Understanding the 30 year Decline in Business Dynamism: a General Equilibrium Approach

Fatih Karahan
Federal Reserve Bank of New York
fatih.karahan@ny.frb.org

Benjamin Pugsley
Federal Reserve Bank of New York
benjamin.pugsley@ny.frb.org

Ayşegül Şahin
Federal Reserve Bank of New York
aysegul.sahin@ny.frb.org

March 30, 2015

Abstract

Recent empirical work has shown that new business formation has declined substantially over the past three decades, while firm growth and survival rates conditional on firm age have remained relatively stable. This paper studies the causes of declining firm entry. First, by exploiting cross-state variation in OLS and IV settings, we establish that declines in the growth rate of the working age population over this period can explain more than half of the decline. We then study a simple model of firm dynamics and show that it can generate a similar response of firm entry to labor supply shifts. When calibrated to the 1980s, the model can explain most of the decline in firm entry, while being consistent with the stability of survival and growth rates over time. Our findings suggest that the decline in the growth rate of the working age population and its general equilibrium effect on labor demand drives the bulk of the decline in firm entry.

PRELIMINARY

1 Introduction

Recent empirical work has drawn attention to an unmistakable shift in U.S. firm dynamics since the late 1970s. Principally, the entry rate, measured as the share of new employer firms out of all employer firms, declined by nearly 40 percent from 1977 to 2007, even before the impact of the Great Recession. Remarkably, this steady decline occurred relatively uniformly within geographic areas and within relatively narrow industry aggregations. Taking this trend decline or "startup deficit" as given, Pugsley and Şahin (2014) show that both its direct effect on business formation and its indirect cumulative effect through a shift in the employer age distribution partly explain the emergence of slower employment recoveries with each business cycle.

An important question is what explains this apparent decline in business dynamism? Understanding the source or sources of the decline is crucial to understand whether it is an efficient response to technological shifts or escalating misallocation from increases, for example, in the costs of starting or running a business. In this paper we provide the first, to our knowledge, quantitative analysis of the set of factors that may have affected business dynamism. Our results are surprising. Rather than significant changes in fixed and entry costs on the firm side, we find that a shift in the growth rate of the working age population that begins in the late 1970s and its general equilibrium effect on labor supply drives the bulk of the declines along the entry margin.

To reach this conclusion, we consider a variety of potential explanations in partial and general equilibrium. The first set of factors that might shift business formation are related to changes in barriers to entry and changes in fixed and variable operating costs. Changes in laws and regulations, market concentration, education and licensing requirements, and shifts in economies of scale might discourage firm entry by creating higher barriers to enter and/or a higher fixed cost of operating. The second set of factors we consider is related to demographic changes. There are various reasons through which the population growth rate can affect business formation. Most directly, an older population might be associated with a lower rate of business formation in the economy if younger workers are more likely to engage in entrepreneurial activity. Changes in population growth also affect the labor supply, which could have important effects on business formation through a general equilibrium channel. In all of these cases, but especially for the general equilibrium effects it is important to differentiate between the effects on incumbent firms and the effects on potential entrants. Shocks to the labor supply that put downward pressure on wages create incentives for incumbent firms to expand, but they also create opportunities for potential entrants. Any effect on the entrant share will depend on how the shocks to the labor supply are accommodated by expanding incumbents and new firms.

We first evaluate the importance of these two sets of factors exploiting pooled cross-state variation in firm entry rates. In particular, we estimate a simple linear regression using OLS and document a strong correlation between the growth rate of working age population and changes in the firm entry rate at the state level, controlling for state and time fixed effects. We also instrument for the variation in the age composition with lagged birthrates and find a startup rate semi-elasticity of roughly 1 to 1.5. Given that the growth rate of working age population went down from around 2 percent in early 1980s to slightly above 1% by 1990s, this estimate suggests that more than half of the decline in the start-up rate can be explained by the decline in the growth rate of working age population. The second part of our empirical analysis focuses on various regulatory and policy changes, which is still in progress.

While the regression based results are informative, they fail to provide us an internally consistent composition of the factors we considered. In addition, it is not possible to have aggregate time-consistent measures of the type of frictions that affect entry costs and fixed costs. To alleviate this problem, the second part of our analysis follows a more structural approach and estimates a variant of the Hopenhayn and Rogerson (1993) model with population growth to evaluate the quantitative

importance of different sets of explanations. Since the model has implications for many other measures of firm dynamics, such as average firm size, survival rates, and employment growth rates, it allows for a more complete evaluation of alternative channels.

With entry and exit in equilibrium, a shock to the labor supply requires either an adjustment along incumbent firms' downward sloping labor demand curve or a shift in the number and possibly size of entrants. The workhorse Hopenhayn and Rogerson (1993) model when adjusted to incorporate a growing population generates patterns consistent with the evidence in the reduced form and instrumented regressions as well as aggregate trends. Negative shocks to the population growth rate reduce the share of new firms required in equilibrium but leave other margins such as age-specific survival rates, average firm sizes, and employment growth rates completely unaffected. The intuition stems from the free entry condition, which implies that supply shocks do not affect real wages. With no change in equilibrium real wages, incumbent hiring or exit is unaffected. The nonresponse of entrants to labor supply shocks also implies that changes in labor supply are fully absorbed by the entry margin.

To test the model, we study empirically the response of various margins to labor supply shocks. Here, we document a contemporaneous response of incumbents and entrants. When the growth rate of the population declines, incumbent firms get slightly smaller and the share of entrants or "startup-rate" declines. Over a slightly longer horizon of two years, only the startup rate adjusts. This is the long-run prediction of the Hopenhayn and Rogerson (1993) model, which places the burden of labor demand adjustment entirely the number of entrants. We take from this analysis that several predictions of our theory is consistent with the empirical evidence.

Our paper is closely related to the emerging literature on the declining dynamism in the U.S. economy. Early work by Reedy and Strom (2012) first called attention to a decline in the aggregate entry rate of new employers. Using more disaggregated data, recent papers by Pugsley and Şahin (2014), Decker, Haltiwanger, Jarmin, and Miranda (2014a), Hathaway and Litan (2014), Gourio, Messer, and Siemer (2014) and Davis and Haltiwanger (2014) all document that these declines in the entry rate are pervasive within geographic areas and relatively narrow industry aggregations. All of these papers have also drawn attention to the relevance of this decline for the ongoing health in labor market. We are the first, to our knowledge, to provide cross-sectional and time-series evidence on the determinants of the declining entry rate. Specifically, we identify the principal role of shifting demographics on the equilibrium quanitity of startup activity, which is also consistent with the predictions of the workhorse model of industry dynamics.

2 Empirical analysis

Our analysis focuses on the period 1980–2007, prior to the Great Recession and its recovery. We start by documenting several aggregate facts that motivate this study and discipline the choice of the model. This is followed by an empirical investigation of various factors. We first investigate the effects of labor supply shifts on startups and then turn to changes in barriers to entry and operating

costs. The analysis of these shifts relies on cross-state variation.

2.1 Data

Our analysis is based on annual data for the period 1980–2007. We use firm-level data from the U.S. Census Bureau Longitudinal Business Database (LBD) and its public use data product the Business Dynamics Statistics (BDS). This dataset covers the vast majority of employers in the U.S. and is based on a longitudinally-linked version of the Census Bureau's business register that includes all private-sector establishments with paid employees. The data report the total employment of each firm on March 12 of each calendar year from 1977 through 2012.[1]

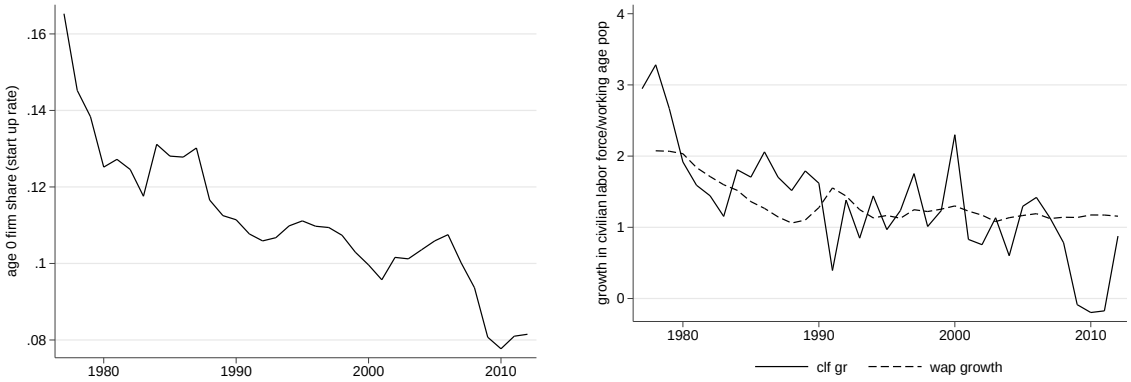
To link changes in firm startup rates to changes in labor supply, we use state-level population estimates by age from the Census Bureau and labor force estimates from the Local Area Unemployment Statistics database maintained by the Bureau of Labor Statistics. We define working age population as the number of workers older than 20. Similar to Shimer (2001) and Karahan and Rhee (2014), we obtain exogenous variation in population growth rates across states by instrumenting with birthrates lagged by 20 years. These are measured in births per thousand residents and are available in the various Statistical Abstracts of the United States.[2]

2.2 Aggregate facts

Figure 1a shows the startup rate, which is the number of newborn firms as a fraction of the overall stock of firms, for the period 1977 to 2012. The startup rate has declined steadily from an average of roughly 13 percent in the early 1980s to around 10 percent before the Great Recession and eventually to 8 percent by 2012. Figure 1b shows that this long-run decline has coincided with a shift in two supply side factors. The solid line shows that over the same period the growth rate of the labor force has declined from around two percent to about 0.8 percent. Similarly, the growth rate of the working age population fell from over two percent to a low of just over one percent.

[1] Multiple establishments owned by the same firm are linked through their ownership records. This is an important detail, since we are interested in true firm startups rather than new establishments of an existing firm.

[2] We are grateful to Rob Shimer for providing us with his data constructed from the Statistical Abstracts for the period 1940–91. Data are unavailable for Hawaii and Alaska prior to 1960. We drop these states entirely from the analysis.



(a) Startup rates

(b) Labor supply shifts

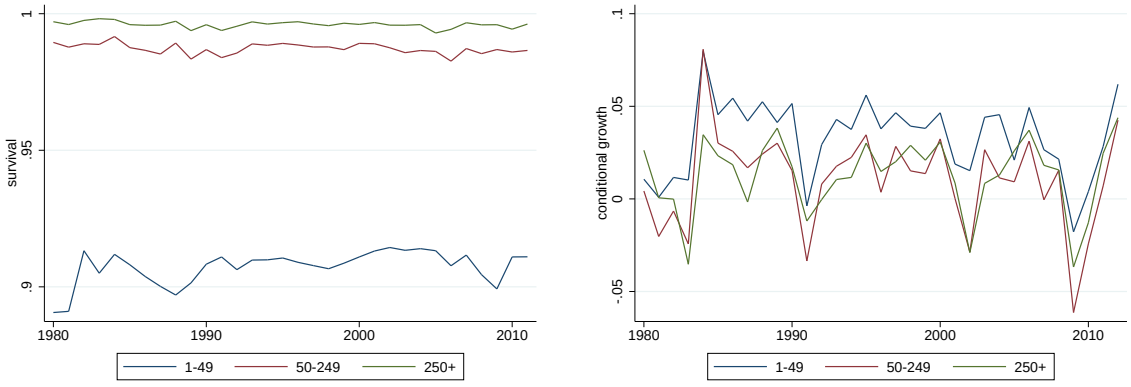
Figure 1: Declines of startup rate and labor supply growth rate from 1977 to 2012

Note: Business Dynamics Statistics and Current Population Survey. Startup rate is number of age 0 employers as share of the total number of employers within a year. Working age population is defined as the noninstitutional population ages 20+.

One immediate question is whether the decline in firm entry affected the quality of entrants. Given that the entry rate declined over time, it is reasonable to expect that entrants have become better over time in terms of their post-entry performances. One way to examine this possibility is to compare the performances of new firms upon enty over time. We consider two distinct metric of performance to shed light on this question. In particular, we report survival rates and conditional employment growth rates by firm size. 2a plots the survival rates of firms by three size categories: 1–49, 50–249 and 250+. While there are business cycle fluctuations in survival rates, these rates have remained surprisingly stable in a period of declining entry. Figure 2b plots the employment growth of firms for the same size categories. Aside from the large swings associated with the business cycles, there is no evidence to support the idea that entrants in later years performed better. As we explain later on, these facts help us discipline our structural model by providing additional margins of variation. More importantly, they provide valuable information in telling apart the relative contributions of labor supply shifts from shifts in operating and entry costs to the decline in the startup rate.

2.3 Assessing the role of labor supply shifts: population and labor force growth

Changes in labor supply—either through changes in population growth or labor force participation—may have effects on firm dynamics through direct and/or general equilibrium effects. At this point, we remain agnostic as to what the economic forces could be and conduct an empirical analysis to measure the effect of this demographic shift. Our main empirical analysis for testing and measuring



(a) Survival rates

(b) Conditional employment growth (excl. startups)

Figure 2: Incumbent survival and conditional growth rates by firm size from 1980 to 2012

Note: Business Dynamics Statistics. Survival rate is the number of incumbent firms within each size group as a fraction of the number of firms in the size group cohort in the previous year. Year 2012 is omitted because of difficulties measuring firm exits in the final year. Incumbent conditional growth rate is the growth rate of average employment size within each size group (excludes all startups) relative to the average employment of the size group cohort in the previous year.

labor supply effects relies on cross-state differences in the growth rate of labor supply, which we proxy with the growth rates of labor force and working age population.

The main empirical specification examines how the startup rate in state i and year t, s_{it}, relates to the growth rate of labor supply, n_{it}:

$$s_{it} = \alpha_i + \beta_t + \gamma n_{it} + \epsilon_{it}. \tag{1}$$

Here, α_i and β_t denote state and time fixed effects, respectively. ϵ_{it} captures other sources of variation in startup rates. State fixed effects are needed to dispose of any unmeasured state-level fixed factor that might simultaneously affect the labor force as well as startup rates in a state. Similarly, time fixed effects are needed to take out the effects of the various aggregate shocks that may have hit the U.S. economy during our sample period. We first estimate equation (1) with OLS and later instrument for the variation in the age composition with lagged birthrates. We report state-clustered standard errors to account for the serial correlation within states.

Columns (1) of table 1 report the OLS estimates using the growth rate of working age population as the proxy for labor supply shifts. According to this estimate, a 10 percentage point slow down in the labor force growth rate leads to an almost 6.7 percentage point decline in firm entry. Since the labor force growth rate slowed down roughly by a full percentage point in our sample period, the OLS results suggest that this labor supply shift can explain about 0.7 percentage points decline in startup rates—about 14 percent of the total decline. Column (2) shows that the results are robust to heterogeneous time trends across states.

Changes in the working age population might fail to fully capture labor supply shifts since labor

Table 1: Labor supply shifts and startup rates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Startup Rate (%)							
WAP GR (20+, %)	0.668***	0.645***			0.925***	1.434***		
	(0.0730)	(0.0716)			(0.292)	(0.302)		
CLF GR (%)			0.295***	0.258***			0.786***	1.141***
			(0.0423)	(0.0437)			(0.255)	(0.259)
Constant	12.15***	12.28***	8.929***	8.830***	8.545***	10.07***	8.983***	10.58***
	(0.0963)	(0.0919)	(0.0730)	(0.0517)	(0.277)	(0.584)	(0.166)	(0.503)
Observations	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316
Adjusted R-squared	0.886	0.910	0.866	0.890	0.880	0.854	0.800	0.681
Instrumented	OLS	OLS	OLS	OLS	IV	IV	IV	IV
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
State Linear Trend	-	Yes	-	Yes	-	Yes	-	Yes
Years	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007
F-stat	-	-	-	-	23.53	25.30	23.70	17.83

Note: Business Dynamics Statistics and Current Population Survey. Stand errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

force participation rate also responds to changes in the economy. In fact, our sample period has witnessed important changes in the participation behavior of young and old workers.[3] To tackle this issue, columns (3) and (4) repeat the analysis using the labor force growth rate. Our findings are qualitatively similar: a 10 percentage point decline in labor force growth rate is associated with 2.5–3 percentage point decline in the startup rate.

One possible explanation for these cross-sectional comovements could be that workers move to more dynamic labor markets with a higher rate of firm entry, generating a spurious correlation between the growth rate of labor supply and the startups rate.[4] Such a bias would arise if a state has a temporary change in its labor market that temporarily attracts more workers. To get at this problem and to establish causal inference, we follow Shimer (2001) and exploit variation in labor supply induced by birthrates in the past. More specifically, our instrument in state i and year t is defined as the birthrate in state i in year $t-20$. Note that if migration rates are high, lagged birthrates may not have enough power to capture the variation in population growth rates. Our first-stage results indicate that the instrument does a good job of inducing variation in the labor supply measures.[5]

Column (5) shows the results for the benchmark specification. According to this estimate, a 10 percentage points decline in the growth rate of the working age population results in a 9 percent decline in startup rates. The corresponding figure for labor force growth rate is about 8 (see column 6). Allowing states to have different linear time trends, the estimated effects go up to 14 and 11 percentage points for the working age population and labor force growth, respectively. These estimates suggest that labor supply shifts over this period can explain 0.9 to 1.4 percentage points of the decline in startup rates. In percentage terms, labor supply shifts explain 30 to 45 percent of the decline in the startup rate.

So far we have established a strong response of startup rates to changes in labor supply. As we will argue in section 4, the Hopenhayn and Rogerson model provides a natural framework to think about the effects of labor supply shocks on firm dynamics. To test the model, we now document the short- and long-run response of startup rates as well as other related measures pertaining to firm dynamics.

Columns (2) of table 2 show the response of startup rates 2 years after the labor supply shock and shows that such shocks have a long-run effect on firm entry.[6] Columns (3) and (4) show the response of average firm size, whereas (5) and (6) display the results for the average size of new entrants. Finally, the last two columns report the response of the average firm size among incumbents. Several remarks are in order. First, the point estimates suggest that the size of entrants decreases in response to positive labor supply shocks, whereas that of incumbents increases. Second, the contemporaneous effect is always larger in magnitude than the effect in the longer run. This suggests that incumbents

[3] See, for example, Aaronson and Davis (2012).

[4] Note that permanent differences across labor markets would be captured by state fixed effects.)

[5] The coefficient on the lagged birthrate in the working age population (labor force) regression is 0.13 (0.15). The first stage yields F-values of 23.5 and 23.7 in these regressions, and together with the fixed effects, lagged birthrate explains 68 percent and 42 percent of the variation, respectively.

[6] Column (1) replicates the contemporaneous response of startup rates for convenience.

Table 2: Labor supply shifts and startup rates

	(1) SR_t	(2) SR_{t+2}	(3) N_t	(4) N_{t+2}	(5) N_t^0	(6) N_{t+2}^0	(7) N_t^{1+}	(8) N_{t+2}^{1+}
WAP GR (20+, %)	0.925***	1.152***	0.630	0.564	-0.358	-0.164	0.955	0.864
	(0.292)	(0.263)	(0.563)	(0.517)	(0.238)	(0.217)	(0.608)	(0.559)
Constant	10.76***	10.05***	19.43***	19.42***	6.342***	6.022***	20.89***	20.85***
	(0.550)	(0.577)	(1.018)	(1.063)	(0.487)	(0.481)	(1.090)	(1.142)
Observations	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316
R-squared	0.887	0.862	0.949	0.947	0.455	0.503	0.945	0.941
Instrumented	IV	IV	IV	IV	IV	IV	IV	IV
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Years	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007	1980-2007
F-stat	23.53	27.09	23.53	27.09	23.53	27.09	23.53	27.09

Note: Business Dynamics Statistics and Current Population Survey. Stand errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Excludes AK, HI, UT and DC.

alter their hiring behavior in the short run, but this effect diminishes over time. Lastly, the effects on firm size are statistically insignificant.

2.4 Supply of young workers or supply of potential entrepreneurs?

[To be completed]

2.5 Assessing the role of costs: Barriers to entry and operating costs

[To be completed]

3 A general equilibrium framework

The cross-state evidence points to a tight link between changes in the growth rate of the working age population and the entry behavior of firms. The link also appears strongest through a labor supply channel rather than a shift in the quantity of potential entrepreneurs or new business. Although the cross-state evidence is weaker, we also see some link between proxies for firm-level costs and declines in the entry rate. To assess these relative contributions quantitatively and in general equilibrium, we turn to an equilibrium model of firm dynamics.

Our starting point is the workhorse Hopenhayn (1992) model set in general equilibrium as in Hopenhayn and Rogerson (1993). In addition to the stationary equilibrium being well-understood, the model's implied behavior of incumbent firms is consistent with the results in Pugsley and Şahin (2014) and further developed in Section 2.2. Specifically, incumbent firm's conditional life-cycle dynamics appear remarkably stable during over the thirty year period we study.

The key divergence between the predictions of the Hopenhayn (1992) model and its variants is in the behavior along the entry margin. To address this margin, to the standard model we introduce non-stationarity through both population and productivity growth. With an appropriate normalization, we can characterize a balanced growth path in a stationary equilibrium, but the equilibrium allocation will not be invariant to demographic shifts. The model allows the entry rate and other key indicators of firm dynamism to shift with demographics and various costs to business operation. We will show that these predictions are consistent with the reduced-form evidence, and we use the model to identify and quantify the sources of the secular shifts in these measures of firm dynamism.

3.1 Preliminaries

The economy consists of a growing population of identical households and a continuum of firms also of growing size. Time is discrete. The population H_t grows at deterministic rate η_t and has preferences over per-household consumption and leisure ordered by

$$\sum_{t=0}^{\infty} H_t \beta^t \left(\log c_t + \alpha l_t \right) . \tag{2}$$

Each household has one unit of time to allocate between leisure l_t and supplying labor to firms.[7] The consumption good is produced by a continuum of firms, each of which has a decreasing returns technology in labor n_t

$$f_t(s_t, n_t) = A_t s_t n_t^{\theta} - c_{ft} \tag{3}$$

that further depends on a firm's idiosyncratic productivity s_t and an aggregate productivity frontier A_t, which grows deterministically at rate g_t. The technology requires a fixed cost c_{ft} be incurred each period.[8] A firm's idiosyncratic productivity evolves as an AR(1) in logs

$$\log s_{t+1} = (1-\rho)\,\bar{s} + \rho s_t + \sigma_s \varepsilon_{t+1}\,. \tag{4}$$

At the end of the period, after production and incurring the fixed cost, firms may exit the market, but only in advance of learning next period's productivity s_{t+1}. Potential firms may freely enter the market by paying a fully sunk entry cost c_{et} units of output and drawing from an initial productivity distribution G. With a continuum of incumbent and entrant firms, and deterministic rates of population and productivity growth there is no aggregate uncertainty. Households may trade in a one period bond with return r_{t+1}, and markets for output and labor are competitive with prices p_t and w_t respectively. Firms are fully owned by the population and any profits are distributed immediately as an aggregate dividend Π_t. We set $w_t = 1$ by choosing labor as the numeraire.

3.2 Firm problem

Given relative price p_t and technology (3) the value $W_t(s_t)$ of a firm with idiosyncratic productivity s_t solves the following Bellman equation

$$W_t(s_t) = \max_{n_t \geq 0} \left\{ p_t A_t s_t n_t^{\theta} - p_t c_{ft} - n_t + \frac{1}{1+r_t} \max_{X_t \in \{0,1\}} \left\{ E_t\left[W_{t+1}(s_{t+1})\right], 0 \right\} \right\}.$$

The inner maximization reflects the end of period exit decision. Let $X_t(s)$ be the optimal exit decision, which is 1 if the firm decides to exit and 0 otherwise. Let $n_t(s)$ be the optimal policy for n. With no adjustment costs n solves a static profit maximization problem

$$n_t(s) = \operatorname*{argmax}_n \left\{ p_t A_t s n^{\theta} - p_t c_{ft} - n \right\},$$

with solution

$$n = (\theta p_t A_t s)^{\frac{1}{1-\theta}}\,.$$

[7] Here we use the preferences from Hopenhayn and Rogerson (1993), but we only require preferences be compatible with a balanced growth path.

[8] Alternatively, we could have introduced an overhead cost in units of labor, i.e., $A_t s_t (n_t - \bar{n})^{\theta}$, where $w_t \bar{n}$ would be the fixed cost with wage w_t.

Gross of fixed costs, profit is

$$\pi_t(s) = (\chi p_t A_t s)^{\frac{1}{1-\theta}}$$

where $\chi \equiv \theta^{\theta}(1-\theta)^{1-\theta}$. Given the optimal choice of n the value of an operating firm simplifies to

$$W_t(s_t) = (\chi p_t A_t s_t)^{\frac{1}{1-\theta}} - p_t c_{ft} + \frac{1}{1+r_t} \max_{X_t \in \{0,1\}} \{E_t W_{t+1}(s_{t+1}), 0\} . \tag{5}$$

3.3 Entrant and incumbent dynamics

Entrants draw an initial productivity from distribution G and are able to operate immediately.[9] The expected value of an entering firm gross of entry costs $p_t c_{et}$ is

$$W_t^e = \int W_t(s) G(ds) .$$

We let M_t be the measure of entrants in period t. We only consider distributions G compatible with entry and exit in equilibrium so that $M_t > 0$ and

$$p_t c_{et} = W_t^e . \tag{6}$$

As in Hopenhayn and Rogerson (1993), let $\mu_t(S)$ for Borel subsets S be the measure of firms producing in period t with idiosyncratic productivity $s \in S$. This measure of firms includes the new entrants M_t and incumbent firms, which are firms that chose not to exit at the end of the previous year. This implies that

$$\mu_t(S') = \underbrace{\int \mathbf{1}\{X_{t-1}(s) = 0\} F(S'|s) \mu_{t-1}(ds)}_{\text{incumbents}} + \underbrace{M_t G(S')}_{\text{new entrants}} , \tag{7}$$

where F is the conditional distribution for productivity implied by the stochastic process for productivity s_t in equation (4). We define the startup rate as the share of new entrants out of all firms

$$SR_t \equiv \frac{M_t}{\int d\mu_t} .$$

3.4 Households

Households may trade in a one period bond b_{t+1}, which for convenience will be denoted in units of labor. The population of measure H_t chooses paths for savings b_t, consumption c_t and leisure l_t to

[9] Given the fixed operating costs, this timing admits the possibility for inital s sufficiently low that entering firms produce and immediately exit at the end of the period with a loss.

maximize (2) subject to a per-capita budget constraint

$$p_t c_t + b_{t+1} + l_t = \frac{1+r_t}{1+\eta} b_t + 1\,.$$

With multiplier λ_t on the budget constraint, an optimal choice for b_t, c_t and l_t requires

$$\frac{1}{c_t} = \lambda_t p_t \qquad \alpha = \lambda_t, \qquad \lambda_t = \beta\,(1+\eta)\,\lambda_{t+1}\frac{1+r_{t+1}}{1+\eta}\,.$$

The constant marginal utility of leisure and choice of units for the bonds require $1+r = \beta^{-1}$. Since bonds are in zero net supply and households are identical then $b_t = 0$ and

$$c_t = \frac{1}{\alpha p_t} \quad \text{and} \quad 1 - l_t = \frac{1}{\alpha} - \frac{\Pi_t}{H_t}\,.$$

3.5 Aggregation

We aggregate over households and firms to define several quantities. First, we aggregate over all firms in a period to measure the aggregate dividend total labor demand

$$N_t^d \equiv \int (\theta p_t A_t s)^{\frac{1}{1-\theta}}\,\mu_t\,(ds)\;.$$

Total profits include the dividends (possibly negative) summed across all firms net of the total sunk entry costs

$$\Pi_t \equiv \int \left((\chi p_t A_t s_t)^{\frac{1}{1-\theta}} - p_t c_{ft}\right)\mu_t\,(ds) - M_t p_t c_{et}\,.$$

Total output is

$$Y_t \equiv \int \left(A_t s\,(\theta p_t A_t s)^{\frac{\theta}{1-\theta}} - c_{ft}\right)\mu_t\,(ds)$$

On the household side total labor supplied is

$$N_t^s \equiv H_t\left(\frac{1}{\alpha} - \frac{\Pi_t}{H_t}\right)\;.$$

Market clearing requires

$$\int (\theta p_t A_t s)^{\frac{1}{1-\theta}}\,\mu_t\,(ds) = H_t\left(\frac{1}{\alpha} - \frac{\Pi_t}{H_t}\right)\;. \tag{8}$$

3.6 An equilibrium with demographic and productivity shifts

We first define a potentially non-stationary competitive equilibrium that admits shifts in population and productivity growth as well as fixed and entry costs.

Definition 1. Given an initial population of measure H_{-1}, technology A_{-1}, a measure of firms μ_{-1}, sequences of population growth, productivity growth, fixed costs and entry costs $\{\eta_t, g_t, c_{ft}, c_{et}\}$ for $t \geq 0$ an equilibrium is a constant real interest rate $r = \beta^{-1} - 1$, a sequence of prices $\{p_t\}$, sequences

of per-capita consumption and leisure $\{c_t, l_t\}$, sequences of individual labor demand and exit rules $\{n_t, X_t\}$ for operating firms, and a sequence of measures $\{\mu_t\}$ such that given prices (i) $\{c_t, l_t\}$ are optimal, (ii) $\{n_t, X_t\}$ are optimal, (iii) $\{\mu_t\}$ satisfies the the law of motion given by equation (7), and (iv) markets clear for all $t \geq 0$.

This definition of an equilibrium allows the number of firms to grow as needed along with the population, as in the data. To compute the comparative statics of a stationary version of this equilibrium we make the necessary assumptions to define a balanced growth path.

3.6.1 Balanced growth path

Assumption 1. *Fixed costs and entry costs are proportional to productivity so that* $c_{ft} = c_f A_t$ *and* $c_{et} = c_e A_t$.

If fixed costs and entry costs grow at the same rate as productivity, a balanced growth path just requires reformulating the law of motion in equation (7) in terms of a normalized measure of firms $\bar{\mu}_t$ where the normalization constant grows at the same rate as the population η_t. We normalize the measure of firms by its total mass so that $\bar{\mu}_t$ is a probability density. If the total measure of firms grows at rate η_t then $\bar{\mu}_t$ satisfies a law of motion

$$\bar{\mu}_t\left(S'\right) = \int \mathbf{1}\left\{X_{t-1}\left(s\right) = 0\right\} F\left(S'|s\right) \frac{\bar{\mu}_{t-1}\left(ds\right)}{1+\eta_t} + SR_t G\left(S'\right) , \tag{9}$$

where the startup rate SR_t is defined as the share of entrants $M_t / \int d\mu_t$.

Definition. Given a constant population growth rate η and aggregate productivity growth g a stationary recursive equilibrium with balanced growth is a set of prices r and $p\left(A\right)$ a value function $W\left(s\right)$ and individual labor demand $n\left(s\right)$, allocations $c\left(A\right)$ and l for each household, a measure $\bar{\mu}$, and a startup rate $\overline{SR}$ such that (i) given relative price $\tilde{p}\left(A\right)$, $n\left(s\right)$ solves $W\left(s\right)$, (ii) $c\left(A\right)$ and l are optimal, (iii) $\bar{\mu}$ satisfies equation (9) and (iii) markets clear.

The solution strategy is the same as in Hopenhayn (1992) and Hopenhayn and Rogerson (1993). First note that the solution to an incumbent firm's problem is invariant to shifts in the population growth rate. An incumbent firm's behavior and value are also invariant to changes in aggregate productivity when $p\left(A\right) = \frac{\bar{p}}{A}$, which will turn out to be the equilibrium relative price. Since entry costs are proportional to productivity in units of output, with entry and exit this price satisfies the free entry condition in equation (6) for any value of A. Similarly, household consumption must be $c\left(A\right) = \bar{c}A$ where $\bar{c} = \frac{1}{\alpha \bar{p}}$. With constant $\bar{\mu}$ and $\overline{SR}$ the total measure of firms and of entrants grow at rate η insuring that profits per household Π / H are constant.

It remains only to determine the equilibrium startup rate $\overline{SR}$ and with it the distribution of firms $\bar{\mu}$. As in Hopenhayn (1992) the right hand side of equation (9) defines a linear operator homogeneous in $\bar{\mu}_{t-1}$ and $\overline{SR}_t$. With both constant then the constant measure $\bar{\mu}$ that satisfies (9) can be as

$$\bar{\mu} = f\overline{SR},$$

where f satisfies

$$f\left(S'\right)=\int \mathbf{1}\left\{X\left(s\right)=0\right\}F\left(S'|s\right)\frac{f\left(ds\right)}{1+\eta}+G\left(S'\right)$$

Clearing the labor market in equation (8)

$$\overline{SR}=\left(\int\left(s\left(\theta ps\right)^{\frac{\theta}{1-\theta}}-c_f\right)f\left(ds\right)-c_e\right)^{-1}\frac{1}{\alpha\bar{p}}.$$

determines $\overline{SR}$ and with it $\bar{\mu}=f\overline{SR}$.

4 Sources of declines in the startup rate

This section provides a quantitative evaluation of the various sources of variation on the decline in the startup rate. While the ultimate goal of this paper is to provide a structural estimation of the model, this version of the paper focuses on providing comparative statistics to help identify major sources of the decline. In particular, our quantitative strategy is to calibrate our model economy to match features of the U.S. economy in early 1980s and then consider variation along three dimensions: growth rate of working age population; fixed costs of production and entry cost. In addition, we present preliminary findings from a quantitative exercise where we vary the growth rate the labor force following its evolution from 1980 to 2012.

4.1 Calibrating the early 1980s economy

We calibrate our model at the annual frequency to match various statistics in early 1980s. We calibrate the time discount rate $\beta^{-1}-1$ and the parameter θ directly to match the real interest rate and labor's share of total revenue. Population growth rate η is set to 0.023, its level in 1980. The persistence ρ of the productivity shock and its volatility σ are set to 0.95 and 0.262 respectively. The fixed cost c_f, entry cost c_e, average idiosyncratic productivity $a/\left(1-\rho\right)$, and the disutility of labor α, are jointly estimated using the simulated method of moments to match the employment-to-population ratio (0.6), the exit rate of startups (0.73), the startup rate (0.13) and the average firm size (20.7) in 1980. These parameters are summarized in the tables.

Table 3: Fixed Parameters

Parameter	Value
Time discount rate, r	0.064
Elasticity of output relative to labor, θ	0.638
Persistence of productivity shock, ρ	0.970
Population growth rate, η	0.023
Standard deviation of productivity process errors, σ_e	0.262

Note: Table 3 reports the values for parameters fixed outside the model.

Table 4: Parameters Calibrated with MSM

Parameter	Value
Fixed cost, c_f	2.604
Entry cost, c_e	11.00
Average log productivity, a	0.003
Disutility of labor, α	1.146

Note: Table 4 reports the values of parameters calibrated through Simulated Method of Moments (SMM).

Table 5 shows the outcome of the model for the distribution of firms and distribution of employment across different firm sizes and compares them with the average in the data for the 1977-1980 period. The model captures the distributions of firms and employment across firm sizes very well.

Table 5: Firm and employment distributions across firms sizes in the data and the model

	Distribution of Employment		Distribution of Firms	
	Data	*Model*	*Data*	*Model*
1-49	0.32	0.16	0.96	0.96
50-249	0.16	0.15	0.03	0.03
250+	0.52	0.69	0.01	0.01

4.2 Comparative statics

This section considers variation in fixed costs of production, entry costs and the growth rate of working age population and discusses the implications of changes along these dimensions on different aspects of firm dynamics as well as the startup rate.

4.2.1 Changes in the entry cost

A natural candidate to account for the decline in firm entry is a rise in the cost of entry. An increase in the entry cost protects incumbent firms and increases the relative price of output through the the free entry condition in 6. The rise in the price increases the profits of incumbents and their optimal employment. Since firm profits are convex in the level of price, incumbents' exit threshold for productivity becomes lower, increasing their survival probability. In addition, the wealth effect of an increase in aggregate profits causes the households to reduce their labor supply. With larger incumbents that are more likely to survive, there is less room for entry in the economy. Even though incumbent firms are on average bigger, aggregate employment demand diminishes from the lower

entry. To summarize, the increase in the entry cost discourages firm entry by making incumbents more profitable and and by reducing household's desire to supply labor.

4.2.2 Changes in the fixed cost of production

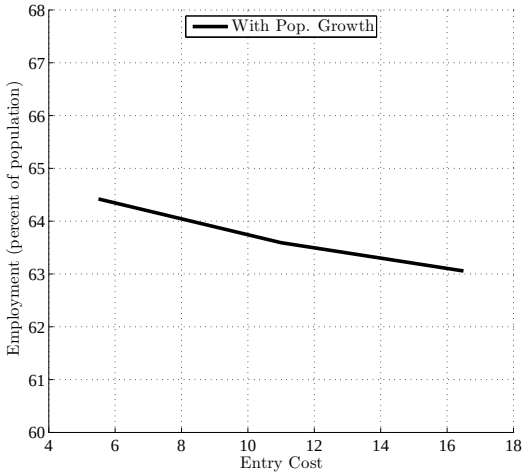
Another possibility is the rise in fixed operating cost. The increase in overhead expenses and its effects on cash flows makes survival difficult for less productive firms, and reduces the profitability of all firms. As values decline, entry becomes less attractive, which puts some upward pressure on the relative price of output. Again because cash flows are convex in the relative price of output, very productive incumbents ultimately become more profitable at the expense of less productive incumbents that are driven closer to exit by an increase in the exit threshold. Aggregate profits decline increasing labor supply through a household wealth effect. The increase in labor supply is accommodated by expanding incumbents and additional entrants.

4.2.3 Changes in the growth rate of population

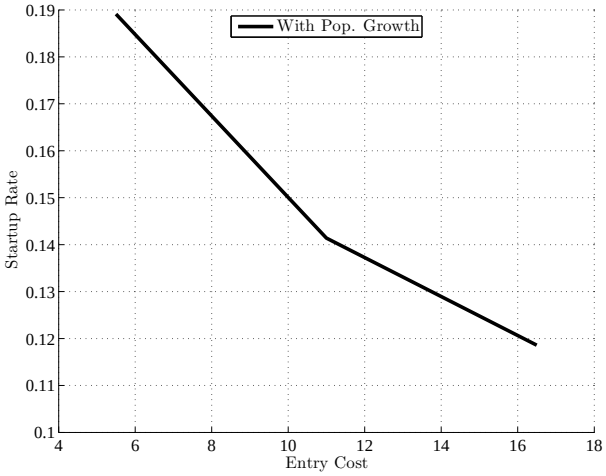
Unlike their response to changes in firm resource costs, equilibrium firm behavior is invariant to changes in the growth rate of the population. This result emerges because of the absence of changes in real wages. Due to free entry, changes in labor supply have no effect on real wages. In the absence of changes in prices, the behavior of incumbents is completely unaffected. The only margin that adjust in response to a labor supply shock is new entry. In particular, a decline in the growth rate of labor supply decreases firm entry.

4.3 The role of labor supply in accounting for the decline in the startup rate

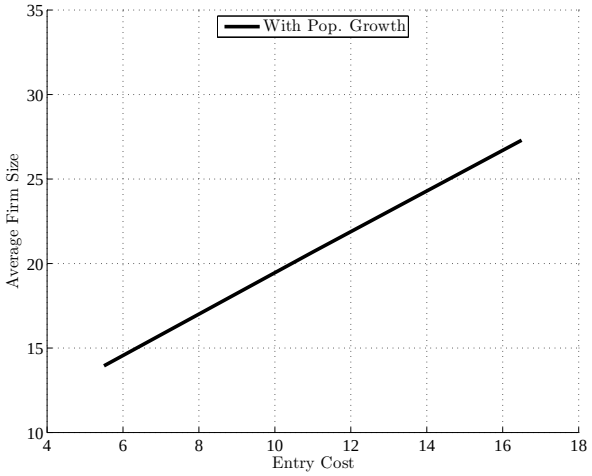
The cross-state empirical analysis and comparative statistics exercises both identified labor supply shifts as a promising source of variation to account for the decline in the startup rate. This subsection presents preliminary findings from a quantitative exercise where we vary the growth rate the labor force following its evolution from 1980 to 2012. Our simulations show that the model captures almost all the decline in the entry rate over timewith only this source of variation. However, it has the counterfactual implication of a drastic increase in the average firm size which suggests that other factors—which might have played offsetting roles on the firm startup rate—might have also played a role in shaping firm dynamics. The goal of our project is to uncover these additional forces and analyze their effects on various features of U.S. firm demographics.



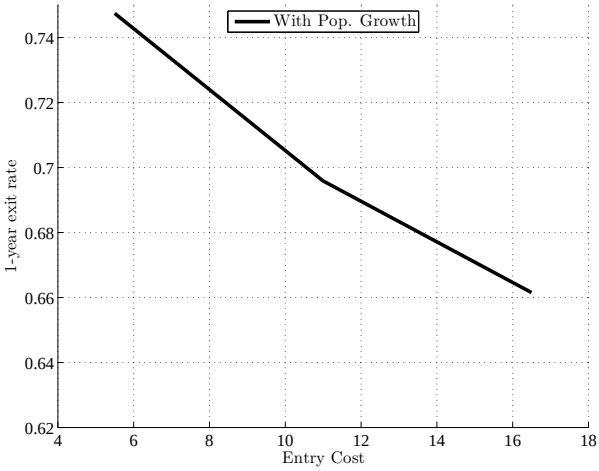
(a) Employment



(b) Startup rate

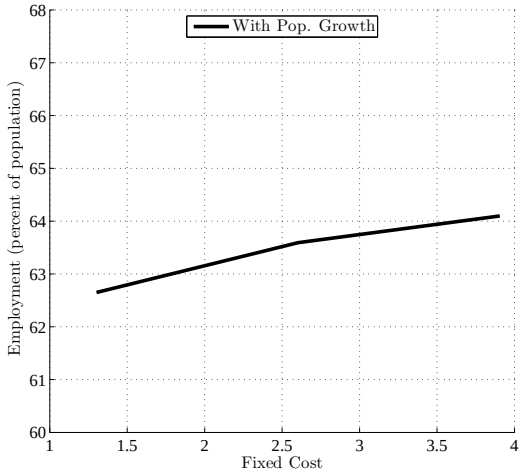


(c) Average firm size

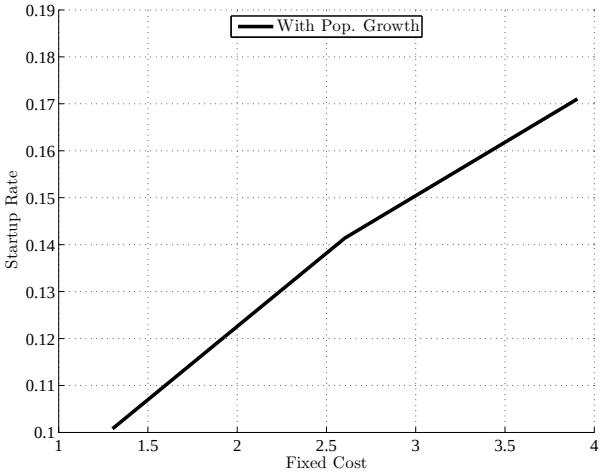


(d) Conditional exit rate

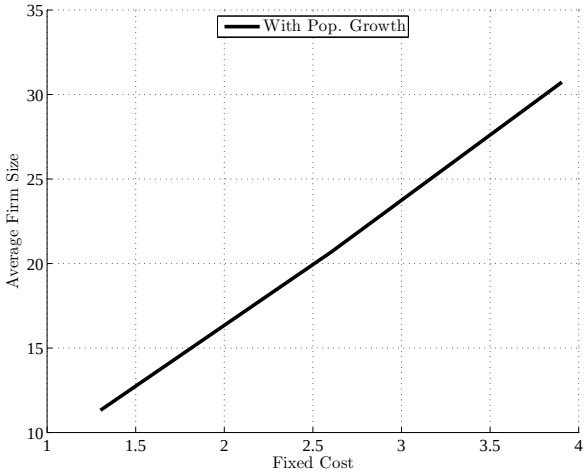
Figure 3: Changes in entry costs c_e



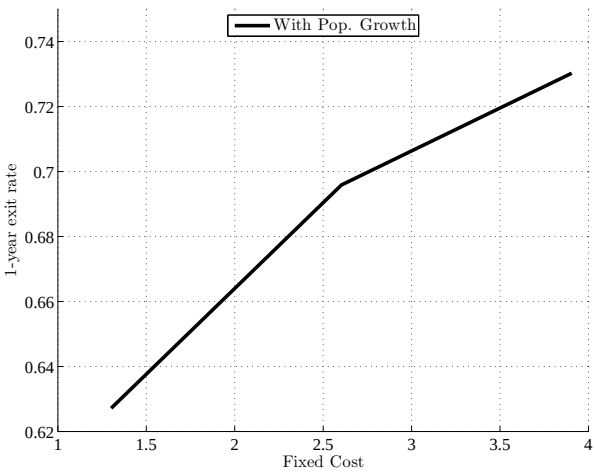
(a) Employment



(b) Startup rate

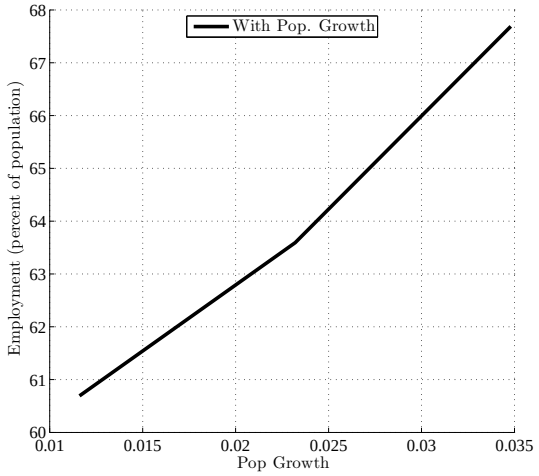


(c) Average firm size

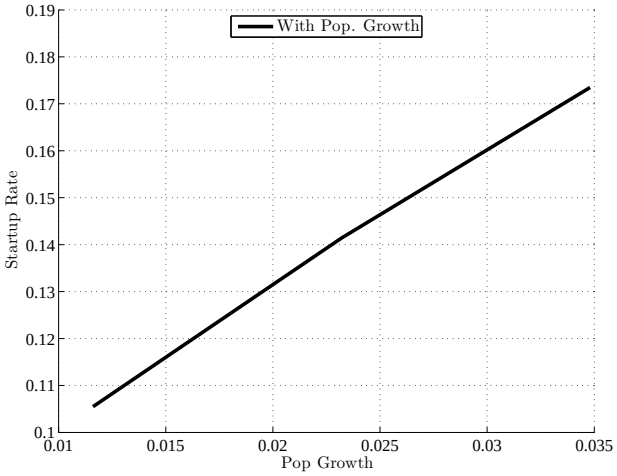


(d) Conditional exit rate

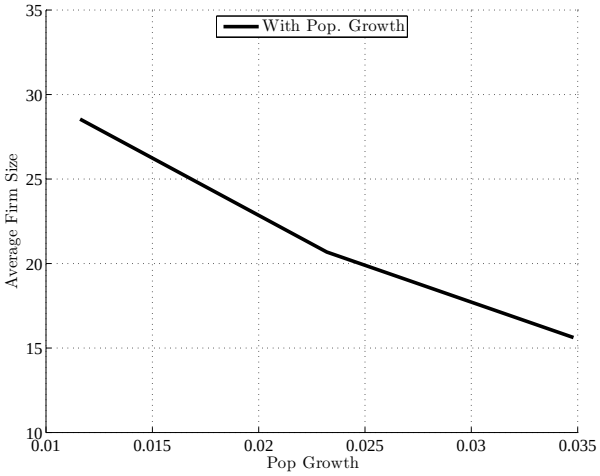
Figure 4: Changes in fixed costs c_f



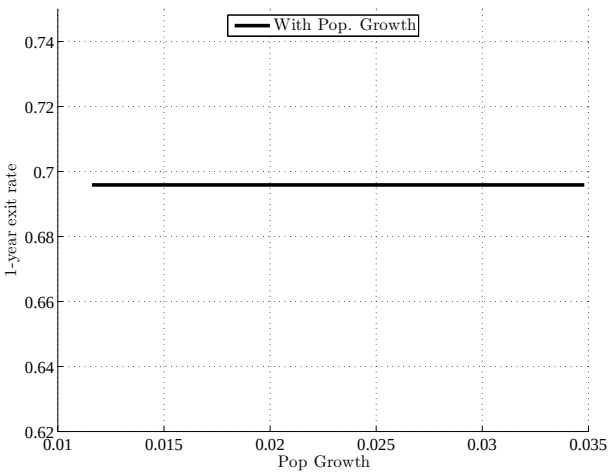
(a) Employment



(b) Startup rate

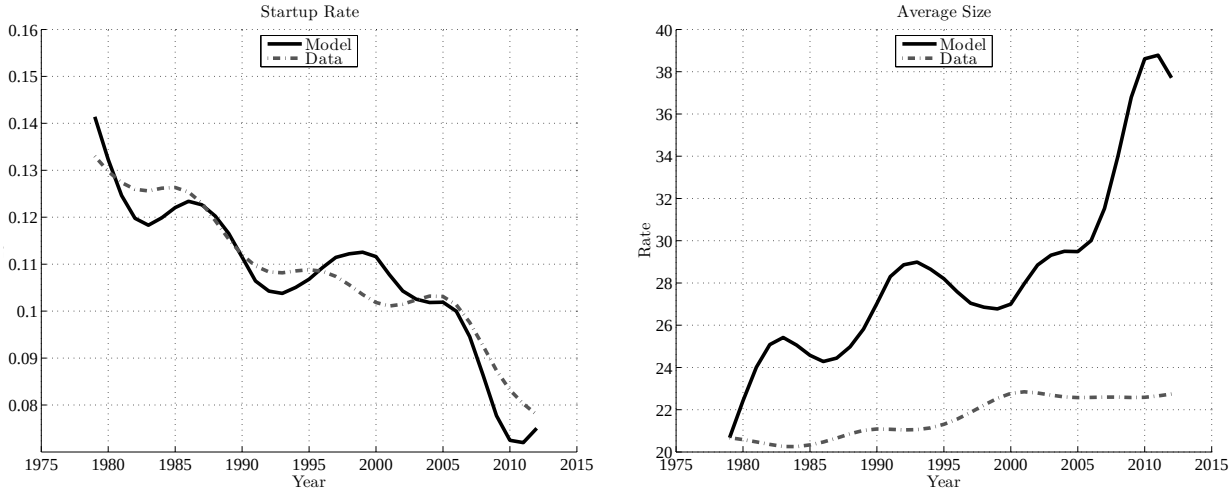


(c) Average firm size



(d) Conditional exit rate

Figure 5: Changes in population growth η



(a) Startup rate

(b) Average firm size

Figure 6: Predicted effects of actual change in population growth rate 1980 to 2012

References

Aaronson, D., and J. Davis (2012): "Explaining the decline in the U.S. labor force participation rate," *Chicago Fed Letter*, 296.

Davis, S. J., and J. Haltiwanger (2014): "Labor Market Fluidity and Economic Performance," Discussion paper, National Bureau of Economic Research.

Decker, R., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2014a): "The Role of Entrepreneurship in US Job Creation and Economic Dynamism," *The Journal of Economic Perspectives*, 28(3), 3–24.

——— (2014b): "The Secular Decline in Business Dynamism in the U.S.," mimeo.

Gourio, F., T. Messer, and M. Siemer (2014): "A Missing Generation of Firms? Aggregate Effects of the Decline in New Business Formation," mimeo.

Hathaway, I., and R. E. Litan (2014): "Declining Business Dynamism in the United States: A Look at States and Metros," mimeo, Brookings Institution.

Hopenhayn, H., and R. Rogerson (1993): "Job turnover and policy evaluation: A general equilibrium analysis," *Journal of Political Economy*, pp. 915–938.

Hopenhayn, H. A. (1992): "Entry, exit, and firm dynamics in long run equilibrium," *Econometrica: Journal of the Econometric Society*, pp. 1127–1150.

Karahan, F., and S. Rhee (2014): "Population Aging, Migration Spillovers and the Decline in Interstate Migration," *FRB of New York Staff Report*.

Pugsley, B. W., and A. Şahin (2014): "Grown-up business cycles," *FRB of New York Staff Report*, (707).

Reedy, E., and R. J. Strom (2012): "Starting Smaller; Staying Smaller: America's Slow Leak in Job Creation," in *Small Businesses in the Aftermath of the Crisis*, pp. 71–85. Springer.

Shimer, R. (2001): "The Impact of Young Workers on the Aggregate Labor Market," *Quarterly Journal of Economics*, Vol. 116, No: 3 (Aug., 2001), 969–1007.