Exchange Rates and UIP Violations at Short and Long Horizons

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Abstract

The much-studied Uncovered Interest Rate Parity (UIP) puzzle, the observation that exchange rates do not adjust sufficiently to offset interest rate differentials, is more complicated than commonly understood. It changes nature with the horizon. I confirm existing short-run evidence that high interest rate currencies depreciate less than predicted by the interest rate differential, but, building on Engel (2012), at longer horizons (4 to 7 years) I find a reverse puzzle: high interest rate currencies depreciate too much. Interestingly, the long-horizon excess depreciation leads exchange rates to converge to the UIP benchmark over the long-run. To address the changing nature of the puzzle, I propose a novel model, based on the mechanism of bond convenience yields, that can explain both the short and the long horizon UIP violations. I also provide direct empirical evidence that supports the mechanism.

JEL Codes: F31, F41, F42, E43, E52, E63

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*Please find the latest version of the paper at http://sites.duke.edu/rosenvalchev/research

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1 Introduction

The Uncovered Interest Rate Parity (UIP) condition, which equates the expected returns on domestic and foreign bonds, is central to exchange rate determination in standard open economy models. It is directly implied by risk-neutrality, and can also be derived by log-linearizing the standard no-arbitrage pricing conditions that obtain in a large class of equilibrium models. The excess return of foreign bonds over home bonds consists of two components: the interest rate difference between the two countries and the exchange rate change over the investment period. Thus, equating expected returns across countries requires that, on average, the exchange rates of high interest rate countries depreciate and offset potential gains that arise from interest rate differentials.

Despite the fundamental role this condition plays in theoretical models, its failure in the data is well established. Numerous papers have documented that currencies fail to depreciate sufficiently to offset interest rate differentials, which opens up profit opportunities (e.g. the carry trade) and violates the UIP condition (see the surveys by Hodrick (1987), Lewis (1995), Engel (1996, 2013)).[1] In fact, most estimates cannot reject the hypothesis that exchange rates follow a random walk, implying that, on average, they do not offset any of the interest rate differential.[2] Overall, at the shorter horizons emphasized by the literature, the empirical evidence has consistently characterized the UIP puzzle as insufficient currency depreciation in response to high domestic interest rates.[3]

In this paper, inspired by the essential empirical insight in Engel (2012), I show that the puzzle is more complicated than commonly understood; I find that UIP violations change nature and direction with the horizon. I confirm the standard result that currencies fail to depreciate sufficiently in response to high interest rate differentials at short horizons (up to 3 years). However, at the longer horizons of 4 to 7 years, I find the reverse puzzle: high-interest rate currencies depreciate *too much*. To address the puzzle in its full complexity, I propose a novel model in which bonds act as convenience assets that help facilitate transactions (similar to money), and I show that the model can closely match both the short and the long horizon UIP violations. In addition, I provide direct empirical evidence in support of the mechanism, by verifying two of its key implications in the data.

[1] See also Bilson (1981), Fama (1984), Froot and Thaler (1990), Canova and Ito (1991), Canova and Marrinan (1993), Bekaert and Hodrick (1993), Backus et al. (1993), Hai et al. (1997), Burnside (2013) and others.

[2] On the one hand, most UIP regressions, cannot reject the hypothesis that exchange rate changes are unpredictable. On the other, a large related literature, following Meese and Rogoff (1983), finds that it is exceedingly difficult to beat the random walk forecast out of sample (see the survey by Rossi (2013))

[3] The UIP puzzle, also known as the Forward Premium puzzle, is separate from the equity risk premium puzzle. The puzzle with currencies is about the time variation in excess returns and in their relationship with interest rate differentials. A constant risk premium, for example, would not resolve the puzzle.

In the first part of the paper, I test the UIP condition at different horizons by exploiting the implication that excess returns on one-month foreign bonds, over one-month home bonds, must be zero in expectation and, hence, unforecastable at all horizons. I examine this with a series of direct predictive regressions, where I regress the one-month excess return k periods into the future on the current interest rate differential, using a panel of 18 OECD currencies. I find that the regression coefficients are significantly different from zero, and hence UIP is violated, at horizons of up to 7 years. I confirm the usual finding that the coefficients are negative, which implies that high interest rate currencies do not depreciate sufficiently, at horizons of up to 3 years. However, I find that the coefficients become positive at horizons of 4 to 7 years, implying that high interest rate currencies eventually start depreciating too much, relative to UIP. This is also a violation of UIP but constitutes the reverse puzzle.

Moreover, I find that the sum of positive UIP violations is roughly equal to the sum of negative violations. This leads to the interesting result that the excess depreciation at longer horizons offsets the initial insufficient depreciation, and thus, exchange rates converge to the UIP benchmark in the long run. This result is qualitatively consistent with and helps provide a new interpretation to previous studies that have looked at the excess returns on long-term bonds (5+ years) and have found some support for UIP in the long run.[4] My findings show that the UIP puzzle is not a short-run phenomenon that averages out over the long-run, but rather it changes its nature. Exchange rate movements violate the UIP condition at both short and long horizons, but the direction of the violations changes, and the effects roughly cancel out in the long-run. This has interesting, non-linear implications about the underlying exchange rate dynamics, and leads me to reject the random walk hypothesis as well.[5]

The second part of the paper develops a novel two-country model of time-varying bond convenience yields and shows that it can closely match both the short-horizon and long-horizon UIP deviations. The key feature of the model is that government bonds offer both financial returns and convenience, or liquidity, benefits. Following the literature on bond convenience yields (e.g. Krishnamurthy and Vissing-Jorgensen (2012)), I refer to the convenience of a bond as the non-pecuniary benefit arising from the fact that safe and liquid government bonds can act as a substitute for money and help facilitate transactions. The convenience yield is the amount of interest investors are willing to forego in exchange for the convenience benefit.

In the model, excess currency returns arise as a compensation for differences in convenience yields across countries, and UIP deviations are driven by endogenous time variation

[4]See Flood and Taylor (1996), Chinn and Meredith (2005), Chinn (2006), Chinn and Quayyum (2012). There is an analytical link between the two sets of results under risk-neutrality, but not necessarily otherwise.

[5]The results in this paper refer to *in-sample* analysis, and no claims are made about out-of-sample predictability. Incorporating the results and insights in an out-of-sample analysis is a future research project.

in the convenience yield differential. A decrease in the convenience value of home bonds, relative to foreign bonds, leads to a corresponding, compensating increase in the excess financial return on home bonds, over foreign bonds. This is achieved through excess return on the home currency, which constitutes a violation of the standard UIP condition. Moreover, when the home convenience yield falls, the home interest rate tends to rise because investors require a higher financial compensation to hold the supply of home government debt, as its convenience benefit has decreased. This generates the classic UIP puzzle relationship of high-interest rate currencies earning excess returns.

The switch in the sign of UIP violations at longer horizons is driven by the interaction between monetary and fiscal policy. The dynamics of the convenience yield differential are closely linked to the relative supply of government debt, and hence the dynamics of UIP violations are determined by the joint dynamics of interest rates and debt.[6] In particular, I am able to show that when monetary policy is independent of fiscal considerations and tax policy is persistent, debt has cyclical, complex root dynamics that are also imparted on the convenience yield differential. This generates a cyclical profile of UIP deviations that are negative at short horizons, but turn positive at longer horizons. If tax policy reacts quickly to debt levels, or if the central bank helps through inflating debt away, the UIP violations, especially at longer horizons, decrease, and we could have a situation in which UIP violations are negative at all horizons.

As a first step, I analyze the mechanism in a stylized model that allows me to analytically characterize the main results and helps illustrate the key intuition. Next, to study the mechanism's quantitative implications, I incorporate it in a benchmark, two-country open economy model. I calibrate the parameters with standard values from the literature, independently of the implied UIP violations, and show that the model closely matches the estimated UIP deviations at both short and long horizons. Lastly, I provide direct empirical support for the mechanism by verifying two of its key implications in the data. First, I show that excess currency returns are indeed closely related to and forecastable by the level of government debt. Second, I document that currencies with a lower degree of capital controls, a common proxy for central bank independence, exhibit stronger UIP violations, especially at longer horizons, as predicted by the monetary-fiscal interplay in the model.[7]

[6] The relationship between the supply of debt and the convenience yield is also emphasized in the previous literature on bond convenience yields.

[7] I use capital controls as a proxy for central bank independence due to the lack of direct and comparable measurements that span the time period and the countries in my data set. On the other hand, a number of papers find that countries with (i) greater central bank independence and/or (ii) lesser dependence on seignorage and inflation tax revenues are associated with lower degrees of capital controls (Alesina and Tabellini (1989), Drazen (1989), Alesina and Grilli (1994), Grilli and Milesi-Ferretti (1995), Leblang (1997), Quinn and Inclan (1997), Bai and Wei (2000)).

My empirical analysis is inspired by and related to recent work on the relationship between the level of the exchange rate and the real interest rate by Engel (2012). One of Engel's findings is that when a country's real interest rate is above its mean, its currency tends to earn positive excess returns in the short-run, and negative excess returns in the longer-run. My empirical results, which are based on predicting returns using nominal interest rates and also exploit the cross-sectional variation in the data, echo his.[8] Additionally, Engel (2012) estimates that the expected cumulative excess returns of high-interest rate currencies, at very long horizons, are negative, rather than positive, as the classic UIP puzzle relation would dictate. I find a different result in my panel analysis using the nominal interest differential as the predictor: cumulative returns at the longest horizons are roughly zero and hence appear to be consistent with UIP.

The theoretical mechanism is novel to the literature on the UIP puzzle, which largely turns to one of two explanations: time-varying risk (Bekaert (1996), Farhi and Gabaix (2008), Alvarez et al. (2009), Verdelhan (2010), Bansal and Shaliastovich (2012), Colacito and Croce (2013) among others), and deviations from rational expectations (e.g. Gourinchas and Tornell (2004), Bacchetta and Van Wincoop (2010), Burnside et al. (2011), Ilut (2012)). In addition, I analyze the changing nature of UIP violations at short, medium and long-horizons, whereas the literature has focused on explaining the short-run negative UIP violations. The new UIP mechanism is also complementary to existing ones and incorporating it in a model together with some of the previous approaches is a promising direction for future work.[9,10,11]

A number of papers have quantified the convenience yield in the data and documented its important role in the determination of equilibrium bond prices (for example Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014)). A related theoretical literature has explored bond convenience yields as a possible explanation for asset pricing puzzles such as the equity risk-

[8] In a new version of his paper, Engel (2014) considers some results based on nominal interest rate differentials as well. An advantage of working with nominal interest rates is that no model (such as a VAR) is needed in order to estimate expected inflation and construct ex-ante real interest rates. This allows me to utilize the method of local projections (Jorda (2005)), which is well-suited for examining long-horizon dynamics.

[9] For example, Lustig et al. (2013) find that transitory risk accounts for the great majority of short-horizon carry trade profits. Thus, building a model that incorporates both time varying risk and convenience yields could potentially deliver even stronger negative UIP coefficients at short horizons while preserving the positive UIP violations at longer horizons.

[10] A number of recent papers find that traditional risk factors cannot explain currency returns (Burnside (2007), Burnside et al. (2010), Burnside (2010, 2011), Menkhoff et al. (2012b)). My model suggests that time-varying convenience yields could have acted as omitted variables in such analyses and incorporating them in future studies could help properly quantify the effects of both risk and convenience factors.

[11] Hassan and Mano (2013) find that a significant portion of carry trade profits is not due to capturing time-variation in excess returns, but rather due to persistent differences in excess returns across currencies. This can also be rationalized by the model, given steady state differences in convenience yields.

premium, the low risk-free rate and the term premium (e.g. Bansal and Coleman II (1996), Bansal et al. (2011), Lagos (2010, 2011)). I extend the theoretical analysis of convenience yields by introducing it to an open economy setting, applying it to the UIP puzzle, and by studying the implications of time-varying convenience yield differentials.

Section 2 presents the main empirical results, Section 3 briefly introduces convenience yields and then Section 4 lays out and analyzes the analytical model, while Section 5 presents the quantitative model. Section 6 provides direct empirical evidence in support of the mechanism and Section 7 concludes.

2 Uncovered Interest Parity

2.1 The UIP Condition in Economic Models

This section gives a brief overview of the Uncovered Interest Parity condition and highlights the important role it plays in the determination of equilibrium exchange rates in standard open economy models.

To fix ideas, I define S_t to be the exchange rate, in terms of home currency per one unit of foreign currency (e.g. 1.25 USD per EUR), and i_t and i_t^* as the nominal interest rates on default-free bonds at home and abroad. For ease of exposition, I will refer to the US dollar as the "home" currency and the Euro as the "foreign" currency. A \$1 investment in US bonds at time t offers a return of $1 + i_t$ dollars next period. The same \$1 invested in Euro denominated bonds would earn $\frac{S_{t+1}}{S_t}(1 + i_t^*)$ dollars next period. That is, we first need to exchange this one dollar for Euros and obtain $\frac{1}{S_t}$ EUR in return. Investing this amount of Euros earns a gross interest rate of $1 + i_t^*$ that next period can be exchanged back into dollars at the rate S_{t+1}, for a total return of $\frac{S_{t+1}}{S_t}(1 + i_t^*)$ dollars.

Assuming that the law of one price holds the fundamental theorem of asset pricing tells us that there exists a stochastic process M_{t+1}, usually referred to as the *stochastic discount factor*, such that[12]

$$E_t(M_{t+1}(1 + i_t)) = 1 \tag{1}$$

$$E_t(M_{t+1}\frac{S_{t+1}}{S_t}(1 + i_t^*)) = 1. \tag{2}$$

[12]The law of one price states that the price of a linear combination of assets must be equal to that same linear combination of the underlying assets' prices. In other words, any unique payoff, whether it is the payoff of a particular asset or formed by constructing a portfolio of distinct assets, must have a unique price.

For example, in standard representative agent models with separatively additive utility over consumption the stochastic discount factor is equal to the marginal rate of substitution (adjusted for inflation because the payoffs considered here are nominal): $M_{t+1} = \frac{U'(C_{t+1})}{U'(C_t)} \frac{1}{\Pi_{t+1}}$.

To obtain the Uncovered Interest Parity condition log-linearize the two equations, subtract them from one another and re-arrange to arrive at

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = 0$$

where lower case letters represent variables in logs and I have used the approximation $i_t \approx \ln(1 + i_t)$.[13]

The condition equates, up to a first order log-approximation, the expected return on foreign bonds, $E_t(s_{t+1} - s_t + i_t^*)$, to the expected return on the home bond, i_t. This restricts the joint dynamics of exchange rates and interest rates, and delivers strong implications for exchange rate behavior. In particular, the UIP condition states that exchange rates are expected to adjust in response to non-zero interest rate differentials and offset them. Thus, if for example the US interest rate is 1% higher than the Euro interest rate, i.e. $i_t - i_t^* = 0.01$, the UIP condition would imply that the Euro is expected to appreciate against the dollar by 1% as well, so that $E_t(s_{t+1} - s_t) = 0.01$. Hence, in expectation, there are no gains to be made by borrowing in one currency and investing in the other.

The limited set of assumptions underlying the UIP condition ensures that it obtains, at least up to a first order approximation, in a large class of models. It is a fundamental part of the overwhelming majority of open economy models and it puts important restrictions on the model-implied joint dynamics of exchange rates and interest rates. Given its theoretical importance, it is not surprising that there has been great interest in examining the condition's validity in the data, and I turn to empirical tests of the UIP condition next.

2.2 The UIP Puzzle

The failure of the UIP condition in the data is one of the longest standing and best documented puzzles in international finance. This section briefly reviews the results in the existing literature and confirms that the condition is similarly violated in the data set used by this paper. The literature documenting the empirical failure of UIP is vast and is still an active area of research, with numerous studies expanding on the seminal contributions by Bilson (1981) and Fama (1984). For excellent surveys, please see Hodrick (1987), Froot and Thaler

[13] The log-linearization is typically done around the symmetric steady state where $S_{t+1} = S_t = 1$ and $i_t = i_t^*$, because this allows us to express the condition in terms of the log-variables themselves. But the log-linearized condition holds for any arbitrary point of approximation.

(1990), Engel (1996, 2013).[14,15]

Examining the UIP condition in the data is typically done by testing whether any variable in the time t information set can help forecast the return on foreign bonds relative to home bonds. As is standard in the literature I will equivalently refer to the relative return on foreign to home bonds as "excess return on foreign bonds" and also as "excess currency return". I denote the one period excess return from time t to $t+1$ as λ_{t+1}:

$$\lambda_{t+1} \equiv s_{t+1} - s_t + i_t^* - i_t.$$

The UIP condition requires $E_t(\lambda_{t+1}) = 0$ and hence $\text{Cov}(\lambda_{t+1}, X_t) = 0$ for any variable X_t in the time t information set. In other words, there should be no variables known this period that can forecast the excess return that will realize next period. The bulk of the empirical work documenting the failure of the UIP condition comes down to showing that the current interest rate differential, $i_t - i_t^*$, can indeed forecast future excess returns.

The vast majority of the literature focuses on some version of the original regression specification estimated by Fama (1984):

$$\lambda_{t+1} = \alpha_0 + \beta_1(i_t - i_t^*) + \varepsilon_{t+1},$$

where typically the base or "home" currency is the USD and i_t is the US interest rate. Under the null hypothesis that the UIP condition holds we should obtain $\alpha_0 = \beta_1 = 0$ so that the average excess return is zero and not forecastable by current interest rates. Contrary to this, numerous papers have found that $\beta_1 < 0$ which implies that currencies which are experiencing high interest rates today are also expected to earn positive excess returns in the future.[16] These findings form the basis for the celebrated"UIP Puzzle".[17]

Next, I turn my attention to the UIP tests considered in this paper. For my empirical analysis I construct a data set of 18 major currencies, all advanced OECD countries, with the US dollar as the base currency and the data spans the period 01/01/1976 - 28/06/2013 at

[14] See also Canova (1991), Canova and Ito (1991), Bekaert and Hodrick (1992), Backus et al. (1993), Bekaert and Hodrick (1993), Canova and Marrinan (1993), Cheng (1993), Hai et al. (1997), Bekaert (1995), Burnside (2013), Burnside et al. (2006)

[15] A related finding is the high profitability of the carry trade, an investment strategy that goes long high-interest rate currencies and short low-interest rate currencies, and that should yield zero average return under UIP. Some papers that document profitable currency trading strategies are Lustig and Verdelhan (2007), Villanueva (2007), Burnside et al. (2008), Brunnermeier et al. (2008), Burnside et al. (2010), Lustig et al. (2011), Menkhoff et al. (2012a)

[16] The typical frequency of the regressions is monthly or quarterly.

[17] The UIP Puzzle is traditionally centered on the β_1 coefficient not being zero, and not α_0 because most studies find that α_0 are indeed insignificantly different from zero.

the daily frequency.[18,19] The data comes from Datastream, and forms an unbalanced panel because the Euro-legacy currencies cease to exist in 1999. As is standard in the literature, I do not enter the Euro as a separate currency but rather attach it at the end of the Deutsche Mark series. The time series for the other Euro-legacy currencies stops on January 1st, 1999, the time of Euro adoption for the currencies in my data set. Per the established convention, the interest rate differentials are computed from forward rates using the Covered Interest Rate Parity (CIP).[20]

The interest rates I use are for one month (30 days), hence with daily observations on the exchange rate the standard UIP regression can be expressed as

$$s_{j,t+30} - s_{j,t} + i^*_{j,t} - i_t = \alpha_{j,0} + \beta_{j,1}(i_t - i^*_{j,t}) + \varepsilon_{j,t+1},$$

and I estimate this equation currency-by-currency, with j indexing the different currencies. The time period is one day, and i_t is the 30 day interest rate on USD and $i^*_{j,t}$ the 30 day interest rate on currency j, hence the left-hand side variable is a 30 day excess return on the foreign currency. I use Newey-West standard errors to correct for the serial correlation induced by the overlapping periods in the dependent variable and report the results in Table 1. My estimates reaffirm the well established UIP Puzzle - I find that all β_1 point estimates are negative and almost all are statistically significant at conventional levels (15 out of 18).[21]

The evidence of negative and significant β_1 is pervasive throughout all 18 currencies. Moreover, the actual magnitudes of the estimated coefficients are quite similar to each other which leads me to consider panel regressions as a good way to summarize the results, that also increases efficiency by leveraging the cross-sectional variation of the data.[22] I re-estimate the UIP regression as a panel in two ways, first by pooling all the data together and imposing $\alpha_{j,0} = \alpha_0$ and $\beta_{j,1} = \beta_1$ and second by allowing the constants to differ among currencies (fixed effects) but still restricting the slope to be the same. The resulting coefficient estimates and

[18]The currencies are for Australia, Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

[19]Results at the monthly frequency, which have been more often used in the previous literature, are available in the Appendix and show that the estimates are extremely similar to the daily data ones.

[20]This is the standard procedure in the literature because data on forward rates is available for more periods and more countries than reliable data on constant maturity, default-free bonds and the CIP has been found to hold very well in the data. Nevertheless, the Appendix shows that the results remain unchanged when considering interest rates on physical assets, although this cuts down on the sample size.

[21]I also re-affirm the standard finding that the constants in the UIP regression are generally found to be insignificant. I find that only 3 out of the 18 estimates are significant at standard levels, the pooled panel estimate is small and not significant, and I cannot reject the hypothesis that the mean of the fixed effects is different from zero in the fixed effects panel regression.

[22]The observation that UIP coefficients tend to be rather similar across countries (at least among developed economies) is a pervasive feature of the data and has been noted as early as Fama (1984)

Table 1: UIP Regression Currency by Currency

Country	Currency	α_0	(s.e.)	β_1	(s.e.)	$\chi^2(\alpha_0 = \beta_1 = 0)$	R^2
Australia	AUD	-0.001	(0.002)	-1.63***	(0.48)	16.3***	0.014
Austria	ATS	0.002	(0.002)	-1.75***	(0.58)	9.5***	0.023
Belgium	BEF	-0.0002	(0.002)	-1.58***	(0.39)	17.5***	0.025
Canada	CAD	-0.003	(0.001)	-1.43***	(0.38)	19.1***	0.013
Denmark	DKK	-0.001	(0.001)	-1.51***	(0.32)	25.4***	0.025
France	FRF	-0.001	(0.002)	-0.84	(0.63)	1.9	0.007
Germany	DEM	0.002	(0.001)	-1.58***	(0.57)	7.9**	0.015
Ireland	IEP	-0.002	(0.002)	-1.32***	(0.38)	12.3***	0.020
Italy	ITL	-0.002	(0.002)	-0.79**	(0.33)	7.0**	0.013
Japan	JPY	0.006***	(0.002)	-2.76***	(0.51)	28.9***	0.038
Netherlands	NLG	0.003	(0.002)	-2.34***	(0.59)	16.0***	0.041
Norway	NOK	-0.0003	(0.001)	-1.15***	(0.39)	10.4***	0.013
New Zealand	NZD	-0.001	(0.002)	-1.74***	(0.39)	28.3***	0.038
Portugal	PTE	-0.002	(0.002)	-0.45**	(0.20)	5.9*	0.019
Spain	ESP	0.002	(0.003)	-0.19	(0.46)	2.8	0.001
Sweden	SEK	0.0001	(0.001)	-0.42	(0.50)	0.9	0.002
Switzerland	CHF	0.005***	(0.002)	-2.06***	(0.55)	13.9***	0.026
UK	GBP	-0.003**	(0.001)	-2.24***	(0.60)	14.2***	0.028
Panel, pooled		0.0002	(0.001)	-0.79***	(0.15)	22.3***	
Panel, fixed eff.				-1.01***	(0.21)	19.1***	

This table presents estimates of α_0 and β_1 from the regression $s_{j,t+1} - s_{j,t} + i^*_{j,t} - i_{j,t} = \alpha_{j,0} + \beta_{j,1}(i_{j,t} - i^*_{j,t}) + \varepsilon_{j,t+1}$. The standard errors in single currency regressions are Newey-West errors robust to serial correlation. The standard errors for the panel estimations are computed according to the Driscoll and Kraay (1998) method that is robust to heteroskedasticity, serial correlation and contemporaneous correlation across equations. The base currency is the USD.

the corresponding Driscoll and Kraay (1998) errors that correct for both cross-sectional and time series dependence in the error term are presented in the last two lines of Table 1.[23] The estimated β_1 coefficients are negative and highly significant and the constants are not statistically different from zero.[24]

Lastly, notice that the panel estimates imply that β_1 is roughly -1, which implies that the exchange rates changes are unpredictable on the basis of the interest rate differential.

[23] Applying the Vogelsang (2012) asymptotic adjustment does not make any perceptible difference to the inference.

[24] In the case of the fixed effects estimation, the overwhelming majority of fixed effect estimates are found to be insignificant. I also cannot reject the hypothesis that the mean of the fixed effects is zero.

This is also a common finding in the literature and is consistent with the hypothesis that exchange rates are a random walk. There is also a large, related literature, following the seminal contribution of Meese and Rogoff (1983), which has shown that beating the random walk forecast out of sample is exceedingly difficult. Such findings have lead to the general view that the random walk is a reasonable description of exchange rate behavior.

In summary, all results point to the fact that β_1 is negative and hence periods of high domestic interest rates tend to forecast positive excess returns on the home bonds versus foreign bonds. This is the crux of the famous "UIP Puzzle" and has given rise to the common saying in the literature that "high-interest rate currencies earn high returns", a fact that is further supported by the evidence on the profitability of the carry trade.[25]

2.3 The UIP at Different Horizons

In this section I test the UIP hypothesis at different horizons and find that while the UIP condition is violated at horizons of up to 7 years, the nature of the violations changes with the horizon. Similarly to the one-step ahead predictive regressions considered in the previous section, I find that high domestic interest rates today tend to forecast positive domestic currency excess return at horizons of up to 3 years. However, the forecastability pattern flips at longer horizons and it turns out that high interest rates today tend to forecast *negative* excess returns at horizons of 4 to 7 years. This finding is also a violation of the UIP condition, but constitutes the *opposite* puzzle, as compared to the standard formulation of the "UIP puzzle". Thus, this section documents that the puzzle is more complex than commonly understood, as it reverses course at longer horizons.[26]

The methodology I follow here is based on forecasting 1-month excess returns k periods into the future. UIP requires that the conditional expectation of the one step-ahead excess return is zero, i.e. $E_t(\lambda_{t+1}) = 0$, and this must be true for all time periods t. Hence, for any arbitrary $k > 0$ we have $E_{t+k}(\lambda_{t+k+1}) = 0$ and by the law of iterated expectations,

$$E_t(\lambda_{t+k+1}) = 0,$$

[25] Recent work by Hassan and Mano (2013) has shown that only a part of the carry profits can be explained by time-variation in the interest rate differential and that a significant portion is actually related to the persistent differences in interest rates across countries. Nevertheless, Burnside (2013) provides additional evidence that a large part of the carry profits is still traceable to time variation in interest rates.

[26] My findings do not contradict earlier work by Flood and Taylor (1996), Chinn and Meredith (2005), Chinn (2006) and Chinn and Quayyum (2012) that has found that UIP appears to hold better at horizons of 5+ years. The differences between my results and the previous studies is in the methodology used to test UIP at longer horizons, and in fact my results could help provide an interpretation to the earlier findings. As Appendix C discusses in more details, one can view the methodology of the previous papers as approximately averaging over the violations I document, which change sign with the horizon and tend to cancel out over the long run. The relationship is exact under risk-neutrality, but not necessarily otherwise.

for all $k > 0$. In other words, the UIP hypothesis requires that any future, one-period excess return must be unforecastable with time t information. This observation provides us with a series of conditions that we can test at any horizon k.

To test them, I implement a straightforward extension of the standard 1-step ahead UIP regression and estimate the equation

$$\lambda_{j,t+k} = \alpha_{j,k} + \beta_k(i_t - i^*_{j,t}) + \varepsilon_{j,t+k} \tag{3}$$

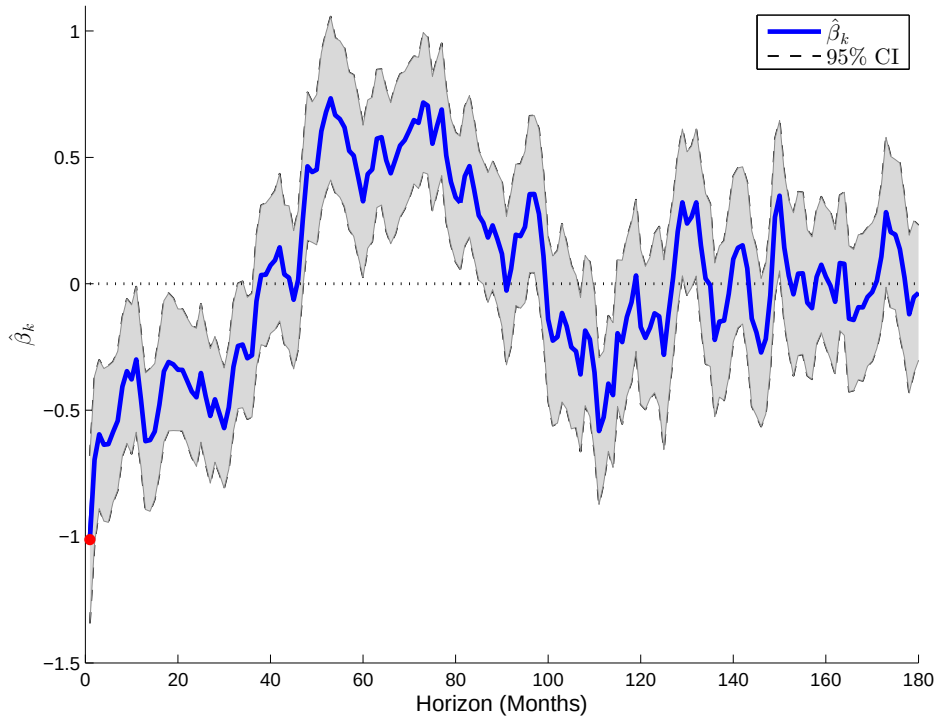
as a panel regression with fixed effects, where j indexes the currencies and $k = 1, 2, \ldots, 180$ indexes the horizon in months.[27] Since the data is at the daily frequency, the left-hand side variable for $k = 1$ is the 30-day return that ends 30 days into the future, for $k = 2$ this is the 30-day return that ends 60 days into the future (i.e. the return between 30 and 60 days into the future) and so on and so forth. Most importantly, the left-hand side variable in the regressions is always precisely a 1-month excess return of foreign bonds over home bonds and the horizon, indexed by k, changes by 1-month increments as well. Thus, each regression tests if a particular 1-month excess return is forecastable by the current 1-month interest rate differential, but the horizon at which the returns are being forecasted changes.

Figure 1 plots the estimated coefficients, $\hat{\beta}_k$, from the above set of regressions on the Y-axis with the horizon k, in months, on the X-axis. The solid blue line plots the point estimates and the shaded region represents the 95% confidence intervals around each estimate, computed with Driscoll and Kraay (1998) standard errors that correct for heteroskedasticity, serial correlation and spatial correlation. The red dot on the plot is the point estimate of the standard UIP regression that looks one month into the future. This dot is also the estimate reported in Table 1, and the focus of many previous papers on the UIP puzzle.

The plot shows three important results. First, the coefficients are negative and statistically significant at horizons of up to 36 months, a finding that corresponds to the typical statement of the UIP puzzle. Second, the graph shows that the coefficients, however, are *positive* and statistically significant at horizons between 48 and 84 months. This is a novel result which indicates that high interest rates today forecast *negative* excess returns 4 to 7 years into the future, contrary to the standard short-horizon result that high-interest rate currencies tend to earn positive excess returns. And third, after a brief spell at horizons of 100 to 120 months where the point estimates turn negative again, but only a handful are significant, the coefficients appear to converge to zero in the long-run. Overall, the UIP violations follow a clear, cyclical pattern, where they are negative at short horizons, positive at longer horizons, and gradually disappear in the long-run.

[27] Similarly to the results in the previous section, running separate regressions for each currency, rather than considering a single panel regression, produces similar results across all currencies.

Figure 1: UIP Regression at horizons from 1 to 180 months



The main takeaway from the estimates is that the nature of UIP violations changes with the horizon - the short-horizon violations are characterized by negative coefficients and the longer horizon violations by positive ones. The difference is not so much in the magnitude of violations, which is roughly the same at short and long horizons, but in their fundamental nature. At short horizons we have the finding that exchange rates fail to depreciate sufficiently to fully offset the interest rate differential, while at longer horizons we have the opposite puzzling behavior, as exchange rates in fact depreciate *too much*.[28] The findings show that UIP is violated at both short and long horizons, and suggest that the UIP puzzle is not driven by short-run frictions that disappear over the long-run but rather that the fundamental nature of the violations changes with the horizon. A comprehensive model of the UIP puzzle should be consistent with the full complexity of the UIP violations at all horizons.

My empirical analysis is inspired by the essential empirical insights of Engel (2012) regarding excess currency returns and real interest rates. He studies the relationship between currency returns, real exchange rate levels and real interest rate differentials, using a VAR on observed exchange rates, price levels and nominal interest rates, for each G7 country (paired with the US). The VAR is used to construct real interest differentials and model-implied

[28]The next section provides a detailed discussion on the implications for exchange rate behavior.

ex-ante expected currency returns. Using results from the VAR, Engel (2012) shows that when a country's real interest rate is high, these expected returns are initially positive, and then, at longer horizons, negative. My results, and those in Engel (2014), show that these qualitative findings extend to the case where nominal interest rate differentials are used to predict realized returns.

There are some distinctions between the two sets of empirical results. For example, Engel (2012) finds that the forecastability pattern in excess returns changes from positive to negative at horizons of about 10 to 12 months, while I estimate that this occurs after approximately 40 months. Additionally, Engel (2012)'s results suggest that UIP is violated at very long horizons in the sense that long-horizon cumulative returns are predicted to be statistically significantly negative. I do not find an equivalent result when the nominal interest rate differential is the predictor. These different quantitative findings could be due to the specific restrictions implied by Engel's VAR system.[29] Alternatively, they could be due to interesting and important differences in how nominal and real interest rates forecast excess returns at horizons longer than one year or due to the fact that my analysis also leverages the cross-sectional variation of the data. I leave disentangling those effects to future work.

2.4 Implications for Exchange Rate Behavior

In this section I examine the implications of the changing nature of UIP violations for exchange rate behavior. I show that, following an increase in the interest rate differential, the short-horizon negative violations lead to a short-run exchange rate appreciation (rather than depreciation as UIP implies), but that the long-horizon positive violations lead to a subsequent strong depreciation. The long-horizon excess depreciation offsets the short-horizon appreciation, and in the long-run the exchange rate depreciates, and converges to the path implied by UIP. This behavior generates significant, predictable swings in the exchange rate and also lead me to reject the Random Walk hypothesis.

To begin, re-arrange the equation $\lambda_{t+1} = s_{t+1} - s_t + i^*_t - i_t$ to express the cumulative exchange rate change from now up to k periods in the future as:

$$s_{t+k} - s_t = \underbrace{\sum_{h=1}^{k} (i_{t+h-1} - i^*_{t+h-1})}_{\text{Sum of Future Int Diffs}} + \underbrace{\sum_{h=1}^{k} \lambda_{t+h}}_{\text{Sum of Future Excess Returns}} . \tag{4}$$

[29] The standard OLS procedure ensures that one step ahead forecast errors are uncorrelated with any of the variables included in the VAR. However, forecast errors at longer horizons are not necessarily unforecastable by the variables included in the VAR. This issue is likely to get larger as the horizon increases, since VARs are known to provide a good approximation of short-run dynamics, but less so for long-run dynamics.

Thus, the cumulative exchange rate change, over any horizon, can be represented as the sum of future interest rate differentials and future excess returns. This is a standard decomposition that can be obtained for any asset price - prices move in response to fundamentals (interest rate differentials in this case) and excess returns.

Consider estimating the impulse response function of the cumulative exchange rate change to an innovation in the interest rate differential by using the Jorda (2005) method of local projections. This amounts to separately projecting each k-periods cumulative exchange rate change on the current interest rate differential so as to obtain

$$\text{Proj}(s_{t+1} - s_t | i_t - i_t^*) = \gamma_k (i_t - i_t^*).$$

The sequence $\{\gamma_k\}$ forms an estimate of the impulse response function (in percentage terms) of the cumulative exchange rate change to a 1% increase in the interest rate differential $i_t - i_t^*$. The method of local projections is especially well suited for estimating long-run responses because of its flexible nature – there are no restrictions on the dynamics from period to period, as each response horizon is estimated via a separate projection.

Let ρ_k be the k-th autocorrelation of the interest rate differential $i_t - i_t^*$ and use the definition of the projection coefficient $\gamma_k = \frac{\text{Cov}(s_{t+k} - s_t, i_t - i_t^*)}{\text{Var}(i_t - i_t^*)}$ to express γ_k as:

$$\gamma_k = \underbrace{\sum_{h=1}^{k} \beta_h}_{\text{Sum of UIP Violations}} + \underbrace{\sum_{h=0}^{k-1} \rho_h}_{\text{Sum of Expected Int Diffs}}.$$

Thus the expected path of the cumulative exchange rate change is equal to the sum of UIP violations plus the sum of expected, future interest rate differentials. This showcases the important role that UIP, and deviations from UIP, play in exchange rate determination.

If UIP held, then we would have $\beta_h = 0$ for all h, and the predicted path for the exchange rate change will be entirely determined by expectations about future interest rate differentials. In this case, exchange rate behavior will be closely linked to the underlying fundamentals, the interest rate differentials. On the other hand, UIP violations disconnect the exchange rate from its fundamentals and can cause it to behave very differently from the UIP benchmark.

For example, the alternative hypothesis of a random walk exchange rate obtains when the UIP violations component of exchange rate dynamics exactly offsets the interest rate differentials component. Since in the data the interest rate differentials are strongly positively autocorrelated, $\rho_h \geq 0$, this would require that β_h are negative and similar in magnitude. As I show below, the short-horizon negative UIP violations indeed contribute to the fact that

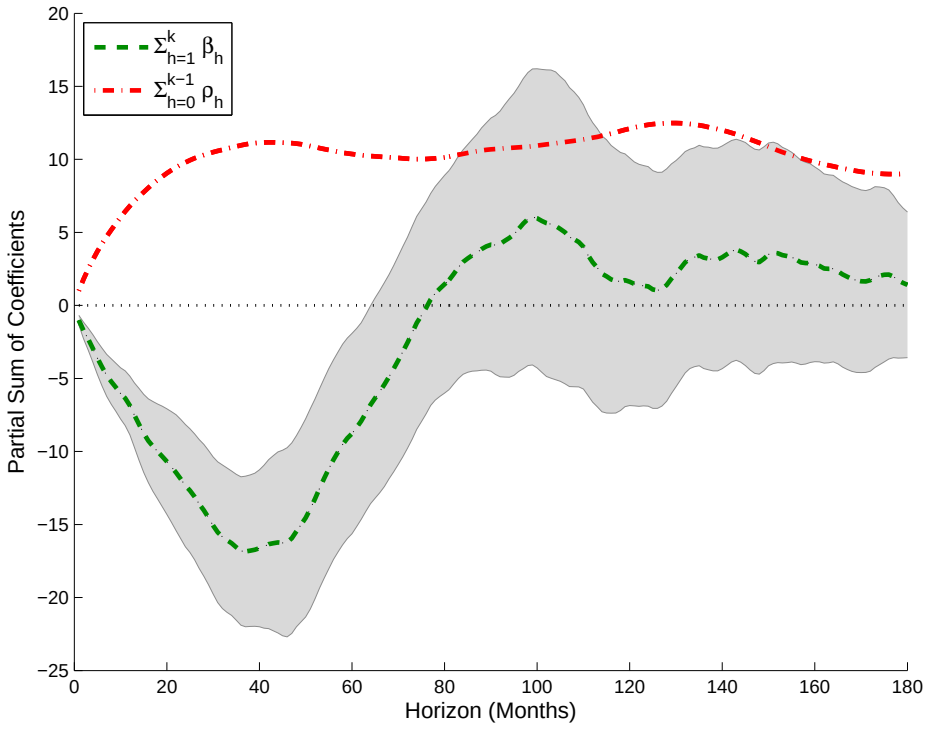
exchange rates tend to look like a random walk in the short-run. However, my results on the changing nature of UIP violations, i.e. $\hat{\beta}_h > 0$ at longer horizons, suggest that at long horizons the two components may no longer offset and may even reinforce each other.

At this point, it is clear that cumulative UIP violations and cumulative expected interest rate differentials both play an important role in exchange rate behavior. To quantify their effects, I start by estimating ρ_k using similar panel regressions

$$i_{t+k} - i^*_{j,t+k} = \alpha_{j,k} + \rho_k(i_t - i^*_{j,t}) + \varepsilon_{j,t+k}$$

and then plot the partial sums of UIP violations and interest rate differential autocorrelations in Figure 2, for horizons ranging from 1 to 180 months.

Figure 2: Cumulative Sums of the UIP Coefficients β_k



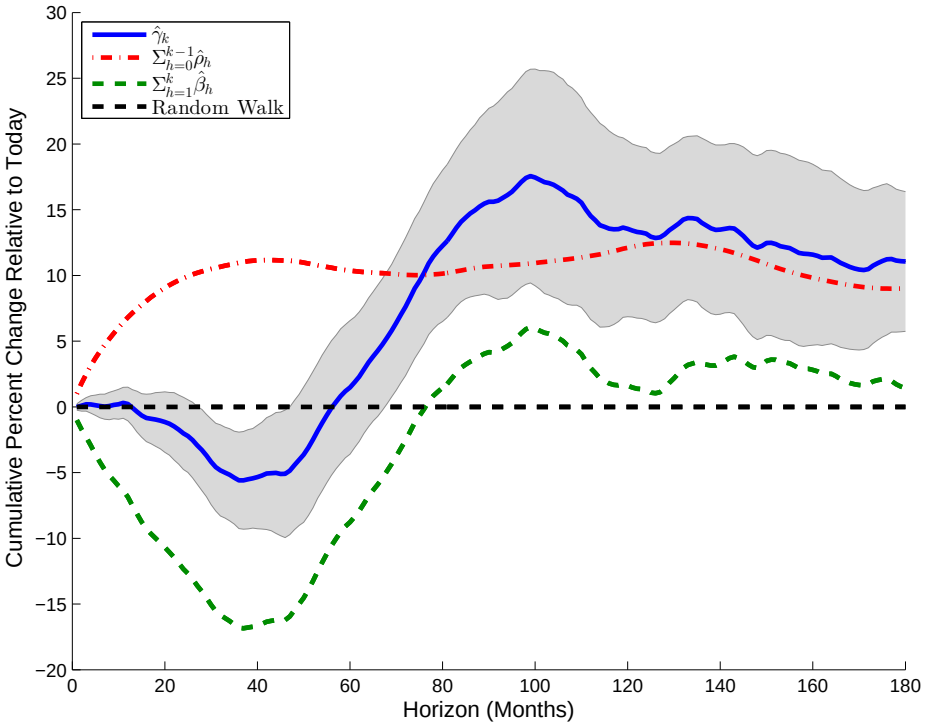
The dashed green line plots the partial sums of $\hat{\beta}_k$ (the estimated UIP violations), with the shaded region representing the corresponding 95% confidence error bands, and the dash-dot red line plots the partial sums of $\hat{\rho}_k$ (I do not include the corresponding error bands to reduce clutter). The figure shows that the two components of exchange rate dynamics go in opposite directions, and thus work against each other, at horizons of up to 36 months. However, at longer horizons the positive UIP violations begin to weigh in, and the sum of $\hat{\beta}_k$ start trending upwards. In fact, it turns out that the cumulative total of positive UIP violations is roughly equal to the cumulative total of negative UIP violations. This is

manifested in the findings that in the long run (10+ years) the sum of UIP violations is roughly zero and is not statistically different from zero at horizons longer than six years.

The fact that the sum of UIP violations converges to zero has the interesting implication that the exchange rate path itself converges to the UIP benchmark. Thus, long-run exchange rate dynamics may appear consistent with UIP, even though UIP is in fact violated. My results suggest that the UIP puzzle is not a short-run phenomenon that disappears over the long-run; it is violated at both short and long horizons, but the nature of violations change in a way that makes long-run exchange rate behavior appear consistent with UIP.

To illustrate how these different effects impact exchange rate behavior at different horizons, in Figure 3 I add the estimated response to an increase in the interest rate differential, γ_k, with a solid blue line and its corresponding 95% error bands as the shaded region surrounding it.

Figure 3: $\text{Proj}(s_{t+1} - s_t | i_t - i_t^*)$



Recall that the red dash-dot line represents the UIP benchmark – the expected path of the exchange rate in case all $\beta_h = 0$. The disparate movements of the blue and the red line at short horizons is a manifestation of the standard formulation of the UIP Puzzle. Instead of steadily depreciating and offsetting the increase in interest rates, as predicted by UIP, the exchange rate fails to depreciate and even tends to appreciate slightly at horizons of up to

36 months. This is due to the accumulation of the short-horizon negative UIP violations, as illustrated by the strong downward trend in the green line over those horizons.

Just as the negative UIP violations do not persist at long horizons, however, the exchange rate appreciation lasts for only three to four years. At that point, the exchange rate starts responding to the accumulation of positive UIP violations and experiences a sharp depreciation at horizons of four to seven years. The depreciation is strong enough to fully offset the initial appreciation and to catch up the exchange rate with the UIP-implied path.[30] This is an illustration of the fact that the cumulative sum of UIP violations is roughly zero, and hence long-run exchange rate behavior is roughly consistent with the UIP hypothesis.

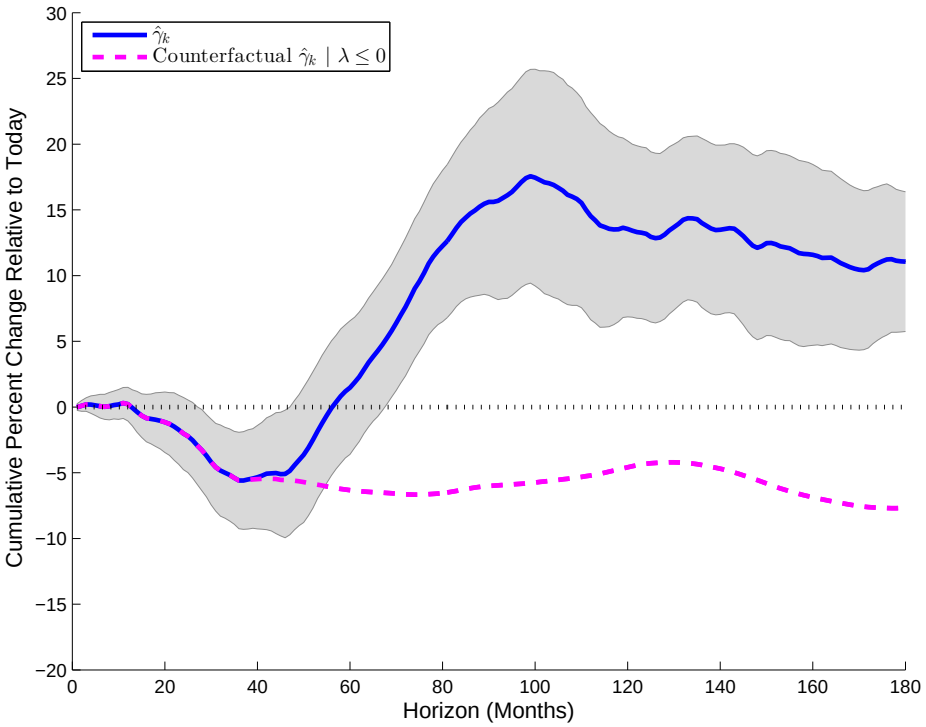
But the path the exchange rate takes to get back to the UIP benchmark in the long-run, is very much in violation of UIP at every step of the way. At first, exchange rates tend to appreciate when we actually expect them to depreciate and later they reverse course and start depreciating too sharply. These cyclical movements are driven by the documented UIP violations that change sign with the horizon. Moreover, this cyclical behavior imparts interesting predictable movements in the exchange rate, especially in the long-run. Notice that the black dashed line at zero, in Figure 3, represents the expected path under the random walk hypothesis. The cyclical movements of the exchange rate move it away from that path, and thus the changing nature of the UIP violations also leads me to reject the random walk hypothesis.

To further illustrate the significance of the changing sign of UIP violations, Figure 4 plots the estimated path of the exchange rate (γ_k) against a counter-factual path obtained by setting $\beta_k = 0$ for all $k \geq 36$. This counterfactual exercise emulates exchange behavior in a world where UIP violations are persistently negative at short horizons (as also estimated in the data), but converge to zero monotonically, without turning positive. The main difference between the two is the long-run behavior of the exchange rate. Ignoring the positive UIP violations leads to the counterfactual implication that, following an increase in interest rates at home, the exchange rate appreciates persistently and stays high.[31] On the contrary, the estimated exchange rate response features a strong depreciation at longer horizons which leaves the exchange rate depreciated, rather than appreciated in the long-run. Thus, the positive UIP violations have important implications about the long-run exchange rate behavior.

[30] The finding of appreciation in the short-run and depreciation in the long-run is in line with the "delayed overshooting" result of Eichenbaum and Evans (1995), which they obtain through a VAR analysis of the response of exchange rates to identified monetary shocks.

[31] Note that all statements about long-run exchange rate behavior are made *relative* to today's value. I am estimating the path of the cumulative change in the exchange rate conditional on today's value, $s_{t+k} - s_t$, and I am not making a statement about the level of the exchange rate itself, i.e. s_t. I proceed in this way because the exchange rate s_t is clearly non-stationary.

Figure 4: Proj$(s_{t+1} - s_t | i_t - i_t^*)$ and Counterfactual



A model that fully captures the empirical regularities must account for both the negative and the positive UIP violations, and feature cyclical dynamics in the exchange rate. Given the novel nature of my empirical results, it is perhaps not surprising that existing models have largely focused on explaining the short-horizon UIP violations and exchange rate behavior, and tend to generate UIP deviations that converge monotonically to zero and cannot deliver the changing nature of UIP violations. To address this, the rest of the paper develops and analyzes a novel model, relying on the mechanism of bond convenience yields, and shows that it can closely match the full complexity of the UIP violations and exchange rate behavior at both short and long horizons.

3 Time-Varying Convenience Yields and Excess Currency Returns

The UIP condition is derived under two important implicit assumptions. First, it abstracts from risk considerations (due to the log-linearization) and second it assumes that financial returns are the only benefit to holding bonds. Risk-premia have been extensively analyzed as a potential resolution to the UIP puzzle in the previous literature and in this paper instead I focus on relaxing the second assumption. I explore the implications of introducing a non-

pecuniary (convenience or liquidity needs motivated) benefit to holding bonds and show that this can help generate UIP violations that closely match the empirical evidence.

A growing empirical literature documents a significant, time-varying "convenience yield" component to government bond yields and show that it is related to the outstanding amount of government debt (see Reinhart et al. (2000), Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2012), Greenwood and Vayanos (2014)). The convenience yield is defined as the market price of the non-pecuniary benefit of owning government debt (e.g. safety, liquidity) and is the amount of interest investors are willing to forego in exchange for the non-pecuniary benefits. The literature motivates and relates the convenience yield to the fact that government debt is extremely (nominally) safe and liquid, and could serve as a substitute to money, a special convenience asset that investors are willing to hold at a zero equilibrium rate of return. The empirical literature has documented that yields on government bonds, and US Treasuries in particular, indeed vary with the degree to which the bonds could provide some of the special services of money (e.g., facilitate transactions, store value, etc.).

There is also a sizable theoretical literature that has incorporated convenience yield mechanisms in equilibrium models and has shown that they can help account for a diverse set of asset pricing puzzles, such as the low equilibrium risk-free rate, the equity risk premium, and the term premium(Bansal and Coleman II (1996), Bansal et al. (2011), Lagos (2010, 2011), Acharya and Viswanathan (2011)). In the following two sections I build on this literature by introducing the convenience yield mechanism into an international framework, and show that it can account for both the short and the long horizon UIP deviations I documented previously. In a later section, I also show that two of the key implications of the model are well supported by the data, by providing novel results that document the significant relationship between excess currency returns and the outstanding amount of government debt, and the relationship between UIP violations and capital controls, which serve as a proxy for monetary policy independence.

I proceed in two steps. First, I develop an intentionally stylized model that allows me to analytically characterize the main results and showcase the key intuition. Next, I relax the simplifying assumptions of the analytical model and set the mechanism in a benchmark, two-country open economy model in the modern tradition of Obstfeld and Rogoff (1995), Chari et al. (2002), Clarida et al. (2002) and the subsequent literature. I calibrate the model using standard parameters from the existing literature and show that it can closely match the documented UIP violations and exchange rate behavior at all horizons.[32]

[32]The model abstracts from trade in forward contracts, however, Appendix B shows that introducing forward markets does not change the results. The intuition is that buying foreign currency forward is long foreign currency and short home currency, hence it creates a synthetic, zero-cost position in the underlying home and foreign bonds, which earns the convenience yield differential and violates UIP. The UIP violations

4 Analytical Model

In this section, I setup and analyze an intentionally stylized model which allows me to derive several analytical results that are useful in understanding and illustrating the main mechanism behind the UIP violations. There are two countries, a large home country and a small (measure zero) foreign country that is negligible in world equilibrium. The assumption of a small foreign country greatly simplifies the model, as the equilibrium is determined entirely by the actions of the home agents (the foreign agents are measure zero).[33]

There is a representative home household that consumes a single consumption good, in fixed supply, and has access to money, and home and foreign nominal bonds. In terms of the preference for liquidity, I keep things simple and transparent by following the standard approach of money-in-the-utility, assuming that real money balances provide utility to the household. Additionally, bond holdings (both home and foreign) also provide convenience (i.e. liquidity) benefits and for simplicity those are also modeled directly through the utility function. The idea is that government bonds, through their safety and liquidity, are a close substitute for money and can provide some of the same transaction benefits. For example, Treasuries are the main type of collateral accepted in repo markets, a key source of short-term financing for financial intermediaries and other investors, and are often used to mitigate counter-party risk, which helps facilitate complex financial transactions.[34]

The goal of this paper is to study the implications of the existence of convenience yields on equilibrium exchange rates, and not the structural microfoundations that generate the convenience yield in the first place. To this end, I follow much of the literature on bond convenience yields and assume a flexible, reduced form specification for the convenience benefits; for a structural approach to money demand see for example Lagos (2010, 2011). Endogenizing the convenience yield is a future research project.

Finally, there is a government that sets the growth rate of money, and finances a fixed level of expenditures by levying lump-sum taxes and issuing government bonds. The government is the sole supplier of the bonds that the households trade, and hence I will use the terms "government debt", "government bonds" and "bonds" interchangeably.[35]

show up in forwards data just as they do in the relationship between exchange rates and interest rates.

[33]The quantitative model relaxes this assumption and studies a fully specified two country economy.

[34]Modeling liquidity demand through preferences allows for cleaner analytical expressions, as compared to modeling it through real transaction costs that affect the agent's budget constraint, but imply virtually the same optimality and equilibrium conditions. The analytical tractability is useful for the purposes of this section, but the quantitative model uses instead a transaction cost specification, as is standard in the existing literature (e.g. Bansal and Coleman II (1996), Bansal et al. (2011)).

[35]The government is kept simple for analytical tractability, but is generalized in the quantitative model.

4.1 The Household

The household is infinitely lived and has additively separable preferences over consumption, real money balances and real holdings of home and foreign bonds, with the instantaneous utility function[36,37]

$$u(c_t, m_t, b_{ht}, b_{ft}) = \ln(c_t) + \alpha_m \ln(m_t) + \alpha_b \ln(b_{ht} + \delta b_{ft})$$

where c_t is consumption, m_t are the real money balances of the household, and b_{ht} and b_{ft} are its holdings of home and foreign bonds respectively, and $\alpha_m > 0$, $\alpha_b > 0$ and $\delta \in [0, 1)$ are constants. The underlying assumption is that through their safety and liquidity, government bonds are a close substitute for money and provide some of the same liquidity benefits. Moreover, $\delta \in [0, 1)$ models the idea that nominal government bonds denominated in foreign currency are not a perfect substitute for home government bonds, and are in general an inferior substitute for domestic money. I will use "liquidity benefits" and "convenience benefits" interchangeably throughout the discussion, as is common in the literature.[38]

The household faces the following budget constraint at date t

$$c_t + m_t + b_{ht} + b_{ft} = y - \tau_t + \frac{m_{t-1}}{\Pi_t} + b_{h,t-1}\frac{(1 + i_{t-1})}{\Pi_t} + b_{f,t-1}\frac{(1 + i^*_{t-1})}{\Pi_t}\frac{S_t}{S_{t-1}}$$

where y is the endowment of the consumption good, which is constant over time, τ_t are the real lump-sum taxes, Π_t is the gross inflation rate, i_t and i^*_t are the domestic and foreign nominal interest rates, and S_t is the exchange rate, expressed in terms of home currency units per unit of foreign currency (i.e. the domestic currency cost of foreign currency). This

[36] Separability of preferences is not important for the results - the quantitative model uses generalized, non-separable preference for liquidity. Separability, however, helps provide tractability and this is the reason it is used in this section.

[37] The log-utility specification of preferences is not important, and is chosen simply for ease of exposition. The Appendix shows the results hold for any concave utility that is separable in the effects of consumption, money and bonds (and not necessarily separable in the two types of bonds).

[38] Again, the focus of the paper is to examine the overall implications of convenience yields for UIP violations, and I leave the study of the particular mechanisms generating the convenience yields to future work.

yields the following three Euler equations:

$$1 = \underbrace{\beta E_t[(\frac{c_{t+1}}{c_t})^{-1}\frac{1}{\Pi_{t+1}}]}_{\text{Discounted Financial Return on Money}} + \underbrace{\alpha_m\frac{c_t}{m_t}}_{\text{Convenience Benefit of Money}} \quad (5)$$

$$1 = \underbrace{\beta E_t[(\frac{c_{t+1}}{c_t})^{-1}\frac{1+i_t}{\Pi_{t+1}}]}_{\text{Discounted Financial Return on Home Bonds}} + \underbrace{\alpha_b\frac{c_t}{b_{ht}+\delta b_{ft}}}_{\text{Convenience Benefit of Home Bonds}} \quad (6)$$

$$1 = \underbrace{\beta E_t[(\frac{c_{t+1}}{c_t})^{-1}\frac{S_{t+1}}{S_t}\frac{1+i_t^*}{\Pi_{t+1}}]}_{\text{Discounted Financial Return on Foreign Bonds}} + \underbrace{\delta\alpha_b\frac{c_t}{b_{ht}+\delta b_{ft}}}_{\text{Convenience Benefit of Foreign Bonds}} \quad (7)$$

Each of the Euler conditions equates the real cost of one unit of investment (in money or bonds) to the discounted, expected benefit. The cost is simply the one unit of foregone consumption today and the payoffs are composed of both financial returns and convenience benefits (i.e. direct utility effect). Take the first Euler equation for example. An additional unit of money offers the financial return $\frac{1}{\Pi_{t+1}}$, which is simply the devaluation due to inflation, and also provides the direct utility benefit (expressed in consumption good terms) $\alpha_m\frac{c_t}{m_t}$. The real payoff to an extra unit of investment is the sum of the two - it includes both financial and non-pecuniary (convenience) benefits.

The other two Euler equations follow a similar structure: each asset offers both financial and direct utility returns. For future reference, I define the following notation for the convenience returns on money, home and foreign bonds:

$$\Psi_{m,t} = \alpha_m\frac{c_t}{m_t}$$

$$\Psi_{b_h,t} = \alpha_b\frac{c_t}{b_{ht}+\delta b_{ft}}$$

$$\Psi_{b_f,t} = \delta\alpha_b\frac{c_t}{b_{ht}+\delta b_{ft}}$$

4.2 The Government

The government controls money supply and taxes. It implements monetary policy by setting the growth rate of the nominal money supply

$$\ln(M_t) - \ln(M_{t-1}) = v_t$$

where v_t is an exogenous, white noise monetary shock. On the fiscal side, it faces a constant level of real expenditures g and the budget constraint

$$b_t^G + \tau_t + m_t^s - \frac{m_{t-1}^s}{\Pi_t} = \frac{(1+i_{t-1})}{\Pi_t} b_{t-1}^G + g$$

where b_t^G is real government debt and m_t^s is real money supply. I follow the literature on the interaction of monetary and fiscal policy and assume that the lump-sum taxes are set according to the simple linear rule,[39]

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)\kappa_b b_{t-1}^G,$$

where $\rho_\tau \geq 0$ is the tax smoothing parameter and $\kappa_b \geq 0$ controls how strongly taxes respond to debt levels.[40] This rule is a simple and tractable way of modeling the general idea that the government adjusts tax revenues to stay solvent, but that tax policy is (possibly) smoothed over time. Neither the monetary policy rule, nor the fiscal policy rule, are meant to capture optimal policy, but rather model government behavior in a tractable and reasonable way.

4.3 Equilibrium Relations

The foreign country is negligibly small and does not affect world markets, hence equilibrium in the goods market implies that the home household's consumption is constant over time:

$$c_t = c + g = y.$$

Constant consumption and the separable utility specification imply that the marginal rate of substitution between consumption today and consumption tomorrow is constant, and hence the equilibrium risk-free rate is also constant. This is useful to remember in interpreting results on the Euler equations that are to follow.

The small size of the foreign country also implies that foreign bonds are in zero net supply, $b_{ft} = 0$, and that home agents must hold the whole supply of home bonds:

$$b_{ht} = b_t^G.$$

Lastly, the money market is in equilibrium when money demand equals money supply so that $m_t = m_t^s$.

I solve the model by log-linearizing around the zero inflation steady state. Start by

[39] See for example Leeper (1991), Chung et al. (2007), Davig and Leeper (2007), Bianchi and Ilut (2013).
[40] Bohn (1998) documents that US primary surpluses indeed respond to the level of government debt.

log-linearizing the monetary policy rule to obtain

$$\hat{\pi}_t = v_t - \Delta\hat{m}_t,$$

where hatted variables represent log-deviations from steady state. Next, log-linearize the Euler equation for money holdings and use the fact that consumption is constant to obtain

$$-\Psi_m\hat{m}_t - \beta E_t(\hat{\pi}_{t+1}) = 0.$$

where Ψ_m is the steady state marginal convenience value of money. The left hand side is the real return on money, in log-deviations from steady state, which is the sum of its convenience value and the devaluation due to inflation. Again, this is the sum of money's non-pecuniary and financial returns, respectively. The right-hand side (RHS) is the log-linearized real risk-free rate, which is constant, hence the RHS is zero.

To solve for equilibrium real money holdings, substitute out expected inflation using the inflation equation and solve forward to arrive at:

$$\hat{\pi}_t = v_t$$

$$\hat{m}_t = 0$$

In this simple economy, inflation is white noise and adjusts one-for-one with movements in the money supply, which leaves real money holdings constant over time.

Next, log-linearize the Euler equation for home bonds to obtain the following expression for the equilibrium interest rate:

$$\underbrace{\hat{i}_t - E_t(\hat{\pi}_{t+1})}_{\text{Real Interest Rate}} + \underbrace{\frac{\Psi_{b_h}}{\beta(1+i)}\hat{\Psi}_{b_h,t}}_{\text{Convenience Yield}} = 0$$

where $\hat{\Psi}_{b_h,t}$ is the log-linearized marginal convenience value of home bonds. The left hand side of the equation is the log-linearized, real return on home government debt: the real interest rate plus the convenience yield. The convenience yield is the amount of interest agents are willing to forego in exchange for the non-pecuniary benefits of the bonds. Naturally, there is a clear negative relationship between the convenience yield and the equilibrium interest rate - the higher the convenience yield, the lower the equilibrium interest rate the household requires in order to hold the outstanding supply of government debt.

Log-linearizing the expression for the convenience benefit of the home bond, using the

equilibrium condition $b_{ft} = 0$, yields

$$\hat{\Psi}_{b_h,t} = -\hat{b}_{ht}$$

which shows two important things. First, the marginal convenience value of bonds is decreasing in the total amount of bonds owned by the household.[41] Second, the convenience yield is entirely determined by the supply of government bonds (recall that in equilibrium $b_{ht} = b_{ht}^G$). This is due to the simplifying assumptions of constant consumption and additive separability in the utility function.[42] While this is an extreme case, it serves to highlight one of the key ingredients of the mechanism: movements in government debt. This is a feature the model shares with the previous literature, which has also emphasized the relationship between the supply of debt and the equilibrium convenience yield.

Furthermore, combining this with the previous result of white noise inflation, $E_t(\hat{\pi}_{t+1}) = 0$, we obtain

$$\hat{i}_t = -\frac{\Psi_{b_h}}{\beta(1+i)}\hat{\Psi}_{b_h,t} = \frac{\Psi_{b_h}}{\beta(1+i)}\hat{b}_{ht}.$$

This equation shows two important and intuitive results. First, we see that the interest rate is negatively related to the convenience yield. The higher the convenience benefits provided by the home bond, the lower is the equilibrium interest rate that investors require to hold the outstanding supply of debt. This negative relationship between the convenience yield and the interest rate differential is a fundamental property of the mechanism and will be important later. Second, the equation also defines a downward sloping demand for government bonds – the higher is the outstanding supply of debt, the higher the equilibrium interest rate.

4.4 Currency Returns and UIP Violations

Log-linearize the Euler condition for foreign bonds around the symmetric, zero inflation steady state where $S = 1$, and combine it with the equation for the home interest rate to obtain a modified version of the UIP condition:

$$E_t(\underbrace{\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t}_{=\lambda_{t+1}}) = \underbrace{\frac{\Psi_{b_h}}{\beta(1+i)}\hat{\Psi}_{b_h,t} - \frac{\Psi_{b_f}}{\beta(1+i^*)}\hat{\Psi}_{b_f,t}}_{\text{Conv Yield Differential}}. \tag{8}$$

Uncovered Interest Rate Parity does not hold in this model and there are predictable, excess returns on cross-country investment. The predictable returns arise as a compensation for differences in the convenience yields on home and foreign bonds. The left hand side

[41] The liquidity preferences exhibit diminishing marginal utility of convenience assets.

[42] Both of which will be relaxed in the quantitative model.

of the equation above gives the expected excess financial return of foreign bonds relative to home bonds. In equilibrium, the expected excess returns are equal to the difference (in log-deviations from steady state) in the convenience values of home and foreign bonds. Thus, when the home bond's convenience value increases, the foreign bond is compensated with higher expected financial returns and vice versa. Furthermore, if we remove the convenience yield mechanism there will be no forecastable currency returns and no UIP violations.

Moreover, since foreign bond holdings are constant, we have that

$$\hat{\Psi}_{b_f,t} = -\hat{b}_{ht}$$

and defining $\eta_b \equiv \frac{\Psi_{b_h}}{\beta(1+i)} - \frac{\Psi_{b_f}}{\beta(1+i^*)}$ we have

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -\eta_b \hat{b}_{ht}.$$

Thus, in this stylized model the convenience yield differential and the predictable excess returns are driven solely by movements in home government debt. Moreover, since $\delta \in [0, 1)$ (i.e. foreign debt is an imperfect substitute for home debt) we can show that $\eta_b > 0$. Thus, the convenience yield differential and the expected excess returns on the foreign bond are decreasing in government debt. This is due to the diminishing marginal convenience value of debt and the fact that the home convenience yield is more responsive to movements in home debt, than the foreign convenience yield. When government debt increases, the convenience value of home bonds drops faster than the convenience value of foreign bonds and this creates a convenience yield differential across countries.

To fully characterize exchange rate dynamics and the implied UIP regression coefficients, we also need to model the foreign interest rate. For simplicity, I assume that the foreign central bank follows a symmetric monetary policy, resulting in an iid inflation process $\hat{\pi}_t^*$. Furthermore, assume that Purchasing Power Parity holds, so that the consumption good prices at home and abroad are equal when expressed in the same currency, i.e. $P_t = S_t P_t^*$, and thus

$$\hat{\pi}_{t+1} = \hat{s}_{t+1} - \hat{s}_t + \hat{\pi}_{t+1}^*.$$

Taking the conditional expectations on both sides yields $E_t(\Delta \hat{s}_{t+1}) = 0$ and hence the exchange rate follows a random walk.[43] Using this result it follows immediately that the

[43]The random walk exchange rate is an artifact of the simple monetary policy, which is assumed here for tractability. The analytical model is meant to illustrate the dynamics of the excess currency returns and UIP deviations, and not necessarily capture the behavior of the exchange rate itself. The quantitative model, on the other hand, relies on Taylor-rule monetary policy and this allows it to closely match both the UIP violations and the cyclical behavior of exchange rates, which appreciate over the short-run after an increase in the interest rate, but then depreciate strongly at longer horizons.

expected excess currency returns are equal to the negative of the interest rate differential:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -(\hat{i}_t - \hat{i}_t^*).$$

This implies that any forecastability in the excess return necessarily comes from the interest rate dynamics, and not the exchange rate. This is an implication of the intentionally stylized nature of the analytical model and is relaxed in the full blown model. The analytical model is meant to capture the key intuition that excess returns are driven by the convenience yield differential and the important role played by government debt dynamics in the convenience yields themselves. Generalizing the framework so that exchange rate adjustments, and not the interest rate differentials, are the main source of forecastability is straightforward, and is accomplished in the quantitative model by introducing Taylor rule monetary policy, which generates interest rate dynamics that match the data. The important takeaway here is the equilibrium relation that equates forecastable excess returns to the convenience yield differential, which is a fundamental part of the mechanism and is exactly the same in the quantitative model.

Combining the results on the equilibrium excess return and interest rate differential, leads us to the observation that they are both proportional to the supply of home bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -\eta_b \hat{b}_{ht}^G$$

$$(\hat{i}_t - \hat{i}_t^*) = \eta_b \hat{b}_{ht}^G$$

But while the interest rate differential increases with the supply of government debt, the excess currency return falls. This induces a negative relationship between excess returns and the interest rate differential (at least at short horizons), and it is immediate that estimating the now familiar set of UIP regressions,

$$\hat{s}_{t+k} - \hat{s}_{t+k-1} + \hat{i}_{t+k-1}^* - \hat{i}_{t+k-1} = \alpha_0 + \beta_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+1},$$

on data generated by this model, would yield $\beta_1 = -1$ regardless of parametrization. Ostensibly, the convenience yield differential acts as an omitted variable that is perfectly negatively correlated with the regressor (at the one period horizon), the interest rate differential, hence high interest rates at home are associated one-for-one with positive excess returns on the home bond. Thus, even though interest rates themselves are not an equilibrium determinant of excess returns, the regression yields a significant negative coefficient.[44]

[44]For ease of exposition, the benchmark model abstracts from trade in forward exchange rate contracts, however, Appendix B shows that the mechanism generates equivalent UIP violations in the forwards data.

The UIP deviations are a direct consequence of the convenience yield mechanism – without it the model would yield $\beta_k = 0$ for all $k \geq 0$. The UIP violations and the corresponding regression coefficients are driven by the time-variation in the convenience yield differential across countries, and the tractable nature of the model allows us to characterize the coefficients at any horizon k:

$$\beta_k = -\frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -\frac{\text{Cov}(\hat{b}_{h,t+k-1}, \hat{b}_{h,t})}{\text{Var}(\hat{b}_{h,t})}$$

However, to solve for the full set of regression coefficients β_k we need to fully characterize the equilibrium dynamics of the model, which is done in the next section.

4.5 Model Solution

Let $\gamma_b \equiv \dfrac{\Psi_{b_h}}{\beta(1+i)} > 0$, enforce the equilibrium conditions $b_{ht}^G = b_{ht}$ and $m_t^s = m_t$, and substitute the inflation, money holdings and interest rate equations into the log-linearized government budget constraint to obtain:

$$\hat{b}_{ht} + \frac{\tau}{b}\hat{\tau}_t = (1+i)(1+\gamma_b)\hat{b}_{h,t-1} - (1+i+\frac{m}{b})v_t$$

Combine this equation with the tax-policy rule to arrive at the following system of two linear difference equations:

$$\underbrace{\begin{bmatrix} b_{ht} \\ \tau_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} (1+i)(1+\gamma_b) - (1-\rho_\tau)\kappa_b & -\rho_\tau \frac{\tau}{b} \\ (1-\rho_\tau)\kappa_b \frac{b}{\tau} & \rho_\tau \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_{h,t-1} \\ \tau_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} -(1+i+\frac{m}{b}) \\ 0 \end{bmatrix}}_{B} v_t. \qquad (9)$$

For simplicity of notation, rewrite the re-labeled system as

$$x_t = Ax_{t-1} + Bv_t.$$

A stationary solution for the two endogenous variables $\{\hat{b}_{ht}, \hat{\tau}_t\}$ exists if and only if the eigenvalues of the autoregressive matrix A are both smaller than 1 in magnitude, which is true when the parameters of the tax rule, κ_b and ρ_τ, obey the restrictions derived in Lemma 1 below. When the solution exists it is also unique.

The intuition is that buying foreign currency forward creates a synthetic position that is long in foreign bonds and short home bonds, and hence earns the convenience yield differential. The mechanism operates in much the same way in forwards data.

LEMMA 1 (Existence and Uniqueness). *A stationary solution to the system of difference equations (9) exists if and only if the following two conditions are satisfied*

(i) $\kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$

(ii) $\rho_\tau \in [0, \frac{1}{K})$

where $K = (1+i)(1+\gamma_b)$. *When the solution exists, it is unique.*

Moreover, the stronger restriction $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$ *ensures* $\hat{b}_t$ *and* $\hat{\tau}_t$ *are positively autocorrelated.*

Proof. See Appendix A.1. □

The conditions impose restrictions on the coefficients of the tax policy rule that have clear and intuitive economic meaning. First, the conditions specify a lower and an upper bound for κ_b, the coefficient that controls how strongly tax policy responds to the outstanding stock of government debt. The lower bound assures that the tax response is sufficiently strong to keep debt levels stationary and the government solvent. This is particularly straightforward to see in the case of $\rho_\tau = 0$ when the system of two equations reduces to

$$\hat{b}_{ht} = (K - \kappa_b) b_{h,t-1} - (1 + i + \frac{m}{b}) v_t.$$

In order for debt to be stationary, we need $\kappa_b > K - 1 > 0$.[45] The constant $K = (1+i)(1+\gamma_b)$ characterizes the cost of servicing outstanding government debt, taking into account the fact that an increase in government debt decreases the convenience yield and hence pushes the equilibrium interest rate up (this works through the term $(1+\gamma_b)$). An extra dollar of debt issued today would require $K > 1$ dollars to repay tomorrow and thus, to stay solvent, the government must increases taxes accordingly.

The upper bound on κ_b ensures that the government does not respond too strongly, and introduce a negative root that is larger than one in absolute value. However, negative roots are not relevant empirically, as the data displays strong positive autocorrelation, and for the rest of the analysis I will restrict attention to $\kappa_b < \frac{K}{1-\rho_\tau}$, which ensures that the endogenous variables are positively autocorrelated.[46]

In addition to the restrictions on κ_b, Lemma 1 also imposes a restriction on the persistence of the tax rule, ρ_τ. This is needed because a policy of smoothing taxes over time,

[45] The last inequality follows because $i > 0$ and $\gamma_b > 0$, hence $K > 1$.

[46] Intuitively, the stricter upper bound ensures that the immediate tax response is always smaller than the change in debt, so that an increase in debt does not lead to an overly large increase in taxes that actually completely reverses the original increase in debt.

$\rho_\tau > 0$, makes taxes an infinite moving average of past debt levels and this introduces feedback effects into the system. A sufficiently high smoothing coefficient would in fact lead to complex roots and oscillating (i.e. cyclical) dynamics which become unbounded when ρ_τ is too high. The distinction between real and complex roots has important implications not only about the dynamics of the system, but also for the nature of the UIP deviations.

To formalize this, I turn attention to Impulse Response Functions (IRF) to a monetary shock, the only exogenous variable. Using the Wold decomposition of x_t,

$$x_t = Bv_t + ABv_{t-1} + A^2Bv_{t-2} + A^3Bv_{t-3} + \dots,$$

and the particular structure of the matrices A and B, I obtain

$$\hat{b}_{ht} = -(1 + i + \frac{m}{b})(v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \dots)$$

$$\hat{\tau}_t = -(1 + i + \frac{m}{b})(a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \dots)$$

where $a_{j1}^{(k)}$ is the (j,1) element of the matrix A^k. Define $a_{11}^{(0)} = 1$, $a_{21}^{(0)} = 0$ and let $\tilde{a}_{j1}^{(k)} = -(1+i+\frac{m}{b^G})a_{j1}^{(k)}$, then the sequence $\{\tilde{a}_{11}^k\}_{k=0}^{k=\infty}$ defines the IRF of government debt and the sequence $\{\tilde{a}_{21}^k\}_{k=0}^{k=\infty}$ is the IRF of taxes. Lemma 2 derives the conditions under which the roots of the system are real, as opposed to complex, and characterizes the differing behavior of the IRFs.

LEMMA 2 (Impulse Response Functions). *Let $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$ and define $\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+1-K)+K-2\sqrt{K\kappa_b(\kappa_b+1-K)}}{(1+\kappa_b)^2} > 0$. Then,*

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ the autoregressive matrix A has two real, non-negative eigenvalues and the Impulse Response Functions never cross the steady state, i.e.

$$\tilde{a}_{j1}^{(k)} \leq 0 \text{ for } j \in \{1,2\}, \text{ and } k = 1,2,3,\dots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$ the autoregressive matrix A has a pair of complex conjugate eigenvalues which can be written as $\lambda_k = a \pm bi$ for $k \in \{1,2\}$, and the corresponding conjugate eigenvectors are of the form $\vec{v}_k = [x \pm yi, 1]'$, where a, b, x, y are real numbers and i is the imaginary unit. Furthermore, the Impulse Response Functions follow increasingly dampened cosine waves:

$$\tilde{a}_{11}^{(k)} = -(1 + i + \frac{m}{b})|\lambda|^k\sqrt{1 + (\frac{x}{y})^2}\cos(k\phi + \psi - \frac{\pi}{2}), \text{ for } k = 1,2,3,\dots$$

$$\tilde{a}_{21}^{(k)} = -(1 + i + \frac{m}{b})|\lambda|^k \frac{x^2 + y^2}{y} \cos(k\phi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \ldots$$

$$\tilde{a}_{12}^{(k)} = (1 + i + \frac{m}{b})|\lambda|^k \frac{1}{y} \cos(k\phi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \ldots$$

where $\phi = \arctan(\frac{b}{a}) = \arctan(\frac{\sqrt{4K\rho_\tau - (K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2}}{K-(1-\rho_\tau)\kappa_b+\rho_\tau}) \in (0, \frac{\pi}{2})$ *and* $\psi = \arctan(\frac{y}{x})$. *Moreover,* $\tilde{a}_{j1}^{(1)} \leq 0$ *for* $j \in \{1, 2\}$.

Proof. See Appendix A.2. □

Lemma 2 tells us several important things. First, the dynamics of the system are governed by real roots as long as taxes are not too persistent, and by complex roots otherwise. Second, the dynamics under real roots are characterized by Impulse Response Functions that converge back to steady state gradually, without overshooting it, while the IRFs under complex roots are cyclical cosine functions that overshoot the steady state before converging. Third, in both cases a positive (expansionary) monetary shock pushes both taxes and debt below steady state for at least two periods.

Consider the dynamics under real roots first. A positive monetary shock does not affect taxes on impact, because they only respond to time $t-1$ variables, but debt falls because of two main forces. First, the increase in money supply raises inflation, which decreases the real-value of outstanding government debt and also lowers debt servicing costs by reducing the real interest rate. And second, the increase in money supply raises seignorage revenues. The two effects combine to reduce government outlays and increase government revenues which improves the budget and leads to a decrease in debt. In the following periods taxes decrease as well, because the government has a lower stock of outstanding debt to service. When the system is characterized by real roots both variables converge back to steady state without overshooting (i.e. going above steady state levels).

In the case of complex roots, the behavior on impact is similar but the transition dynamics back to steady state are different. The dynamics of both variables are characterized by cosine curves with a frequency of oscillation controlled by $\phi = \arctan(\frac{b}{a}) \in (0, \frac{\pi}{2})$, where a and b are the real and the imaginary parts of the complex roots. The larger ϕ is, the higher the frequency of oscillations, but its upper bound of $\frac{\pi}{2}$ ensures that each cycle takes at least 4 periods to complete and that the IRFs remain negative for at least one period after the shock. The cyclical nature of the cosine dynamics, however, guarantees that the endogenous variables will turn positive at some point in the future, with the number of periods needed for this first crossing to occur depending on the magnitude of ϕ.

This cyclical behavior arises when tax policy is smoothed over time, which makes taxes an infinite moving average of past debt levels, with higher levels of ρ_τ putting more weight

on debt levels further in the past. When taxes are sufficiently smooth, as defined by the condition $\rho_\tau > \underline{\rho}(\kappa_b)$, they remain low even as debt approaches steady state because taxes responds to past, lower debt levels. The persistently low taxes keep pushing debt higher and it overshoots steady state, giving rise to the cyclical dynamics formalized by the cosine curves specified in Lemma 2.

With these results in hand, we can now characterize the UIP regression coefficients implied by the model. Given the cyclical nature of the IRFs, it is perhaps not surprising that the regression coefficients β_k are negative at all horizons when the roots of the system are real, but have a cyclical profile and are negative at short horizons, and positive at longer horizons when the roots are complex. Theorem 1 formalizes this result.

THEOREM 1 (UIP Violations). *Let $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$. The UIP regression coefficients β_k are equal to*

$$\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -(a_{11}^{(k-1)} + \delta a_{12}^{(k-1)})$$

where $a_{jl}^{(k)}$ is the (j,l) element of the matrix A^k for $k = 1, 2, 3, \ldots$, $a_{11}^{(0)} = 1$ and $a_{12}^{(0)} = 0$, and $\delta = (1-\rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1+\rho_\tau)-\kappa_\tau)}{1+\rho_\tau(1+2\kappa_\tau-K(1+\rho_\tau))}$. Furthermore,

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$, the roots of the system are real and

$$\beta_k < 0, \text{ for all } k = 1, 2, 3, \ldots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$, then the eigenvalues of the autoregressive matrix A are complex and of the form $\lambda_k = a \pm bi$, $k \in \{1, 2\}$ with corresponding eigenvectors of the form $\vec{v}_i = [x \pm yi, 1]'$, and

$$\beta_k = \begin{cases} -1 & , k = 1 \\ -|\lambda|^{k-1}\sqrt{1+\chi^2}\cos((k-1)\phi + \psi - \frac{\pi}{2}) & , k = 2, 3, \ldots \end{cases}$$

where $\chi = \frac{x-\delta(x^2+y^2)}{y}$, $\phi = \arctan(\frac{b}{a})$, $\psi = \arctan(\frac{1}{\chi}) + \pi\mathbb{I}(\chi < 0)$.

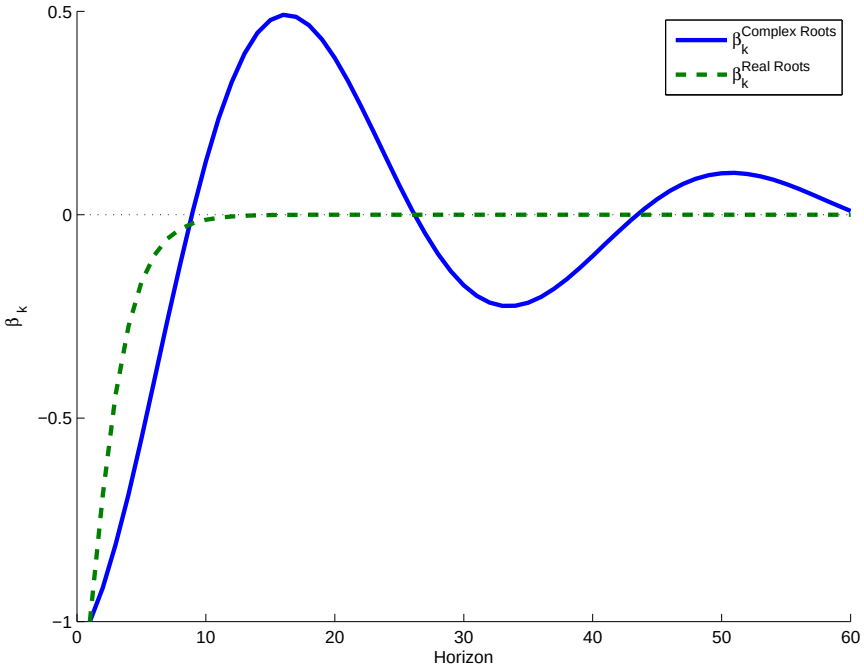
Proof. See Appendix A.3. □

Theorem 1 derives the main results of the analytical model. First, it shows that the model always implies $\beta_k < 0$ at short horizons and is thus qualitatively consistent with the standard formulation of the UIP Puzzle under all parametrization (as long as a solution exists).[47] Second, it shows that when the persistence of the tax rule is small and the dynamics

[47] Theorem 1 can also be extended to the case where the solutions are not necessarily positively autocorrelated and we would still have $\beta_1 = -1$.

are governed by real roots, the model cannot generate positive β_k at any horizon but rather implies $\beta_k < 0$ for all k. Under those conditions, the model is similar to existing models of the UIP puzzle and implies that the UIP coefficients are negative at all horizons. This pattern is illustrated by the green dashed line in Figure 5.

Figure 5: Model Implied UIP Regression Coefficients



Third, Theorem 1 shows that when the tax rule is sufficiently persistence, i.e. $\rho_\tau \geq \underline{\rho}_\tau(\kappa_b)$, the model implies a pattern of UIP regression coefficients that starts out negative and then follows a cyclical profile along an increasingly dampened cosine curve. In particular, this mean that the model generates $\beta_k < 0$ at short horizons and $\beta_k > 0$ at longer horizons, with the coefficients converging to zero on a vanishing cyclical path. This alternative pattern of UIP coefficients is illustrated by the solid line in Figure 5. The curve starts out negative, becomes positive at longer horizons and briefly dips back into negative territory before largely dieing out. The pattern is strongly reminiscent of the empirical estimates and the next section shows that the quantitative model closely matches the data.

In this stylized model, the UIP deviations are entirely determined by the dynamics of government debt $\hat{b}_t$. Recall that today's interest rate is negatively related to $\hat{b}_t$ while the expected excess foreign bond return is positively related to $\hat{b}_t$. Under both real and complex roots, a contractionary (negative) shock to money growth leads to an increase in government debt. The increased supply of home government debt reduces its marginal convenience

value (each bond is now less special) and thus increases the home interest rate, due to the downward sloping demand for debt. Moreover, the decrease in the home convenience yield leads to compensating excess financial returns on the home currency (recall equation (8)).[48] This highlights the fact that the negative UIP coefficients are a fundamental part of the mechanism, as movements in the convenience yield push interest rates and excess currency returns in opposite directions. Lastly, the effects are persistent, under both real and complex roots, and we have $\beta_k < 0$ for several periods after the initial shock.

In the case of real roots, debt levels, and hence the convenience yield, converge monotonically back to steady state and $\beta_k < 0$ for all k. On the other hand, under complex roots the debt dynamics are cyclical. Similarly to the case of real roots, the initial shock increases debt levels on impact and taxes follow suit the next period. However, with $\rho_\tau \geq \underline{\rho}(\kappa_b)$ taxes sluggish and slow to adjust, hence it takes a longer time for debt to be brought back down. Moreover, taxes can be represented as an infinite moving average of past debt levels and with a higher ρ_τ debt levels further into the past play a larger role. Thus, even when debt approaches its steady state level taxes remain high, as they are responding to the high debt levels further in the past. This leads debt to overshoot its steady state level, go below it, and converge to steady state on a cyclical path. The convenience yield follows a similar trajectory, and eventually goes above steady state (when debt is below steady state), which pushes excess home currency returns below steady state (while they were above steady state at short horizons) and generates positive UIP regression coefficients, $\beta_k > 0$, at longer horizons. The oscillations eventually die out and hence $\lim_{k\to\infty} \beta_k = 0$.

The UIP violations in the model occur because monetary and fiscal policy do not coordinate to keep the supply of government debt constant.[49] In particular, when fiscal policy does not accommodate monetary policy, monetary shocks lead to movements in the real supply of government debt which affects the supply of convenience assets to the economy. This causes movements in the convenience yield differential across countries, which in turn lead to predictable, compensatory excess currency returns and thus UIP violations.[50]

Monetary policy shocks are not unique in generating cyclical UIP violations – a large variety of policy shocks (expenditure, tax, etc.) have similar implications. For example, a positive government expenditures shock increases government debt, and unless the central

[48] There is a corresponding move in the foreign convenience yield as well, because home and foreign debt are substitutes, but $\delta < 1$ ensures that the convenience yield differential moves proportionally to the home convenience yield.

[49] Note that studying optimal policy is beyond the scope of this paper and is left to future research. This discussion only intends to highlight how monetary and fiscal policy interaction (or lack of) can affect exchange rate movements and deviations from the UIP.

[50] The key is actually movements in the *relative* supply of convenience assets across countries, but here the foreign country is negligible.

bank coordinates with the fiscal authority, and loosens monetary policy to inflate the new debt away and offset the fiscal shock, this generates UIP violations. Moreover, the violations behave in exactly the same manner, with $\beta_k < 0$ at all horizons when the roots are real and $\beta_k < 0$ at short horizons and $\beta_k > 0$ at longer horizons when the roots are complex. For the sake of brevity, this section analyzed only monetary shocks but different types of policy shocks can generate the results, as long as monetary and fiscal policy do not coordinate to keep debt constant. In this sense, the model is robust to the source of exogenous shocks.

5 Quantitative Model

To examine the quantitative performance of the mechanism I start with a benchmark nominal, two country model (Obstfeld and Rogoff (1995), Chari et al. (2002), Clarida et al. (2002)) and add convenience benefits to holdings bonds and fiscal policy. I calibrate the model using standard parameters from the literature and then show that it can closely match the empirical evidence on UIP deviations at both short and long horizons. I start by giving a brief overview of the model.

There are two symmetric countries, home (H) and foreign (F). Households have access to a complete set of Arrow-Debreu securities and consume both a domestically produced final good and a foreign final good. Each country produces both a final good and a continuum of intermediate goods. The final good is produced by a representative firm that aggregates the intermediate goods produced domestically. There is also a mass of intermediate goods firms equal to the total population in each country, that produce differentiated goods, compete monopolistically and face Calvo (1983) frictions in setting nominal prices. Lastly, there is also a government that implements monetary policy by setting the interest rate and finances spending via lump-sum taxation and issuing government bonds.

Next I present the decision problems of the different agents in the home country, with the stipulation that the agents in the foreign country are symmetric.

5.1 Households

I use the same household preferences as in Clarida et al. (2002) and Gali and Monacelli (2005). The representative household maximizes the expected infinite stream of utility:[51]

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\phi}}{1+\phi}\right)$$

[51] Complete asset markets allows me to appeal to a representative household structure

Consumption is a CES aggregate of home (H) and foreign (F) final goods,

$$C_t = \left(a_h^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution between the two goods and the weights a_h and a_f, normalized to sum to 1, determine the degree of home bias in consumption. C_{Ht} and C_{Ft} are the amount of the home final good and the foreign final good that the household purchases. It follows that the corresponding consumption price index is

$$P_t = \left(a_H P_{Ht}^{1-\eta} + a_F P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

where P_{Ht} and P_{Ft} are the prices of the home and foreign good in terms of the home currency.

To motivate the demand for liquidity, I assume that the household incurs transaction costs, Ψ_t, in purchasing consumption, a standard approach in both the monetary literature (e.g. Feenstra (1986)) and the literature on bond convenience yields (Bansal and Coleman II (1996), Bansal et al. (2011)). I model the transaction costs with a flexible CES function that includes both real money balances and real bond holdings as convenience assets:

$$\Psi(C_t, m_t, b_{ht}, b_{ft}) = \bar{\psi} c_t^{\alpha_1} h(m_t, b_{ht}, b_{ft})^{1-\alpha_1}$$

The transaction cost function has two components, the level of transactions C_t and a bundle of transaction services $h(m_t, b_{ht}, b_{ft})$, which is generated by the three convenience assets: real money balances m_t and the real holdings of home and foreign nominal bonds b_{ht} and b_{ft}. The elasticity parameter $\alpha_1 > 1$, hence transaction costs are increasing in the level of purchases (C_t) and decreasing in the level of transaction services. The transaction services $h(\cdot)$ are a CES aggregator of real money balances and a bundle of transaction services generated by bonds:

$$h(m_t, b_{ht}, b_{ft}) = (m_t^{\frac{\eta_m-1}{\eta_m}} + \tilde{h}(b_{ht}, b_{ft})^{\frac{\eta_m-1}{\eta_m}})^{\frac{\eta_m}{\eta_m-1}}$$

where

$$\tilde{h}(b_{ht}, b_{ft}) = \kappa_b (a_b b_{ht}^{\frac{\eta_b-1}{\eta_b}} + (1-a_b) b_{ft}^{\frac{\eta_b-1}{\eta_b}})^{\frac{\eta_b}{\eta_b-1}}$$

The dual structure of the transaction services function aims to capture the idea that money and bonds are two separate classes of convenience assets and allows for different elasticity of substitution between money and the bundle of bonds, and between home and

foreign bonds.[52] The elasticity of substitution between money balances and the bundle of bonds is given by η_m, while η_b is the elasticity of substitution within bonds. The parameter κ_b shifts the overall level of transaction services provided by bonds and allows me to control the relative importance of bonds as convenience assets. Lastly, the parameter a_b controls the relative importance of home to foreign bonds.

The budget constraint of the household is

$$C_t + \int \Omega_{H,t}(z_{t+1}) x_t(z_{t+1}) dz_{t+1} + \Psi(c_t, m_t, b_{ht}, b_{ft}) + m_t + b_{ht} + b_{ft} =$$
$$w_t N_{it} + \frac{x_{t-1}(z_t)}{\Pi_t} - \tau_t + d_t + \frac{m_{t-1}}{\Pi_t} + b_{h,t-1}\frac{(1+i_{t-1})}{\Pi_t} + b_{f,t-1}\frac{(1+i^*_{t-1})}{\Pi_t}\frac{S_t}{S_{t-1}}$$

where $\Omega_{H,t}(z_{t+1})$ is the price, in terms of home currency, of the Arrow-Debreu security that pays off in the state of nature z_{t+1} tomorrow and $x_t(z_{t+1})$ is the amount of this security that the home household has purchased. The household spends money on consumption, Arrow-Debreu securities, transaction costs, money holdings, and home and foreign nominal bonds. It funds its purchases with the money balances it carries over from the previous period, the real wages w_t it receives for its labor, the profits of the intermediate good firms d_t, and payoffs from its holdings of contingent claims, and home and foreign bonds. Lastly, it also pays lump-sum taxes τ_t to the domestic government.

This first-order necessary conditions for home and foreign nominal bond holdings are:

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht,} b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1,} b_{f,t+1})} \frac{1}{\Pi_{t+1}} \frac{1+i_t}{1 + \Psi_{b_h}(c_t, m_t, b_{ht}, b_{ft})} \tag{10}$$

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht,} b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1,} b_{f,t+1})} \frac{S_{t+1}}{\Pi_{t+1} S_t} \frac{1+i^*_t}{1 + \Psi_{b_f}(c_t, m_t, b_{ht}, b_{ft})} \tag{11}$$

where the term $\Psi_x = \frac{\partial \Psi}{\partial x}$ is the derivative of the transaction costs in respect to the variable x. The terms Ψ_{b_h} and Ψ_{b_f} are the marginal transaction benefit of holding an extra unit of home and foreign bonds respectively. Similarly to the analytical model, these marginal benefits determine the convenience yields and will generate UIP deviations.

The foreign household faces a symmetric set of first order conditions, and since the Arrow-

[52] The structure is flexible enough to also allow me to treat all convenience assets as equally substitutable. None of the results I will report later depend on the flexible structure of transaction costs and can also be delivered by a simple Cobb-Douglas function of the type $\Psi_t(C_t, m_t, b_{ht}, b_{ft}) = \bar{\psi} c_t^{\alpha_1} m_t^{\alpha_2} b_{ht}^{\alpha_3} b_{ft}^{\alpha_4}$. However, a Cobb-Douglas formulation has counter-factual implication about certain features of the implied money demand and I consider it only as a robustness check.

Debreu securities are traded internationally we can combine the optimality conditions of the two of agents to arrive at the following risk-sharing condition:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1+\Psi_c(c_t, m_t, b_{ht}, b_{ft})}{1+\Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1}, b_{f,t+1})} = \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma} \frac{1+\Psi_c(c^*_t, m^*_t, b^*_{ft}, b^*_{ht})}{1+\Psi_c(c^*_{t+1}, m^*_{t+1}, b^*_{f,t+1}, b^*_{h,t+1})} \frac{Q_t}{Q_{t+1}}$$

where $Q_t = \frac{S_t P^*_t}{P_t}$ is the real exchange rate. Complete financial markets ensure that the marginal rates of substitution (MRS) are equalized across countries, however in this model the MRS is also adjusted for the liquidity needs of the households. This adjustment acts as a wedge between the consumption growth differential and the real exchange rate depreciation, and breaks the typical result that those two variables are perfectly correlated. In particular, a log-linear approximation of this equation around the steady state yields

$$\sigma(\Delta c_t - \Delta c^*_t) = \Delta q_t + \frac{\Psi_c}{1+\Psi_c}(\Delta \hat{\Psi}^*_{c,t} - \Delta \hat{\Psi}_{c,t})$$

which clearly shows that the consumption growth differential is a function of both real exchange rate changes and the differential in marginal transaction costs. This suggests that the model, which is built with an eye towards explaining the UIP Puzzle, a phenomenon in nominal exchange rates, can also shed some light on the imperfect correlation between consumption growth differentials and changes in the real exchange rate, a major puzzle in the dynamics of the real exchange rates.

5.2 Final Goods Firms

There is a home representative final goods firm which uses the domestic continuum of intermediate goods and the following CES technology to produce total output $Y_{H,t}$:

$$Y_{H,t} = \left(\int_0^1 Y_{it}^{\frac{\lambda_H - 1}{\lambda_H}} di\right)^{\frac{\lambda_H}{\lambda_H - 1}}.$$

Profit maximization yields the standard type of demand for intermediate goods

$$Y_{it} = \left(\frac{P_{it}}{P_{Ht}}\right)^{-\lambda_H} Y_{Ht},$$

and the price index (GDP Deflator)

$$P_{H,t} = \left(\int_0^1 P_{it}^{1-\lambda_H} di\right)^{\frac{1}{1-\lambda_H}}.$$

5.3 Intermediate Goods Firms

Intermediate goods firms use a production technology linear in labor input N_{it}^D,

$$Y_{it} = A_t N_{it}^D,$$

where A_t is an exogenous productivity process. The firms face a Calvo (1983) friction in setting prices and have a probability $1-\theta$ of being able to adjust prices in any given period. Firms that can adjust choose their optimal price $\bar{P}_{it}$ so as to satisfy the first-order condition

$$\sum_{j=0}^{\infty} E_t \Omega_{Ht}(z_{t+j}) Y_{i,t+j} \theta^j \left(\bar{P}_{it} - \frac{\lambda_H}{\lambda_H - 1}(1-\tau)\frac{w_{t+j}P_t}{A_{t+j}} \right) = 0. \tag{12}$$

Firms that do not get to re-optimize keep their prices constant and adjust their labor input to satisfy demand. The law of large numbers then implies that the price of the home final good evolves according to

$$P_{Ht} = (\theta P_{H,t-1}^{1-\lambda_H} + (1-\theta)\bar{P}_{it}^{1-\lambda_H})^{\frac{1}{1-\lambda_H}} \tag{13}$$

5.4 Government

The government consists of a Monetary Authority (MA), which I will also refer to interchangeably as the Central Bank (CB), and a Fiscal Authority (FA).[53]

The Monetary Authority sets monetary policy according to a standard Taylor rule (in log approximation to steady state):

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)\phi_\pi \hat{\pi}_t + v_t$$

where π_t is CPI inflation and v_t is an iid monetary shock.

The MA issues and underwrites the supply of domestic currency, M_t^s, and backs a fraction $\frac{1}{\mu}$ of it with holdings of domestic government bonds,

$$M_t^s \frac{1}{\mu} = B_{ht}^M,$$

[53] Because the mechanism of the model is tightly linked to the net amount of bonds available to the private agents, I model the two authorities as separate in order to account for the Central Bank's balance sheet and any effects coming from open market operations. The distinction between Monetary and Fiscal Authorities turns out to not be quantitatively important, however, and the Appendix shows that the main results remain largely unchanged if we move to a single, consolidated government.

where B_{ht}^M is the nominal amount of domestic government bonds held by the CB.[54] As is true in reality, the Monetary Authority transfers all seignorage revenues to the fiscal Authority and faces the following budget constraint

$$T_t^M = M_t^s - M_{t-1}^s + B_{h,t-1}^M(1+i_{t-1}) - B_{ht}^M,$$

where T_t^M is the money transferred to the Fiscal Authority.

The Fiscal Authority collects taxes, the transfer of seignorage from the MA and issues government bonds to fund government expenditures and has the the budget constraint

$$B_{ht}^G + T_t + T_t^M = B_{h,t-1}^G(1+i_{t-1}) + G_t$$

where B_{ht}^G is the nominal amount of government bonds issued and T_t are the nominal, lump-sum taxes collected. For brevity and simplicity, in much of the analysis I will abstract from movements in government spending and assume it is constant, $G_t = G$.[55]

Lastly, I follow the literature on the interaction between monetary and fiscal policy (e.g. Leeper (1991), Davig and Leeper (2007), Bianchi and Ilut (2013)) and assume that the fiscal authority follows a simple taxation rule which adjusts lump-sum taxes (as a percentage of GDP) in response to the government's debt to GDP ratio[56]

$$\frac{P_t\tau_t}{P_{H,t}Y_{H,t}} = \rho_\tau \frac{P_{t-1}\tau_{t-1}}{P_{H,t-1}Y_{H,t-1}} + (1-\rho_\tau)\kappa_b \frac{P_{t-1}b_{h,t-1}^G}{P_{H,t-1}Y_{H,t-1}}$$

5.5 Excess Bond Returns and and UIP violations

Log-linearize equations (10) and (11) around the symmetric, zero-inflation steady state and subtract them from one another to obtain

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \left|\frac{\Psi_{b_h}}{1+\Psi_{b_h}}\right| \hat{\Psi}_{b_h,t} - \left|\frac{\Psi_{b_f}}{1+\Psi_{b_f}}\right| \hat{\Psi}_{b_f,t} \tag{14}$$

where hatted variables represent log-deviations from steady state. The terms $\hat{\Psi}_{b_h,t}$ and $\hat{\Psi}_{b_f,t}$ are the log-deviations from steady state of the marginal transaction benefits of home and foreign bond holdings, and Ψ_{b_h} and Ψ_{b_f} are the corresponding steady state values. Note that at the

[54] I allow for $\mu > 1$ to be able to match the fact that the bond holdings of the Central Bank are in fact less than the total amount of money available in the economy at any given point in time (primarily because of the money multiplier).

[55] The Appendix discusses an extension with both fiscal and tax shocks and shows that these type of shocks also generate empirically relevant UIP deviations. Since results are so similar, in the interest of conciseness I relegate this discussion to the Appendix.

[56] The results do not change materially whether I express the tax rule in terms of Tax-to-GDP or simply in terms of real taxes τ_t.

symmetric steady state $\Psi_{b_h} = \Psi_{b_f}$ and define $\gamma_b \equiv \left|\frac{\Psi_{b_h}}{1+\Psi_{b_h}}\right| = \left|\frac{\Psi_{b_f}}{1+\Psi_{b_f}}\right|$.

If there was another home currency denominated nominal bond that did not provide convenience benefits, its interest rate $\tilde{i}_t$ would be

$$\tilde{\hat{i}}_t = \hat{i}_t + \gamma_b \hat{\Psi}_{b_h,t}$$

Hence $\gamma_b \hat{\Psi}_{b_h,t}$ is the equilibrium convenience yield (in log-deviations from steady state) - the amount of interest the agents are willing to forego because home government bonds also offer convenience benefits. A similar relationship can also be derived for the foreign country and hence $\gamma_b \hat{\Psi}_{b_f,t}$ is the convenience yield on the foreign bond.

Thus, equation (14) shows us that the expected excess currency return is equal to the convenience yield differential; an increase in the home convenience yield would result in a compensating, expected increase in the excess financial return on the foreign bond. The excess return is a function of time t variables (the convenience yields), and is thus forecastable which constitutes a violation of the UIP condition. The violations at different horizons are driven by the dynamics of the convenience yield differential, in a similar manner to the analytical results derived earlier. Moreover, we can further simplify equation (14) to show that

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \frac{\gamma_b}{\eta_b}(\hat{b}_{ft} - \hat{b}_{ht}),$$

which showcases the close connection between the relative holdings of government bonds and the convenience yield differential. The more abundant are home bonds, relative to foreign bonds, the lower is the marginal convenience value of holding an extra unit of the home bonds, relative to holding an extra unit of foreign bonds, and thus the lower is the convenience yield differential. This follows from the fact that the equilibrium convenience yield is equal to the marginal convenience value of each bond, and the marginal convenience value itself is decreasing in the holdings of the bond.

The more home bonds, relative to foreign bonds, an investor is holding, the lower is the convenience yield of the home bond relative to the foreign bond. This follows from the fact that the marginal benefit of each bond is decreasing in the holdings of that bond.

We can also rewrite equation (14) as

$$E_t(\hat{s}_{t+1} - \hat{s}_t) = \hat{i}_t - \hat{i}_t^* + \gamma_b(\hat{\Psi}_{b_h,t} - \hat{\Psi}_{b_f,t})$$

to highlight how exchange rate dynamics are affected. Unlike in a model where the standard UIP condition holds, in this model exchange rates need to offset not just the interest rate differential across countries but the combined effect of the differentials in interest rates and convenience yields. A positive interest rate differential would not necessarily imply a depreciating interest rate, because we would need to factor in the current value of the convenience yield differential as well. Clearly,

the convenience yield differential acts as an omitted variable in the standard UIP regression. In the next section, I calibrate the model, quantify the introduced bias, and show that the model is able to closely match the full complexity of UIP violations at both short and long horizons.

5.6 Main Results

I log-linearize the model's equations around the symmetric, zero-inflation steady state and solve the resulting system of difference equations using standard techniques. Then I calibrate the model's parameters, using standard values from the literature whenever possible and matching relevant relevant moments of the data otherwise (but always moments independent of the UIP regressions), and compute the UIP regression coefficients implied by the model.

5.6.1 Calibration

The benchmark calibration is summarized in Table 2 and below I detail the calibration process. In general, the results are very robust to the great majority of coefficients.

As is standard in the literature, one period in the model represents one quarter. In terms of consumer preferences, I set the coefficient of risk aversion σ equal to 2 and the inverse Frisch elasticity of labor supply $\phi = 1.5$, both of which are standard values in the RBC literature. Estimates of the elasticity of substitution between home and foreign goods vary a lot, but most estimates fall in the range from 1 to 2 and I follow Chari et al. (2002) and set $\eta = 1.5$. I choose the degree of home bias $a_h = 0.8$, a common value in the literature that is roughly in the middle of the range of values for the G7 countries.

To calibrate the transaction cost function I proceeds as follows. I calibrate $\alpha_1, \eta_m, \kappa_b, \bar{\psi}$ to match the interest rate semi-elasticity of money demand, the income elasticity of money demand, money velocity and the average convenience yield. I target an interest rate semi-elasticity of money demand of 7, which is in the middle of most estimates which range from 3 to 11 (see discussion and references in Burnside et al. (2011)). I set the income elasticity of money demand to 1, a widely accepted value, and then set the money velocity equal to 7.7, which is the average value for the $M1$ money aggregate in the US for the time period $1976 - 2013$.[57] Next, I target a steady state annualized convenience yield of 1%, which is in the middle of the range of estimates in the literature.[58] Finally, I choose a_b so that foreign bonds constitute 10% of the total bond portfolios of the agents, implying a strong home bias in accordance with the data.[59]

[57] It makes little difference if I calibrate the model instead to money velocity as calculated from the $M2$ or MZM money aggregates.

[58] The estimates of Krishnamurty and Vissing-Jorgensen (2012) imply that over 1969-2008 the average convenience yield difference between Treasuries and AAA and BAA corporate bonds has been 85 and 166 basis points respectively. Using a different methodology, Krishnamurty (2002) estimates an average Treasury convenience yield of 144 basis points.

[59] On the home bias in international bond portfolios see Warnock and Burger (2003), Fidora et al. (2007), Kyrychenko and Shum (2009) and Coeurdacier and Rey (2013)

Lastly, there is little prior literature guidance in choosing η_b, the elasticity of substitution between home and foreign bonds in the transaction cost function. I set it equal to 0.25 in order to match the volatility of foreign bond holdings to GDP. In the model, increasing η_b makes the home and foreign bonds better substitutes and increases the overall volatility of foreign bond holdings, as agents are more likely to substitute into foreign holdings following shocks. The value of 0.25 is also about 2.5 times higher than the elasticity of substitution between money and bonds as a whole, indicating that bonds are better substitutes for each other, than they are for cash.

Table 2: Calibration

Param	Description	Value	Param	Description	Value
σ	Risk Aversion	2	$\frac{G}{Y}$	Gov Exp to GDP	0.22
ϕ	Inv Labor Elast	1.5	$\frac{b_h^D}{Y}$	Gov Debt to GDP	0.5
η	Elast Subst Cons	1.5			
a_h	Home Bias in C	0.8	ϕ_π	TR Infl Resp	1.5
β	Time Discount	0.9901	ρ_i	TR Smoothing	0.9
			σ_v	StdDev Mon Shock	0.0033
α_1		19			
η_m		0.091	ρ_τ	Tax Smoothing	0.92
κ_b		0.35	κ_b	Tax Resp to Debt	0.48
$\bar{\psi}$		6.75E-19			
a_b		0.9998	ρ_a	Autocorr TFP	0.97
η_b		0.25	σ_a	StdDev TFP	0.0078
μ	Money Multiplier	2.17			

Next, I set the money multiplier $\mu = 2.17$, the average ratio of the money base to the M1 money aggregate in the US for the 1976-2013 time period. This parametrization also has the independent implication that the Central Bank holds 15% of government bonds in steady-state, which is in line with the US data. This gives further support for the overall parametrization of the liquidity preferences of the households. Lastly, I choose $\beta = 0.9901$ which implies a steady state real interest rate of 3% per year.

I calibrate the steady state ratio of government spending to GDP to 22% and the ratio of government debt to GDP to 50%, the average values of total federal spending to GDP and total federal debt to GDP, respectively, in the US data. For the Taylor rule response to inflation I pick $\phi_\pi = 1.5$, the original value proposed by Taylor which has become a common choice in the literature. I then set $\rho_i = 0.9$ to match the high persistence of interest rate differentials and the high relative

volatility of exchange rate changes to interest rate differentials.[60] I estimate the coefficients on the taxation rule, ρ_τ and κ_b, using US data on federal taxes to GDP and the federal debt to GDP, and obtain $\rho_\tau = 0.92$ and $\kappa_b = 0.48$. Following Corsetti et al. (2010) the Calvo parameter is set to $\theta = 0.75$, a standard value in the literature.

In order to parametrize the technology process I used John Fernald's TFP data and estimated an AR(1) process in logs which yielded $\rho_a = 0.97$ and $\sigma_a = 0.0078$. I back out the standard deviation of the Taylor rule shock from the US data as well, using data on the federal funds rate, CPI inflation and the calibrated parameters of the Taylor rule to construct a series of residuals. The standard deviation of the residuals gives me $\sigma_v = 0.0033$. This value also implies that a one standard deviation monetary shock results in a 76 basis points (in annualized terms) response on impact by the nominal interest rate, which matches the empirical estimate in Eichenbaum and Evans (1995).[61].

5.6.2 Quantitative Results

In this section I examine the model's quantitative ability to match the UIP deviations in the data. I do this by computing the implied regression coefficient of the series of UIP regressions:

$$\hat{\lambda}_{t+k} = \alpha_k + \beta_k(\hat{i}_t - \hat{i}^*_t) + \varepsilon_{t+k}$$

where as always $\hat{\lambda}_{t+k} = \hat{s}_{t+k} - \hat{s}_{t+k-1} + \hat{i}^*_{t+k-1} - \hat{i}_{t+k-1}$ is the one period excess foreign bond return. These are the same regressions that were also considered in the empirical section and I will compare the model implied sequence of coefficients β_k with the empirical estimates.

I start by considering the implied regression coefficients conditional on the model being driven by only one type of shock at a time. I will show that both monetary and technology shocks are able to generate empirically relevant UIP deviations at both short and long horizons. Both types of shocks are able to match the cyclical profile of UIP deviations and the timing at which the UIP deviations switch signs. However, monetary shocks are in general able to generate stronger UIP violations at all horizons, as compared to technology shocks. After establishing these results, I will show that a model driven by both shocks is able to closely match the magnitude of the empirical estimates of UIP deviations at both short and long horizons.

The solid line in Figure 6 plots the β_k coefficients as implied by the model when only monetary shocks are active, and the dashed line plots the empirical estimates.[62] Figure 7 shows the same plot, but for the case when the model is only driven by technology shocks. The main message

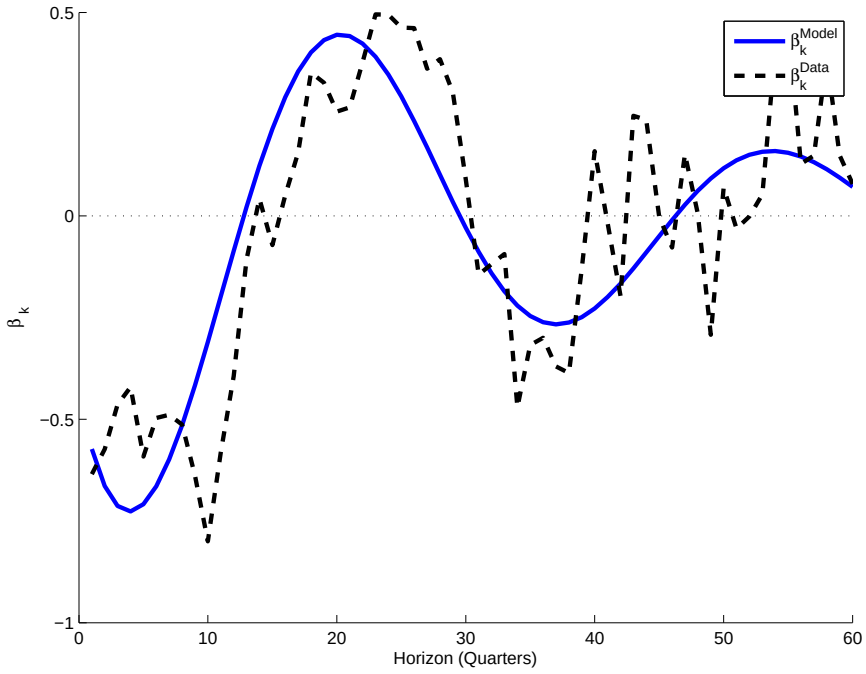
[60] Bianchi and Ilut (2013) also estimate a value of 0.9 for the Taylor Rule smoothing parameter.

[61] $\sigma_v = 0.0033$ is also among the range of common estimates obtained by the prior literature, e.g. Davig and Leeper (2007) estimate 0.0036 and Galí and Rabanal (2005) estimate 0.003

[62] The empirical estimates plotted here use 3-month interest rates as the predictive variable, rather than 1-month interest rates as in the empirical section, in order to be directly comparable with the model-implied regressions. The model is calibrated to a quarterly frequency and hence the one-period interest rate in the model itself is a 3-month interest rate.

of the two figures is that the cyclical profile of UIP deviations are fundamental feature of the model's mechanism and are robust to the source of exogenous variation. Both types of shocks are able to generate negative UIP coefficients at short horizons and positive coefficients at longer horizons, with the switch in signs occurring around 3 years (12 quarters) in the future, just as in the data. Monetary shocks are relatively more successful at matching the exact amplitude in the UIP deviations, implying stronger violations at both short and long horizons that align closely with the exact empirical estimates.

Figure 6: UIP Regression Coefficients, Monetary Shock



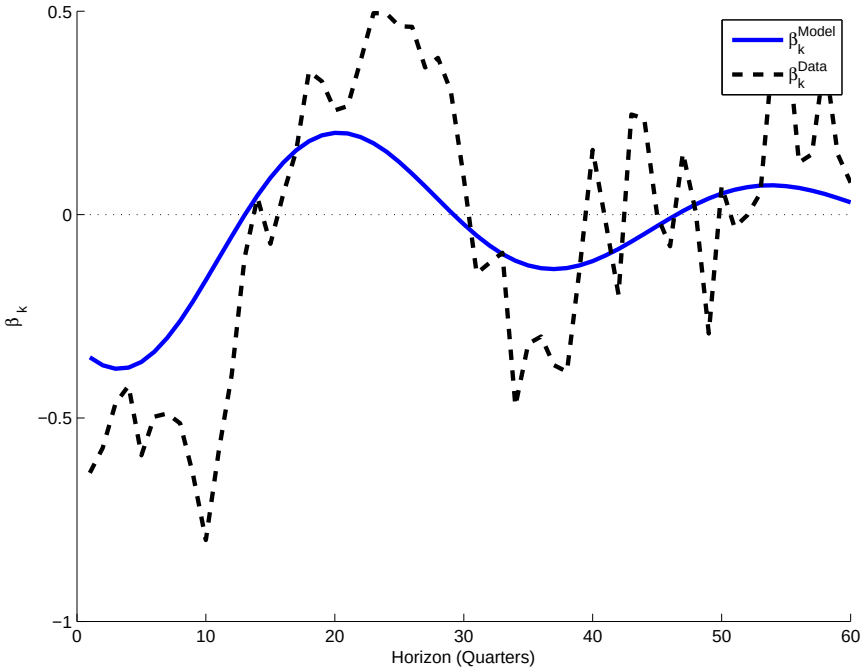
As will be explained in greater detail in the next section, the cyclical profile of UIP deviations is a fundamental feature of the model that is closely linked to the cyclical dynamics of government debt, which is in turn driven by the smoothing in tax policy. The cyclicality of UIP deviations is not unique to just technology and monetary shocks, but can be generated by a number of other shocks, such as fiscal, tax, and liquidity shocks. In the interest of conciseness, here I have only detailed the effects of monetary and technology shocks and have relegated analyzing other sources of exogenous variation to the Appendix.

Lastly, Figure 8 plots the resulting UIP coefficients when both shocks are active, and shows that in its full complexity, the model is able to closely match the empirical estimates at all horizons.

I conclude the section by examining the model's implications about exchange rate behavior. In particular, I estimate direct projections of cumulative exchange rate changes on the interest rate differential as was done in the empirical section:

$$\hat{s}_{t+k} - \hat{s}_t = \alpha_k + \gamma_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+k}$$

Figure 7: UIP Regression Coefficients, Technology Shock



Recall that the sequence $\{\gamma_k\}$ provides an estimate of the Impulse Response Function of the cumulative exchange rate change, relative to today, to an innovation in the interest rate differential, i.e. $\text{Proj}(s_{t+k} - s_t | i_t - i_t^*) = \gamma_k(i_t - i_t^*)$. The model-implied coefficients γ_k are plotted against their empirical counterparts in Figure 9. They show that the model does an excellent job of matching the initial appreciation of the exchange rate, the turning point at which it starts depreciating and also the level of the long-run depreciation.[63]

5.6.3 The Mechanism Explained

The mechanism behind the cyclical UIP violations is very similar to the one in the analytical model considered earlier, with the difference that the quantitative model features two extra channels in additional to cyclical government debt dynamics. Namely, the Open Market Operations of the central bank and the international spillover of shocks also contribute to the UIP deviations. This section will describe all three channels at work and how they propagate monetary and technology shocks. To help frame this discussion, recall that the equilibrium expected excess returns on foreign bonds is driven by changes in the households' portfolio holdings of home and foreign bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \frac{\gamma_b}{\eta_b}(\hat{b}_{f,t} - \hat{b}_{h,t})$$

[63] Given that the model matches the UIP deviations well, this graph tells us that it also matches the dynamics of the underlying interest rate differentials. The forecastability pattern of the excess returns arises from the cyclical movements in the exchange rate and not from the interest rate differential. The Appendix provides further details on this point.

Figure 8: UIP Regression Coefficients, Policy and Technology Shocks

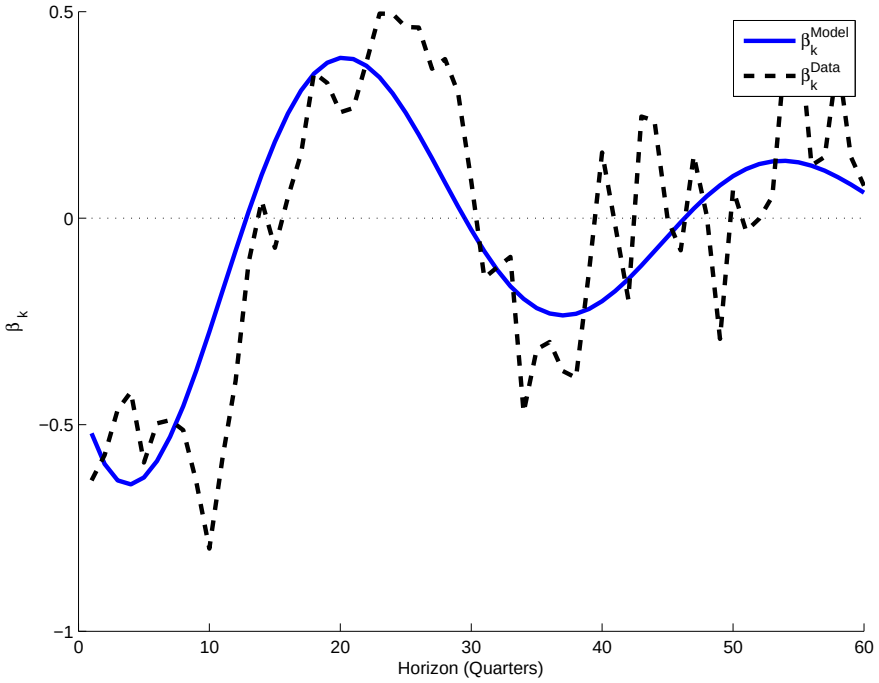
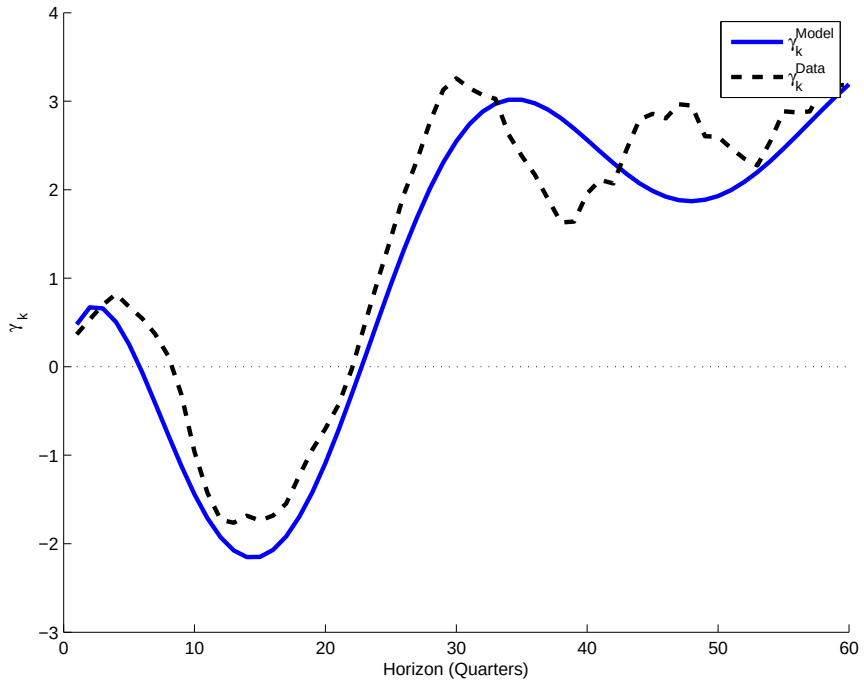


Figure 9: Direct Projection: $\gamma_k = \frac{\partial E_t(\hat{s}_{t+k} - \hat{s}_t)}{\partial(\hat{i}_t - \hat{i}_t^*)}$



I start by discussing the effects of monetary shocks. A contractionary home country monetary shock raises interest rates and lowers inflation and GDP. The drop in inflation works the same way as before: it re-values existing real debt upwards, lowers seignorage revenues and increases the real

cost of servicing the existing debt. On the other hand, the drop in GDP lowers tax revenues. All of these forces tighten the government's budget constraint and contribute to an increase in debt. The rise in the supply of home debt leads to a proportionate increase in the private agents' holdings of home government bonds, which lowers the home convenience yield relative to the foreign one. This leads to compensating, financial returns on the home bonds, which are achieved by an exchange rate appreciation at short horizons and this generates negative UIP coefficients.

On the other hand, taxes are quite sluggish ($\rho_\tau = 0.92$) which introduces a complex root into the system and generates cyclical government debt dynamics. The reasons for the oscillations is the same as in the analytical model - persistent taxes can be expressed as a moving average of past debt levels, hence they are relatively slow to adjust to current government debt conditions. The cyclical dynamics of government debt also transfer to the private holdings of government bonds, home bond holdings eventually fall below steady state and generate a positive convenience yield differential. This gives rise to a corresponding positive excess return on the foreign bond and thus positive UIP violations at longer horizons.

The monetary shock also creates UIP deviations through Open Market Operations. A contractionary monetary shock that increases the interest rate today is accompanied by Open Market Operations in which the central bank reduces money supply by selling some of its holdings of government bonds. This, increases the net supply of home government bonds and through similar reasoning as above, leads to a negative convenience yield differential and negative UIP violations at short horizons. Equilibrium money holdings, however, do not display strong cyclicality and this channel does not contribute much to the positive violations at long horizons.

Lastly, the effects of the monetary shock also spill over to the foreign country. A contractionary home monetary shock leads to a persistent exchange rate appreciation, which increases inflation and output abroad and leads to an increase in the foreign interest rate. The rise in inflation and output loosen the foreign government's budget constraint and leads to a decrease in foreign debt. The decrease in net supply of foreign debt carries over to the private agents' portfolios, and the holdings of foreign bonds decrease as the holdings of home bonds are increasing due to the effects operating through the other two channels. Thus, the spillovers reinforce the domestic channels and make the convenience yield differential even more negative. The persistence in the foreign taxation policy also leads to similar reinforcing effects to the positive UIP deviations at longer horizons.

Now consider a negative technology shock. It decreases output and thus taxes, and increases inflation and the interest rate today. The tax and inflation effects on the government budget constraint roughly cancel out and lead to a negligible change in government debt on impact. However, the technology shock is persistent and leads to a prolonged drop in output, and hence taxes, and thus an eventual increase in government debt. Similarly to before, the increase in home government debt leads to a negative convenience yield differential and generates negative UIP coefficients. The systematic component of tax policy again reacts only sluggishly to the increase in debt and leads to

the type of cyclicality in government debt that leads to positive UIP deviations at longer horizons.

The spillover of the shock operates in a similar manner and again reinforces the UIP deviations at both short and long horizons. The Open Market Operations channel, however, only generates negative UIP deviations in the case of technology shocks. The negative TFP shock generates a persistent drop in money supply, but does not impart any cyclicality on the money process. The sum total of all three effects lead to UIP deviations that are similarly cyclical, but dampened in magnitude, as compared to the monetary shock.

In this generalized framework it is also true that the UIP deviations can be traced back to the fact that monetary policy does not accommodate fiscal policy (and vice versa), and this leads to cyclical fluctuations in the relative supply of liquid assets to the economy. If the central bank was committed to stabilizing the net supply of domestic government debt this would eliminate the two domestic channels that generate UIP deviations. The international spillover channel will still be operative, however, and some UIP deviations would be present unless there was cooperation between policy institutions internationally as well.

6 Testing the Model's Implications

6.1 Government Debt and UIP Violations in the Data

Government debt dynamics are central to the main mechanism of the model and in this section I provide direct empirical evidence that government debt affects excess currency returns in the way predicted by the model.

Recall that the excess return of foreign over home bonds in the model are equal to the convenience yield differential, which is in turn a function of the relative holdings of home and foreign bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = |\frac{\Psi_{b_h}}{1 + \Psi_{b_h}}|\frac{1}{\eta_b}(\hat{b}_{f,t} - \hat{b}_{h,t}).$$

In the model, forecastable excess returns and hence UIP deviations stem from movements in the relative portfolio composition of the private agents. A direct way to test this implication would be to re-estimate the standard UIP regression and control for the portfolio composition of investors. We would expect to find a significant negative coefficient on home bond holdings and a significant positive coefficient on foreign bond holdings, and an insignificant coefficient on the interest rate differential.

Data on private portfolio compositions is not readily available, but in the model household bond holdings are highly correlated with the supply of government bonds. Moreover, the supply of government debt is a common proxy for the convenience yield in the empirical literature (e.g. Krishnamurthy and Vissing-Jorgensen (2012). This considerations lead me to estimate an augmented version of the UIP regression by adding the US Debt-to-GDP ratio to the right hand

side.[64]

I also consider two additional control variables: the outstanding amount of USD denominated Commercial Paper (standardized by US GDP) and the VIX index of implied volatility. I include the stock of dollar denominated Commercial Paper to control for possible substitution effects between public and high quality private debt.[65] A number of papers argue, both theoretically and empirically, that private debt could act as a substitute for government debt in the demand for safe and liquid assets (e.g. Bansal et al. (2011), Krishnamurthy and Vissing-Jorgensen (2012)). In fact, even a cursory look at the data is quite suggestive of this substitution effect. See for example Figure E.2 in the Appendix, which plots the evolution of both US government debt-to-GDP and commercial paper-to-GDP and shows a clear negative relationship, with a correlation coefficient of -0.87.

On the other hand, the VIX index is commonly seen as a proxy for the "global risk appetite", with higher values indicating the market is less willing to accept risk, and I include it to proxy for any possible risk-premia that could be generating UIP deviations. A potential solution to the standard UIP puzzle at short horizons is that high interest rate currencies are riskier than lower interest rate currencies, and hence earn higher excess returns. To control for this effect, I sign the VIX index with the sign of the interest-rate differential, i.e. $\text{sign}(i_t^* - i_t)VIX_t$, so that the risk-factor is always properly signed and goes in the direction of the higher interest rate currency.

The general version of the updated UIP regression I consider is

$$\lambda_{j,t+1} = \alpha_j + \beta(i_{j,t} - i_{j,t}^*) + \gamma \ln(\frac{DEBT_t^{US}}{GDP_t^{US}}) + \delta_1 \ln(\frac{CP_t^{US}}{GDP_t^{US}}) + \delta_2 \text{sign}(i_t^* - i_t)VIX_t + \varepsilon_{j,t+1}.$$

Due to the availability of data on the stock of Commercial Paper the time period under consideration is shortened to January 1993 to July 2013. Moreover, I drop all Euro legacy currencies (e.g. French Franc, Italian lira etc.) except the Deutsch Mark because now there is only at most 6 years of data for each. I keep the Deutsch Mark because the Euro is appended to it and hence there is a long time series in this case.[66]

Table 3 presents the estimation results, with each column representing a different specification. The first column estimates a regression where the interest rate differential is the only regressor, the second column adds $\ln(\frac{DEBT_t^{US}}{GDP_t^{US}})$, and so on. Column (1) shows us that the UIP puzzle is very much true in this subsample as the standard UIP coefficient is -2.18 and highly significant. Look

[64] In the benchmark estimation I only include US Government debt because of data availability issues for higher frequency (i.e. quarterly) series on foreign government debt. Adding foreign debt (at the annual frequency) does not affect results significantly.

[65] Commercial Paper is very short short-term (less than 1 year in maturity) unsecured debt of large banks and corporations with excellent credit ratings. It is also a very safe investment, with default rates that are basically zero.

[66] Including the short-sample currencies makes no significant difference, just decreases the efficiency of the estimates. More details are available in the Appendix.

Table 3: Modified UIP Regression

Variable	(1)	(2)	(3)	(4)	(5)	(6)
$i_t - i_t^*$	-2.18***	-2.29***	-2.26***	-1.24	-1.54***	-0.82
	(0.76)	(0.76)	(0.83)	(0.81)	(0.74)	(0.82)
$\ln \frac{\text{DEBT}_t}{\text{GDP}_t}$		-0.53		-3.54**		-3.36**
		(0.62)		(1.60)		(1.62)
$\ln \frac{\text{CP}_t}{\text{GDP}_t}$			0.13	-2.93**		-2.75**
			(0.54)	(1.31)		(1.32)
$sign(i_t - i_t^*)VIX_t$					0.009**	0.007
					(0.0045)	(0.0045)
T	245	245	245	245	245	245
N	10	10	10	10	10	10

This table presents estimates of versions of the modified UIP regression $\lambda_{j,t+1} = \alpha_j + \beta(i_{j,t} - i_{j,t}^*) + \gamma \ln(\frac{DEBT_t^{US}}{GDP_t^{US}}) + \delta_1 \ln(\frac{CP_t^{US}}{GDP_t^{US}}) + \delta_2 \text{sign}(i_t^* - i_t)VIX_t + \varepsilon_{j,t+1}$. The regression is estimated as a panel with fixed effects and the reported standard errors are robust to both contemporaneous correlation (within time period and across currencies) and serial correlation.

next in column (4), which shows that augmenting the regression with the two measures of public and private debt changes the results. The two debt variables have significant negative coefficients, as predicted by the theory, while the coefficient on the interest rate differential is cut roughly in half and becomes insignificant. This suggests that the interest rate differential itself is not intricately linked to the UIP deviations, but rather shows up significant in standard UIP regressions because it is correlated with debt.

The table also shows that controlling for the private supply of debt is indeed important. Neither government debt nor Commercial Paper are significant regressors when included by themselves, but are both significant and of the right sign when included together. This suggests that the substitution effect might indeed be playing a significant role.

Lastly, the addition of the signed VIX index has no major effect on the estimated coefficients of the debt variables, which remain negative and significant. The coefficient of the VIX itself is positive, which is the expected sign, but is not significant. Including it, however, does help to reduce the coefficient on the interest rate differential even more, which is suggestive evidence that there may be a small risk-premium effect as well. But the forecastability in the excess returns appear to be mainly driven by the debt variables.[67]

[67] The Appendix also presents results where I estimate the regression on a currency-by-currency basis. The resulting coefficients tell very much the same story as the panel estimates presented above, but as could be expected the estimates are noisier and less significant.

6.2 Monetary Policy Independence and UIP Violations in the Data

Another strong prediction of the model is that central bank independence is positively associated with UIP deviations. In this section, I will show that this is also true in the data, by using the degree of capital controls as a proxy for monetary policy independence. I use this proxy for a lack of high quality, objective measurements of monetary policy independence that are comparable across currencies and span a significant portion of my data set (both in the cross-section and in the time dimension). On the other hand, a number of papers have shown that the degree of capital controls is a good proxy for monetary policy independence, see for example Alesina and Tabellini (1989), Drazen (1989), Alesina and Grilli (1994), Grilli and Milesi-Ferretti (1995), Leblang (1997), Quinn and Inclan (1997), and Bai and Wei (2000). Moreover, data on capital controls is readily available, has a long time series and is directly comparable across all currencies. I document that UIP violations are strongly negatively associated with the degree of capital controls, i.e. the countries that are most open financial exhibit the largest UIP violations, especially in regards to the positive violations at long horizons. Note that, if anything, a priori we might have expected UIP violations to be positively associated with capital controls because one of the implicit assumptions of the UIP hypothesis is free movement of capital across countries.[68]

In both the analytical and the quantitative models, the dynamics of government debt are a key channel through which UIP violations are generated, especially the positive violations at longer horizons. In the analytical model, in particular, government debt dynamics are the only such channel and the central bank could fully eliminate UIP violations by committing to a monetary policy that stabilizes government debt. In the big model there are two additional channels, but monetary policy could still greatly weaken UIP violations by stabilizing government debt.[69] In Appendix D I discuss two different experiments in which the foreign central bank is committed to helping the foreign fiscal authority manage its debt, and does so through lowering interest rates in response to high debt, which lowers the debt servicing costs and also helps inflate debt away. In both experiments the home central bank remains independent of the home fiscal authority, and the results show that it is enough to just change the behavior of the foreign central bank to greatly reduce UIP violations. Please see Appendix D for more details on how the model works; for the rest of this section I will focus on the empirical results.

As discussed earlier, I will use the degree of capital controls as an empirical proxy for monetary policy independence and will measure them with the Chinn and Ito (2006) index of de jure capital controls, a standard procedure in the literature. The index is constructed from the binary dummy

[68] Flood and Rose (1996), one of very few existing papers that have looked at the relationship between UIP violations and capital controls, also finds evidence that countries with fixed exchange rate regimes tend to exhibit smaller UIP violations at horizons of one and three months.

[69] By doing so, the central bank will in fact be reducing two channels: government debt dynamics themselves and the channel operating through the international spillover of shocks. This is because the spillovers mainly serve as an amplification of the effects coming from cyclical debt dynamics.

variables that characterize a country's restriction on capital flows as reported by the IMF's "Annual Report on Exchange Arrangements and Exchange Restrictions" and higher values mean less capital controls. It is a comprehensive measure, which accounts for both impediments to the free float of exchange rates and varying frictions in actual movements of financial capital in and out of the particular country.

Even though my sample is composed mainly of advanced economies there is a significant amount of cross-sectional variation in the degree of capital controls between countries, especially in the early part of the sample. Moreover, since the index is annual and not at the daily frequency as the exchange rates data, I focus on exploiting the cross-sectional variation in the degree of capital controls. In particular, I take the time-average of the Chinn-Ito index for each country and differentiate between currencies in their average level of capital controls.[70] To start, in Figure 10 I plot the average Chinn-Ito index of each currency against its estimated 1-month horizon UIP regression coefficient β_1.

The figure displays a strong negative relationship with a correlation coefficient of -0.72 that is significant at the 1% level. It shows that countries with higher degrees of capital controls exhibit smaller UIP violations than countries with open capital markets. Prima facie, this result is counter-intuitive as one of the implicit assumptions underlying the derivation of the UIP condition is that capital is free to chase the highest available return.[71] However, this negative correlation is fully consistent with the prediction of the convenience yield model, when capital controls are viewed as a proxy for monetary policy independence. In the convenience yield model considered earlier, UIP violations are a direct result of the interaction between independent monetary and fiscal policies, and are greatly diminished in situations where the monetary policy is subordinate to the goals of managing the government's budget.

Next, I examine how the degree of capital controls is related to UIP deviations at different horizons. To do this, I split the sample of currencies into two subgroups based on their average Chinn-Ito index. I split the sample around around a Chinn-Ito value of 1, because there is a large gap between the two currencies to each side of 1 which makes for a natural break point.[72] As can be seen from Figure 10 this leaves me with a subsample of five currencies with "more capital controls" and another subsample of 13 currencies with "less capital controls".[73]

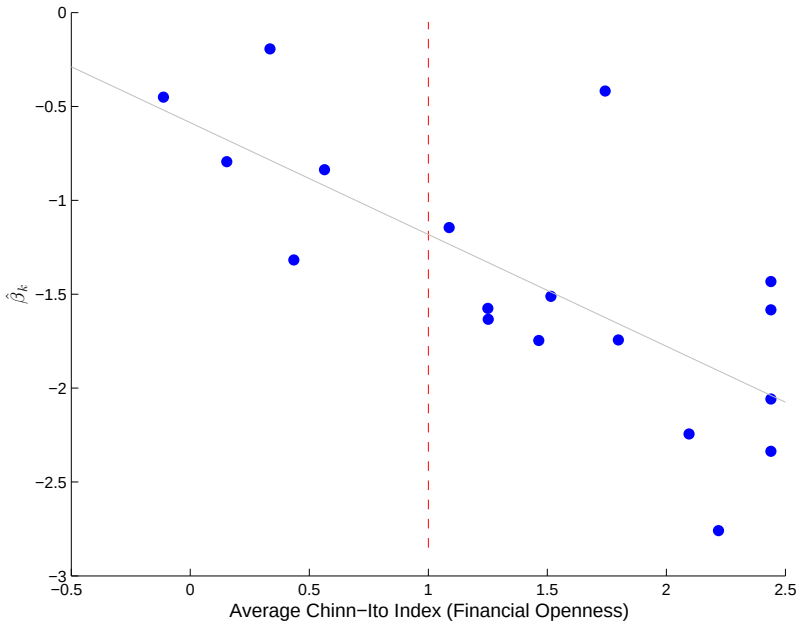
[70] I also consider a couple of different ways to control for capital controls dynamically despite the differences in frequencies of the observations. The results and the overall conclusion are very similar and are relegated to the Appendix for the sake of brevity.

[71] The results are also counter-intuitive from a risk-premium perspective, considering that historically many of the largest losses to the carry trade strategy stem from the re-valuation of fixed exchange rates.

[72] This is illustrated in Figure 10 where the average Chinn-Ito index is on the X-axis and the red dashed line marks the index value of 1.

[73] I have considered a number of robustness checks, including ways of controlling for capital controls dynamically by including them directly in the regression specification, and the results do not change. Splitting the sample is most straightforward and I present it as the benchmark here, and relegate the alternative specifications to the Appendix.

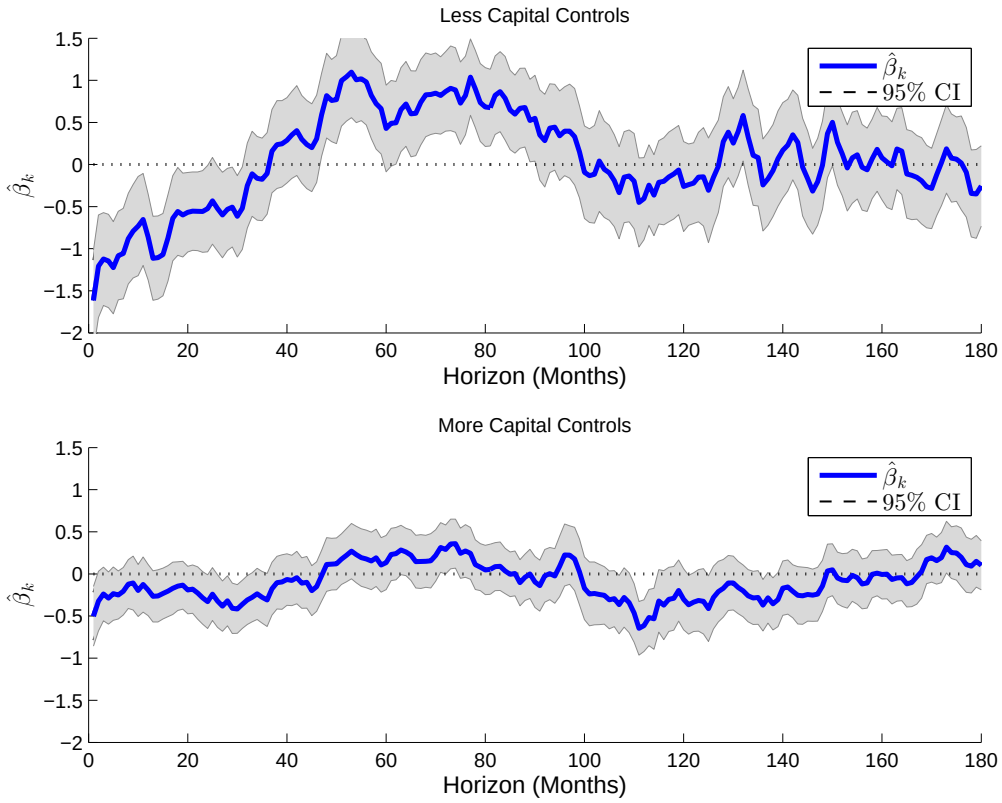
Figure 10: UIP Coefficients and Financial Openness



I re-estimate the whole set of UIP regressions (3) on the two different subsamples of currencies. The estimated coefficients are plotted in the two panels of Figure 11, with the top panel displaying estimates for the subsample of the currencies with fewer capital controls. The currencies with freer capital accounts clearly display stronger UIP violations at all horizons. In fact, the group of currencies with "more capital controls" exhibit statistically significant UIP violations only at very short-horizons and are most consistent with the standard formulation of the UIP Puzzle, which emphasizes the negative short-horizon violations. Even at the short horizons, the magnitudes of the deviations are almost three times as small as the short-horizon deviations for the subset of currencies with open capital markets. The difference is even starker at longer horizons, where the currencies with stronger capital controls display no statistically significant UIP violations.

Overall, the results point to a strong, negative relationship between capital controls and UIP violations. The higher the degree of capital controls, the lower are the estimated UIP violations, especially at longer horizons. If we interpret the capital controls as a proxy for monetary policy independence, the findings support the implications of the model that UIP violations are increasing in the independence of monetary policy. Moreover, a different interpretation of the results is not straightforward, as ex-ante we would expect capital controls to actually be positively, and not negatively related to UIP violations.

Figure 11: UIP Regressions, 1 to 180 months



7 Conclusion

This paper makes two main contributions. First, it shows that the famous Uncovered Interest Parity puzzle is more complex than typically understood. The main findings confirm previous results at short horizons which show that exchange rates fail to depreciate sufficiently to offset interest rate differentials. However, I document new evidence that at the longer horizons of 3 to 7 years exchange rates exhibit the opposite puzzling behavior - they depreciate too much. The changing nature of UIP deviations have important implications about exchange rate behavior, as they introduce cyclical dynamics and also lead to exchange rates converging to the UIP benchmark over the long-run.

The second contribution of the paper is to develop a novel model that can capture the full complexity of the UIP deviations at both short and long horizons. Unlike previous models which have primarily focused on explaining the short horizon negative UIP violations, I explore a novel mechanism that can also deliver the changing nature of UIP deviations at longer horizons. The model relies on the mechanism of the bond convenience yield and the interaction of monetary and fiscal policies.

The negative short-horizon UIP violations are a fundamental feature of the convenience yield mechanism, which implies a negative, contemporaneous relationship between interest rates and

convenience yields. The long-horizon UIP violations are in turn tied to the interaction between monetary and fiscal policy. In the model, the central bank is independent of the fiscal authority and raises interest rates to fight inflation without regard to the upward pressure this puts on government debt. On the other hand, the fiscal authority smooths taxes over time, which introduces cyclical dynamics in government debt that play a key role in the positive long-horizon violations. Lastly, I also show direct empirical evidence that supports the model's mechanism.

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Appendix

Incomplete and in progress - please see http://sites.duke.edu/rosenvalchev/research for the latest version of the paper and appendix.

A Proofs

A.1 Proof of Lemma1:

LEMMA 1 (Existence and Uniqueness). *A stationary solution to the system of difference equations (9) exists if and only if the following two conditions are satisfied*

(i) $\kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$

(ii) $\rho_\tau \in [0, \frac{1}{K})$

where $K = (1+i)(1+\gamma_b)$. *When the solution exists, it is unique.*

Moreover, under the stronger restriction $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau}]$ *the solutions to* $\hat{b}_t$ *and* $\hat{\tau}_t$ *have non-negative autocorrelation.*

Proof. There exists a covariance stationary solution for x_t if and only if the eigenvalues of the autoregressive matrix A are inside the unit circle, i.e. are less than 1 in magnitude. Thus, in the proof I will show that conditions (i) and (ii) are necessary and sufficient for the eigenvalues of A to be smaller than 1 in absolute value.

I will prove the if direction first. To this end, let $\kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$ and $\rho_\tau \in [0, \frac{1}{K})$ and define the useful notation $K = (1+i)(1+\gamma)$. The two eigenvalues of A are

$$\lambda_{1,2} = \frac{K - (1-\rho_\tau)\kappa_b + \rho_\tau \pm \sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau}}{2}.$$

The eigenvalues are complex conjugates when $(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau < 0$. The left-hand side of this equation defines a quadratic expression in ρ_τ that is convex and crosses zero at the following two points

$$\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+1-K) + K - 2\sqrt{K\kappa_b(\kappa_b+1-K)}}{(1+\kappa_b)^2} < 1$$

$$\overline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+1-K) + K + 2\sqrt{K\kappa_b(\kappa_b+1-K)}}{(1+\kappa_b)^2} > 0$$

For $\kappa_b = K-1$ we have $\underline{\rho}(\kappa_b) = \overline{\rho}(\kappa_b) = \frac{1}{K}$ and for $\kappa_b > K-1$ we have $\overline{\rho}(\kappa_b) > \frac{1}{K}0$. To prove the second fact we will proceed in two steps. First, we will evaluate $\overline{\rho}(\kappa_b)$ at $\kappa_b = (K+1)\frac{1+\rho_\tau}{1-\rho_\tau}$ and show that it is greater than $\frac{1}{K}$. Next, we will solve for the first order conditions for the minimization of $\overline{\rho}(\kappa_b)$ in respect to κ_b and show that the interior solution is also greater than $\frac{1}{K}$.

Start with evaluating $\overline{\rho}(\kappa_b)$ at $\kappa_b = (K+1)\frac{1+\rho_\tau}{1-\rho_\tau}$

$$\overline{\rho}((K+1)\frac{1+\rho_\tau}{1-\rho_\tau}) - \frac{1}{K} = \frac{2(K(K-1+(K^2-1)\rho_\tau + K(1+K)\rho_\tau^2 + (1-\rho_\tau)\sqrt{2K(1+K)(1+\rho_\tau)(1+K\rho_\tau)}) - 2)}{K(2+K+K\rho_\tau^2)}$$

Call the numerator of the above quantity D and notice that

$$\frac{\partial D}{\partial \rho_\tau} = K(K^2-1)+2K(1+K)-\sqrt{2K(1+K)(1+\rho_\tau)(1+K\rho_\tau)}+(1-\rho_\tau)\frac{2(K(1+K)(1+2\rho_\tau K+K))}{\sqrt{2K(1+K)(1+\rho_\tau)(1+K\rho_\tau)}}$$

and since $\sqrt{2K(1+K)(1+\rho_\tau)(1+K\rho_\tau)} \leq 2\sqrt{K}(1+K) \leq 2K(1+K)$ it follows that $\frac{\partial D}{\partial \rho_\tau} > 0$. Therefore, it is enough to show that $D > 0$ for $\rho_\tau = 0$ to know that $D > 0$ for all $\rho_\tau \in (0,1)$. Computing the expression with $\rho_\tau = 0$ yields:

$$D = 2(K(K-1) + K\sqrt{2K(1+K)} - 2) \geq 0$$

where the inequality follows from $K\sqrt{2K(1+K)} > 2K\sqrt{K} > 2$, and these results imply $\overline{\rho}((K+1)\frac{1+\rho_\tau}{1-\rho_\tau}) > \frac{1}{K}$.

Now evaluate the derivative $\frac{\partial \overline{\rho}(\kappa_b)}{\partial \kappa_b}$ to find that the only solution to $\frac{\partial \overline{\rho}(\kappa_b)}{\partial \kappa_b} = 0$ is $\kappa_b = K$. At the local optimum $\overline{\rho}(K) = \frac{4K}{(1+K)^2}$ and notice that

$$\frac{4K}{(1+K)^2} - \frac{1}{K} = \frac{(K-1)(2K+1)}{K(1+K^2)} > 0$$

Thus, we have shown that the minimum of $\overline{\rho}(\kappa_b)$ is obtained at $\kappa_b = K-1$ and is equal to $\frac{1}{K}$. Hence for $\kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$ we have that $\overline{\rho}(\kappa_b) > \frac{1}{K}$.

Now go back to the discriminant in the expression for the eigenvalues $(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau$, and notice that the quadratic polynomial in ρ_τ is convex and thus

$$(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau \geq 0, \text{ for } \rho_\tau \in [0, \underline{\rho}_\tau]$$

$$(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau \leq 0, \text{ for } \rho_\tau \in [\underline{\rho}_\tau, \overline{\rho}_\tau]$$

If $\rho_\tau \in [\underline{\rho}_\tau, \overline{\rho}_\tau]$ the eigenvalues of A are complex and in that case their magnitude is

$$\begin{aligned}
|\lambda_k| &= \frac{1}{2}\left((K-(1-\rho_\tau)k+\rho_\tau)^2 + [4K\rho_\tau - (K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2]\right)^{\frac{1}{2}} \\
&= \frac{1}{2}\sqrt{4K\rho_\tau} \\
&= \sqrt{K\rho_\tau}
\end{aligned}$$

and hence in this case $|\lambda_k| < 1$ if and only if $\rho_\tau < \frac{1}{K}$. Thus $\rho_\tau \in [\underline{\rho}_\tau, \frac{1}{K})$ results in eigenvalues that are inside the unit circle.

Now consider the case $\rho_\tau \in [0, \underline{\rho}_\tau]$, which results in real eigenvalues. For $\kappa_b = K-1$ we have

$$\begin{aligned}\lambda_1 &= \frac{1}{2}(1+\rho_\tau K + \sqrt{(1+\rho_\tau K)^2 - 4K\rho)}) \\ &= \frac{1}{2}(1+\rho_\tau K + \sqrt{(1-\rho_\tau K)^2}) \\ &= \frac{1}{2}(1+\rho_\tau K + (1-\rho_\tau K)) \\ &= 1\end{aligned}$$

Next, notice that when $\kappa_b < \frac{K+\rho_\tau}{1-\rho_\tau}$ we have $K-(1-\rho_\tau)\kappa_b+\rho_\tau > 0$ and thus $\lambda_1 > 0$. Moreover,

$$\frac{\partial\lambda_1}{\partial\kappa_b} = -\frac{1-\rho_\tau}{2\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2-4K\rho_\tau}}(\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2-4K\rho_\tau}+K-(1-\rho_\tau)\kappa_b+\rho_\tau) < 0$$

and since at $\kappa_b = K-1$ we have $\lambda_1 = 1$ it follows that for $\kappa_b \in (K-1, \frac{K+\rho_\tau}{1-\rho_\tau}$ we have $\lambda_1 \in (0,1)$. Meanwhile since $(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau \geq 0$, it follows that

$$0 < \lambda_2 < \lambda_1$$

and hence $\lambda_2 \in (0,1)$ as well.

If on the other hand $\kappa_b \in [\frac{K+\rho_\tau}{1-\rho_\tau}, \frac{(K+1)(1+\rho_\tau)}{1-\rho_\tau}$ and

$$\begin{aligned}\lambda_1 &= \frac{K-(1-\rho_\tau)\kappa_b+\rho_\tau+\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2-4K\rho_\tau}}{2} \\ &\leq \frac{K-(1-\rho_\tau)\kappa_b+\rho_\tau+|K-(1-\rho_\tau)\kappa_b+\rho_\tau|}{2} \\ &\leq 0\end{aligned}$$

and thus $\lambda_2 < \lambda_1 \leq 0$. Moreover,

$$\frac{\partial\lambda_2}{\partial\kappa_b} = -\frac{1-\rho_\tau}{2\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2-4K\rho_\tau}}(K-(1-\rho_\tau)\kappa_b+\rho_\tau-\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2-4K\rho_\tau}) < 0$$

and at $\kappa_b = \frac{(K+1)(1+\rho_\tau)}{1-\rho_\tau}$ we have

$$\lambda_2 = -1$$

Therefore, for $\kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$ and $\rho_\tau \in [0, \underline{\rho_\tau}]$ the eigenvalues of A are real and $|\lambda_k| < 1$. And for $\rho_\tau \in [\underline{\rho_\tau}, \frac{1}{K}]$ the eigenvalues are complex and again less than 1 in modulus. This completes the proof that the two conditions are sufficient for a stationary solution.

To prove they are necessary, notice that if $\rho_{tau} \in [\frac{1}{K}, \overline{\rho}_\tau]$ then the resulting complex

eigenvalue will be outside of the unit circle. On the other hand, now I will show that $\rho_\tau \in [\overline{\rho}_\tau, 1)$ results in real eigenvalues that are bigger than 1 in absolute value.

At $\rho_\tau = \overline{\rho_\tau}$ the bigger eigenvalue is

$$\lambda_1(\overline{\rho_\tau}) = \frac{K + \sqrt{K\kappa_b(\kappa_b + 1 - K)}}{1 + \kappa_b}$$

and I will show that $\lambda_1(\overline{\rho_\tau}) \geq 1$. Start with observing that since $K \geq 1$ we have $K\kappa_b \geq \kappa_b \geq 1 + \kappa_b - K$ and since $1 + \kappa_b - K \geq 0$ for $\kappa_b \geq K - 1$ we have

$$\begin{aligned} & K\kappa_b \geq 1 + \kappa_b - K \\ \Rightarrow & K\kappa_b(1 + \kappa_b - K) \geq (1 + \kappa_b - K)^2 \\ \Rightarrow & \sqrt{K\kappa_b(1 + \kappa_b - K)} \geq (1 + \kappa_b - K) \end{aligned}$$

and therefore

$$\begin{aligned} \lambda_1(\overline{\rho_\tau}) &= \frac{K + \sqrt{K\kappa_b(\kappa_b + 1 - K)}}{1 + \kappa_b} \\ &\geq \frac{1 + \kappa_b}{1 + \kappa_b} \\ &\geq 1 \end{aligned}$$

Therefore, there exist no stationary solutions for $\rho_\tau \geq \overline{\rho_\tau}$. This proves that $\rho_\tau \in [0, \frac{1}{K})$ is a necessary condition.

Now consider $\kappa_b < K - 1$. Above we showed that $\frac{\partial \lambda_1}{\partial \kappa_b} < 0$ and that $\lambda_1 = 1$ when $\kappa_b = K - 1$, hence $\lambda_1 \geq 1$ for any $\kappa_b \leq K - 1$. Similar reasoning and the facts that λ_2 is a decreasing function of κ_b for $\kappa_b > \frac{K+\rho_\tau}{1-\rho_\tau}$ and $\lambda_2 = -1$ for $\kappa_b = (K + 1)(1 + \rho_\tau)1 - \rho_\tau$ lead to the conclusion that $\kappa_b = (K + 1)(1 + \rho_\tau)1 - \rho_\tau$ implies $\lambda_2 < -1$. This completes the necessary direction of the proof.

To prove the second result let $\kappa_b \in (K - 1, \frac{K}{1-\rho_\tau}]$ and start by computing the variance on both sides of the tax policy rule to obtain

$$Var(\tau_t) = \frac{b^2 k_\tau^2}{(1 + \rho)\tau^2}(1 - \rho_\tau)Var(b_t) + 2\frac{bk\rho_\tau}{(1 + \rho_\tau)\tau}Cov(\tau_t, b_t)$$

and then combine with

$$\begin{aligned} \mathrm{Cov}(\hat{\tau}_t, \hat{b}_t) &= \mathrm{Cov}(\rho\tau_{t-1} + a_{21}b_{t-1}, a_{11}b_{t-1} + a_{12}\tau_{t-1} + b_{11}v_t) \\ &= -\rho_\tau^2 \frac{\tau}{b} \mathrm{Var}(\hat{\tau}_t) + a_{11}^{(1)}(1 - \rho_\tau)\kappa_b \frac{b}{\tau} \mathrm{Var}(\hat{b}_t) + a_{11}^{(1)}\rho_\tau - (1 - \rho_\tau)\kappa_b\rho_\tau \mathrm{Cov}(\hat{b}_t, \hat{\tau}_t) \end{aligned}$$

to obtain

$$\mathrm{Cov}(\hat{\tau}_t, \hat{b}_t) = \underbrace{(1-\rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1+\rho_\tau)-\kappa_\tau)}{1+\rho_\tau(1+2\kappa_\tau - K(1+\rho_\tau))}}_{=\delta} \mathrm{Var}(\hat{b}_t).$$

Now work with $\mathrm{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t)$ to obtain

$$\begin{aligned}\mathrm{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t) &= \mathrm{Cov}(\rho_\tau \hat{\tau}_t + (1-\rho_\tau)\frac{b}{\tau}\kappa_b \hat{b}_t, \hat{\tau}_t) \\ &= \rho_\tau \,\mathrm{Var}(\hat{\tau}_t) + (1-\rho_\tau)\frac{b}{\tau}\kappa_b \delta \,\mathrm{Var}(\hat{b}_t)\end{aligned}$$

Combine again with the expression for $\mathrm{Var}(\hat{\tau}_t)$ and simplify to arrive at

$$\mathrm{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t) = \frac{b^2 k^2}{\tau^2} \frac{(1-\rho_\tau)\rho_\tau(1+K-(1-\rho_\tau)(\kappa_b - K\rho_\tau))}{1+\rho_\tau(1+2\kappa_b - K(1+\rho_\tau))} \mathrm{Var}(\hat{b}_t$$

Let

$$\tilde{\rho} = \frac{b^2 k^2}{\tau^2} \frac{(1-\rho_\tau)\rho_\tau(1+K-(1-\rho_\tau)(\kappa_b - K\rho_\tau))}{1+\rho_\tau(1+2\kappa_b - K(1+\rho_\tau))}$$

Taxes are non-negatively autocorrelated as long as $\tilde{\rho} \geq 0$. Notice that the denominator is always positive and hence this expression is positive only if the numerator is positive. The numerator itself is positive if and only if

$$\kappa_b < \frac{K+1}{1-\rho_\tau} + \rho_\tau K$$

which satisfies our initial assumption.

Now lets turn our attention to $\mathrm{Cov}(\hat{b}_{t+1}, \hat{b}_t)$. It is straightforward to show that

$$\mathrm{Cov}(\hat{b}_{t+1}, \hat{b}_t) = a_{11} \,\mathrm{Var}(\hat{b}_t) + a_{12} \,\mathrm{Cov}(\hat{\tau}_t, \hat{b}_t) = (a_{11} + \delta a_{12}) \,\mathrm{Var}(\hat{b}_t)$$

where a_{kl} is the (k,l) element of the matrix A. The coefficient in front of $\mathrm{Var}(\hat{b}_t)$ can be simplified down to

$$(a_{11} + \delta a_{12}) = K - \frac{\kappa_b(1-\rho_\tau)(1+\rho_\tau+\kappa_b\rho_\tau)}{1+\rho_\tau(1+2\kappa_b - K(1-\rho_\tau))}$$

which is non-negative if and only if

$$K(1+\rho_\tau(1+2\kappa_b - K(1-\rho_\tau))) - \kappa_b(1-\rho_\tau)(1+\rho_\tau+\kappa_b\rho_\tau) \geq 0$$

the above expression is a convex quadratic polynomial in κ_b, which crosses zero at the points

$$\underline{\kappa} = \frac{\rho_\tau^2 + 2K\rho_\tau - 1 - \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau}$$

$$\overline{\kappa} = \frac{\rho_\tau^2 + 2K\rho_\tau - 1 + \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau}$$

First, I will show that $\underline{\kappa} < 0$ so that under my assumed conditions $\kappa_b \geq \underline{\kappa}$. This is trivially true if $\rho_\tau^2 - 2K\rho_\tau - 1 < 0$. Assume that ρ_τ is high enough so that is positive and assume

$$\rho_\tau^2 + 2K\rho_\tau - 1 > \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}$$

Take squares on both sides, re-arrange and simplify to arrive at the condition

$$4K\rho_\tau(1-\rho_\tau^2)(K\rho_\tau - 1) > 0$$

but since $\rho_\tau < \frac{1}{K}$ we have reached a contradiction, and thus $\underline{\kappa} < 0 < \kappa_b$.
Now, work with $\overline{\kappa}_b$:

$$\begin{aligned}
\overline{\kappa}_b &= \frac{\rho_\tau^2 + 2K\rho_\tau - 1 + \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau} \\
&= \frac{\rho_\tau^2 - 1 + \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau} + \frac{K}{(1-\rho_\tau)} \\
&\geq \frac{\rho_\tau^2 - 1 + (1-\rho_\tau^2)}{2(1-\rho_\tau)\rho_\tau} + \frac{K}{(1-\rho_\tau)} \qquad\qquad \geq \frac{K}{(1-\rho_\tau)}
\end{aligned}$$

therefore under our assumptions $\kappa_b < \overline{\kappa}_b$ and hence, rolling everything back, this implies that

$$\mathrm{Cov}(\hat{b}_{t+1}, \hat{b}_t) \geq 0$$

and we are done. □

A.2 Proof of Lemma2:

LEMMA 2 (Impulse Response Functions). *Let $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$ and define $\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+1-K)+K-2\sqrt{K\kappa_b(\kappa_b+1-K)}}{(1+\kappa_b)^2} > 0$. Then,*

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ the autoregressive matrix A has two real, non-negative eigenvalues and the Impulse Response Functions never crosses the steady state, i.e.

$$\tilde{a}_{k1}^{(j)} \leq 0 \text{ for } k \in \{1,2\}, \text{ and } j = 1,2,3,\ldots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$ the autoregressive matrix A has a pair of complex conjugate eigenvalues which can be written as $\lambda_k = a \pm bi$ for $k \in \{1,2\}$, and the corresponding conjugate eigenvectors are of the form $\vec{v}_k = [x \pm yi, 1]'$, where a, b, x, y are real numbers and i is the imaginary unit. Furthermore, the Impulse Response Functions follow the

increasingly dampened cosine waves:

$$\tilde{a}_{11}^{(j)} = -(1+i+\frac{m}{b^G})|\lambda|^j\sqrt{1+(\frac{x}{y})^2}\cos(j\phi+\psi-\frac{\pi}{2}),\ \textit{for } j=1,2,3,\ldots$$

$$\tilde{a}_{12}^{(j)} = (1+i+\frac{m}{b^G})|\lambda|^j\frac{1}{y}\cos(j\phi-\frac{\pi}{2}),\ \textit{for } j=1,2,3,\ldots$$

$$\tilde{a}_{21}^{(j)} = -(1+i+\frac{m}{b^G})|\lambda|^j\frac{x^2+y^2}{y}\cos(j\phi-\frac{\pi}{2}),\ \textit{for } j=1,2,3,\ldots$$

where $\phi = \arctan(\frac{b}{a})$ *and* $\psi = \arctan(\frac{y}{x})$. *Moreover,* $\tilde{a}_{k1}^{(1)} \leq 0$ *for* $k \in \{1,2\}$.

Proof. (i) The first part follows directly from the proof of Lemma 1. The eigenvalues of A are real when

$$(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2 - 4K\rho_\tau \geq 0$$

which is true for $\rho_\tau \in [0, \underline{\rho}_\tau(\kappa_v)]$, using the fact that we restrict attention to $\rho_\tau \geq 0$. Moreover, when $\kappa_b \leq \frac{K}{1-\rho_\tau}$ it is immediate that $K-(1-\rho_\tau)\kappa_b+\rho_\tau > 0$ and hence $\lambda_1 > 0$. Moreover,

$$\begin{aligned}\lambda_2 &\geq K-(1-\rho_\tau)\kappa_b+\rho_\tau+\sqrt{(K-(1-\rho_\tau)\kappa_b+\rho_\tau)^2}\\ &\geq 0\end{aligned}$$

Hence both eigenvalues are non-negative.

To characterize the Impulse Response Function not that the Wold Decomposition of x_t is

$$x_t = Bv_t + ABv_{t-1} + A^2Bv_{t-2} + \ldots$$

Use the fact that

$$B = \begin{bmatrix} -(1+i+\frac{m}{b^G}) \\ 0 \end{bmatrix} v_t$$

to obtain

$$\hat{b}_t = -(1+i+\frac{m}{b^G})(v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \ldots)$$

$$\hat{\tau}_t = -(1+i+\frac{m}{b^G})(a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \ldots)$$

where $a_{kl}^{(j)}$ is the (k,j) element of the matrix A^j. Define $a_{11}^{(0)} = 1$ and $a_{11}^{(0)} = 0$ and the transformation

$$\tilde{a}_{kl}^{(j)} = -(1 + i + \frac{m}{b^G})a_{kl}^{(j)}.$$

The sequences $\{\tilde{a}_{k1}\}_0^\infty$ define the Impulse Response Functions of $\hat{b}_t$ (for $k = 1$) and $\hat{\tau}_t$ (for $k = 2$).

Firs, I will show that $a_{k1} \geq 0$ for all $j = 1, 2, 3, \ldots$ and $k = 1, 2$ when the matrix A is diagonalizable, and then I will handle the case when the eigenvalue is repeated and A is not diagonalizable (the only other case we need to worry about for a two by two matrix).

Assuming that A is diagonalizable, define

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

as a matrix with the two eigenvalues of A on its diagonal ordered like $\lambda_1 > \lambda_2$ (remember we are handling the case of real eigenvalues right now) and P as a matrix that has the eigenvectors of A as its columns. Since we have assumed A is diagonalizable, we have $A = P\Lambda P^{-1}$ and in particular

$$\begin{aligned} A^j &= (P\Lambda P^{-1})^j \\ &= (P\Lambda P^{-1})(P\Lambda P^{-1})(P\Lambda P^{-1})\ldots \\ &= P\Lambda^j P^{-1} \end{aligned}$$

and since Λ is diagonal

$$\Lambda^j = \begin{bmatrix} \lambda_1^j & 0 \\ 0 & \lambda_2^j \end{bmatrix}$$

and thus if we expand the expression for A^j we obtain that

$$a_{11}^{(j)} = \frac{p_{11}p_{22}\lambda_1^j - p_{12}p_{21}\lambda_2^j}{|P|}$$

where $|P|$ is the determinant of the matrix of eigenvectors P and p_{kl} is its (k, l)-th element. Computing the eigenvectors of A I obtain

$$\begin{aligned} p_{1k} &= \frac{\tau\lambda_k}{\sqrt{\tau^2\lambda_k^2 + b^2(K - \lambda_k)^2}} \\ p_{2k} &= \frac{b(K - \lambda_k)}{\sqrt{\tau^2\lambda_k^2 + b^2(K - \lambda_k)^2}} \end{aligned}$$

Since both of the eigenvalues are positive and are ordered so that $\lambda_1 > \lambda_2$ it follows that $|P| > 0$ and hence

$$\frac{p_{11}p_{22}\lambda_1^j - p_{12}p_{21}\lambda_2^j}{|P|} > 0.$$

This proves that $a_{11}^{(j)} \geq 0$ for all j and hence $\tilde{a}_{11}^{(j)} \leq 0$ for all j. On the other hand,

$$a_{21}^{(j)} = p_{21}p_{22}(\lambda_1^j - \lambda_2^j)$$

and since $\lambda_k < 1 < K$ it follows that $p_{21}p_{22} > 0$, $a_{21}^{(j)} \geq 0$ and thus $\tilde{a}_{21}^{(j)} \leq 0$, which completes the proof for diagonalizable A.

Now assume that A is not diagonalizable. A can still be written as $A = P\Lambda P^{-1}$ but now

$$\Lambda = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}$$

and the columns of P are the generalized eigenvectors of A. In this case, there is only one linearly independent eigenvector associated with the eigenvalue of λ, call it $\vec{p}$, and thus the second generalized eigenvector, call it $\vec{u}$, is a 2x1 vector that solves

$$(A - \lambda I)\vec{u} = \vec{p}$$

Thus $P = [\vec{p}, \vec{u}]$. We know that the $\vec{p}$ is a regular eigenvector and hence must satisfy the relationship we derived before:

$$p_1 = \frac{\tau\lambda}{(K - \lambda)b}p_2.$$

Using this fact, we can solve $(A - \lambda I)\vec{u} = \vec{p}$ for $\vec{u}$ and show that the generalized eigenvector is of the form $[1, 0]$. With these results in mind we can show that for $j \geq 1$:

$$\begin{aligned} a_{11}^{(j)} &= \lambda^{j-1}(\lambda - j\frac{p_2p_1}{|P|}) \\ &= \lambda^{j-1}(\lambda + j\frac{p_1}{u_1}) \qquad\qquad > 0 \end{aligned}$$

where the second equality follows from the fact that $|P| = -u_1p_2$ (remember $P = \begin{bmatrix} p_1 & u_1 \\ p_2 & u_2 \end{bmatrix}$) and the inequality on the third line follows from the fact that $u_1 \geq 0$, $p_1 \geq 0$, $\lambda \geq 0$. On the other hand,

$$\begin{aligned} a_{21}^{(j)} &= -j\frac{p_2^2}{|P|}\lambda^{j-1} \\ &= j\frac{p_2}{u_1}\lambda^{j-1} \qquad\qquad > 0 \end{aligned}$$

and thus we are done with the case when A is not diagonalizable. This completes the proof of part [(i)].

(ii) From the proof of Lemma 1 we know that $\rho_\tau \in (\underline{\rho}_\tau, \frac{1}{K})$ implies that the eigenvalues of A are complex. We can express them as $\lambda_1 = a + bi$ and $\lambda_2 = a - bi$ where $a = \frac{1}{2}(K - (1-\rho_\tau)\kappa_b + \rho_\tau) > 0$, $b = \frac{1}{2}\sqrt{4K\rho_\tau - (K-(1-\rho_\tau)\kappa_b + \rho_\tau)^2} \geq 0$ and i is the imaginary unit. The two conjugate egeinvectors can be written as $\vec{p}_k = [x \pm yi, 1]'$, where .

$$x = \frac{(K - (1-\rho_\tau)\kappa_b - \rho_\tau)\tau}{2b(1-\rho_\tau)\kappa_b}$$

$$y = \frac{\tau\sqrt{4K\rho_\tau - (K-(1-rho_\tau)\kappa_b + \rho_\tau)^2}}{2b(1-\rho_\tau)\kappa_b}$$

With two conjugate complex eigenvalues A is diagonalizable and can be expressed as $A = P\Lambda P^{-1}$ where again P is a similarity matrix with the eigenvectors of A as its columns and Λ is a diagonal matrix with the eigenvalues on the diagonal. Next remember that Euler's formula allows us to express complex numbers in exponential form, so that we can write $\lambda_1 = a + bi = |\lambda|e^{\phi i}$ where $\phi = \arctan(\frac{b}{a})$. This formulation is convenient because it is easy to take powers of the eigenvalues, e.g.

$$\lambda_1^j = |\lambda|^j e^{j\phi i}$$

and hence it is easy to compute powers of the eigenvalue matrix Λ. Using this, Euler's formula and the fact that $A^j = P\Lambda^j P^{-1}$ it is straightforward to compute

$$a_{11}^{(j)} = |\lambda|^j(\cos(j\phi) + \frac{x}{y}\sin(j\phi))$$

$$a_{12}^{(j)} = -|\lambda|^j\frac{x^2+y^2}{y}\sin(j\phi)$$

$$a_{21}^{(j)} = |\lambda|^j\frac{1}{y}\sin(j\phi)$$

where $\phi = \arctan(\frac{b}{a})$.

Moreover we can rewrite these expressions as

$$
\begin{aligned}
a_{11}^{(j)} &= |\lambda|^j(\cos(j\phi) + \frac{x}{y}\sin(j\phi)) \\
&= |\lambda|^j\sqrt{1+(\frac{x}{y})^2}\sin(j\phi+\psi) \\
&= |\lambda|^j\sqrt{1+(\frac{x}{y})^2}\cos(j\phi+\psi-\frac{\pi}{2})
\end{aligned}
$$

where $\psi = \arctan(\frac{y}{x}) + \pi\mathbb{I}(\frac{y}{x} < 0)$. The second equality follows from the formula for linear combinations of trig functions, and the third is simply an application of $\cos(\theta - \frac{\pi}{2}) = \sin(\theta)$. And similarly

$$
\begin{aligned}
a_{12}^{(j)} &= -|\lambda|^j\frac{x^2+y^2}{y}\cos(j\phi-\frac{\pi}{2}) \\
a_{21}^{(j)} &= |\lambda|^j\frac{1}{y}\cos(j\phi-\frac{\pi}{2})
\end{aligned}
$$

By the definition of the arctan$(\cdot)$ function and the virtue of $a \geq 0, b \geq 0$ it follows that $\phi \in [0, \frac{\pi}{2})$. Therefore, $j\phi - \frac{\pi}{2} < \frac{\pi}{2}$ for $j \leq 2$ and hence

$$
\cos(j\phi - \frac{\pi}{2}) \geq 0, \text{ for } j \in \{1, 2\}
$$

and hence $\tilde{a}_{21}^{(j)} \leq 0$ for $j \in \{1, 2\}$. On the other hand, if $x \geq 0$ then $\psi \leq \frac{\pi}{2}$ and this case $\cos(j\phi + \psi - \frac{\pi}{2}) \geq 0$ for at least $j = 1$. On the other hand $x < 0$ implies that $\kappa_b > \frac{K-\rho_\tau}{1-\rho_\tau}$. Then using the formula for addition of arctangent I obtain,

$$
\arctan(\frac{b}{a}) + \arctan(\frac{y}{x}) = \arctan(\frac{\frac{b}{a}+\frac{y}{x}}{1-\frac{by}{ax}}).
$$

where $1 - \frac{by}{ax} > 0$. Working with the numerator I get $\frac{y}{x} = \frac{b}{a-\rho_\tau}$ and given $\kappa_b \in (\frac{K-\rho_\tau}{1-\rho_\tau}, \frac{K}{1-\rho_\tau})$ it follows that $\frac{\rho_\tau}{2} < a < \rho_\tau$ and thus

$$
|\frac{y}{x}| > \frac{b}{a}.
$$

This shows that $\frac{b}{a} + \frac{y}{x} < 0$ and therefore $\arctan(\frac{\frac{b}{a}+\frac{y}{x}}{1-\frac{by}{ax}}) \in (-\frac{\pi}{2}, 0)$. Therefore, we again reach the conclusion that $\cos(j\phi + \psi - \frac{\pi}{2}) \geq 0$ for at least $j = 1$. This completes the proof of Lemma 2.

□

A.3 Theorem 1:

THEOREM 1 (UIP Violations). *Let* $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$. *The UIP regression coefficients* β_j *are equal to*

$$\beta_j = \frac{\text{Cov}(\hat{\lambda}_{t+1}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -(a_{11}^{(j)} + \delta a_{12}^{(j)})$$

where $a_{kl}^{(j)}$ *is the (k,j) element of the matrix* A^j *for* $j = 1, 2, 3, \ldots$, $a_{11}^{(0)} = 1$ *and* $a_{12}^{(0)} = 0$, *and* $\delta = (1-\rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1+\rho_\tau)-\kappa_\tau)}{1+\rho_\tau(1+2\kappa_\tau-K(1+\rho_\tau))}$. *Furthermore,*

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$, *the roots of the system are real and we have*

$$\beta_j < 0, \text{ for all } j = 1, 2, 3, \ldots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K}]$, *then the eigenvalues of the autoregressive matrix A are complex and of the form* $\lambda_k = a \pm bi$, $k \in \{1, 2\}$ *with corresponding eigenvectors of the form* $\vec{v}_i = [x \pm yi, 1]'$, *and we have*

$$\beta_j = \begin{cases} -1 & , j = 1 \\ -|\lambda|^j \sqrt{1+\chi^2} \cos(j\phi + \psi - \frac{\pi}{2}) & , j = 2, 3, \ldots \end{cases}$$

where $\chi = \frac{x-\delta(x^2+y^2)}{y}$, $\phi = \arctan(\frac{b}{a})$, $\psi = \arctan(\frac{1}{\chi}) + \pi\mathbb{I}(\chi < 0)$. *Thus, while* $\beta_1 < 0$, *there exists a* $j > 0$ *such that* $\beta_j > 0$.

Proof. To derive the expression for the UIP regression coefficients notice that $E_t(\hat{b}_{t+j}) = [1, 0]A^j x_t$, hence

$$\text{Cov}(E_t(\hat{b}_{t+j}), b_t) = a_{11}^{(j)} \text{Var}(\hat{b}_t) + a_{12}^{(j)} \text{Cov}(\hat{\tau}_t, \hat{b}_t). \tag{15}$$

Compute the variance on both sides of the tax policy rule to obtain

$$Var(\hat{\tau}_t) = \frac{b^2 k_\tau^2}{(1+\rho_\tau)\tau^2}(1-\rho_\tau)Var(\hat{b}_t) + 2\frac{bk\rho_\tau}{(1+\rho_\tau)\tau}Cov(\hat{\tau}_t, b_t)$$

and then combine with

$$\begin{aligned}\text{Cov}(\hat{\tau}_t, \hat{b}_t) &= \text{Cov}(\rho\hat{\tau}_{t-1} + a_{21}\hat{b}_{t-1}, a_{11}\hat{b}_{t-1} + a_{12}\hat{\tau}_{t-1} + b_{11}v_t) \\ &= -\rho_\tau^2 \frac{\tau}{b} \text{Var}(\hat{\tau}_t) + a_{11}^{(1)}(1-\rho_\tau)\kappa_b \frac{b}{\tau} \text{Var}(\hat{b}_t) + a_{11}^{(1)}\rho_\tau - (1-\rho_\tau)\kappa_b\rho_\tau \text{Cov}(\hat{b}_t, \hat{\tau}_t)\end{aligned}$$

to obtain

$$\text{Cov}(\hat{\tau}_t, \hat{b}_t) = \underbrace{(1-\rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1+\rho_\tau)-\kappa_\tau)}{1+\rho_\tau(1+2\kappa_\tau-K(1+\rho_\tau))}}_{=\delta} \text{Var}(\hat{b}_t).$$

Substituting this back in (15) yields $\text{Cov}(E_t(\hat{b}_{t+j}), b_t) = (a_{11}^{(j)} + \delta a_{12}^{(j)}) \text{Var}(\hat{b}_t)$ and hence

$$\beta_j = -(a_{11}^{(j)} + \delta a_{12}^{(j)}).$$

This gives us the general expression for the UIP regression coefficients, next I characterize them further under real and complex roots specifically.

To prove (i) let $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ and remember that

$$\hat{b}_t = \tilde{a}_{11}^{(0)} v_t + \tilde{a}_{11}^{(1)} v_{t-1} + \tilde{a}_{11}^{(2)} v_{t-2} + \tilde{a}_{11}^{(3)} v_{t-3} + \dots$$

and then

$$Cov(E_t(\hat{b}_{t+j}), \hat{b}_t) = \tilde{a}_{11}^{(j)} \tilde{a}_{11}^{(0)} + \tilde{a}_{11}^{(j+1)} \tilde{a}_{11}^{(1)} + \tilde{a}_{11}^{(j+2)} \tilde{a}_{11}^{(2)} + \tilde{a}_{11}^{(j+3)} \tilde{a}_{11}^{(3)} + \dots.$$

Lemma 2 gives us $\tilde{a}_{11}^{(j)} \leq 0$ for $j = 0, 1, 2, 3, \dots$ and thus all pairs in the above sum are non-negative, and hence $Cov(E_t(\hat{b}_{t+j}), \hat{b}_t) \geq 0$. This gives us that

$\beta_j = -\frac{\text{Cov}(E_t(\hat{b}_{t+j-1}), \hat{b}_t)}{\text{Var}(\hat{b}_t)} \leq 0$, for $j = 1, 2, 3 \dots$

To prove (ii) let $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K}]$ and notice that Lemma 2 tells us that the eigenvalues of A are of the form $\lambda_k = a \pm bi$ for $k \in \{1, 2\}$, and the corresponding conjugate eigenvectors are of the form $\vec{v}_k = [x \pm yi, 1]'$, where a, b, x, y are real numbers and i is the imaginary unit. Using the expressions for $a_{11}^{(j)}$ and $a_{12}^{(j)}$ found in Lemma 2 I get that for $j > 1$

$$\begin{aligned}\beta_j = -(a_{11}^{(j)} + \delta a_{12}^{(j)}) &= -|\lambda|^j (\cos(j\phi) + \frac{x}{y}\sin(j\phi) - \delta\frac{x^2+y^2}{y}\sin(j\phi)) \\ &= -|\lambda|^j \sqrt{1+\chi^2} cos(j\phi + \psi - \frac{\pi}{2})\end{aligned}$$

with $\chi = \frac{x - \delta(x^2+y^2)}{y}$, $\phi = \arctan(\frac{b}{a}) > 0$, $\psi = \arctan(\frac{1}{\chi}) + \pi\mathbb{I}(\chi < 0)$. For $j = 1$, trivially $\beta_j = \frac{\text{Cov}(\hat{b}_t, \hat{b}_t)}{\text{Var}(\hat{b}_t)} = -1$. Since the cosine function is cyclical at some $j > 1$ it will turn negative and hence $\beta_j > 0$, and this completes the derivation of the regression coefficients in the case of complex roots. □

B Forward Exchange Rate Contracts and UIP Violations

In this section, I augment the model to include trade in forward contracts on currencies, and show that trading in forward contracts creates a synthetic position long one country's bond and short the other. Hence, it does not matter whether one implements carry trades through forward contracts, or through trades in the bonds themselves, as both trading strategies earn the same convenience yield differential, which violates UIP. In other words, the convenience yield mechanism generates UIP violations that emerge both when looking at exchange rates and interest rates data only, and when only looking at forward and spot exchange rates.

The key to the result is that in a model with bond convenience yields, the Covered Interest Parity (CIP) holds if and only if a covered position in foreign currency is equivalent to a position in home bonds, both in financial terms and in convenience benefits. Why is that? Notice that a covered position in EUR risk-free bonds, where the future interest rate $(1 + i_t^*)$ has been sold forward for dollars at the equilibrium rate F_t, generates a risk-free USD payoff, and not a risk-free EUR payoff. As such, it has a comparable convenience value to the other risk-free USD asset - US Treasuries. Being a risk-free USD asset, it carries the convenience benefits of USD risk-free assets, because it allows the investor to pledge a sure, future amount of USD, and not a sure future amount of EUR. Another way to think about it, is that a covered position in EUR bonds is in fact long USD, and not long EUR. Similarly, buying foreign currency forward is a strategy long in foreign currency and short home currency. It simultaneously increases the pledgeable amount of foreign currency proceeds and decreases the pledgeable amount of home currency, hence it creates a synthetic, zero-cost position that is long home bonds and short foreign bonds, and thus in equilibrium, on average it earns the convenience yield differential.

The CIP condition states that investing in foreign risk-free bonds and using forward contracts to eliminate the exchange rate risk must yield the same rate of return as investing in home bonds. CIP has been shown to hold in the data very well, outside of a few, short-lived episodes during times of extraordinary financial markets turbulence (e.g. some days during the recent financial crisis). The intuition behind the condition is that the covered position in foreign bonds is a risk-free asset denominated in home currency, and not in foreign currency, hence in equilibrium it must have the same rate of return as the domestic bond (another domestic risk-free asset), or otherwise there will be an arbitrage opportunity.

To be more concrete, let F_t denote the equilibrium USD-EUR forward rate, so that today we can agree to trade 1 EUR tomorrow in exchange of F_t USD. Imagine then that an investor borrows \$1 today at the interest rate $1 + i_t$, changes it into $\frac{1}{S_t}$ EURs and invests it at the interest rate $1 + i_t^*$, and at the same time has sold forward the proceeds at the forward rate F_t. Thus, his payoff from the covered foreign position is $\frac{F_t}{S_t}(1 + i_t^*)$ and the cost of the 1 USD is $1 + i_t$ and CIP states:

$$1 + i_t = \frac{F_t}{S_t}(1 + i_t^*),$$

so that a position in a US Treasury has an equivalent financial return to a covered position in EUR denominated government bonds (e.g. German Bunds). A position in US Treasuries also

carries the convenience benefit $\Psi_{b_h,t}$ and the covered position in foreign bonds is another risk-free USD asset which carries the (possibly different) convenience benefit $\tilde{\Psi}_{b_h,t}$. Conditional on CIP, the convenience benefits of the two positions must be the same:

$$\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}.$$

This follows from the fact that an investment in US Treasuries carries a total return of $1 + i_t + \Psi_{b_h,t}$, the sum of the financial return and the convenience benefit, and an investment in a covered position in EUR denominated bonds similarly carries a total return of $\frac{F_t}{S_t}(1+i_t^*)+\tilde{\Psi}_{b_h,t}$. The two risk-free returns must be equal, otherwise there is an arbitrate opportunity. Given that CIP restricts the financial returns to be equal, it follows that the convenience benefits must be equal as well: $\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}$.

Thus, when CIP holds (as it does in the data) and bonds offer convenience benefits (as is also true in the data), in equilibrium, covered position in foreign bonds, which yield a risk-free payoff in the home currency and not a payoff in foreign currency, must offer the same convenience benefits as an equivalent position in home currency bonds.

This leads to the important result that (in log-approximation) the expected return on buying foreign currency forward (a popular way of implementing the carry trade without the need to transact in bond markets) is:

$$E_t(s_{t+1} - f_t) = E_t(\Delta s_{t+1} + i_t^* - i_t) = \hat{\Psi}_{b_h,t} - \hat{\Psi}_{b_f,t}.$$

This shows that taking positions in the forwards market is akin to creating a synthetic position that is simultaneously long foreign currency bonds and short home currency bonds. This is of course, very intuitive, as buying foreign currency forward is a contract long in the foreign currency and short in the home currency. Entering into this contract reduces the amount of future USD the investor is able to pledge today as collateral (since he has already sold this USD for EURs) and at the same time increase the pledgeable amount of EUR. At the end of the day, the strategy implemented through forwards market has equivalent financial and convenience returns to a trade in the home and foreign bonds themselves, hence the forwards data would display equivalent UIP violations and the mechanism works in the same way. Due to this equivalence and for simplicity, the benchmark model abstracts from trade in forward contracts.

C Relationship to existing Long-Run UIP Tests

Previous work by Chinn and Meredith (2005), Chinn (2006) and others has found that UIP tends to hold better over the long run, especially at horizons of five or more years. It is important to note that my results, which find that UIP is violated at horizons of up to 7 years but with a change in the direction of violations, are not contradictory but rather complementary to the previous results. The difference comes from a difference in methodologies, and in fact my results can help interpret and provide context to the existing findings.

The previous literature on the UIP hypothesis at long horizons has focused on testing whether long-run exchange rate changes (say 5+ years) tend to offset the corresponding long-run interest rate differential. The UIP condition equates the rates of return on bonds

of equivalent maturities, hence if $i_t^{(k)}$ is the yield (per month) on a k-months to maturity US Treasury bond, and $i_t^{(k),*}$ is the corresponding yield on a foreign k-months to maturity bond then for any k the UIP condition requires:

$$E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) = 0$$

Under UIP we would expect the long-term exchange rate change $s_{t+k} - s_t$ to offset the corresponding k-month interest rate differential. The existing literature on UIP in the long-run has focused on testing this condition by employing bonds with maturities of up to ten years. The common result is that the empirical evidence against the UIP condition is weaker when using long term bonds with maturities of five or more years. Thus, there is some evidence that the long-run movements in exchange rates tend to offset the corresponding long-term interest rate differential and hence UIP has appeared to hold better over the long run.

In contrast, in this paper I always use one month interest rates and forecast one month excess returns, but I vary the horizon of the forecast. Rather than asking whether the cumulative, five year change in the exchange rate offsets the five year interest rate and hence the five year excess return is zero in expectation,

$$E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) = 0,$$

I test whether all sixty, intermediate one-month excess returns in the next five years are individually zero in expectation

$$E_t(s_{t+h} - s_{t+h-1} + i_{t+h-1}^{(1),*} - i_{t+h-1}^{(1)}) = 0 \text{ for } h = 1, 2, \ldots, k$$

The two approaches are clearly related to each other. In particular, as I will show below, if the expectations hypothesis (EH) on the yield curve held, then the two approaches would have a very clear and intuitive link, where the methodology of the previous papers would amount to averaging over the UIP violations I document. Thus, my results can help us understand why the previous long-run UIP tests have tended to find support for the UIP hypothesis. My findings suggest that this happens because looking at long-run bonds and the corresponding long-run exchange rate changes tends to average over intermediate, higher-frequency UIP violations that change sign and cancel each other out when cumulated over a sufficiently long horizon. Hence, the UIP is violated at both short and long horizons, and this does not appear to be due to a short-term phenomenon that disappears over the long run. Rather, the UIP violations themselves change nature with the horizon, and as I have shown in the main body of the text, the positive violations roughly offset the negative violations in the long-run which makes it appear as if the UIP held in the long-run when only long-run evidence is considered.

To illustrate the connection between the two empirical methodologies, consider the following predictive regression that forecasts the excess return on long term, k-month bonds held to maturity

$$\frac{s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})}{k} = \alpha_k + \delta_k(i_t^{(1)} - i_t^{(1),*}) + \varepsilon_{t+k} \tag{16}$$

To emphasize the link with my main empirical results, I use the one month interest rate differential as the predictive variable, and for the same reason I have standardized (i.e. divided by k, the holding period) the left hand side to turn it into the *per month* excess return on the k-months to maturity bonds. It is worth emphasizing that the predictive variable I use is not the same as in the previous literature, which has rather used the corresponding long-term interest rate differential $i_t^{(k)} - i_t^{(k),*}$ as the regressor. Here I consider instead a regression on the short-term, one-month interest rate in order to highlight the link with the main empirical results, which always the one-month interest rate differential as the regressor.

As mentioned earlier, the link between my empirical strategy and the existing studies is particularly straightforward when the expectations hypothesis on the term structure of the interest rate differential across countries holds, at least up to a constant risk-premium, so that

$$i_t^{(k),*} - i_t^{(k)} = \frac{1}{k}\left(\sum_{h=0}^{k-1} E_t(i_{t+h}^* - i_{t+h}) + c_k\right).$$

Intuitively, this means that the yield differential on long-term k-period bonds is equal to the expected average of future 1-period short-term interest rate differentials, up to a constant risk-premium.[74] In this case, the long holding period excess returns are equal (up to a constant) to the expected sum of the future, one-period excess returns from now until maturity:

$$\begin{aligned} E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) &= E_t\left(\sum_{h=0}^{k-1}(s_{t+h+1} - s_{t+h} + i_{t+h}^{(1),*} - i_{t+h}^{(1)} + c_k)\right) \\ &= E_t(\sum_{h=0}^{k-1}\lambda_{t+h+1}) + c_k. \end{aligned}$$

In other words, the k period excess return on foreign bonds relative to the home bonds is akin to cumulating the k one period, intermediate excess returns. With this result in mind, it is not surprising that we can re-express the regression coefficient δ_k as:

[74] Bekaert et al. (2007) show that this may not be a bad assumption. They show that in international data the hypothesis is rejected statistically but the actual numerical deviations are quite small and they argue that they are insignificant from an economic point of view.

$$
\begin{aligned}
\delta_k &= \frac{1}{k}\frac{\text{Cov}(E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})), (i_t^{(1)} - i_t^{(1),*}))}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \frac{1}{k}\frac{\text{Cov}(E_t(\sum_{h=0}^{k-1} \lambda_{t+h+1}) + c_k, (i_t^{(1)} - i_t^{(1),*}))}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \frac{1}{k}\sum_{h=0}^{k-1}\frac{\text{Cov}(\lambda_{t+h+1}, i_t^{(1)} - i_t^{(1),*})}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \sum_{h=1}^{k}\frac{\beta_h}{k}
\end{aligned}
$$

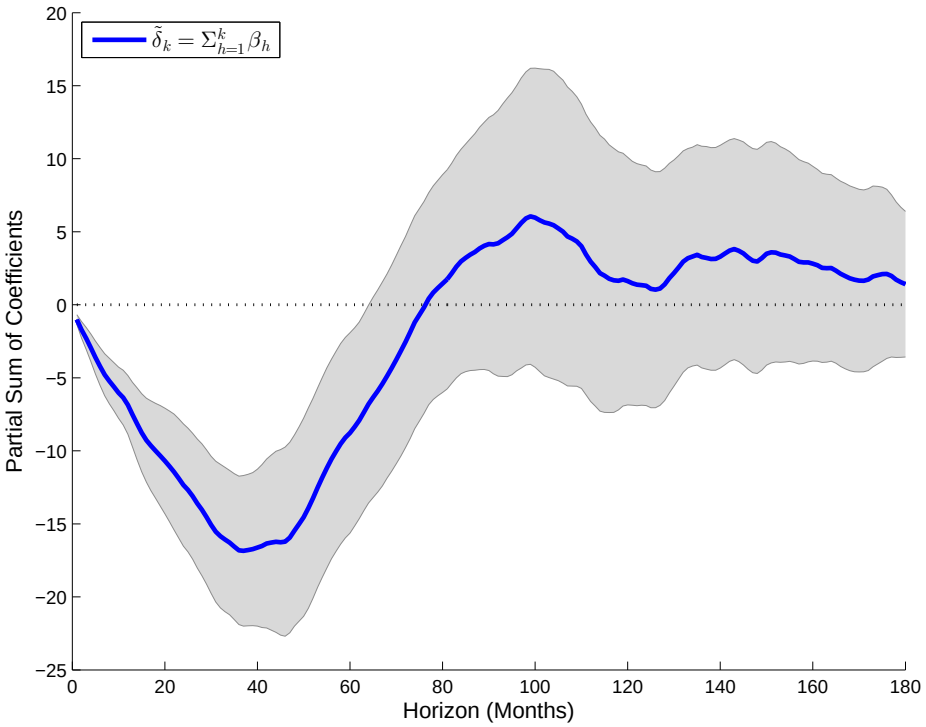
In the above expression β_h are the coefficients obtained in the the main set of regressions, defined in equation (3), where we regressed one-month excess returns at different horizons on the current one-month interest rate differential. Thus, the coefficient in a long-run UIP regression that uses long-term, k-period excess returns as the dependent variable, can be viewed as the average of the k regression coefficients from the one-period UIP regressions at the intermediate horizons from 1 to k periods in the future.

This suggests that UIP regressions using long-term bonds and returns tend to average over the high-frequency UIP violations within the full holding period of the long-term bond. Combined with my previous finding that the high-frequency violations change sign at longer horizons, this can help rationalize why previous studies have found strong evidence against the UIP in the short-run but much weaker evidence in the long-run. Moreover, the weaker evidence in long-run studies is not because the UIP condition tends to hold over horizons of five years or more – Figure 1 shows that in fact the UIP is violated at horizons of up to 7 years. Instead, my results suggest that long-run UIP test might find weaker evidence against the UIP because the fundamental nature of the underlying, higher-frequency UIP violations change sign at longer horizons. A regression with a sufficiently long-period cumulative return on the left hand side would average over negative and positive violations that cancel each other out and it may look like UIP holds.

To illustrate the fact that the positive and negative violations roughly cancel out at long horizons, Figure 2 plots $\tilde{\delta}_k = \sum_{h=1}^{k} \beta_h$, the cumulative sum of the UIP coefficients β_k against the horizon in months on the X-axis. The solid blue line shows the value of the partial sum up to horizon k and the shaded area represents the 95% confidence interval. The partial sums are at first decreasing, because the short-horizon violations are all negative, but eventually start rising and at the longest horizons appear to be roughly zero. This shows that the cumulative total of the positive UIP violations is approximately equal to the cumulative total of the negative violations.

Thus, the results of this paper support the earlier findings that the exchange rate behavior in the long run, on average, is consistent with the UIP hypothesis. But this is not because UIP violations manifest themselves only in the short-run and get washed out in the longer-run. The reason is instead that the high-frequency violations change sign at

Figure C.1: Cumulative Sums of the UIP Coefficients β_k



different horizons, and thus tend to cancel each other over the long-run. Interestingly, the UIP condition is still violated in both the short and the long run, but nevertheless exchange rate behavior in the long run appears to be consistent with Uncovered Interest Parity.

D Monetary Independence and UIP Violations in the Model

In this section I will illustrate how monetary independence is related to UIP violations in the (quantitative) model. The relationship is trivial in the analytical model, as keeping debt constant in that model removes any time variation in excess returns. For the quantitative model, I will consider two different experiments, and in both I will only be changing the behavior of the foreign central bank but not the home one. In the first experiment, I will augment the Taylor rule of the bank to include the outstanding stock of government debt and in the second I will consider an Active Fiscal/Passive Monetary policy mix, in which case the fiscal authority does not raise taxes to pay off debt and the central bank is left to induce inflation and keep the government solvent by inflating it away.

For the first experiment, I will keep the assumption that the home central bank follows a standard Taylor rule and is committed to fighting inflation, but will assume that part of the mandate of the foreign central bank is to keep foreign government debt constant. I only change the policy in the foreign country because in the data I take the USD as the benchmark exchange rate and the Federal Reserve's actions have been apparently independent of fiscal considerations for the time period at hand. I implement this in a straightforward way,

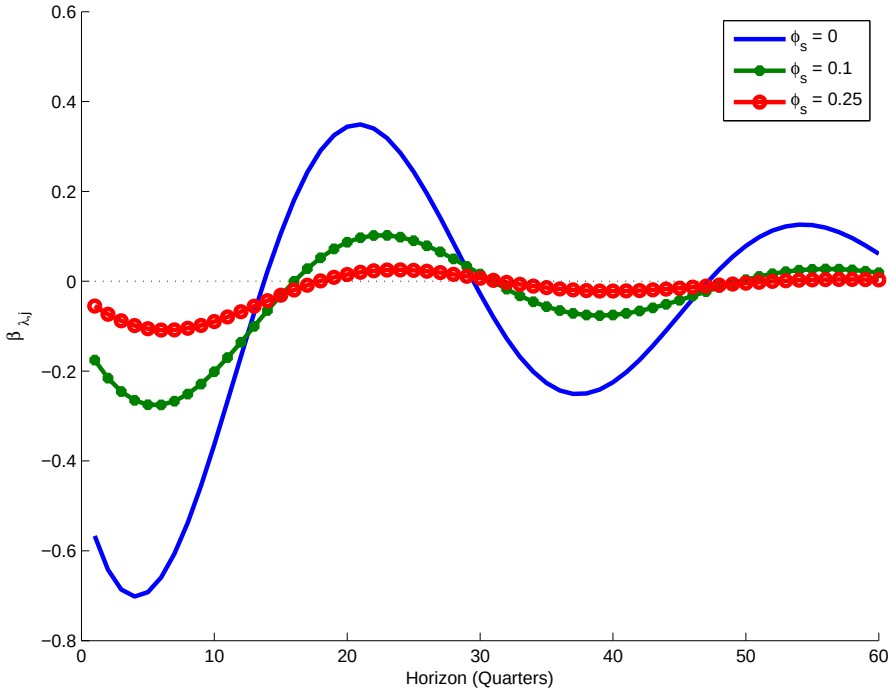
by augmenting the foreign Taylor rule to respond to the outstanding amount of foreign government debt

$$\hat{i}_t^* = \rho_i \hat{i}_{t-1}^* + (1-\rho_i)\phi_\pi^* \hat{\pi}_t^* - (1-\rho_i)\phi_b^* \hat{b}_{f,t}^* + v_t^*,$$

where $\phi_b^* > 0$. With this parametrization, when foreign government debt rises, the foreign central bank lowers interest rates which helps reduce the cost of servicing the debt and also puts an upward pressure on inflation, and helps inflate some of the debt away. Overall, increasing ϕ_b^* decreases the volatility of government debt.

I plot the implied UIP regression coefficients from the augmented version of the model in Figure D.1 for three different values of ϕ_b. As we can see, increasing ϕ_b leads to lower UIP deviations at all horizons, but the effect is particularly visible for the positive long-horizon violations. With $\phi_b = 0.25$ the longer horizon positive violations virtually disappear, while there are still short-horizon negative violations, although they are small.

Figure D.1: UIP Regression Coefficients, Augmented Taylor Rule



The intuition for the results is that when the monetary authority is actively trying to stabilize debt it achieves two things. First, it makes government debt less sensitive to shocks and second its actions help any new debt to be repaid quicker, which brings debt back to steady state faster and also greatly reduces the cyclicality in its dynamics. Debt is less responsive to shocks, because concurrent with the shocks (not only monetary, but any type of shock, e.g. technology, fiscal etc.) there is a systematic, endogenous response in the monetary policy that counteracts the shock. Moreover, any new debt ends up being repaid much faster, as compared to when the fiscal authority is on its own, because the endogenous monetary policy is persistently helping the fiscal authority pay off its debt through both reduced interest rates and increased inflation. This second effect is especially important

at long horizons because the quicker repayment of debts reduces the cyclical dynamics of government debt, and this explains the complete disappearance of positive UIP violations at longer horizons. Interestingly, it is enough to have only one of the two central banks (home and foreign) targeting government debt in their policy to achieve a significant reduction in the generated UIP violations.

Another exercise I consider is moving to an Active Fiscal and Passive Monetary policy in the foreign country. I use the terms Active and Passive policy as in Leeper (1991), hence an Active Fiscal and Passive Monetary policy mix is one where the fiscal authority does not adjust taxes sufficiently to stay solvent on its own, but instead the monetary authority allows inflation to rise and inflate debt to keep the government solvent. To implement this experiment, I set $\phi_\pi^* = 0$ and $\kappa_b = 0$, the common parametrization that characterizes Active Fiscal/Passive Monetary policy mix in the literature. This parametrization implies that in fact the fiscal authority does not respond to debt at all, and the monetary authority does not respond to inflation but rater adjusts interest so that inflation is allowed to rise and keep the government solvent.

The resulting implied coefficients from this experiment are plotted in D.2 and they show that UIP violations virtually disappear again. The reason is very much the same as before, the actions of the central bank are again stabilizing foreign debt supply, which makes it less responsive to shocks and also helps pay it down faster and removes the cyclical dynamics. This greatly weakens through key mechanisms through which UIP violations are generated: dynamics of government debt and spillover of shocks abroad. As a result, the implied UIP deviations become almost negligible.

Figure D.2: UIP Regression Coefficients, Active Fiscal Policy

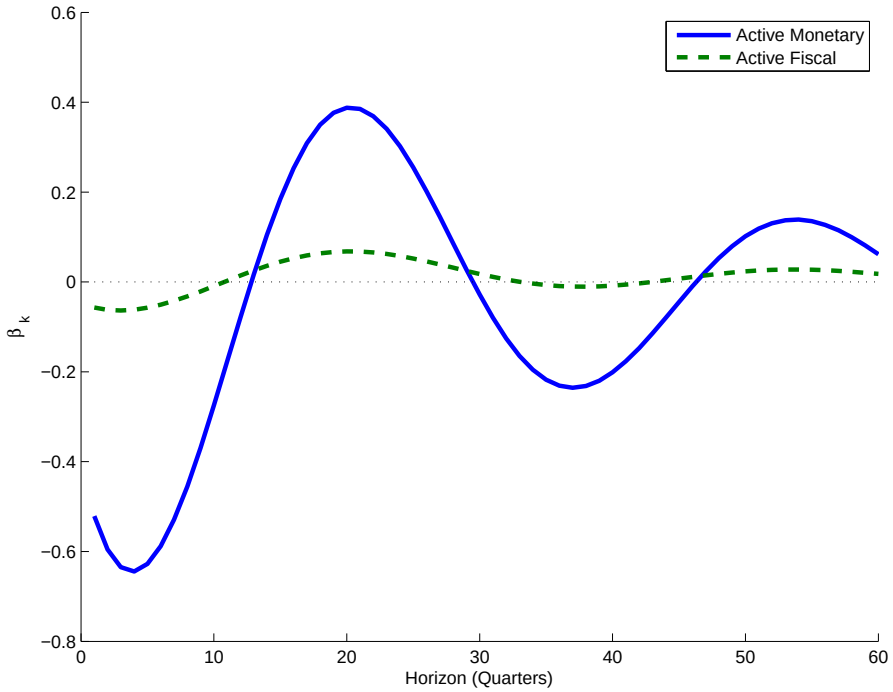
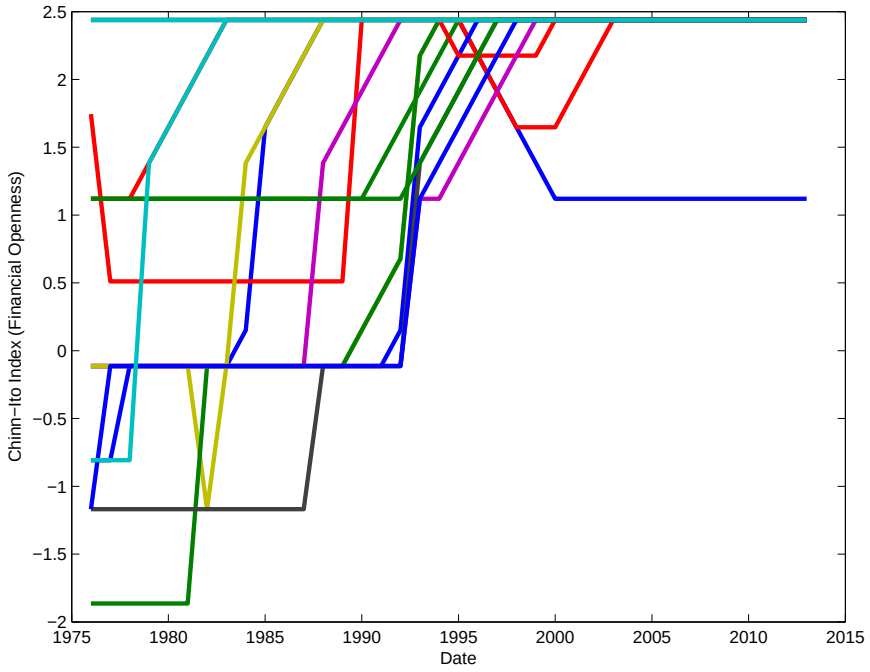


Figure E.1: Evolution of Capital Openness



E Extra Figures

Figure E.2: The Evolution of Public and Private Debt

