Sticky Prices and Costly Credit*

Lucy Qian Liu
International Monetary Fund

Liang Wang†
University of Hawaii - Manoa

Randall Wright
University of Wisconsin - Madison, FRB Chicago, FRB Minneapolis, NBER

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Abstract

This paper has two goals: (i) construct a framework where money and credit coexist as alternative payment instruments; (ii) pursue in this setting a theory of endogenously sticky prices that we take to the data. In the model, frictions generate monetary equilibria with price dispersion. This allows sellers to set prices in nominal terms they may keep fixed when aggregate conditions change. Buyers use cash and credit, with the former (latter) subject to inflation (transaction costs). We deliver strong analytic results – e.g., existence and uniqueness – plus a novel closed-form for money demand. The calibrated model matches pricing data quite well, generating large average price changes, many small changes, many negative changes, and a decreasing hazard. It is also consistent with macro and micro data on money and credit. Policy implications are discussed.

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†Corresponding Author: Liang Wang, Department of Economics, University of Hawaii Manoa, Honolulu, HI 96822, USA. Email: lwang2@hawaii.edu.

1 Introduction

This paper has two related goals: (i) construct a model where money and credit coexist as alternative means of payment; (ii) pursue within this setting a theory of endogenous price stickiness that we can take to the data. We build on the analysis of price dispersion in frictional goods markets by Burdett and Judd (1983), integrated with the general equilibrium monetary framework of Lagos and Wright (2005). The resulting model generates monetary equilibria with an endogenous distribution of prices. As in some other settings discussed below, this means sellers can set prices in nominal terms that they may keep fixed when aggregate conditions change. Buyers use both cash and credit because the former is subject to the inflation tax while the latter involves fixed or variable transaction costs, making the choice of payment instrument nontrivial. At the same time, costly credit allows us to develop a tractable model of nominal rigidity while avoiding an indeterminacy of equilibrium than plagues similar models, as we now explain.

To begin, consider Diamond (1971), where each seller (firm) of an indivisible good posts a price p, then buyers (households) sample one at a time until finding p below their reservation price p^*. Clearly, for any seller, the profit maximizing strategy is $p = p^*$. Hence, there is a single price in the market and it turns out to give all the gains from trade to sellers. The Burdett-Judd model makes one change in Diamond's specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. This implies there cannot be a single p, nor even a set of sellers with positive measure charging the same p, since sellers would have a profitable deviation to $p - \varepsilon$ (see fn. 7). One can compute explicitly the Burdett-Judd distribution $F(p)$, where any p in the support $\mathcal{F}$ yields the same profit, because lower-price sellers earn less per unit but are more likely to make a sale.

If one embeds Burdett-Judd pricing in a monetary economy, it makes sense for sellers to post prices in dollars. If the money supply M increases, $F(p)$ shifts so that the real distribution stays the same. Some sellers can keep p fixed, however, and hence prices can be sticky with increasing M, even though changing p is costless. While real prices fall for sellers that stick to their p, profits do not because they increase sales. This is true in Head et al. (2012), too, but there is a technical problem: the combination of indivisible goods and posting in monetary economies entails an indeterminacy (multiplicity) of equilibria. Hence, Head et al. use divisible goods, but then another problem pops up, since it is not obvious what firms should post. That paper imposes linear menus, where a seller posts p and lets buyers get any q they like as long as they pay pq, but there is no reason to think that this maximizes profit.

Giving agents a choice between cash and credit eliminates the indeterminacy that obtains with indivisible goods and posting without the need for restrictions like linear pricing. Intuitively, consider a fixed cost of using credit. Given $F(p)$, random matching implies buyers might spend a little or spend a lot, and they pay the cost of credit in larger transactions. Holding more cash reduces the probability of needing to use credit, which delivers a well-behaved money demand function and a unique stationary equilibrium. Thus, sticky prices arise endogenously with fewer "delicate" assumptions.[1] However, in contrast to theories where stickiness is imposed exogenously, here an increase in M is neutral (does not affect real activity). Whether or not there is neutrality in actuality, the point is that sticky nominal prices do not imply nonneutrality.

Head et al. (2012) calibrate their model to match the observed price-change distribution, and then show it matches other facts reasonably well, including

[1] We do not take a stand on whether divisible or indivisible goods are more "realistic," as that depends on the context, but would argue that indivisibility is an assumption on the physical environment and hence less "delicate" than a restriction on pricing strategies.

the average duration between price changes. We perform a similar exercise, but in addition to the price-change distribution, and different from Head et al., we also match the fractions of cash and credit purchases in the micro data plus macro observations on money demand. One substantive conclusion is this: the ability of search-based theories to account for salient features of empirical pricing is not compromised by changing assumptions about divisibility, by giving buyers a choice between cash and credit, or by bringing to bear more quantitative discipline. The rest of the paper involves making this precise.

2 Literature

Many sticky-price papers follow Taylor (1980) or Calvo (1983) by only letting sellers adjust p at certain points in time. Others follow Rotemberg (1982) or Mankiw (1985) by letting them change only at a cost. We allow sellers to change any time for free. A few papers push imperfect-information or rational-inattention theories, including Mankiw and Reis (2002), Woodford (2002), Sims (2003) and Mackowiak and Wiederhold (2009), which are related to search theory in that both concern scarce or costly-to-process information. We concentrate on search, mainly because when Burdett and Menzio (2014) combine this with menu costs, the analysis is more complicated and delivers similar results, and because they find the majority of price dispersion in the data (about 70%) is due to search. Based on these considerations, we abstract from menu costs and related devices in this project.[2]

The literature on Burdett-Judd pricing is large, including the labor-market applications in Burdett and Mortensen (1998) and Mortensen and Pissarides (1999). In monetary economics, prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd pricing in Lagos and Wright (2005), Head and

[2]Relatedly, nonmonetary search models with menu costs, where prices are sticky in unit of account, as in Benabou (1988,1992) or Diamond (1993), are special cases of Burdett and Menzio (2014). See also Craig and Rocheteau (2008).

Kumar (2005) and Head et al. (2010) put it in the related model of Shi (1997). Alternatives models of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where buyers differ not in terms of what they observe, but in their intrinsic (e.g., preference) type. The monetary version of that model in Curtis and Wright (2004) generically delivers a two-point p distribution, for any the number of types, which is less useful for our purposes. Also, as in Shi (1995) or Trejos and Wright (1995), and diametrically from us, that paper assumes goods are divisible while money is not, which limits the discussion of many policy and empirical issues.

Caplin and Spulber (1987) and Eden (1994) have sticky-price models that are similar in spirit yet also very different. In addition using Burdett-Judd, we build on the foundation for monetary economics in Lagos-Wright, which builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001), etc. See Wallace (2010), Williamson and Wright (2010), Nosal and Rocheteau (2011) or Lagos et al. (2014) for surveys of this approach, sometimes called New Monetarist Economics, trying to provide relatively clean descriptions of how agents trade and specifications for specialization, commitment and information that give money and related institutions essential roles. We believe it is desirable to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are there to ameliorate. If one prefers short-cuts, many of the points made here could also be made in, say, CIA or MUF models, but we find it natural to use monetary theory based on search, as is this same friction that drives price dispersion

As regards empirical work, Campbell and Eden (2014) report for a large grocery-store data set an average duration between price changes of 10.3 weeks, but we do not want to focus exclusively on groceries. In BLS data, Bils and Klenow (2005) find that at least half of prices last less than 4.3 months, or 5.5 months excluding sales. Klenow and Kryvtsov (2008) find durations from

6.8 to 10.4 months, with an mean of 8.6. Nakamura and Steinsson (2008) get 8 to 11 months by excluding substitutions and sales. These papers also find a large fraction of negative and small price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 6 months for "reference" prices, which are those most often quoted in a quarter (presumably this avoids, e.g., recording Saturday Night Specials as two changes in week). In earlier work, Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices from 5.9 months for household appliances to 19.2 for chemicals, and also finds many small changes, with about 2/3 below 2%.[3]

One troubling feature in the data for menu cost models is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low costs. Midrigan (2011) accounts this in a model where firms sell multiple goods, and paying a cost to change one price lets them change the rest for free. This is nice, but it is still worth considering alternatives. We account for average duration, large average price changes, many small changes, and pricing behavior that depends on inflation. While extending other models in various ways can help, even a very simple version of our setup easily accounts for the facts. It also generates price dispersion at low or even 0 inflation, consistent with the evidence (e.g., Campbell and Eden 2014), while dispersion arises in many other models only because of inflation.

In terms of interpretations and implications, we think it is fair to highlight Ball and Mankiw (1994), who champion the position that stickiness leads to

[3]More empirical work is surveyed by Klenow and Malin (2010). In conversations with people in the area, they more or less agree on the following list of stylized facts: (1) Prices change slowly, although durations vary across studies. (2) The frequency and size of price changes vary a lot across goods. (3) Two sellers that change at the same time do not typically change to the same price. (4) About 1/3 of changes are negative. (5) Hazard rates are relatively flat but slightly declining. (6) There are many small changes (below 5%), and many big changes (above 20%). (7) The frequency, size and the fraction of negative changes vary with inflation. (8) There is much price dispersion even at low inflation.

nonneutrality: "We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky." Moreover: "As a matter of logic, nominal stickiness requires a cost of nominal adjustment." Golosov and Lucas (2003) similarly say: "[M]enu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing." We agree that prices are in fact sticky, but the other statements are incorrect: menu costs may or may not be important, and money may or may not be neutral, but logically sticky prices imply neither menu costs nor nonneutrality.

The paper may also be considered a contribution to pure monetary theory. Our environment delivers money demand as a square- or cube-root function of the nominal interest rate. This relates to classic results in Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966), although those are partial-equilibrium models, or, more accurately, decision-theoretic analyses of how to manage one's money assuming that it (as opposed to credit, barter or something else) must be used for transactions. While such models are still being applied to good effect (e.g., Alvarez and Lippi 2009), we admit to a preference for theories where the roles of money and credit are not taken for granted, for several reasons, including the notion that this is important for analyzing monetary policy (e.g., Wallace 2013).[4]

The above-mentioned multiplicity of monetary equilibria in models with indivisible goods and posting occurs in a series of papers spawned by Green and Zhou (1998). See Jean et al. (2010) for citations and further discussion, but consider the following intuition: If all sellers post p then buyers' best

[4] Also, incorporating this kind of pricing (Burdett-Judd with indivisible goods) in the New Monetarist framework is a further step in understanding its properties. As discussed in the surveys cited above, previous analyses of this framework have used Nash, Kalai or strategic bargaining, price posting with random or directed search, competitive price taking and auctions in versions with multilateral meetings, and pure mechanism design.

response is to bring $m = p$ dollars to the market as long as p is not too big. If all buyers bring m then sellers' best response is post $p = m$ as long as m is not too small. Hence, any $p = m$ in some range is an equilibrium, and payoffs depend on the one we select. Sometimes this multiplicity can be refined away, but it is not clear how to do this with Burdett-Judd pricing, and even if it were, the outcome tends to be sensitive to details, like who moves first. Head et al. (2012) avoid this because, even if buyers think all sellers post p, they need not bring $m = p$ when q is variable, but again that leads to other issues. Our alternative is based on a venerable idea that credit is costly.[5]

Before moving to the model, we emphasize how seller heterogeneity may matter, even though this is fairly well understood in other applications (e.g., see Mortensen and Pissarides 1999). When firms are homogeneous, as they are here, theory does not pin down which one charges what p, but only the distribution $F(p)$. With heterogeneity, however, lower cost firms prefer lower p because they are more interested in high volume. Still, for any subset with the same marginal cost, it does not matter which firm in the subset posts what p. So there is still dispersion, and with that we can get stickiness, within sets of similar sellers. This is perhaps especially relevant for retail, where the marginal cost is the wholesale price, and even if a few retailers get better deals – e.g., it could be that Kmart gets a quantity discount – many others face the same wholesale terms. Even with different fixed costs, these sellers are homogeneous from our perspective, and our theory of stickiness applies to any group of retailers facing the same wholesale price.

[5] Papers with money and costly credit include Prescott (1987), Freeman and Huffman (1991), Lacker and Schreft (1996), Freeman and Kydland (2000) and Lucas and Nicolini (2013). Work on models with multiple payment methods that are closer to our setup include Ferraris and Watanabe (2008), He et al. (2008), Telyukova and Wright (2008), Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2013), Li and Li (2013), Lotz and Zhang (2013), Araujo and Hu (2014), Gu et al. (2014) and Bethune et al. (2015). None of those papers consider the substantive application that is the focus of this project: a theoretical and empirical analysis of sticky nominal prices.

3 Model

Each period in discrete time has two subperiods: first there is a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms interpreted as retailers with measure 1, and a set of households with measure $\bar{b}$. Households consume a divisible good x_t in the AD market and an indivisible good y_t in the BJ market, but supply labor ℓ_t only in AD. The BJ good is produced at cost c by firms. Agents are anonymous in the BJ market, and hence cannot use credit, unless they access a technology to authenticate identities and record transactions at cost δ. By incurring δ, households can get BJ goods in exchange for commitments to deliver d_t dollars in the next AD market; otherwise, they need cash at point of sale.[6] Let M_t be the money supply per capita, with $M_{t+1} = (1+\pi) M_t$. Changes in M occur in AD via lump-sum transfers if $\pi > 0$ or taxes if $\pi < 0$.

For simplicity, preferences within a period are separable, $U(x_t) + u\mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, while $u > 0$ is a parameter and $\mathbf{1}(y_t)$ is an indicator function giving 1 iff the indivisible BJ good is consumed. Let $\beta = 1/(1+r)$, $r > 0$, be a discount factor between today's AD market and tomorrow's BJ market. We impose $\pi > \beta - 1$, where in stationary equilibrium π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1 + i = (1+\pi)(1+r)$. Thus, $\pi > \beta - 1$ iff $i > 0$, and the Friedman rule is the limiting case $i \to 0$. As usual, $1+i$ is the amount of money agents require in the next AD market to give up a dollar in the current AD market, whether or not such trades occur in equilibrium. Let x_t be the AD numeraire, produced one-for-one with ℓ_t, so the real wage is 1. The AD price of money in

[6] While δ is paid by households, it is not hard to show the allocation is identical if it is instead paid by firms, as in elementary tax-incidence theory. Also, it does not matter if debt due in the frictionless AD market is denominated in dollars or numeraire.

numeraire is ϕ_t, so $1/\phi_t$ is the nominal price level.

Following Liu et al. (2011), firms enter the BJ market for free, but households must pay cost k, which in general determines participation $b_t \leq \bar{b}$. However, in the baseline model $k = 0$, so $b_t = \bar{b}$ (see fn. 16 for a proof). Firms use BJ profits to buy AD goods, over which they have linear utility, but nothing of interest would change if instead they disburse profits to households as dividends. A firm posts a price taking as given household behavior and the CDF of other firms' prices, $F_t(p)$, with support $\mathcal{F}_t$. Every household in BJ randomly samples n firms – i.e., they see n independent draws from F_t – with probability $\alpha_n(b_t)$ that generally depends on the buyer-seller ratio, or market tightness, b_t. We assume $\alpha_1(b_t), \alpha_2(b_t) > 0$ and $\alpha_n(b_t) = 0 \ \forall n \geq 3$ for simplicity, but this is not crucial (Burdett et al. 2014, e.g., show how to proceed when $\alpha_n > 0$ $\forall n$). We also assume $\delta < u - c$, which guarantees that when $k = 0$ and there is no money, there exists an equilibrium where $b = \bar{b}$ and all transactions use credit – basically, the original Burdett-Judd equilibrium.

3.1 Firms

Expected real profit for a firm posting p at date t is

$$\Pi_t(p) = b_t \left\{ \alpha_1 (b_t) + 2\alpha_2 (b_t) \left[1 - F_t (p) \right] \right\} (p\phi_t - c) . \tag{1}$$

Thus, revenue per unit is $p\phi_t - c$, and the number of units is determined as follows: The probability a household contacts this firm and no other is $\alpha_1 (b_t)$. In this case the firm makes a sale for sure. The probability a household contacts this firm plus another is $2\alpha_2 (b_t)$, since it can happen in two ways, this one first and the other second, or vice versa. In this case the firm makes a sale iff it beats the other's price, which occurs with probability $1 - F_t (p)$. This is all multiplied by tightness b_t to convert buyer probabilities into seller probabilities. Equilibrium in price posting means every $p \in \mathcal{F}_t$ yields the same profit, and no $p \notin \mathcal{F}_t$ yields higher profit.

As is standard in Burdett-Judd models, there is a unique outcome, and it has $F_t(p)$ continuous with $\mathcal{F}_t$ an interval.[7] Taking as given for now the upper limit of the interval $\bar{p}_t$, profit from any $p \in \mathcal{F}_t$ must equal the profit from $\bar{p}_t$, which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1(b_t)(\bar{p}_t \phi_t - c). \tag{2}$$

Equating (1) and (2), for every $p \in \mathcal{F}_t$, we can solve for

$$F_t(p) = 1 - \frac{\alpha_1(b_t)}{2\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - c}. \tag{3}$$

This is the BJ distribution. Notice

$$F_t'(p) = \frac{\alpha_1(b_t)\phi_t}{2\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^2} > 0 \tag{4}$$

$$F_t''(p) = -\frac{\alpha_1(b_t)\phi_t^2}{\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^3} < 0. \tag{5}$$

Also, since the lower limit of $\mathcal{F}_t$ satisfies $F(\underline{p}_t) = 0$,

$$\underline{p}_t = \frac{\alpha_1(b_t)\phi_t \bar{p}_t + 2\alpha_2(b_t)c}{\phi_t[\alpha_1(b_t) + 2\alpha_2(b_t)]}. \tag{6}$$

As regards $\bar{p}_t$, there are different possibilities, depending on real balances $z_t = \phi_t m_t$, where m_t denotes nominal balances. If $0 < z_t < u - \delta$ the highest price is $\bar{p}_t = (u - \delta)/\phi_t > m_t$.[8] In this case buyers use credit if they sample $p \in (m_t, \bar{p}_t]$ and cash otherwise. If $u - \delta < z_t < u$ then $\bar{p}_t = m_t$. In this case, buyers always use cash. If $u < z_t$ then $\bar{p}_t = u/\phi_t$, but this never happens, since no one carries more money than they need. To translate from dollars to numeraire, let $q_t = \phi_t p_t$ and write the distribution of real BJ prices as

$$G_t(q) = F_t(\phi_t p) = 1 - \frac{\alpha_1(b_t)}{2\alpha_2(b_t)} \frac{\bar{q}_t - q}{q - c}. \tag{7}$$

[7] There cannot be a mass of firms posting the same p in equilibrium, because any one of them would have a profitable deviation to $p - \varepsilon$, since he would lose only ε per unit and make discretely more sales by undercutting others at the mass point. Similarly, if there were a gap between say p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$, and earn more per unit without losing sales.

[8] This follows because no buyer would pay more than $(u - \delta)/\phi_t$, and if $\bar{p}_t < (u - \delta)/\phi_t$ is the highest price then a firm would have a profitable deviation to $(u - \delta)/\phi_t$.

There is also the possibility of nonmonetary equilibrium, where $z = 0$, firms post q and all transactions use credit, and the bounds are $\bar{q}_t = u - \delta$ and

$$\underline{q}_t = \frac{\alpha_1(b_t)(u-\delta) + 2\alpha_2(b_t)c}{\alpha_1(b_t) + 2\alpha_2(b_t)}. \tag{8}$$

We focus on stationary equilibrium, where real variables are constant and all nominal variables grow at rate π. There are three cases: a nonmonetary equilibrium NME, where $z = 0$; a mixed monetary equilibrium MME, where $0 < z < u-\delta$, so cash and credit both used in BJ; and a pure money equilibrium PME, where $u-\delta < z$, so only cash is used in BJ. In NME, the BJ prices must be described in terms of numeraire; in the other cases they can be described in numeraire or dollars.

3.2 Households

In the interest of stationarity we frame the household problem in real terms. The state variable in AD is net worth, $A = \phi(m-d) - \delta\mathbf{1}(d) + \bar{A}$, where money m and debt d are brought in from the previous BJ market, $\mathbf{1}(d)$ is an indicator function equal to 1 when $d > 0$ and 0 otherwise, while $\bar{A}$ is any other source of purchasing power, including transfers. Since preferences are linear in ℓ, wlog we can impose all debt is settled in AD. The state variable in the BJ market is z. Let the AD and BJ value functions be $W(A)$ and $V(z)$.

For a household that decides to enter the next BJ market,

$$W^1(A) = \max_{x,\ell,z} \{U(x) - v\ell + \beta V(z)\} \text{ st } x = A - k + \ell - (1+\pi)z,$$

where k is the entry cost, and due to inflation the cost of having z next period in terms of current numeraire is $(1+\pi)z$. Eliminating ℓ, we get

$$W^1(A) = A - k + U(x^*) - x^* + \beta \max_z O_i(z), \tag{9}$$

where $U'(x^*) = 1$ and the objective function for the choice of z is $O_i(z) \equiv V(z) - (1+i)z$, where again i is given by the Fisher equation. For a household

that skips the next BJ market,

$$W^0(A) = A + U(x^*) - x^* + \beta W\left(\bar{A}\right). \tag{10}$$

Then $W(A) \equiv \max\{W^1(A), W^0(A)\} = W^1(A)$ if the BJ market is active.

As in Lagos and Wright (2005), $W'(A) = 1$, and this keeps the analysis tractable. In particular, it implies

$$V(z) = W\left(z + \bar{A}\right) + [\alpha_1(b) + \alpha_2(b)][u - \mathbb{E}_J q - \delta + \delta J(z)], \tag{11}$$

where $\mathbb{E}_J q = \int_{\underline{q}}^{\bar{q}} q dJ(q)$ and

$$J(q) = \frac{\alpha_1(b) G(q) + \alpha_2(b)\left\{1 - [1 - G(q)]^2\right\}}{\alpha_1(b) + \alpha_2(b)} \tag{12}$$

is the CDF of transaction prices, which differs from $G(q)$, because buyers seeing multiple posted prices pick the lowest.[9] From (11), the benefit of z is that it reduces the probability of needing to use credit, $1 - J(z)$.

Households also need to make a BJ entry decision. Algebra implies $\Phi \equiv (1+r)[W^1(A) - W^0(A)]$ is independent of A and satisfies

$$\Phi = [\alpha_1(b) + \alpha_2(b)][u - \mathbb{E}_J q - \delta + \delta J(z)] - \kappa - iz, \tag{13}$$

where $\kappa = k/\beta$. The first term is the expected return to participating in BJ, while the others are the costs. The equilibrium entry condition is

$$b = \bar{b} \implies \Phi \geq 0;\ b = 0 \implies \Phi \leq 0;\ b \in (0, \bar{b}) \implies \Phi = 0. \tag{14}$$

[9] To derive (11), first write

$$V(z) = W(z) + \alpha_1(b)\int_{\underline{q}}^{z}(u-q)\,dG_1(q) + \alpha_1(b)\int_{z}^{\bar{q}}(u-q-\delta)\,dG_1(q)$$
$$+ \alpha_2(b)\int_{\underline{q}}^{z}(u-q)\,dG_2(q) + \alpha_2(b)\int_{z}^{\bar{q}}(u-q-\delta)\,dG_2(q),$$

where $G_n(q) = 1 - [1 - G(q)]^n$ is the CDF of the lowest of n independent draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \leq z$, so cash is used, times the expected surplus. The third is the probability of meeting a seller with $q > z$, so credit is used, which has cost δ. The fourth or fifth term are similar except the buyer meets two sellers. Then use algebra.

3.3 Equilibrium

Definition 1 *A stationary equilibrium is a list* $\langle G(q), b, z\rangle$ *solving (7), (14) and the money demand problem in (9).*

Definition 2 *A PME is an equilibrium with* $z > u - \delta$ *and* $\bar{q} = z$. *A MME is one with* $u - \delta > z > 0$ *and* $\bar{q} = u - \delta$. *A NME is one with* $z = 0$ *and* $\bar{q} = u - \delta$.

These definitions involve only real variables, but in monetary equilibrium the nominal price distribution is $F_t(p) = G(\phi_t p)$. Other variables can be computed, include AD consumption, which is $x = x^*$, and labor supply ℓ, which comes from the budget equation, but we do not need these below.

4 Analytic Results

For now $k = 0$, so $b = \bar{b}$ is fixed and we omit the argument from the meeting probabilities α_1 and α_2. Now consider these preliminary results:

Lemma 1 *For* $\underline{q} < z < \bar{q}$, $V(z)$ *is smooth, with* $V'(z) > 0$ *and* $V''(z) < 0$. *For* $z < \underline{q}$ *or* $z > \bar{q}$, $V'(z) = 1$.

Proof: For $z \in (\underline{q}, \bar{q})$,

$$V'(z) = 1 + (\alpha_1 + \alpha_2)\,\delta J'(z) \tag{15}$$

$$V''(z) = (\alpha_1 + \alpha_2)\,\delta J''(z). \tag{16}$$

by virtue of (11). Using (12), we have

$$J'(q) = \frac{\alpha_1 G'(q) + 2\alpha_2 [1 - G(q)] G'(q)}{\alpha_1 + \alpha_2} > 0 \tag{17}$$

$$J''(q) = \frac{\alpha_1 G''(q) + 2\alpha_2 [1 - G(q)] G''(q) - 2\alpha_2 G'(q)^2}{\alpha_1 + \alpha_2} < 0 \tag{18}$$

where G is smooth by virtue of (7). The rest is obvious. ■

Intuitively, these results deliver a unique stationary MME, in contrast to the above-mentioned indeterminacy of stationary monetary equilibrium in models with indivisible goods, posting and no credit. Indeed, $V''(z) < 0$ implies a unique $z_i = \arg\max_{z \in [\underline{q}, \bar{q}]} O_i(z)$, satisfying the FOC

$$(\alpha_1 + \alpha_2)\, \delta J'(z_i) = i, \tag{19}$$

but to check if $z_i \in (\underline{q}, \bar{q})$, which we need for MME, there is more work to do. Let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. We first check $O_i^+(\underline{q})$. If $O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$ we check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = z_i$, as in the right panel. This leads to the following characterization of the (stationary) equilibrium set.

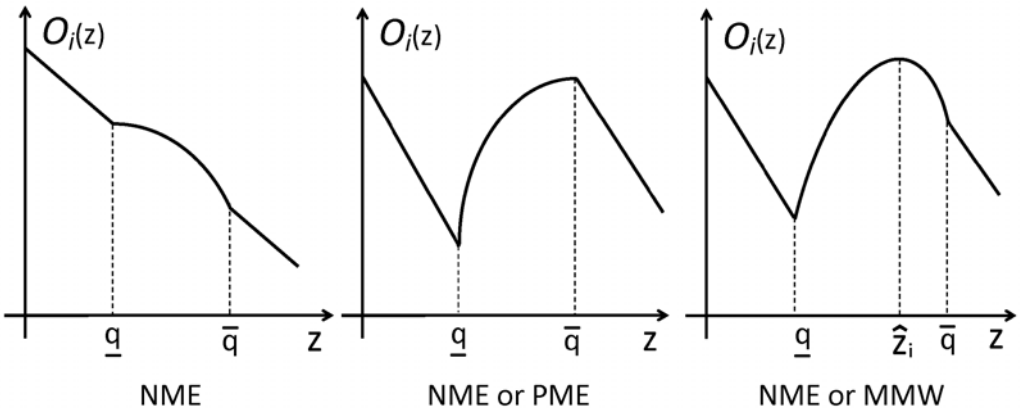


Figure 1: Possible Types of Equilibria

Proposition 1 *There exists a unique NME.*

Proof: With fiat currency $\phi = 0$ is always self-fulfilling, so we simply set $G(q)$ according to (7), as in the original Burdett-Judd model. This implies positive payoffs (with $b = \bar{b}$) by the maintained assumption $\delta < u - c$. ■

Proposition 2 *There exists a unique MME iff*

$$\delta < \bar{\delta} \equiv u - \frac{2\alpha_2^2 + 2\alpha_1\alpha_2}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2} c \tag{20}$$

and $i \in (\underline{i}, \bar{\imath})$, *where* $\underline{i} = \delta\alpha_1^2/2\alpha_2 (u - \delta - c)$ *and* $\bar{\imath} \in (\underline{i}, \infty)$.

Proof: As is clear from Figure 1, necessary and sufficient conditions for MME are: (i) $O_i^-(\bar{q}) < 0$; (ii) $O_i^+(\underline{q}) > 0$; and (iii) $O_i(z_i) > O_i(0)$. Condition (i) is equivalent to $(\alpha_1 + \alpha_2)\,\delta J^-(\bar{q}) < i$. A calculation implies this holds iff $i > \underline{i}$. Condition (ii) is equivalent to $(\alpha_1 + \alpha_2)\,\delta J^+(\underline{q}) > i$, which holds iff $i < \tilde{\imath}$ where

$$\tilde{\imath} = \frac{\delta (\alpha_1 + 2\alpha_2)^3}{2\alpha_1\alpha_2 (u - \delta - c)} > \underline{i}. \tag{21}$$

Condition (iii) is equivalent to $(\alpha_1 + \alpha_2)\,\delta J(z_i) - iz_i > (\alpha_1 + \alpha_2)\,\delta J(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -ic + \frac{\delta (\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}}\delta^{\frac{1}{3}}\alpha_1^{\frac{2}{3}}\alpha_2^{-\frac{1}{3}} (u - \delta - c)^{\frac{2}{3}} \left(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}\right).$$

Notice $\Delta(0) > 0 > \Delta(\tilde{\imath})$ and $\Delta'(i) < 0$. Hence $\exists!\ \bar{\imath}$ such that $\Delta(\bar{\imath}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{\imath}$. It remains to verify that $\bar{\imath} > \underline{i}$, so that (i) and (iii) are not mutually exclusive. It can be checked that $\bar{\imath} > \underline{i}$ iff $\delta < \bar{\delta}$. Hence a MME exist under the stated conditions. It is unique because $\bar{q} = u - \delta$ in MME, which pins down $G(q)$, and given this there is a unique $\hat{z}_i = \arg\max_{z \in (\underline{q}, \bar{q})} O_i(z)$. ■

The simplicity of NME is no surprise, since it basically boils down to the original Burdett-Judd model. The simplicity of MME is more interesting. This comes from households having a unique best response $\hat{z}_i$ to $G(q)$, and we do not have to worry about firms' best responses because $\bar{q} = u - \delta$ and hence $G(q)$ are independent of $\hat{z}_i$ in MME (this is a big simplification over Head et al. 2012). Although the interest below is in MME, for completeness, consider PME. As in the related models discussed above, there is a continuum of PME, but we can still provide sharp conditions for existence.

Proposition 3 *PME exists iff either:* $\delta > \bar{\delta}$ *and* $i < \hat{\imath} = \delta(\alpha_1 + \alpha_2)/(u - \delta)$*; or* $\delta < \bar{\delta}$ *and* $i < \underline{\imath}$.

Proof: To construct a PME, as in the middle panel of Figure 1, necessary and sufficient conditions are: (i) $O_i^-(\bar{q}) > 0$; (ii) $O_i^+(\underline{q}) > 0$; and (iii) $O_i(\bar{q}) > O_i(0)$. Now (i) holds iff $i < \underline{\imath}$ and (ii) holds iff $i < \tilde{\imath}$, as given above. Condition (ii) holds iff $i < \hat{\imath}$. For $\delta > \bar{\delta}$, it is easily checked that $\hat{\imath} < \underline{\imath}$, and $\underline{\imath} < \tilde{\imath}$ by (21), so the binding condition is $i < \hat{\imath}$. For $\delta > \bar{\delta}$, it is easily checked that $\hat{\imath} > \underline{\imath}$, and $\underline{\imath} < \tilde{\imath}$, so the binding condition is $i < \underline{\imath}$. ■

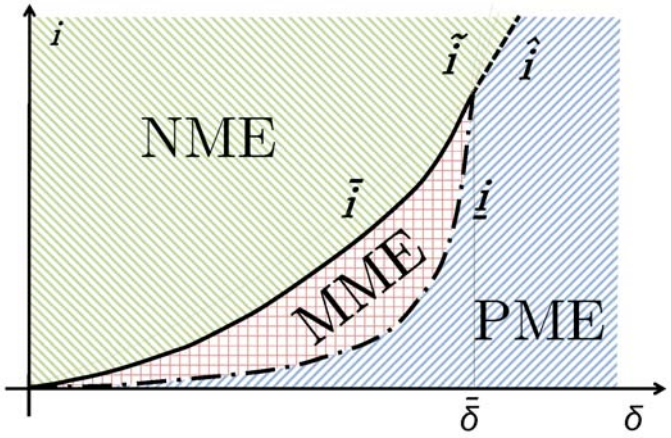


Figure 2: Different Equilibria in Parameter Space

Figure 2 partitions parameter space into three regions. For intermediate values of (δ, i) there exists a MME, and we focus on this case. Inserting the closed-form solution for the BJ distribution into the FOC for $\hat{z}_i$ and rearranging, in MME we get a nice closed-form solution for money demand:

$$\hat{z}_i = c + \left[\frac{\alpha_1^2 \delta (u - \delta - c)^2}{2\alpha_2 i}\right]^{1/3} \tag{22}$$

This expresses real balances in terms of the cube-root of $1/i$, reminiscent of the money demand functions in Baumol (1952), Tobin (1956), Whalen (1966) or Miller and Orr (1966), except the first two get a square- rather than cube-root

(we get something more like a square-root function in Section 6.2, when the fixed cost is replaced by a variable cost of credit). Although the models are very different, we claim that the results derive from similar economic forces.

Consider the standard story behind Baumol-Tobin, where an agent sequentially incurs expenses requiring money, by assumption, and has a fixed cost of rebalancing z. The decision rule compares i, the opportunity cost of holding cash, with the benefit of reducing the number of financial transactions, usually interpreted as trips to the bank. Buyers here make at most one transaction in BJ before rebalancing z in AD, but its size is random, and credit (spending beyond one's money balances) is costly. Our agents also compare i, again the cost of z, with the benefit of reducing the number of financial transactions, which can also be interpreted as trips to the bank, but now they go to get a loan and not to make a withdrawal.

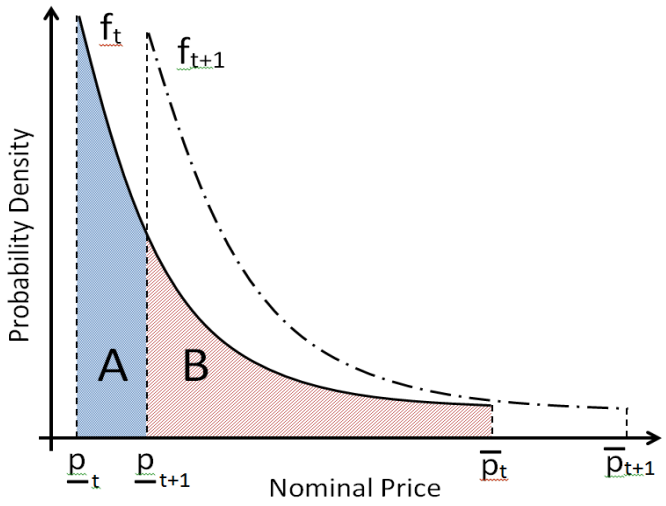


Figure 3: Changing Nominal Price Density with Inflation

In terms of nominal prices, $F_t(p)$ is uniquely determined $\forall t$ but individual-firm price dynamics are not. Consider Figure 3, drawn for the calibrated parameters in Section 5. With $\pi > 0$, the density $f_{t+1} = F'_{t+1}$ is a rightward shift of $f_t = F'_t$. Firms with $p < \underline{p}_{t+1}$ at t (Region A) must reprice at $t+1$,

because, while p maximized profit at t, it no longer does so at $t+1$. But as long as the supports $\mathcal{F}_t$ and at $\mathcal{F}_{t+1}$ overlap, there are firms with $p > \underline{p}_{t+1}$ at t (Region B) that can keep the same price at $t+1$ without reducing profit – again, they earn less per unit but make it up on the volume. Since equilibrium puts weak restrictions on individual price behavior, two additional conditions are imposed: (i) we add a payoff-irrelevant tie-breaking rule; and (ii) we focus on symmetric equilibria.

According to (i), firms use strategies of the following class: if $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1-\sigma \end{cases} \tag{23}$$

so those that are indifferent stick with probability σ to their current p.[10] Then (ii) simply says all changers pick their new $\hat{p}$ from the same repricing distribution $H_{t+1}(\hat{p})$. Given σ and $F_t(p)$, the unique $H_{t+1}(\hat{p})$ generating the equilibrium $F_{t+1}(p)$ is given by

$$H_{t+1}(p) = \begin{cases} \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[F_t(p) - F_t(\underline{p}_{t+1})]}{1-\sigma+\sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t) \\ \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[1 - F_t(\underline{p}_{t+1})]}{1-\sigma+\sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \tag{24}$$

Different σ capture a range of price dynamics, and it is routine to compute statistics comparable to those in the empirical literature, including duration, the distribution of changes and the hazard.[11]

5 Quantitative Results

While one can in principle examine $F_t(p)$ directly, our focus is on the distribution of changes $(p_{t+1} - p_t)/p_t$, because that is what is emphasized in the

[10] This is *very* different from Calvo pricing, where firms desperate to change p are simply not allowed; here they are indifferent and hence happy to randomize. The calibration below has $\sigma = 0.9$, so indifferent firms change rarely. One can imagine this emerging from small menu costs, too, but that is more complicated (Burdett and Menzio 2014).

[11] See Head et al. (2012) for derivations of formulae for these statistics and (24).

literature. We proceed by first calibrating the real economy and then studying the implied nominal price change distribution. For the real part, in terms of preferences, the only functional form needed is the AD utility function, which is taken to be $U(x) = \log(x)$. The parameter u then governs the trade off between x and the indivisible BJ good. We can also specify a disutility of labor parameter and set it to match average hours $\bar{\ell}$, as in standard business cycle theory, but there is no need to bother since the results below are independent of $\bar{\ell}$. Also, we focus on MME, and always check the relevant existence conditions in Proposition 2 at the calibrated parameters.

Here the matching probabilities α_1 and α_2 are calibrated directly, rather than imposing a parametric form, but see Section 6.1. Given the α's, the real posted-price and transaction-price distributions from (7) and (12) are

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2}\left(\frac{u-\delta-q}{q-c}\right) \\ J(q) &= 1 - \frac{\alpha_1^2(u-\delta-q)(u-\delta+q-2c)}{4\alpha_2(\alpha_1+\alpha_2)(q-c)^2}, \end{aligned}$$

with bounds $\bar{q} = u - \delta$ and $\underline{q} = [\alpha_1(u-\delta) + 2\alpha_2 c]/(\alpha_1 + 2\alpha_2)$. The closed-form for real balances in (22) implies the fraction of transactions that do not need credit is

$$J(\hat{z}_i) = \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1\alpha_2(u-\delta-c)\,i]^{2/3}(4\delta)^{-2/3}}{\alpha_2(\alpha_1+\alpha_2)}. \tag{25}$$

Since the data discussed below come from retail surveys, we consider markups defined using posted rather than transaction prices,

$$\frac{\mathbb{E}_G q}{c} = \frac{\alpha_1(u-\delta-c)\log(1+2\alpha_2/\alpha_1) + 2\alpha_2 c}{2\alpha_2 c}. \tag{26}$$

As in many other papers (e.g., Lucas 2000), our empirical notion of money demand is $L(i) = M/PY = \hat{z}_i/Y$, where $Y = x + (\alpha_1+\alpha_2)\mathbb{E}_J q$ is real output evaluated at transaction prices. Simplification yields

$$L(i) = \frac{c + \left[\alpha_1^2\delta(u-\delta-c)^2/2\alpha_2\right]^{1/3} i^{-1/3}}{1 + \alpha_1(u-\delta) + \alpha_2 c}, \tag{27}$$

which implies an elasticity

$$\varepsilon(i) = \frac{-1}{3 + 3c\left[\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2\right]^{-1/3} i^{1/3}}. \tag{28}$$

Below we target the fraction of monetary transaction $J(\hat{z}_i)$, average markup $\mathbb{E}_G q/c$, plus two moments of money demand, $L(i)$ and $\varepsilon(i)$ evaluated at $\mathbb{E}i$.

5.1 Data

We focus on 1988-2004, because we are going to evaluate the theory by its ability to account for the empirical price-change distribution during that period. However, longer series can be used to calibrate some parameters. For money we use the M1J definition in Lucas and Nicolini (2012), which adjusts M1 for money market deposit accounts, related to the way M1S adjusts it for sweeps, as discussed in Cynamon et al. (2006). The Lucas-Nicolini data provide an annual M1J series from 1915 to 2008, and a quarterly series from 1984 to 2013, and the authors make the case that there is a stable relationship between these data and nominal interest rates measured using 3-month T-Bills. Here we focus on the quarterly series, which better corresponds to the price-change sample period. In these data, the average annualized nominal rate is $\mathbb{E}i = 4.80\%$, implying $L(\mathbb{E}i) = 0.279$ and $\varepsilon(\mathbb{E}i) = -0.149$.[12]

As in several related studies, markup data comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008 (https://www.census.gov/retail). At the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target is for the gross margin is 1.3, right in the middle of these data. A gross margin of 1.3 implies a markup 1.39 (as in

[12] In the annual data $\mathbb{E}i = 3.83\%$, $L(\mathbb{E}i) = 0.257$ and $\varepsilon(\mathbb{E}i) = -0.105$. Using this instead of quarterly data does not affect the results too much. We also considered truncating the sample in 2004, to better match the price-change sample, and to eliminate the recent financial crisis, but again this did not affect the results much.

Bethune et al. 2014). However, the exact value for this target does not matter very much over a reasonable range, similar to the findings in Aruoba et al. (2011) in a different context (they use bargaining, not price posting).

On the fraction of transactions using money and credit, there are various micro data sources. First, in terms of concept, we follow the literature that interprets monetary transactions broadly to include cash, check and debit card purchases. As a rationale, first, checks and debit cards use demand deposits, which very much like currency are liquid and pay basically no interest, and it does not matter for the theory whether one's money is in one's pocket or bank account. Second, for us a key feature of credit is that it allows one to pay for BJ goods by working in the *next* AD market, while cash, check and debit purchases require working in the *previous* market, and this matters a lot when transaction are random, as they are here. Third, this notion of money in the micro data is consistent with our use of M1J in the macro data.

Older calibrations of business cycle models (see Cooley 1995, chapter 7) use a target of 16% for credit purchases. More recent information includes Klee (2008), where in grocery-store scanner data from 2001, credit cards account for 11.6% of transactions. In 2012 Boston Fed data discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 21.6% of purchases in the survey and 17.3% in the diary sample. Differences from Klee are due to the fact that the Boston Fed numbers are not just for groceries, and they are more recent (changes in payment methods over time are discussed in Schuh and Briglevics 2014). In Bank of Canada data discussed by Arango and Welte (2012), in 2009 the fraction of credit card purchases is 19.3%. Averaging between the Boston Fed and Bank of Canada numbers, we target 20% credit transactions.[13]

[13]To be clear, these numbers are for transactions by volume, not value. In Bank of Canada data, the fraction of credit card purchases by value is 40.1% , about double the fraction by volume, since cash gets used for smaller transactions consistent with theory:

Our tie-breaking rule σ is set to match average price duration, different from Head et al. (2012), where the comparable parameter is calibrate jointly with the curvature of $u(y)$ to match the empirical price-change distribution, and then they examine what the model predicts for duration. We prefer to calibrate σ to duration and evaluate model performance by it's ability to match the price-change distribution, which arguably provides more discipline, but it turns out not to affect the results very much – since the model fits well, it does not matter dramatically if one matches duration and checks the distribution, or vice versa. We use Klenow and Kryvtsov's (2008) price-change data, where the average duration of a price ranges between 6.8 and 10.4 months, depending on details, with an average of 8.6. We benchmark 8.6, but also report results with alternatives, since there is variability across studies, e.g., Nakamura and Steinsson (2008) report durations between 8 and 11 months. We also target the average price change in absolute value, which is 11% in Klenow-Kryvtsov data, well above inflation because there are many negative changes.

5.2 Findings

Since the price data are monthly, we choose the period in the model to be a month, and aggregate the model-generated money demand to a quarterly frequency in order to compare it to the Lucas-Nicolini series. The results of the calibration are in Table 1. Points to note include the result that the transaction cost δ is very low and u quite high, which is needed to match the fraction of credit transactions and average money demand. The value of c is primarily driven by the markup. The probability of sampling one price is $\alpha_1 = 0.15$, and the probability of sampling two is $\alpha_2 = 0.16$. As usual in

"Cash [accounts] for 76 per cent of all transactions below $15, and for 49 per cent of those in the $15 to $25 dollar range ... whereas credit cards clearly dominate payments above $50" (Arango and Welte 2012). However, in Boston Fed data the fraction by value is 16.4%, about the same by volume. There is no consensus as to why American and Canadian data differ on transactions by value; in any case we use volume, where they basically agree.

search-based models, trading probabilities adjust with frequency, and since we are using a monthly model the α's are fairly low. The probability of sticking to the same price for a firm that is indifferent is high, $\sigma = 0.90$.

	BJ utility	BJ cost	credit cost	$pr(n=1)$	$pr(n=1)$	tie-break
Var	u	c	δ	α_1	α_2	σ
Val	28.76	16.54	0.14	0.15	0.16	0.90

Table 1: Baseline Calibration

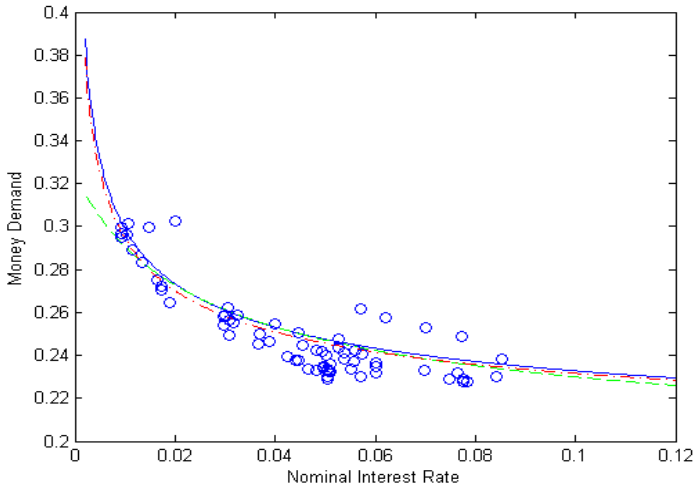


Figure 4: Money Demand for Different Specifications

In Figure 4 the model's predicted money demand is shown the solid curve (the other curves are for alternative specifications in Section 6.2 but they are all very similar). Clearly the fit is good, which is important for evaluating the model from a macro perspective. In terms of micro data, the model's price change distribution is shown in Figure 5 together with its empirical counterpart from Klenow and Kryvtsov (2008). The model captures the overall shape as well as some details deemed important in the literature. The average absolute price change is 11%, which we hit by calibration. The fraction of small price changes (less than 5%) is 44% in the data and 33% in the model, not wildly

different.[14] The fraction of negative changes is 37% in the data and 43% in the model, not far off. As is well understood, these observations are difficult to match simultaneously with simple menu cost or Calvo models, although Midrigan (2011) manages in a generalized version. We do quite well in a rudimentary search model without adding complications, idiosyncratic shocks, etc. A shifting F_t combined with our simple tie-breaking rule simultaneously generates large average, many small, and many negative p adjustments.

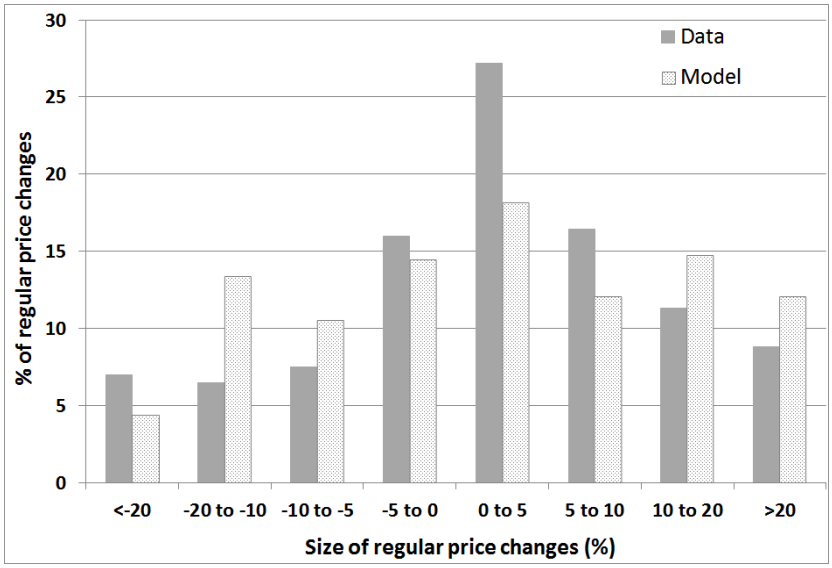


Figure 5: Distribution of Price Changes

Figure 6 shows the hazard – the probability of changing p as a function of the duration since the last change – over 18 months, from the model and from the data in Nakamura and Steinsson (2008).[15] The model does not generate enough action in the first few months, clearly, but what we emphasize is our hazard slopes downward, something Nakamura and Steinsson argue does not

[14] Eichenbaum et al. (2014) find the fraction of price changes below 5% is 5.2% after correcting for measurement error, and suggest that other evidence on small price changes is partly due to not taking this into account. For the purpose of this exercise, we take the Klenow-Kryvtsov numbers at face value.

[15] We also looked at the hazard in the Klenow-Kryvtsov data, but emphasize Nakamura-Steinsson on this point but they argue that it is important.

happen in some other models. Now, one should not expect to explain every detail, since there is probably a lot going on in the market that is not in the model – e.g., experimentation by sellers trying to learn about demand conditions. Still, it is interesting that the hazard decreases at least for a while, before eventually increasing, because with continuing inflation any p falls off the support $\mathcal{F}_t$ in the long run. However, at 10 years, the hazard is only up to 14%, which means some prices can be sticky for a very long time, even as long as Carlton's (1986) chemicals and Cecchetti's (1986) mucilaginous magazines mentioned in Section 2.

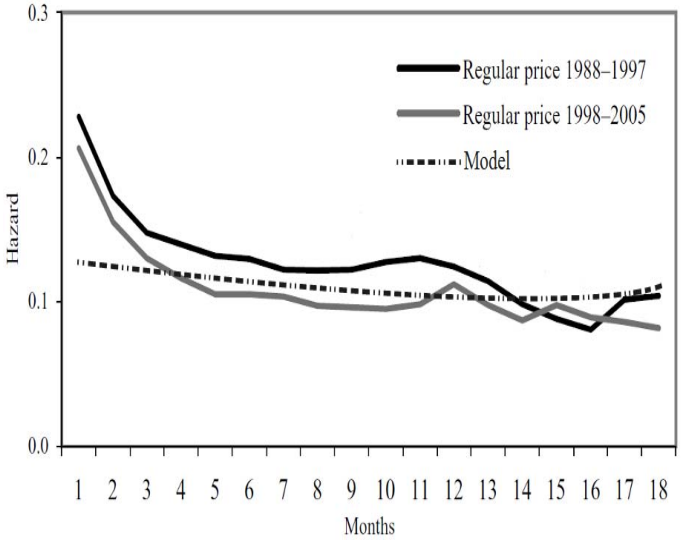


Figure 6: Price Change Hazard over 18 Months

Because there is a range of durations found in the empirical literature, Figure 7 shows results of matching average durations of 1, 4, and 18 months, as well as choosing σ to minimize the sum of squared errors between the model-predicted and empirical distributions, which implies a duration of 15.8 months. The overall shape of the distribution does not change a lot, although the fraction of small changes goes up with duration and the fraction of negative changes goes down.

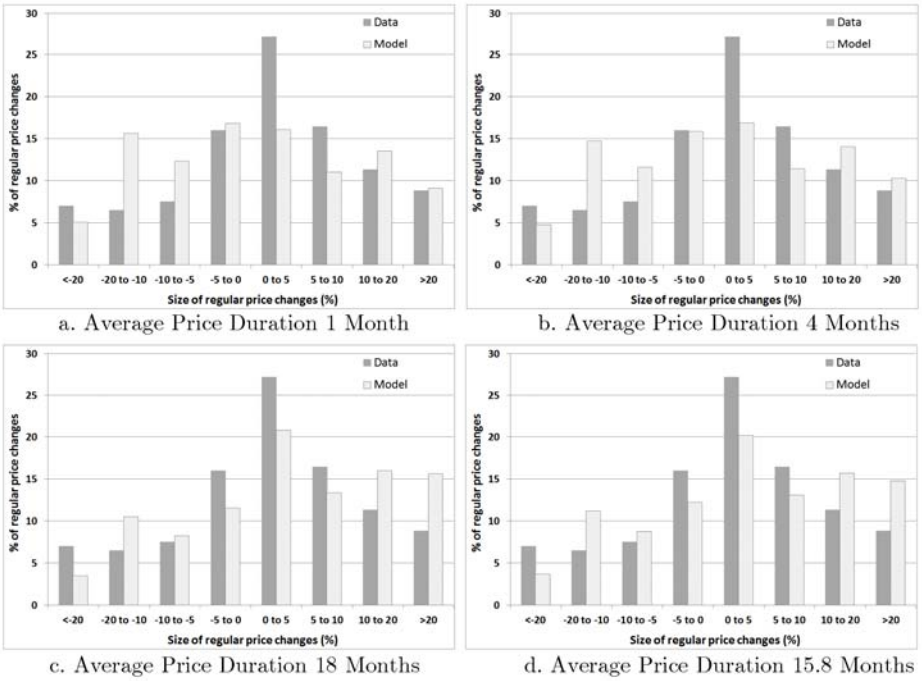


Figure 7: Distribution of Price Changes for Different σ

We conclude from all this that, while the model misses some details, it generally performs well. Certainly it would be hard to say there is anything especially puzzling about the behavior of price changes – it is pretty much what one should expect from simple search theory. It would be even harder to argue there is anything informative about menu costs or Calvo parameters in these data, given the outcomes generated by the model without these ingredients. Also, we emphasize that there is quantitative discipline imposed here by matching both macro money demand observations as well as micro evidence on consumers' use of cash and credit.

6 Extensions

Two alternative specifications are entertained. The first is the case where buyers must pay to participate in the BJ market. The second is a version with a variable, rather than fixed, cost of credit.

6.1 Participation

With $k = 0$ all households enter the BJ market, $b = \bar{b}$. Hence the arrival rates $\alpha_n\left(\bar{b}\right)$ are fixed exogenously, and since y is indivisible, BJ output is fixed, too. Since AD output is always fixed by $U'(x) = 1$, changes in the level of M, or in the inflation and nominal interest rates π and i, have no impact on output. In this sense monetary policy is neutral, by design. The specification can be amended in various ways, however, to make output endogenous. A simple way is to allow $k > 0$, so the measure b of BJ buyers is determined by having arrival rates adjust until the marginal participant is just indifferent, similar to Pissarides' (2000). Then $\exists \bar{k} > 0$ such that $b < \bar{b}$ iff $k > \bar{k}$.[16]

This allows us to make some observations that we consider interesting. First, note that if prices were sticky due to Calvo pricing or Mankiw menu costs, e.g., a one-time unanticipated increase in M can have nonzero real effects. The reason is that at least some firms could not (with Calvo pricing) or would not (with Mankiw pricing) adjust p, and so the nominal distribution $F(p)$ may not change by enough to keep the same real distribution $G(q)$. With a jump in M, one should expect the real distribution to turn in favor of buyers, increasing b – a shopping spree – and thus increasing production. By contrast, with endogenous sticky prices, as modeled here, jumps in M have no impact on $G(q)$ and hence no effect on entry or output. A policy pundit in our economy seeing stickiness in data, with only a fraction of sellers adjusting p each period, could conclude that a change in M would have real effects. That would be a mistake. And note that recognizing this as just another example of the Lucas critique does not make it any less interesting.

[16] Before proceeding we verify $k = 0 \implies b = \bar{b}$. Consider first NME, $z = 0$. Given $\delta < u - c$, buyers' BJ payoff is $V > 0$ and $b = \bar{b}$ if $k = 0$. Generally V falls with b, due to increased market tightness, but we still get $V > 0$. In MME there is a new cost iz of participating in BJ, but buyers can choose $z = 0$ to avoid this, so again $b = \bar{b}$. Allowing $z \neq 0$ can only increase $V - iz$, so again $b = \bar{b}$. Now z and $V - iz$ fall with i, but still $V - iz > 0$ and $b = \bar{b}$. The difference with k is that it is a fixed cost.

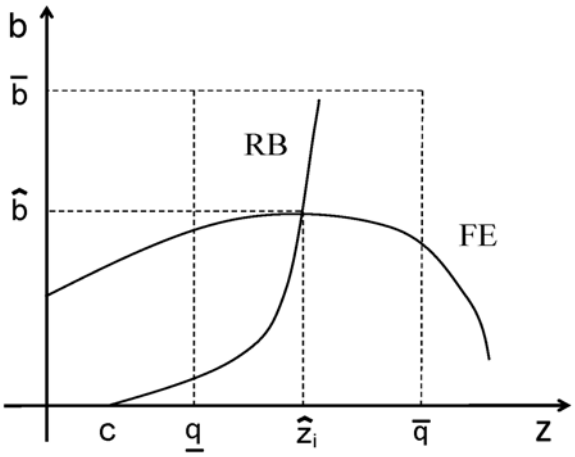


Figure 8: Real Balances and Free Entry

With endogenous entry, M is irrelevant but π and i are not: in the vernacular, money is neutral but not superneutral. To see this, consider parameterizing the search process as in Head et al. (2012): a buyer in BJ attempts to solicit 2 price quotes, and succeeds in each try independently with probability $\lambda(b)$, where $\lambda(0) = 1$, $\lambda(\bar{b}) = 0$, $\lambda'(b) < 0$ and $\lambda''(b) > 0$. Then $\alpha_1(b) = 2\lambda(b)[1 - \lambda(b)]$ and $\alpha_2(b) = \lambda(b)^2$. Inserting these into (22), we get

$$\hat{z}_i = c + \left\{2\delta \left[1 - \lambda\left(b\right)\right]^2 \left(u - \delta - c\right)^2\right\}^{1/3} i^{-1/3}. \tag{29}$$

This defines a relation between $\hat{z}_i$ and b called the RB (real balance) curve. Similarly, (14) defines a relation called the FE (free entry) curve,

$$\kappa = \lambda(b)^2 \left(u - \delta - c\right) - \frac{\delta \left[1 - \lambda(b)\right]^2 \left(u - \delta - c\right)^2}{\left(\hat{z}_i - c\right)^2} + \delta - i\hat{z}_i. \tag{30}$$

As shown in Figure 8, RB is increasing and convex, with $\hat{z}_i = c$ at $b = 0$, while FE is upward sloping to the left and downward sloping to the right of RB, with $b \in (0, \bar{b})$ at $z = 0$. Hence there is a unique intersection of FE and RB.[17]

[17]For this to constitute a MME, we need to verify $\phi_t \underline{p}_t < \hat{z}_i < \phi_t \bar{p}_t$), which is equivalent to having the FE curve below the RB curve at $z = \phi_t \bar{p}_t$ and above it at $z = \phi_t \underline{p}_t$, and we need to verify $V(\hat{z}_i) - i\hat{z}_i \geq V(0)$. We take these for granted for the purpose of this discussion, but check them in the quantitative work below.

Consider an increase in i, which ultimately comes from higher inflation and monetary expansion, π. This shifts both curves toward the origin, reducing b and hence output. Monetary policy matters, although this has nothing to do with nominal rigidities – it is in fact due to higher i taxing households' participation in decentralized exchange.

To make another point, consider calibrating parameters as in Section 5, with one exception: Since households now choose to enter BJ, instead of setting α_1 and α_2 directly, we set the entry cost κ, which determines b and hence the α's according to some matching function – e.g., $\lambda(b) = 1/(1+b)$, but this is not actually important since all that matters is the implied λ (it could be relevant for other exercises, e.g., changing policy). Since lose one parameter after dropping the α's and adding κ, we drop one target. Suppose first that we drop the average price change. Then the results are not good. While money demand still fits well – see Figure 4, where the curves are virtually indistinguishable – the model-generated price change distribution shown in Figure 9 looks nothing like the data.

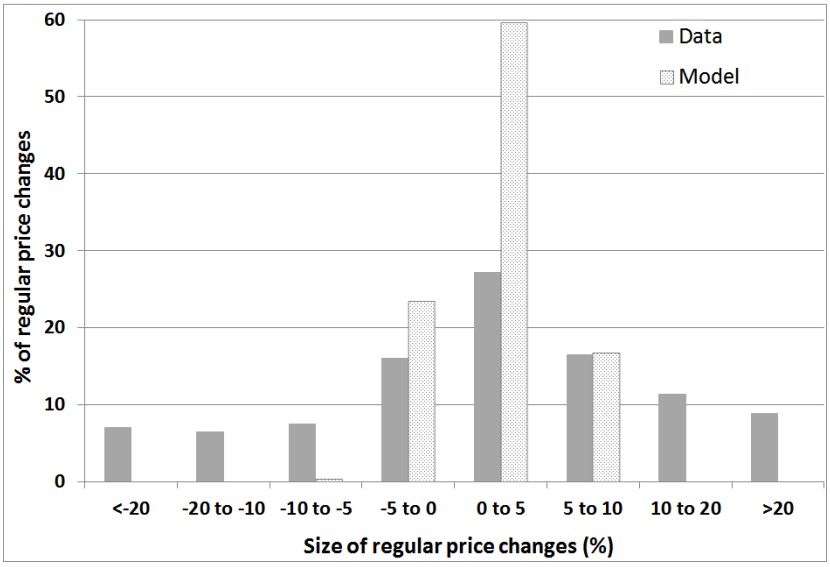


Figure 9: Distribution of Price Changes: Endogenous Participation

If instead we reinstate the average price change as a target and drop average real balances, Figure 4 would look fine but money demand would be way off. Thus we can either account for money demand. or the price change distribution, but not both. This illustrates that it is not trivial to simultaneously match the data. While the model in Section 5 can manage, an ostensibly reasonable special case cannot, something that did not come up in Head et al. (2012) because that exercise did not look at money demand. To be clear, the problem is not free entry per se, as one can in principle come up with a matching technology and entry cost that reproduce α_1 and α_2 in the earlier calibration. The problem in this example comes from having fewer parameters. It is not surprising – of course – that we do worse fewer parameters, but the point is that it is not automatic that any model can simultaneously account for the facts. In any case, perhaps a more important message from this extension is to show that observations of sticky prices can lead policy makers astray.

6.2 Variable Credit Cost

Now consider a variable cost of credit, say τ, so that on debt d carried into the AD market, τd replaces the fixed cost δ. There are several reasons for this. First, it is a basic check on robustness. Second, one interpretation (not the only one) is that τ is a proportional sales tax that can be avoided by using cash. Third, a variable cost is in some respects easier. Fourth, it delivers a different but still nice money demand function. Finally, it avoids a technical problem that we waited until now to mention. As is well known from monetary theory, or economic models, more generally, with nonconvexities, it can sometimes be desirable to use *lotteries* (e.g., Berentsen et al. 2002). Thus, a seller may want to post this: "you can get my good for sure if you pay p using money or credit; if you pay $p' < p$ then you get my good with probability $\gamma = \gamma(p')$." If, e.g. a buyer with $m = p - \varepsilon$ meets a seller posting p, in the model analyzed above,

the buyer pays $p - \varepsilon$ in cash, ε in credit, and δ as a fixed cost. But it can be preferable for the pair to trade only using cash, avoid the credit and hence the fixed cost, and have the seller deliver the good with probability $\gamma < 1$.[18]

Now, one could try to argue that lotteries are infeasible or unrealistic or – but that would be awkward, for us, because ruling out randomized exchange might be judged uncomfortably close to the linear menu restriction in the divisible goods models criticized in the Introduction. We chose to impose a fixed cost in our benchmark model, because fixed costs of credit have a long history in the literature, including many papers cited in fn. 5, but we also wanted to avoid the complication of introducing lotteries. While we think that setup is interesting, in case it is considered problematic, this variable cost version has no such technical problem.

The price distribution emerging from the firms' problem is similar to the baseline model. For households, as usual they never choose $z \geq u$, and $z < u$ now implies the surplus from a transaction at real price q is $u - q - (q - z)\tau$. This leads to an upper bound $\bar{q} = (u + z\delta)/(1 + \tau) > z$, and as before we can back out the lower bound

$$\underline{q} = \frac{\alpha_1 (u + z\delta) + 2\alpha_2 c (1 + \tau)}{(\alpha_1 + 2\alpha_2)(1 + \tau)}.$$

Conveniently, the objective function $O_i(z) = V(z) - (1 + i)z$ is now differentiable, even at $\bar{q}$ and $\underline{q}$, with $V'(z) = 1 + (\alpha_1 + \alpha_2)\tau[1 - J(z)]$ and hence $V'' < 0$. Therefore, $O_i(z)$ is smooth, strictly concave in $(\underline{q}, \bar{q})$, and otherwise linear with $O_i'(z) = -i \ \forall z > \bar{q}$ and $O_i'(z) = (\alpha_1 + \alpha_2)\tau - i \ \forall z < \underline{q}$. As shown in Figure 10, there are two types of equilibria, NME and MME – intuitively, there cannot be a PME because buyers always want to use at least some credit.

If $i > (\alpha_1 + \alpha_2)\tau$, holding money is too costly, and we get a NME. If $i < (\alpha_1 + \alpha_2)\tau$, we get a unique MME where the choice of real balances

[18] To be clear, it is not the indivisibility of the good that creates the problem here, it is the nonconvexity in the transaction cost.

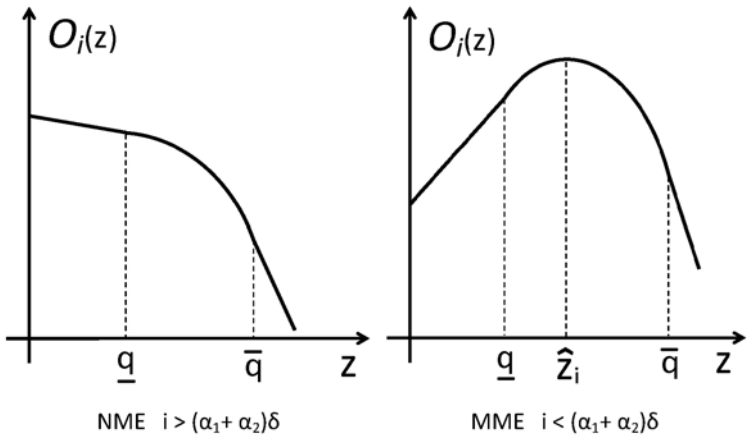


Figure 10: Different Types of Equilibrium: Variable Cost

satisfies the FOC $i = \tau(\alpha_1 + \alpha_2)[1 - J(\hat{z}_i)]$. Inserting $J(z)$, we get the inverse money demand function

$$i = \frac{\alpha_1^2 \tau \left(u - \hat{z}_i\right) \left[\left(1 + 2\delta\right) \left(\hat{z}_i - c\right) + u - c\right]}{4\alpha_2 \left(1 + \tau\right)^2 \left(\hat{z}_i - c\right)^2}, \tag{31}$$

a quadratic in $\hat{z}_i$ for a given i. This is different from the model with a fixed cost, but equally nice.[19] One can check there is a unique $\hat{z}_i$ satisfying (31). We know that $\hat{z}_i < \bar{q}$ iff $\hat{z}_i < u$, and still need to check $\hat{z}_i > \underline{q}$, which one can check holds iff $i < \tau(\alpha_1 + \alpha_2)$. This proves the following result.

Proposition 4 *With a variable cost of credit τ, there exists a unique MME iff $0 < i < \tau(\alpha_1 + \alpha_2)$.*

The parameters are calibrated, with τ with replacing δ, as in Section 5. As shown by the dashed curve in Figure 4, this model also matches money demand

[19] To solve this for $\hat{z}_i$, first rewrite (31) as a quadratic form in $(\hat{z}_i - c)$:

$$\left[1 + 2\tau + 4\alpha_2 \left(1 + \tau\right)^2 i/\alpha_1^2 \tau\right] \left(\hat{z}_i - c\right)^2 - 2\tau \left(u - c\right) \left(\hat{z}_i - c\right) - \left(u - c\right)^2 = 0$$

Then, solve for

$$\hat{z}_i = \frac{\left(u - c\right) \left[\tau + \left(1 + \tau\right) \sqrt{1 + 4\alpha_2 i/\alpha_1^2 \tau}\right]}{1 + 2\tau + 4\alpha_2 \left(1 + \tau\right)^2 i/\alpha_1^2 \tau} + c,$$

which is like a classic square-root rule except i also appears in the denominator.

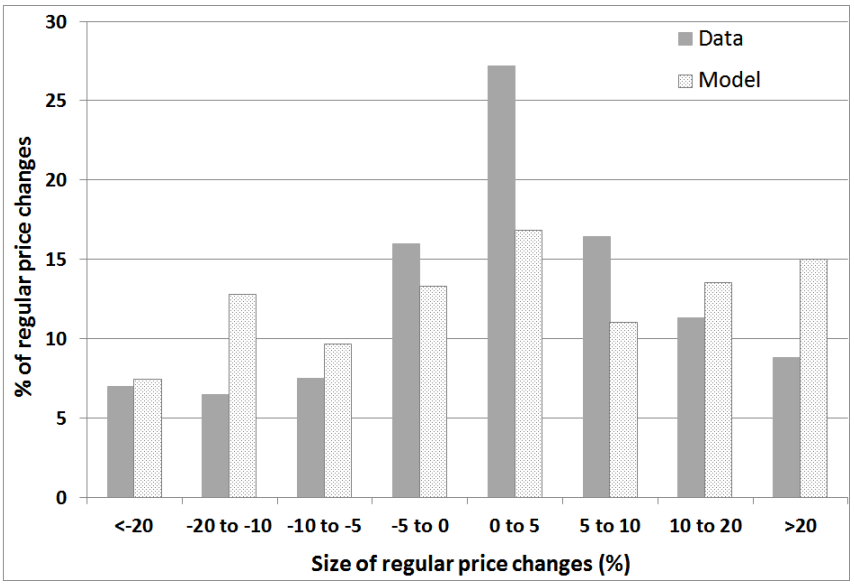


Figure 11: Distribution of Price Changes: Variable Cost

well, although it predicts somewhat lower z at small i. The distribution of price changes shown in Figure 11 is again similar to the empirical distribution, although based on casual inspection it is perhaps slightly worse than the other model: it predicts an average price change of 12.4%, larger than 11% in the data and in the benchmark model; it has 30.3% of price changes less than 5%, slightly worse than the benchmark model; and it has 43.4% of price changes negative, slightly better than that model and identical to the data. Overall, the interpretation of the results in Section 5 survives.

7 Conclusion

A contribution of search theorists in general is to show how some observations that appear anomalous from the perspective of standard theory can be readily understood once frictions are modeled explicitly. A leading example is price dispersion, with deviations from the law of one price emerging naturally in search models. It turns out something similar is true for price rigidity. It is

a huge puzzle for standard theory when sellers let their real prices vary in arbitrary ways by not adjusting nominal prices in response to monetary or real shocks. Yet once one understands how frictions lead to dispersion the puzzle goes away: letting one's real price vary in arbitrary ways over some range does not affect one's profit. This paper exploits that insight by studying a general equilibrium monetary framework where sticky nominal prices emerge endogenously.

Constructing rigorous monetary models in the first place is nontrivial – indeed, money is another classic anomaly for standard theory that can be better understood once frictions are introduced carefully. Our approach was based on lessons from the literature. We used divisible money and indivisible goods, then added costly credit, to avoid technical problems as well as to match the facts concerning payment methods. This gave rise to nice money demand functions that fit the macro data well. Calibrated versions of the model did a reasonable job emulating the empirical price-change distribution, generating large average changes, many small changes, many negative changes, and a decreasing hazard, while staying consistent with macro and micro evidence on money and credit. Although we did not emphasize it, another desirable feature is that pricing behavior in the model obviously depends on inflation, which can be further explored in future research. Additionally, the paper showed by example how money can be neutral even when prices are sticky, and provided another example where policy makers can be led astray by certain observations, in this case observations of apparent nominal rigidity.

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