following default and the probability of the sunspot that coordinates on alternative schedules. Following default, the borrower is cut off from international credit markets. To the extent that integration to world markets is associated to the possibility of rapid growth, it is natural to think that default could substantially reduce the probability to draw from the high-endowment distribution. Following this notion, we set the value of endowment following default to be equal to $y^d = \mu_1 = 4$.

Finally, we assume that the sunspot distribution is *i.i.d.* $p(s'|s) = p$, set the probability of the sunspot that coordinates on the high-interest rate schedule to be 0.2. All results we show are essentially the same if we set that probability to be 0.4.

Characterization of Equilibria Figure 7 plots the schedule of yearly interest rates as a function of the debt level.

Clearly, for debt levels between 1.8 and 2.2, there are two possible interest rates. Note that when there is multiplicity, rates range from 1.8% per year to 5.6%, so this example delivers a spread of about 3.8% a year. This number is close to the maximum value of Spanish and Italian spreads, but much smaller that the ones of Portugal, Ireland or Greece.[26] One reason for the observed high spreads could be a run up to default - as it already happened in Greece

and lower that a decade for the bad state.

[26]This does depend on our choice of a key parameter: the probability of entering a period of stagnation, $\pi = 0.3$. If we set $\pi = 0.5$, the model generates a spread of 9.5%.

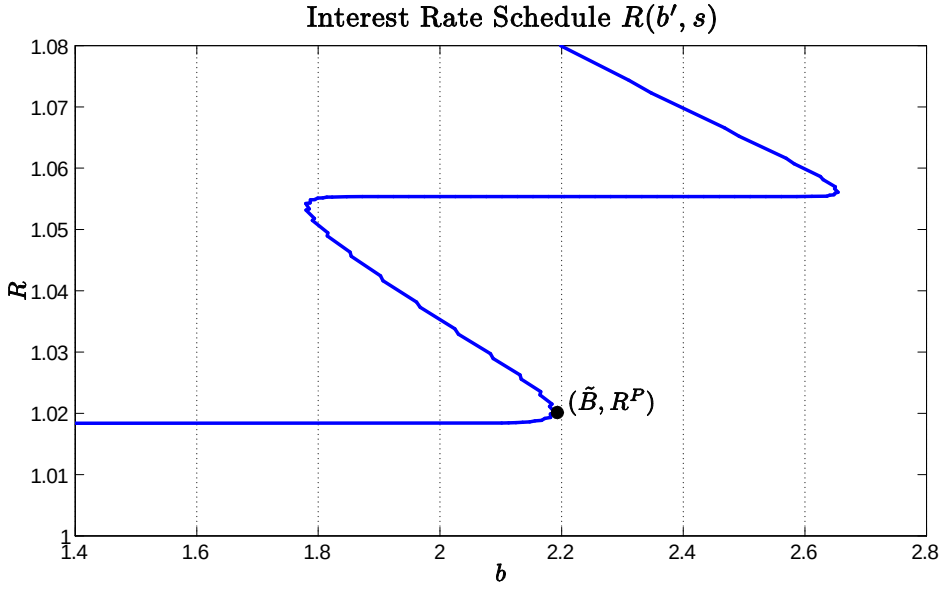


Figure 7: Interest rate schedule $R(b', s)$

- that our long-run calibration cannot capture.

A particular feature of the increasing schedule is the apparently flat sections.[27] This is the result of having two normal distributions with relatively large differences in means, and very small standard deviations. Note that the "good" distribution has most of the mass between 5.8 and 6.2, so that, if the threshold is below 5.8, but not too far away, increases in the threshold have a negligible effect on the probability of default, so they barely affect the interest rates.

Figure 8 plots the policy functions for the debt levels as a function of wealth, for different realizations of the sunspot. The dashed red (solid blue) line corresponds to the case in which the sunspot selects, for each value of the debt, the high (low) interest rate. The horizontal axis is the wealth at the beginning of the period, which is equal to the realization of the endowment minus debt gross of interest payments. For values of the wealth above 2.8, the

[27] The schedules are not exactly flat.

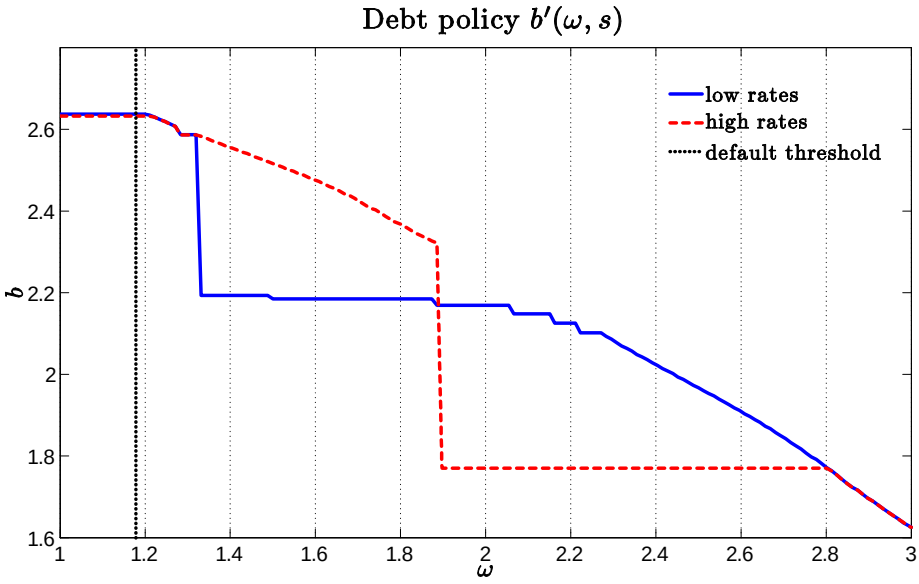


Figure 8: Debt policy function $b'(\omega, s)$

two policy functions coincide. This corresponds to choices of debt that are below the value beyond which there is multiplicity. Thus, for this region, the realization of the sunspot is inessential and as wealth goes down, the amount borrowed goes up - the standard consumption smoothing result. However, for values of wealth close but lower than 2.8, the behavior critically depends on the realization of the sunspot. If the sunspot selects the lower schedule, debt keeps on increasing as wealth goes down. Instead, if the sunspot selects the high schedule, debt is invariant with wealth. The reason is that borrowing is close to 1.8, a value such that (see Figure 7) the interest rate schedule exhibits a discontinuous jump in the interest rate. Faced with such a high effect on the interest rate, the borrower reduces consumption one to one with wealth. Eventually, however, when wealth is sufficiently low - and so is current consumption - the borrower is willing to pay the fixed cost of the high interest rate on all the debt, at which point debt increases discontinuously. From there on, increases in the debt have very marginal effects on the rate,

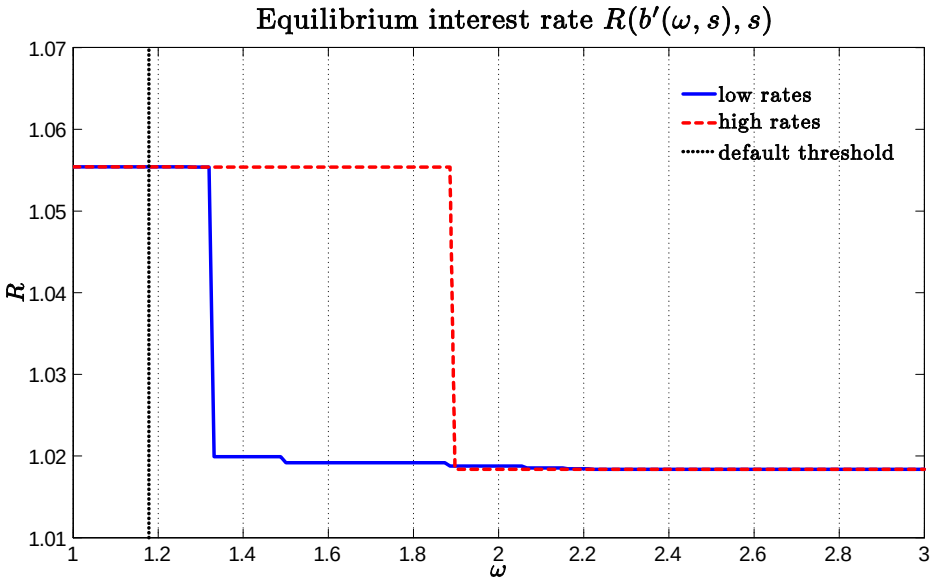


Figure 9: Equilibrium interest rates $R\left(b'\left(\omega, s\right), y, s\right)$

so debt again goes up, one to one with the reduction in wealth.

The behavior of the policy function when the sunspot pins down the low interest rate schedule is similar, except that the effects occur for lower values of wealth: The policy function flattens when wealth reaches around 2.2 and jumps-up discontinuously once wealth is around 1.9.

Figure 9 plots the equilibrium interest rates, as a function of wealth, for the two different realizations of the sunspot. As before, the dashed red (solid blue) line is the interest rate if the sunspot selects the high (low) interest rate schedule.

It is interesting to highlight how the borrower's choices are key to understand equilibrium outcomes. The region of multiplicity is roughly the one where wealth is between 1.3 and 2.8. And notice (see Figure 9) that, contrary to what one could have expected, the equilibrium interest rate when in the bad sunspot (high interest rate) is lower, even if by a very small amount, than the interest rate in the good sunspot for values of wealth between 1.9

and 2.8, close to two thirds of the multiplicity region. The reason is simple: It is precisely because the borrower faces the high interest rate schedule that it is willing to adjust consumption and avoid those high interest rates, a form of endogenous austerity. This rationalizes the notion that the probability of a crisis may have a disciplinary effect. This effect however is present only up to a point: Once wealth is below 1.9, the borrower, facing the high interest rate schedule has such a pressing motive to borrow that is willing to borrow at very high rates. When shocks bring the borrower to this region, debt levels and interest rates go up in the data, also a feature of the data through the European sovereign debt crisis.

This endogeneity of debt implies that interest equilibrium interest rates are less revealing of the existence of multiplicity than borrowing choices. In Figure 10 we plot the ratio of optimal debt choices (left vertical axes) and the interest rate differential (right vertical axes) for the two values of the sunspot. We focus in the range of values of wealth for which there is multiplicity, between roughly 1.4 and 2.8. While debt choices are very different for the whole range, the interest rate differentials are barely different for a large fraction of the range.

Policy intervention and multiplicity. We now use the solution of the model to illustrate how a sovereign debt crisis can unfold, shedding light on the role of the sunspot realization and the role of policy. The first step is to define the policy intervention. We assume there exists an institution with enough funds that can offer by itself an amount larger than the value $\widetilde{B}$, in Figure 7. A policy consists of a pair $(\widetilde{B}, R^P)$ such that the institution is willing to lend funds to the borrower at a rate R^P, up to a maximum value of $\widetilde{B}$. Let $P^* = (\widetilde{B} + \delta, \widetilde{R} + \varepsilon)$ for low enough values of ε, δ where $\widetilde{R}$ is depicted in Figure 9 and $\varepsilon > 0, \delta > 0$.

Then, by the same logic as the ones explained in Section 2.2, policy P^*eliminates the high interest rate schedule. In addition, in equilibrium,

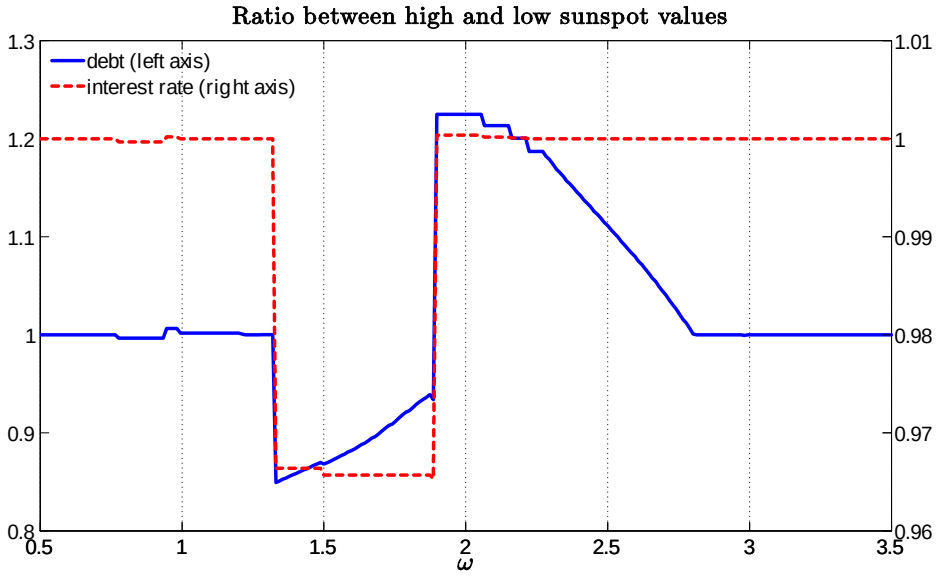


Figure 10: Debt and interest rate comparison

the institution lends no funds. Note the importance of the maximum level $\widetilde{B}$: if δ is too large and ε small enough, it may be optimal to borrow from the institution amounts larger than $\widetilde{B}$. But at those values, the institution's expected return is lower than R^*.

Policy P^*, by removing the high equilibrium schedule, is equivalent to setting the probability of the sunspot to zero. To put it differently, assume that the institution implements policy P^* with probability ϕ and implements no policy with probability $(1-\phi)$. This is equivalent to an economy without any policy intervention and probability of the bad sunspot to be $p(1-\phi)$. By reinterpreting the parameters above, we can simulate a sovereign default crisis and the policy intervention: We let $\widetilde{p} = 0.4$ and $\phi = 0.5$, so $\widetilde{p}(1-\phi) = p = 0.2$.

In Figure 11 we plot the time series of interest rates and debt choices after the following a sequence of shocks: We start the economy with wealth equal to 3.2, a value for which there is no multiplicity, and assume that the endowment shock is equal to 4 every period. We assume that the good

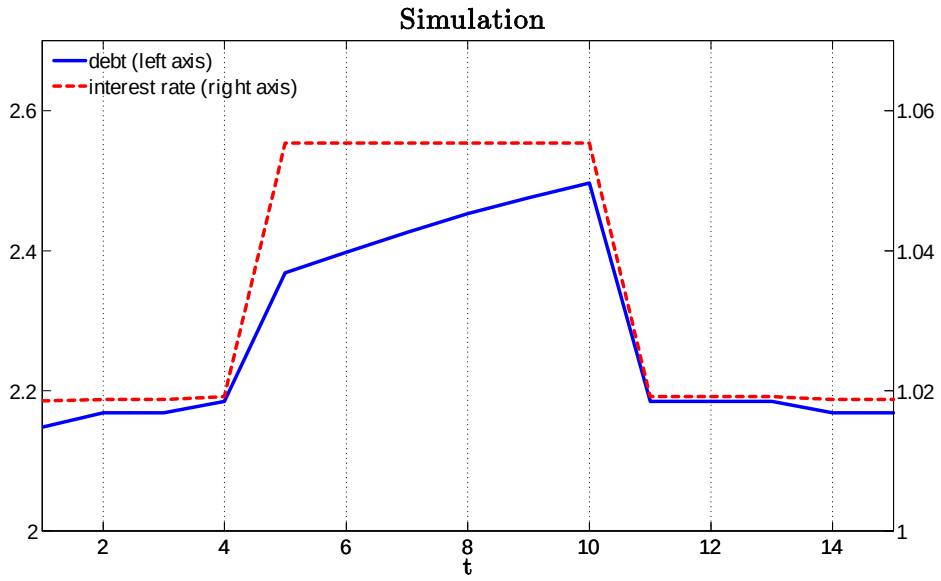


Figure 11: Simulation

sunspot realizes for 4 periods, after which, the bad sunspot realizes every period. Policy is only implemented at period $t = 11$ and it remains in place thereafter. As it can be seen from the Figure 11, spreads go up once the bad sunspot is realized and they come down once policy is implemented. Note also that debt goes up when the spreads go up and then it comes down when the spreads come down, induced by the policy. Notice that "austerity" arises endogenously once the policy is implemented.

5 Concluding remarks

In models with sovereign debt default, interest rates are high because default probabilities are high. The object of this paper is to investigate conditions under which the reverse is also true, that default probabilities are high because interest rates are high. This means that there can be equilibrium outcomes in which interest rates are unnecessarily high, and in which policy

arrangements can bring them down. This exploration is motivated by the recent sovereign debt crisis in Europe, but it is also motivated by a literature that does not seem to be consensual on this respect. Indeed, while Eaton and Gersovitz (1981) claim that there is a single equilibrium, Calvo (1988) using a similar structure shows that there are both high and low interest rate equilibrium schedules. Aguiar and Gopinath (2006) and Arellano (2008) building on Eaton and Gersovitz, modify an important assumption on the choice of debt by the large player and find a single equilibrium. We show that small changes in timing assumptions and actions of agents, that cannot be directly disciplined by empirical evidence, can explain these conflicting results.

Assumptions on whether the country chooses the debt net of interest payments or gross of those payments, or whether the borrower moves first or the creditors do, are not assumptions that can be obtained directly from empirical evidence. But there is indirect evidence. The multiplicity of equilibria that arises under some of those assumptions is consistent with the large and abrupt movements in interest rates that are observed in sovereign debt crises, while the single equilibrium is not.

We also simulate a dynamic version of the model, in which a sunspot variable can induce high frequency movements in interest rate equilibria. We believe this can be a reading of a sovereign debt crisis. If so, then policies of large purchases of sovereign debt, at penalty rates, in the spirit of the ones announced by the ECB back in 2012, can have the effect that they seem to have had, of bringing down sovereign debt spreads.

According to this view, the ruling of the German constitutional court in early 2014, that "found that the central bank had overstepped its mandate and that OMT was a backdoor to "monetary financing" of governments outlawed under European treaties", is not founded.[28] This conclusion is by no means warranted: Alternative interpretations based on the concept that the OMT actually implies future transfers to countries experiencing high spreads

[28] The Economist, February 15th, 2014.

from countries not experiencing the high spreads, that will eventually be used to pay for the debt, may also explain the data.

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Appendix 1.

In this appendix, we perturb the game described in Section 2 by introducing a second stage at the end of period 1, that allows for partial renegotiation. We then explore the robustness of the equilibria described in Section 2 to "very small" perturbations, in a sense we make precise below.

Specifically, and given any outcome (b, R) in the first stage of period 1, nature allows the borrower, with probability $\pi \in (0, 1)$, to make a take it or leave it offer of an alternative interest rate equal to $R - \delta$, to a coalition of a fraction α of lenders, where $\delta > 0$ is exogenously given.[29] The coalition then chooses to accept the new rate or to keep the one of the first stage. Period 2 is the same as before: Given the amount owed by the borrower and the realization of the endowment shock, the borrower decides to default or pay the debt in full. The payoff following default is as before.

We denote by $\alpha \in (0, 1)$ the measure of lenders that may be called into the coalition. It is important to emphasize that in this second stage it is the coalition, rather than each individual lender, making decisions. Each agent in the coalition is treated equally. It is this assumption that imposes an "α-limited" degree of coordination. As $\alpha \to 0$, there is no degree of coordination and as we will show, the refinement requires α to be strictly bounded above zero. The first stage is exactly as before: all lenders - the α ones that can be called into the coalition and $(1 - \alpha)$ who cannot - then compete among each other, so they all charge the same rate in the first stage.[30]

Let the so perturbed game be denoted by $G^{\alpha}(\delta, \pi)$.[31] We first characterize equilibria in the games $G^{\alpha}(\pi, \delta)$. In the spirit of trembling-hand perfection we explore, given α, which of the equilibria described in Section 2 are the limit of the sequence of equilibria of the games $G^{\alpha}(\pi, \delta)$ when $\pi \to 0, \delta \to 0$.

[29] Considering only reductions in interest rates is without loss of generality. Had the borrower the option of chosing higher interest rates, he will never do so.

[30] Note that the perturbation introduces ex-ante heterogeneity. We will focus on the limiting cases where $\delta \to 0$ and $\pi \mapsto 0$, so the heterogeneity is vanishing in the limit.

[31] The original game is equivalent to $G^{\alpha}(\delta, 0)$ or $G^{\alpha}(0, \pi)$.

Definition 1 *Given $\alpha \in (0,1)$, an equilibrium (R,b) in the game $G^{\alpha}(0,0)$ is robust to an $\alpha-$degree of coordination if it is the limit of the sequence of equilibria in the games $G^{\alpha}(\delta,\pi)$ when $\delta \to 0$, and $\pi \to 0$.*

We will show that the equilibria in the decreasing part of the zero-profit schedule

$$R^{*} \equiv R(b)\left[1-F\left(1+bR(b)\right)\right] \tag{11}$$

do not survive a refinement based on this perturbation, while equilibria in the increasing part do. Two assumptions are key to obtain the results: First, the auction in the first stage must be anonymous (Lemma 1).[32] Second, a strictly positive degree of coordination of lenders is required (Result 2).

We prove the results in a series of steps. First, we show that, as long as the auction in the first-stage is anonymous, there is no equilibrium in the perturbed game in which the offer is accepted.

Lemma 1 *In any equilibrium of the perturbed game in which nature allows the borrower to make the offer $R-\delta$, the offer is rejected by the coalition if the auction in the first stage of the game is anonymous.*

Proof. Assume there is one equilibria with $\delta > 0$ where the offer is accepted. The $\alpha-$members of the coalition get an interest rate of R^{c} with probability π and an interest rate of $R^{c}-\delta$ with probability $(1-\pi)$, while the lenders that cannot be part of the coalition get an interest rate of R^{n}. The expected return for lenders within the coalition is

$$\begin{aligned} R^{*} \;=\; & (1-\pi)R^{c}[1-F(1+b\left[\alpha R^{c}+(1-\alpha)R^{n}\right])] \\ & +\pi(R^{c}-\delta)[1-F(1+b[\alpha R^{c}+(1-\alpha)R^{n}-\alpha\delta])] \end{aligned} \tag{12}$$

[32] Anonimity is irrelevant for the game in Section2, where all agents are homogeneous. But the perturbation introduces heterogeneity, so anonymity is important in the perturbed game.

while the condition for the $(1-\alpha)$ fraction of agents that do not get to be part of the coalition, it is

$$\begin{aligned} R^* &= (1-\pi)R^n[1-F(1+b\left[\alpha R^c+(1-\alpha)R^n\right])] \\ &\quad +\pi R^n[1-F(1+b[\alpha R^c+(1-\alpha)R^n-\alpha\delta])]. \end{aligned} \tag{13}$$

As $\delta > 0$, $R^n < R^c$. It is immediate that this cannot be an equilibrium in a second price auction, where $R^n = R^c$. Consider now a first price auction, where each lender receives the interest rate offered. Note that for the borrower, borrowing from each lender type implies the same expected payment. However, the lenders not in the coalition imply a payment with certainty, so the borrower will choose those lenders first. Thus, a fraction $(1-\alpha)$ receives an interest rate of R^n and the fraction α receives the interest rate $R^c > R^n$. It immediately follows that in an anonymous auction, non coalition lender's best interest is to offer their funds at the rate R^c, where profits are higher than R^*. ■

If the auction is not anonymous, the borrower can fully discriminate in a first price option and conditions (12) , (13) fully characterize the equilibrium interest rates. Once the borrower cannot discriminate the lender's type, the second price auction provides incentives for truthful revelation, but implies a unique interest rate, which breaks down the equilibrium. On the other hand, in a first price auction, agents not in the coalition do not have incentives to reveal their type which also brakes the proposed equilibrium.

Note also that the perturbation we consider is a simple one, in which agents ex-ante know if they belong in the potential coalition or not. A more general perturbation would allow for each lender to have a probability $\alpha(j)$ of belonging to the coalition, with $\int_0^b \alpha(j)dj = \alpha$. The only case in which there can be an equilibrium with an anonymous auction in which an offer is accepted is the knife-edge case in which $\alpha(j) = \alpha$ for all j, so (13) is not an equilibrium condition anymore. In this case all agents are ex-ante identical

and anonymity plays no role.

We now characterize the conditions under which an offer will be accepted by the coalition for δ small enough.

Let the outcome of the first stage be a point in the schedule (R, b), defined by (11) . Assume that nature allows the borrower to offer $R - \delta$ to the coalition. If it accepts the new rate, their return will be given by

$$(R - \delta)\left[1 - F\left(1 + (1 - \alpha)bR + \alpha b\left(R - \delta\right)\right)\right] \equiv E(\delta).$$

The new rate reduces the payment in case of no default, but it reduces the probability of default, so the net effect depends on which effect dominates. We now find a condition such that the second effect dominates so reductions in the interest rates (positive values for δ) can increase the expected payment to the coalition, for small enough perturbation, so δ is close to zero. For that, we derive the expression above $E(\delta)$ with respect to δ and evaluated it at $\delta = 0$, to obtain

$$E'(\delta = 0) = -\left[1 - F\left(1 + b\right)\right] + R[f\left(1 + bR\right)\alpha b]$$

This expression is positive when

$$Rb\frac{f\left(1 + bR\right)}{\left[1 - F\left(1 + b\right)\right]} \equiv H(R, b) > \frac{1}{\alpha}.$$

in which case the coalition would accept the offer, for δ small enough.[33] We now show the results mentioned above.

Result 1: If the pair (R_1, b_1) is an equilibrium in the original game and it is in the *increasing* part of the schedule defined by (11) , then it is robust to an $\alpha-$degree of coordination for any α.

[33]Note that the smaller the coalition, the strongest is this condition. This is whay the coalition is important.

Pf: Differencing the identity (11) with respect to b we obtain

$$R^{'}(b) = \frac{R^2 f(1+bR)}{[[1-F(1+bR)] - Rbf(1+bR)]}.$$

If (R_1, b_1) is in the increasing part of the schedule, $R^{'}(b_1) > 0$. As the numerator is positive, this implies that the denominator must be negative, which implies that

$$R_1 b_1 \frac{f(1+b_1 R_1)}{[1-F(1+b_1 R_1)]} \equiv H(R_1, b_1) < 1$$

Thus, $H(R_1, b_1) < 1 < \frac{1}{\alpha}$ for any $\alpha \in (0, 1]$, which means that the offer is not accepted for any degree of coordination, for some δ_1 which is small enough. This means that the equilibrium in the original game, (R_1, b_1), is also an equilibrium in the perturbed game $G^{\alpha}(\pi, \delta)$ for any $\alpha >$, any π and any $\delta < \delta_1$. It therefore follows that (R_1, b_1) is the limit of this sequence of games when $\delta \to 0, \pi \to 0$.

Result 2: If the pair (R_2, b_2) is an equilibrium in the original game and it is in the *decreasing* part of the schedule defined by (11), then it is not robust to an $\alpha-$degree of coordination for any $\alpha > \alpha^{\min}$, where $\alpha^{\min} < 1$ is the minimal required degree of coordination.

Pf: If (R_2, b_2) is in the increasing part of the schedule, $R'(b_2) < 0$. As the numerator is positive,

$$R_2 b_2 \frac{f(1+b_2 R_2)}{[1-F(1+b_2 R_2)]} \equiv H(R_2, b_2) > 1.$$

Let

$$\alpha^{\min} = \frac{1}{H(R_2, b_2)} < 1.$$

Assume that (R_2, b_2) is robust to a $\alpha^{\min}-$ degree of coordination. It means that there is an equilibrium arbitrarily close to (R_2, b_2) in the game $G^{\alpha}(\pi, \delta)$ for $\alpha > \alpha^{\min}$ and small enough values for π and δ. By continuity of the function $H(R, b)$, it follows that if nature lets the borrower make an offer

$R_2 - \delta$, it will be accepted by a coalition larger than $\alpha^{\min}$, which contradicts Lemma 1.

Note that the value of $\alpha^{\min}$ is related to the value of $H(R_2, b_2)$ relative to 1. When the slope of the schedule defined by (11) becomes very close to $-\infty$, which happens when the equilibria in the decreasing schedule get arbitrarily close to the equilibria in the increasing schedule, $H(R_2, b_2) \to 1$, and $\alpha^{\min} \to 1$, requiring an arbitrarily large degree of coordination.

Appendix 2.

In this appendix, we show how small perturbations to the uniform distribution can give rise to multiple equilibria of the type obtained with the bimodal distribution.[34] Consider a perturbation $g(y)$ of the uniform distribution, so that the density would be $f(y) = \frac{1}{Y-1} + \gamma g(y)$, with $\int_1^Y g(y)dy = 0$. In particular the function g can be $g(y) = \sin ky$, with $k = \frac{2\pi}{Y-1}N$, where N is a natural number. If $N = 0,$ the distribution is uniform, so there is a single increasing schedule. If $N = 1$, there is a single full cycle added to the uniform distribution. The amplitude of the cycle (relative to the uniform distribution) is controlled by the parameter γ. The number of full cycles of the $\sin ky$ function added to the uniform, is given by N. As $\gamma \to 0$, so does the perturbation.

Given a value for γ, the closer the debt to its maximum value, the larger the degree of multiplicity. The equation

$$\frac{1}{R} - \frac{1}{R^*}\left[1 - \frac{1+bR}{Y-1} - \gamma \sin kbR\right] = 0$$

has more than two solutions for R, for γ that can be made arbitrarily small, as long as b is close enough to $b^{\max}$. On the other hand, if b is lower than $b^{\max}$, there is always a $\gamma > 0$, but small enough, such that there are only two zeros to the function above. An illustration is presented in Figure 7, for two levels of the debt and for two values of γ.

As can be seen, when the debt is low, a positive value of γ is not enough to generate multiplicity, but multiplicity arises as the level of the debt goes up.

Note that if γ is small, it may take a very long series to identify it in the data. Thus, it is hard to rule out this multiplicity based on calibrated versions of the distribution of output if the debt is close enough to its maximum.[35]

[34] The uniform distribution is used only as an example.

[35] This resembles the result in Cole and Kehoe (2000), where the fraction of short term

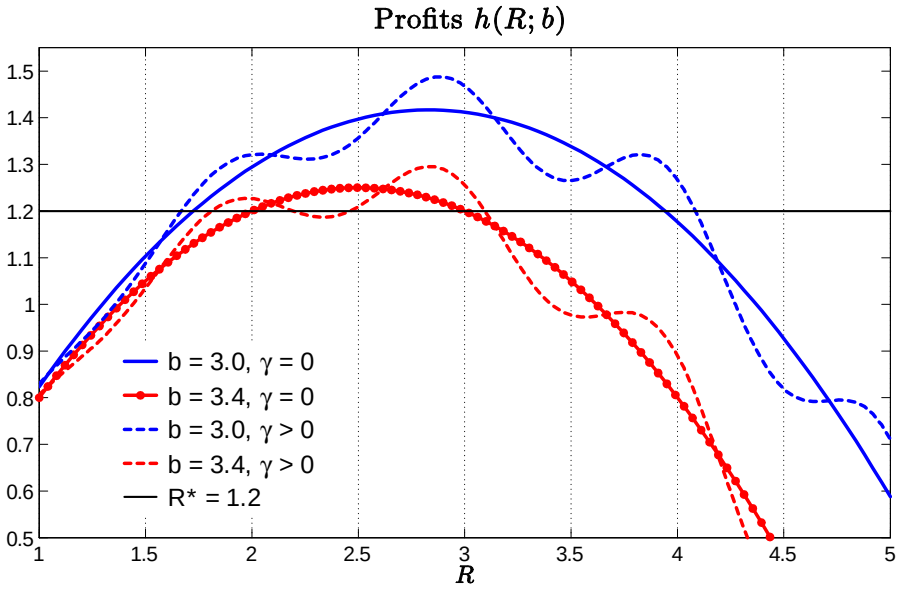


Figure 12: Perturbing the uniform distribution

debt affects the chances of multiplicity.