Self-Fulfilling Unemployment Crises*

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PRELIMINARY AND INCOMPLETE

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Abstract

This paper proposes a theory of self-fulfilling unemployment fluctuations driven by an aggregate demand externality. Expectations of high unemployment, lead to low income and low aggregate demand. In turn, low aggregate demand leads to expectations of deflation and high unemployment. I show that this feedback between expectations and aggregate demand can lead to the emergence of self-fulfilling fluctuations in involuntary unemployment. Quantitative analysis calibrated to Spain shows that episodes of large unemployment are a result of pessimistic expectations. We discuss the policy implications of this novel source of multiplicity.

Keywords: Multiple equilibria, sunspots, unemployment, secular stagnation
JEL Classification Codes: D62, E32, E44, F32, F41

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"The main enemy for Europe is unemployment....this is caused by a broader sentiment of lack of confidence" Interview of Mario Draghi, President of the ECB, (conducted by Jean-Pierre Elkabbach (Europe 1) on Tuesday 23)

1 Introduction

Fear of a prolonged period of unemployment has been at the center of the recent European recession. By itself, this lack of confidence is perceived to be causing high unemployment, and it is argued that reverting the confidence slump is crucial to bring down unemployment levels (as the quote from Draghi illustrates). Indeed, Figure 1 shows a strong negative comovement between consumer confidence and unemployment. However, if the lack of confidence is a mere symptom of high unemployment, rather than a fundamental cause of it, the only way to restore confidence might be to bring down unemployment in the first place. Is lack of confidence a symptom of high unemployment, or a fundamental cause of it?

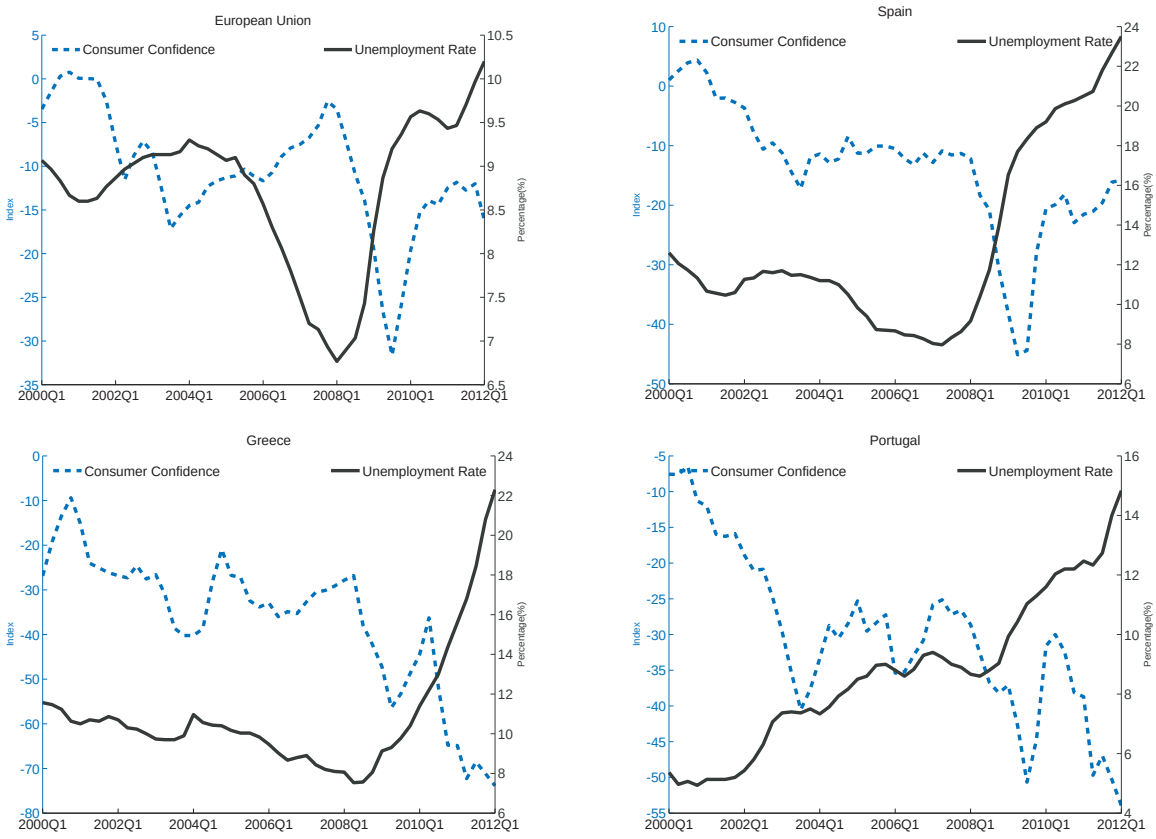
In this paper, we argue that restoring confidence per se can be an important factor causing high unemployment. We study a small open economy model, with a tradable and a non-tradable sector and two key ingredients: downward wage rigidity and a fixed exchange rate, as in Schmitt-Grohé and Uribe (2013). The motivation for these two features is empirical. Indeed, despite the severe recession and high unemployment in Europe, nominal wages remained high and being part of a currency union ruled out a reduction of real wages via a devaluation. The main result of the paper is that the combination of these two features generate a novel source of multiple equilibria.

We show that in the model there is a "good equilibrium", where households have high consumption, financed by borrowing, and the economy is at full employment or close to it. There is a "bad equilibrium", however, with high unemployment. In this equilibrium, low unemployment leads to low income and low aggregate demand, which in turn generates deflation and unemployment.

The mechanism that generates multiplicity is as follows. When agents expect high unemployment, there is little aggregate demand, and this leads to a low price of non-tradable goods, which lead to low employment in the non-tradable sector. As a result, consumption of non-tradables has to be low in equilibrium, which may lead to a decline in the demand for tradable goods—this occurs provided that tradable and non-tradable goods are complements in the utility function—and a decline in the price of non-tradable goods. Since wages are downard rigid, this leads to a decline in labor demand, which rationalizes the expectations of high unemployment in the first place.

To assess the quantitative importance of this fragility problem, we calibrate our model using Spanish data. Specifically, we calibrate our model to reproduce several features of the Spanish business cycle including wage and unemployment data, and investigates how pervasive is the phenomenon by which pessimistic expectations can induce a sudden unemployment crisis.

In our dynamic model, given shocks and the endogenous debt position, the economy may be



in a multiple equilibria region or in a unique equilibrium region. In the simulations, the economy is in the multiple equilibria region a significant amount of time. Interestingly, this occurs for intermediate values of debt positions: For low values of debt, the economy features high aggregate demand and low unemployment. For high values of debt, the economy features low aggregate demand and always experiences higher unemployment. It is for intermediate values of debt that the economy is exposed to a sudden shift in expectations.

Finally, we investigate the policy implications of our model. A key element behind the multiple equilibria is that the model rationalizes government initiatives to restore confidence in the midst of an episode of high unemployment, but only when this is due to a shift in expectations. Notable, a policy that subsidizes debt and boosts aggregate demand can take the economy out of an unemployment trap. The welfare gains of these policies are substantial reaching about 0.8 percentage points of permanent consumption and the reduction in unemployment can reach several percentage points.

1.1 Related Literature

There is a large and growing literature on the role of self-fulfilling expectations for aggregate economic fluctuations. To the best of my knowledge, this is the first paper that shows that downward wage rigidity can lead to multiple equilibria.

- Classic papers on multiple equilibria: Diamond (1982) Benhabib and Farmer (1994), Kiyotaki (1988)
- Interest-rate feedback rules: Benhabib, Schmitt-Grohe, and Uribe (2001,2002), Schmitt-Grohe, and Uribe (2014), Aruoba-Cubas-Shorfeide (2014)
- Secular stagnation: Eggerston and Mehrota (2014), Benigno and Fornaro (2015)
- Other related papers: Heathcote and Perri (2014), Kaplan and Menzio (2014), Schmitt-Grohe and Uribe (2014), Farmer (2013, 2014), Chamley (2014)

RELATIONSHIP TO BE DESCRIBED

The paper is organized as follows. Section 2 presents the model. Section 3 presents a simple two period example to illustrate the self-fulfilling unemployment crises. Section 4 presents the quantitative analysis (incomplete).

2 Model

We present a model with downward wage rigidity for an economy that has a fixed exchange rate regime, or equivalently an economy that belongs to a currency union. The model follows closely Schmitt-Grohé and Uribe (2013).

The economy is populated by a continuum of infinitely lived agents. Preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C^{1-\sigma} - 1}{1-\sigma} \right)$$

In this expression, $\mathbb{E}(\cdot)$ is the expectation operator, and β is the discount factor. The period utility function $u(\cdot)$ is a standard concave, twice-continuously differentiable function that satisfies the Inada condition.

Consumption C is a CES of private consumption c and public consumption g with elasticity ϕ.

$$C = [\theta c^{1-\frac{1}{\phi}} + (1-\theta) g^{1-\frac{1}{\phi}}]^{\frac{\phi}{\phi-1}}$$

Private and public consumption follow a CES aggregator with elasticity of substitution η between tradable goods and nontradable goods, given by:

$$\begin{aligned} c &= \left[\omega \left(c^T \right)^{1-\frac{1}{\eta}} + (1-\omega) \left(c^N \right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ g &= \left[\omega \left(g^T \right)^{1-\frac{1}{\eta}} + (1-\omega) \left(g^N \right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \eta > 0, \omega \in (0,1). \end{aligned}$$

In each period t, agents receive an endowment of tradable goods y_t^T and labor income wn. Because households do not value leisure, they supply all available hours, which are normalized to one.

Because of downward wage rigidity, however, the effective number of hours supplied could be lower than their available time, leading to involuntary unemployment.

We assume that the law of one price holds for tradable goods $P_t^T = P_t^{*T} E_t$ and that the price of foreign tradable is constant and equal to one. An important assumption that we make is that $E_t = 1$, which corresponds to a fixed exchange rate policy or equivalently an economy that belongs to a currency union. These assumptions together imply that the price of tradables is fixed $P_t^T = 1$ in domestic currency.

As a result, we obtain the following budget constraint expressed in terms of tradable goods:

$$\frac{b_{t+1}}{R_t} + c_t^T + p_t^N c_t^N = b_t + y_t^T + w_t n_t + \pi_t^N,$$

where p_t^N and w_t^N denotes the price of non-tradable goods and wages, both expressed in terms of tradable goods.[1] Profits are denoted by π_t and b_{t+1} denotes non-state contingent bond holdings denominated in foreign currency and R_t is the gross interest rate on bonds, which is determined exogenously in world capital markets.[2]

2.1 Firms

Denoting by w the wage in terms of tradable goods, we have that firms solve

$$\pi^N = \max_h p^N z h_t^\alpha - w h_t$$

$$\alpha p_t^N z_t h_t^{\alpha-1} = w_t$$

This optimality condition will be key for multiplicity. Changes in the price of non-tradable represent demand shifters in the demand for labor. In turn, we will show that the price of non-tradables will depend on collective borrowing decisions of the households. When households borrow little, this would lead to low price of non-tradables and low labor demand. If wages were flexible, they would adjust so that the economy is at full employment. We introduce next downward wage rigidity.

2.2 Downward Wage Rigidity

We assume that there is a process for minimum wages $\overline{W}_t$ and that equilibrium wages need to satisfy

$$W_t \geq \overline{W}_t$$

[1] This is also the price in terms of domestic or foreign currency since the exchange rate is fixed and the law of one price holds.

[2] Because the exchange rate is fixed, it is equivalent if bonds are denominated in domestic currency.

Using $w_t = W_t/E_t$ and that E_t is fixed, we have that the nominal wage rigidity is translated into a real wage rigidity condition:

$$w_t \geq \overline{w_t}$$

In addition, following Schmitt-Grohé and Uribe (2013), we impose the condition

$$(w_t - \overline{w_t})(h_t - 1) = 0$$

This condition, says that if labor market are below full employment $h = 1$, then $w_t \geq \overline{w_t}$. Conversely, if wages are below the minimum

2.3 Recursive Equilibrium

In each period, the value of an exogenous sunspot variable ζ_t is realized. This variable is independently and uniformly distributed on the interval [0, 1]. As we will show next, this variable will determine the probability of a bad equilibrium or an "unemployment trap".

We start the definition of the equilibrium in recursive form by setting up the recursive problem of households. We denote by $\mathbf{X} = \{B, s\}$ the aggregate state where s collects all the exogenous states $\mathbf{s} = \{R, \overline{w}, y^T, \zeta\}$. We follow the convention of denoting current variables without subscript and denoting next period variables with the prime superscript the exogenous states. Let $\Gamma(\cdot)$ the forecast function of aggregate bond holdings, i.e., $B' = \Gamma(\mathbf{X})$.

The problem of a representative household can then be written as:

$$V(\mathbf{X}) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}_{\mathbf{s}'|\,\mathbf{s}} V(b', B', \mathbf{X}') \tag{1}$$

subject to

$$\frac{b'}{R} + p^N(\mathbf{X})c^N + c^T = y^T + b(1+r) + w(\mathbf{X})n(\mathbf{X}) + \pi(\mathbf{X})$$

$$B' = \Gamma(\mathbf{X})$$

In this problem prices $w(\mathbf{X}), p^N(\mathbf{X})$, employment $n(\mathbf{X})$ and profits $\pi(\mathbf{X})$are taken as given by the representative agent . The solution to the household problem yields decision rules for individual bond holdings $\hat{b}(\mathbf{X})$, tradable consumption $\hat{c}^T(\mathbf{X})$ and nontradable consumption $\hat{c}^N(\mathbf{X})$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings to an actual law of motion, given by the representative agent's choice $\hat{b}(B, \mathbf{X})$. In a rational expectations equilibrium, as defined below, these two laws of motion must coincide.

Definition. ***(Decentralized Recursive Competitive Equilibrium)***
Given a process for $\mathbf{s} = \{R, \overline{w}, y^T, \zeta\}$*, a recursive competitive equilibrium for our SOE is defined by pricing functions* $p^N(\mathbf{X}), w^N(\mathbf{X})$*, profits* $\pi(\mathbf{X})$ *a perceived law of motion* $\Gamma(B, \mathbf{y})$ *and decision*

rules $\left\{\hat{b}(b,\mathbf{X}),\hat{c}^T(b,\mathbf{X}),\hat{c}^N(b,\mathbf{X})\right\}$ *with associated value function* $V(b,\mathbf{X})$ *such that the following conditions hold:*

1. *Household optimization:* $\left\{\hat{b}(b,\mathbf{X}),\hat{c}^N(b,\mathbf{X}),\hat{c}^N(b,\mathbf{X}),V(b,\mathbf{X})\right\}$ *solve the recursive optimization problem of the household.*

2. *Firm optimization* $h(\mathbf{X})$ *satisfy* $p^N(\mathbf{X})\alpha h(\mathbf{X})^{\alpha-1}=w(\mathbf{X})$

3. *Rational expectation condition: the perceived law of motion is consistent with the actual law of motion:* $\Gamma(\mathbf{X})=\hat{b}(B,\mathbf{X})$.

4. *Markets clearing in non-tradable goods:* $y^N=\hat{c}^N(b,\mathbf{X})$ *and the resource constraint for tradable goods holds* $\hat{b}(B,\mathbf{X})+\hat{c}^T(b,\mathbf{X})=y^T+B(1+r)$.

5. *Labor market conditions:* $n=h$

$$w(\mathbf{X})\geq\overline{w}$$

In addition, there is a condition

$$(w(\mathbf{X})-\overline{w})(h(\mathbf{X})-1)=0$$

$$h(\mathbf{X})\leq 1$$

2.4 Analytical Properties

We establish the following propositions to highlight the ingredients behind the multiple equilibria results.

Proposition 1 *If* $\overline{W}=\infty$ *there is a unique equilibrium.*

Proposition 2 *With closed capital markets, there is a unique equilibrium.*

Proposition 1 imply that if wages are not constrained by the minimum wage $\overline{w}$ there is a unique equilibrium. This represents the case of flexible wages. With flexible wages, the welfare theorems hold, and there is a unique Pareto optimum allocation, which has to be the competitive equilibrium.

Proposition 2 indicates that in a closed economy there is a unique equilibrium. Intuitively, in this case consumption of the two goods equal output, and this eliminates endogenous movements in tradable consumption, which lead to relative price movements that create the source of multiplicity.

The next section clarifies the existence of multiplicity in a two-period version of the model.

3 Two-Period Example

To illustrate the source of multiplicity, we take a short detour and present a two-period version of the model. In particular, we assume $\beta = 1, R = 1$, no uncertainty, and that $\overline{w}_2$ is low enough so that the economy is in full employment in period 2. We also assume that the government spends g^T and g^N in periods 1 and 2 this is financed with lump sum taxes.

The problem of the household is as follows:

$$\begin{aligned} & \max_{c_1,c_2,n_1\leq 1,n_2\leq 1} u(C_1) + u(C_2) \\ c_1^T + c_2^T + c_1^N p_1^N + c_2^N p_2^N &= y_1^T + y_2^T + w_1 n_1 + \pi_1 + w_2 n_2 + \pi_2 \end{aligned}$$

The solution of the model can be reduced to a system of two equations and two unknowns c_1^T and h_1^T—the complete description of this two period model is described in the appendix. The consumption allocations c_2^T, c_1^N are determined using the budget constraint and the production function, and the prices p_1^N, p_2^N are determined using condition **??**.

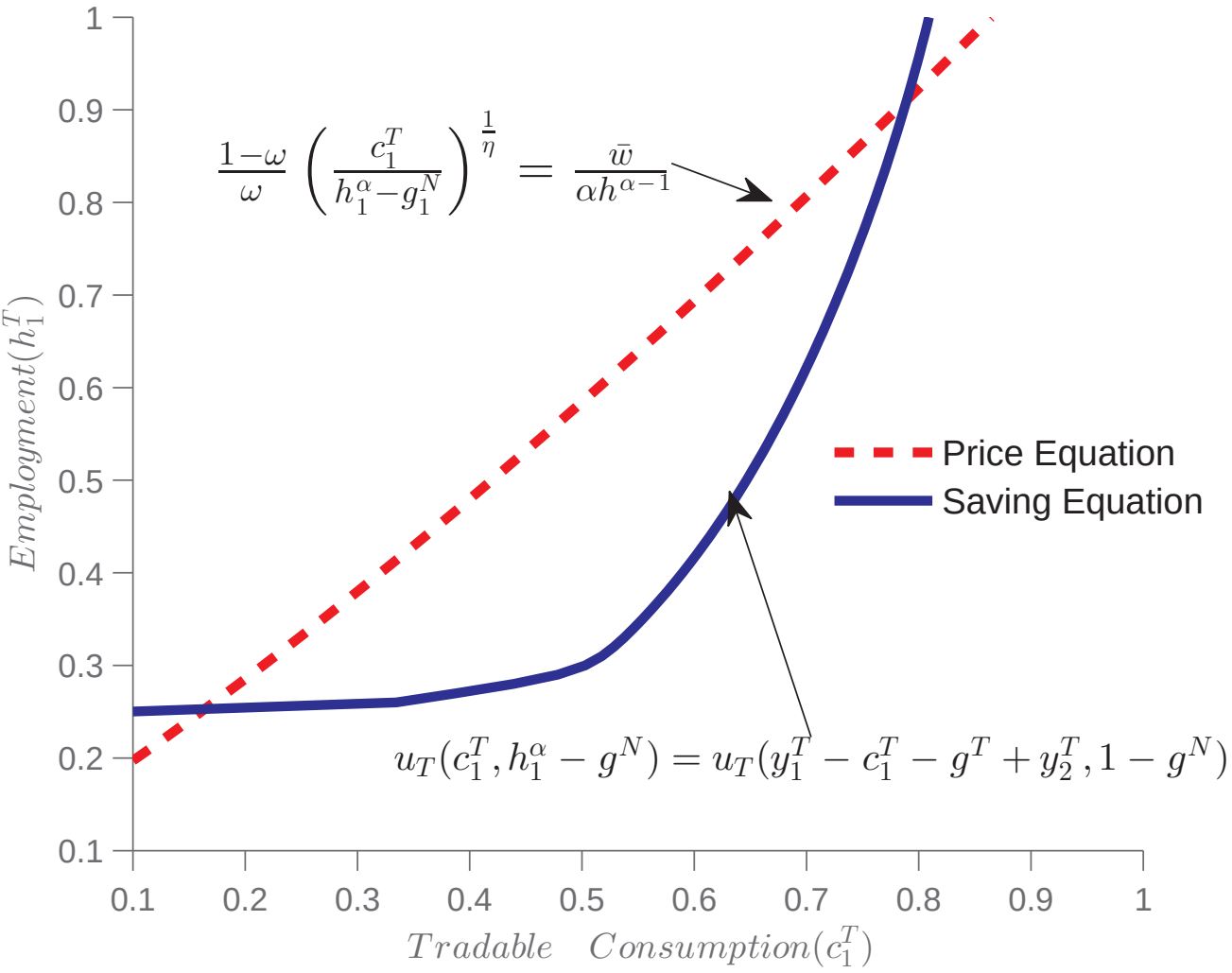
$$\begin{aligned} \text{Price Equation} \qquad \frac{1-\omega}{\omega}\left(\frac{c_1^T}{h_1^\alpha - g_1^N}\right)^{1/\eta} &= \frac{\overline{w}}{\alpha h_1^{\alpha-1}} \\ \text{Saving Equation} \qquad u_T(c_1^T, h_1^\alpha - g^N) &= u_T\big(\big(y_1^T - c_1^T - g^T\big) + y_2^T, 1 - g^N\big) \end{aligned}$$

We make the following assumption, which is a necessary condition for multiple equilibria.

Assumption 1. *The utility function satisfied $u_{TN} > 0$, a condition that requires $\eta < \frac{1}{\sigma}$.*

Figure shows that there are two solutions for the system as the Saving Equation and the Price equation intersects twice. First, notice that the Price equation is upward sloping: A higher consumption of tradables in period 1 raises the price of non-tradables, which in turn leads to higher demand for employment in period 1. Second, notice that the Saving Equation is upward sloping. . That is, if the marginal utility of tradable consumption is increasing in the amount of non-tradable consumption—which is guaranteed under Assumption 1, the Saving Equation implies a positive relationship between period 1 tradable consumption and period 1 employment. When employment is higher in period 1, this leads to a higher supply of non-tradable goods, higher consumption of non-tradables, and in turn a higher demand for tradable consumption. Depending on the rest of the parameters, these curves may intersect multiple times leading to multiple equilibria.

Why does multiple equilibria exist? Expectations of higher employment leads to higher non-tradable consumption, and higher demand for tradable consumption. In turn, higher tradable consumption leads to higher price of non-tradables, raising the value of the marginal product of labor and the demand for employment. Hence, this rationalizes the expectations of higher employment.


$\frac{1-\omega}{\omega}\left(\frac{c_1^T}{h_1^\alpha - g_1^N}\right)^{\frac{1}{\eta}} = \frac{\bar{w}}{\alpha h^{\alpha-1}}$
Price Equation
Saving Equation
$u_T(c_1^T, h_1^\alpha - g^N) = u_T(y_1^T - c_1^T - g^T + y_2^T, 1 - g^N)$
$Employment(h_1^T)$
$Tradable \quad Consumption(c_1^T)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

3.1 Special Case

We provide a special case where we show analytically there is a continuum of equilibria with different rates of unemployment. Specifically, we consider the following set of assumptions

Assumption 2.

- *Linear production function,* $y^N = h$
- *Linear utility in the aggregate consumption bundle and Leontief preferences over tradables and non-tradables. Preferences are given by:*

$$\begin{aligned} U &= C_1 + C_2, \\ C_t &= \min(c_t^T, c_t^N) \end{aligned}$$

- *Fixed rigid wages:* $\bar{w}_1 = \bar{w}_2 = \bar{w}$*, tradable endowment* $y_1^T = y_2^T = y^T$*, and* $R = 1$

Proposition: Under Assumption 2, there is a continuum of equilibria where $p_1^N = p_2^N \equiv p = \bar{w}$, and allocations satisfy $(c_1^N, c_1^T, c_2^N, c_2^T, n_1, n_2) = \phi(2y^T, 2y^T, 0, 0, 2y^T, 0) + (1-\phi)(0, 0, 2y^T, 2y^T, 0, 2y^T) \forall \phi \in [0, 1]$

4 Calibration

5 Quantitative Results

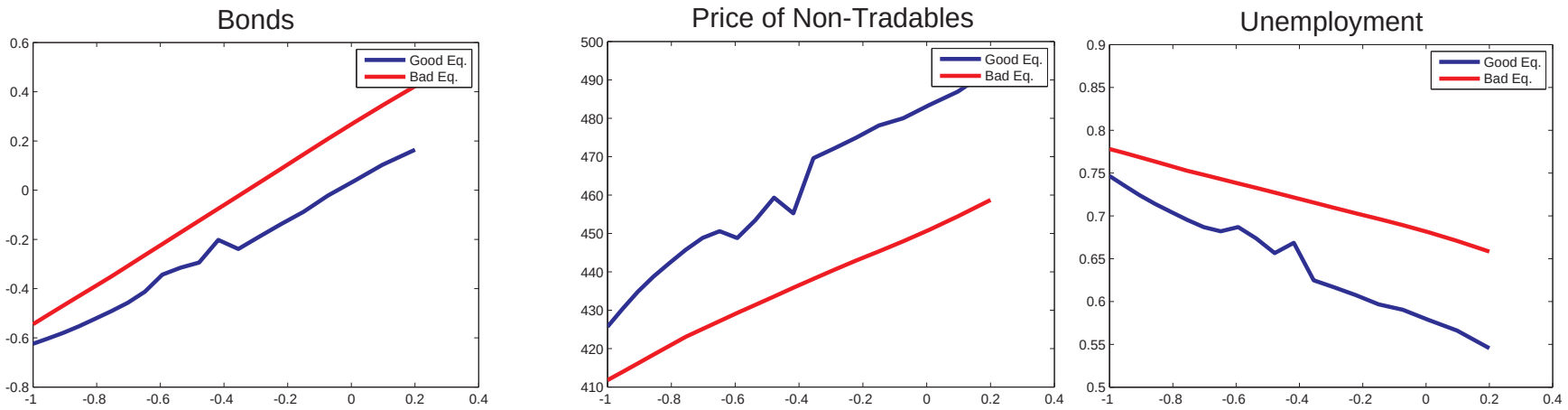


Figure 1: Policy Functions—Saving, Price of non-tradables and Unemployment for current bond choices good vs. bad equilibrium. Policies conditioned on average shocks for TFP, interest rate and a high minimum wage.

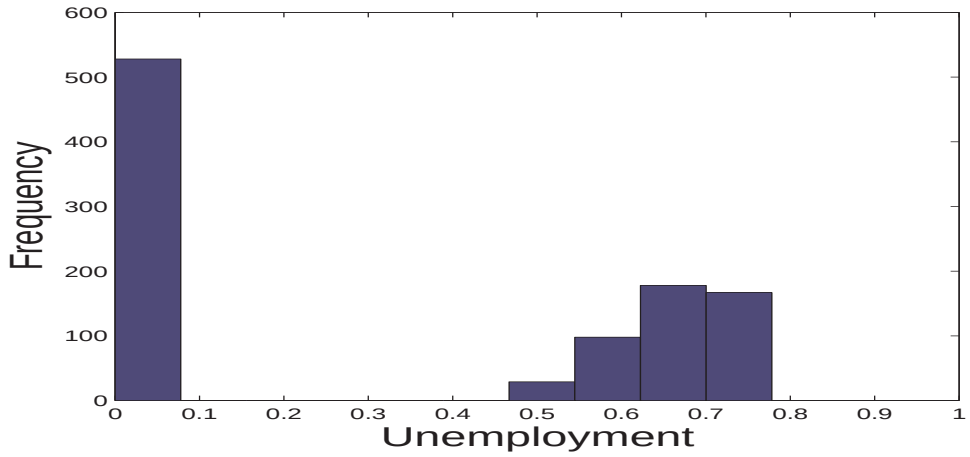


Figure 2: Distribution of Unemployment

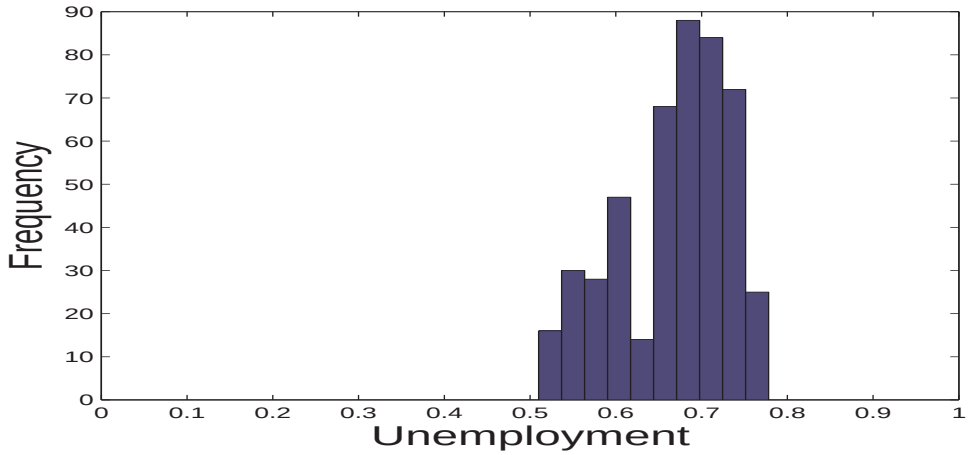


Figure 3: Distribution of Unemployment conditional on high minimum wage

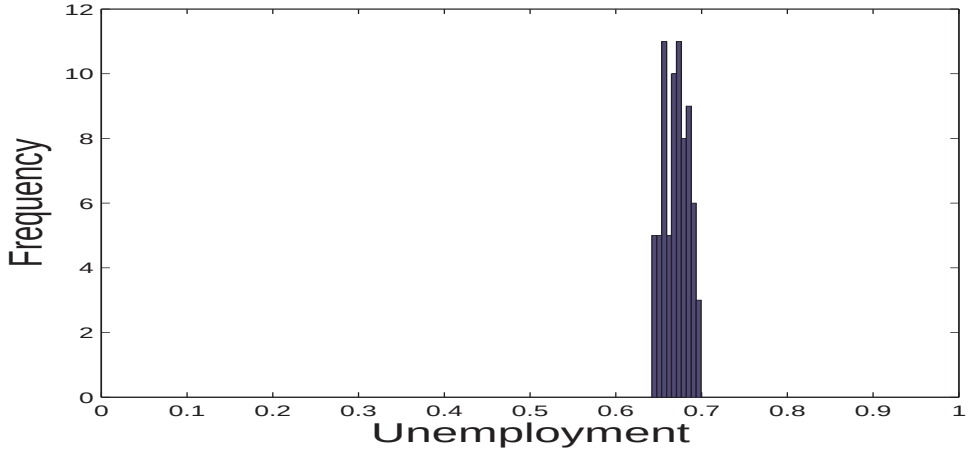


Figure 4: Distribution of Unemployment conditional on bad equilibrium

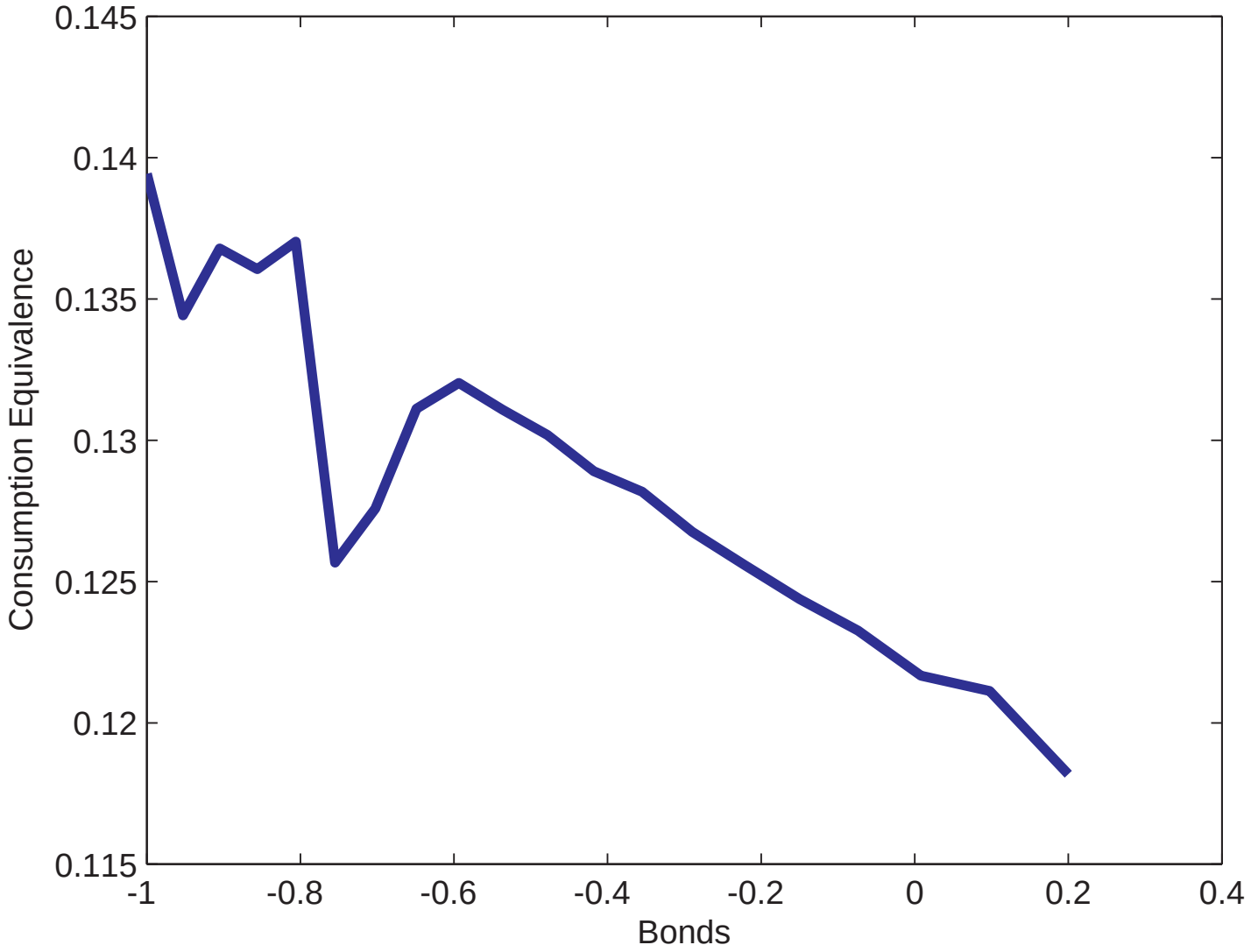


Figure 5: Welfare Gains of Restoring Good Equilibrium

6 Conclusions

References

Bengino, G. and Fornaro, L. (2015). 'Twin Traps in a Keynesian Growth Model'.

Benhabib, J. and Farmer, R. E. (1999). 'Indeterminacy and sunspots in macroeconomics'. *Handbook of macroeconomics*, vol. 1, 387–448.

Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2001). 'The perils of Taylor rules'. *Journal of Economic Theory*, vol. 96, no. 1, 40–69.

Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2002). 'Avoiding liquidity traps'. *Journal of Political Economy*, vol. 110, no. 3, 535–563.

Eggertsson, G. B. and Krugman, P. (2012). 'Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach*'. *The Quarterly Journal of Economics*, vol. 127, no. 3, 1469–1513.

Eggertsson, G. B. and Mehrotra, N. R. (2014). 'A model of secular stagnation'. Technical report, National Bureau of Economic Research.

Eggertsson, G. B. et al. (2003). 'Zero bound on interest rates and optimal monetary policy'. *Brookings Papers on Economic Activity*, vol. 2003, no. 1, 139–233.

Farmer, R. E. (2012). *The evolution of endogenous business cycles*. Cambridge Univ Press.

Farmer, R. E. (2013). 'Animal Spirits, Financial Crises and Persistent Unemployment*'. *The Economic Journal*, vol. 123, no. 568, 317–340.

Hall, R. E. (2005). 'Employment fluctuations with equilibrium wage stickiness'. *American economic review*, pages 50–65.

Hansen, A. H. (1939). 'Economic progress and declining population growth'. *The American Economic Review*, pages 1–15.

Heathcote, J. and Perri, F. (2014). 'Wealth and volatility'. Mimeo, Minneapolis Fed.

Kaplan, G. and Weidner, J. (2014). 'The Wealthy Hand-to-Mouth'. *Brookings Papers on Economic Activity*.

Kocherlakota, N. (2009). 'Bursting bubbles: Consequences and cures'.

Schmitt-Grohé, S. and Uribe, M. (2011). 'Pegs and pain'.

Schmitt-Grohé, S. and Uribe, M. (2013). 'Downward nominal wage rigidity, currency pegs, and involuntary unemployment'. *Unpublished manuscript, Columbia University*.

Shimer, R. (2012). 'Wage rigidities and jobless recoveries'. *Journal of Monetary Economics*, vol. 59, S65–S77.

7 Appendix

8 Proofs (incomplete)

8.1 Proof of Proposition 1

Flexible wages imply that $h_t = 1$. In this case, welfare theorems hold and the existence of a unique Pareto optimum allocation imply that the equilibrium has to be unique. ■

8.2 Proof of Proposition 2.

In a closed economy

$$c_t^T = y_t^T$$

Lemma 1 (Uniqueness of full employment equilibrium) *The full employment equilibrium is unique.*

Proof. Fully employment implies $h = 1$, $c^N = y^T$, $c^N = 1, p^N = \frac{1-\omega}{\omega} y_1^{T1/\eta}$, $w = \alpha \frac{1-\omega}{\omega} y_1^{T1/\eta}$ ■

Lemma 2 *The unemployment equilibrium is unique.*

Proof. Unemployment steady state requires $\overline{w} < \alpha \frac{1-\omega}{\omega} y_1^{T1/\eta}$. Employment is given by $h = \left(\frac{\overline{w}_t}{\frac{1-\omega}{\omega} \left(c_1^T\right)^{1/\eta}} \right)^{\frac{\eta}{\alpha+\eta(1-\alpha)}}$ ■

Proof of Proposition 2.

Suppose that there are two distinct equilibria $(h_t, p_t^N, w_t, c_t^T, c_t^N)$,$(\tilde{h}_t, \tilde{p}_t^N, \tilde{w}_t, \tilde{c}_t^T, \tilde{c}_t^N)$.Lemma 1 and Lemma 2 imply that if there are two equilibria, one is the full employment and the other one is the unemployment employment. Without loss of generality

suppose $h_t = 1$ and $\tilde{h}_t = \left(\frac{\overline{w}_t}{\frac{1-\omega}{\omega} \left(c_1^T\right)^{1/\eta}} \right)^{\frac{\eta}{\alpha+\eta(1-\alpha)}}$. But $h_t = 1$ imply $\overline{w} > \alpha \frac{1-\omega}{\omega} y_1^{T1/\eta}$, which contradicts that $\tilde{h}_t < 1$. Hence $(h_t, p_t^N, w_t, c_t^T, c_t^N)$=$(\tilde{h}_t, \tilde{p}_t^N, \tilde{w}_t, \tilde{c}_t^T, \tilde{c}_t^N)$ ■

Proof of Proposition 3.

8.3 Two-Period Example

We consider a small open economy with a tradable and a non-tradable sector, a fixed exchange rate regime and downward wage rigidity. We start the analysis with a two-period example to illustrate the source of multiplicity. There are two time periods $t = 1, 2$, and a continuum of identical households of measure one. There is no uncertainty.

Preferences are given by:

$$u(c_1) + u(c_2)$$

Households receive an endowment of tradables y^T, provide labor $n \leq 1$ and receive profits π from firms. Households trade a zero coupon non-state contingent bond denominated in units of tradables. The interest rate on this bond is assumed to be zero. Accordingly, their lifetime budget constraint is given by

$$P_1^T c_1^T + c_1^N P_1^N + P_2^T c_2^T + c_2^N P_2^N = P_1^T y_1^T + W_1 h_1 + \Pi_1 + P_2^T y_2^T + W_2 h_2 + \Pi_2 - T$$

where P^T (P^N) denote the price of tradable (non-tradable) goods in domestic currency, π_t denotes firm's profits, w denotes wages, and n denotes labor supply and T is a lump sum tax from the government.

The law of one price of one price holds for tradable goods, so that $P_t^T = P^{T*} E_t$, where P^{T*} is the price of tradable goods in foreign currency and E_t is the exchange rate defined as the domestic-currency price of one unit of foreign currency. Because the exchange rate is fixed, this implies that $P_1^T = P_2^T$. Denoting by lower case, all prices expressed in units of tradables, we have that households solve the following problem:

$$\begin{aligned} & \max_{c_1, c_2, n_1 \leq 1, n_2 \leq 1} u(c_1) + u(c_2) \\ c_1^T + c_1^N p_1^N + c_2^T + c_2^N p_2^N &= y_1^T + \pi_1 + y_2^T + w_2 n_2 + \pi_2 \end{aligned}$$

Optimality conditions:

$$\begin{aligned} \frac{\omega}{1-\omega} \left(\frac{c_t^T}{c_t^N} \right)^{1/\eta} &= p_t^N \quad t = 1, 2 \\ u_T(c_1) &= u_T(c_2) \end{aligned}$$

where $u_T = \frac{\partial u(c)}{\partial c} \frac{\partial c(c^T, c^N)}{\partial c^T}$. The first condition equates the marginal rate of substitution to the price of non-tradables. Because $\mu > -1$, this implies that a higher price of non-tradables leads to a reduction in the ratio of tradable to non-tradable consumption. Both consumption goods as well as the price of non-tradables are determined in equilibrium. The second condition implies that households at the optimum are indifferent between consuming one more unit of tradables today or saving it and consuming it tomorrow. Notice also that because households do not value leisure, they provide one unit of hours in the market, although in equilibrium they may not be employed full time.

Firm Problem

Firms have a production function h_t^α, with $\alpha < 1$, and maximize profits

$$\max_{h_t \leq 1} P_t^N h_t^\alpha - W_t h_t$$

Firms' first order condition is:

$$P^N \alpha h^{\alpha-1} = W_t$$

Downward wage rigidity—There is a lower bound on equilibrium wages at period 1, so that $W_1 \geq \overline{W}$ (for simplicity wages are flexible in period 2). This constraint captures the fact that wages are downward sticky due to minimum wage regulations or because of the reluctance of employers on reducing wages. This constraint produces involuntary unemployment whenever the wage that would clear the labor market is lower than the minimum wage. That is,

$$(W_1 - \overline{W})(h_t - 1) = 0$$

Government budget constraint—Government taxes in the first period and conducts wasteful government spending in the first period:

$$T_1 = P^T g_1^T + P^N g_1^N$$

Non-tradable goods market must clear

$$c_t^N = h_t^\alpha - g^N$$

A non-walrasian equilibrium in this economy is a collection of prices and allocations so that:

$$\begin{aligned}
p^N \alpha h^{\alpha-1} &= w_t \\
c_t^N &= h_t^\alpha - g_t^N \\
\frac{\omega}{(1-\omega)} \left(\frac{c_t^T}{c_t^N} \right)^{1/\eta} &= p_t^N \\
u_T(c_1) &= u_T(c_2) \\
(w_1 - \overline{w})(h_t - 1) &= 0 = 0 \\
c_1^T + g^T + b &= y_1^T \\
c_2^T &= y_2^T - b' \\
h &\leq 1
\end{aligned}$$

Solution of the Model—

If the wage constraint is not binding and assuming $g^T = g^N$ and $y_t^T = y^T$, the equilibrium is $h_t = 1$, $c_t^N = 1 - g^N$, $c^T = y^T$. The interesting case is when the wage constraint is binding. In this case, the model can be solved in a block recursive way. Specifically, we can express the solution of the model in terms of the system of two equations and two unknowns c, h.

$$\begin{aligned}
\frac{1-\omega}{\omega}\left(\frac{c_1^T}{h^\alpha - g_1^N}\right)^{1/\eta} &= \frac{\overline{w}}{\alpha h^{\alpha-1}} \\
u_T(c_1^T, h_1^\alpha - g^N) &= u_T(\left(y_1^T - c_1^T - g^T\right) + y_2^T, 1 - g^N)
\end{aligned}$$

The first condition follows from combining the firm first order condition (), market clearing condition (), and the demand condition (). The second condition follows from combining the Euler condition, with $c_2 = \left(y_1^T - c_1^T - g^T\right) + y_2^T$ from the budget constraint and market clearing conditions for non-tradable goods.