In Search of a Nominal Anchor: What Drives Inflation Expectations?*

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Abstract

According to both central bankers and economic theory, anchored inflation expectations are key to successful monetary policymaking. Yet, we know very little about the determinants of those expectations. We explore a theory of expectations formation that can produce episodes of unanchoring. Its key feature is state-dependency in the sensitivity of long-run inflation expectations to short-run inflation surprises. Price-setting agents act as econometricians trying to learn about average long-run inflation. They set prices according to their views about future inflation, which hence feed back into actual inflation. When expectations are anchored, agents believe there is a constant long-run inflation rate, which they try to learn about. However, in the spirit of Marcet and Nicolini (2003), a long enough sequence of inflation suprises leads agents to doubt the constancy of long-run inflation, and switch to putting more weight on recent developments. As a result, long-run inflation expectations become unanchored, and start to react more strongly to short-run inflation surprises. Shifts in agents' views about long-run inflation feed into their price-setting decisions, imparting a drift to actual inflation. Hence, inflation can show persistent swings away from its long-run mean. We estimate the model using actual inflation data, and **only short-run** inflation forecasts from surveys, for various countries. The estimated model produces long-run forecasts that track survey measures extremely well.

The estimated model has several uses:

1) It can tell a story of how inflation expectations got unhinged in the 1970s; it can also be used to construct a counterfactual history of inflation under anchored long-run expectations.

2) At any given point in time, it can be used to compute the probability of unanchoring episodes, that may produce inflation or deflation scares.

3) When embedded into an environment with explicit monetary policy, it can also be used to study the role of policy in shaping the expectations formation mechanism.

JEL classification codes:

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"Public confidence that inflation will remain low in the long run has numerous benefits. Notably, if people feel sure that inflation will remain well controlled, they will be more restrained in their wage-setting and pricing behavior, which (in something of a virtuous circle) makes it easier for the Federal Reserve to confirm their expectations by keeping inflation low. At the same time, by reducing the risk that inflation will come loose from its moorings, well-anchored inflation expectations may afford the central bank more short-term flexibility to respond to economic disturbances that affect output and employment." Ben Bernanke, former Governor of the Federal Reserve, October 7, 2004.[1]

1 Introduction

According to both central bankers and economic theory, anchored inflation expectations are key to successful monetary policymaking. As long as long-run expectations are anchored, monetary policy has more leverage to respond to short run disturbances to the economy. Yet, we know very little about the determinants of those expectations. While policymakers may take some comfort in the stability of long-run inflation expectations, it should not be taken for granted, as it is not an inherent feature of the economy.

What does it take for expectations to become unanchored? In this paper we explore a theory of expectations formation based on learning, which embeds a well-defined notion of anchored and unanchored expectations. Its key feature is state-dependency in the sensitivity of long-run inflation expectations to short-run inflation surprises. When estimated on actual inflation and **short-run** inflation forecasts from surveys, the model captures long-term inflation expectations extremely well. Hence, despite its simplicity, the model provides a useful framework to think about the actual expectations formation process.

In the model, price-setting agents act as econometricians trying to learn about average long-run inflation. In response to short-run inflation surprises, they update their views about future inflation, and set prices accordingly – hence, expectations about the whole path of inflation feed back into actual inflation. The key feature of the model is the fact that the sensitivity of long-run inflation expectations to short-run inflation surprises is state-dependent. This allows for an explicit criterion to determine whether expectations are anchored.

When expectations are anchored, agents believe there is a constant long-run inflation rate, which they try to learn about. Hence, their estimates of long-run inflation move relatively more slowly, as they keep expanding the sample they use in estimation. For the reader familiar with the jargon from the learning literature, this amounts to a decreasing-gain algorithm. As a result, the sensitivity of long-run expectations to short-run inflation surprises decreases over time, along with the learning gain.

In the spirit of Marcet and Nicolini (2003), however, a sizeable enough sequence of inflation suprises leads agents to doubt the constancy of long-run inflation and start putting more weight on recent inflation developments – i.e., they switch to a constant-gain learing algorithm. As a result, long-run inflation expectations start to react more strongly to short-run inflation surprises – i.e.,

[1] Speech available here: http://www.federalreserve.gov/Boarddocs/speeches/2004/200410072/default.htm.

they become unanchored. Shifts in agents' views about long-run inflation feed into their price-setting decisions, imparting a drift to actual inflation. Hence, actual inflation can show persistent swings away from its long-run mean.

The reduced-form representation of the model features a time-varying drift in inflation. For that reason, our framework makes contact with the literature on inflation dynamics that assumes an exogenous, time-varying inflation drift (e.g., Cogley and Sbordone 2006, Cogley, Primiceri, and Sargent 2007). However, in our model that drift is determined endogenously, and its persistence and volatility depend on the state of the economy. Specifically, innovations to the drift are related to innovations to actual inflation, and so is the time-variation in inflation persistence.

The restrictions alluded to in the previous paragraph constrain our model significantly, relative to similar models with an exogenous inflation drift. More fundamentally, however, we discipline our model through our estimation exercise. Specifically, we estimate the model using actual inflation data, and **only short-run** inflation forecasts from surveys. We do so for various coutries for which we have inflation forecasts from survey data. Hence, the dynamics of anchoring and unanchoring, which depend on the sequence of inflation surprises, is pinned down by observation of short-term forecast errors from surveys. These restrictions could be expected to handicap the model in terms of fitting data on inflation and inflation expectations, relative to models with an exogenous inflation drift. Perhaps surprisingly, our estimated model fits the data essentially as well as specifications with an exogenous drift.

More fundamentally, our model fits long-run inflation forecasts from survey data – which we do not use in the estimation – extremely well. In that sense, it provides a framework to think about the actual expectations formation process. The estimated models provide stories for the joint evolution of inflation and the term structure of inflation expectations for different countries, which can enhance our understanding of the role of long-run expectations in determining actual inflation. This is so because of the feedback from agents' expectations into actual inflation, through their price-setting decisions. So, for example, the model suggests that the unanchoring of inflation expectations was key to the high inflation of the 1970s. It also provides an estimate of when inflation expectations became anchored again, which accords with common wisdom. Despite the reduced-form nature of the model that we take to the data, we also find surprising that the few parameters that call for a "structural" interpretation are somewhat comparable across countries.

While in this paper we focus on "inflation surprises" as the fundamental inovations to the expectations formation process, this model of expectations formation can be embedded into an environment with structural shocks and explicit monetary policy. This would allow us to evaluate the role of policy in shaping the response of long-run expectations to different shocks.

1.1 Literature review

[To be added]

2 A simple model of expectations formation and inflation determination

In this section we present a simple reduced-form model of expectations formation and inflation determination. Because of nominal price rigidities, agents pricing decisions reflect their expectations of future inflation. This creates a link from the latter to actual inflation. In subsection 2.2 we derive this link explicitly in a model with Calvo (1983) pricing. That model also illustrates how our assumptions on agents' views about inflation amount to them being quite "close" to rational expectations – departing from them only because of the need to estimate the long-run average rate of inflation.

Price-setting agents perceive the law of motion of inflation to be

$$\text{PLM:} \qquad \pi_t = \bar{\pi}_t + \varphi_t, \tag{1}$$

where π_t is inflation, φ_t is a zero mean stationary "short-run component", and $\bar{\pi}_t$ is the average level of inflation that they expect to prevail in the long run. Agents need to learn about (i.e., estimate) this long-run mean and, possibly, the law of motion for φ_t.

For simplicity, we adopt the usual "anticipated utility assumption": Although agents realize that they will be updating their estimate of $\bar{\pi}_t$ as data come in, they expect $\bar{\pi}_t$ to remain constant. Hence, they form expectations of inflation in the future as

$$\hat{E}_t \left[\pi_{t+T}\right] = \bar{\pi}_t + \hat{E}_t \left[\varphi_{t+T}\right],$$

where the operator $\hat{E}_t$ denotes their subjective expectations.

These agents set prices that will remain in place for at least a few periods. So, whenever an agent is resetting her price at a given time t, she embeds her expectations of inflation going forward into her price-setting decision. This makes actual inflation depend on price-setters' expectations of future inflation. Suppose that this dependence arises only through the perceived long-run inflation average, so that the actual law of motion of inflation is:

$$\text{ALM:} \qquad \pi_t = T\bar{\pi}_t + \varphi_t, \tag{2}$$

where φ_t and $\bar{\pi}_t$ have been defined before, and $T < 1$ is the parameter that controls the feedback from inflation expectations to actual inflation – it depends on the details of firms' optimal price-setting policies. Notice that agents' perceived law of motion and the actual law of motion for inflation differ only in that the time-varying intercept in the actual law of motion for inflation is $T\bar{\pi}_t$ rather than $\bar{\pi}_t$. Hence, if T is close to unity these differences will usually be small.

It remains to specify the process by which agents update their estimates of average long-run inflation. We borrow from Marcet and Nicolini (2003), and assume the following learning algorithm:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1}, \tag{3}$$

where

$$f_t = \pi_t - \hat{E}_{t-1}\pi_t.$$

As usual in models with learning of this kind, we avoid simultaneity issues by assuming that the $\bar{\pi}_t$ estimate (which affects current decisions) depends on the previous period's forecast error, f_{t-1}. The learning gain $k_t > 1$ is determined by

$$k_t = \begin{cases} \bar{g}^{-1}, \text{ if } \frac{\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right|}{\sqrt{\mathbb{MSE}}} > v \\ \\ k_{t-1} + 1, \text{ otherwise} \end{cases},$$

where $\hat{E}_{t-1}\pi_t$ is the agent's one-period-ahead forecast, $\mathbb{E}_{t-1}\pi_t$ is the model-consistent one-period-ahead expectation, and $\mathbb{MSE}$ is the mean squared error associated with one-period-ahead model-consistent expectations – i.e. $\mathbb{MSE} = \mathbb{E}\left[\pi_t - \mathbb{E}_{t-1}\pi_t\right]^2$.

The state-dependent gain k_t captures agents' attempts to protect against "structural change". A constant gain $\bar{g}$ produces better forecasts when the economic environment changes, but it does not converge in a stationary environment. A decreasing gain (OLS-type) estimator, on the other hand, converges in stationary environments. In the spirit of Marcet and Nicolini (2003), the proposed learning algorithm uses OLS in periods or relative stability and switches to constant-gain when instability is "detected" – i.e., when $\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right| > v\ \sqrt{\mathbb{MSE}}$. A stable environment here is an economy where inflation does not vary too much. So that the distance between agents' forecast and the model-consistent forecast tends to be small. The updating rule (3) can also interpreted in terms of the Kalman filter. Agents' model of the inflation drift is

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \eta_t \tag{4}$$

where the variance of η_t can take two values corresponding to different "regimes": $\sigma_\eta^2 = \{\bar{\sigma}_\eta^2, 0\}$. In the first regime $\bar{\pi}_t$ drifts according to a random walk. The constant gain algorithm can be interpreted as the Kalman updating of this model. The second regime corresponds to a constant mean for inflation, giving a decreasing gain – see Bullard (1992) for a discussion. The updating rule (3) is, however, not optimal because it does not fully internalize the regime switching in the variance of η_t.

We now make two simplifying assumption that we maintain throughout the paper.

Assumption 1. The process φ_t is exogenous and, in particular, is determined by

$$\begin{aligned} \varphi_t &= s_t + \mu_t \\ \\ s_t &= \rho_s s_{t-1} + \epsilon_t, \end{aligned}$$

where μ_t is an i.i.d. disturbance, normally distributed with mean zero and variance σ_μ^2.

Assumption 2. Agents conditional expectations for φ_t coincide with the model-consistent expectation, that is

$$\hat{E}_t\left[\varphi_{t+T}\right] = \mathbb{E}_t\left[\varphi_{t+T}\right] \text{ for } T \geq t.$$

To obtain more intuition on the properties of the postulated learning algorithm, we can use the above assumptions to write:

$$\begin{aligned}
\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right| &= (1-T)\,\bar{\pi}_t \\
&= (1-T)\left(\bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1}\right) \\
&= (1-T)\left[\bar{\pi}_0 + \sum_{\tau=0}^{t} k_\tau^{-1} f_\tau\right];\ \text{given } \bar{\pi}_0,\ f_0, k_0.
\end{aligned}$$

Hence, the distance $\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right|$ tends to be large when forecast errors happen to be of the "same sign" for a few periods. This pushes $\bar{\pi}_t$ away from its long-run value (of zero or otherwise), and drives a larger wedge between agents' model of the economy and the truth.

One may wonder how agents can switch to a constant gain algorithm based on a criterion that involves model-consistent expectations – which they are assumed not to know. This makes this assumption belong to the "as if" tradition in Economics. Agents indeed do not know $\mathbb{E}_{t-1}\pi_t$, but they conclude that there must be something unusual happening when their forecasts "go astray" in the sense that $\left|\hat{E}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t\right|$ becomes too large. This assumption captures agents' efforts to deter model instability by using statistical tools to detect time-variation in their model's intercept. They key here is that this term depends on past forecast errors. In turn, the size of forecast errors depends on the gain being used by agents. This opens the door for multiple "learning equilibria", as discussed below.

We consider parameters' restrictions for which agents' learning converges to a constant inflation rate. The model thus implies a unique steady state. Our third assumption is then that the economic environments we study have a time invariant long-run mean of inflation that is eventually learned by market participants.

Assumption 3. The data generating process for inflation has a time-invariant mean $\pi = 0$.

2.1 Relationship with models with an exogenous "inflation drift"

We now relate our model to models with an *exogenous* "inflation drift". These include reduced-form models as in Stock and Watson (1993, 2007) and Cogley and Sargent (2006), Kozicki and Tinsley (2013), models of the Phillips curve (Cogley and Sbordone, 2006) and DSGE models (Smets and Wouters 2003 and Del Negro et. al, 2014 among the others). In these models $\bar{\pi}_t$ evolves exogenously according to

$$\bar{\pi}_t = \rho_{\bar{\pi}}\bar{\pi}_{t-1} + e_t; \rho_{\bar{\pi}} \approx 1. \tag{5}$$

In contrast, in the model studied here the inflation drift is determined *endogenously* and it depends on agents past inflation forecast errors. To see this, we can re-write the agents' algorithm (3) as

$$\begin{aligned} \bar{\pi}_{t+1} &= \bar{\pi}_t + k_t^{-1}\left(\pi_t - \hat{E}_{t-1}\pi_t\right) \\ &= \rho_{\bar{\pi},t}\bar{\pi}_t + \tilde{e}_t, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \rho_{\bar{\pi},t} &= \left[1 + k_t^{-1}\left(T_\pi - 1\right)\right]; \\ \tilde{e}_t &= k_t^{-1}\left(\varphi_t - \mathbb{E}_{t-1}\varphi_t\right) = k_t^{-1}\left(\epsilon_t + \mu_t\right). \end{aligned}$$

Notice that the second equation is obtained by substituting (2) for inflation and by using assumptions 1-2 described in the previous section. The evolution of $\bar{\pi}_t$ under (6) differs in two key aspects from (5). First, its autocorrelation coefficient $\rho_{\bar{\pi},t}$ and the volatility of its innovation are time-varying and depend on the state of the economy. Second, the innovations to $\bar{\pi}_t$ depend on inflation's forecast errors. For example, persistent shocks to the economy and policy mistakes creating unexpected movements in inflation can trigger movements in the inflation drift.

2.2 A model with explicit price-setting decisions

This section develops a model where the link from expectations to actual inflation assumed above arises explicitly in the context of Calvo (1983) pricing, as in Preston (2005). Firm i maximizes the present discounted value of profits

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T}\left[Y_T(i)\left(\frac{P_t(i)}{P_T} - MC_T\right)\right],$$

where $Q_{t,T}$ is the discount factor, MC_t is the real marginal cost and

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_{p,t}} Y_t$$

is the demand curve that each firm faces. Its elasticity $\theta_{p,t}$ is time-varying. Each period the firm's price is reset optimally with probability α, or it is set to evolve according to a weighted average of past inflation and the perceived long-run inflation rate:

$$\bar{\Pi}_t^p = \bar{\pi}_t^{1-\gamma_p} \pi_{t-1}^{\gamma_p}.$$

To keep things simple, we assume a constant mean of zero inflation. In log-linear deviation from this steady state, the first-order conditions of this problem deliver the following price-setting decision

for price re-setting firms:

$$\hat{p}_t^* = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1-\alpha\beta)(mc_T + u_T) + \alpha\beta \left(\pi_{T+1} - \gamma_p \pi_T - (1-\gamma_p)\bar{\pi}_t \right) \right], \tag{7}$$

where u_t denotes a cost push shock arising from time-varying elasticity of demand. Aggregating across firms delivers the following data generating process for inflation:

$$\pi_t = \gamma_p \pi_{t-1} (1-\gamma_p)\bar{\pi}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa (mc_T + u_T) + (1-\alpha)\beta \left(\pi_{T+1} - \gamma_p \pi_T - (1-\gamma_p)\bar{\pi}_t \right) \right], \tag{8}$$

where $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$.

We close the model by specifying the evolution of marginal cost, which is assumed to follow

$$\lambda \pi_t + mc_t = \tau_t, \tag{9}$$

where $\lambda > 0$ is a parameter, and τ_t is exogenous and evolves according to

$$\tau_t = \rho_\tau \tau_t + \epsilon_t^\tau. \tag{10}$$

This setup can be mapped into a popular specification in the literature. In the simple new Keynesian model we can interpret (9) as deviations from a targeting rule under discretion with a constant target of zero inflation. The looser interpretation is that we allow some feed-back from inflation to the marginal costs, as a reduced-form way to capture some policy responses. This specification (with $\lambda > 0$) is useful to control the inflation to drift (discussed below).

What do agents know? To keep the framework as simple as possible, and in line with the previous section, we assume they know everything except for the long-run mean of inflation. Agents estimate a long-run inflation mean $\bar{\pi}_t$ in every period. They form expectations of marginal costs and inflation using the correct law of motion for the marginal cost:

$$\lambda \pi_t + mc_t = \tau_t \quad \rightarrow \quad mc_t = \tau_t - \lambda \pi_t,$$

and the following equilibrium *perceived* law of motion (PLM) for inflation:

$$\pi_t = \left(1-\gamma_p\right)\bar{\pi}_t + \gamma_p \pi_{t-1} + \omega \tau_t + e_t,$$

where $\omega = \tilde{\kappa}\left(1 - \tilde{\beta}\rho_\tau\right)^{-1}$ denotes the rational expectations coefficient (solved for by matching coefficients) and e_t is an *i.i.d.*innovation. Substituting agents' forecasts in (8) under the usual "anticipated utility" assumption for $\bar{\pi}_t$ we obtain the *actual* law of motion (ALM):

$$\pi_t = T\bar{\pi}_t + \tilde{\kappa}\left(1 - \tilde{\beta}\rho_\tau\right)^{-1} \tau_t + \tilde{u}_t,$$

where

$$T = \left(\frac{1}{1+\lambda\kappa} - \frac{\tilde{\beta}(1-\alpha)}{1-\alpha\beta}\right) + \left(\tilde{\beta}(1-\alpha) - \tilde{\kappa}\alpha\beta\lambda\right) \times \\ \times \frac{1-\gamma_p\alpha\beta}{1-\gamma_p}\left(\frac{1}{1-\alpha\beta} - \frac{\gamma_p}{1-\alpha\beta\gamma_p}\right).$$

The ALM displays a time-varying intercept that differs from the PLM's. Notice that a stronger marginal cost response to inflation (higher λ) implies less feed-back from beliefs to inflation – see Ferrero (2007), Orphanides and Williams (2005). Similarly, a lower degree of indexation to $\bar{\pi}_t$ (higher γ_p) implies smaller feedback effects from learning.

3 Data and estimation

As mentioned in the Introduction, the estimation strategy involves using data on actual inflation and measures of short-term inflation expectations to estimate the model parameters. We then evaluate its predictions for long-term forecasts (*not used* in the estimation). The model is estimated both for the US and a selected number of foreign countries, using quarterly data. For all countries, inflation is measured by CPI. This choice is driven by the availability of survey-based forecasts. Short-term expectations are measured using consensus (mean) forecast from different surveys of professional forecasters.

For the US, we use two sources for short-term CPI inflation forecasts. The Livingston survey provides the longest-series for short-term forecasts. The dataset contains forecasts for the price level. From those, two series measuring 6-months ahead CPI inflation forecasts are used. The first series, available starting in 1955, computes the inflation forecasts from a base period, which is the last monthly price level known at the time the survey was fielded. As this might not capture the information set of the forecasters, we use an additional series for the six-months ahead forecast (only available starting in 1992). For this series, the forecasted inflation rate is computed in deviation from the price level forecast for the month in which the survey was taken. These forecasts are available twice a year (in the second and fourth quarter). We also use one- and two- quarters ahead CPI inflation forecasts from the Survey of Professional Forecasters, which are available at a quarterly frequency since 1981Q3. **Figure 1** shows the available data used for the estimation.

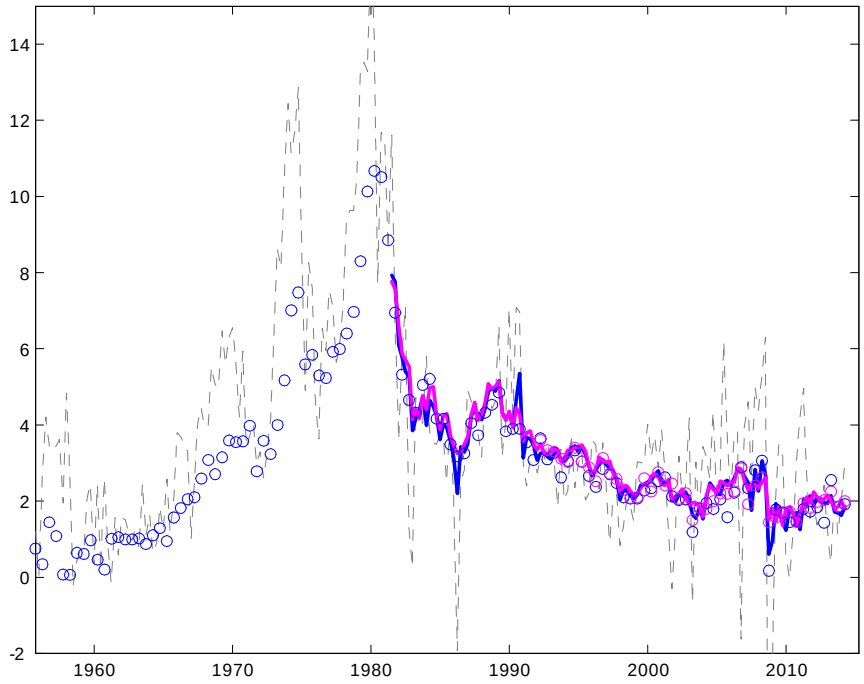


Figure 1: The figure shows the evolution of CPI inflation (dashed grey line) and consensus (mean) survey-forecasts for inflation. In detail, the one- and two-quarters ahead forecasts from SPF are shown by the solid blue and magenta lines, respectively. Te blue and magenta circles show the six months-ahead forecasts from the Livingston survey.

Based on the discussion in the previous section, we estimate the following model

$$\begin{aligned}
\pi_t &= \left(1-\gamma_p\right) T\bar{\pi}_t + \gamma_p \pi_{t-1} + \varphi_t \\
\bar{\pi}_t &= \bar{\pi}_{t-1} + k_{t-1}^{-1} \times f_{t-1} \\
f_t &= \left(1-\gamma_p\right)(T-1)\,\bar{\pi}_t + \mu_t + \epsilon_t \\
\varphi_t &= s_t + \mu_t \\
s_t &= \rho_s s_{t-1} + \epsilon_t,
\end{aligned}$$

where ϵ_t and μ_t are normally distributed with zero mean and variances σ_s^2 and σ_μ^2. The model is estimated by casting the system in state-space form:

$$\boldsymbol{\xi}_t = F(\mathbf{k}_{t-1}^{-1})\boldsymbol{\xi}_{t-1} + S_C\boldsymbol{\epsilon}_t,$$

where the *time-varying gain* $\mathbf{k}_t$ *is pre-determined at time* t. The model can be then estimated

with full-information Bayesian estimation methods by using the standard Kalman filter recursions (details will be added to the Appendix). The observation equation for US data is then

$$Y_t^{US} = \begin{bmatrix} \pi_t \\ \mathbf{E}_t^{SPF}\pi_{t+1} \\ \mathbf{E}_t^{SPF}\pi_{t+2} \\ \mathbf{E}_t^{LIV_1}\left(\frac{1}{2}\sum_{i=1}^{2}\pi_{t+i}\right) \\ \mathbf{E}_t^{LIV_2}\left(\frac{1}{2}\sum_{i=1}^{2}\pi_{t+i}\right) \end{bmatrix} = \pi^* + H_t'\xi_t + R_t o_t,$$

where the survey-forecasts labelled "LIV_1" uses the actual price level as a base, and "LIV_2" uses the forecast (see above). The vector o_t includes observation errors for all variables. We include an observation error on CPI inflation as well as on the survey-based forecasts as it allows the model to filter a measure of underlying inflation that drives short-term inflation expectations. The matrices H_t and R_t are time-varying because of missing observations. We seek to estimate the following vector of structural parameters $\bar{\theta} = \begin{pmatrix} \pi^* & \nu & \bar{g} & \gamma_p & T & \rho_s & \sigma_s^2 & \sigma_\mu^2 \end{pmatrix}'$, together with the variance of the observation errors.

We use the same model to study the evolution of inflation expectations in a few countries. In this draft we discuss results for the following countries: France, Germany, Japan and Sweden, but more countries will be added in later versions. We measure inflation expectations using data available from Consensus Economics. The dataset include short and long-term professional survey forecasts for a wide set of countries. Unfortunately the dataset for most countries starts around 1991, so we have a limited set of forecasts. As second issue arises with the available forecasts. In contrast with the datasets used for the US, Consensus Economics forecasts are made on a year-over-year basis. This formulation may prevent a clean identification of the mechanism of the model, which links one-step ahead forecast errors to the beliefs about $\bar{\pi}_t$.

We use available forecasts for one-year and two-years ahead. Because of the year-over-year specification, these forecast give most weight to quarterly forecasts up to four quarters ahead. They are thus classifiable as short-term forecast. The section below shows that these forecasts differ significantly from the long-term forecasts we seek to explain. To give a bit more detail, year-over-year forecasts can be approximated as a weighted average of quarterly forecasts at different horizons, with "tent-shaped" weights. For the estimation we use six sets of forecasts. The first two are forecasts for the current year, made in the first and second quarter of the year. These forecasts can be expressed as

$$\begin{aligned} \mathbf{E}_t^{Cons}\pi_{year1,Q2} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-3+i} \\ \mathbf{E}_t^{Cons}\pi_{year1,Q1} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-4+i}, \end{aligned}$$

where the vector $w = \left(\frac{1}{16} \quad \frac{2}{16} \quad \frac{3}{16} \quad \frac{4}{16} \quad \frac{3}{16} \quad \frac{2}{16} \quad \frac{1}{16} \right)$ includes the appropriate weights. The forecasts for the next calendar year are taken in each quarter of the current year. They can be expressed as

$$\begin{aligned}
\mathbf{E}_t^{Cons}\pi_{year2,Q4} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-2+i}; \quad \mathbf{E}_t^{Cons}\pi_{year2,Q3} \approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t-1+i} \\
\mathbf{E}_t^{Cons}\pi_{year2,Q2} &\approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t+i}; \quad \mathbf{E}_t^{Cons}\pi_{year2,Q1} \approx \mathbf{E}_t^{Cons}\sum_{i=0}^{6} w(i)\pi_{t+1+i},
\end{aligned}$$

where the weights remain the same. The observation equation is then

$$Y_t^F = \begin{bmatrix} \pi_t \\ \mathbf{E}_t^{Cons}\pi_{year1,Q2} \\ \mathbf{E}_t^{Cons}\pi_{year1,Q1} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q4} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q3} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q2} \\ \mathbf{E}_t^{Cons}\pi_{year2,Q1} \end{bmatrix} = \pi^* + H_t'\xi_t + R_t o_t.$$

Because forecasts are different in each quarter, we have only one observation for each forecast per year. Looking at the weights, a key observation is that what is common to all these forecasts is that most of the weight (a fraction ranging from from 10/16 to 12/16) is given to quarterly forecasts ranging from one to four-quarters ahead.

4 Results

4.1 US

The following table reports the prior and posterior distribution of the parameters. [Comments to be added].

Table 1. *Prior Distributions and Posterior Parameter Estimates for the US (Baseline Model)*

Parameter	Dist.	Prior (mean, std)		Posterior mean	Posterior (5,25,50,75,95 percentiles)				
$400 * \pi^*$	Normal	2.5	1.2	2.65	2.3	2.5	2.65	2.80	3.01
ν	Gamma	0.02	0.006	0.021	0.013	0.017	0.021	0.025	0.031
g	Gamma	0.15	0.03	0.14	0.11	0.13	0.14	0.15	0.18
T	Beta	0.6	0.25	0.92	0.86	0.90	0.92	0.94	0.96
ρ_s	Beta	0.7	0.15	0.88	0.83	0.86	0.88	0.89	0.92
γ_p	Beta	0.5	0.08	0.13	0.09	0.12	0.13	0.16	0.19

Recall that we discipline the estimation of the model with *short-run inflation forecasts only.*

Hence, a comparison of the long-run inflation forecasts produced by the model with long-run survey forecasts is a legitimate "test" of the model as a description of long-run inflation expectations.

Results of the estimated model are presented in **Figure 2**, showing long-run inflation forecasts produced by the model, and various measures of long-run inflation expectations that were *not* used in the estimation.

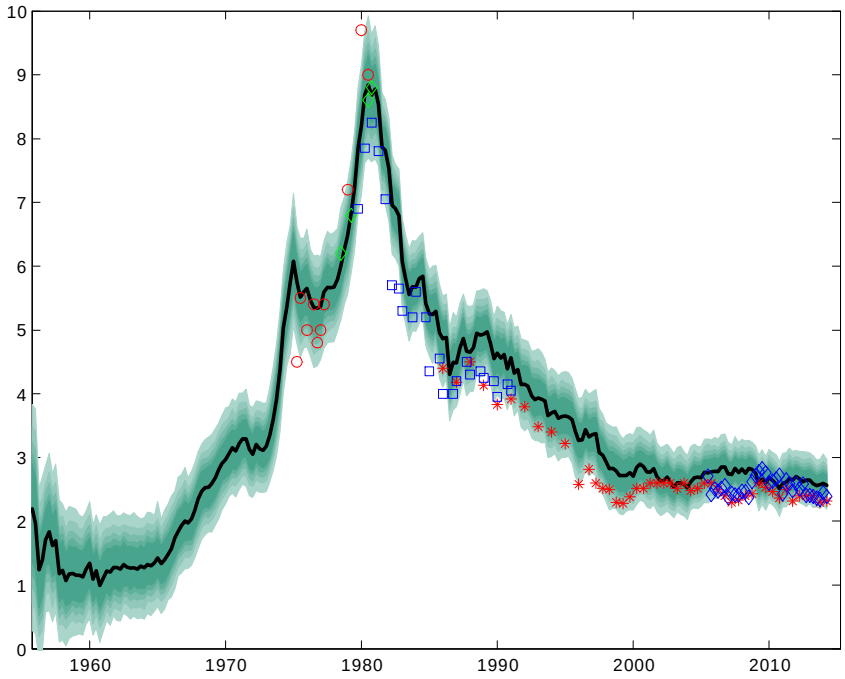


Figure 2: The figure compares the long-term forecast predicted by the model to different measure of survey-based long-term forecasts. In detail, the black line is the (median) predicted five-to-ten years forecasts from the model's PLM, with the green shaded area showing the 90% bands. Turning to the survey data. The red circles show 5-10 years CPI forecasts from the Michgan households survey, the green diamonds show the 1-10years CPI inflation forecasts from the Decision-Makers Poll of Portfolio Managers, the blue squares show the 1-10years CPI from Blue Chip Economic Indicators. Finally the red stars show the 5-10years forecasts from Blue Chip Financial Forecasts and the blue diamond are 5-10years implied forecasts from SPF.

The figure reveals that the model captures surprisingly well the dynamics of long-term inflation expectations as measured by survey data. The result is remarkable in that the model has very little degrees of freedom since the forecasts errors, f_t, driving long-term forecasts are "observed". In fact, observation errors on the survey forecasts are relatively small, with annualized standard deviations in the range of $0.08 - 0.3$ (modal estimates).

Figure 3 shows the path of long-term forecasts (and $\bar{\pi}_t$) together with the path of the learning gain implied by the estimated model at the parameters' posterior mode. The estimated model tells a story in which inflation expectations got unanchored in the 1970s, after a series of short-run inflation surprises. Agents kept using a constant gain learning algorithm for quite a while – until the late 1990s. At that point, they reverted back to a decreasing gain procedure, and as

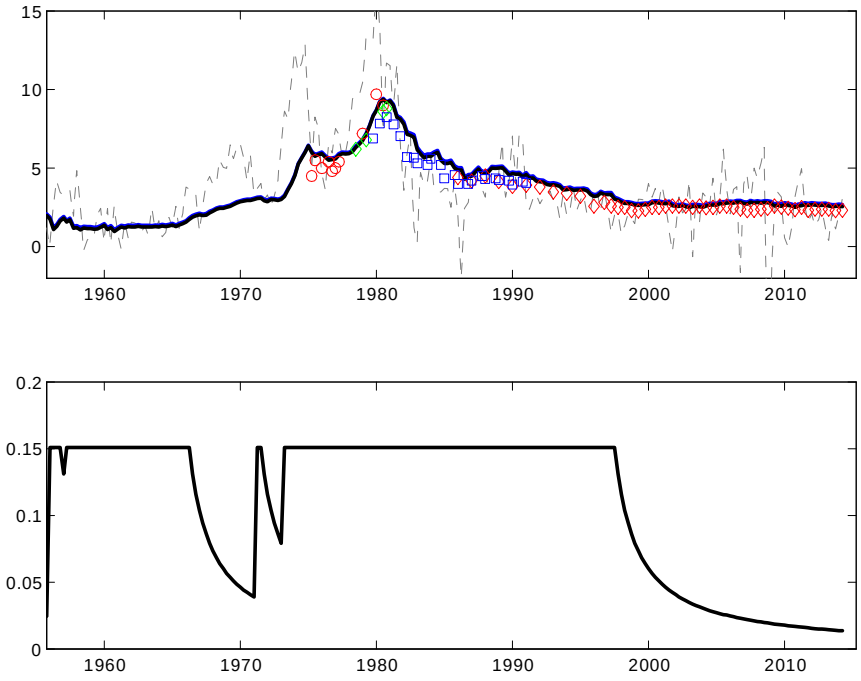


Figure 3: The figure shows the modal predictions for long-term forecasts and the learning gain k_t^{-1}. The top panel shows the model predictions for $\bar{\pi}_t + \pi^*$ (blue line) and the 5-10years forecast (black line) compared with different measure of long-term forecasts. The dashed grey line is CPI inflation. The bottom panel shows the evolution of te learning gain k_t^{-1}.

a result the sensitivity of long-run inflation expectations decreased. This explains why long-run inflation expectations have been remarkably stable in the face of relatively volatile inflation in the last decade.

4.1.1 Are households forecasts consistent with the model?

[To be added]

4.2 International evidence

Because of the limited survey data available on foreign countries, the model parameters posterior distribution is obtained using the US posterior distribution as a prior [more to be added]. **Table 2** shows the modal estimates for the different countries. The key observation is that, with the exception of the constant gain estimate for France, the parameters estimate are remarkably close to the ones for the US.

The following set of figures shows that the model does a remarkably good job at predicting the evolution of long-term forecasts as measured by the 5-10 years CPI forecasts from Consensus. The figures also provide a measure of the degree of anchoring of long-term expectations in each country. For each country we show two figures. The first figure focuses on the post 1985 period. The top panel displays the evolution of short-term forecast, while the bottom panel shows the

model's prediction for long-term forecasts at the parameters' mode. The second figure shows the models' prediction starting in 1960 (the estimation uses CPI data starting at the end of 1950s) and the evolution of the learning gain during that period. [more to be added].

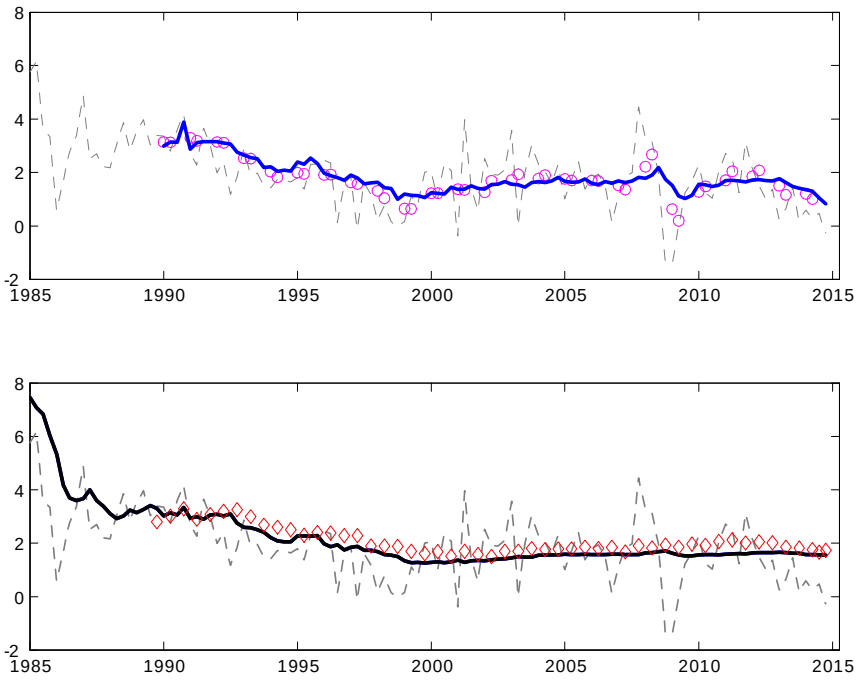


Figure 4: **France**. The figure shows the evolution of survey-based short-term forecasts (top panel) and the model predictions for long-term forecasts (bottom panel). In more detail, the grey dashed line is CPI. In the top panel, the magenta circles groups forecasts for the current year taken in the first and second quarter of the year. The solid blue line groups forecast for the next year, taken each quarter. In the bottom panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$,while the red diamonds display the 5-10years forecast from Consensus economics.

Table2. *Key Parameters at posterior mode*

Parameter	US	France	Germany	Japan	Sweden
$400 * \pi^*$	2.40	1.60	1.84	1.52	1.92
ν	0.018	0.016	0.016	0.019	0.019
g	0.15	0.30	0.16	0.16	0.14
T	0.95	0.96	0.92	0.96	0.96
ρ_s	0.88	0.70	0.88	0.93	0.94
γ_p	0.12	0.06	0.15	0.10	0.10

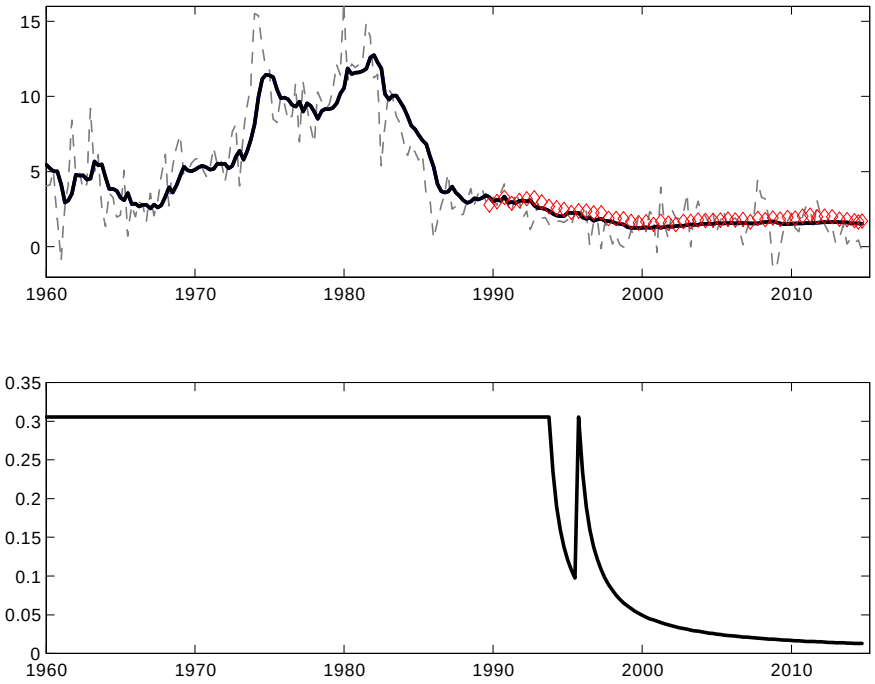


Figure 5: **France.** The figure shows the model predicted long-term forecasts and the learning gain. On the top panel, panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$, while the red diamonds display the 5-10years forecast from Consensus Economics. The bottom panel displays the learning gain.

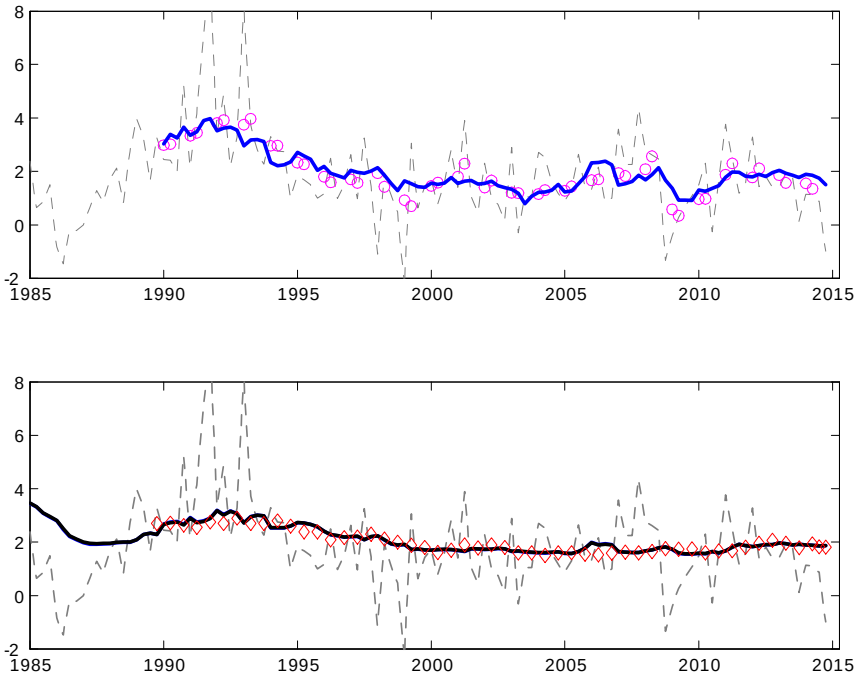


Figure 6: **Germany**. The figure shows the evolution of survey-based short-term forecasts (top panel) and the model predictions for long-term forecasts (bottom panel). In more detail, the grey dashed line is CPI. In the top panel, the magenta circles groups forecasts for the current year taken in the first and second quarter of the year. The solid blue line groups forecast for the next year, taken each quarter. In the bottom panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$,while the red diamonds display the 5-10years forecast from Consensus economics.

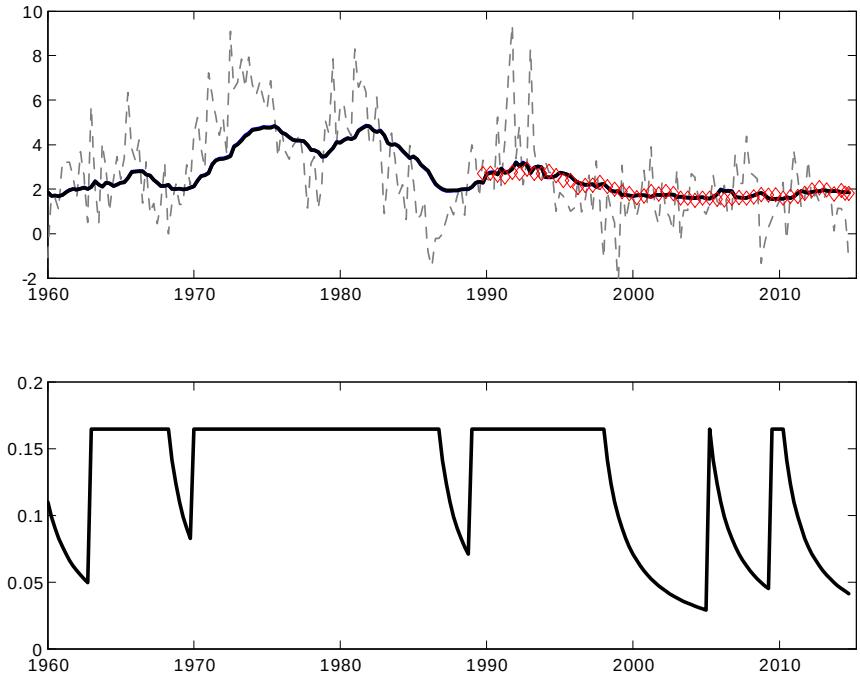


Figure 7: **Germany.** The figure shows the model predicted long-term forecasts and the learning gain. On the top panel, panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$, while the red diamonds display the 5-10years forecast from Consensus Economics. The bottom panel displays the learning gain.

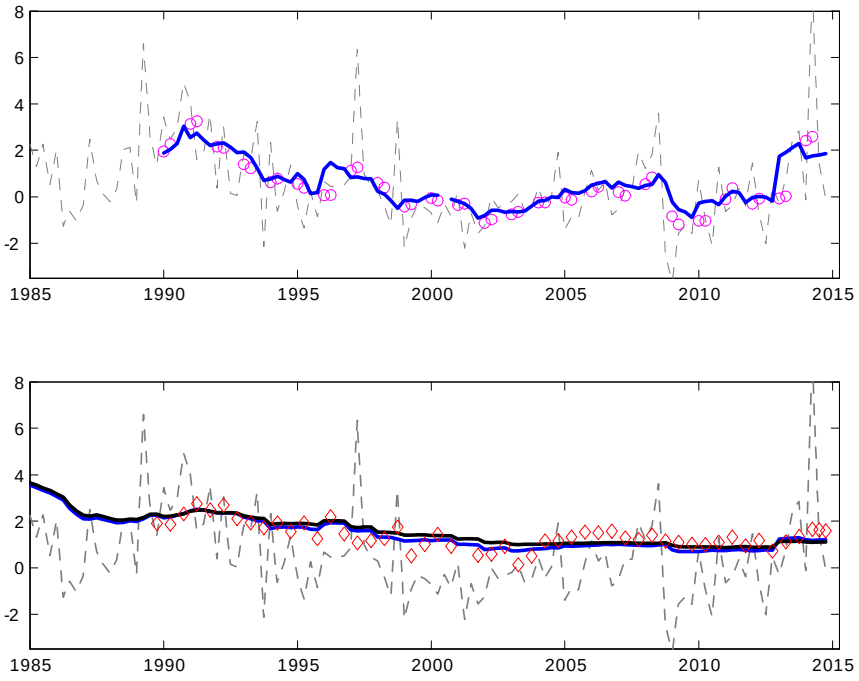


Figure 8: **Japan**. The figure shows the evolution of survey-based short-term forecasts (top panel) and the model predictions for long-term forecasts (bottom panel). In more detail, the grey dashed line is CPI. In the top panel, the magenta circles groups forecasts for the current year taken in the first and second quarter of the year. The solid blue line groups forecast for the next year, taken each quarter. In the bottom panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$,while the red diamonds display the 5-10years forecast from Consensus economics.

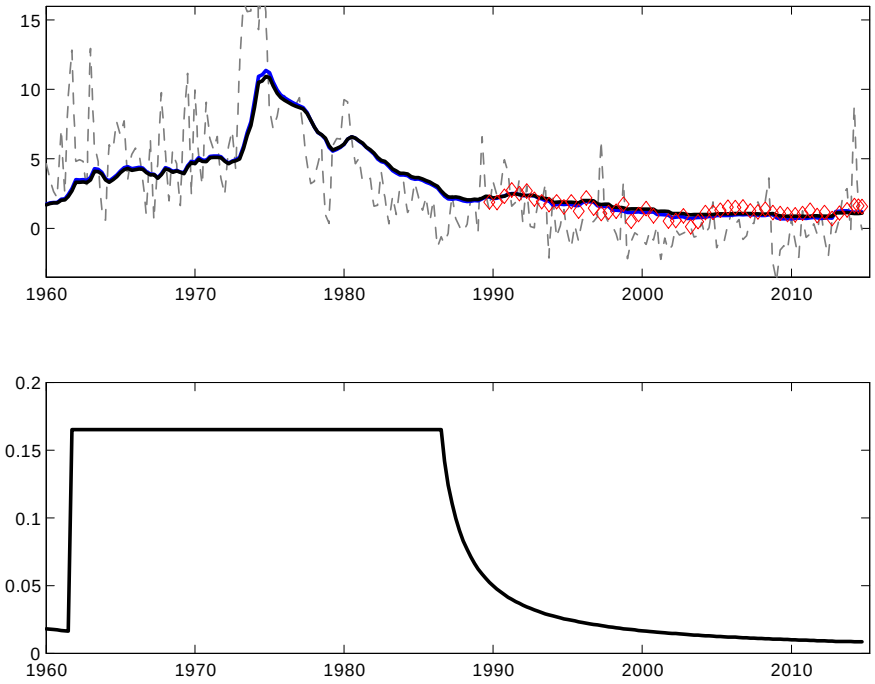


Figure 9: **Japan.** The figure shows the model predicted long-term forecasts and the learning gain. On the top panel, panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$, while the red diamonds display the 5-10years forecast from Consensus Economics. The bottom panel displays the learning gain.

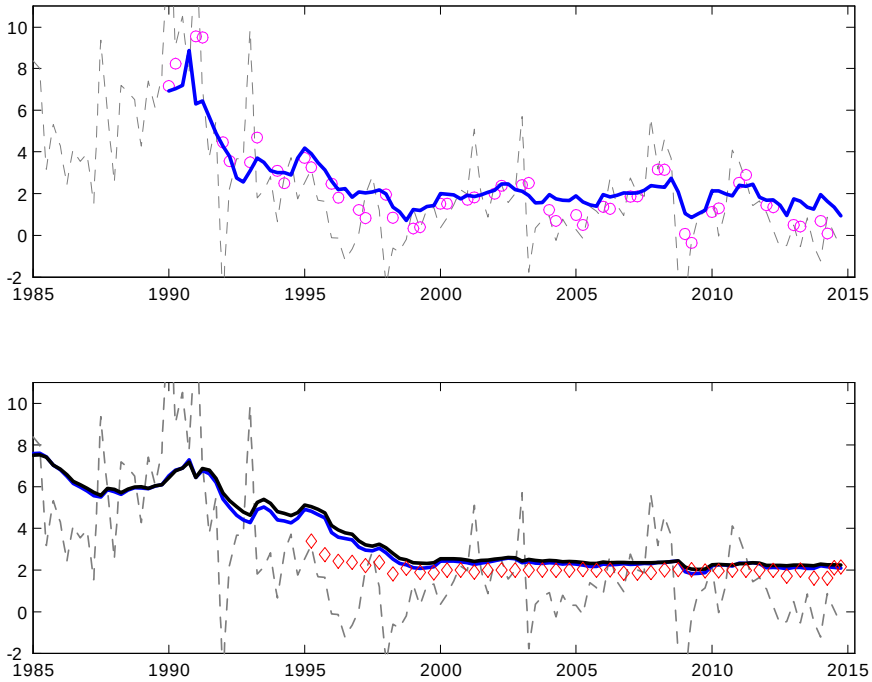


Figure 10: **Sweden**. The figure shows the evolution of survey-based short-term forecasts (top panel) and the model predictions for long-term forecasts (bottom panel). In more detail, the grey dashed line is CPI. In the top panel, the magenta circles groups forecasts for the current year taken in the first and second quarter of the year. The solid blue line groups forecast for the next year, taken each quarter. In the bottom panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$,while the red diamonds display the 5-10years forecast from Consensus economics.

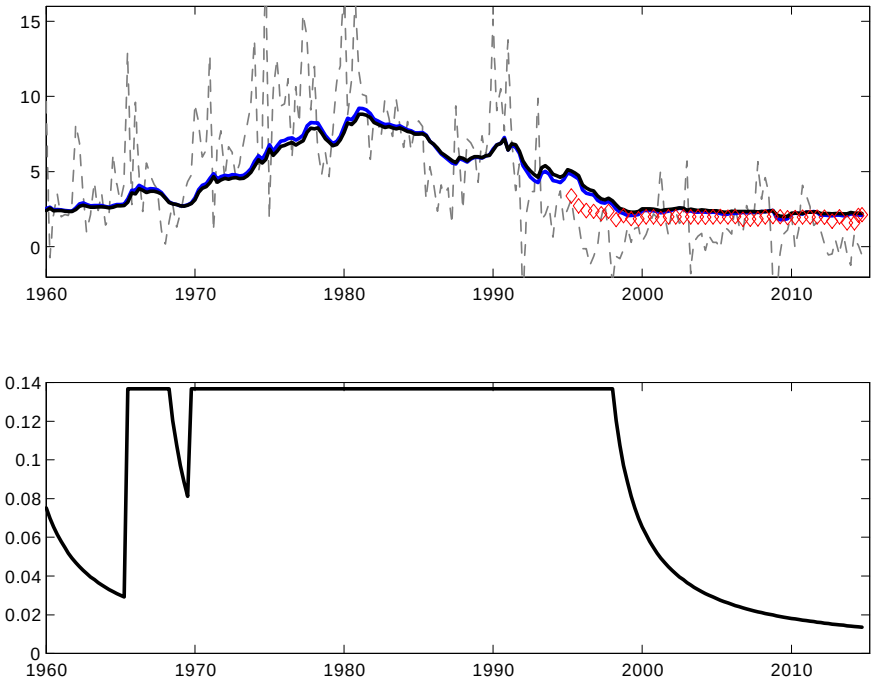


Figure 11: **Sweden.** The figure shows the model predicted long-term forecasts and the learning gain. On the top panel, panel, the solid blue line shows the evolution of the 5-10 years ahead forecast from the model, the solid black line shows predicted $\bar{\pi}_t$, while the red diamonds display the 5-10years forecast from Consensus Economics. The bottom panel displays the learning gain.

4.3 Counterfactuals

[To be added]

5 Discussion

5.1 Role of policy

5.2 Individual rationality

A desirable feature of the learning algorithm describe above is that it should asymptotically converge to the truth $\bar{\pi}_t = 0$, given that no actual regime switch occurs in the model. This is guaranteed provided the switching parameter ν is not "too small". From the convergence properties of the model described above, for t sufficiently large it is possible to find a ν such that the condition $\left(1-\gamma_p\right)(1-T)\,\bar{\pi}_t > v$ is satisfied almost surely. As ν is estimated, we would need to verify that the parameter in fact satisfies this restriction. This corresponds to the *Asymptotic Rationality* requirement in Marcet and Nicolini (2003). What happens if ν is too large? In that case agents never switch to constant gain, preventing what we call learning equilibria. However, during the transition, agents make large and persistent forecast errors. In the paper we estimate ν and, predictably enough, it is consistent with switches to the constant gain algorithm. Another property of the learning algorithm that we imposed is what Marcet and Nicolini (2003) refer to as *Internal Consistency*. This refers to the fact that agents have to be satisfied with the parameters of the learning algorithm (in particular the focus in of the gain $\bar{g}$). It is going to be formalized in later version of the notes but the idea is that $\bar{g}$ is a best response for each agent in the model. Any agent facing the actual law of motion for inflation and choosing an alternative gain should not be able improve her forecasting performance meaningfully. Hence, she would like to stick with the "equilibrium gain". If such a gain exists (and we shall verify that the estimated gain satisfies this property), then as the algorithm switches to constant gain, the economy is in a learning equilibrium where inflation expectations drift and agents have no incentive to deviate from the learning rule they use. Intuitively, the closer T is to one, the more likely the economy has a learning equilibrium other than $\bar{g}^{-1} \to 0$ (which corresponds to the rational expectations equilibrium).

Summing up, agents' estimate eventually converges to the correct inflation mean. However, during the transition the economy can switch to a period of volatile long-term inflation expectations as agents' algorithm switches to a constant gain In detail, a sufficiently large shock to inflation (or a sequence) is likely to induce a switch to constant gain. This in turn generates strong feedbacks from inflation expectations to inflation, increasing volatility and reinforcing the choice of a constant gain algorithm (learning equilibrium). Eventually, a sequence of shocks leads the long-term estimate of inflation close to the truth. The algorithm switches to OLS, inducing stability into the inflation process. As the gain becomes sufficiently low (and $\bar{\pi}_t$ sufficiently close to its true value), the condition $\left(1-\gamma_p\right)(1-T)\,\bar{\pi}_t$ is satisfied at all times and the learning process converges. However, during the transition process, agents can switch few times between OLS and tracking, as a large

shock can hit inflation for still relatively high values of the decreasing gain, prompting a switch back to the constant gain.

Learning equilibria. In this section we evaluate whether the estimated gain is consistent with a specific equilibrium concept, which we define as a learning equilibrium (this is the Internal consistency requirement in Marcet and Nicolini, see above). We evaluate this through simulation.

To make the whole business simple, let us focus on the model where there is not switching. In principle we should evaluate the learning equilibrium for both ν and $\bar{g}$, but for simplicity we focus on the constant gain. Then what matters is how good the predictor is once you switch to the constant gain. (more on this.)

One can then compute or simulate and compare the mean square errors associated to different predictors. As in Marcet and Nicolini, the learning algorithm is Internally Consistent if

$$\mathbf{E}\left[f_t\left(\bar{g}\right)\right]^2 \leq \min_{\bar{g}'} \mathbf{E}\left[f_t'\left(\bar{g},\bar{g}'\right)\right]^2 + \epsilon$$

for ϵ small. It is easy enough to show that it holds in our paper, as shown in the figure below (we can also show through simulation of the nonlinear algorithm but the result would be fairly similar). So learning equilibria exists for these parameter values. These equilibria depend on the strength of feedback between expectations and actual inflation, which in this case is captured by the evolution of $\bar{\pi}_t$

$$\bar{\pi}_t = \left[\bar{g}^{-1}\left(1-\gamma_p\right)\left(\tilde{T}_\pi - 1\right) + 1\right]\bar{\pi}_{t-1} + \bar{g}^{-1}\cdot \text{Shocks}.$$

with $\left[\bar{g}^{-1}\left(1-\gamma_p\right)\left(\tilde{T}_\pi - 1\right) + 1\right]$ close to unity the *actual drift* of inflation is very close to a random-walk, the agents' *perceived drift*. As mention before, the drift depends on of strongly the marginal cost responds to inflation. [Results to be added]

6 Conclusion

[To be added]

References

[1] [To be added]

7 Appendix

[To be added]