Approximating time varying structural models with time invariant structures

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Abstract

The paper studies what parameter variations does to the decision rules of a DSGE model and to structural inference. We provide diagnostics to detect parameter variations and to ascertain whether variations are exogenous or endogenous. Identification and estimation distortions when a constant parameter model is incorrectly assumed are examined; likelihood-based and VAR-based estimates of the structural dynamics when the DGP features neglected parameter variations are compared. The features of time variations in the financial frictions of a Gertler and Karadi (2010) model are studied.

Key words: Structural model, time varying coefficients, endogenous variations, misspecification.

JEL Classification: C10, E27, E32.

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1 Introduction

In macroeconomics, it is standard to study models that are structural in the sense of Hurwicz (1962); that is, model where the parameters characterizing the preference and the constraints of the agents and the technologies to produce goods and services are invariant to changes in the parameters describing the policy actions by the government. Such a requirement is fundamental to distinguish structural models from reduced form ones, and it is of fundamental importance to conduct correctly designed policy counterfactuals in dynamic stochastic general equilibrium (DSGE) models.

Recent work by Dueker et al. (2007), Fernandez Villaverde and Rubio Ramirez (2007), Canova (2009), Rios Rull and Santaeularia Llopis (2010), Liu et al. (2011), Galvao, et al. (2014), Vavra (2014), Meier and Sprengler (forthcoming) among others, has convincingly indicated that the parameters of DSGE are not time invariant, and in many cases, smoothly evolve over time. The evidence these papers provide agrees with Stock and Watson' (1996) conclusion that reduced economic relationships show small but persistent time variations.

The presence of time variations in DSGE parameters can not be taken as direct evidence that these models are not structural. For example, Cogley and Yagihashi (2010), and Chang et al. (2013) have shown that parameter variations may result from the misspecification of a model with time invariant structure. On the other hand, parameter variations may be needed in certain small open economy models to insure the existence of a stationary equilibrium, see e.g. Schmitt Grohe and Uribe (2003).

The approach the DSGE literature has taken to model parameter variations follows the VAR literature, pioneered by Cogley and Sargent (2005) and Primiceri (2005), and makes parameters exogenously drifting over time as independent random walks, see e.g. Fernandez and Rubio (2007). Many economic questions, however, hint at the possibility that parameter variations may instead be endogenous. For example, is it reasonable to assume that the Federal Reserve reacts in the same way to inflation in an expansion or in a contraction? Davig and Leeper (2006) analyze a situation where the policy rule could be state dependent and describe how such a rule may impact on the dynamics induced by different structural shocks. Does the propagation of shocks depend on the state of private and government debt? Are the multipliers of fiscal expansions driven by the amount of inequality present in the economy, see e.g. Brinca et al. (2014)? Are household as risk averse when they are wealthy as when they are poor or as impatient in their consumption needs when the capital stock is high or low? Questions of this type are potentially numerous and the answer is crucial for policymakers' decisions. Clearly, counterfactual analyses and optimal monetary policy conclusions derived assuming time invariant parameters or an inappropriate forms of time variations may be incorrect; comparisons of the welfare costs of business cycles are likely to be biased; growth prescriptions may be invalid; and standard impulse responses,

historical and variance decompositions tools may provide a distorted picture of the dynamics of an economy in real time.

This paper has a number of goals. First, we want to characterize the decision rules of a DSGE when parameter variations are exogenous or endogenous, and in the latter case, when agents internalize or not the effects that their optimal decisions may have on parameter variations. By characterizing their structure and their differences, we hope to shed light on the consequences that alternative modelling assumptions may have for our understanding of how the economy works. Second, we wish to measure how distorted are standard statistics when a researcher erroneously assumes a time invariant structure but the data generating process (DGP) features time varying parameters and provide diagnostics to detect the misspecification driven by parameter variations. Third, we are interested in studying the consequences of using time invariant models when the parameters are time varying in terms of identification, estimation, inference, and policy analyses. Finally, we want to compare likelihood-based and SVAR-based estimates of the dynamic responses to structural shocks when time variations in the parameters are neglected. In particular, in the same vein as Canova and Paustian (2011), we wish to ask whether in the presence of misspecification driven by parameter variations, approaches that take a less structural approach are as good as likelihood based methods in describing the dynamics induced by structural shocks.

The existing literature is generally silent on the issues of interest in this paper. Kulish and Pagan (2014) study how to characterize the decision rules of a DSGE model when structural breaks which are partly predictable occur. Magnusson and Mavroedis (2014) and Huang (2014) examine how time variations in the certain parameters may affect the identification of other structural parameters, the asymptotic theory of maximum likelihood estimators, and standard break tests. Andreasan (2012) studies how time variations in the variance of the exogenous shocks affect risk premia in models approximated to the second and third order and Fernandez et al. (2013) investigate to what extent variations in volatility of shocks matter for real variables. Ireland (2007) assumes that trend inflation in a standard New Keynesian model is driven by structural shocks and estimates the model by likelihood techniques, while Ascari and Sbordone (2014) highlight that trend inflation may be a function of monetary policy decisions.

The next section characterizes the decision rules in a general DSGE setup where both exogenous and endogenous parameter variations are possible. We consider time variations in the parameters regulating preferences, technologies, and constraints and disregard variations in the auxiliary parameters regulating the persistence and the volatility of structural shocks and of the parameters. We consider both first order approximations and higher order solutions. We present a simple RBC example to illustrate our results and to provide some intuition for our modelling choices of time variations.

We show that if parameter variations are purely exogenous, the contemporaneous impact and the dynamics induced by structural shocks are the same as in a model

with no parameter variations. Thus, if one considers a time invariant version of the model and correctly identifies the structural disturbances, she would make no mistakes in characterizing the dynamics in response to these shocks. Clearly, variance decompositions exercises will be distorted, since some sources of variations (the disturbances to the parameters) will be omitted.

On the other hand, if parameter variations are endogenous, the instantaneous impact and the dynamics in responses to structural shocks may be different from the one of a constant coefficient model. The extent of the differences in the two specifications depends on two matrices. We show situations when these matrices are zero (making the dynamics of time varying and time invariant models identical) and situations when they are not. We show that the conclusions obtained when the model solution is approximated with a first order expansion around the steady state carry over when a second order expansion is constructed.

Section 3 provides diagnostics to detect the misspecification induced by employing a time invariant model when the data has been generated by a time varying coefficient model. We present statistics constructed using the optimality wedges of Chari et al. (2007) and the forecast errors and apply them to the RBC example previously considered. We also describe a marginal likelihood diagnostics which can help us to recognize whether the time variations detected with these statistics are of exogenous or endogenous nature.

Section 4 deals with parameter identification. We are interested in examining whether ignoring time variations in the parameters may have important repercussions for the way time invariant parameters can be identified from the likelihood of the model. Since the likelihood is constructed using the forecast errors of the model, which are generally misspecified when a time invariant model is used, one expects the shape of the likelihood function to be altered in a somewhat unpredictable way. Once again, in the context of the RBC example, we show that the likelihood function can be strongly altered when a time invariant model is considered, but its curvature does not acquire pathological characteristics when time variations in the parameters are neglected. Overall, our conclusions broadly agree with Huang (2014) in the sense that weakly identified parameters do not become better identified when time variations are present.

Section 5 considers the structural estimation of a model with time invariant parameter when the data is generated by models with time varying coefficients. The forecast errors used to construct the likelihood generally differ from the structural shocks because the dynamics assumed by the constant coefficient model are incorrect and because shocks misaggregation is present. Thus, one should expect important difference between the parameters of the DGP and the estimated ones. Indeed, we show that distortions in parameter estimates are present, are more likely to be significant when parameter variations are exogenous, and are generally produced because the parameters regulating the income and substitution effects are poorly estimated. As a consequence, estimated impulse responses differ from the true ones in terms of impact magnitude, shape and persistence, and the contribution

of structural shocks to the variability of the endogenous variables is generally biased, with technology shocks absorbing to a large extent the contribution of missing shocks.

Section 6 studies whether less structural time invariant VAR methods can help to recover the dynamics induced by structural shocks when time varying parameters are present. We endow the researcher with the population autoregressive matrices of the time invariant and of the time varying model and we ask whether it is possible to "invert" the information present in VAR errors using model based restrictions. When the model displays parameters with exogenous variations, SVAR methods are right on target, since VAR shocks contain the relevant information about the structural shocks. The picture still very good when there are endogenous time variation: most of the qualitative features (impact effect, shape and persistence) of the structural dynamics are well captured by the SVAR methods. The good performance of SVAR methods is due to two features of the DGP: the fact that the dynamics in response to shocks are not very different when the model has fixed or time varying parameters; and the fact that shocks to the parameters, which are disregarded when a constant coefficient model is used, have low persistence.

Section 6 estimates a version of the Gertler and Karadi model of unconventional monetary policy, applies the diagnostics we propose to detect time variations in the parameters regulating the financial frictions and estimates a version of the baseline model where certain parameters are allowed to vary over time (TO BE DONE). Section 7 concludes.

2 The setup

The equilibrium conditions of a dynamic stochastic general equilibrium (DSGE) model can be represented as:

$$E_t \left[f(X_{t+1}, X_t, X_{t-1}, Z_{t+1}, Z_t, \Theta_{1t+1}, \Theta_{1t}) \right] = 0 \tag{1}$$

where X_t is an $n_x \times 1$ vector of endogenous variables, Z_t is an $n_z \times 1$ vector of strictly exogenous variables, Θ_{1t} is an $n_{\theta 1} \times 1$ vector of possibly time varying structural parameters and f is a continuous function, differentiable up to order q, mapping onto a $\mathbb{R}^{n_x}$ space.

The law of motion of the strictly exogenous processes Z_{t+1} is given by

$$Z_{t+1} = \Psi(Z_t, \sigma \Sigma_\epsilon \epsilon^z_{t+1}) \tag{2}$$

where Ψ is a continuous function, differentiable up to order q, mapping onto a $\mathbb{R}^{n_z}$ space; ϵ^z_{t+1} is a $n_e \times 1$ vector of i.i.d. structural disturbances with mean zero and identity covariance matrix, $n_z \geq n_e$; $\sigma \geq 0$ is an auxiliary scalar, Σ_ϵ is a known $n_z \times n_e$ matrix.

Let $\Theta_t = [\Theta_{1t}, \Theta_{2t}]$, where Θ_{2t} is a $n_{\theta 1} n_{x1} \times 1$ vector of parameters, $n_x \geq n_{x1}$,appearing in the case agents internalize the effects that their decisions have on the parameters. The law of motion of the structural parameters Θ_t is given by

$$\Theta_{t+1} = \Phi(\Theta, X_t, U_{t+1}) \tag{3}$$

where Φ is a continuous function, differentiable up to order q, mapping onto the $\mathbb{R}^{n_\theta}$ space; U_t is a $n_u \times 1$ vector of exogenous disturbances, $n_\theta = n_{\theta 1}(1 + n_{x1}) \geq n_u$; Θ is a vector of constants.We assume that the law of motion of U_{t+1} is given by

$$U_{t+1} = \Omega(U_t, \sigma \Sigma_u \epsilon^u_{t+1}) \tag{4}$$

where Ω is continuous and differentiable up to order q,mapping onto the $\mathbb{R}^{n_u}$ space; ϵ^u_t is a $n_u \times 1$ vector of i.i.d. disturbances with mean zero and identity covariance matrix, uncorrelated with the ϵ^z_{t+1},and Σ_u is a known $n_u \times n_u$ matrix.

Three features of the setup need some discussion. First, the vector of structural disturbances ϵ^z_{t+1} may be smaller than the vector of exogenous variables Z_{t+1}, in which case there are common sources of variations in the exogenous variables Similarly, the dimension of ϵ^u_{t+1} may be smaller than the dimension of the structural parameters. Second, we allow for time variations in the parameters regulating preferences, technologies and constraint but we not consider variations in the auxiliary parameters regulating the law of motion of the Z_t and the U_t because we are not interested in stochastic volatility, GARCH or rare events phenomena (as in e.g. Andreasan, 2012), nor in time variations driven by evolving persistence of the exogenous processes. Third, (1) makes no distinction between states and controls. Thus, the solution we derive has to be thought of as final form (endogenous variables as a function of the exogenous variables and the parameters) rather than a state space form (control variables as a function of the states and of the parameters). As it will be clear in the next subsection, our representation implies that the first order approximate solution will have a standard VARMA format.

2.1 The first order approximate decision rules

We start by studying the implications of structural parameters variation for agents' decision rules when a first order approximate solution is considered. Taking a linear expansion of (1) around the steady states leads to

$$0 = E_t \left[F x_{t+1} + G x_t + H x_{t-1} + L z_{t+1} + M z_t + N \theta_{t+1} + O \theta_t \right] \tag{5}$$

where $F = \partial f / \partial X_{t+1}$, $G = \partial f / \partial X_t$, $H = \partial f / \partial X_{t-1}$, $L = \partial f / \partial Z_{t+1}$, $M = \partial f / \partial Z_t$, $N = \partial f / \partial \Theta_{t+1}$ $O = \partial f / \partial \Theta_t$ all evaluated at the steady states values of (X_t, Z_t, Θ_t) and lower case letters indicate deviations from the steady states. Let $\phi_u = \partial \Phi / \partial U_{t+1}$, $\phi_x = \partial \Phi / \partial X_t$, $\psi_z = \partial \Psi / \partial Z_t$, $\omega_u = \partial \Omega / \partial U_t$ and assume that all the eigenvalues of ψ_z and of ω_u are strictly less than one in absolute value.

Proposition 2.1. *Assume that equations (2)-(4) hold and that there exists a recursive equilibrium law of motion solving* (5)*. Then, the unique recursive law of motion is*

$$x_t = Px_{t-1} + Qz_t + Ru_t \tag{6}$$

where

- *P solves $FP^2 + (G + N\phi_x)P + (H + O\phi_x) = 0$.*
- *Given P, Q solves $VQ = -vec(L\psi_z + M)$ and $V = \psi_z' \otimes F + I_{n_z} \otimes (FP + G + N\phi_x)$ where vec denotes the columnwise vectorization.*
- *Given P, R solves $WR = -vec(N\phi_u\omega_u + O\phi_u)$ where $W = \omega_u' \otimes F + I_{n_\theta} \otimes (FP + G + N\phi_x)$*

Proof. The proof is straightforward. Substituting (6) into (5), we obtain

$$\begin{aligned} 0 =& [FP^2 + (G + N\phi_x)P + (H + O\phi_x)]x_{t-1} + [(FP + G + N\phi_x)Q + FQ\psi_z + L\psi_z + M]z_t \\ &+ [(FP + G + N\phi_x)R + FR\omega_u + N\phi_u\omega_u + O\phi_u]u_t \end{aligned}$$

Since the solution must hold for every realization of x_{t-1}, z_t, u_t, we need to equate their coefficient to zero and the result obtains. □

Corollary 2.2. *Assume that the condition stated in Proposition 2.1 hold and $\phi_x = 0$. Then, variations in the j-th structural parameter, θ^j, have no impact on the dynamics induced by structural shocks. Their instantaneous effect on the endogenous variables will be non-zero if and only if the j^{th} column of $N\phi_u\omega_u + O\phi_u$ is non zero.*

Corollary 2.3. *Assume that the conditions stated in Proposition 2.1 hold and $\phi_u = 0$. If the matrices $N\phi_x$ and $O\phi_x$ are zero, variations in the j-th structural parameter θ^j have no impact on the endogenous variables x_t.*

Corollaries 2.2 and 2.3 constitute the main results of this section. If parameter variations are purely exogenous, $\phi_x = 0$, and the dynamics induced by structural shocks are the same as in a constant coefficient model. The intuition for this result is simple: when $\phi_x = 0$,parameter variations play the role of additional strictly exogenous variables. As long as the shocks to Z_t and Θ_t are uncorrelated, the dynamics induced by the structural disturbances are not affected by the presence of time variations in the parameters. In other words, suppose an economy is perturbed by a technology shocks. Then, the dynamics induced by these shocks do not depend on whether the discount factor is constant or time varying, provided that its innovations unrelated to the innovations in the technological process.

This result implies that if one considers a time invariant version of the model and correctly identifies the structural disturbances ϵ_t^z, she would make no mistakes in characterizing the dynamics in response to these shocks. Clearly, variance decompositions exercises will be distorted, since certain sources of variations (the ϵ_t^u disturbances) will be omitted from the model. One interesting question is whether standard procedures

allow a researcher employing a time invariant setup to recover the correct ϵ_t^z from the data when the DGP is a model with time varying structural parameters. If this is not the case, it is also interesting to know which structural disturbance absorbs the effect of missing shocks.

On the other hand, if parameter variations are purely endogenous, $\phi_u = 0$, and the dynamics in response to structural shocks may be altered. In this case, a researcher employing a time invariant version of the model is likely to incorrectly characterize both the dynamics in response to structural shocks and the relative importance of different sources of disturbances for the variability of the endogenous variables. The extent of the distortion crucially depends on whether the matrices $N\phi_x$ and $O\phi_x$are identically equal to zero or not.

Exogenous variations in the structural parameters typically have an instantaneous effect on the endogenous variables of the model, i.e. R$\neq 0$, but purely endogenous parameter variations have zero instantaneous effect on x_t. Thus, to avoid identification problems in the latter case, it is preferable to have the relationship between parameters and the endogenous variables disturbed by shocks.

2.2 The second order approximate decision rules

One may be curious as to whether the conclusions we have reached are affected when higher order approximations are considered. We know from Schmitt-Grohe and Uribe (2004) that the first order terms in first and higher order approximations will be the same. To examine whether quadratic terms will be affected by the presence of time variations we need to simplify the notation. Let $W_t = [Z_t', U_t']'$, $Y_{t+1} = [X_{t+1}', X_t']'$, $\epsilon_{t+1} = [\epsilon_{t+1}^{z\prime}, \epsilon_{t+1}^{u\prime}]', \Sigma = diag[\Sigma_z, \Sigma_u]$ so that (1) is

$$0 = E_t[F(Y_t, W_t, \sigma\Sigma\epsilon_{t+1}, \Theta)] \tag{7}$$

once the solution $Y_{t+1} = h(Y_t, W_t, \sigma\Sigma\epsilon_{t+1})$ is taken into account. The second order approximation of (7) is

$$E_t[(F_y y_t + F_w w_t + F_\sigma \sigma) + 0.5(F_{yy}(y_t \otimes y_t) + F_{ww}(w_t \otimes w_t) + F_{\sigma\sigma}\sigma^2) \quad + \tag{8}$$

$$F_{yw}(y_t \otimes w_t) + F_{y\sigma} y_t \sigma + F_{w\sigma} w_t \sigma] \quad = \quad 0 \tag{9}$$

Note that the terms $F_\sigma\sigma$, $F_{y\sigma}y_t\sigma, F_{w\sigma}w_t\sigma$ are all zero, see Schmitt Grohe and Uribe (2004). The second order expansion of the decision rule is

$$\begin{aligned} y_{t+1} &= g_y y_t + g_w w_{t+1} + 0.5(g_{yy}(y_t \otimes y_t) + g_{ww}(w_{t+1} \otimes w_{t+1}) + g_{\sigma\sigma}\sigma^2) \\ &+ g_{yw}(y_t \otimes w_{t+1}) + g_{y\sigma} y_t \sigma + g_{w\sigma} w_{t+1}\sigma \end{aligned} \tag{10}$$

Proposition 2.4. *Consider the second order approximation of the optimality condition* (9)*. Time variations in the structural parameters affect g_{yy} and g_{yw} the if only if they affect g_y and g_w.*

Proof. The proof is strightforward. Matching coefficient and requiring, for example, $F_{yy} = 0$ implies that $g_{yy} = -(F_{y'})^{-1}J_1$ where

$$J_1 = 0.5F_{y'y'}(g_y \otimes g_y) + F_{y'y}g_y + F_{y'w'}g_y h_y + 0.5F_{yy} + F_{yw'}h_y + 0.5F_{w'w'}h_y h_y{}' \quad (11)$$

where primes indicate future values. Thus g_{yy} depends only on the first order term g_y, the matrix of derivatives of the optimality conditions with respect to the argument evaluated at the steady state ($F_{y'y'}, F_{w'w'}$, etc.) and h_y. Similarly $F_{ww} = 0$ implies that $g_{ww} = -(F_{y'})^{-1}J_2$ where

$$J_2 = 0.5F_{y'y'}(g_w \otimes g_w) + F_{y'w'}h_w g_w + F_{yw}g_w + 0.5F_{w'w'}h_w h_w{}' + F_{w'w}h_w + 0.5F_{ww} \quad (12)$$

which also depends on first order terms (g_w),the matrix of derivatives of the optimality conditions with respects to the argument evaluated at the steady state ($F_{y'y'}, F_{w'w'}$, etc.) and h_w.The proof for the other terms of the expression is analogous. □

The intuition for proposition 2.4 is simple: since second order terms are function of first order terms, of the parameters of the law of motion of the W_t, and of the gradient and the Hessian of the optimality conditions evaluated at the steady states, they do not feature independent variations. Thus, the second order dynamics will be affected by time variations only when the first order dynamics are affected.

2.3 Discussion

The results derived in this section requires time variations to be continuous. This is in line with the evidence produced by Stock and Watson (1996) and with the standard practice employed in SVAR. Note that our framework is flexible and can accommodate once-and-for-all breaks (at a known date) as long as transition between the two states is smooth. For example, a smooth threshold switching specification such as $\theta_{t+1} = (1-\rho)\theta + \rho\theta_t + exp(t-T_0)/(1+exp(t-T_0))$, $t = 1, \ldots, T_0-1, T_0, T_0+1, \ldots T$ is an acceptable form of exogenous time variations. Similarly, endogenous forms of time variations of the type $\theta_{t+1} = (1-\rho)\theta + \rho\theta_t + a\exp(-(K_t - K + U_{\theta,t+1}))/(b + exp(-(K_t - K + U_{\theta,t+1}))$ where a and b are vectors of parameters can also be used. What the framework does not allow is for Markov switching variations in the parameters which occur at unknown dates or for abrupt changes, such as those considered in Davig and Leeper (2006), since, in these cases, the smoothness conditions on the f function may not hold.

Kulish and Pagan (2014) have developed solution and estimation procedures for models with abrupt breaks and learning between the states. Their solution for the pre and post break period is a constant coefficient VAR, while for the learning period is a time varying coefficient VAR. Since (6) is a constant coefficient VAR with an extended set of shocks, a few words of comparison are needed. First, they are interested in characterizing the solution during the learning period, when the structure is unchanged but expectations move, while we are interested in the

solution when parameters are continuously varying. Second, their modelling of time variations is abrupt and the solution is designed to deal with that situation, while in our case variations are continuous. Third, in our setup expectations are varying with the variations of the structure, while Kulish and Pagan have expectations varying only in anticipation of a (foreseeable) break.

While we are able to establish what parameter variations do to the decision rules in the first and second order approximate solutions, the conclusions for higher order approximations will generally depend on the details of the model specification. One should expected third and higher order dynamics to differ in time varying coefficient and time invariant models, regardless of whether time variations are exogenous or endogenous, because third order terms depend on the variance of the shocks to the parameters and thus they will be different from those in constant coefficient models which do not feature these additional shocks. For the parameterization employed in the example considered in the next subsection, the dynamics in exogenously varying and fixed coefficient model differ only in the seventh digit, but in general one can conceive examples where second and third order approximations will not be the same.

One way of thinking about the differences between exogenous and endogenous parameter variations is that in the former each parameters evolve independently and covariations can be modelled by selecting the matrix Σ_u to be of reduced rank. With endogenous variations, there is an observable factor (the X's) which drives the common variations in the parameters. Thus, Σ_u will be diagonal and will feature reduced rank if for some parameter variations are purely endogenous.

2.4 An example

To convey some intuition into the mechanics of corollaries 2.2-2.3, we use a simple, closed economy, RBC model. The representative agent maximizes the discounted stream of future utilities given by

$$\max E_0 \sum_{t=1}^{\infty} \beta_t \left(\frac{C_t^{1-\eta}}{1-\eta} - A\frac{N_t^{1+\gamma}}{1+\gamma}\right) \tag{13}$$

subject to the sequence of constraints

$$\begin{aligned} Y_t(1-g_t) &= C_t + K_t - (1-\delta_t)K_{t-1} \\ Y_t &= \zeta_t K_{t-1}^{\alpha} N_t^{1-\alpha} \end{aligned}$$

where Y_t is output, C_t consumption, K_t the stock of capital and N_t is hours worked and $g_t = \frac{G_t}{Y_t}$ is the share of government expenditure in output. The system is perturbed by two exogenous structural disturbances: one to the technology Z_t and one to the government spending share, g_t, assumed to follow time invariant AR(1) processes

$$\ln \zeta_t = (1-\rho_\zeta)\ln \zeta + \rho_\zeta \ln \zeta_{t-1} + e_t^{\zeta} \tag{14}$$

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + e_t^{g} \tag{15}$$

where variables without time subscript denote steady state quantities. There are 12 parameters in the model: 6 structural ones (α is the capital share, η the risk aversion coefficient, γ the inverse of the Frisch elasticity of labor supply, A the constant in front of labor in utility, β_t the time discount factor and δ_t the depreciation rate), and 6 auxiliary ones (the steady state values of the government expenditure share and of TFP, (ζ, g), their autoregressive parameters, (ρ_ζ, ρ_g), and their standard deviations (σ_ζ, σ_g)). We assume that all parameters but β_t and δ_t are time invariant. Dueker et al. (2007), Liu et al (2011) and Meier and Sprenger (forthcoming) provide evidence that these two parameters are indeed varying. The first order approximation to their law of motion is described below.

The optimality conditions of the problem are:

$$\begin{aligned} AC_t^\eta N_t^\gamma &= (1-\alpha)(1-g_t)Y_t/N_t && (16)\\ \beta_t C_t^{-\eta} &= E_t \frac{\partial \beta_{t+1}}{\partial K_t} u(C_{t+1}, N_{t+1}) + E_t\left(\beta_{t+1} C_{t+1}^{-\eta}\left(\frac{\alpha(1-g_{t+1})Y_{t+1}}{K_{t+1}} + (1-\delta_{t+1}) - \frac{\partial \delta_{t+1}}{\partial K_t} K_t\right)\right) && (17)\\ (1-g_t)Y_t &= C_t + K_t - (1-\delta_t)K_{t-1} && (18)\\ Y_t &= \zeta_t K_{t-1}^\alpha N_t^{1-\alpha} && (19) \end{aligned}$$

Time variations in β_t and δ_t affect optimal choices in two ways. First, there is a direct effect in the Euler equation and in the resource constraint when β_t and δ_t are time varying. Second, if agents take into account the fact that their decisions may affect parameter variations, there will be a second (endogenous) effect coming from variations in the derivatives of β_{t+1} and δ_{t+1} with respect to the endogenous states - see the Euler equation (17).

We specialize this general setup to consider various possibilities.

2.4.1 Model A: Constant coefficients.

As a benchmark, we let $\beta_t = \beta^t$ and $\delta_t = \delta$. The optimality conditions are

$$\begin{aligned} AC_t^\eta N_t^{\gamma+1} &= (1-\alpha)(1-g_t)Y_t\\ C_t^{-\eta} &= E_t \beta C_{t+1}^{-\eta}\left(\alpha(1-g_{t+1})Y_{t+1}/K_t + 1 - \delta\right)\\ (1-g_t)Y_t &= C_t + K_t - (1-\delta)K_{t-1}\\ Y_t &= \zeta_t K_{t-1}^\alpha N_t^{1-\alpha} \end{aligned}$$

In the steady state, it must be the case that:

$$\frac{K}{Y} = \frac{\alpha(1-g)}{\delta - 1 + 1/\beta};\ \frac{C}{Y} = 1 - \delta\frac{K}{Y} - \frac{g}{Y};\ \frac{N}{Y} = \zeta^{\frac{1}{1-\alpha}}\left(\frac{K}{Y}\right)^{\frac{\alpha}{\alpha-1}};\ Y = \left[\frac{A}{(1-\alpha)(1-g)}\left(\frac{C}{Y}\right)^\eta\left(\frac{N}{Y}\right)^{1+\gamma}\right]^{-\frac{1}{\eta+\gamma}}. \quad (20)$$

2.4.2 Model B: Exogenous parameter variations

Here we set $d_t = \beta_{t+1}/\beta_t$ and let $\Theta_{t+1} - \Theta \equiv (d_{t+1}(1-\rho_\beta)\beta, \delta_{t+1} - (1-\rho_\delta)\delta)' = U_{t+1}$. We postulate

$$u_{d,t+1} = \rho_d u_{d,t} + e_{d,t+1} \quad (21)$$

$$u_{\delta,t+1} = \rho_\delta u_{\delta,t} + e_{\delta,t+1} \quad (22)$$

Since Θ_{t+1} is independent of the capital stock, $\partial\beta_{t+1}/\partial K_t = \partial\delta_{t+1}/\partial K_t = 0$ and the f function becomes

$$E_t\left[f(X_{t+1}, X_t, X_{t-1}, Z_{t+1}, Z_t, \Theta_{t+1}, \Theta_t)\right] = E_t\begin{pmatrix} AC_t^\eta N_t^{\gamma+1} - (1-\alpha)(1-g_t)Y_t \\ 1 - d_t C_{t+1}^{-\eta}/C_t^{-\eta}\left(\alpha(1-g_{t+1})Y_{t+1}/K_t + 1 - \delta_{t+1}\right) \\ (1-g_t)Y_t - C_t - K_t + (1-\delta_t)K_{t-1} \\ Y_t - \zeta_t K_{t-1}^\alpha N_t^{1-\alpha} \end{pmatrix} = 0 \quad (23)$$

where $X_t = (K_t, Y_t, C_t, N_t)'$, $Z_t = (\zeta_t, g_t)'$ and $\Theta_t = \Theta_{1t}$.

It is easy to verify that the steady state for $(\frac{K}{Y}, \frac{C}{Y}, \frac{N}{Y}, Y)$ coincide with those of the constant coefficient model. In addition, since $\phi_x = 0$, time variations in (d_{t+1}, δ_{t+1}) leave the decision rule matrices P and Q as in model A. Thus, as far as impulse responses to structural shocks are concerned, models A and B are observationally equivalent.

To examine whether variations in Θ_t have an instantaneous impact on the endogenous variables, we need to check whether the columns of $N\phi_u\omega_u + O\phi_u$ are zero. The relevant matrices of partial derivative evaluated at the steady state are

$$N = \frac{\partial f}{\partial \Theta_{t+1}} = \begin{pmatrix} 0 & 0 \\ 0 & 1/\beta \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad O = \frac{\partial f}{\partial \Theta_t} = \begin{pmatrix} 0 & 0 \\ -1/\beta & 0 \\ 0 & -K \\ 0 & 0 \end{pmatrix}, \quad \phi_u = \frac{\partial \Phi}{\partial U_{t+1}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (24)$$

Thus:

$$N\phi_u\rho_u + O\phi_u = \begin{pmatrix} 0 & 0 \\ 0 & 1/\beta \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_\delta \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -1/\beta & 0 \\ 0 & -K \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -1/\beta & -\rho_\delta/\beta \\ 0 & -K \\ 0 & 0 \end{pmatrix}$$

Hence, time variations in (d_{t+1}, δ_{t+1}) will have an impact effect on X_t - at least the entry of $N\phi_u\omega_u + O\phi_u$ is non zero. Note that if d_t is a fast moving variable, the impact effect on X_t depends on the persistence of shocks to the growth rate of the discount factor. For example, if $\rho_d = 0$, shocks to the growth rate of the time discount factor have no effects on X_t. In this case if only the discount factor is time varying and variations in its growth rate are i.i.d., the decision rules of the models A and B will be identical.

2.4.3 Model C: State dependent parameter variations without internalization

Assume that the time variations in the growth rate of the discount factor and in the depreciation rate are driven by the aggregate capital stock. We specify

$$\Theta_{t+1} = [\Theta_u - (\Theta_u - \Theta_l)e^{-\phi_a(K_t - K + U_{\theta,t+1})}] + [\Theta_u - (\Theta_u - \Theta_l)e^{\phi_b(K_t - K + U_{\theta,t+1})}] \quad (25)$$

where $\phi_a, \phi_b, \Theta_u, \Theta_l$ are vectors of parameters and let $u_{\theta,t+1}$ be a zero mean, i.i.d. vector shocks. To insure that model C has the same steady state as model A, we let $\Theta_l = (\beta/2,\ \delta/2)$, so that

$$\begin{pmatrix} d_{t+1} \\ \delta_{t+1} \end{pmatrix} = \Phi(\Theta, K_t, U_{t+1}) = \begin{pmatrix} 2d_u - (d_u - \beta/2)[e^{-\phi_1(K_t - K + U_{\beta,t+1})} + e^{\phi_2(K_t - K + U_{\beta,t+1})}] \\ 2\delta_u - (\delta_u - \delta/2)[e^{-\phi_3(K_t - K + U_{\delta,t+1})} + e^{\phi_4(K_t - K + U_{\delta,t+1})}] \end{pmatrix} \quad (26)$$

This specification is quite flexible: depending on the choice of $\phi's$,we can accommodate linear or quadratic relationships, which are symmetric or asymmetric. We assume that agents treat the capital stock appearing in (26) as an aggregate variable. This assumption is similar to the 'small k -big k' situation encountered in standard rational expectations models or to the distinction between internal and external habit formation. Thus, agents' first order condition do not take into account the fact that their optimal capital choice of K_t changes d_t and δ_t and $\partial\beta_{t+1}/\partial K_t = \partial\delta_{t+1}/\partial K_t = 0$. Hence, the equilibrium conditions are then as in (23) Since the f function is the same as in model B, the matrices N and O are unchanged.

We examine first whether parameter variations affect the matrices P and Q regulating the dynamics induced by the structural shocks. We have,

$$N\phi_x = \begin{pmatrix} 0 & 0 \\ 0 & 1/\beta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (d_u - \beta/2)(\phi_1 - \phi_2) & 0 & 0 & 0 \\ (\delta_u - \delta/2)(\phi_3 - \phi_4) & 0 & 0 & 0 \end{pmatrix} \quad (27)$$

$$O\phi_x = \begin{pmatrix} 0 & 0 \\ -1/\beta & 0 \\ 0 & -K \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (d_u - \beta/2)(\phi_1 - \phi_2) & 0 & 0 & 0 \\ (\delta_u - \delta/2)(\phi_3 - \phi_4) & 0 & 0 & 0 \end{pmatrix} \quad (28)$$

Thus, unless $\phi_1 \neq \phi_2$ and/or $\phi_3 \neq \phi_4$, endogenous variations in d_t, δ_t leave the decision rule matrices P and Q unaffected, i.e. symmetries in the law of motion for time variations in d_t, δ_t are needed to produce structural dynamics which are different from those of a constant coefficient model.

To verify whether parameter variations have an impact effect on X_t, note that $\omega_u = 0_{2\times 2}$, $\rho_d = \rho_\delta = 0$ and that

$$\phi_u = \frac{\partial\Phi}{\partial U_{t+1}} \begin{pmatrix} (\beta_u - \beta/2)(\phi_1 - \phi_2) & 0 \\ 0 & (\delta_u - \delta/2)(\phi_3 - \phi_4) \end{pmatrix}, \quad (29)$$

$$\phi_x = \frac{\partial\Phi}{\partial X_t} \begin{pmatrix} (\beta_u - \beta/2)(\phi_1 - \phi_2) & 0 & 0 & 0 \\ (\delta_u - \delta/2)(\phi_3 - \phi_4) & 0 & 0 & 0 \end{pmatrix}. \quad (30)$$

Thus,

$$N\phi_u\omega_u + O\phi_u = \begin{pmatrix} 0 & 0 \\ 1/\beta(d_u - \beta/2)(-\phi_1 + \phi_2) & 0 \\ 0 & K(\delta_u - \delta/2)(-\phi_3 + \phi_4) \\ 0 & 0 \end{pmatrix} \quad (31)$$

Thus, as long as $\phi_1 \neq \phi_2$or $\phi_3 \neq \phi_4$, endogenous parameter variations have an instantaneous impact on X_t. Note that this is true regardless of whether shocks to the parameters are i.i.d. or not.

2.4.4 Model D: State dependent parameter variations and internalize internalization.

We still assume that time variations in the discount factor and in the depreciation rate are driven by the aggregate capital stock and by an exogenous shock, as in equation (25). Contrary to case C, however, we assume that agent internalize the effects that their capital decisions have on parameter variations. The relevant derivatives are

$$d'_{t+1} \equiv \partial d_{t+1}/\partial K_t = -(\beta_u - \beta/2)[-\phi_1 e^{-\phi_1(K_t - K + u_{\beta,t+1})} + \phi_2 e^{\phi_2(K_t - K + u_{\beta,t+1})}] \quad (32)$$

$$\delta'_{t+1} \equiv \partial \delta_{t+1}/\partial K_t = -(\delta_u - \delta/2)[-\phi_1 e^{-\phi_1(K_t - K + u_{\delta,t+1})} + \phi_2 e^{\phi_2(K_t - K + u_{\delta,t+1})}] \quad (33)$$

and the Euler equation becomes

$$C_t^{-\eta} = E_t d'_t \; u(C_{t+1}, N_{t+1}) + E_t \left(d_t \; C_{t+1}^{-\eta}(\alpha Y_{t+1}/K_{t+1} + 1 - \delta_{t+1} - \delta'_{t+1} K_t) \right) \quad (34)$$

In order for the steady states of model D to equal to those of model A, we restrict $\phi_1 = \phi_2 = \phi_1$, $\phi_3 = \phi_4 = \phi_3$. Note that in model D, the $f(.)$ function differs from the one obtained in model C. In particular, we have

$$0 = E_t \left[f(X_{t+1}, X_t, X_{t-1}, Z_{t+1}, Z_t, \Theta_{t+1}, \Theta_t) \right] = E_t \begin{pmatrix} AC_t^{\eta} N_t^{\gamma+1} - (1-\alpha)(1-g_t)Y_t \\ 1 - d'_t \; u(C_{t+1}, N_{t+1})/C_t^{-\eta} - d_t \; C_{t+1}^{-\eta}/C_t^{-\eta}(\alpha(1-g_{t+1})Y_{t+1}/K_{t+1} + 1 - \delta_{t+1} + \delta'_{t+1}K_t) \\ (1-g_t)Y_t - C_t - K_t + (1-\delta_t)K_{t-1} \\ Y_t - \zeta_t K_{t-1}^{\alpha} N_t^{1-\alpha} \end{pmatrix} \quad (35)$$

where as before $X_t = (K_t, Y_t, C_t, N_t)'$, $Z_t = (\zeta_t, g_t)'$ but now $\Theta_t = (d_t, \delta_t, d'_t, \delta'_t)'$ and its law of motion is

$$\begin{pmatrix} d_{t+1} \\ \delta_{t+1} \\ d'_{t+1} \\ \delta'_{t+1} \end{pmatrix} = \Phi(\Theta, k_t, u_{t+1}) = \begin{pmatrix} 2d_u - (d_u - \beta/2)[e^{-\phi_1(K_t - K + u_{\beta,t+1})} + e^{\phi_1(K_t - K + u_{\beta,t+1})}] \\ 2\delta_u - (\delta_u - \delta/2)[e^{-\phi_3(K_t - K + u_{\delta,t+1})} + e^{\phi_3(K_t - K + u_{\delta,t+1})}] \\ -(d_u - \beta/2)\phi[-e^{-\phi_1(K_t - K + u_{\beta,t+1})} + e^{\phi_1(K_t - K + u_{\beta,t+1})}] \\ -(\delta_u - \delta/2)\phi[-e^{-\phi_3(K_t - K + u_{\delta,t+1})} + e^{\phi_3(K_t - K + u_{\delta,t+1})}] \end{pmatrix} \quad (36)$$

The relevant matrices of derivatives evaluated at the steady states are $\omega_u = 0_{2\times 2}$

$$N = \frac{\partial f}{\partial \Theta_{t+1}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/\beta & -u(C,N)/C^{-\eta} & -\beta K \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$O = \frac{\partial f}{\partial \Theta_t} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1/\beta & 0 & 0 & 0 \\ 0 & -K & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\phi_x = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2(\beta_u - \beta/2)\phi_1^2 & 0 & 0 & 0 \\ -2(\delta_u - \delta/2)\phi_3^2 & 0 & 0 & 0 \end{pmatrix}$$

$$\phi_u = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -2(\beta_u - \beta/2)\phi_1^2 & 0 \\ 0 & -2(\delta_u - \delta/2)\phi_3^2 \end{pmatrix}$$

Clearly, $N\phi_x \neq 0$, $O\phi_x = 0$ and $N\phi_u\omega_u + O\phi_u = 0$. Thus, a shock to the law of motion of the parameters alters the dynamics of the endogenous variables (the matrix P and Q matrices are affected) but has no instantaneous effect on the x_t, i.e. $R = 0$.

In sum, to have time variations mattering for the dynamics to structural shocks we need endogenous parameter variations and either that the relationship between parameters variations and the states is asymmetric; or that agents internalize the consequences their decisions have on the law of motion of the parameters.

2.4.5 Impulse responses

Assume that in model A : $\alpha = 0.30$, $\beta = 0.99$, $\delta = 0.025$, $\gamma = 2$, $\eta = 2$, $A = 4.50$, ζ=1;$\rho_\zeta = 0.90$, $\sigma_\zeta = 0.00712$, $g = 0.18$, $\rho_g = 0.50$ and $\sigma_g = 0.01$. For the parameter specific to the other models we choose:

- Model B: $\rho_\beta = 0.985, \rho_\delta = 0.95$ and $\sigma_\beta = 0.002$ $\sigma_\delta = 0.07$.
- Model C : $\phi_{1\beta} = 0.01, \phi_{2\beta} = 0.03, \phi_{1\delta} = 0.2, \phi_{2\delta} = 0.1$, $\sigma_d = \sigma_\delta = 0.5$, $\beta_u = 0.999, \delta_u = 0.025$.
- Model D : $\phi_{1\beta} = 0.0001, \phi_{2\beta} = 0.016, \phi_{1\delta} = 0.2, \phi_{2\delta} = 0.1$, $\sigma_d = 0.0001$;$\sigma_\delta = 0.1$, $\beta_u = 0.999, \delta_u = 0.025$.

Figures 1 and 2 report the responses to the two structural shocks for each of the four model. In the first column are the responses to technology shocks; in the second responses to government expenditure shocks. Figure 1 has the responses obtained with a first order approximation; figure 2 responses obtained with a second order approximation.

There are a few facts worth commenting upon. First, as expected, models A and B have identical dynamics. Thus, if technology and government expenditure shocks could be correctly identified, one would be able recover the true dynamics to structural disturbances using a constant coefficient model even when the DGP displays exogenous parameter variations. Second, the responses of models C and D differ from those of a constant coefficient model. With both shocks, it is primarily the shape and the persistence of consumption and capital responses which is altered because income and substitution effects are not the same as in model A. For instance, in responses to technology shocks, agents work less, save less and consume more in models C and D than in the constant coefficients model, while in response to government expenditure shocks consumption fall more and capital fall less relative to the constant coefficients case.

Since the response of hours and output to government expenditure shocks are somewhat different from what the conventional wisdom indicates, a few words of explanations are needed. In a standard RBC in response to government expenditure shocks hours and output typically increase because of the wealth effect that the shocks generates. However, the shock here affects the share of government expenditure in GDP. Thus, the positive wealth effect on labor supply is absent because government expenditure will increase exactly in the same proportion as output thus disincentivating agents to try to increase private output.

Third, confirming the results of proposition 2.4, second order approximation will deliver responses that differ across models only if they different in a first order approximation. The magnitude of the responses and the direction of the differences in the two systems is similar, indicating that the magnitude of second order terms is small - nonlinearities in RBC models are known to become important only for extreme values of certain parameters - see e.g. Fernandez Villaverde and Rubio Ramirez (2004).

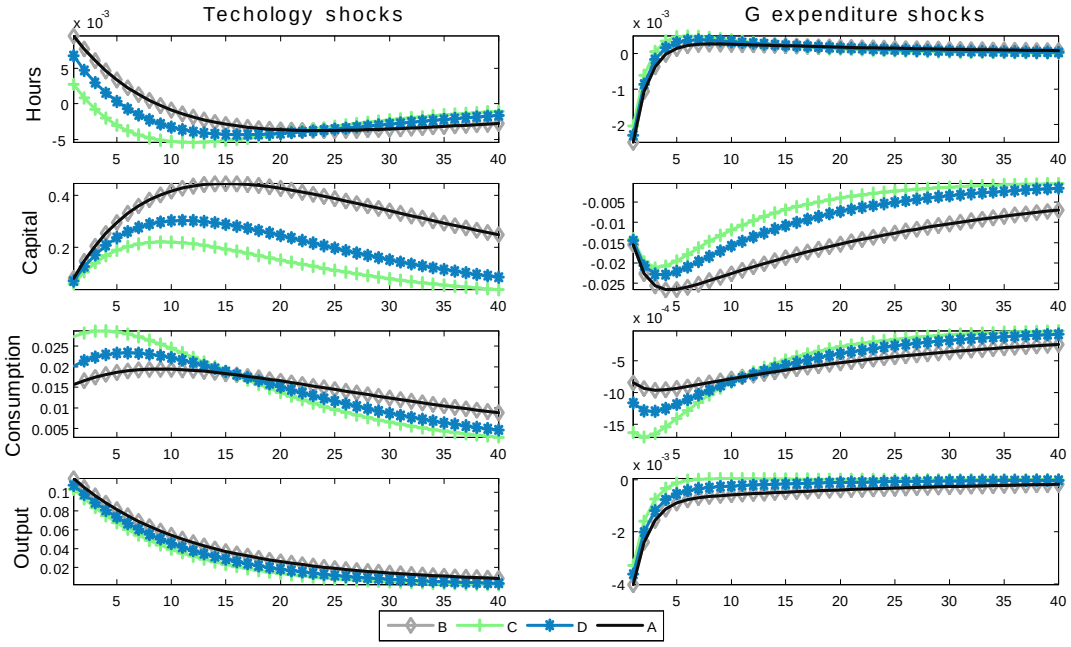


Figure 1: Impulse responses, first order approximation

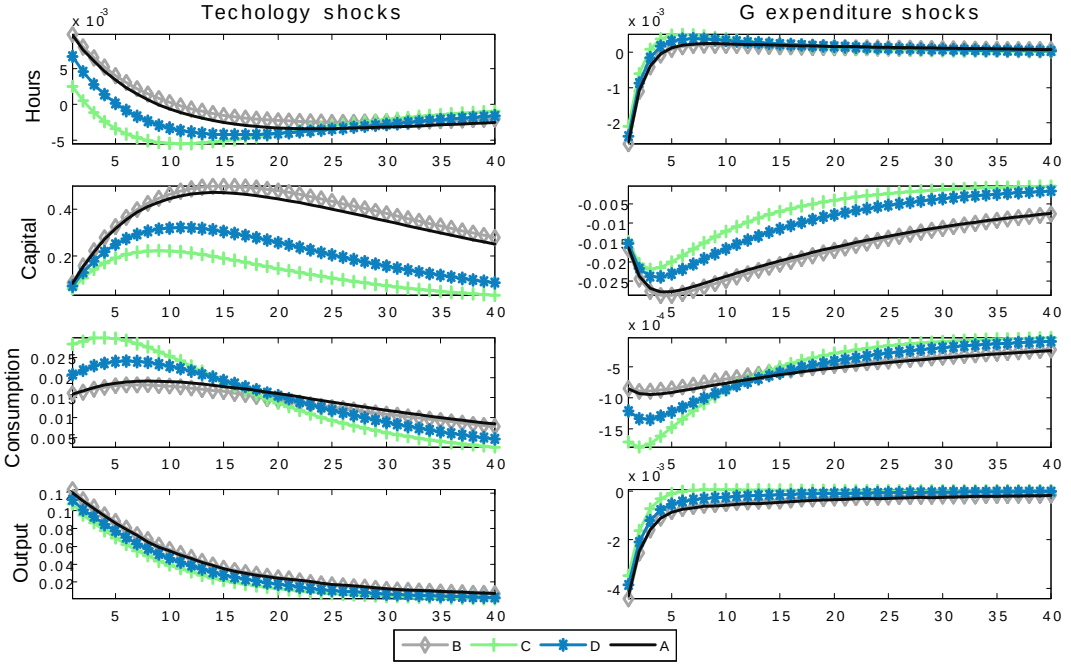


Figure 2: Impulse responses, second order approximation

3 Characterizing misspecification due to time variations

One way to characterize the misspecification produced by a constant coefficient model when the true DGP has time varying coefficients is to use "wedges" (see Chari et al., 2008). In our case, wedges are calculated using the optimality conditions of the model with constant coefficient and the available data for X_t. Thus, while $\mathrm{E}f(X_{t+1}, X_t, X_{t-1}, Z_{t+1}, Z_t, \Theta_{t+1}, \Theta_t) = 0$ when (Z_t, X_t) are generated by f, it will have expected value different from zero when this is not the case. Since $f(X_{t+1}, X_t, X_{t-1}, Z_{t+1}, Z_t, \Theta_{t+1}, \Theta_t)$ has a different format depending on the time varying specification one considers, wedges will be non-zero in general. In the RBC example previously considered, the components of f corresponding to the Euler equation and the resource constraint will differ in models with fixed and time varying coefficients. Furthermore, additional terms will appear in the Euler equation when agents internalize the effects their capital decisions have on the parameters.

Because of the relevance of the Euler equation in determining income and substitution effects and the dynamics in response to structural shocks, we focus attention on the Euler wedge. Clearly, any component of the optimality conditions can used for diagnostic purpose. The expression for the Euler wedge is

$$E_t w^j_{t+1} = E_t[\beta(\frac{c^j_{t+1}}{c^j_t})^{-\eta} r^j_{t+1}] \tag{37}$$

where $r^j_{t+1} = (\frac{\alpha y^j_{t+1}}{k^j_{t+1}} + 1 - \delta)$, j refers to data generated by model j. Let $PK^i_{t+1} = \beta(\frac{c^i_{t+1}}{c^i_t})^{-\eta}$ and $r^i_{t+1} = (\frac{\alpha y^i_{t+1}}{k^i_{t+1}} + 1 - \delta)$ be the pricing kernel and the real rate obtained when the DGP has constant coefficients. Equation (37) can be rewritten as

$$E_t w^j_{t+1} = E_t[PK^i_{t+1} r^i_{t+1}][(\frac{c^j_{t+1}}{c^j_t})(\frac{c^i_t}{c^i_{t+1}})]^{-\eta}[\frac{r^j_{t+1}}{r^i_{t+1}}] \tag{38}$$

The first term in (38) has expected value of 1; the other two terms are the wedges produced because consumption growth and the real rate may differ from those of the constant coefficient model. Because of these wedges, $E_t w^j_{t+1} \neq 1$, unless i=j.

In the RBC economy and using a long realization (T=1000) of (Z_t, Θ_t), we find that $E_t w^j_{t+1}$ is equal to 1.012 (DGP model B), to 1.008 (DGP model C) and to 1.011 (DGP model D). The time path of the Euler wedges and of their components are plotted in figure 3.

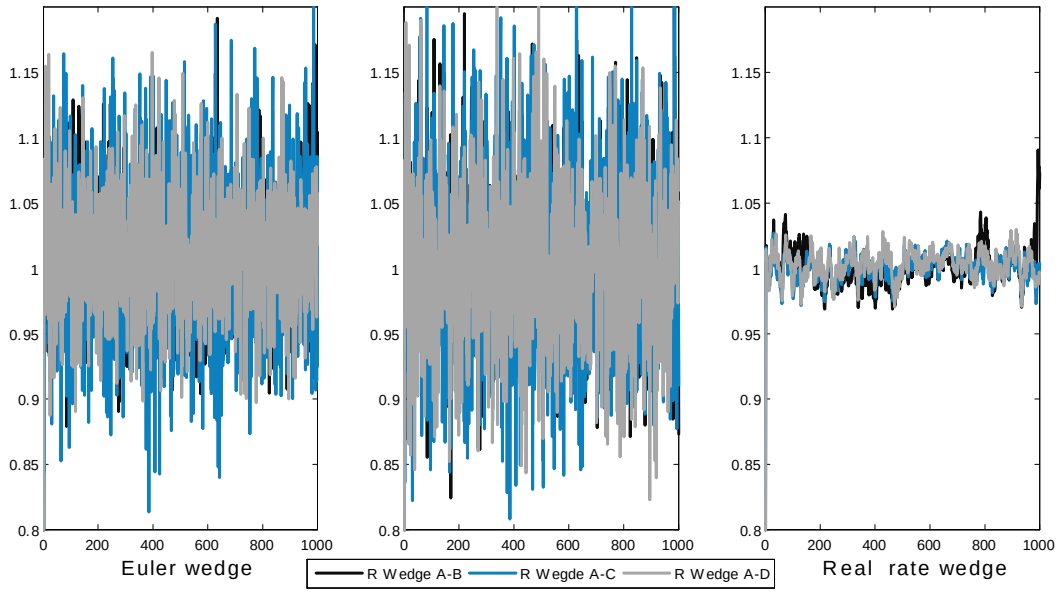


Figure 3: Simulated wedges

The means of the consumption growth wedge are 1.0015, 1.0021 and 1.0013. Since the volatilities are substantial the consumption growth profile generated by the constant coefficients and the time varying coefficients models are different. The means of the real rate wedge are 0.9995, 0.9998, 1.0001, respectively and the volatilities are small. Furthermore, a scatterplot of w_{t+1}^j against the consumption growth and the real rate wedges suggests that there is a tight positive relationship between w_{t+1}^j and the consumption growth wedge $[(\frac{c_{t+1}^j}{c_t^j})(\frac{c_t^A}{c_{t+1}^A})]^{-\eta}$ and a more blurred negative relationship between w_{t+1}^j and the real rate wedge $[\frac{r_{t+1}^j}{r_{t+1}^A}]$. Hence, neglecting covariance terms (which are small in this case), deviations of $E_t w_{t+1}^j$ from unity are mainly due to the fact that the consumption growth series produced by a constant coefficient model and by time varying coefficient models are different.

The above arguments suggest a simple way to assess the misspecification due to time variations. If the DGP is a model with constant coefficients, $E_t w_{t+1}^j - 1$ should be a martingale difference and thus unpredictable given the information available at time t. However, if the data has been generated by a model with time varying coefficients and a researcher employs a constant coefficient model in the analysis, $E_t w_{t+1}^j - 1$ will not be a martingale difference and variables contained in the information set may help to predict it. Thus, a regression of w_{t+1}^j on a constant and on the lags of the observable variables should have non-zero and significant coefficients.

Applying this reasoning to the data generated by the RBC model, we find that the coefficients on the first lag of the real rate are 1.06 (DGP model B) 1.81 (DGP model C) and -0.59 (DGP model D) all significant different from zero, while the

coefficients on the first lag of consumption growth are -0.0008 (DGP model B), 0.06 (DGP model C) and -0.05 (DGP model D) and the second is significantly different from zero.

Note that here we are jointly examining the hypothesis that the model is correctly specified under the null and that the coefficients are constant. To check what would happen to our diagnostic when the model is incorrectly specified but coefficients are constants, we simulate data from an RBC model with constant coefficients and one period time to build and consider our baseline model with no time to build and constant coefficients. We find that the coefficients on the first lag of the real rate and of consumption growth are -0.03 and 0.06 and both insignificantly different from zero.

An alternative way to characterize the misspecification generated by employing a constant coefficient specification when the DGP has time varying coefficients is via forecast errors. Recall that the linearized decision rule in a constant coefficient model is $x_t^i = P^i x_{t-1}^i + Q^i z_t$, while in a time varying coefficients model is $x_t^j = P^j x_{t-1}^j + Q^j z_t + R^j u_t$. Let v_t^j be the forecast error in predicting x_t^j using the decision rules of the constant coefficient model and the data from model j. The forecast error can be decomposed into

$$v_t^j = x_t^j - P^i x_{t-1}^j = Q^j z_t + R^j u_t + (P^j - P^i) x_{t-1}^j \tag{39}$$

Thus, the forecast errors are functions of the lags of the observables x_{t-1}^j. This is obvious when P^j is different from P^i. However, even if $P^j = P^i$, the forecast error could be function of the lags of the observables as long as u_t is serially correlated. Thus, a simple diagnostic for model misspecification involves regressing the forecast errors v_t^j on lagged values of the observables and checking the significance of the regression coefficients.

For the RBC model, and focusing for illustration on the forecast error produced in the hour equation, we find that lagged values of the endogenous variables are significant regardless of the process generating time variations (see table 3) and that an F-test strongly rejects the null hypothesis that all the coefficient are jointly zero.

DGP	n_{t-1}	k_{t-1}	y_{t-1}	c_{t-1}	Ftest, P-value
B	0.08	-0.40	0.05	0.51	0.00
	(0.004)	(0.006)	(0.007)	(0.002)	
C	0.08	-0.28	-0.15	0.43	0.00
	(0.002)	(0.007)	(0.003)	(0.29)	
D	0.27	0.09	0.33	-1.93	0.00
	(0.06)	(0.01)	(0.02)	(0.21)	

Table 1: Regression coefficients, hours equation; dependent variable forecast error using the decisions rules of model A.In parenthesis standard errors.

3.1 Detecting exogenous vs. endogenous parameter variations

If the wedge and the forecast error diagnostics indicate the presence of parameter variations, one may interested in knowing whether they are of exogenous or of endogenous type. One way to distinguish what type of time variations is present is to use the DGSE-VAR methodology of Del Negro and Schorfheide (2004). In a DSGE-VARs one uses the DSGE model as a prior for the VAR of the observable data and employs the marginal likelihood to measure the value of the information contained in the DSGE. Intuitively, a DSGE prior can be thought as a set additional observations added to the VAR model. If the additional observations come from the DGP generating the data, the quality of the estimates will be improved, standard errors will be reduced, and the marginal likelihood, which measures the fit of the specification, increased. On the other hand, if the additional observations comes from a DGP different from the one generating the data, noise will be added, and the precision of the estimates and the fit of the model reduced. Thus, for a given data set, a researcher comparing the marginal likelihood produced adding data generated by the exogenous and the endogenous specifications should be able to detect whether the available data is more likely to be generated by one of the two models.

Formally, let $L(\alpha|y)$ the likelihood of the VAR model for data y and let $g_j(\alpha|\gamma_j, M_j)$ be the prior induced by the DSGE model M_j using parameters γ_j on the VAR parameters α .The marginal likelihood is $h_j(y|\gamma_j, M_j) = \int L(\alpha|y)g_j(\alpha|\gamma_j, M_j)d\alpha$ which, for given y,is a function of M_j. Since $L(\alpha|y)$ is fixed, $h_j(y|\gamma_j, M_j)$ reflects the plausibility of $g_j(\alpha|\gamma_j, M_j)$ in the data. Thus, if g_1 and g_2 are two DSGE-based priors and $h_1(y|\gamma_1, M_1) > h_2(y|\gamma_2, M_2)$, there is better support for in the data for g_1.

Applying this technology to the RBC example, and using a sample size of $T = 150$, we report in table **??** the log marginal likelihood in the various cases when T_1 simulated data from the DSGE are added to the actual data. Both when T_1 is large relative to T, and when it is of the same magnitude, the marginal likelihood is able to clearly pick the correct DGP in all specifications. The differences within columns are quite large, even when $T_1 = T$, indicating that the diagnostic conclusively selects the model which has generated the available data.

	T_1=150			T_1=750		
DGP	Model B	Model C	Model D	Model B	Model C	Model D
Simulated from B	1586	-6709	-5108	9714	-3478	-12597
Simulated from C	1421	2005	-855	7480	4828	-409
Simulated from D	697	-2649	1864	6083	622	11397

Table 2: Log marginal likelihood obtained using data produced by the model on the first row and simulated data from the model in the first column.

4 Parameter identification

Since forecast errors are typically used to construct the likelihood function via the Kalman filter, one should expect the misspecification present in the forecast errors to spread to the likelihood function, making parameter estimate problematic. In this section we examine whether the time invariant parameters of a model can be identified from a potentially misspecified likelihood function. Canova and Sala (2009) have shown that standard DSGE models feature several population identification problems, intrinsic to these models and to the solution method employed. These problems typically show up because several structural parameters turn out to be weakly or partially identified and others close to be underidentified. The issue we are concerned with here is whether parameters which could be identified if the likelihood function would be correctly specified became weakly or underidentified because the likelihood function is constructed using the wrong forecast errors. In other words, we ask whether weak or underidentification of time invariant parameters may emerge as a by-product of the assuming that the model is structural when it is not. Magnusson and Mavroedis (2014) have shown that when GMM is used, time variations in certain parameters may help rather than hurt in the identification of fixed parameters. Huang (2014) qualifies the result by showing that time variations in weakly identified parameters have no effect on the asymptotic distribution of strongly identified parameters.

Figures 4 plots the likelihood function of the RBC model in the risk aversion coefficient γ and the share parameter η and figure 5 plots the likelihood function of the RBC model in the labor share α and the autoregressive parameter of the technology ρ_ζ when the forecast errors of the correct model (top row) and of the constant coefficient model (bottom row) are used to construct the likelihood function. The first column considers data generated by the model B, the second data generated by model C and the third data generated by model D.

While the curvature of the likelihood constructed using the correct model in not large, it is easy to verify that the maximum occurs at $\gamma = 2$, $\eta = 2, \alpha = 0.30$, $\rho_\zeta = 0.9$ for all three specifications displaying time varying discount factor and time varying depreciation rate. When the decision rules of the constant coefficient model are used to construct the likelihood function and the true DGP is model B, the distortions are present and the risk aversion coefficient γ become very weakly identified. When the true model features endogenous time variations, distortions are larger - likelihood function become convex ρ_ζ, γ and α become very weakly identified - and the maximum in the ρ_ζ is shifted away from the true value.

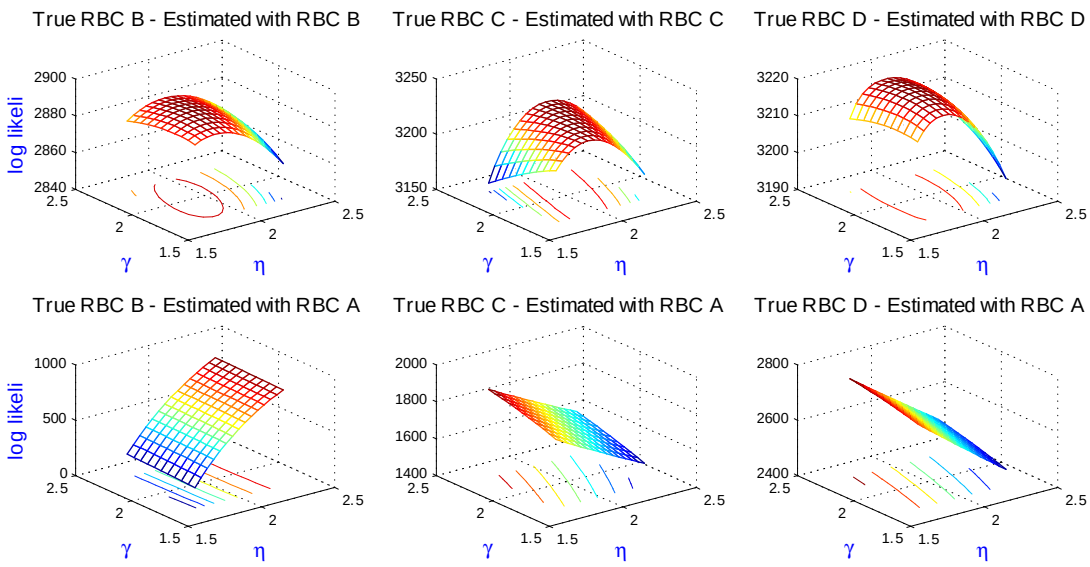


Figure 4: Likelihood surfaces

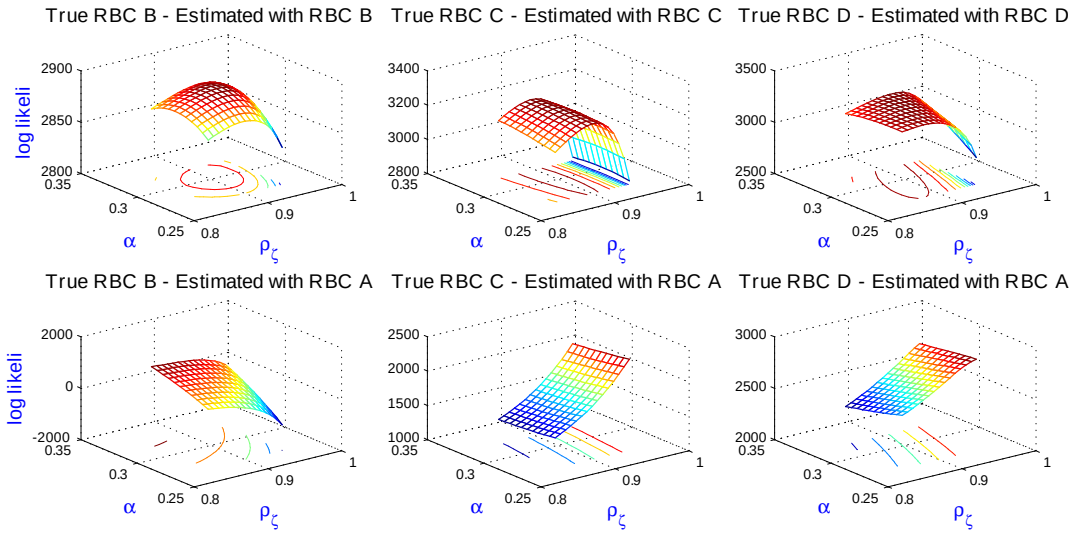


Figure 5: Likelihood surfaces

Hence, at least in the example we consider, weak identification problems may be the result of neglected time variations in certain parameters. Overall, the likelihood has more curvature when parameter variations are neglected, even though the increased curvature comes at the cost of centering the likelihood in the wrong place and introducing ridges in certain dimensions that may complicate inference about the parameters.

Koop, Pesaran, Smith diagnostic							
Parameter	T=150	T=300	T=500	T=750	T=1000	T=1500	T=2500
	DGP Model B, Estimated model A						
η	15.9	17.8	17.2	18.8	18.4	19.3	17.9
γ	28.5	45.7	108.4	81.4	93.6	104.2	90.17
ρ_z	1.8e+4	2.6e+4	4.2e+4	4.2e+4	4.5e+4	4.9e+4	4.37e+4
ρ_g	209.2	655.5	2741	2190	2860	3417	2802
δ	927.3	973.8	1.7e+4	1.7e+4	2.4e+4	2.3e+4	2.5e+4
α	140.2	156.2	264.2	215.5	239.1	252.1	229.3
A	28.42	30.67	7.99	10.99	9.15	7.83	9.83
	DGP Model C, Estimated model A						
η	822	1033	743	785	759	746	752
γ	2261	3147	2682	2809	2720	2579	2566
ρ_z	3073	2673	2952	2909	2799	2806	2877
ρ_g	1.74	2.23	2.44	2.96	3.17	2.82	2.90
δ	4.6e+5	4.4e+5	4.3e+5	4.0e+5	3.8e+5	4.4e+5	4.3e+5
α	1.8e+4	1.1e+4	1.4e+4	1.2e+4	1.1e+4	1.6e+4	1.5e+4
A	351	493	441	505	500	449	444
	DGP Model D, Estimated model A						
η	550	575	592	610	545	542	494
γ	3577	2442	2660	2870	2564	2711	2430
ρ_z	1613	1243	1120	1162	1068	1189	1074
ρ_g	1.22	1.28	1.44	1.53	1.60	1.62	1.67
δ	5.2e+5	6.7e+5	6.5e+5	6.0e+5	5.7e+5	5.8e+5	5.7e+5
α	1.1e+4	2.5e+4	2.4e+4	1.9e+4	2.1e+4	2.0e+4	2.1e+4
A	488	276	340	382	349	395	334

Table 3: Koop et al diagnostic. Different sample sizes

These observations are confirmed when one uses the Koop et al. (2013) statistic, which we report in table 4. Koop et al. show that asymptotically the precision matrix of the estimates grows at the rate T for identified parameters and at rate less than T for underdentified parameters. Thus, the precision of the estimates scaled by the sample size should converge to a constant for identified parameters and to zero for underidentified parameters. Furthermore, the magnitude of the constant can be used to measure the strength of the identification: a large value indicates a strongly identified parameter; a small value a weakly identified one.

The table shows that when the DGP is model B and a fixed coefficient model is estimated, all parameters are identified even though there is some weak identification issues for A and η. When the DGP are models C and D, all parameters but ρ_g seem identifiable even when the sample size is small. The curvature of the likelihood in the ρ_g dimension is small but the maximum can be easily found.

It is interesting to note, that when the DGP are models B and C, ρ_g is weakly identified when the correct likelihood is used. Thus, the presence of time variations in β_t and δ_t does not help in the identification of ρ_g, a result which is in line with the conclusions of Huang (2014), since both β and δ are weakly identified in the constant coefficient version of the model, see Canova and Sala, (2009).

5 Likelihood estimation with a misspecified model

To study the properties of likelihood based estimates of a misspecified fixed coefficient model we conduct a Monte Carlo exercise. We generate either 150 or 1000 data points from versions B, C, D of the RBC model previously considered, assuming that β and δ are time varying, estimate the structural parameters using the decision rules of the time invariant model A and repeat the exercise 150 times using different realizations of the shocks. We also estimate the structural parameters using the equations of the correct model (i.e. model B if the data has been generated by model B, and so on), to account for numerical difficulties one may encounter in estimating the model.

We consider two setups, one where time variations in the parameters are small (the variance of output explained by the shocks to the parameters is 2-5 percent, henceforth DGP1) and one where time variations in the parameters are substantial (the variance of output explained by the shocks to the parameters is about 20 percent, henceforth DGP2). Tables 4 and 5 report the parameters used to generate the data (column 1), the mean estimate (across replications) obtained with the correct model (column 2), the mean posterior estimate, the 5th and the 95 percentile of the distribution obtained with the time invariant model when T=150 (columns 3-5) and when T=1000 (columns 6-8). Table 4 has the results for DGP1 and table 2 for DGP2. Figures 6-7 present the distributions of estimates for the DGP cases: the vertical line represent the true parameter, solid black lines represent distributions obtained with the correct model; in solid blue lines are distributions obtained with the incorrect constant coefficient model when T=150, and in solid red lines the distributions obtained with the incorrect constant coefficient model and T=1000. When the model is correctly specified, the distribution of estimates should collapse around the true value. Thus, if the mean is away from the true parameter value and the spread of the distribution is large, one can conclude that likelihood based methods have difficulties in recovering the constant parameters of the data generating process. Figure 8-9 present the impulse responses for the two DGP: in the first three columns are the responses to technology shocks and in the last three the responses to government spending shocks. In each box we report the estimate obtained using the correct model and the mean value of the estimated parameters and the interval obtained with the incorrect constant coefficient model and the 16th and 84th percentiles of the distribution of estimated parameters. Finally, table 6 present the long run variance decomposition with the first DGP (first four columns) and the second DGP (last four columns) when T=150 and the mean estimate is used. In each block we report in the first two columns the variance

decomposition to technology and government spending shocks in the correct model; in the last two columns the variance decomposition to technology and government spending shocks when the incorrect time invariant model is used.

True Parameter	Correct Mean T=150	Time invariant Mean T=150	Time invariant 5th percentile T=150	Time invariant 95th percentile T=150	Time invariant Mean T=1000	Time invariant 5th percentile T=1000	Time invariant 95th percentile T=1000
DGP Model B							
$\eta = 2.0$	2.00	2.03	1.47	2.88	2.32	1.55	3.37
$\gamma = 2.0$	2.02	1.23	-0.14	2.07	0.96	-0.38	2.04
$\rho_z = 0.98$	0.97	0.99	0.97	1.00	0.99	0.96	1.00
$\rho_g = 0.5$	0.47	0.74	0.60	0.96	0.87	0.77	0.98
$\delta = 0.025$	0.03	0.01	0.01	0.02	0.01	0.01	0.05
$\alpha = 0.3$	0.30	0.19	0.11	0.28	0.23	0.15	0.40
$A = 4.5$	4.55	2.79	1.33	4.12	2.68	1.23	4.06
DGP Model C							
$\eta = 2.0$	2.00	2.42	1.63	3.85	2.85	1.73	6.14
$\gamma = 2.0$	2.00	0.64	-0.26	1.77	0.60	-0.50	1.79
$\rho_z = 0.98$	0.98	0.99	0.97	1.00	0.97	0.85	1.00
$\rho_g = 0.5$	0.48	0.43	-0.10	0.96	0.65	0.27	0.98
$\delta = 0.025$	0.03	0.01	0.01	0.02	0.02	0.01	0.09
$\alpha = 0.3$	0.30	0.22	0.13	0.34	0.29	0.18	0.47
$A = 4.5$	4.49	2.14	1.18	3.47	2.37	1.18	3.66
DGP Model D							
$\eta = 2.0$	2.00	2.58	1.69	3.34	2.40	1.74	3.26
$\gamma = 2.0$	2.01	0.29	-0.28	1.54	1.09	-0.30	1.99
$\rho_z = 0.97$	0.96	0.99	0.94	1.00	0.96	0.91	1.00
$\rho_g = 0.5$	0.48	0.51	-0.26	0.96	0.66	0.39	0.98
$\delta = 0.025$	0.02	0.01	0.01	0.03	0.01	0.01	0.02
$\alpha = 0.3$	0.30	0.22	0.14	0.35	0.22	0.15	0.30
$A = 4.5$	4.52	2.32	1.42	3.68	3.45	1.37	4.51

True Parameter	Correct Mean	Time invariant Mean	Time invariant 5th percentile	Time invariant 95th percentile	Time invariant Mean	Time invariant 5th percentile	Time invariant 95th percentile
	T=150	T=150			T=1000		
DGP Model B							
$\eta = 2.0$	2.00	2.29	1.53	3.87	2.45	1.61	3.09
$\gamma = 2.0$	2.01	1.11	-0.33	2.06	0.25	-0.27	1.95
$\rho_z = 0.9$	0.94	0.99	0.96	1.00	0.99	0.97	1.00
$\rho_g = 0.5$	0.47	0.76	0.62	0.96	0.91	0.79	0.98
$\delta = 0.025$	0.03	0.01	0.01	0.03	0.01	0.01	0.01
$\alpha = 0.3$	0.30	0.19	0.11	0.41	0.21	0.10	0.34
$A = 4.5$	4.53	2.73	1.33	4.14	1.80	1.14	4.16
DGP Model C							
$\eta = 2.0$	2.00	3.40	1.56	7.51	5.19	1.77	22.90
$\gamma = 2.0$	2.00	-0.08	-0.32	0.73	-0.19	-0.35	0.35
$\rho_z = 0.9$	0.88	0.99	0.93	1.00	0.99	0.90	1.00
$\rho_g = 0.5$	0.48	0.56	0.08	0.97	0.91	0.59	0.98
$\delta = 0.025$	0.02	0.02	0.01	0.07	0.02	0.01	0.07
$\alpha = 0.3$	0.30	0.26	0.15	0.34	0.26	0.19	0.35
$A = 4.5$	4.50	1.71	1.25	2.77	2.27	1.24	8.17
DGP Model D							
$\eta = 2.0$	2.00	3.05	1.68	4.59	2.40	1.98	4.81
$\gamma = 2.0$	2.00	-0.06	-0.28	0.54	1.63	-0.27	1.98
$\rho_z = 0.9$	0.88	0.98	0.90	1.00	0.92	0.91	1.00
$\rho_g = 0.5$	0.47	0.42	-0.46	0.96	0.50	0.32	0.97
$\delta = 0.025$	0.02	0.01	0.01	0.03	0.01	0.01	0.01
$\alpha = 0.3$	0.30	0.23	0.15	0.32	0.21	0.13	0.27
$A = 4.5$	4.49	1.91	1.45	3.57	4.10	1.65	4.51

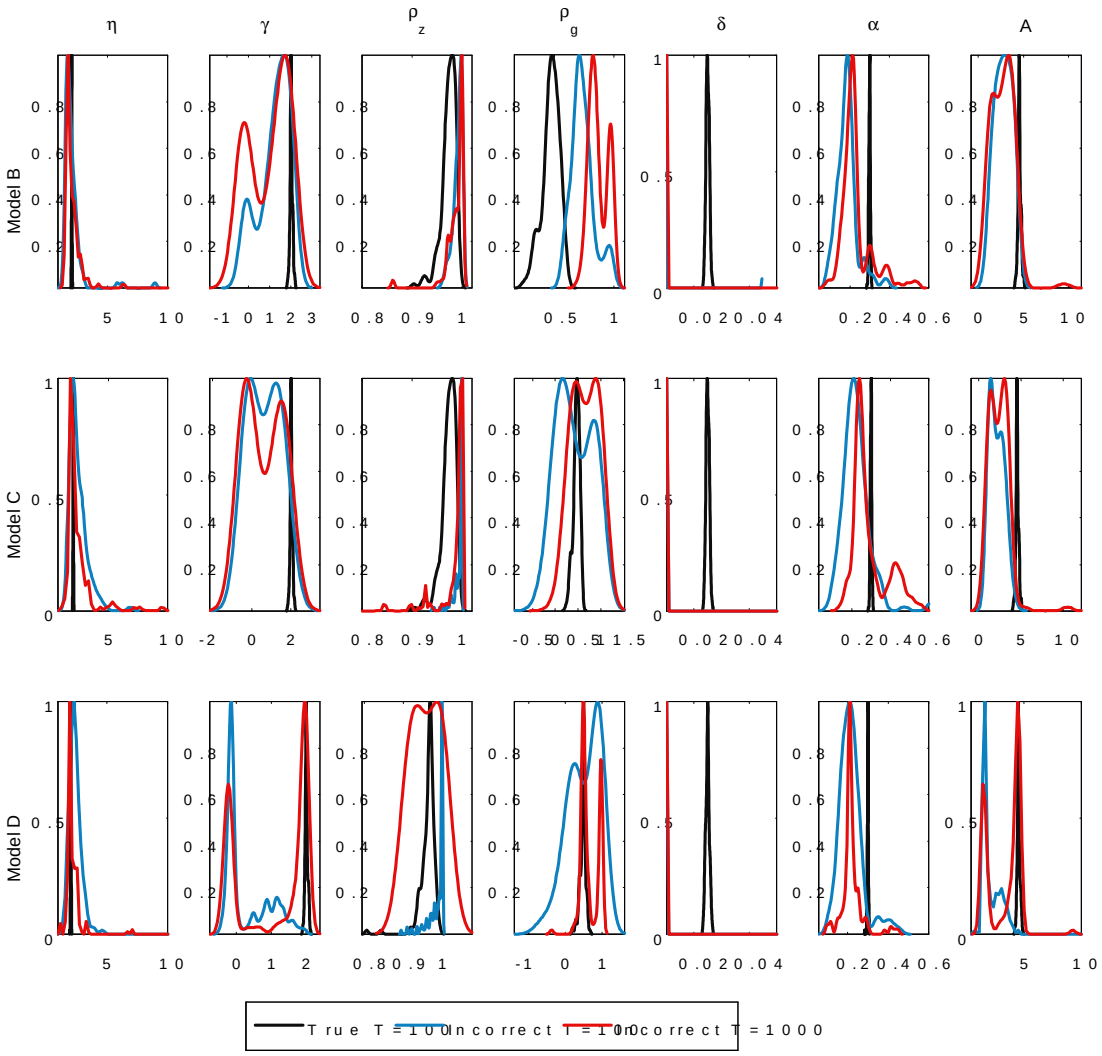


Figure 6:Density of estimates; DGP1 (time variations explain little of output variance).

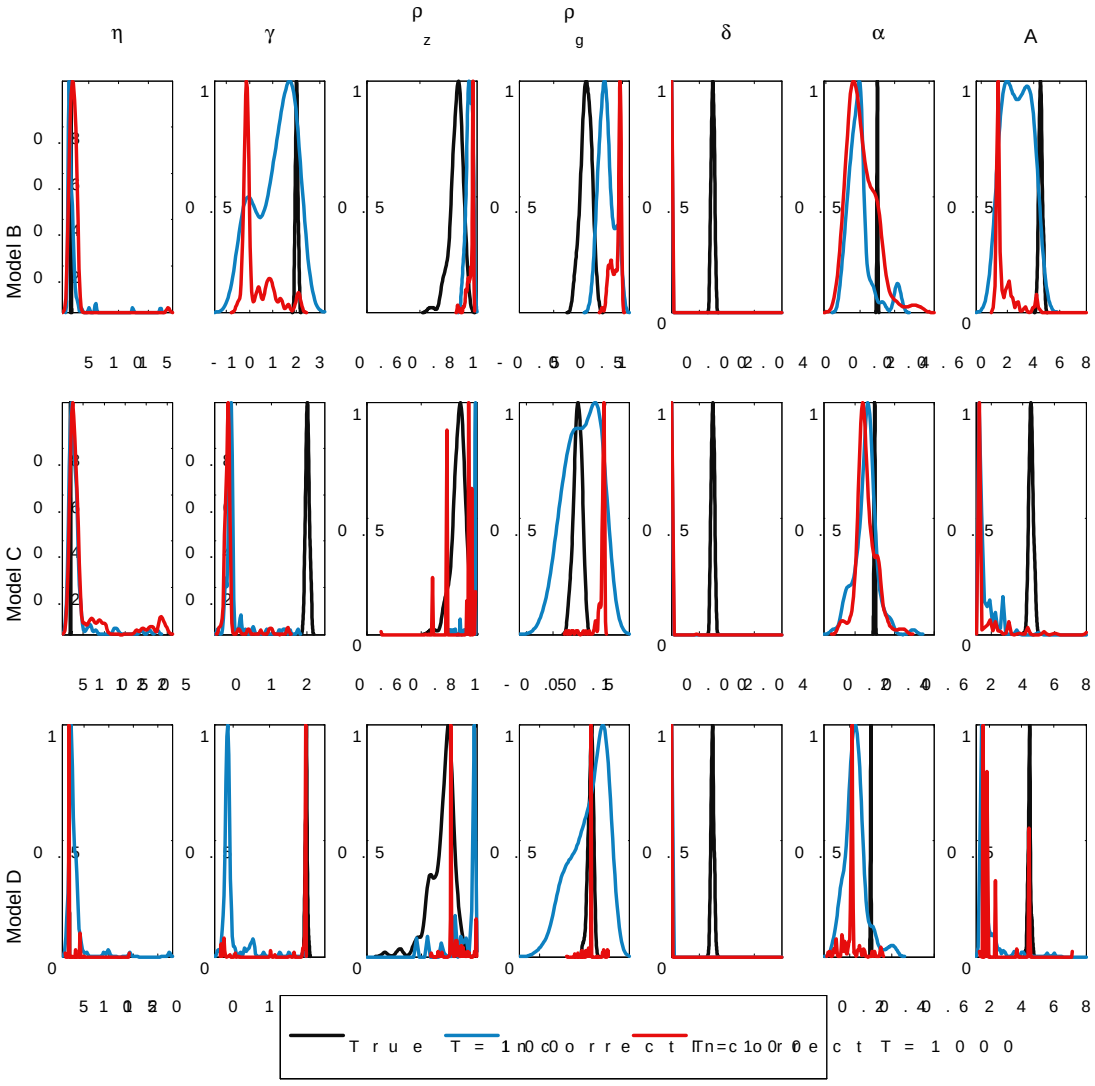


Figure 7:Density of estimates; DGP1 (time variations explain little of output variance).

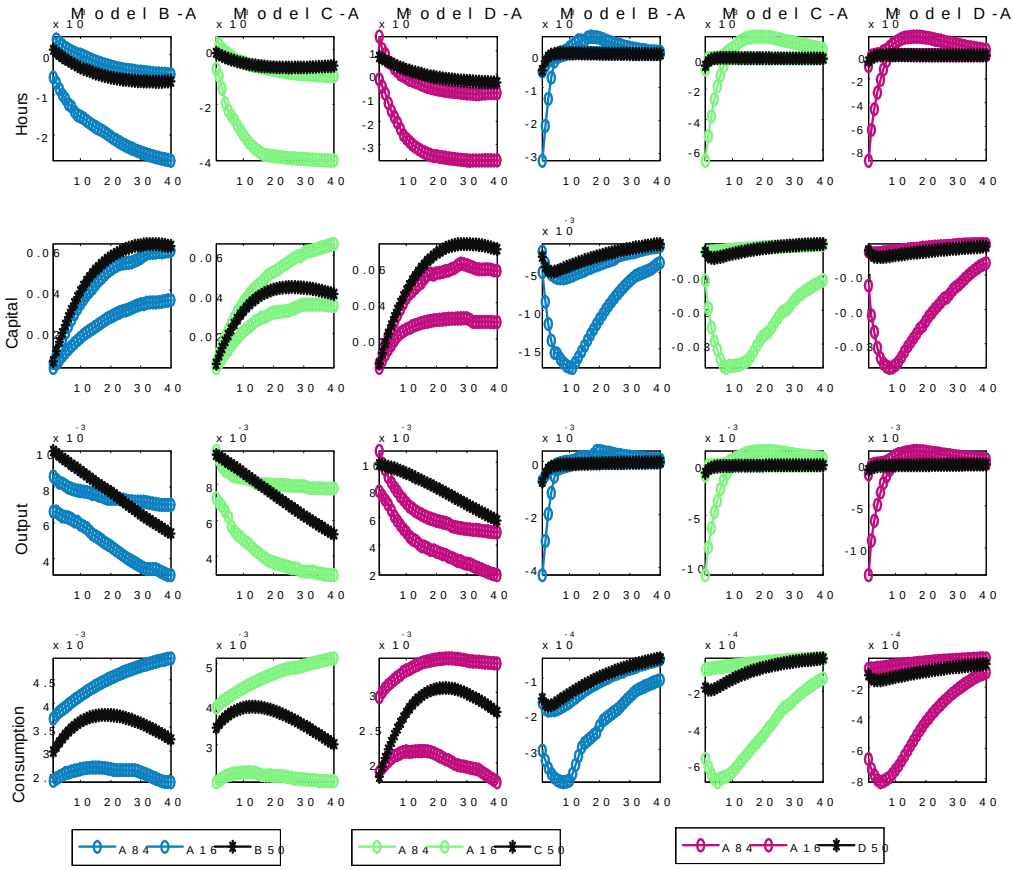


Figure 8:Impulse responses, DGP1

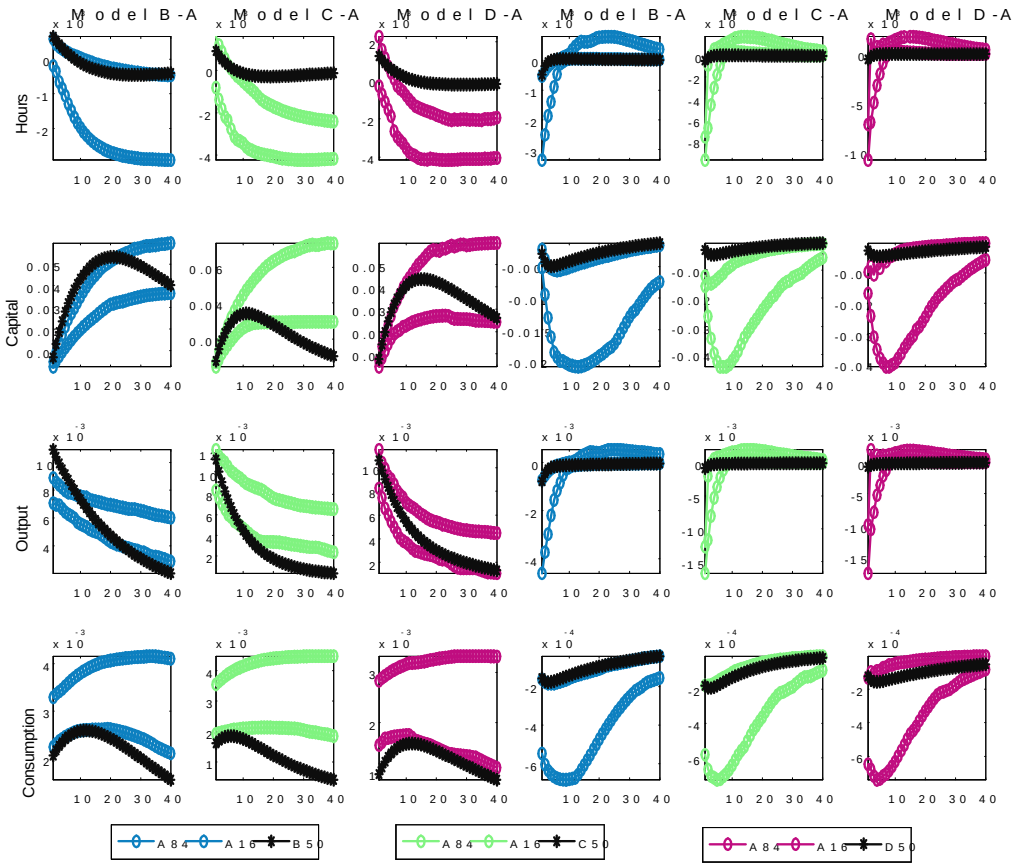


Figure 9:Impulse responses, DGP2

	Variance decomposition							
	DGP: Small time variations				DGP: Large time variations			
Variable	Technology	Government	Technology	Goverment	Technology	Government	Technology	Goverment
	Model B		Time invariant		Model B		Time invariant	
Y	94.1	0.3	99.7	0.4	81.3	0.1	99.8	0.6
C	89.5	0.2	99.9	0.1	55.3	0.1	99.8	0.2
N	60.2	0.5	98.6	1.4	15.6	0.4	97.8	2.3
K	70.2	0.4	99.5	0.6	40.6	0.1	99.4	0.8
	Model C		Time invariant		Model C		Time invariant	
Y	97.2	0.3	98.8	1.6	81.9	0.1	92.7	8.2
C	88.1	0.3	99.9	0.1	26.5	0.1	99.9	0.1
N	44.6	0.6	99.0	1.2	5.4	0.4	96.6	3.9
K	84.4	0.2	99.0	1.4	37.4	0.1	97.4	3.0
	Model C		Time invariant		Model C		Time invariant	
Y	98.0	0.1	99.3	1.5	82.2	0.1	93.6	7.2
C	92.2	0.2	99.8	0.3	32.8	0.1	99.6	0.8
N	35.9	0.5	97.3	3.4	10.2	0.5	92.8	7.9
K	96.6	0.3	99.2	1.2	60.0	0.4	97.9	2.8

Table 4: Long run variance decomposition

First of all, notice that when the correct model is employed, estimation is very successful even when the smaller sample size, regardless of the DGP and of whether time variations are exogenous or endogenous. Thus, in principle, there are no numerical distortions one should worry about. Second, with the DGP featuring exogenous time variations explaining a small portion of output variance, a number of distortions occur. For example, the persistence of government spending shock is poorly estimated (mean persistence is about 50 percent larger than the true one), while estimates of δ, α and A are severely biased downward. Third, the distortions are smaller when the time variations are endogenous (model C and D): nevertheless significant downward bias exists for the inverse of the Frish elasticity γ, for $\delta and \alpha$. Notice that the performance of the time invariant model is similar regardless of whether the DGP features externalized or internalized endogenous time variations. Fourth, the performance of the time invariant model does not improve, and if anything worsens, when T=1000 for all three specifications. Thus, as sample size increases, failure to converge to the true DGP becomes even more obvious.

When the model feature time variations which explain a significant portion of the variability of output, the above tendencies become more striking. Take, for example a model with exogenous time varying parameters: here estimating a time invariant model leads to an overestimation of the persistence of the structural shocks. Basically, the only way the time invariant model can accommodate the (additional) serial correlation

present in the endogenous variables is by increasing the persistence of both technology and government spending shocks. In models C and D the distortions become considerably larger and for example, the mean estimate of inverse of the Frish elasticity is now negative. Furthermore, as it is clear from figure 10, the distributions one obtain are typically skewed and multimodal. Overall, as one would expect, neglecting time variations in the parameters if more detrimental when they account for a significant portion of the time variations of the endogenous variables.

The impulse responses we present confirm our previous results. When time variations in the parameters explain a small fraction of the variability of output, we find that true responses to technology shocks are off in terms of impact magnitude (in particular for output), tend to be on the upper side of the estimated 68 percent band, but their shape is consistent with the shape of the bands produced with the time invariant model. Interestingly, output responses are those which characterized more poorly with the time invariant model and, consistent with previous discussion, the misspecification obtained when the true model features exogenous time variations is larger. On the contrary, the responses to government expenditure shocks obtained with a time invariant model are different from the true ones in terms of magnitude, shape and persistence. The reason is simple: since the signal that government expenditure produces in the model is weak, it tends to be obscured by the presence of time variations. The distortions obtained when time variations in the parameters explain a considerable amount of the variance of output are generally larger, but the pattern of results is similar. To be noticed is the fact that, consistent with the previous discussion, the persistence of the responses to technology shocks is not very well characterized and while true responses tend to zero after 10 years, the bands obtained with the time invariant model do not include the zero line at that horizon.

Is it possible to correctly characterized the contribution of structural shocks to the variability of the endogenous variables when the forecast errors of the time invariant model are used to construct the likelihood function? In general, one should expect the structural shocks of the time invariant model to be a contaminated version of the structural shocks of the time varying DGP for two reasons. First, the wrong P matrix is used to compute forecast errors. As we have previously seen this creates a correlation between the forecast errors and the lags of the endogenous variables and thus changes the timing of the innovations. Second, we are aggregating m (structural and parameter) shocks into n (structural) shocks. This aggregation is known to generate complicated VARMA structures where the n structural shocks are functions of the leads and lags of the original disturbances (see e.g. Canova and Paustian, 2011). Thus, even if the P matrix is correctly specified, one should expect the distortions to occur, unless the shocks we are omitting (those to the parameters) are unimportant and feature low persistence.

Table (5) indicates that when time variations in the parameters do not explain a large portion of the variance of output, technology shocks absorb the variability that is missing in the time invariant model. This is true regardless of whether time variations are exogenous or endogenous and the effect seems particularly strong for hours worked.

When parameter variations explain a larger portion of the variance of output, we find that technology shock still absorb a large amount of the missing variability. However, in some cases, the missing variability is captured by government spending shocks. For example, while government spending shocks explain only 0.1 percent of the variability of output in the long run, it explains 7-8 percent when a time invariant model is used and parameter variations are endogenous.

In sum, for the RBC model we consider and the parameterization employed, we find that estimating a constant parameter model when the DGP features time varying parameters leads to distortions, regardless of the sample size and of whether time variations in the parameters are important in explaining the variability of output or not. In general, the parameters that are most distorted are those regulating the estimated persistence of the exogenous shocks and the parameters controlling the income and substitution effects. Since the Euler equation and the resource constraints are the optimality conditions which are different in the constant and time varying coefficient models, it is clear where these distortions come from. We have also performed a Monte Carlo exercise allowing also the labor share to be time varying. Variations in the labor share have been documented in the literature (see e.g. Rios Rull and Santaeularia Llopis, 2010) and there is evidence that variations in this parameter are strongly countercyclical. This is relevant for our exercise because, in this case, all four optimatility conditions are affected by time variations. Thus the strength of the income and substitution effect distortions will likely to be larger. Indeed, we do find that distortions in this case become quite large and it many cases it is very difficult to estimate the time invariant model regardless of the DGP (results for this case are available on request). Thus, the relevant case seems to be one where some but not all of the equilibrium conditions are affected by time variations since the main consequence of using a time invariant model seems to be an incorrect characterization of the income and substitution effects present in the DGP.

6 Recovering structural dynamics with SVAR methods

We have seen that with certain forms of parameter variations, the decision rules of the time invariant model are distorted and this generates erroneous forecast errors and misspecification of the likelihood function. This misspecification leads to distorted impulse response dynamics when likelihood based methods are used to estimate the structural parameters of the time invariant model. Because of these problems, one may wonder whether less structural and computationally less demanding methods can be employed to recover the dynamics induced by structural shocks. Canova and Paustian (2011) have shown that when the structural model is misspecified VAR methods which employ robust sign restrictions can be effective in capturing the dynamics induces by the structural disturbances. The misspecification they consider is different from the one analyzed here - there the model fails to

consider certain features of the DGP, while misspecification here comes from the presence of time variations in the parameters. In this section we ask whether VAR methods can also be a useful also in this case.

The exercise we conduct is as follows. Using the illustrative RBC model, we simulate data using the decision rules of models B, C, and D. We then compute VAR residuals using the population P matrix of the constant coefficient model, rotate the residuals using an orthonormal matrix, and keep the impulse responses if technology shocks generate a positive response of hours, capital, output and consumption on impact and if government expenditure shocks generate a negative response of hours, output, consumption and capital (these signs are those present in figure 3 and hold for variations of the (constant) structural parameters within a reasonable range).

We compare the responses we obtain with those generated computing VAR residuals using the correct population P matrix and rotating them with the same orthonormal matrices previously employed until the same sign restrictions are satisfied. To keep the interpretation of the graphs clear we present the median response at each horizon for the true model (red line in each box) and the one standard errors bands which take into account rotation uncertainty (blue and green lines in each box) . Figure 14 has the responses when the DGP is model B, figure 15 the responses when the DGP is model C, and figure 16 the responses when the DGP is model D.

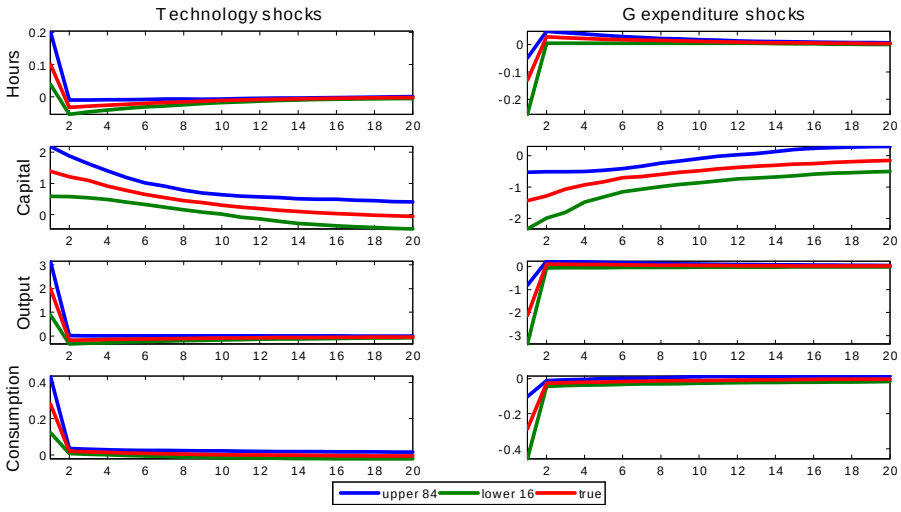


Figure 14: Impulse responses, exogenously varying and constant coefficient RBC models

Overall, SVAR methods seem to do very well when parameter variations are neglected. Since neither the impact response nor the lagged dynamics are misspecified when the model features exogenous time varying parameters, it is not surprising to find that in figure 14, the correct impulse falls exactly at the median

of the estimated band. This is simply a reality check that the method has indeed power to detect the truth if endowed with the correct model. Figures 15-16, on the other hand, indicate that VAR methods have excellent properties even when endogenous time variations in the parameters are neglected: all qualitative features of the true responses are reproduced and, quantitatively, true responses fall around the median of the band computed using the incorrect model except for consumption responses.

Recall that here, two sources of misspecification are present: the P matrix is incorrect; aggregation problems are present. What figures 25-26 show is i) the distortion in the P matrix are small and only the entries corresponding to consumption are altered; ii) the Q matrix is not very strongly affected; iii) shock misaggregation is a minor problem with the parameterization we have used. Because shocks to the parameters are i.i.d., timing distortions in the VAR innovations are likely to be small.

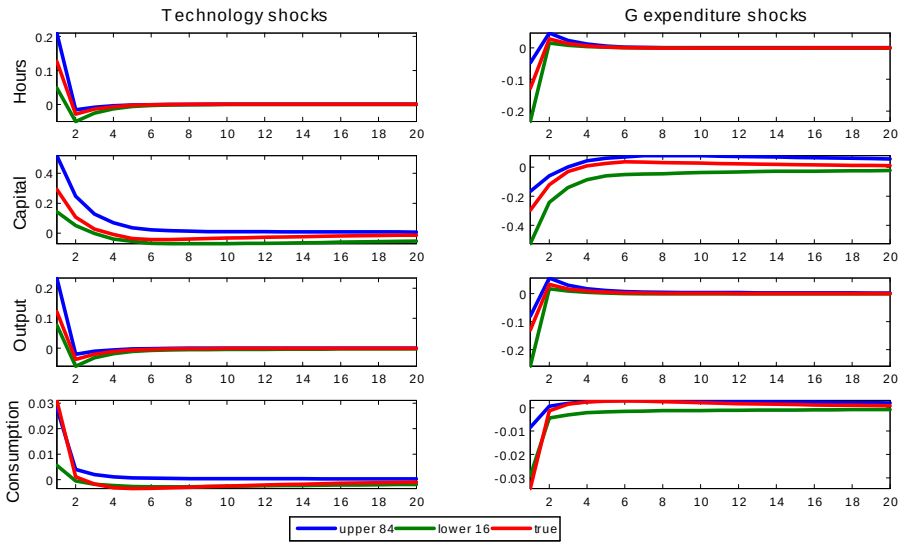


Figure 15: Impulse responses, endogenously varying (no internalization) and constant coefficients RBC models

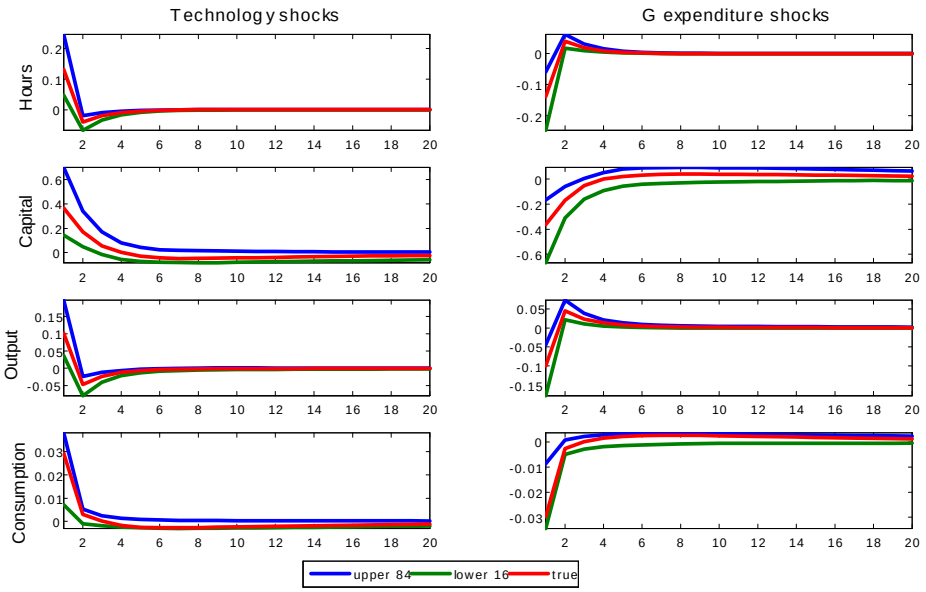


Figure 16: Impulse responses, endogenously varying (with internalization) and constant coefficients RBC models

In sum, the performance of SVAR seems to be superior to the one of likelihood based methods in the presence of misspecification driven by parameter variations. When parameter variations are exogenous, estimating a constant coefficient VAR identified with sign restrictions derived from theory is sufficient, since the dynamics in response to structural disturbances are not distorted. When there are endogenous variations, the features of the responses are generally well captured with a constant coefficient SVAR model. However, high persistence in the shocks driving parameter variations may alter the quality of the approximation to the responses to the structural shocks.

7 An application: are financial friction parameters time varying

(TO BE ADDED)

8 References

Andreasen, M. (2012). On the effects of rare disasters and uncertainty shocks for risk premia in non-linear DSGE models. Review of Economic Dynamics, 15, 293-316.

Ascari, G. and A. Sbordone (2014). The macroeconomics of Trend Inflation. Journal of Economic Literature, 52, 679-739.

Brinca, P., Holter, H., P. Krusell, and L. Malafry (2014). Fiscal multipliers in the 21st century. EUI Schuman center working paper, 2014-119.

Canova, F. (2009). What Explains the Great Moderation in the U.S?: A Structural Analysis. Journal of the European Economic Association, 7, 697-721.

Canova, F. and L. Sala (2009). Back to square one: Identification issues in DSGE models. Journal of Monetary Economics, 56, 431-449.

Canova, F. and M. Paustian (2011). Business cycle measurement with some theory. Journal of Monetary Economics, 48, 345-361.

Chang, Y., Kim, S. and F. Schorfheide (2013). Labor market heterogeneity and the policy-(in)variance of DSGE model parameters'. Journal of the European Economic Association, 11,193-220.

Chari, V.V., Kehoe, P. and Mc Grattan, E. (2007). Business Cycle Accounting. Econometrica, 75, 781-836.

Cogley, T. and T. Yagihashi, (2010). Are DSGE approximating models invariant to policy shifts? The B.E. Journal of Macroeconomics (Contributions), Article 27.

Davig, T. and E. Leeper (2006). Endogenous monetary policy regime changes. NBER International Seminar in Macroeconomics, 345-391. National Bureau of Economic Research.

Del Negro, M. and F. Schorfheide (2004). Prior from general equilibrium models for VARs. International Economic Review, 45, 643-673.

Dueker, M. Fisher, A. and Dittman, R. (2006). Stochastic Capital Depreciation and the Co-movements of Hours and Productivity. Berkeley Journals: Topics in Macroeconomics, vol 6,3, article 6.

Fernandez Villaverde, J. and J. Rubio Ramirez (2004). Comparing dynamic equilibrium economies to the data. Journal of Econometrics, 123, 153-187.

Fernandez Villaverde, J. and J. Rubio Ramirez (2007). How structural are structrual parameters. NBER Macroeconomics Annual, 22, 83-132.

Galvao, A. , Giraitis, L., Kapetanios, G., and K. Petrova (2014) Local Bayesian estimation and forecasting with a TVP model, manuscript

Gertler, M. and Karadi, P. (2010) A Model of unconventional monetary policy. Journal of Monetary Economics, 58, 17-34.

Huang, N. (2014). Weak Inference for DSGE models with time varying parameters. Boston College, manuscript.

Hurwicz, L. (1962). On the structural form of interdependent systems, in E. Nagel, P. Suppes, A. Tarski (eds.) Logic, Methodology and Philosophy of science: proceedings of a 1960 international congress. Stanford University Press.

Koop, G., Pesaran, H., and R. Smith (2013). On the identification of Bayesian DSGE models. Journal of Business and Economic Statistics, 31, 300-314

Ireland, P. (2004). Technology shocks in a New keynesian model. Review of Economics and Statistics, 83, 926-933.

Ireland, P. (2007). Changes in the Federal Reserve Inflation target: Causes and Consequences. Journal of Money Credit and Banking, 39, 1851-1882.

Liu, Z., Waggonner, D. and T. Zha (2011). Sources of Macroeconomic fluctuations: A regime switching DSGE approach. Quantitative Economics, 2, 251-301.

Magnusson, L. and S. Mavroeidis (2014). Identification using stability restrictions. Econometrica 82, 1799-1851.

Meier S. and C.Sprengler (forthcoming). Temporal Stability of Time preferences. Review of Economics and Statistics.

Mutschler, W. (2014) Identification of DGSE Models - the effect of higher order Approximation and Pruning, University of Munster, manuscript.

Rios Rull J. V. and R. SantaEularia-Llopis, (2010). Redistributive shocks and productivity shocks. Journal of Monetary Economics,37, 931-948.

Schmitt Grohe, S. and M. Uribe (2003). Closing small open economy models. Journal of International Economics, 62, 161-185.

Schmitt Grohe, S. and M. Uribe (2004). Solving Dynamic General Equilibrium Models using second order approximation to the policy function. JOurnal fo Economic Dynamics and Control, 28, 755-775.

Stock, J. and Watson, M. (1996). Evidence of Structural Instability in Macroeconomic Relationships. Journal of Business and Economic Statistics,14, 11-30.

Vavra, J. (2014). Time varying Phillips curve, NBER working paper 19790.