How Modern Dictators Survive: Co-optation, Censorship, Propaganda, and Repression

Sergei Guriev* Daniel Treisman†

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Abstract

We develop an informational theory of dictatorship. Dictators survive not because of their use of force or ideology but because they convince the public—rightly or wrongly—that they are competent. Citizens do not observe the dictator's type but infer it from signals inherent in their living standards, state propaganda, and messages sent by an informed elite via independent media. If citizens conclude the dictator is incompetent, they overthrow him in a revolution. The dictator can invest in making convincing state propaganda, censoring independent media, co-opting the elite, or equipping police to repress attempted uprisings—but he must finance such spending with taxes that depress the public's living standards. We show that incompetent dictators can survive as long as economic shocks are not too large. Moreover, their reputations for competence may grow over time. Censorship and co-optation of the elite are substitutes, but both are complements of propaganda. Repression of protests is a substitute for all the other techniques. In some equilibria the ruler uses propaganda and co-opts the elite; in others, propaganda is combined with censorship. The multiplicity of equilibria emerges due to coordination failure among members of the elite. We show that repression is used against ordinary citizens only as a last resort when the opportunities to survive through co-optation, censorship, and propaganda are exhausted. In the equilibrium with censorship, difficult economic times prompt higher relative spending on censorship and propaganda. The results illuminate tradeoffs faced by various recent dictatorships.

*Professor of Economics, Sciences Po, Paris. E-mail: sergei.guriev@sciencespo.fr
†Professor of Political Science, UCLA. E-mail: treisman@polisci.ucla.edu

1 Introduction

How do dictators hold onto power? The totalitarian tyrannies of Stalin, Hitler, Mao, Pol Pot, and others relied largely—although not exclusively—on mass terror and indoctrination. Although less ideological, many 20th Century military regimes—from Franco's Spain to Pinochet's Chile—used considerable violence to intimidate opponents of the regime.[1] However, in recent decades, a less carnivorous form of authoritarian rule has emerged, one better adapted to the globalized media and sophisticated technologies of the 21st Century. From the Peru of Alberto Fujimori to the Hungary of Viktor Orban, illiberal regimes have managed to consolidate power without isolating their countries from the world economy or resorting to mass killings.

Instead of inaugurating "new orders," such regimes simulate democracy, holding elections that they make sure to win, bribing and censoring the private press rather than abolishing it, and replacing ideology with an amorphous anti-Western resentment.[2] Their leaders often enjoy genuine popularity—albeit after eliminating plausible rivals—that is based on "performance legitimacy," a perceived competence at securing prosperity and defending the nation against external threats. State propaganda aims not to re-engineer human souls but to boost the leader's ratings, which, so long as they remain high, are widely publicized. Political opponents are harassed and humiliated, accused of fabricated crimes, and encouraged to emigrate.

The new-style dictators can brutally crush separatist rebellions and deploy paramilitaries against unarmed protesters. But, compared to previous regimes, they use violence sparingly. They prefer the ankle bracelet to the Gulag. Maintaining power, for them, is less a matter of terrorizing victims than of manipulating beliefs about the world. Of course, shaping beliefs was also important for the old-style dictatorships, but violence came first. "Words are fine things, but muskets are even better," Mussolini quipped (Odegard 1935, p.261). Recent tyrannies reverse the order. "We live on information," Fujimori's security chief Vladimir Montesinos confessed in one interview. "The addiction to information is like an addiction to drugs." Montesinos paid million dollar bribes to television stations to skew their coverage. But killing members of the elite struck him as foolish: "Remember why Pinochet had his problems. We will not be so clumsy"(McMillan and Zoido 2004, p.74). When dictators are accused of political murders these days, it often augurs the fall of the dictatorship.

Some bloody military regimes and totalitarian states remain—for instance, in Egypt and Syria, or North Korea—but the balance has shifted. As Table 1 shows, far more of the undemocratic orders around today have elected legislatures in which non-government parties occupy a significant place. And fewer are currently involved in mass atrocities against their populations. Besides Fujimori's Peru and Orban's Hungary, other regimes that share some or all of these characteristics include Vladimir Putin's Russia, Mahathir Mohamad's Malaysia, and Recep Tayyip Erdogan's Turkey. One might even see Lee Kuan Yew's Singapore as a pioneer

[1] More than 30,000 people are believed to have been killed by Pinochet's agents, "most of them taken away to secret detention centers and camps, tortured, tossed still alive from airplanes into the sea or shot and buried in unmarked graves" (Roht-Arriaza 2005, p.viii).

[2] On the use of elections and partially democratic or pseudo-democratic institutions in dictatorships, see Gandhi (2008), Gandhi and Lust-Okar (2009) and Levitsky and Way (2010). Although our focus is on "dictatorships," which we see as synonymous with "authoritarian regimes," we consider the dividing line between soft authoritarian regimes and illiberal democracies to be a fuzzy one. The model also applies to most illiberal democracies.

Table 1: Changing characteristics of non-democracies, 1980-2012.

	Percent with ongoing mass killings	Percent with no elected legislature	Percent with legislatures where government parties have more than 95 percent of seats
1980	22	33	70
2012	6	15	35

Sources: Database of Political Institutions 2010 (see Beck et al. 2001); Polity IV database; Mass Killings Database (see Ulfelder and Valentino 2008, and updated data at https://dartthrowingchimp.wordpress.com/2013/07/25/trends-over-time-in-state-sponsored-mass-killing/).

Notes: "Non-democracies" are countries with Polity2 scores of less than 6. A "mass killing" is "any event in which the actions of state agents result in the intentional death of at least 1,000 noncombatants from a discrete group in a period of sustained violence."

of such soft autocracy.

We develop a model of dictatorship to capture the logic that governs the survival of such regimes. As in "career concerns" models of democratic politics, the ruler may be either competent or incompetent.[3] Only the dictator and a subset of the public—"the informed elite"—observe his type directly. But ordinary citizens update their beliefs about this based on the information available to them from the state media, independent media, and their own living standards. Citizens' living standards depend on the tax rate set by the dictator and on economic performance, itself a function of the leader's competence and a stochastic shock. If enough citizens infer, based on these various signals, that the incumbent is incompetent, they rise up and overthrow him in a revolution. Members of the elite—if not co-opted—would also prefer to replace an incompetent incumbent but cannot do so without the masses to back them up.[4]

The dictator can affect all the channels of information. He can spend on making the propaganda broadcast via state media more convincing. He can bribe (or impose costs—such as fines or violence—on) the informed elite to prevent the latter from sending critical messages. And he can censor those messages that they do send. The dictator can also spend on hiring and equipping the police with the tools of repression, thus increasing the cost of revolution. However, all these actions require money, which must come from taxing the citizens, depressing their living standards, and indirectly lowering their estimate of his competence. Hence the tradeoff.

We show how the dictator's strategy and his equilibrium survival odds, as well as the path of citizens' beliefs, change with the various exogenous parameters—the size of the stochastic shocks, the distribution of competence, and the technology of censorship, propaganda, and repression. We identify two equilibria: in one the dictator co-opts the elite, in the other he censors the private media. Multiple equilibria exist because

[3]Competence here refers to the ability to promote—or at least not undermine—economic growth and defend the country against external threats. The contrast between the rapid, sustained growth in the Singapore of Lee Kuan Yew and the consistent economic decline in the Zaire of Sese Seko Mobutu suggests that some dictators may indeed be more competent than others.

[4]Leaders of authoritarian states are more often removed by coups than by mass uprisings (Geddes 1999), but we focus here on regime change, which is less frequent than leader turnover.

members of the elite must coordinate on a strategy. When both equilibria exist, the one with co-optation always yields the dictator higher survival odds than the one with censorship. We also find that force is used against the general public only as a last resort after co-optation, censorship, and propaganda have failed.

The model offers insight into a number of observations and puzzles about the functioning of dictatorships. First, it shows how modern autocracies can survive while employing relatively little violence against the public. Repression is not necessary if mass beliefs can be manipulated sufficiently by means of censorship, co-optation, and propaganda. Indeed, since in our model major repression is only used if equilibria based on co-optation and censorship have disappeared, violence signals to the general public that the regime is incompetent and therefore vulnerable. Conversely, when co-optation and censorship have failed and opposition to the regime becomes overt, state repression is all that remains–and is sure to be used. As Davenport (2007, pp.7-8) notes, one result that emerges with "astonishing consistency" from studies of repression is that: "When challenges to the status quo take place, authorities generally employ some form of repressive action to counter or eliminate the behavioral threat."

Second, the effectiveness of propaganda in authoritarian regimes is a prima facie puzzle. Given that citizens know the dictator has an incentive to lie about his type, why do they ever listen? In our model, propaganda works because whereas competent leaders can costlessly show themselves to be competent, incompetent ones must invest resources to fake the evidence that will make this claim convincing—and sometimes they choose to spend their budget on other actions instead. Thus, observing a dictator claim convincingly to be competent increases the odds that he actually is—especially given the resources he could shift from propaganda to consumption and public goods which are directly observed by the public.

Third, why do some clearly incompetent dictators nevertheless hold on to power for long periods? The model shows how rulers whom most or all citizens—if fully informed—would prefer to overthrow can still survive in many circumstances simply by manipulating information. And we focus on a mechanism other than the well-studied one of blocking communication among potential protesters. The dictator's survival depends here not on preventing citizens from expressing their willingness to rebel (in our model, we abstract from the issue of coordination among the protesters), but on manipulating public beliefs about the state of the world and the incumbent's type.

Moreover, we show that over time incompetent leaders, if they survive, may acquire a reputation for competence as a result of rational Bayesian updating by citizens. Such reputations can withstand temporary economic downturns if these are not too large. This is consistent with the empirical finding that dictators that last through their first few years are less likely to be overthrown (Svolik 2009, Bueno de Mesquita and Smith 2010, Treisman 2014). However, the mechanism we propose here—of rational updating of beliefs about the leader's type—differs from those of Svolik (2009), who emphasizes power consolidation by the incumbent, and Treisman (2014), who emphasizes selection effects.

Fourth, the multiple equilibria associated with different leader strategies illuminate why, among dictatorships that seem otherwise quite similar, some focus on censoring independent media while others censor much less but co-opt the elite with patronage. For example, while Iran has the world's strictest limits on internet

content, according to Freedom House, Morocco has among the least restrictive internet controls, on a par with those of Japan (Freedom House 2013). The Moroccan royal family has consistently viewed co-optation "as a much more effective tool than confrontation and repression," given the country's traditional system in which "patronage and accommodation were deeply ingrained" (Willis 2014, p.444).

Fifth, the model predicts that as economic conditions worsen, a dictatorship may switch to using censorship in place of co-optation of the elite, and boost relative spending on censorship and propaganda. This is consistent with a noted increase since the global financial crisis in efforts to limit opposition media in a range of countries, from Hungary and Turkey to Russia. As Turkey's growth rate fell from 7.8 percent in 2010 to 0.8 percent in 2012, the number of journalists in jail increased from 4 to 49.[5] Between 2008 and 2011, Hungary fell 15 percentage points on Freedom House's press freedom index. Conversely, the result makes sense of the increasing focus on co-optation in China during its long period of rapid growth. Starting in the 1980s, Beijing replaced the Mao-era system of comprehensive control with one that relied increasingly on commercialized self-censorship; media owners, editors, and journalists are rewarded for loyalty with state advertising contracts and well-paid jobs, thus incentivizing them to censor themselves. "The desire to win performance bonuses tends to result in journalism that steers well clear of dangerous political controversy and meets the party's propaganda requirements," according to one analyst (Esarey 2005, pp.57-9). Censorship of the internet, meanwhile, has focused on blocking collective action rather than on suppressing criticism of the government and party (King, Pan, and Roberts 2013). In Singapore as well, "forsaken profits and stiff legal penalties have been more effective in fostering self-censorship than earlier methods of intimidation" (Rodan 1998, p.69).

Sixth, the model offers a variety of reasons why—as widely noted—modernization makes dictatorship harder to sustain, at least in the absence of vast resource rents. By increasing the size of the "informed elite," economic development increases the cost for incompetent dictators of either co-opting potential critics or censoring their media messages. Even relatively small economic shocks then become sufficient to threaten the incumbent's rule. If modernization also increases the technological sophistication of opposition media, then censorship may fail for this reason as well.

Yet, if modernization makes it harder to sustain an information-based dictatorship, we believe it increases the costs of mass repression even more. An autocrat's greatest nightmare is that his security services, when ordered to violently suppress opposition, will split or simply disobey, igniting a power struggle that undermines the regime (Francisco 2005, p.66). The more educated, globally integrated, and professionalized are the dictator's agents, the greater is the chance they will shrink from committing a massacre. At the same time, harsh repression is both more public and more economically harmful in a more developed society. A knowledge economy, dependent on innovation, stands to lose its entrepreneurs to emigration if the dictator uses force. In a world of global media and capital flows, political violence can depress foreign investment, stimulate capital flight, and even puncture confidence in the currency (Blanton and Blanton, 2007, Fielding and Shortland, 2005). All these factors help explain the modern dictator's preference for less overt methods.

[5]Data from the Committee to Protect Journalists, https://www.cpj.org/imprisoned/2014.php.

We develop the model assuming the simplest, linear technologies for propaganda, censorship, co-optation, and repression. This allows us to study the interactions between the dictator's techniques that arise due to the inference problem of citizens rather than due to assuming non-linear functional forms. We point out that propaganda (positive reports by the leader about his competence) is complementary to both the co-optation of the informed elite and the censorship of independent media, both of which are techniques that aim to prevent news of the incumbent's incompetence from getting out. Censorship and co-optation are substitutes. Repression against potential revolutionaries is a substitute for all the information-based techniques for maintaining power. Under some parameter values, equilibria exist in which no repression against the public is needed at all. Under other parameter values for which there are no equilibria based on co-optation, censorship, or propaganda, an equilibrium based on repression of potential protests may exist. To focus on the less studied aspects of the problem, we assume the uninformed public is homogeneous and we do not examine the problem of coordination among protesters. For treatments of the latter issue, see Edmond (2014) and Lohmann (1994).

The next section reviews related literature. Section 3 outlines the elements of the model. Section 4 derives the equilibria. Sections 5 and 6 discuss comparative statics and several extensions, including a dynamic version of the game and one with continuous economic shocks. Section 7 concludes.

2 Literature

Wintrobe (1990) pioneered the formal analysis of dictators' behavior, modeling, in a reduced form, decision-theoretic framework, the tradeoff between investing in repression and co-opting citizens with material benefits. He examined the choices of two types of dictators—"totalitarians," who maximized power, and "tin-pot" dictators, who maximized their own consumption subject to a minimum power constraint. In Wintrobe (2007), he generalized the dictator's objective function to include both power and consumption, allowing the equilibrium levels to be determined by cost parameters.

Subsequent formal analyses of dictatorship focused on several different— although related—questions. One set of works considered the economic policies autocrats would choose and how these compare to those prevailing under either anarchy or democracy. Olson (1993) argued that dictators' incentives to adopt growth-promoting policies depend on their time horizon and how encompassing an interest they have in national output.

A second direction of research examined the role of institutions in authoritarian states, interpreting them as mechanisms by which the ruler solved time inconsistency problems. By creating institutions that constrained him in the short run, a dictator could enable himself to commit credibly to certain policies—repaying state debts and respecting property rights (North and Weingast 1989, Gehlbach and Keefer 2011), redistributing income to the poor (Boix 2003, Acemoglu and Robinson 2006), or sharing power with members of his ruling group (Myerson 2008, Svolik 2012, Boix and Svolik 2013). Models showed how such credible commitments could increase borrowing power and private investment in the first case, prevent revolutions

in the second, and avoid coups or elite defection in the third.

Another literature models the relationship between dictators and their support group when such interactions are not mediated by institutions. These works examine how the dictator chooses the number and characteristics of his inner circle (Egorov and Sonin 2011, Bueno de Mesquita et al. 2003).

A fourth set of works analyzes the effects of information problems under dictatorship. By monopolizing the media and criminalizing dissent, rulers are assumed to prevent citizens from sharing information. Some papers in this set have explored how costly protest actions by individuals communicate facts to other individuals, triggering—or failing to trigger—informational cascades (Kuran 1991, Lohmann 1994). The greater is the repression, the less likely are protests to occur, but the greater is the discontent that they communicate if they do (Kricheli et al. 2011). Dagaev et al (2013) examine how new information technologies facilitate coordination by potential opponents of a regime.

A few papers explore more complicated ways, in which dictators may seek to manipulate the information environment. Egorov, Guriev, and Sonin (2009) and Lorentzen (2014) model the tradeoff for a dictator in setting the level of censorship, where a free media, on the one hand, provides the information necessary for the dictator to discipline his bureaucrats, but, on the other hand, assists the expression and coordination of opposition. Edmond (2013) examines the strategic use of propaganda. In his game, citizens want to overthrow an existing regime but are imperfectly informed about the regime's "strength." The ruler sends costly, biased signals about the regime's strength in the attempt to prevent citizens from coordinating against him. Citizens, although understanding the ruler's incentive to deceive, cannot fully separate the true information from the bias. In this sense, propaganda can work in equilibrium, and it is more effective, the more precise are the signals. Huang (2014) also models state propaganda as the sending of signals of repressive capacity.[6] In his model, the content of the message is irrelevant; what constitutes the signal is the amount of propaganda sent, which citizens are assumed to observe. His key assumption is that the capacity to broadcast propaganda and the capacity to repress rebellions are correlated, so more propaganda signals greater repressive capacity.

Our model explores the tradeoffs for rulers who choose a portfolio of techniques—censorship, propaganda, co-optation, and repression—to preserve their power. Our approach differs from those of Edmond and Huang in that our incumbents vary in "competence," whereas their incumbents vary in "strength," understood as the ability to repress mass protests. (In our model, as in Edmond's, the "incompetent"/"weak" leaders try to pool with the "competent"/"strong" ones and are able to do so in some states of the world.) Neither paper considers censorship of private messages or the tradeoffs between propaganda, censorship, and positive and negative rewards. They focus on the way flows of information among the mass public influence protest. We bracket this question in order to explore the game between the dictator and the "informed elite" over propaganda, private media messages, and censorship.

[6]Propaganda may also feature as part of loyalty rituals in authoritarian states—the leader's ability to get citizens to repeat the propaganda shows the extent of his political control, deterring challenges, as discussed in many analyses of totalitarian regimes. We abstract from such loyalty rituals here in order to focus on the type of state propaganda that aims to influence citizens' beliefs about the competence of the ruler (at providing economic or security benefits).

A considerable literature examines state repression by itself (see Davenport, 2007). However, since Wintrobe's (1990) decision-theoretic analysis, few papers have explored the tradeoffs between repression and other techniques in the dictator's repertoire. One recent exception is Frantz and Kendall-Taylor (2014), who argue that dictators who co-opt potential opposition by means of partially-free parties and legislatures tend to rely less on censorship and more on torture and political imprisonment. This is consistent with our model, in which efforts to (a) censor the free media, and (b) secure the loyalty of the elite with a mix of carrots and sticks occur in two distinct equilibria. However, we posit a different mechanism. In our model, co-optation/repression and censorship are alternative ways that dictators prevent disillusioned elites from communicating with the masses; for Frantz and Kendall-Taylor, allowing a public sphere of controlled politics enables the regime to better target harsh tactics against its true opponents.

3 Setting

3.1 Players

There is a dictator and a continuum of citizens of unit mass. The citizens are exogenously divided into informed (elite) and uninformed (general public). The mass of informed citizens, I, is small (so the elite cannot overthrow the dictator by themselves; the general public can). Citizens maximize their current consumption plus the net present value of future consumption.

The dictator receives an exogenous rent each period he remains in power. He maximizes the net present value of expected future rents. In a one-period version of the model, the dictator simply maximizes the probability of staying in power π.

All agents are risk-neutral but also have limited liability (i.e. cannot pay large fines).

3.2 Timing

1. The dictator learns his type $\theta \in \{0, 1\}$. Nature picks I citizens who observe the type of the dictator, θ. Both the informed citizens and the dictator observe the realization of economic output, Y.

2. The dictator chooses the level of censorship, $x \in [0, 1]$. The dictator offers a level of rewards, $r \geq 0$, to members of the elite who commit not to send anti-regime messages. The dictator chooses a level of investment in propaganda, P.

3. The informed citizens choose whether (i) to support the regime and receive the reward, r, or (ii) to join the opposition and to send a signal to the public. The share of those who support the regime is α; the share of those who join the opposition is $\beta = 1 - \alpha$.

4. Contracts for the elite are implemented. Censorship blocks x per cent of the independent signals, so the public receives the opposition's signals with probability $(1 - x)\beta$. Payoffs are realized.

5. Citizens observe their consumption, C, propaganda signal, $p = \{0, 1\}$, and any independent signals that get through censorship, $z = \{0, 1\}$. The citizens update their beliefs about θ and decide whether to overthrow the dictator.

3.3 The economy

Total output (GDP), Y, may take two values: Y^L and Y^H, where $\Delta Y \equiv Y^H - Y^L > 0$. The probability ξ_θ of the high state, $Y = Y^H$, depends on the quality of the dictator, θ.

For simplicity, we assume that θ can take two values: $\theta = 0$ (the dictator is incompetent) and $\theta = 1$ (the dictator is competent). The dictator is competent with probability $\bar{\theta}$ (the parameter $\bar{\theta}$ is therefore also the expected value of θ).

The probability ξ_θ of the high output $Y = Y^H$ is higher if the dictator is competent:

$$\xi_1 > \xi_0.$$

The dictator can use part of GDP for funding propaganda, P, censorship, X, and for rewarding the elites, R. His budget constraint is $Y = C + P + X + R$, where C is consumed by the population. For simplicity, we assume that C is distributed equally among citizens—so C is also per capita consumption, since the number of citizens is normalized to 1—and is observed by all. By contrast, we assume that Y, X, P, and R are not observed by the general public. Y may include both official and unofficial income sources, while C may include consumption by citizens of both private goods and non-excludable public goods provided by the government.

By spending P on propaganda, the dictator increases the probability that the public gets a convincing propaganda message: $p = 1$. (The content of the message is: "the dictator is competent, $\theta = 1$"; the value of p indicates whether it is convincing or not.) We assume that the competent dictator can send the signal $p = 1$ costlessly. If the dictator is incompetent, the probability of public getting the positive message $p = 1$ is

$$\Lambda(P) = \min\left\{\frac{P}{\widehat{P}}, 1\right\}. \tag{1}$$

Here $\widehat{P}$ is a parameter that represents the cost to an incompetent dictator of generating fully convincing propaganda.

By spending X on censorship, the dictator blocks x per cent of the opposition's messages:

$$x = \min\left\{\frac{X}{\beta\widehat{X}}, 1\right\}.$$

Here $\widehat{X}$ is a cost parameter that captures how much it costs to block all the messages if all informed citizens join the opposition.

The co-optation of the elites works as follows: the dictator pays r to each informed citizen if the citizen

does not send negative messages. The total cost of rewarding the elites is $R = r\alpha I$; both α and r are endogenously determined in equilibrium.

3.4 Information

There are four kinds of signals in the model. First, each informed citizen learns the type of the dictator, θ, and GDP, Y, precisely. Second, all citizens observe per capita consumption, C. Third, the dictator disseminates propaganda—which is a public signal, $p = \{0, 1\}$, received by every citizen. Given that the competent dictator can send $p = 1$ at no cost, he always does so. Therefore if the public observes $p = 0$, it knows with certainty that the dictator is incompetent.

Finally, the informed citizens can send a signal $z = \{0, 1\}$ to the public. The signal z gets through the censorship with probability $(1 - x)$, where x is the level of censorship. If the public observes the signal $z = 0$ from the opposition, it knows for sure that the dictator is incompetent. If the public observes the absence of a negative signal from the opposition (we denote this outcome as $z = 1$), it needs to infer whether this is because the signal was censored (probability x), or because the true state is $\theta = 1$, or because nobody wanted to join the opposition ($\alpha = 1$) as all the informed citizens were co-opted.

3.5 Payoffs

3.5.1 Dictator's payoff

The dictator minimizes the probability of regime change. He does not benefit from higher GDP directly, just through increased resources to buy support and fund propaganda and censorship.

3.5.2 Citizens' payoff

The public maximizes current consumption, C, plus expected future consumption. Future consumption is either $\delta\theta$, if the current dictator stays, or $\delta\left(\bar{\theta} - K\right)$, if the regime is changed. Here, δ is the discounted payoff of having a competent (rather than incompetent) dictator in the future. If the regime is changed, the new dictator is drawn from the same distribution; hence the expected quality is $\bar{\theta}$. Regime change involves a non-trivial, additional cost to citizens, K.

The co-opted informed citizens (pro-regime elite) receive additional current consumption of r per capita.

3.6 Assumptions

We assume that $\widehat{X}$ and $\widehat{P}$ are sufficiently large relative to ΔY that $X/\widehat{X} < 1$ and $P/\widehat{P} < 1$ in equilibrium. This assumption is realistic but not innocuous. The dictator cannot fully silence the elites through censorship while he may still fully silence them through co-optation.

To focus on more interesting cases, we also assume that $K < \bar{\theta}$, so that if the public knows with certainty that the dictator is incompetent, it prefers to replace the dictator.

4 Analysis

In this Section, we solve for equilibria. Given the parameters $\bar{\theta}$, ξ_1, ξ_0, ΔY, I, $\widehat{X}$, $\widehat{P}$ δ, and K, we find the equilibrium strategies of the dictator (C, x, P, r), of the informed citizens (to get co-opted or to join the opposition), and of the public (to protest or to support the regime).

All agents are rational and maximize their expected payoffs given the available information. In particular, the informed citizens' choice is contingent on the rewards, r, offered for loyalty.

The general public observes consumption, C, and the signals p and z. If at least one of these two signals is low ($p = 0$ or $z = 0$), the public knows with certainty that the dictator is incompetent and protests. If both signals are high $p = z = 1$, then the decision depends on the consumption level, C: the public protests if consumption is low and supports the regime if consumption is high. Therefore, its strategy is fully described by the consumption threshold C^* such that the public supports the regime if and only if it observes $p = z = 1$ and $C \geq C^*$.

We first describe the best response functions of the three players and then solve for the equilibria.

4.1 Players' best response strategies

4.1.1 Public's choice

The general public maximizes

$$U^P = C + \delta \max\left\{E(\theta|C,p,z); \bar{\theta} - K\right\}.$$

where $E(\theta|C,p,z)$ is the expected value of θ given the public's inference of the equilibrium strategies of other players and the observed values of C, p, z. Therefore, the public chooses C^* as the lowest level of consumption that satisfies

$$E(\theta|C^*, p = z = 1) \geq \bar{\theta} - K. \tag{2}$$

The public does not protest if and only if $C \geq C^*$ and $p = z = 1$.

4.1.2 Informed citizens' choice

The informed citizens choose whether to join the opposition or to get co-opted. They make decisions given the strategies of the dictator and of the general public. They infer the probability of the regime surviving as a function of the size of the pro-regime elite and of the opposition, $\Pi(\alpha)$. The latter is a monotonically increasing function $\Pi'(\alpha) > 0$: the fewer people are in the opposition, the less likely is the opposition signal to reach the public, and (for a given realization of Y and the dictator's choice of propaganda, P, censorship, x, and rewards, r) the less likely is the public to overthrow the dictator.

If an informed citizen of mass $d\alpha$ accepts to be co-opted, she gets:[7]

$$U^C = C + rd\alpha + \Pi(\alpha + d\alpha)\delta\theta + (1 - \Pi(\alpha + d\alpha))\delta(\bar{\theta} - K).$$

If she joins the opposition, she gets:

$$U^O = C + \Pi(\alpha)\delta\theta + (1 - \Pi(\alpha))\delta(\bar{\theta} - K).$$

The trade-off is straightforward. By joining the opposition, the informed citizen decreases the odds of the incumbent staying in power—but loses the reward, r. The returns to regime change are: $\delta\left(\theta - (\bar{\theta} - K)\right)$.

Therefore, an informed citizen joins the opposition if and only if $r < \delta\Pi'(\alpha)\left[(\bar{\theta} - K) - \theta\right]$. It is immediately clear that if the true type is high $\theta = 1$, the right hand side is negative and nobody wants to join the opposition. A competent dictator does not need to offer any rewards.

If the dictator is incompetent ($\theta = 0$), then all the informed public joins the opposition if and only if the co-optation rewards are sufficiently low: $r < r^*$. Here

$$r^* \equiv \delta\Pi'(\alpha)(\bar{\theta} - K). \tag{3}$$

Therefore, if the dictator is incompetent,

$$\alpha(r) = \mathbf{1}\{r \geq r^*\}.$$

4.1.3 Dictator's choice: censorship, rewards, propaganda

The dictator learns his type, θ, observes Y and chooses the strategies: censorship (x), rewards (r), and propaganda (P). He maximizes his probability of staying in power, π. Given the decisions made by the dictator (r, x, and P), the expected probability of staying in power depends on the type of the dictator. We will consider the competent and incompetent dictator separately.

4.1.3.1 Solving the competent dictator's problem

If the dictator is competent, there is no opposition in equilibrium: $\beta = 0$. A competent dictator therefore never uses rewards, censorship or propaganda: $P = R = X = 0$. For him, the propaganda signal is $p = 1$ with probability 1 and consumption is $C = C_1(Y) \equiv Y$. The probability of staying in power is therefore

$$\pi_1 = \mathbf{1}\{Y \geq C^*\}. \tag{4}$$

[7] As we show later, in equilibrium $\Pi(\alpha)$ is continuous and differentiable.

4.1.3.2 Solving the incompetent dictator's problem

If the dictator is incompetent, the probability of staying in power is:[8]

$$\pi_0 = [1 - (1 - x)(1 - \alpha(r))]\,\Lambda(P)\mathbf{1}\{C \geq C^*\} \tag{5}$$

Indeed, the probability of $z = 1$ is $1 - (1 - x)(1 - \alpha(r))$, and the probability of $p = 1$ is $\Lambda(P)$.

Now consider the incompetent dictator's optimization problem. We will maximize (5) in two steps. First, the dictator will choose the level of consumption $C = C_0(Y)$ and therefore the total budget, $B = Y - C_0(Y)$, for propaganda, censorship and rewards. Second, given the choice of consumption and budget, B, he will choose the x, P, and r that maximize the probability, ρ, of $p = z = 1$ as a function of the budget:

$$\rho(B) = \max_{X+R+P\leq B}[1 - (1 - x)(1 - \alpha(r))]\Lambda(P). \tag{6}$$

4.1.3.3 Incompetent dictator's strategies given the choice of consumption

The dictator solves (6) taking into account $R = \alpha(r)rI$ and $X = \widehat{X}(1 - \alpha(r))\min\{x, 1\}$.

The choice of rewards for co-optation is binary: either (i) high rewards $r = r^*$ (and $R = R^* \equiv r^*I$), or (ii) no rewards $r = 0$ (and $R = 0$). Indeed, the rewards $0 < r < r^*$ are suboptimal for the dictator relative to $r = 0$—they do not reduce the number of opposition activists but are costly. The rewards $r > r^*$ are also suboptimal for the same reason.

The dictator can choose high rewards only if $R^* \leq B$. In this case, he spends $R = R^*$; censorship is useless (as there is no opposition), so all remaining resources are spent on propaganda: $P = B - R^*$. The probability of $p = z = 1$ is:

$$\rho(B) = \Lambda(B - R^*) = \frac{B - R^*}{\widehat{P}}.$$

If the dictator chooses not to co-opt the elites ($r = 0$), he solves:

$$\max_{X+P=B} \frac{X}{\widehat{X}}\frac{P}{\widehat{P}}. \tag{7}$$

The solution is $P = X = \frac{B}{2}$. and the probability of $p = z = 1$ is:

$$\rho(B) = \frac{B^2}{4\widehat{P}\widehat{X}}.$$

The analysis implies:

$$\rho(B) = \max\left\{\frac{B - R^*}{\widehat{P}}; \frac{B^2}{4\widehat{P}\widehat{X}}\right\}. \tag{8}$$

[8]Notice that the probability of survival is a continuous, monotonic and differentiable function of α. Therefore $\Pi'(\alpha)$ (which is part of expression (3)) is well-defined.

Therefore the regime with co-optation is chosen if and only if R^* is sufficiently low:

$$R^* < B\left(1 - \frac{B}{4\widehat{X}}\right). \tag{9}$$

4.1.3.4 Incompetent dictator's choice of consumption

In equilibrium, the choice of consumption $C_0(Y)$ should solve:

$$C_0(Y) = \arg\max_C \rho(Y - C)\mathbf{1}\{C \geq C^*\} \tag{10}$$

Since $\rho(B)$ is strictly increasing in C, the dictator will always choose $C = C^*$ and $B = Y - C^*$.

4.2 Equilibria

As the competent dictator always sets $C = Y$, it is easy to show that there can be two kinds of equilibria.

First, $C^* = Y^L$, and the competent dictator always stays in power. The incompetent dictator is overthrown if $Y = Y^L$. However, if $Y = Y^H$ he stays in power and spends $B = Y^H - Y^L = \Delta Y$ on rewards, censorship and propaganda and therefore achieves a non-trivial probability of $p = z = 1$.

Second, $C^* = Y^H$, and the competent dictator only stays in power if $Y = Y^H$; the incompetent dictator is overthrown with probability 1.

Can there be equilibria with $C^* < Y^L$? This is impossible as the public knows that if it observes $C < Y^L$, the dictator must be incompetent. So it makes sense to replace this dictator with probability 1.

Similarly, there cannot be equilibria with $C^* \in (Y^L, Y^H)$. In this case, if the public observes such a C^* and $p = z = 1$, it knows with certainty that the dictator is incompetent (and lucky).

4.2.1 Equilibrium with $C^* = Y^H$

The equilibrium with $C^* = Y^H$ exists whenever the inferred quality of a dictator with $C = Y^L$ is below the potential alternative $\bar{\theta} - K$.

Proposition 1. *The equilibrium with $C^* = Y^H$ exists if and only if*

$$\Delta Y \geq \widehat{P}\left(\frac{1}{\bar{\theta} - K} - 1\right)\frac{\bar{\theta}}{1 - \bar{\theta}}\frac{1 - \xi_1}{\xi_0}. \tag{11}$$

In equilibrium, none of the informed citizens joins the opposition, the co-optation rewards are infinitesimal, there is no censorship, and the incompetent dictator with $Y = Y^H$ spends ΔY on propaganda.

In this equilibrium, the incompetent dictator survives with probability 0 while the competent dictator survives with probability ξ_1. The intuition for the structure of equilibria is as follows: when the public observes $p = z = 1$ and $C = Y^L$, it understands that it is very likely that the dictator is incompetent and is spending a great deal of resources (ΔY, which must be sufficiently large) on propaganda. Notice that there

is no opposition: the informed citizens, understanding that the dictator will be overthrown anyway, accept to be co-opted even if the rewards are close to zero. Given that there is full co-optation, the dictator spends nothing on censorship. Finally, as the total co-optation rewards are trivial, the dictator spends all of ΔY on propaganda.

4.2.2 Equilibrium with $C^* = Y^L$

In this equilibrium, the competent dictator always survives, while the incompetent dictator only survives if $Y = Y^H$. In the latter case, he spends ΔY on propaganda, censorship and co-optation.

The properties of this equilibrium depend critically on the relative costs of co-optation and propaganda, which is captured by the following parameter:

$$a \equiv \frac{\delta(\bar{\theta} - K)I}{\widehat{P}}. \tag{12}$$

The co-optation rewards are endogenous in equilibrium; one of their key determinants is the informed citizens' long-term cost of not revealing the dictator's incompetence. There are I informed citizens, and each knows that replacing an incompetent dictator would improve her own welfare by $\delta(\bar{\theta} - K)$. Therefore, $\delta(\bar{\theta} - K)I$ is the opportunity cost for the informed citizens of remaining silent—which will be the minimum reward they will require for co-optation. The denominator $\widehat{P}$ is how much it costs the incompetent dictator to generate a positive propaganda signal with probability 1; in other words, raising the probability of $p = 1$ by 1 percentage point costs the dictator $0.01\widehat{P}$.

In this equilibrium, the general public should believe that the average type of the dictator with $C = C^* = Y^L$ and $p = z = 1$ is weakly better than regime change:

$$E(\theta | C = Y^L, p = z = 1) = \frac{\bar{\theta}(1 - \xi_1)}{\bar{\theta}(1 - \xi_1) + (1 - \bar{\theta})\xi_0 \rho(\Delta Y)} \geq \bar{\theta} - K. \tag{13}$$

Proposition 2. *The equilibrium with $C^* = Y^L$ exists only if ΔY is sufficiently small*

$$\Delta Y < 4\widehat{X} \max\left\{ \frac{1}{a+1}, \frac{\frac{a}{2} - 1}{a - 1} \right\} \tag{14}$$

There can be two kinds of stable equilibria :

If $\Delta Y < 4\widehat{X}\frac{1}{a+1}$ and

$$\Delta Y \leq (1 + a)\widehat{P}\left(\frac{1}{\bar{\theta} - K} - 1 \right) \frac{\bar{\theta}}{1 - \bar{\theta}} \frac{1 - \xi_1}{\xi_0}. \tag{15}$$

then there is an equilibrium with co-opted elites: $\alpha = 1$. In this equilibrium, the dictator uses co-optation and propaganda, but not censorship. The probability of survival is:

$$\rho(\Delta Y) = \frac{\Delta Y}{\widehat{P}(1 + a)} \tag{16}$$

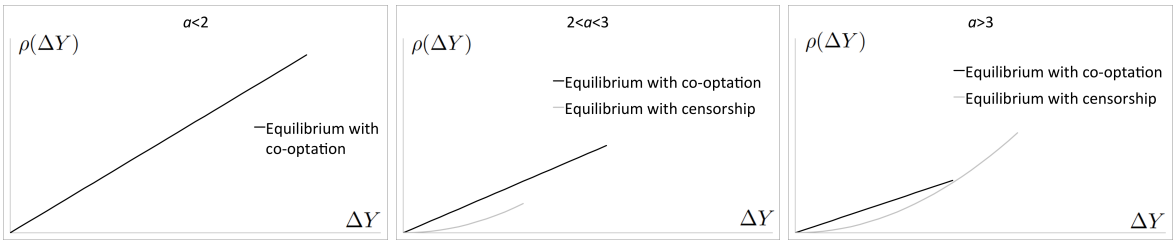


Figure 1: Structure of equilibria for different values of parameters. The graphs represent the probabilities of survival $\rho(\Delta Y)$ for the following parameter values: $\widehat{X} = \widehat{P} = 1$, and $a = 1.5$ (left), $a = 2.5$ (middle), $a = 3.5$ (right).

If $a > 2$ *and* $\Delta Y < 4\widehat{X}^{\frac{\frac{a}{2}-1}{a-1}}$ *and*

$$\Delta Y \leq \sqrt{4\widehat{P}\widehat{X}\left(\frac{1}{\bar{\theta}-K}-1\right)\frac{\bar{\theta}}{1-\bar{\theta}}\frac{1-\xi_1}{\xi_0}}, \tag{17}$$

there is an equilibrium with censorship in which all informed citizens join the opposition: $\alpha = 0$. *In this equilibrium, the dictator uses censorship and propaganda, but not co-optation. The probability of survival is:*

$$\rho(\Delta Y) = \frac{(\Delta Y)^2}{4\widehat{P}\widehat{X}} \tag{18}$$

Equilibria with $C^* = Y^L$ only exist if ΔY is sufficiently small. Indeed, if ΔY is large, the public knows that the incompetent dictator can spend substantial resources for assuring $p = z = 1$. Hence observing $p = z = 1$ and $C = C^*$, citizens still decide to protest.

Multiple equilibria. If $a > 2$ and if ΔY is sufficiently small (satisfying both (14), (15) and (17)), then we have two stable equilibria — one with propaganda and co-optation ($\alpha = 1$) and one with propaganda and censorship ($\alpha = 0$). Comparing (16) and (18) we find that whenever both equilibria exist, the probability of survival is higher in the equilibrium with co-optation.

Figure 1 presents the structure of equilibria under different values of parameters (if conditions (15) and (17) are not binding—e.g. if $(1 - \xi_0)/\xi_1$ is very high).

These equilibria (where $C^* = Y^L$) can also co-exist with the equilibrium with $C^* = Y^H$ (indeed, for some parameter values both conditions (11) and (15) are satisfied). In the latter equilibria, the public, observing a positive signal $p = z = 1$ and low consumption, infers that the expected quality of the dictator is too low and protests. As the informed citizens understand that the dictator will be removed anyway, they know that their information will have no impact and are therefore happy to be co-opted even if rewards are low. Thus, achieving the outcome with $p = z = 1$ and $C^* = Y^L$ is relatively cheap for the dictator. So the public's inference is justified. For the same parameter values, there may exist an equilibrium with $C^* = Y^L$ where informed citizens do have an impact on the dictator's survival probability; in this case, co-opting them is costly. Hence, observing the outcome $p = z = 1$ and $C^* = Y^L$, the public infers that the expected type of the dictator is relatively high.

4.3 Discussion of the results

The previous analysis yields four main results.

First, if economic volatility (the difference between good and bad economic states, ΔY) is not very high, the incompetent dictator can survive in equilibrium. The logic is as follows. If ΔY is low, that means that an incompetent dictator who is lucky enough to experience a good economic outcome ($Y = Y^H$) will have only a small amount of extra resources, ΔY, to spend on propaganda, censorship, and co-optation. As a result, the messages received by the public should be relatively accurate. Thus, if the public does receive the positive signals $p = z = 1$, it will tend to believe them and think the incumbent is competent, even if consumption is low: $C = Y^L$.

However, if volatility is high, then no equilibrium with $C = Y^L$ exists and the incompetent dictator is overthrown. The impact of economic volatility is especially important in the dynamic setting (see section 5.2): we show that when volatility is low, the dictator is less likely to survive the initial period; but, if he does, he is then more likely to remain in power for a long time.

This result may also be interpreted in a cross-sectional sense. If the public notices that other comparable countries are doing much better (ΔY is high), they infer that their dictator may well have diverted a lot of resources (ΔY) into propaganda, censorship, and co-optation. As a result, they stop believing the positive signals, conclude that the dictator is probably bad, and take to the streets to protest.

The second result is the multiplicity of equilibria. If ΔY is small there is always an equilibrium with co-optation, in which the elite agrees to stay silent in return for rewards. For some parameters, there is also an equilibrium with censorship, in which all informed citizens join the opposition but the dictator attempts to block their media communications.

The multiplicity of equilibria is driven by the 'fiscal externality' of the informed elites. Consider the equilibrium with co-optation $\alpha = 1$. Every informed citizen is happy to stay in the pro-regime camp, receiving r^* for not joining the opposition. But now imagine that some informed citizens ($\tilde{\beta} > 0$) join the opposition. In this case, the dictator spends $r^*\tilde{\beta}$ less on rewards and reallocates the money to propaganda and censorship, increasing x and P. How does that change the returns to joining the opposition for the remaining $1 - \tilde{\beta}$ members of the elite? These returns are related to $\Pi'(\alpha)$, which in turn is proportional to $(1 - x)P$: the payoff to joining the opposition decreases in censorship and increases in propaganda.

Thus, the defection of some informed citizens has two opposite effects. On the one hand, the increase in censorship reduces the odds that an anti-regime message will get through. This *lowers* the payoff to joining the opposition. On the other hand, the increase in propaganda makes it less likely the general public will learn of the dictator's incompetence—and thus all the more important for informed citizens to try to communicate it. This *raises* the payoff to joining the opposition. Which effect dominates will depend on the parameters. If the former effect is stronger, no additional members of the elite defect, and the co-optation equilibrium holds. But if the latter effect dominates, then the initial departures set off a spiral of defections, ending in the equilibrium with censorship and no co-optation.

The third result concerns how the dictator's various instruments interact. As seen in (5), censorship and co-optation of the elite are substitutes, while propaganda is complementary to both of them. Returns to censorship and co-optation increase with propaganda. But higher spending on censorship decreases incentives for co-optation. These interactions emerge endogenously due to the public's statistical inference based on the information it receives.

Finally, we see that whenever both the equilibrium with co-optation and that with censorship exist, the dictator's survival probability is always higher in the former. This result may be driven by the specific linear forms selected for the propaganda and censorship cost functions (and by the assumption that $\widehat{X}$ and $\widehat{P}$ are sufficiently large). We chose linear technologies here for simplicity and to explore how non-linearities can emerge simply due to the properties of statistical inference. Of course, in reality there may be increasing or decreasing returns to scale in propaganda and censorship—although we lack strong intuitions about which way these would go–and such nonlinearities could alter the relative survival odds in the different equilibria. But the multiplicity of equilibria is a more robust finding, and one that suggests a novel insight. A dictator can get stuck in an inferior yet stable equilibrium, relying on a relatively ineffective technique to stay in power.

The properties of equilibria also help to understand the role of our assumption $r \geq 0$ that the dictator can only reward the loyal members of the elites but cannot punish the disloyal ones. This assumption rules out the possibility of silencing the opposition through repression rather than through co-optation or censorship. We believe that this is a reasonable assumption for the following reason. In equilibrium, the competent dictator would never use repressions against the disloyal members of the elite (because there are none). Thus, if such punishments are observed by the public, they reveal any dictator who uses them to be incompetent, rendering the tactic self-defeating. A similar argument of course applies to co-optation and censorship. These are are also never applied by a competent dictator. Hence, co-optation and censorship are used only to the extent that they can be concealed. By definition, censorship allows hiding information about the very fact of censorship. What about co-optation? We suppose that concealing payments to loyal elite members (or at least concealing the true reason for such payments) is generally much easier than concealing repression of disloyal elite members. The jailing of a political activist is far more visible than a wire transfer to a secret bank account. For this reason, we expect positive rewards to the elite to be used much more often than repression.[9] The need to conceal repression against the elite contrasts with the situation with regard to repression against ordinary citizens, which is used only once the information-based equilibria no longer exist (see section 5.3 below). For the threat of repression against ordinary citizens to be effective, it should be public.

[9]In section 5.4 we discuss an extension where the dictator can use limited repression against disloyal members of the elite.

5 Comparative statics and extensions

5.1 Comparative statics

How do the exogenous parameters affect the dictator's survival probability? While the model has nine parameters (ΔY, $\bar{\theta}$, K, I, δ, $\widehat{P}$, $\widehat{X}$, ξ_0 and ξ_1), the structure and existence of equilibria depends on just two combinations of them: $a = \frac{\delta(\bar{\theta}-K)I}{\widehat{P}}$ and $\frac{\Delta Y}{4\widehat{X}}$.

The parameter ΔY is the ex ante economic volatility. From (16) and (18), we see that the dictator's survival probability in both the possible equilibria, $\rho(\Delta Y)$, increases in ΔY (although this is not necessarily true in a multi-period model, see Section 5.2). The reason is simple: the higher is ΔY, the more resources an incompetent dictator can spend on censorship, co-optation and propaganda in a given period when $Y = Y^H$. Not surprisingly, this translates into a higher probability of survival—as long as the equilibrium exists.

Parameter a captures the cost of co-optation relative to the cost of propaganda. If a increases then the equilibrium with censorship is more likely to exist, while the equilibrium with co-optation is less likely to exist. (This can be seen as we shift the curves in Figure 1 in response to change in a; indeed $\frac{1}{1+a}$ decreases in a and $\frac{\frac{a}{2}-1}{a-1}$ increases in a).

If $\widehat{X}$ increases then both equilibria are more likely to exist—the public understands that censorship is not cost-effective so a good signal, if received, is more likely to be true. If the censorship equilibrium holds, higher $\widehat{X}$ reduces the dictator's survival odds (see 18): his censorship becomes less effective. But if the censorship cost rises too high, this equilibrium no longer exists and instead the equlibrium with co-optation will hold. Recall that in the co-optation equilibrium, the dictator's survival probability is always higher. Paradoxically, an incompetent dictator is better off if he has an extremely ineffective apparatus of censorship. High censorship costs serve as a commitment device, allowing the dictator to achieve an equilibrium that he prefers.

If the alternative regime is good and revolution is not too costly (high $(\bar{\theta} - K)$), then the equilibrium with censorship is more likely and that with co-optation less likely. The intuition is straightforward: for members of the elite, the returns to removing the incompetent dictator are high, and so to silence them with rewards is expensive. As a result, the dictator prefers to use censorship. By the same logic, if the elite cares more about the future (higher δ), bribing it to accept an incompetent incumbent costs more, so censorship is favored.

What happens if the cost of propaganda, $\widehat{P}$, falls? Interestingly, a goes up and the equilibrium with co-optation becomes less likely (while the equilibrium with censorship becomes more likely). The lower the cost of propaganda, the more likely the public will get a convincing propaganda signal. This makes the signals of the opposition more important, and renders the informed elite harder to co-opt, increasing the odds the dictator will use censorship instead. If an external enemy threatens the state's security, this may help the incumbent to dodge blame for poor economic performance, rendering his propaganda claims more believable. If so, the equilibrium with co-optation could disappear, leaving only that with censorship.

One consequence of economic modernization is to broaden the informed elite, increasing I. In the current setting, this makes co-opting the elite more expensive, and thus favors the equilibrium with censorship. However, it may be more intuitive to suppose that the cost of censorship, $\widehat{X}$, rather than remaining fixed, also increases with I. If, for instance, $\widehat{X}$ is proportional to I, it is easy to check that an increase in I makes *both* equilibria less likely to exist. Indeed, higher I raises the costs of both co-opting and censoring. Since propaganda cannot substitute for censorship and rewards (propaganda is a *complement* rather than a substitute for both), incompetent dictators face more problems whatever they do. As suggested by "modernization theory," broader access to information (or education) renders dictatorships less likely to survive.

5.2 Dynamics

We consider here a multi-period version of the model. We assume that the economic and informational environment does not change over time. The only state variable that does evolve over time, t, is the public's belief, $\widetilde{\theta}_t$, regarding the public's beliefs about the type of the dictator at time t. Initially, the beliefs are the same as in the one-period model above $\widetilde{\theta}_t = \bar{\theta}$. After each period, if the dictator survives, the public rationally updates its beliefs about him. We will assume that the alternative regime's quality is constant over time $\bar{\theta} - K$. The informed citizens do not update their views as they learn the dictator's type precisely in the initial period.

We first consider the dynamics of equilibrium with $C^* = Y^H$. In this equilibrium, if the dictator survives after the first period, the public knows with certainty that the dictator is competent. Therefore in all future periods $\widetilde{\theta}_t = 1 > \bar{\theta} - K$, there are no protests.

The dynamics of the equilibrium with $C^* = Y^L$ are more complicated. With probability $\xi_0\rho(\Delta Y)$ the incompetent dictator survives. Each period, the public compares its current expectation of the dictator's type $\widetilde{\theta}_t$ with the alternative $\bar{\theta} - K$. If the dictator survives, the expectation $\widetilde{\theta}_t$ evolves over time according to the following Bayesian updating formula:

$$\widetilde{\theta}_{t+1} = \frac{\widetilde{\theta}_t(1-\xi_1)}{\widetilde{\theta}_t(1-\xi_1) + (1-\widetilde{\theta}_t)\xi_0\rho(\Delta Y)} = \widetilde{\theta}_t + \frac{\widetilde{\theta}_t(1-\widetilde{\theta}_t)\left(1-\xi_1-\xi_0\rho(\Delta Y)\right)}{\widetilde{\theta}_t(1-\xi_1) + (1-\widetilde{\theta}_t)\xi_0\rho(\Delta Y)} \tag{19}$$

Therefore, $\widetilde{\theta}_t$ increases over time if and only if the following condition holds

$$\xi_0\rho(\Delta Y) \leq 1 - \xi_1 \tag{20}$$

If this is the case, the probability of survival in each particular period $\xi_0\rho(\Delta Y)$ is rather low. However, if the dictator does survive, next period the public raises its expectation regarding the type of the dictator.

If $\xi_0\rho(\Delta Y) > 1 - \xi_1$, the public downgrades its beliefs every period according to (19). Eventually, at a certain (finite) time $t = t^*$ the belief $\widetilde{\theta}_t$ falls below $\bar{\theta} - K$, and the incompetent dictator is removed with probability 1.

Proposition 3. *Suppose that at time $t = 0$ the economy is in an equilibrium with $C^* = Y^L$ and $\rho(\Delta Y)$.*

If condition (20) holds, the incompetent dictator survives each period with probability $\rho(\Delta Y)$. His expected lifetime in office is therefore $\frac{\rho(\Delta Y)}{1-\rho(\Delta Y)}$.

If condition (20) does not hold, then there exists such finite time $t^* > 1$ that $\widetilde{\theta}_{t^*} < \bar{\theta} - K \leq \widetilde{\theta}_{t^*-1}$. In the periods $t < t^*$, the incompetent dictator survives each period with probability $\rho(\Delta Y)$, and at the period $t = t^*$ he is removed with probability 1. His expected lifetime is $\frac{\rho(\Delta Y)}{1-\rho(\Delta Y)}\left(1 - [\rho(\Delta Y)]^{t^*}\right)$.

The distinction between these two cases is especially interesting given that $\rho(\Delta Y)$ is different in the equilibria with co-optation and censorship. For the same values of parameters, there may co-exist two equilibria: an equilibrium with co-optation (where $\rho(\Delta Y)$ is high) and an equilibrium with censorship (where it is low). It may be the case that in the censorship equilibrium, condition (20) holds while in the co-optation equilibrium, it does not. In this case, the dynamics will be as follows: in the co-optation equilibrium, the probability of surviving in each given period is higher (conditional on a particular belief) but the beliefs are downgraded each period and eventually the dictator is overthrown with certainty. In the censorship equilibrium, per-period survival is less likely, but beliefs are always adjusted upwards, so this equilibrium will continue indefinitely.

The ex ante economic volatility ΔY is also important. If ΔY is low, the public knows that condition (20) holds, and a positive public signal $p = z = 1$ implies that the probability of the dictator being competent is high. So if he survives for another period, his reputation improves and he is even likelier to last for yet another period. Conversely, if ΔY is high, the public knows that an incompetent dictator could be spending a lot on propaganda and censorship. Hence, each period, the public downgrades the reputation of the surviving incumbent. Eventually, this reputation sinks so low that the public protests against the dictator even if it observes a positive signal $p = z = 1$.

The binary nature of the shocks renders the dynamics simple to analyze but it also rules out any dynamic effect of change in beliefs on the choice of consumption. In this model, consumption is always equal either to Y^H (if the dictator is competent and lucky) or to Y^L. In section 6, we consider a model with continuous economic shocks. There, an increase in $\bar{\theta}$ results in lower consumption in equilibrium. If the expected type of the dictator is high, the incompetent dictator can get away with lower C. In such a setting, increases in $\widetilde{\theta}_t$—i.e. a trend towards greater competence in government—are accompanied by falling consumption.

5.3 Repressing the protesters

In the model developed above, the elite is not repressed in the literal sense of the word; it is only censored and not rewarded (or fined) if it refuses to be co-opted. Essentially, economic aspects of repression are included into the parameter r (which reflects the difference between the material well-being of the members of the pro-regime elite and that of the opposition activists). In Section 5.4 we also discuss extensions related to jailing the opposition activists.

However, another important element of dictatorial rule is the repression of protests. By allocating certain resources to the riot police and military, the dictator can increase the cost of regime change K. The potential

protesters do not observe this cost but can infer the dictator's optimal choice.

What is the effect of adding a fourth instrument to the dictator's repertoire? Now the dictator can choose to reduce spending on censorship, rewards and propaganda, $B = X + R + P$, and use the savings, $\Delta Y - B$, to increase K. The latter matters as it directly affects the parameter a, which, in turn, determines the cost of rewards and therefore the structure of equilibria (see Figure 1).

Suppose that the dictator spends G on repressive capacity. He sets the level at the same time that he allocates the budgets for censorship, X, rewards, R, and propaganda, P. We will assume that the repression technology is linear, $K = G/\widehat{G}$. We will assume that $\widehat{G} > \Delta Y/\bar{\theta}$; this implies that repression is sufficiently costly so that the dictator cannot achieve $K = \bar{\theta}$. This assumption is reasonable—indeed, if the dictator could achieve $\bar{\theta} - K = 0$, he would always do so and would never use other instruments as the public would never have an incentive to replace him.

We now need to reproduce the analysis of the equilibria replacing $B = \Delta Y$ with $B = \Delta Y - G$ and a with $a(G) = \delta I(\bar{\theta} - G/\widehat{G})$.

Consider first the equilibria with rewards ($\alpha = 1$). This equilibrium exists if $\Delta Y - G < \frac{4\widehat{X}}{1+a(G)}$. The higher the level of repression G, the likelier this inequality is to hold. Therefore, repressions make this equilibrium more likely to exist. However, given that it exists, the the probability of survival $\rho(\Delta Y - G) = \frac{\Delta Y - G}{\widehat{P}(1+a(G))})$ monotonically decreases with G. This is because repression is not cost-effective—it makes more sense to spend money on rewards and propaganda.[10]

Therefore if the equilibrium without repression exists, the dictator does not spend anything on repressions. However, if the equilibrium without repressions does not exist, the dictator is better-off spending on repressions just enough to assure that this equilibrium exists. Suppose that $a(0) < 2$ and $\Delta Y > \frac{4\widehat{X}}{1+a(0)}$. Then without any repression, the equilibrium unravels and the dictator is removed from office with probability 1. In this case, he would benefit from choosing the minimum level of repression that satisfies:

$$\Delta Y - G \leq \frac{4\widehat{X}}{1 + \delta I\left(\bar{\theta} - \frac{G}{\widehat{G}}\right)}. \tag{21}$$

This level of repression is the smaller root of the respective quadratic equation.[11] This solution always exists (at $G = 0$, the left-hand side of (**??**) is greater than the right-hand side, while at $G = \Delta Y$ the left-hand side is zero while the right-hand side is positive).

Given this level of repression, the dictator survives with a non-trivial probability $\rho(\Delta Y - G) = \frac{4\widehat{X}}{\widehat{P}(1+a(G))^2}$. The intuition is simple. The dictator understands that without any investment in repressive capacity, the elite is too costly to silence. So he uses some repression to make sure that the elite is less interested in regime change and is therefore cheaper to bribe. This allows him to free up additional resources for propaganda and therefore make ends meet in equilibrium.

[10] This result holds as the repression technology is linear. In the non-linear case, there may be a non-trivial amount of repression in equilibrium.

[11] The solution is $G = \frac{\widehat{G}}{2\delta I}\left(1 + \delta I\left(\bar{\theta} + \frac{\Delta Y}{\widehat{G}}\right) - \sqrt{\left(1 + \delta I\left(\bar{\theta} + \frac{\Delta Y}{\widehat{G}}\right)\right)^2 - 4\delta I\left(1 + \delta I\left(\bar{\theta} + \frac{\Delta Y}{\widehat{G}}\right) - \frac{4\widehat{X}}{\widehat{G}}\right)}\right)$.

Now consider the equilibrium with censorship ($\alpha = 0$). In this equilibrium, the probability of survival decreases in the level of repression: $\rho(\Delta Y - G) = \frac{(\Delta Y - G)^2}{4\widehat{P}\widehat{X}}$. There are no incentives to spend on repressive capacity in this equilibrium.

However, it may also be the case that this equilibrium does not exist without repressions. Consider the situation where $a(0) > 2$ and $\Delta Y > 4\widehat{X}\frac{\frac{a(0)}{2}-1}{a(0)-1} = 2\widehat{X}\left(1 - \frac{1}{a(0)-1}\right)$. Again, by spending certain amount of resources on repressions, the dictator can assure that the equilibrium does exist. This level of repression is the minimum G that satisfies:

$$\Delta Y - G \leq 2\widehat{X}\left(1 - \frac{1}{\delta I\left(\bar{\theta} - \frac{G}{\widehat{G}}\right) - 1}\right).$$

This level of repression is also the smaller root of the quadratic equation.[12] Unlike in the case of the equilibrium with co-optation, the solution does not always exist. If parameters are such that the quadratic equation has no roots, then no level of repression will assure the existence of equilibrium.

Thus, spending on repressions against the protesters cannot help to raise the probability of survival in equilibrium. However, by raising the cost of revolution, they can assure the existence of equilibrium (in case the equilibrium does not exist without repressions). This rationale for repression works when ΔY is sufficiently high. In this case, the economic decline is so large that the public does not believe the propaganda any more; hence the dictator prefers to spend money on repression.

5.4 Other extensions

Our model assumes linear technologies for propaganda, censorship, and co-optation. In particular, there are no non-linearities in the responses of informed citizens to the amount spent on propaganda. If the dictator spends less on propaganda, the informed citizens understand that their opposition messages will have a lower impact, and so the cost to co-opt them declines proportionally. This is why both equilibria with co-optation and with censorship exist when ΔY is small (so that spending on propaganda, $P \leq B \leq \Delta Y$, is also small).

In reality, the informed citizens' choice may be non-linear. There could be a fixed cost of getting co-opted (due to the "moral cost of opportunism") or a fixed cost of joining the opposition (e.g., a jail term). A non-trivial moral cost would make the equilibrium with co-optation disappear for small ΔY. Indeed, if the total budget for co-optation and propaganda $B = R + P = \Delta Y$ is small, the dictator would not be able to afford to compensate all members of the elite for the moral cost of co-optation.

Meanwhile, a jail term for opposition activists might eliminate the equilibrium with censorship at low values of ΔY. In this case, as the total budget for co-optation and propaganda $B = X + P = \Delta Y$ is low, spending on propaganda $P = \Delta Y/2$. Therefore for each member of the informed elite the benefits from joining opposition are low — they are are proportional to the impact on dictator's survival probability, and

[12] The solution is $G = \frac{\widehat{G}}{2\delta I}\left(-1 + \delta I\left(\bar{\theta} + \frac{\Delta Y - 2\widehat{X}}{\widehat{G}}\right) - \sqrt{\left(-1 + \delta I\left(\bar{\theta} + \frac{\Delta Y - 2\widehat{X}}{\widehat{G}}\right)\right)^2 - \frac{4\delta I}{\widehat{G}}\left(-1 + \delta I\bar{\theta} + 2\widehat{X}\right)}\right)$.

this impact is in turn proportional to the level of propaganda. Hence if ΔY is sufficiently low, the benefits of joining the opposition fall below the fixed cost of a jail term; therefore the equilibrium unravels.

We have implicitly assumed that the acts of both censorship and repression cannot be directly observed by the public. Since competent incumbents never censor or repress, observing the dictator doing one of these things would immediately reveal that the incumbent was incompetent. One might think of this as endogenously increasing the size of I. If enough observed such behavior, it might set off a spiral of opposition activism and protest. There are, indeed, various cases in which the use of excessive violence by the regime, or the discovery of a grizzly murder for which the regime was blamed, catalyzed opposition to it rather than intimidating the public. While it is implausible that a single act of violent repression would automatically undermine the dictatorship, one can imagine introducing sufficient noise into the model that only a level of violence that rose above a certain threshold would trigger a strong public reaction.

6 The impact of economic shocks

The model sketched so far allows only limited analysis of comparative statics with regard to economic shocks. In the setting with two output levels, if the incompetent dictator's output decreases, he is overthrown with probability one, so his spending on propaganda, censorship, and co-optation are not relevant. In this section, we consider a setting with a continuous distribution of economic shocks, in which such shocks can have a non-trivial impact on co-optation, censorship and propaganda.

6.1 Setting

Let us assume that Y is distributed on $[Y^L, Y^H]$ with the density function $f_1(Y)$ if the dictator is competent and $f_0(Y)$ if he is incompetent. We assume that the density is positive for all $Y \in [Y^L, Y^H]$ and that there are no mass points. Higher output is more likely for the competent types and less likely for the incompetent types; we assume that $f_0(Y)$ is a (weakly) monotonically decreasing function and $f_1(Y)$ is a (weakly) monotonically increasing function.

As the informed citizens observe both the output and the type of the dictator, the game between the dictator and the informed citizens remains the same. In this game, there is no asymmetric information. Therefore, the structure of equilibria is similar to that in the previous model: there can be equilbria with co-optation ($\alpha = 1$) and with censorship ($\alpha = 0$). We assume that equilibrium existence conditions (14) hold so these equilibria exist. We will denote by $B(Y)$ the total budget for censorship, co-optation and propaganda as a function of the output (in the model with binary output , $B(Y^H) = \Delta Y$ and $B(Y^L)$ did not matter as the dictator was overthrown anyway).

The model with continuous output has no equilibrium in pure strategies. We therefore consider equilibria in mixed strategies. In such equilibria, the general public chooses a probability of supporting the dictator as a function of the observed consumption level $\gamma(C) \in [0, 1]$ (if $p = z = 1$). If $p = 0$ or $z = 0$, the public knows with certainty that the dictator is incompetent and protests with probability 1.

In equilibrium, the public's choice of $\gamma(C)$ is optimal:

$$\begin{aligned} \gamma(C) = 0 \quad &\text{if} \quad E(\theta|C, p = z = 1) \leq \bar{\theta} - K, \\ 0 < \gamma(C) < 1 \quad &\text{if} \quad E(\theta|C, p = z = 1) = \bar{\theta} - K, \\ \gamma(C) = 1 \quad &\text{if} \quad E(\theta|C, p = z = 1) \geq \bar{\theta} - K. \end{aligned}$$

As in the model with binary output, it is easy to show that the competent dictator always chooses $C_1(Y) = Y$.

The incompetent dictator faces the following trade-off: if he spends more on propaganda, co-optation and censorship, this results in a higher probability of the positive signal $p = z = 1$ by the public, however as this spending comes at the cost of lower consumption C, it implies lower probability of survival *given* the positive signal. His choice of consumption level, $C_0(Y)$, solves the following optimization problem:

$$C_0(Y) = \arg \max_{0 \leq C \leq Y} \gamma(C)\rho(Y - C). \tag{22}$$

6.2 Equilibrium

Lemma 1. *If the distribution density $f_1(\cdot)$ does not have mass points, there is no equiLibrium in pure strategies. The equilibrium in mixed strategies has the following properties:*

(i) Public's strategy $\gamma(C)$ is weakly monotonic. If $\gamma(C) < 1$, it is strictly monotonic. The incompetent dictator's strategy $C_0(Y)$ is also strictly monotonic.

(ii) There exists such $C^{**} \in (Y^L, Y^H]$ that $\gamma(C) = 1$ for all $C \geq C^{**}$;

(iii) $\gamma(Y^L) = 0$;

(iv) $0 < \gamma(C) < 1$ for all $C \in (Y^L, C^{**})$.

(v) $C_0(Y)$ is a differentiable, invertible function. In equilibrium, its first derivative equals

$$C_0'(Y) = \frac{f_0(Y)\rho(Y - C_0(Y))}{f_1(C_0(Y))} \frac{(\bar{\theta} - K)(1 - \bar{\theta})}{(1 - \bar{\theta} + K)\bar{\theta}}. \tag{23}$$

(vi) In the worst economic outcome, $Y = Y^L$, the incompetent dictator spends all resources on consumption:

$$C_0(Y^L) = Y^L. \tag{24}$$

(vii) The total spending on co-optation, censorship and propaganda $B(Y) = Y - C_0(Y)$ monotonically increases with Y.

The Lemma implies that both $\gamma(C_0(Y))$ and $B(Y)$ increase with Y; therefore the higher the realization of the economic shock, Y, the higher the probability of survival, $\gamma(C_0(Y))\rho(B(Y))$.

In order to find the equilibrium, we need to solve equation (23) given the initial condition (24). To complete the description of the equlibrium, we should find $\gamma(C)$ using the first-order condition from (22), which yields the following differential equation: $\gamma'(C_0(Y)) = \gamma(C_0(Y))\frac{\rho'(Y-C_0(Y))}{\rho(Y-C_0(Y))}$. This equation determines $\gamma(\cdot)$ up to a scalar multiplier. To find this multiplier, we need to use the boundary condition: $\gamma(C_0(Y^H)) = 1$.

6.3 Comparative statics

In this section, we carry out the comparative statics analysis with regard to the parameters of the model. We also compare the behavior of an incompetent dictator in an equilibrium with co-optation with the one in an equilibrium with censorship.

Proposition 4. *For given distributions $f_0(\cdot)$ and $f_1(\cdot)$, and for a given realization of the economic shock Y, an incompetent dictator spends more on censorship, co-optation and propaganda and less on consumption if K increases, $\widehat{P}$ increases, $\widehat{X}$ increases (in the equilibrium with censorship), δI increases (in the equilibrium with co-optation), or if both $\bar{\theta}$ and K increase, holding $(\bar{\theta} - K)$ constant;*

The intuition for the comparative statics is straightforward. If $\widehat{P}$ increases, a dollar spent on propaganda is less effective than before at rendering a false message convincing. The public therefore increases its estimate of the incumbent's competence by more when it receives a convincing message, $p = z = 1$, even if the consumption level is somewhat lower. Understanding this, an incompetent dictator has an incentive to provide a lower level of consumption and spend more on increasing the probability of a positive signal, $p = z = 1$. Similar logic applies for the cost of censorship, $\widehat{X}$, (in the equilibrium with censorship), and for the parameter δI (that determines the size of rewards in the equilibrium with co-optation). An increase in K lowers the expected value of revolution; hence the public will accept a lower consumption level from the incumbent.

Finally consider the impact of an increase in the expected quality of dictators, $\bar{\theta}$, holding the quality of the alternative regime $\bar{\theta} - K$ constant. The higher is $\bar{\theta}$, the stronger is the public's expectation that the incumbent is competent, and so he can afford a lower level of consumption. This result is especially important for the analysis of the dynamics. If the dictator is lucky enough to survive the first few periods, his reputation, $\bar{\theta}$, increases, while the expected quality of the alternative, $\bar{\theta} - K$, does not change. In this case, the equilibrium trajectory involves the surviving dictator's reputation growing over time and consumption level declining over time.

We should emphasize that the comparative statics results in Proposition 4 are very different from those that a naive 'partial equilibrium' model would produce. For example, if we did not take into account the endogenous equilibrium inference of the public, an increase in the cost of propaganda should result in lower spending on propaganda and higher spending on consumption. However, as the public is rational and can decipher this logic, the 'general equilibrium' result is that a higher cost of propaganda results in *lower*

consumption and *higher* spending on propaganda.

Equation (23) also allows us to compare the reaction to economic shocks in equilibria with co-optation (where $\rho(Y - C_0(Y))$ is proportional to $Y - C_0(Y)$) and equilibria with censorship (where $\rho(Y - C_0(Y))$ is proportional to $(Y - C_0(Y))^2$). Consider two countries, with the same ex ante distributions of economic shocks, $f_0(\cdot)$ and $f_1(\cdot)$. Suppose that we observe the same output $\widetilde{Y} > Y^L$ and same $\widetilde{C}$ in both countries, but that the first country is in the equilibrium with co-optation, while the second one is in the equilibrium with censorship. Our theory predicts that if these countries are hit by similar economic shocks (e.g., Y declines to a certain level $\bar{Y} \in (Y^L, \widetilde{Y})$), then the first country will respond with a lower decrease in consumption (and a greater decrease in the total budget for co-optation, censorship and propaganda) than the second country. In other words, regimes with censorship react to negative economic shocks with a greater reallocation of resources towards censorship, co-optation and propaganda than do regimes with co-optation.

Proposition 5. *Consider an equilibrium where $C_0(\widetilde{Y}) = \widetilde{C}$. For any $\bar{Y} \in (Y^L, \widetilde{Y})$, the decrease in consumption, $C_0(\widetilde{Y}) - C_0(\bar{Y})$, is smaller if the economy is in the equilibrium with co-optation (both before and after the decline in Y) than in the case in which it is in the equilibrium with censorship (both before and after the decline in Y).*

The intuition for this result is as follows. In the equilibrium with censorship, a given economic deterioration decreases citizens' consumption—and therefore their belief in the dictator's competence—by more than the same economic deterioration does in the equilibrium with co-optation (see Figure 1). Thus, in the censorship equilibrium, the dictator has to spend relatively more on censorship and propaganda to offset the induced loss of public confidence when the economy declines.

6.4 Example

To illustrate the propositions stated above, we use a simple example. Let us assume that Y is distributed on $[0, \Delta Y]$.[13] The distribution is uniform for the bad type: $f_0(Y) = 1/\Delta Y$ and has linear increasing density for the good type $f_1(Y) = 2Y/(\Delta Y)^2$.

Let us assume that ΔY is sufficiently small so that the equilibrium existence conditions (14) are satisfied (indeed, in this case for all realizations of Y the budget, $B(Y) \leq Y \leq \Delta Y$, also satisfies these conditions).

We shall solve (23) for the equilibrium with co-optation ($\alpha = 1$) and then for the equilibrium with censorship ($\alpha = 0$). In the equilibrium with co-optation, $\rho(B) = \frac{B}{\widetilde{P}(1+a)}$. Using the boundary condition (24) (which takes the form $C_0(0) = 0$), we immediately find a solution for (23):

$$C_0(Y) = \xi Y; \quad \gamma(C) = \left(\frac{C}{\xi \Delta Y}\right)^{\frac{\xi}{1-\xi}}. \tag{25}$$

[13] The lower bound of zero is a normalization to the lowest level of GDP above which income can be diverted to propaganda, censorship, and co-optation.

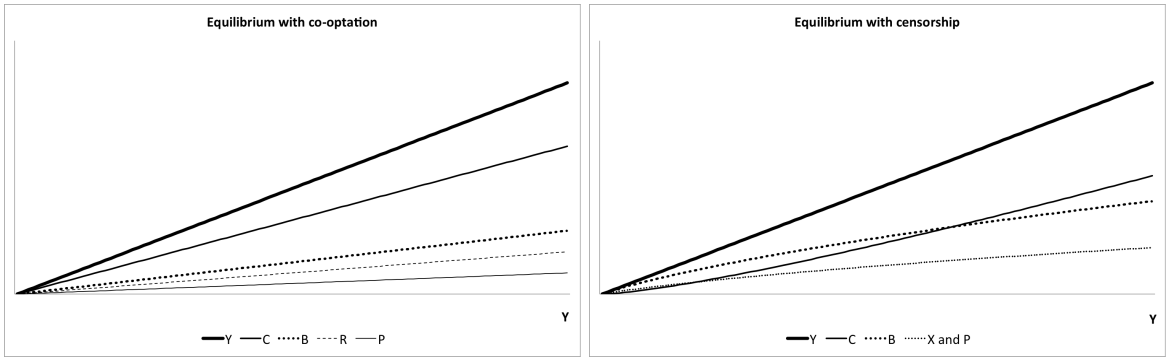


Figure 2: Consumption, C, and spending on censorship, X, co-optation, R, and propaganda, P, as a function of the economic shock Y.

where $\xi \in (0,1)$ is given by

$$\xi = \frac{\sqrt{1 + \frac{8\widehat{P}(1+a)}{\Delta Y}\frac{(1-\bar{\theta}+K)\bar{\theta}}{(\bar{\theta}-K)(1-\bar{\theta})}} - 1}{\frac{4\widehat{P}(1+a)}{\Delta Y}\frac{(1-\bar{\theta}+K)\bar{\theta}}{(\bar{\theta}-K)(1-\bar{\theta})}}.$$

The solution is therefore simple: if output decreases, the incompetent dictator proportionally decreases total spending on co-optation and propaganda $B(Y) = (1-\xi)Y$. Moreover, as follows from the Proof of Proposition 2, spending on co-optation and spending on propaganda also change proportionally: $R(Y) = \frac{a}{a+1}(1-\xi)Y$ and $P(Y) = \frac{1}{a+1}(1-\xi)Y$.

Now let us consider the equilibrium with censorship. In this case, $\rho(B) = \frac{B^2}{4\widehat{P}\widehat{X}}$. Therefore, (23) turns into a non-linear differential equation that has no analytical solution. However, we can approximate the solution in the neighborhood of $Y = 0$:

$$C = \sqrt{\frac{\Delta Y}{12\widehat{P}\widehat{X}}\frac{(1-\bar{\theta})(\bar{\theta}-K)}{\bar{\theta}(1-\bar{\theta}+K)}}Y^{\frac{3}{2}} + o\left(Y^{\frac{3}{2}}\right). \tag{26}$$

This means that in the equilibrium with censorship, the incompetent dictator spends relatively more on censorship and propaganda as the economy worsens: $B(Y)/Y$ increases as Y falls to zero. In the limit $Y \to 0$, the share of output spent on censorship and propaganda actually reaches 100 percent: $\lim_{Y\to 0}\frac{B(Y)}{Y} = 1$. Conversely, as output grows, the share spent on consumption increases: $C_0(Y)/Y$ is increasing with Y. The amounts spent on censorship and propaganda in this equilibrium are equal to each other $X = P = B(Y)/2$. In proportion to total income, Y, their shares therefore also increase as output falls.

In Figure 2 we present the numerical solution and show that these properties hold for the whole range of Y and not just in the neigborhood of $Y = 0$.

The example also illustrates Proposition 4. Indeed, in the equilibrium with co-optation, ξ increases in $\frac{\Delta Y}{\widehat{P}(1+a)}\frac{(1-\bar{\theta})(\bar{\theta}-K)}{\bar{\theta}(1-\bar{\theta}+K)}$. And it is easy to show, both analytically and numerically, that in the equilibrium with censorship, for any given Y, consumption $C_0(Y)$ increases in $\frac{\Delta Y}{\widehat{P}\widehat{X}}\frac{(1-\bar{\theta})(\bar{\theta}-K)}{\bar{\theta}(1-\bar{\theta}+K)}$.

7 Conclusions

The totalitarian tyrants of the past used mass violence, ideological indoctrination, and closed borders to monopolize power. In a world of economic interdependence and modern communications technology, many authoritarian leaders choose a different strategy—that of manipulating information to convince the public that they are doing a good job. We model the tradeoffs this generates, as dictators choose from a repertoire of techniques, and citizens exploit the signals inherent in the dictator's actions to infer his type.

We show that, in this setup, incompetent leaders can survive by manipulating the information environment so long as economic shocks are not too large. In such equilibria, no violent repression—or at most a very little—is needed. Over time, an incompetent leader who survives may acquire a reputation for competence by virtue of rational Bayesian updating on the part of the public. Major economic downturns destroy such equilibria, exposing the leader's incompetence, and generating protests that can only be suppressed by force.

The coordination problem of members of the informed elite leads to multiple equilibria, in one of which the dictator focuses on co-optation and in the other of which he relies instead on censorship. Co-optation and censorship are substitutes—both prevent the elite from publicizing an incumbent's failures—but state propaganda is complementary to both. The multiplicity of equilibria may explain why we observe very different levels of censorship and of patronage in otherwise similar regimes.

We also show that a dictator focused on co-optation reacts to economic shocks differently from one who is commited to censorship. The latter is more likely to boost propaganda and censorship in response to an economic downturn than the former. Although we do not explicitly model the use of institutions such as legislatures, parties, and elections, it is clear that a dictator who succeeds in convincing the public that he is competent can enjoy the benefits of such "democratic" elements without fear of losing power. Such institutions—which are quite compatible with a strategy of co-optation of the elite—can be used to amplify the (accurate) perception of the dictator's popularity.

Our model also explains why in information-based dictatorships violence either is a last resort when all else has failed, or is used sparingly when it is possible to conceal it. The reason is that the competent dictators do not need to use repressions. Hence, reverting to repressions immediately reveals the dictator's incompetence to the public and ultimately results in his downfall. This is why dictators often find or create some non-political pretext to imprison leaders of the political opposition. This also explains why when dictators who arrange for the murder of political opponents, usually go to great lengths to deny responsibility. If the goal were merely to intimidate the opposition, then such denials would be baffling—an open acknowledgement would have a far greater deterrent effect.[14]

[14]This also explains why those—usually in the security services—who stand to benefit from a regime based on raw repression may seek to compromise a dictator by committing acts of violence for which he will be blamed. The true goal in such cases may be to force the dictator to switch from an information-based survival strategy to one based on sheer violence. It also suggests why an incompetent security apparatus can be so dangerous for a dictator. After troops shot the Philippine opposition leader Benigno Aquino on the tarmac of Manila Airport, it was impossible for President Marcos to deny complicity. This murder ignited the "People's Power" movement that eventually prompted a split within Marcos's military support group and the collapse of his regime.

The survival strategies we formalize are more compatible with a modernized society than totalitarian ones or those of monarchs who rely on traditional legitimacy. Yet, we also show how modernization ultimately undermines such informational equilibria. As education and information spread to a broader segment of the population, it becomes harder to control how this informed elite communicates with the masses — either through co-optation or through censorship.

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Appendix: Proofs

Proof of Proposition 1. Let us calculate the expected value of the type of the dictator that the public infers when observing $p = z = 1$ and $C = Y^L$. This outcome is possible if the dictator is good (which happens with probability $\bar{\theta}(1-\xi_1)$) or if the dictator is incompetent wiht Y^H and spends ΔY on censorship, repression and propaganda.

Let us calculate the probability of the incompetent dictator having $p = z = 1$ and $C = Y^L$. The informed citizens understand that the incompetent dictator is overthrown with probability 1. Therefore they expect $\Pi(\alpha) = 0$ and $\Pi'(\alpha) = 0$. Hence $r^* = 0$ and $R^* = 0$ (informed citizens join pro-regime elite even for an infinitesimal reward). Thus $\tilde{B} = 0$, and the dictator does not use censorship $x = 0$ and spends all his resources on propaganda $P = \Delta Y$. Therefore $\rho(\Delta Y) = \Delta Y/\widehat{P}$. Therefore the the probability of the incompetent dictator having $p = z = 1$ and $C = Y^L$ is $(1-\bar{\theta})\xi_0 \Delta Y/\widehat{P}$.

The equilibrium exists whenever

$$E(\theta | C = Y^L, p = z = 1) = \frac{\bar{\theta}(1-\xi_1)}{\bar{\theta}(1-\xi_1) + (1-\bar{\theta})\xi_0 \rho(\Delta Y)} \leq \bar{\theta} - K$$

which can be rewritten as (11).

Notice that which $\rho(\Delta Y) = \Delta Y/\widehat{P}$ is the maximum probability for the incompetent dictator to have $p = z = 1$ and $C = Y^L$, in this equilibrium the incompetent dictator is overthrown anyway so he is indifferent whether to maximize this probability or to choose another strategy. However, if he spends less on propaganda and $C > Y^L$, this equilibrium does not exist. Also, if instead of spending ΔY, the dictator spends it on censorship or co-optation — and therefore may have a lower $\rho(\Delta Y)$, the condition above may not hold and the equilibrium stops to exist. Therefore, (11) is a necessary and sufficient condition for equilibrium with $C^* = Y^H$ to exist. ■

Proof of Proposition 2. Let us first calculate $\Pi'(\alpha)$. The expression (6) implies that the derivative of probability of achieving $p = z = 1$ with regard to α is $(1-x)\Lambda(P)$. Therefore

$$\Pi'(\alpha) = E\left[(1-x)\Lambda(P)\mathbf{1}\{C \geq C^*\}\right]. \tag{27}$$

There can be three cases. First, $\Delta Y > \widetilde{B}$ (no opposition $\alpha = 1$). In this case, $x = 0$, $\Lambda(P) = \frac{\Delta Y - R^*}{\widehat{P}}$. Hence, $\Pi'(\alpha) = \frac{\Delta Y - R^*}{\widehat{P}}$.

Second, if $\Delta Y < \widetilde{B}$ (all informed join the opposition $\alpha = 0$). In this case $x = \Delta Y/(2\widehat{X})$ and $\Lambda(P) = \Delta Y/(2\widehat{P})$. In this case $\Pi'(\alpha) = \left(1 - \frac{\Delta Y}{2\widehat{X}}\right)\frac{\Delta Y}{2\widehat{P}}$.

Third, $\Delta Y = \widetilde{B}$ (the informed citizens are indifferent whether to join the opposition). In this case $\Pi'(\alpha)$ is not defined but the equality $\Delta Y = \widetilde{B}$ is sufficient to solve for the equilibrium.

1. Equilibrium with co-optation. Let us first consider the case $\alpha = 1$. In this case $\Pi'(\alpha) = \frac{\Delta Y - R^*}{\widehat{P}}$.

Substituting $r^* = R^*/I$ and $\Pi'(\alpha)$ from (3), we find an equation for R^*:

$$R^* = r^* I = \delta \Pi'(\alpha)(\bar{\theta} - K)I = a(\Delta Y - R^*)$$

Solving for $R^* = \Delta Y \frac{a}{a+1}$ and substituting into (9) we find the following condition: $\frac{a}{a+1} < \left(1 - \frac{\Delta Y}{4\widehat{X}}\right)$. Therefore, this equilibrium exists only if ΔY is sufficiently low

$$\Delta Y < 4\widehat{X}\frac{1}{1+a} \tag{28}$$

Finally, we need to check whether $\rho(\Delta Y) = \frac{\Delta Y - R^*}{\widehat{P}} = \frac{\Delta Y}{\widehat{P}(1+a)}$ satisfies the condition (13). This gives us the condition (15).

2. Equilibrium with censorship and no co-optation $\alpha = 0$. In this case $R = 0$, and $\Pi'(\alpha) = (1-x)P/\widehat{P} = \left(1 - \frac{\Delta Y}{2\widehat{X}}\right)\frac{\Delta Y}{2\widehat{P}}$. Hence, the equation for R^* is

$$R^* = \delta(\bar{\theta} - K)I\left(1 - \frac{\Delta Y}{2\widehat{X}}\right)\frac{\Delta Y}{2\widehat{P}} = a\Delta Y\left(\frac{1}{2} - \frac{\Delta Y}{4\widehat{X}}\right)$$

Since in this equilibrium $\Delta Y < \widetilde{B}$, the condition (9) implies that $R^* > \Delta Y\left(1 - \frac{\Delta Y}{4\widehat{X}}\right)$ hence

$$a\left(\frac{1}{2} - \frac{\Delta Y}{4\widehat{X}}\right) > \left(1 - \frac{\Delta Y}{4\widehat{X}}\right).$$

Since R^* must be non-negative, the left-hand side in the inequality above must be non-negative; this also implies that the right-hand side is also non-negative ($1 - \frac{\Delta Y}{4\widehat{X}} > 1 - \frac{\Delta Y}{2\widehat{X}} \geq 0$). Hence this equilibrium exists only if $a > 2$ and if $\Delta Y < 4\widehat{X}\frac{\frac{a}{2}-1}{a-1}$,

This equilibrium exists if $\rho(\Delta Y) = \frac{(\Delta Y)^2}{4\widehat{P}\widehat{X}}$ satisfies the condition (13); this is the case whenever (17) holds.

3. Intermediate equilibrium $\alpha \in (0,1)$ There is also an intermediate (unstable) equilibrium: the case where $\Delta Y = \widetilde{B}$ which is equivalent to

$$R^* = \Delta Y\left(1 - \frac{\Delta Y}{4\widehat{X}}\right). \tag{29}$$

In this equilibrium, the informed citizens are indifferent between accepting or rejecting the rewards. Some of them (α) are in the pro-regime elite and some (β) joint the opposition. The budget spent on rewards is αR^* so the dictator solves

$$\max_{X+P \leq \Delta Y - \alpha R^*; X \geq 0}\left(1 - (1-\alpha)\left(1 - \frac{X}{\beta\widehat{X}}\right)\right)\frac{P}{\widehat{P}} = \max_{X+P \leq \Delta Y - \alpha R^*; X \geq 0}\frac{(\alpha\widehat{X} + X)P}{\widehat{P}\widehat{X}}$$

This equilibrium is unstable. If in equilibrium α informed citizens are co-opted then if $d\alpha$ of them decide to deviate and join the opposition, then the dictator will spend less on co-optation and more on propaganda. This will immediately increase the informed citizens' returns to joining opposition in equilibrium (higher level of propaganda will increase $\Pi'(\alpha)$, so the remaining $\alpha - d\alpha$ will now strictly prefer joining the opposition. ■

Proof of Lemma 1. *Non-existence of equilibrium in pure strategies.* Suppose there is an equilibrium in pure strategies. In this equilibrium, if $p = z = 1$ the public protests with probability 1 if $C < C^*$ and does not protest with probability 1 if $C \geq C^*$. Then there exists a certain threshold Y^* such that the incompetent dictators with $Y \geq Y^*$ choose $C_0(Y) = C^*$ and survive with probability $\rho(Y - C^*)$. Indeed, if an incompetent dictator with Y_1 chooses $C = C^*$, then any luckier incompetent dictator $Y_2 > Y_1$ will also choose $C = C^*$ and spend the additional resources $(Y_2 - C^*) - (Y_1 - C^*)Y_2 - Y_1$ on censorship, co-optation and propaganda. There is no rationale for Y_2 to increase consumption — as it does not increase probability of not being overthrown.

The competent dictators still choose $C = Y$; there is no incentive for them to spend any money on censorship, co-optation and propaganda.

Now let us calculate the expected type of the dictator given observed consumption. If the public observes C^* and $p = z = 1$, it knows that this may be the case if the dictator is good and $Y = C^*$ (which happens with infinitesimal probability $f_1(C^*)dY$) or the dictator is incompetent and $Y \geq Y^*$ (which happens with probability $\int_{Y^*}^{Y^H} \rho(Y - C^*)f_0(Y)dY$). Unless there is a mass point of competent dictators at $Y = C^*$, the inferred probability that the dictator is good is actually zero — and the public strictly prefers to overthrow the dictator. Therefore this is not an equilibrium.

Properties of the equilibrium in mixed strategies.

(i) First, let us show that $\gamma(\cdot)$ is monotonically increasing function. Suppose there are two values of consumptions C_1 and C_2 such that $C_1 < C_2$ but $gamma(C_1) > \gamma(C_2)$. If this is the case, no incompetent dictator will choose $C = C_2$. Indeed, if a dictator chooses C_2, he can do better by choosing $C = C_1$ assuring a higher $\gamma(C)$ and higher ρ (by spending additional $C_2 - C_1$ on propaganda, censorship and co-optation). Hence, if the public observes $C = C_2$, it knows that the dictator cannot be bad. Therefore $\gamma(C_2)$ must be equal to 1. But this is impossible since $gamma(C_2) < \gamma(C_1) \leq 1$.

Similarly, we can prove that $\gamma(C_1) = \gamma(C_2)$ is only possible if both are equal to 1.

The fact that $C_0(Y)$ is monotonic directly follows from the monotonic comparative statics of the optimisation problem (22). As $\rho(\cdot)$ is a linear or quadratic function of $Y - C$ (depending on whether the equilibrium is the one with co-optation or the one with censorship, respectively), it is easy to show that the solution of the optimizatio problem increases with Y. Moreover, if $\gamma(C) < 1$, the function $C_0(Y)$ strictly increases with Y.

(ii) If $C = Y^H$ and $p = z = 1$, the public understands that the dictator is good with probability 1 (if dictator is incompetent, he needs to spend a non-trivial amount on propaganda to send a signal $p = 1$ so he cannot assure $C = Y^H$). Therefore $\gamma(Y^H) = 1$.

Let us denote C^{**} the choice of consumption by the incompetent dictator with the highest $Y = Y^H$, i.e. $C^{**} = C_0(Y^H)$. Then by definition for all $C \geq C^{**}$ the public infers that the dictator is good with probability 1 so that $\gamma(C)$ must be equal to 1.

(iii) Now we shall prove that $\gamma(Y^L)$ must be equal to zero? If this is not the case, then there exists certain $\widetilde{Y} > Y^L$ so that for a bad ruler with such $\widetilde{Y}$ it is optimal to spend $\widetilde{Y} - Y^L$ on censorship, co-optation and propaganda. (Indeed, the incompetent dictator with $Y = Y^L$ cannot provide consumption $C = Y^L$ and assure a positive signal $p = z = 1$). Let us now consider the choices of incompetent dictators with $Y \in (Y^L, \widetilde{Y})$. Monotonicity of $C_0(Y)$ implies that they should choose $C_0(Y) \leq Y^L$. If they choose $C < Y^L$, they are ousted with probability 1. Therefore they choose $C = Y^L$ as this choice gives a positive probability $\gamma(Y^L)\rho(Y - Y^L)$ to stay in office. Therefore if the public observes $C = Y^L$ and $p = z = 1$ it infers the dictator may be good with $Y = Y^L$ or he may be bad with $Y \in (Y^L, \widetilde{Y})$. Since there are no mass points, the conditional probability of the dictator being good is infinitesimal. Therefore $E(\theta|C = Y^L, p = z = 1) = 0$, so $\gamma(Y^L)$ cannot be positive.

(iv) Now let us prove that there cannot be an equilibrium in which $\gamma(C) = 0$ for $C > Y^L$. If it were the case, there would be a certain $\widetilde{C} > Y^L$ such that $\gamma(C) = 0$ for all $C \in [Y^L, \widetilde{C}]$ and $\gamma(C) > 0$ for $C > \widetilde{C}$. In this case all incompetent dictators with $Y > \widetilde{C}$ would choose $C_0(Y) > \widetilde{C}$ as this would give them a positive probability to stay in office while $C \leq \widetilde{C}$ guarantees losing office with probability 1. Let us now consider the choice of the incompetent dictator with $Y = \widetilde{C}$. If he does not spend anything on propaganda, censorship, and co-optation, then probability to achieve $p = z = 1$ is trivial; hence the public observing $C = \widetilde{C}$ and $p = z = 1$ knows with certainty that dictator is good. Therefore, $\gamma(\widetilde{C})$ must be equal to 1 rather than 0. Alternatively, if the incompetent dictator with $Y = \widetilde{C}$ does spend a non-trivial amount $B(\widetilde{C}) > 0$ on propaganda, censorship, and co-optation then $C_0(\widetilde{C}) < \widetilde{C}$. But this implies that there is no incompetent dictator that chooses consumption $C \in (C_0(\widetilde{C}), \widetilde{C})$. Therefore if the public observes C in the range $(\widetilde{C} - B(\widetilde{C}), \widetilde{C})$, the public infers that dictator must be good. Hence, $\gamma(C)$ for $C \in (\widetilde{C} - B(\widetilde{C}), \widetilde{C})$ must be equal to 1 rather than 0. We arrive at a contradiction which proves that $\gamma(C) = 0$ only for $C = Y^L$.

(v) We have already proven in (i) that incompetent dictators with different economic shocks cannot choose the same consumption. Therefore, the function $C_0(Y)$ is invertible, and we can introduce the inverse function $y(C) = C_0^{-1}(C)$ (in other words, $Y = y(C)$ if and only if $C = C_0(Y)$).

Let us now calculate the $E(\theta|C, p = z = 1)$ given the equilibrium beliefs of the general public about $f_\theta(Y)$ and $C_0(Y)$. Suppose that the public observes consumption to be in the interval between C and $C + dC$. This can be the case if the dictator is good and has output $Y \in [Y, Y + dY]$; or it can be the case if the dictator is incompetent and the output is $Y \in [y(C), y(C) + y'(C)dC]$. Therefore

$$E(\theta|C, p = z = 1) = \frac{\bar{\theta} f_1(C)}{\bar{\theta} f_1(C) + (1 - \bar{\theta}) f_0(y(C))\rho(y(C) - C)y'(C)} = \frac{1}{1 + \frac{1-\bar{\theta}}{\bar{\theta}} \frac{f_0(y(C))y'(C)}{f_1(C)} \rho(y(C) - C)} \tag{30}$$

As $E(\theta|C, p = z = 1) = \bar{\theta} - K$ for all $C \in [Y^L, C^{**}]$, we can obtain a differential equation for $y(C)$:

$$y'(C) = \frac{f_1(C)}{f_0(y(C))\rho(y(C) - C)} \frac{(1 - \bar{\theta} + K)\bar{\theta}}{(\bar{\theta} - K)(1 - \bar{\theta})}.$$

This equation implies that $y'(C)$ is continuous which implies that $C_0(Y)$ is also differentiable and its derivative is continuous. Using the inverse function theorem $y'(C)C_0'(y(C)) = 1$, we can use the differential equation for $y(C)$ above to obtain the differential equation (23).

(vi) Suppose $y(Y^L) = \widetilde{Y} > Y^L$, this would be a suboptimal choice for the incompetent dictator with $Y = \widetilde{Y}$. By choosing consumption $C = Y^L$, he got a zero probability to stay in power while he could do better by choosing certain $C \in (Y^L, \widetilde{Y})$.

(vii) In order to prove that $B(Y)$ never decreases with Y, we need to prove that $C_0'(Y) = 1 - B'(Y)$ cannot be above. As $C_0(Y^L) = Y^L$, in the neighborhood of $Y = Y^L$ the difference $B(Y) = Y - C_0(Y)$ cannot decrease with Y, therefore $C_0'(Y) < 1$ for all Y sufficiently close to Y^L.

Therefore if $C_0'(Y)$ at least for some Y, there must exist Y such that $C_0'(Y)$ increases with Y and $C_0'(Y) > 1$. However, this is not possible. Indeed, for such Y the left-hand side of (23) increases in Y while the right-hand side decreases with Y (as $B(Y)$ is falling for such Y, the term $\rho(Y - C_0(Y)$ is decreasing, also, $f_0(Y)$ is decreasing by definition, and $f_1(C_0(Y))$ is decreasing well).

■

Proof of Proposition 4. Let us consider a differential equation

$$C_0'(Y) = M\Phi(Y, C_0(Y)) \tag{31}$$

where M is a constant and $\Phi(\cdot)$ is a continuous function. Then a solution of this differential equation on $Y \in [Y^L, Y^H]$ with the initial condition (24) has the following comparative statics property: if $\widetilde{M} > M$ then for each $Y > Y^H$ the respective solution $\widetilde{C}_0^(Y)$ is above the original solution $C_0^(Y)$. Indeed, the equation (31) and the initial condition (24) jointly imply that $\widetilde{C}_0^(Y) > C_0^(Y)$ at least in some neighborhood of Y^L. Suppose that there exist $Y > Y^L$ such that $\widetilde{C}_0^(Y) = C_0^(Y)$. Let us denote $\breve{Y}$ the intersection point of $\widetilde{C}_0^(Y)$ and $C_0^(Y)$ which is closest to Y^L. Since $\widetilde{C}_0^(Y) > C_0^(Y)$ for all $Y \in (\widetilde{Y}, \breve{Y})$, we should have $\widetilde{C}_0'(\breve{Y}) < C_0'(\breve{Y})$. But since $\widetilde{C}_0(\breve{Y}) = C_0(\breve{Y})$ (31) and $\widetilde{M} > M$ imply that $\widetilde{C}_0'(\breve{Y}) > C_0'(\breve{Y})$.

Therefore for each Y, an increase in M implies an increase in $C_0(Y)$ and decrease in $B(Y) = Y - C_0(Y)$. In the equilibrium with co-optation, we can substitute (16) into (23):

$$C_0'(Y) = \frac{(\bar\theta - K)(1-\bar\theta)}{(\widehat{P} + \delta(\bar\theta - K)I)(1 - \bar\theta + K)\bar\theta} \frac{f_0(Y)(Y - C_0(Y))}{f_1(C_0(Y))}.$$

which has the form (31) for $M = \frac{(\bar\theta - K)(1-\bar\theta)}{(\widehat{P} + \delta(\bar\theta - K)I)(1 - \bar\theta + K)\bar\theta}$.

Similarly, (18) and (23) imply the equation (31) for the equilibria with censorship:

$$C_0'(Y) = \frac{(\bar\theta - K)(1-\bar\theta)}{4\widehat{P}\widehat{X}(1 - \bar\theta + K)\bar\theta} \frac{f_0(Y)(Y - C_0(Y))^2}{f_1(C_0(Y))}.$$

In this case $M = \frac{(\bar{\theta}-K)(1-\bar{\theta})}{4\widehat{P}\widehat{X}(1-\bar{\theta}+K)\bar{\theta}}$.

Differentiating the expressions for M with regard to parameters of the model, we obtain the comparative statics results.

■

Proof of Proposition 5. The equation (23) immediately implies that if both countries have the same Y and C then ratio of slopes of $C_0(Y)$ for the two countries is as follows

$$\frac{C_0^{(2)\prime}(Y)}{C_0^{(1)\prime}(Y)} = \frac{Y-C}{M} \tag{32}$$

where M is a constant depending on parameters of propaganda, censorship, and distributions of economic shocks.

Let us first consider $Y = Y^L$. At this level of output the two countries have the same consumption $C_0^{(1)}(Y^L) = C_0^{(2)}(Y^L) = Y^L$. Therefore (32) implies that in the vicinity of $Y = Y^L$ the slope $C_0^{(2)\prime}(Y)$ is below $C_0^{(1)\prime}(Y)$. Therefore there exists a range of Y for which $C_0^{(2)}(Y) < C_0^{(1)}(Y)$.

Suppose that at some Y the two curves $C_0^{(1)}(Y)$ and $C_0^{(2)}(Y)$ intersect. Let us denote $\widetilde{Y}$ the intersection point closest to Y^L. At this point $C_0^{(1)}(Y)$ intersects $C_0^{(2)}(Y)$ from the above, so we should have $C_0^{(1)\prime}(\widetilde{Y}) < C_0^{(2)\prime}(\widetilde{Y})$. Using (32) we find that $\widetilde{B} > M$.

Let us now prove that cannot be another intersection. Indeed, suppose that there exist other intersection points. Let us denote $\breve{Y}$ the intersection point which is closest to $\widetilde{Y}$. As $C_0^{(2)}(Y) > C_0^{(1)}(Y)$ for all $Y \in (\widetilde{Y}, \breve{Y})$, we should have $C_0^{(1)\prime}(\breve{Y}) > C_0^{(2)\prime}(\breve{Y})$. This implies $\breve{B} < M < \widetilde{B}$. But this contradicts monotonicity of $B(Y)$.

Therefore there can only be one intersection point $\widetilde{Y}$. To the left of this intersection point $Y \in (Y^L, \widetilde{y})$, the first country has a higher consumption C and lower budget B for propaganda, censorship and co-optation than the second country. To the right of this intersection point $Y \in (\widetilde{Y}, Y^H)$, the second country spends more on consumption and less on for propaganda, censorship and co-optation than the first country.

■