No-Bubble Condition: Model-Free Tests in Housing Markets

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Abstract

We test for the existence of classic rational bubbles in housing markets by directly measuring failures of the transversality condition that requires the present value of payments occurring infinitely far in the future to be zero. We study housing markets in the U.K. and Singapore, where residential property ownership takes the form of either leaseholds or freeholds. Leaseholds are finite maturity, pre-paid, and tradable ownership contracts with maturities often exceeding 700 years. Freeholds are infinite maturity ownership contracts. The price difference between leaseholds with extremely long maturities and freeholds reflects the present value of a claim to the freehold after leasehold expiry, and is thus a direct empirical measure of the transversality condition. We estimate this price difference, and find no evidence for classic rational bubbles in housing markets in the U.K. and Singapore, even during periods when a sizeable bubble was regularly thought to be present.

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The existence of bubbles in asset markets is one of the fundamental debates in economics and finance, yet challenges to designing an appropriate test for bubbles have prevented an empirically-driven resolution to this debate. In this paper, we propose a model-free and direct empirical test for the existence of classic rational bubbles in housing markets. Our test overcomes many of the challenges that pervade the existing empirical literature.

The classic rational bubble is the workhorse model of bubbles in macroeconomics and finance. It is based on a failure of the transversality condition that requires the present value of a payment occurring infinitely far in the future to be zero. In this paper, we call the claim to this infinitely-delayed payment the "bubble claim," and denote its price by B_t. The transversality condition, sometimes also called the "no-bubble condition," requires that:

$$B_t = \lim_{T \to \infty} E_t[\xi_{t,t+T} P_{t+T}] = 0, \tag{1}$$

where $\xi_{t,t+T}$ is a model-implied discount factor between dates t and $t+T$, and P_{t+T} is the price of the payment at time $t+T$. The bubble claim, B_t, is the present discounted value of the claim to the payment at time $t+T$, for T infinitely large. Theories about classic rational bubbles provide a sharp null and alternative hypothesis: $B_t = 0$ if there is no bubble, and $B_t > 0$ if there is a bubble, because it implies a positive price today for a claim that postpones payments indefinitely and, therefore, has zero fundamental value.

The classic rational bubble has received widespread attention in the theoretical literature, with classic papers by Samuelson (1958), Blanchard and Watson (1982), Tirole (1982, 1985) and Froot and Obstfeld (1991). It has also become the dominant model of bubbles in macroeconomics (e.g., Caballero and Krishnamurthy, 2006; Arce and López-Salido, 2011; Martin and Ventura, 2012, 2014; Doblas-Madrid, 2012; Giglio and Severo, 2012; Galí, 2014; Galí and Gambetti, 2014; Caballero and Farhi, 2014). Despite the central role that classic rational bubbles play in theoretical models of macroeconomics and finance, empirical evidence on their existence has remained elusive. Yet, the empirical relevance of classic rational bubbles is central not only to the debate about whether there are bubbles in financial markets, but also to discussions about what kind of bubbles might be present. This latter question is important because the positive and normative implications of models with bubbles crucially depend on the features of the particular type of bubble that is considered. In this light, our empirical results can help to guide future theoretical work on the design of bubble models.

The natural test for classic rational bubbles is to verify whether claims to payments at infinite maturity do, in fact, have zero or positive present value. This direct test, however, has been impossible to conduct, because we normally do not observe traded claims to

payments that only occur at (even approximately) infinite maturity. We generally either observe prices of claims to cash flows at all horizons (e.g., equities), or prices of claims to cash flows for finite but relatively short horizons (e.g., bonds).[1] Due to the challenges with directly measuring the price of extremely long-run financial claims, the literature has resorted to indirect model-dependent tests of bubbles, thus incurring the *joint hypothesis problem*: every test of a bubble is a joint test of the presence of the bubble in the data and the validity of the model applied by the econometrician.[2]

A connected literature has tried to rule out the classic rational bubble on theoretical grounds, arguing that it is inconsistent with infinitely-lived agents' optimization, backward induction, and no-trade theorems (see Tirole, 1982; Milgrom and Stokey, 1982; Santos and Woodford, 1997). For each theoretical challenge to the classic rational bubble, subsequent theoretical papers have restored the plausibility of its existence by showing that it can occur in the presence of imperfections such as overlapping generations, financial frictions, and market structures that prevent prices from being informative about the bubble (e.g., Tirole, 1985; Kocherlakota, 1992, 2008; Farhi and Tirole, 2012; Doblas-Madrid, 2012). It seems unlikely, therefore, that the active debate over the existence of classic rational bubbles can be settled on purely theoretical grounds; on the contrary, it is an inherently empirical question.

We provide direct tests of classic rational bubbles by exploiting a unique feature of housing markets in the U.K. and Singapore, where property ownership takes the form of either very long-term leaseholds or freeholds. Leaseholds are *finite maturity*, pre-paid and tradable ownership contracts, often with initial maturities of 999 years, while freeholds are *infinite maturity* ownership contracts. The price difference between leaseholds with extremely long maturities (e.g., more than 700 years) and freeholds for otherwise identical properties captures the present value of rental income starting at leasehold expiry, and thus approximates the price of the bubble claim, B_t. By estimating this price difference in the data, we can obtain a direct estimate of the price of the bubble claim, and test whether it is indeed positive. Such an empirical strategy has the advantage of being both *model free* and *direct*. It is model free in the sense that all structural models agree that the fundamental value of the claim B_t is zero. It is direct because we test the very condition that defines the bubble, the no-bubble condition, rather than deriving and testing indirect,

[1] For example, the U.K. government has issued some (callable) perpetuities; however, the longest finite-maturity bond that could be used to back out the equivalent zero-coupon price has a maturity of only 40 years, a horizon unsuitable to approximate the infinity limit required by the theory.

[2] Both sides of the debate on the efficient market hypothesis agree on this fundamental difficulty. Shleifer (2000) remarks: "The dependence of most tests of market efficiency on a model of risk and expected return is Fama (1970)'s deepest insight, which has pervaded the debates in empirical finance ever since."

model-implied necessary or sufficient conditions for the existence of a bubble.

Our empirical analysis is based on proprietary information on the universe of property sales in the U.K. and Singapore between 1995 and 2013. These data contain information on transaction prices, leasehold terms and property characteristics such as location and structural attributes. We estimate the price of the bubble claim by comparing the prices of leaseholds with maturities between 700 years and 1,000 years to the prices of freeholds across otherwise identical properties. We use hedonic regression techniques to control for possible heterogeneity between leasehold and freehold properties. We find that extremely long leaseholds are valued identically to otherwise similar freeholds. Our results, therefore, show no violation of the no-bubble condition in these markets. This is true even during time periods and in geographic regions when people argued that housing bubbles were likely to be present (e.g., U.K. in 2007 or 2013, or Prime Central London).

We address a number of potential challenges to our methodology of directly testing for classical rational bubbles in these housing markets. For example, one might be concerned that freeholds are, in fact, inferior on unobservable property characteristics, and that their price parity with leaseholds is therefore masking a bubble.[3] We show that this is not the case, by documenting that annual rents in Singapore are identical across leasehold and freehold properties. Since any relevant differences in property characteristics should be reflected in these rents, this minimizes concerns that the flow utility from inhabiting either type of property might be different. This finding is not surprising, since properties with extremely long leaseholds and freeholds located in the same geography look identical on all observable characteristics, making it unlikely that they would differ substantially on unobservable characteristics. We also show that freehold and leasehold contracts are similarly liquid and have a similar "time on market" when listed for sale, addressing concerns that differences in liquidity might mask a bubble.

In addition, we discuss a number of institutional features in these housing markets that might affect our interpretation. We consider the possibility of a psychological "full ownership premium," covenants in leasehold contracts, lease extensions, options and restrictions to redevelopment, taxes, and the differential timing of sales and origination of different types of contracts. Importantly, we conclude that our empirical set-up is, if anything, biased toward finding a bubble. For example, the possibility of contractual

[3]Potential challenges to our testing strategy have to conjecture (i) the possibility that a bubble is present, thus making the freehold more valuable than the 999-year leasehold, but that (ii) some confounding factor increases the price of the 999-leasehold exactly by the amount of the bubble. This would result in the two contracts trading for the same price, as we estimate in the data, and would thus mimic an equilibrium without the bubble.

restrictions such as covenants, which could reduce the flow utility of a leaseholder relative to a freeholder, would bias our tests toward finding a bubble. In addition, the fact that we obtain the same results in two markets with different institutional and economic environments, the U.K. and Singapore, minimizes the concern that our no-bubble result is due to market-specific institutional frictions.

We focus on bubbles that require a failure of the transversality condition because they are an important workhorse model of bubbles and the dominant model in macroeconomics, and because we can offer a clear test for such bubbles. We call this type of bubble the "classic rational bubble" because the most prominent models that generate a failure of the transversality condition are models of rational bubbles. These are not the only models of bubbles, and our paper and test methodology are silent on the possible presence of bubbles that can occur in finite horizon economies. Section 1 provides more details on the terminology adopted in this paper and the scope of our test.

Similarly, our paper focuses on residential real estate because the institutional set-up in the U.K. and Singapore provides a clear test of the theory of classic rational bubbles; we are silent on the possibility of bubbles in other asset classes and at other points in time. Nevertheless, residential real estate has been the subject of enormous bubble-related attention during the recent boom-bust cycle. Figure I shows the behavior of real house prices in the U.K. and Singapore. Both countries experienced strong increases in house prices from the late 1990s to 2007, a pronounced reversal during the financial crisis 2007-2009, and a renewed boom after that. These periods have often been described to exhibit a housing bubble. We also plot a "bubble index," constructed by counting the number of references to "real estate bubbles" in major newspapers. For both countries, this index generally increases with house prices, spiking particularly in 2013.

Motivated by this evidence, a sizable academic literature has speculated about the presence of bubbles in housing markets over this period (Case and Shiller, 2003; Himmelberg, Mayer and Sinai, 2005; Glaeser, Gyourko and Saiz, 2008; Piazzesi and Schneider, 2009; Glaeser, Gottlieb and Gyourko, 2010; Mayer, 2011; Glaeser and Nathanson, 2014; Pavlidis et al., 2013; Nathanson and Zwick, 2014). Galí (2014) and Galí and Gambetti (2014) motivate their rational bubble model with the recent boom-bust pattern in house prices, and Kocherlakota (2009), Arce and López-Salido (2011), Basco (2014) and Miao, Wang and Zhou (2014) provide a classic rational bubble model tailored to explain these patterns. We therefore conclude that our focus on the housing markets in Singapore and the U.K. during a period of such extreme boom-bust cycles provides a setting with an ex-ante very good chance of detecting a bubble; in contrast, our results show that no classic rational bubble was actually present.

Our negative result on the presence of rational bubbles impacts not only the ongoing effort to understand real estate prices and the theoretical effort to distinguish between rational and behavioral models of bubbles, but also the theory of general equilibrium in the presence of borrowing limits. Indeed, Hellwig and Lorenzoni (2009) show the equivalence between the existence of equilibria with endogenous self-enforcing private debt and the presence of a rational bubble. Relatedly, Scheinkman and Weiss (1986); Kocherlakota (1992); Santos and Woodford (1997) connect equilibria with exogenous borrowing constraints to rational bubbles. Trough the lenses of these theoretical equivalence results our empirical finding that the transversality condition holds or, equivalently, that there are no classic rational bubbles cast doubt on the empirical relevance of models of credit constraints that, while superficially unrelated, have been shown to rely on the same features of rational bubbles that are tested in this paper.

1 Theoretical and Empirical Review

The theoretical literature on bubbles is vast and richly varied.[4] The focus of this paper is the classic rational bubble whose existence is associated with a failure of the transversality condition of the asset valuation equation. Consider an asset that pays dividend D_t in each period, and denote its price at time t by P_t. If $\xi_{t,t+T}$ is a valid stochastic discount factor for this asset, we have:[5]

$$P_t = E_t[\xi_{t,t+1}(P_{t+1} + D_{t+1})].$$

The price today equals the present discounted value of the price and dividend tomorrow. Applying a recursive argument and the law of iterated expectations, we obtain:

$$P_t = \sum_{s=1}^{\infty} E_t[\xi_{t,t+s} D_{t+s}] + B_t, \qquad B_t \equiv \lim_{T\to\infty} E_t[\xi_{t,t+T} P_{t+T}], \tag{2}$$

where we have defined $\xi_{t,t+s} \equiv \prod_{j=0}^{s-1} \xi_{t+j,t+j+1}$, so that $\xi_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$. The price of the asset is decomposed into its fundamental value (i.e., the present discounted value of dividends), and a bubble component B_t. The absence of a classic rational bubble requires that $B_t = 0$, independently of the model-specific discount factor (ξ) or the asset under consideration. If instead $B_t \neq 0$,

[4] See Brunnermeier and Oehmke (2013) and Shleifer (2000) for reviews.

[5] Note that while the existence of a stochastic discount factor requires that the law of one price holds, this mild restriction is not strictly necessary for our empirical tests. We adopt it here for expositional convenience and review its implication for our empirical tests in Section 2.2.

there exists a bubble.[6] If present, the bubble evolves according to $B_t = E_t[\xi_{t,t+1} B_{t+1}]$.

In the classic papers of Blanchard and Watson (1982) and Froot and Obstfeld (1991) the failure of the transversality condition occurs because the bubble grows faster than the discount rate applied to the asset, which in these models is assumed to be a constant rate of return.[7] There has been a critical literature verifying the theoretical conditions under which a classic rational bubble can arise. For example, standard neoclassical theories rule out this bubble because it is inconsistent with the optimization problem of an infinitely-lived representative agent, or inconsistent with backward induction in a finite horizon setting (Brock, 1982; Tirole, 1982; Santos and Woodford, 1997). In settings with asymmetric information and a finite number of infinitely-lived agents, the "no-trade theorems" also preclude the existence of the classic rational bubble (Tirole, 1982; Milgrom and Stokey, 1982).[8]

These theoretical challenges have inspired a literature demonstrating that the classic rational bubble can occur in economies with combinations of overlapping generations, incomplete markets, and financial frictions (Tirole, 1985; O'Connell and Zeldes, 1988; Kocherlakota, 1992, 2008; Farhi and Tirole, 2012; Doblas-Madrid, 2012).[9] This literature reaffirmed the theoretical plausibility of the classic rational bubble as an equilibrium phenomenon. Recent contributions have imported the classic rational bubble into macroeconomics and finance to study its effects on the accumulation of capital as a store of value, and the resulting possibility of dynamic inefficiency due to over-accumulation (Caballero and Krishnamurthy, 2006; Arce and López-Salido, 2011; Martin and Ventura, 2012, 2014; Giglio and Severo, 2012; Farhi and Tirole, 2012; Galí, 2014; Galí and Gambetti, 2014; Caballero and Farhi, 2014).[10]

The theoretical literature has emphasized that while the classic rational bubble can occur on an asset that pays no dividends, like money, it is essential that the asset has infinite maturity. The classic rational bubble cannot occur on an asset of arbitrarily long but finite maturity. Recall that these bubbles require expectations of ever increasing prices at least along some equilibrium path, with each agent expecting to sell the bubble for a higher price to the next agent. The finite maturity of the asset breaks this loop because

[6] In what follows, we focus on the case $B_t > 0$, because a negative bubble can be easily ruled out if there is free disposal of rents. Nonetheless, our test would detect negative bubbles if they were present.

[7] These models are a particular case of equation (2) in which $\xi_{t,t+s} = R^{-s}$, and R is the gross expected rate of return on the asset. Subsequent literature has allowed for time-varying discount rates.

[8] See Campbell, Lo and MacKinlay (1997), Chapter 7.1.2, for a review of these arguments.

[9] This work builds on earlier contributions of Samuelson (1958) and Diamond (1965) regarding the value of fiat money and government debt. See also Scheinkman and Weiss (1986), Woodford (1986), Woodford (1990) and Miao and Wang (2011).

[10] See also Basco (2014) and Ikeda and Phan (2014) for an open economy perspective.

no agent would want to hold the bubble in the last period before maturity; backward induction then makes it impossible for the bubble to be present in any earlier period.

A second necessary condition for bubbles to occur on assets such as real estate, which are in positive supply, is that the bubble cannot grow faster than the growth rate of the economy. Intuitively, if the bubble were to grow faster than the economy, it would eventually overtake the entire wealth of the economy. As long as wealth and output are cointegrated, it would therefore have to grow at the same rate in the long run. However, this does not mean that a classic rational bubble cannot occur in real estate markets. In fact, the literature has shown that bubbles can exist (and grow at a low rate) even on assets that, on average, have high returns, for two main reasons. First, the bubble does not have to have the same risk properties as the asset it pertains to; hence, it is quite possible that even a risky, high-return asset such as housing has a bubble component that is relatively safe and grows at a lower pace.[11] Second, in an economy with frictions, the rate of return on the asset can be higher than the rate at which the bubble grows, even if the bubble and the underlying asset are equally risky. Indeed, Farhi and Tirole (2012) show that even in a deterministic economy the marginal product of capital can be high, due to frictions that prevent its equalization with the risk-free rate, while the bubble on capital grows at the risk-free rate of return. These results do not overturn the necessary condition for the existence of a bubble derived by Tirole (1985), namely that the long-run growth rate of the bubble should be less than that of the economy, but they restore the theoretical case for bubbles by showing that such condition could easily be satisfied in realistic set-ups.

While $B_t > 0$ is a sufficient condition for the existence of a bubble, it is not a necessary condition. In fact, an interesting literature has created models of bubbles that can occur even in finite horizons (Allen, Morris and Postlewaite, 1993; Abreu and Brunnermeier, 2003; Scheinkman and Xiong, 2003; Hong, Scheinkman and Xiong, 2006, 2008; DeMarzo, Kaniel and Kremer, 2008). In this paper, we only focus on the workhorse classic rational bubble that requires a failure of the transversality (no-bubble) condition, not only because it is a benchmark and popular case, but also because we can provide a clean testing ground for its existence. Our tests, and this theoretical review, are silent on other types of bubbles whose presence in the data remains an open question for future research.

Similarly, our tests do not distinguish between rational and irrational or behavioral bubbles. If $B_t > 0$, there is a bubble, but it could be either rational or irrational. For most of the paper we call B_t a "classic rational bubble" because in the theoretical literature

[11] This is the case, for example, in Galí (2014)'s economy, where the bubble is attached to capital, a risky asset with a positive risk premium, but the bubble itself is purely idiosyncratic and therefore earns the risk-free rate of return.

rational bubbles, in the tradition of Tirole (1982, 1985), are almost exclusively associated with failures of the transversality condition. We clarify here, however, that in principle a behavioral bubble could generate a similar failure. On the other hand, $B_t = 0$ does not imply the absence of behavioral bubbles, as pointed out in the previous paragraph, or even rational bubbles (Conlon, 2004; Doblas-Madrid, 2014).[12]

Determining whether classic rational bubbles are an empirically relevant phenomenon is not only crucial to the overall debate on the presence of bubbles, but is also important within the bubble literature to understand which types of bubbles theoretical models should focus on. Both the positive and normative implications of bubbles depend strongly on the type of bubble being modeled. Since we find no evidence for the classic rational bubble that dominates the macroeconomics literature, it will be important going forward to understand how the positive and normative conclusions of this literature would change in the presence of a (possibly-irrational) finite-horizon bubble. It is often argued that the dominance of the classic rational bubble in the macroeconomics literature is due to its theoretical simplicity and tractability, rather than to its realism. However, the modeling choice is not innocuous: for example, the implication in Galí (2014) that an increase in interest rates by the central bank stimulates the bubble depends on the fact that rational bubbles grow at that rate. The same is not necessarily true for other types of bubbles.

1.1 Existing Empirical Tests

The empirical literature on bubbles is vast but, despite the notable research effort, also largely inconclusive. In their recent survey of the literature on bubbles, Brunnermeier and Oehmke (2013) emphasize that "*identifying bubbles in the data is a challenging task. The reason is that in order to identify a bubble, one needs to know an asset's fundamental value, which is usually difficult to measure.*" By focusing on bubbles that require a failure of the transversality condition our tests are model independent – all infinite-horizon models that preclude bubbles agree that the bubble claim should have zero value.

However, even the literature focusing on bubbles with failures of the transversality condition has been plagued by a number of econometric or interpretational issues. The earliest tests focused on bubbles in the price level during hyper-inflations. In this line, Flood and Garber (1980) test the rational expectation model of Sargent and Wallace (1973) and find no evidence of bubbles. Rather than reviewing each subsequent

[12]The distinction between rational and behavioral bubbles can be blurry and depends on the exact definition adopted by each author. Here we only stress the difference between bubbles that manifest themselves as a failure of the trasnversality condition and other types of bubbles that can occur even in finite horizon set-ups. See also: Morris, Postlewaite and Shin (1995); Allen, Morris and Shin (2006).

paper in detail, we focus on the main difference between our approach and that of the previous literature.[13] Previous tests focused on *indirect* measures of a failure of the no-bubble condition by testing the cointegration between prices and some transformation of current dividends (Diba and Grossman, 1988*a*), imposing a structural model of the present discounted value of dividends (Shiller, 1981), or testing model-implied necessary conditions for the existence of a bubble (Abel et al., 1989). Given the dependence of the results on the different assumptions imposed in each case, these tests – while influential – were overall inconclusive. For example, Evans (1991) shows that the cointegration test of Diba and Grossman (1988*a*) may fail to detect bubbles that periodically collapse; Merton et al. (1985) highlights that Shiller (1981)'s test is sensitive to the specification of the appropriate fundamental model; and Farhi and Tirole (2012) show that the dynamic-inefficiency conditions tested in Abel et al. (1989) are no longer necessary for the existence of bubbles in the presence of financial frictions.[14] Our approach, as explained in detail in the next section, differs from the previous papers because we *directly* measure deviations from the no-bubble condition.[15]

A different, but complementary, approach in the literature has been to document deviations from arbitrage and interpret such mispricing in the light of bubble theory.[16] For example, Lamont and Thaler (2003) document that the Palm/3Com carve-out during the height of the internet boom led to mispricing in the shares of Palm and 3Com that is hard to attribute to fundamental differences. The authors offer an interpretation based on an irrational premium for internet stocks at the time. Similarly, Xiong and Yu (2011) document that warrants on Chinese stocks traded at prices inconsistent with the value of the underlying replicating portfolio, the fundamental value, which they argue to be zero due to the extreme out-of-the-money nature of the warrants. The authors offer an explanation based on the "resale-option value" theory of bubbles in the presence of heterogeneous beliefs and short sale constraints (Scheinkman and Xiong, 2003). Our approach is complementary, because we focus on a different type of bubble by testing failures of the no-bubble condition rather than testing for irrational theories in finite time.

[13] See Flood and Hodrick (1990) for a survey of the econometric difficulties in the previous literature. See also: Flood and Garber (1980), Diba and Grossman (1988*b*) and West (1987).

[14] On the possible misspecification of Shiller (1981)'s volatility bounds as a test for classic rational bubbles see also Mankiw, Romer and Shapiro (1985) and Flood and Hodrick (1986). Following Abel et al. (1989), Geerolf (2014) uses new data to find more favorable evidence for dynamic inefficiency of advanced economies, and links it to the empirical plausibility of asset bubbles.

[15] Our test also differs from the predictability test for bubbles suggested by Cochrane (2011) because we test the existence of a bubble directly in the level of an asset price. Cochrane's test focuses on whether the bubble accounts for any variation in price-dividend ratios over time and therefore cannot rule out the existence of a bubble that accounts for a constant, and possibly sizable, fraction of the asset price.

[16] The mispricing is generally assumed to persist due to limits to arbitrage (Shleifer and Vishny, 1997).

We also analyze an aggregate asset class, real estate, that constitutes a large fraction of household portfolios rather than individual securities. Finally, a vast and interesting literature pioneered by Smith, Suchanek and Williams (1988) has tested for bubbles in experimental settings. We differ from this literature by analyzing market prices of traded assets with both finite and infinite maturity.

To sum up, classic rational bubbles are a workhorse model of bubbles and the dominant type of bubble in macroeconomic theory. Theoretical restrictions to rule them out have been shown to not be binding in practice. Yet, direct empirical tests to verify their existence in the data have been lacking because of the difficulties of observing market prices for cash flows at approximately infinite horizons. Our contribution is to provide the first such test. Our result of no classic rational bubbles in the data can guide the theoretical literature towards developing new models of bubbles with potentially important differences in their positive and normative implications.

2 Institutional Setting and Empirical Test

In this section we outline our empirical approach to directly estimating the value of the bubble claim, B_t. We begin by describing the institutional setting of the U.K. and Singapore housing markets that we exploit to do so. The next section discusses our data and presents the empirical results.

2.1 Institutional Setting

Residential real estate ownership in both the U.K. and in Singapore comes in two forms: infinite maturity ownership, called a freehold, and long-duration, finite maturity ownership, called a leasehold.[17] A leasehold is a grant of exclusive possession of the property for a clearly defined, finite period of time during which the tenant has the right to exclude all other people from the property, including the freeholder (Burn, Cartwright and Cheshire, 2011). In the U.K., common initial leasehold maturities are 99, 125, 150, 250 and

[17]This contract structure is not unique to the U.K. and Singapore. Other papers in the real estate literature have studied the pricing of leasehold and freehold contracts in a variety of settings and countries (e.g., Capozza and Sick, 1991; Wong et al., 2008; Iwata and Yamaga, 2009; Tyvimaa, Gibler and Zahirovic-Herbert, 2014; Bracke, Pinchbeck and Wyatt, 2014; Gautier and van Vuuren, 2014). None of these papers focus on the implications for studying bubbles. Giglio, Maggiori and Stroebel (2014) also exploit this institutional set-up, but use it to address a different economic question: the estimation of very long-run discount rates. The level of long-run discount rates is not relevant for the present paper because any positive net-discount rates, net of the growth rate of rents, are uninformative about the presence of a bubble. Due to the different economic question, their use of the data also differs from the present paper because they focus on shorter (0-300 year) leases on flats as their main source of variation, while we focus on extremely long (700+ years) leases and freeholds on houses.

999 years. In Singapore, initial lease lengths are either 99 or 999 years. During the life of the leasehold, the lessee is entitled to similar rights as a freeholder would be, including the right to mortgage and rent out the property. Leaseholds and freeholds are also treated equally for tax purposes. Unlike for commercial leases, the vast majority of the costs associated with a residential leasehold come through the up-front purchase price; annual payments, the so-called "ground rents," are small to non-existent, and do not significantly affect the prices paid for leaseholds. Leasehold properties are traded in liquid secondary markets, where the buyer purchases the remaining term of the lease. The markets for freehold and leasehold properties are fully integrated, and the two types of contract are advertised side-by-side by real estate agents and on online platforms. Once the leasehold expires, the ownership reverts back to the residual freeholder.

In the U.K., the set of residual freeholders is widely varied, and includes large private corporations, aristocratic estates, the Church of England, Oxford and Cambridge colleges, and the royal family. In Singapore, by far the largest residual freeholder is the government of Singapore, represented by the Singapore Land Authority (SLA). In Singapore, leaseholders have no statutory right to lease extensions or to acquire the underlying freehold interest, a process called enfranchisement. In the U.K., the Leasehold Reform Act 1967 has provided owners of houses with the right to extend the lease or enfranchise at market prices. Such transactions entail significant costs, including those for engaging a valuer and a solicitor, as well as the uncertainty and costs of a possible court trial. We discuss the impact of lease extensions and enfranchisements on our tests in Section 4.

One might be also concerned about a number of factors that could differentially affect the flow utility of leasehold and freehold properties such as potentially restrictive covenants, taxes, maintenance costs, and redevelopment rights. Section 3 discusses these features and shows them to be quantitatively small. In addition, these features would, if anything, bias our tests in favor of finding a classic rational bubble.

2.2 Empirical Test

The institutional setting of the U.K. and Singapore is uniquely suited to directly testing the no-bubble condition, Equation 1, since it allows us to estimate the present value of a claim to rents occurring at extremely long horizons (e.g., $T > 700$ years). Let us define by P_t the price of the freehold contract at time t and by P_t^T the price of the leasehold contract with maturity T at time t. The freehold is the claim to the infinite stream of rents generated by the property, while the leasehold is the claim to the rents for the first T years. A simple algebraic substitution, detailed in the appendix, shows that $P_t - P_t^T = E_t[\xi_{t,t+T} P_{t+T}]$;

intuitively, the difference in value between a freehold and a T-maturity leasehold is the present value of the claim to rents starting T years from today and extending to the infinite future. This is also the present value of the claim to the freehold T years from now.

We focus on maturities of the leasehold in excess of 700 years, a horizon sufficiently long to well approximate the infinity limit of the transversality condition:

$$P_t - P_t^T \approx B_t = \lim_{T\to\infty} E_t[\xi_{t,t+T} P_{t+T}], \qquad \text{for } T > 700 \text{ years.}$$

We can test whether the no-bubble condition, Equation 1, holds on average by testing whether the price difference between extremely-long leaseholds and freeholds is zero. To make the interpretation of this difference easier, we normalize the price discount by the price of the freehold: $Disc_t^T \equiv \frac{P_t^T}{P_t} - 1$. Then, $-Disc_t^\infty$ is the fraction of the current price of the asset (the freehold) that is due to the classic rational bubble. We correspondingly formulate our null hypothesis of no-bubbles as: $Disc_t^T = 0$ for $T > 700$ years.

The test exploits the theoretical restriction that classic rational bubbles cannot exist on any asset of arbitrarily long but finite maturity (see Section 1). The bubble could affect the price of a freehold, an infinite maturity asset, but cannot affect the price of a leasehold, a finite maturity asset. Based on this restriction, the test makes two identifying assumptions: that leasehold and freehold cash flows only differ in their maturity, and that maturities greater than 700 years are a close approximation to the infinite limit, i.e. that the present value of rents more than 700 years in the future is close to zero. Testing the first assumption is a key focus of our empirical estimation (Section 3) and robustness checks (Section 4); the assumption is validated in the data. We next discuss the second assumption.

Intuitively, a horizon between 700 and 999 years is sufficiently long to approximate the infinite limit of the no-bubble condition. We (informally) quantify the approximation error, by considering the simple Gordon (1982) growth environment where rents grow at rate g and are discounted at a constant rate r. A simple derivation provided in the appendix shows that $Disc_t^T = -e^{-(r-g)T}$. In the absence of a bubble, even a very low net discount rate $(r-g)$ of 1% would only imply leasehold discounts of -0.09% and -0.001% at 700 and 900 years, respectively. A net discount rate $(r-g)$ of 1% is much lower than the values normally estimated in the literature, which has found average housing returns r to be above 6% and real rent growth g to be below 1%, leading to a more plausible calibration of $r-g$ above 5%.[18] This confirms that our horizon is sufficiently long for the

[18]See for example Flavin and Yamashita (2002) and Favilukis, Ludvigson and Nieuwerburgh (2010). Giglio, Maggiori and Stroebel (2014) find a long horizon net discount rate of $r - g = 1.9\%$ to be consistent with housing data. This discount rate is consistent, in the absence of bubbles, with zero leasehold discounts

approximation to hold well, even in the presence of very low net discount rates.[19]

Finally, while it is convenient in the theoretical exposition to employ the concept of a stochastic discount factor, $\xi_{t,t+T}$, the existence of which requires no deviations from the law of one price (Cochrane, 2005), our empirical test does not strictly require even this mild restriction. Even if the law of one price was violated, both in the data and in a model of bubbles with frictions, a classic rational bubble would still be defined as a positive price for an asset that postpones indefinitely making any payments and our test would correctly identify it in the real estate market.

3 Testing the No-Bubble Condition

In this section, we present results from our tests of the no-bubble condition. We begin by describing the data used in our empirical analysis. We then provide aggregate results where we pool transactions from all years. We also present year-by-year estimates to test whether there was a bubble at any point during our sample period. We finally address a number of possible concerns about confounding explanations of our results.

Estimating the relative price of leaseholds and freeholds is challenging because the underlying properties are heterogeneous assets, and we do not observe the same properties transacting both as leaseholds and as freeholds. Therefore, to estimate $Disc_t^{\infty}$, we need to compare different properties that are either freeholds or leaseholds. Since leasehold and freehold properties could differ on important dimensions such as property size and location, comparing prices across properties requires us to control for these differences. To do so, we use hedonic regression techniques (Rosen, 1974), which allow us to consider the variation in price over time and across contract type for different properties while controlling for key characteristics of each property. To conduct such hedonic regressions, detailed data on transaction prices and property characteristics are required.

3.1 U.K. Residential Housing Data

We obtain transaction-level administrative data on all residential housing sales between 1995 and 2013 from the England and Wales Land Registry. The data include the price paid, information on the property type (flat, detached house, semi-detached house or terraced house), the full address, and a "new construction" indicator. In addition, the

at horizons greater than 700 years.

[19] Pástor and Veronesi (2003) point out that market valuations might appear bubbly, despite the absence of bubbles, if there is sufficient uncertainty about growth prospects of dividends, with the possibility of having $r - g$ close to 0 in the long run. This possible confounding effect is not a concern for our estimates, because we find valuations to not be consistent with the presence of a classic rational bubble.

Land Registry has provided us with an indicator of whether the transaction was for a freehold or a leasehold property, as well as information on leasehold characteristics such as origination date and lease length. For a large subset of properties, we also obtain proprietary information on property characteristics such as the number of bedrooms, bathrooms, and the size, age and condition of the property. These are collected by Rightmove.co.uk from "for sale" listings and other data sources.[20] We observe a full set of hedonic characteristics for approximately 52% of the properties transacting since 1995.

We focus on houses in the U.K., because this market is dominated by freeholds and extremely long leaseholds, whereas the market for flats is dominated by shorter leaseholds. Given that our focus is on estimating the price difference between freeholds and extremely long leaseholds (with 700 years or more of remaining maturity at the time of transaction), Table I provides key summary statistics for these two types of contracts in the U.K. transaction sample. Our final dataset contains more than 7.6 million transactions between 1995 and 2013 for houses with a full set of hedonic characteristics.

Extremely long leaseholds account for 4.7% of our transaction sample, freeholds account for 94.3%, and shorter leaseholds constitute the remaining transactions. Figure II plots a heatmap of the share of all transactions that are of extremely long leaseholds across 3-digit postcodes.[21] A white postcode indicates an area with no extremely long leasehold transaction; a black postcode indicates an area where at least 2% of the transactions are for extremely long leaseholds. While 1% or 2% may seem like a small percentage, given the large size of our dataset it is large enough in absolute terms to provide us with good identification. While we find transactions of extremely long leaseholds everywhere in the U.K., there is a clear concentration in the North of England (around Manchester and Liverpool) and the South-West. Importantly, there are several postcodes in Greater London with a material fraction of extremely long leaseholds; some of our analysis will focus exclusively on this area, depicted in Figure III.

Panel A of Table I reports the mean, median and standard deviation of the log price, the number of bedrooms, the number of bathrooms, the age of the property and property size for extremely long leaseholds and freeholds in the sample of U.K. houses. Column (7) reports the average unconditional difference between the two contract types (Δ). Extremely long leaseholds properties are, on average, cheaper, smaller and older than freehold properties. Column (9) shows the average difference in each characteristic between extremely long leaseholds and freeholds, conditional on 3-digit postcode $\times$

[20] Rightmove.co.uk is the U.K.'s largest property portal, with more than 13 million unique monthly visitors.

[21] These postcodes, which are also called "postcode districts" constitute the level of geographic fixed effects for our analysis. On average, there are 24,700 inhabitants per postcode district.

transaction year × property type fixed effects. Once we control for spatial and temporal heterogeneity, extremely long leaseholds and freeholds are much more similar, with all the differences in means economically small. We conclude that variation in the geographic distribution of leasehold and freehold properties is the key driver of the unconditional differences in characteristics.

Similar evidence can also be seen in Figure IV, which plots the residuals of a regression of each property characteristic on 3-digit postcode × transaction year × property type fixed effects separately for extremely long leaseholds and freeholds. Panel A shows that, conditional on geographic and time fixed effects, leaseholds and freeholds have the same price distribution; in other words, the price similarity in Table I is not just true on average, but across the entire distribution. Panels B - E show that the distribution of hedonic characteristics is also very similar for leaseholds and freeholds, once we condition on the fixed effects.

3.2 Singapore Residential Housing Data

We also obtain transaction-level administrative data for all private residential housing sales in Singapore from the Urban Redevelopment Authority. We observe approximately 379,000 private market arms-length transactions between 1995 and 2013. For each transaction there is information on the transaction price and date, the lease terms, property characteristics and the precise location of the property. Panel B of Table I provides an overview of the extremely long leaseholds and freeholds in our sample. As in the U.K., leaseholds in Singapore tend to be cheaper than freeholds; they are also slightly larger and younger. The absolute difference between leaseholds and freeholds in terms of observable characteristics is economically small, whether or not we control for a set of 5-digit postcode × transaction year × property type fixed effects. Approximately 6.6% of our sample consists of extremely long leaseholds, 47.7% of freeholds; the remaining transactions are for shorter leaseholds.

Figure V shows the distribution of residuals from a regression of the transaction price and each characteristic on the postcode × transaction year × property type fixed effects separately for extremely long leaseholds and freeholds. As was the case in the U.K., conditional on the fixed effects the distribution of transaction price and observable characteristics are nearly identical for freeholds and extremely long leaseholds.

3.3 No-Bubble Condition: Aggregate Results

The summary statistics in Sections 3.1 and 3.2 show that, conditional on being in the same postcode, extremely long leaseholds and freeholds transact at very similar average prices in both the U.K. and in Singapore. However, this could be either because there was no classic rational bubble, or because leasehold properties are more attractive and have better characteristics, thus masking the presence of a bubble. To control for possible differences in observable characteristics across freeholds and extremely long leaseholds, we study the price difference between extremely long leaseholds and freeholds by estimating the following hedonic regression for the U.K. and Singapore (Rosen, 1974):

$$log(Price_{i,p,h,t}) = \alpha + \beta ExtremelyLongLease_{i,t} + \gamma Controls_{i,t} + \phi_p \times \rho_h \times \psi_t + \epsilon_{i,p,h,t}, \quad (3)$$

where the unit of observation is a transaction i of a property of type p in postcode h at time t. The variable $ExtremelyLongLease$ is an indicator of whether transaction i is a leasehold with more than 700 years of maturity remaining – the excluded category in the regression is freeholds. The coefficient β captures the logarithmic price discount of extremely long leaseholds relative to otherwise similar freeholds. We control for average prices in a property's geography by including property type (ϕ_p) by postcode (ρ_h) by time of sale (ψ_t) fixed effects.[22] For the U.K., we use 3-digit postcodes, also known as postcode districts. For Singapore, we use 5-digit postcodes. These levels of fixed effects correspond to areas that are both sufficiently large to have variation across contract type, but also sufficiently small for the housing stock to be relatively homogeneous. In Singapore, we also interact the fixed effects with whether the property is a "land" or "strata" title.[23] We control for various characteristics of the property using standard hedonic variables.[24]

[22]For U.K. houses, the property types are "detached," "semi-detached," and "terraced." For Singapore, they are "apartment," "condominium," "detached house," "executive condominium," "semi-detached house" and "terraced house."

[23]Land title properties occupy land that is exclusive to the owner (e.g., a detached house), whereas a strata title comprises units in cluster housing (e.g., an apartment). Owners of strata properties enjoy exclusive title only to the airspace of their individual unit. The land that the development is built on is shared by all the owners of the project, based on the share of the strata title unit owned by each owner.

[24]For both the U.K. and Singapore, we control for the size and age of the property in a flexible way. For property size in the U.K. we include 50 equally-sized groups by including corresponding dummy variables; for Singapore we include 40 equally-sized groups. Age is controlled for by adding a dummy variable for each possible outcome. In addition, for the U.K. we include as controls: number of bathrooms, bedrooms, property condition, whether there is parking, whether the property is new, and the type of heating. In Singapore, we include the number of units in the development. Finally, we include transactions of shorter leaseholds (with less than 300 years remaining) in our sample, controlling for the lease length remaining at the time of transaction by including dummies for buckets of maturities as in Giglio, Maggiori and Stroebel (2014). We include short leaseholds in our sample – even though they are not the focus of this paper – to maximize power, since they help identify the hedonic coefficients and the fixed effects. All results are

Standard errors are clustered at both the postcode and the transaction quarter level.

Tables II and III report the results for the U.K. and Singapore, respectively. Our preferred specification, reported in the first column of each table, uses transaction year in the time fixed effects (ψ_t). For both the U.K. and Singapore, there is no significant difference between the price of a 700+ year leasehold and that of a freehold. While standard errors are larger for Singapore, where we have fewer data, for the U.K. the discount is a precisely estimated zero. This suggests that there was no classic rational bubble in house prices in the U.K. and Singapore over the last 20 years.

To confirm the quality of our data, and lend further credibility to our hedonic regression approach, Panels A to D of Figure VI and Panels A and B of Figure VII show the estimated coefficients on the key hedonic characteristics from column (1). As one might expect, larger properties, younger properties, and those with more bedrooms and bathrooms all trade at higher prices. Columns (2) and (3) of each table use transaction quarter and month, respectively, to account for time fixed effects (ψ_t); results are robust to these variations.

The remaining columns of each table explore robustness to features specific to each country. For the U.K., we explore whether the price discount varies by geographic area. In particular, in column (4) of Table II, we estimate the discount just for London. We find the discount economically small (-1.7%) and statistically indistinguishable from zero. In column (5) we further zoom into Prime Central London (PCL), defined as the areas of Mayfair, Knightsbridge, Belgravia, Chelsea and Kensington. This area has experienced particularly strong growth in house prices over the past decade, and is often described in the media to exhibit the features of a housing bubble (e.g., The Guardian, 2013). Even in this geography, we find no evidence of a classic rational bubble because the price difference between extremely long leaseholds and freeholds is indistinguishable from zero (and, indeed, even slightly positive). However, the small number of house transactions in London, particularly in PCL, diminishes our statistical power.

For Singapore, in column (4) of Table III, we restrict the sample to private buyers (as opposed to the public authority, the Housing and Development Board), and in columns (5) and (6) we restrict the sample to the two types of title, "land" or "strata". In all cases, the estimates of the discount of extremely long leaseholds relative to freeholds are economically small and statistically insignificant.

We labeled our tests as being model-free in the sense that they do not require imposing a structural model to compute the fundamental value of the asset. Our regression equation 3, however, does impose an empirical specification in computing how various

robust to excluding them.

hedonic characteristics are related to prices. We have already shown in Tables II and III that our flexible regression specification is robust to permutations in the fixed effects. In addition, by including separate dummy variables for the various percentiles of the distribution of each hedonic variable, we allowed the functional form of the hedonic variables to be very flexible (see Figures VI and V). To confirm that our results are robust to a variety of empirical specifications, we next report results of a propensity score matching estimation (Rosenbaum and Rubin, 1983), for both the U.K. and Singapore.

We start by computing, for each observation, a propensity score that indicates the likelihood that the property is treated (i.e., is a 700+ year leasehold as opposed to a freehold), as a function of the characteristics of the property. The propensity score is obtained from a logit regression of the 700+ year leasehold indicator on the property's hedonic characteristics.[25] We then examine the average price difference across pairs of treated (700+ year leasehold) and control (freehold) properties. We select the matching freehold for each leasehold transaction by considering the freehold transactions that occurred in the same geographic area (postcode) and in the same year as the leasehold transaction, and within these we select the one with the closest propensity score. This is the freehold which, given its hedonic characteristics, had a similar ex-ante probability of being an extremely long leasehold.

The results of our matching estimator are consistent with those of the hedonic regression analysis: the average logarithmic price difference between a 700+ year leasehold and a matching freehold in the U.K. is -0.0006 (standard error 0.0028) and 0.0040 in Singapore (standard error 0.0337). In both cases, the difference is economically and statistically indistinguishable from zero, confirming the results of our regression analysis. We conclude that there is no evidence in our data supporting the presence of a classic rational bubble.

3.4 No-Bubble Condition: Time Series

The tests reported in the previous subsection show that *on average* the prices of freeholds and extremely long leaseholds are very close to each other, thus rejecting the presence of a classic rational bubble *on average*. It is, however, possible that a bubble in the housing market might have originated only late in the sample, or even originated and then collapsed within our sample period. Therefore, the bubble may only be present for a short period of time, and thus be undetectable when looking at an average over 20 years.[26]

[25]For the U.K. the hedonic regressors are: new property indicator, bedrooms, bathrooms, size, property style, age, condition indicator, heating indicator, parking indicator. For Singapore they are property size and age.

[26]Within theory models the origin of bubbles remains unclear and bubbles are often assumed to be present at the start of the model. Similarly, within many models, a bubble cannot arise again after it has

As a more stringent test, Figure VIII reports the price difference between extremely long leaseholds and freeholds for properties in the U.K. and Singapore estimated separately for each year. While our statistical power decreases and standard errors increase, there is no evidence of a classical rational bubble at any point in time between 1995 and 2013. For the U.K., the point estimate of a bubble never accounts for more than 1.2% of the level of house prices and is never statistically significant at the 5% level.

This latter time series test is of particular interest, because it shows the absence of a classic rational bubble even at the peak of the housing market in 2007, after years of strong house price growth. Figure I plots the time series of the real price indices of the U.K. and Singapore together with a "bubble index" constructed from newspaper article mentions of real-estate bubbles. The figure shows that the fast increase in house prices in the two countries was accompanied (and partly followed) by a large increase in the mention of house-price bubbles in newspapers.[27] Indeed, many commentators directly ascribed the run-up in house prices before the financial crisis to the presence of a large bubble in this market (Case and Shiller, 2003; Caballero and Krishnamurthy, 2006; Galí, 2014). Glaeser and Nathanson (2014) review the debate on housing bubbles and point to a number of models that could rationalize the price dynamics due to (combinations of) search frictions, heterogeneous beliefs, and rational bubbles. Within models of real estate bubbles, the closest theoretical papers to our empirical tests are Kocherlakota (2009), Arce and López-Salido (2011), Basco (2014) and Miao, Wang and Zhou (2014) that develop a rational bubble model of real estate prices. Similarly, Galí (2014) and Galí and Gambetti (2014) use the boom-bust pattern in house prices to motivate their analysis of classic rational bubbles. In contrast to this view, we find no evidence of this type of bubble, precisely where and when we would most expect to see it.

3.5 Market liquidity, and unobservable property characteristics

When estimating leasehold discounts using hedonic regressions, one might be worried about other factors that may affect the observed discounts and that are not fully captured by our hedonic regressions. In particular, any factors that would make the freehold less attractive relative to the leasehold would bias us against finding a bubble.

One concern could be that freehold properties are less liquid than long-term lease-

collapsed. In our empirical work, we take the agnostic view that the bubble might only have been present for a temporary period in our data.

[27] For the U.K., the "bubble index" counts the number of times the phrases "real estate bubble" and "U.K." appear in the 11 largest U.K. newspapers. For Singapore, it counts the number of times the phrases "real estate bubble" and "Singapore" appear in the world's most widely-circulated English-language newspapers and periodicals. The data source is ProQuest.

holds in the resale market, and that they might therefore trade at a discount relative to leaseholds in the absence of a bubble. To test whether this hypothesis confounds our rejection of the presence of a classic rational bubble, we obtain for-sale listing information for about 1.8 million transactions of houses in the U.K. since September 2001 from Rightmove.co.uk. For these transactions we calculate the time between first listing and sale, i.e. the "time on the market," which provides a useful proxy for the liquidity of the asset (see Genesove and Han, 2012; Piazzesi, Schneider and Stroebel, 2013). To test whether liquidity differs between extremely long leaseholds and freeholds, Column (6) of Table II repeats the analysis of regression 3 using the log of the time on the market as the dependent variable. On average, freeholds sit on the market for about 2 days longer than extremely long leaseholds. This difference is small relative to an overall average time on market for houses of about 140 days. Differences in liquidity between the two contract types are therefore unlikely to confound our results.

A potential second concern is that, despite controlling for many observable property characteristics, extremely long leaseholds and freeholds could differ on unobservable characteristics. In particular, if leaseholds tended to be on more attractive properties, this could mask the presence of a bubble. This is unlikely for a number of reasons. First, as shown in Section 3.1, freehold houses are somewhat better on all observable characteristics; it is then unlikely that they are sufficiently worse on the unobservable characteristics to mask a possible bubble. Second, we argue that if freehold properties were truly worse on unobservable characteristics, this should affect their rental value. In fact, the rental value of properties provides a sufficient statistic for the flow utility from that property, capturing even characteristics that are unobservable to the econometrician. To test whether extremely long leaseholds and freeholds differ on unobservable characteristics, we were able to obtain more than 100,000 rental listings for Singapore from iProperty.[28] These rental listings cover the period 2010-2013. About 60,000 of the rental listings are for freehold properties, and about 7,000 are for properties with extremely long leaseholds. We repeat regression 3, but substitute the logarithm of the annual rent for the properties as the dependent variable. The results are shown in columns (7) and (8) of Table III. There is no difference in the annual rents between the two types of contracts. To provide confidence in the quality of these rental price data, Panels C and D of Figure VII show the coefficients on the key hedonic characteristics in the regression; as with transaction prices, larger and younger properties rent for higher annual amounts. Overall, these results show that unobservable property characteristics are unlikely to explain our result of zero price difference between extremely long leaseholds and freeholds.

[28] iProperty.com.sg is the leading real estate website in Singapore.

4 Robustness

In this section we describe and address a number of additional possible concerns over institutional differences between extremely long leaseholds and freeholds, the stability of property rights, and the market-timing of participants, that could affect our test for classic rational bubbles. We show that none of these provide a challenge to our conclusion that no classic rational bubble was present in the U.K. and Singapore housing markets.

4.1 Institutional Differences Across Contract Types

We begin by discussing a number of important institutional features of leasehold contracts. We show that these would, if anything, lead us to overestimate the size of the bubble, and therefore make our empirical result of no bubble conservative. See Giglio, Maggiori and Stroebel (2014) and Bracke, Pinchbeck and Wyatt (2014) for a description of these institutional details in the United Kingdom. In particular, buyers might perceive an intrinsic difference between owning a leasehold and owning a freehold contract. For example, one could be concerned with maintenance costs imposed by the freeholder on the leaseholder, restrictions on leaseholders to redevelop the property, or a pure psychological preference for freehold ownership. Intuitively, these concerns would make the leasehold worth *less* than the freehold, thus biasing our test toward finding a bubble. Nonetheless, we briefly review that such concerns are minimal in the data. We also show below that the tax treatment of leaseholds and freeholds is identical and has no impact on our results.

4.1.1 Ground Rents and Service Charges

In the U.K., leaseholders sometimes have to pay annual ground rents to the freeholder, since the purchase price of the lease only covers the temporary ownership of the structure. The land technically still belongs to the residual freeholder who in principle has the right to request that the lessee make regular payments for the use of the land. Ground rents are set separately for each property, and no comprehensive database exists. If such payments significantly affected the value of leaseholds, we would expect those leaseholds to trade at a price discount to freeholds, which would be observationally equivalent to a classic rational bubble. The absence of a price difference between leaseholds and freeholds suggests the quantitative importance of ground rents to be small. Consistent with this, the 2011-2012 English Housing Survey shows that the amounts involved are usually very small (£10-£100 per year) and in many cases are either zero or a symbolic amount ("a peppercorn"). In particular, amongst those households that report paying ground rents,

the median household reported annual rents of about £25. Even in cases where the ground rent is in principle positive, it is often zero in practice, because for the rent to be collected the freeholder has to make a specific written request to the leaseholder. Such requests are often not made because the amounts collected would be too small to cover the administrative costs.

For many leasehold apartment buildings, the freeholder provides services such as maintenance and repairs, insurance of the building and, in some cases, central heating, lifts, porterage, estate staff, lighting and the cleaning of common areas. The leaseholder pays a service charge to the freeholder in return for such payments. While maintenance costs can be non-trivial, as long as the maintenance is carried out at fair value (the private market cost of the works) service charges do not represent a problem for our analysis, since freeholders would also have to pay for the maintenance of the property. If the maintenance was performed at above-market cost, this might bias our test towards finding a bubble. However, the Commonhold and Leasehold Reform Act 2002 gives leaseholders the right to take over the management of the property from the freeholder, severely restricting the freeholder's ability to extract unreasonable service charges.[29] More importantly, while common for leasehold flats, service charges are almost never levied on leasehold houses, where the leaseholder is generally responsible for the maintenance of the property. These leasehold houses are the focus of our analysis.

Similarly, in some cases the lease requires that the leaseholder insures the property through an insurer nominated or approved by the residual leaseholder. The tenant may consider that she can get cheaper insurance from different companies and may be concerned as to the coverage provided. The provisions of Section 164 of the Commonhold and Leasehold Reform Act 2002 provide a right for the leaseholder to arrange her own insurance, provided he notifies the residual freeholder. Again, the absence of a significant price difference between extremely long leaseholds and freeholds suggests that the quantitative importance of this insurance requirement is small.

4.1.2 Structural Changes to the Property

Since the leasehold contracts considered in our tests span several hundred years, it is important to understand the provisions that regulate the ability of leaseholders to make adjustments and improvements to the property. Leaseholders are generally allowed to

[29]Under the Commonhold and Leasehold Reform Act 2002 an application can be made to a Leasehold Valuation Tribunal to challenge the reasonableness of service charges. Secondly, the Commonhold and Leasehold Reform Act 2002 provides a right for leaseholders of flats to force the transfer of the landlord's management functions to a special company set up by them - a "right to manage" company. This does not require the residual freeholder's consent.

make non-structural improvements without the consent of the freeholder, unless explicitly restricted by covenants. For example, the leaseholder can renovate the house.

For structural changes to the property (like demolishing and redeveloping the building) the leaseholder generally has to obtain the consent of the freeholder. This restriction is unlikely to significantly affect our estimated discounts. A covenant against the making of improvements without consent is subject to the provision that consent shall not be unreasonably withheld (see Section 19(2) of the Landlord and Tenant Act 1927). A freeholder would have to show that the redevelopment significantly affects her rights to ownership after the leasehold expires. In the case of extremely long leases, almost all reasonable redevelopments would not tend to infringe on the rights of the freeholder. In practice, zoning and other city regulations, which affect both leaseholders and freeholders equally, are likely to be the more restrictive factor in any redevelopment plan. In addition, as before, any binding restrictions would bias us in favor of finding a price difference that we would attribute to a classic rational bubble, since they would make the leasehold less valuable relative to the freehold.

4.1.3 Leasehold Covenants

A further concern might be that contractual covenants in leasehold deeds place restrictions on leaseholders that reduce the value of the leasehold relative to the freehold; the fact that the two contract types trade at identical prices suggest that any such restrictions are unlikely to be quantitatively important in practice. This is consistent with evidence in Giglio, Maggiori and Stroebel (2014), who analyze a large number of covenants from deed titles individually downloaded from the U.K. Land Registry website to determine that such contractual restrictions do not quantitatively affect leasehold prices. Many leaseholds do not carry restrictions; others contain only old, out-of-date restrictions, which, while technically still in place, are generally not enforced. The most common type of covenant involves restrictions on the broad type of land use, such as restricting structures to be residential rather than commercial. Usually these are consistent with council zoning regulations, which would also apply to the owners of freeholds. Furthermore, while these covenants usually restrict the use of land and future structures to be residential, there are often no further restrictions on the construction of such new residential properties. Some leasehold covenants do place restrictions on the construction of a new structure. To the extent that there are such restrictions on new structures, many times they again relate to restrictions placed by the council, and would equally apply to all new structures that freeholders might want to erect. Other covenants require the permission of the freeholder for certain redevelopments (i.e. placing a structure close to the property edge). In any

case, the presence of restrictive covenants on leasehold would bias the results in favor of finding a difference between the price of a leasehold and that of a freehold.

4.1.4 Taxation and the Stamp Duty

Purchases of property in the U.K. are subject to a transaction tax (stamp duty). The tax applies equally to freehold or leasehold purchases. The tax schedule is progressive: for example, a purchase of a property up to £125,000 is tax exempt, while a purchase of a property between £125,001 and £250,000 is taxed at 1% of the total purchase price. There are no taxes on the ongoing ownership of the property.

Similarly, the purchase of property in Singapore is subject to stamp duty irrespective of the form and duration of ownership. The tax schedule is also progressive: the first SG$180,000 are assessed at 1%, the next $180,000 at 2%, and each additional increase in the sale price at 3%. In addition, Singapore also taxes the ongoing ownership of a property, again irrespective of the form of ownership. Property taxes are levied on the Annual Value (AV), the tax-authority assessed 1-year rental income of the property. For rental properties, the tax rate is set at 10% of AV; for owner-occupied properties, it rises from 0% on the first SG$6,000 to a marginal rate of 6% for AVs exceeding SG$65,000.[30] As we show in Section 3.5, the rental income, and therefore the Annual Value, of a property is unaffected by the form of ownership.

If a bubble were present, these quasi-progressive tax schedules would make leases more attractive, because for similar properties the leasehold might avoid incurring the higher tax bracket because it does not include the price of the bubble. While this would bias against finding a bubble, the effect is most likely negligible. Only buyers of properties that are very close to the boundaries of the tax bracket would be affected, and since the brackets are relatively large, especially in the U.K., the effect on the average discount is unlikely to be quantitatively important. Furthermore, under the null hypothesis of our statistical test that there is no classic rational bubble in the data, taxation introduces no bias to our test.

4.2 Heterogeneous Buyers and Frictions

One additional possible concern is that the market for leasehold and freehold contracts might be segmented in the presence of heterogeneous buyers.[31] Neither in the U.K. nor

[30] Starting from January 1, 2014, property taxes were made more progressive. This does not affect any of the transactions during our sample period.

[31] From a theoretical perspective the main concern is segmentation. Our tests are robust to the presence of heterogeneous agents as long as they can all trade the freehold and leasehold contract. In equilibrium, these agents would have to agree on the price of all traded assets, including the bubble asset. Our test

in Singapore are there any legal restrictions that prevent agents from transacting any of the two contracts. Even in the absence of legal restrictions, one could be concerned about implicit restrictions. We find this concern to be minimal given that Giglio, Maggiori and Stroebel (2014) show that, in the U.K., households who own a leasehold or a freehold do not differ on observable characteristics such as income, age of the head of household, size of the household, and propensity to have a mortgage. For example, conditional on the type of property and its location, freeholders have on average £5.6 less in income than leaseholders. Leaseholders are also only 1.3 years younger than freeholders on average.

Financing frictions are also not a concern for our tests. While it is possible that leaseholds with particularly short maturity, less than 60 years, are less valuable as collateral for mortgages, leases of long maturities are treated identically to freeholds for financing purposes.[32] Intuitively, while a 999-year lease will eventually run down to 60 years of maturity left and might incur a loss in its collateral value, this loss is 939 years into the future and has no impact, even at low discount rates, on the current value of the lease.

4.3 Stability of Property Rights

One obvious concern when analyzing contracts with horizons that span several hundred years is whether agents today expect such property rights to be enforced in the future. Ex ante, such a concern might be more relevant for Singapore, a relatively young hybrid democracy.[33] Yet, despite Singapore's relatively recent independence in 1965, it consistently tops the ranks of countries with the strongest property rights around the world.[34] The enforcement of property rights is of paramount importance for a small open economy whose main business is being a world financial center. The U.K. arguably has the strongest and longest track record of any country in upholding and often promoting property rights. Concerns over the stability of its laws and institutions are as close to non-existent as can be found in real world application of economic theory.

From a theoretical perspective we can distinguish two effects of property rights on our tests. A general concern with the enforceability of all contracts, in our case of both

would then correctly detect the equilibrium pricing of the bubble asset. Segmentation is a possible concern precisely because it could prevent this equalization of valuations in market prices.

[32]Banks in the U.K. typically require 30 years of remaining lease length after the expiry of the mortgage. Since the most common length of U.K. mortgages is 25 years, this restriction starts binding for leases shorter than 60 years. Similarly, in Singapore households are not allowed to use their pension contributions for a property downpayment if the lease has less than 60 years of maturity.

[33]The Economist Democracy Index currently classifies Singapore as a "hybrid regime", the third best rank out of four on their scale of representative democratic institutions.

[34]For example, it is ranked number 7 worldwide in the 2013 International Property Rights Index compiled by The Property Rights Alliance, and ranked number 2 worldwide in the 2014 Index of Economic Freedom compiled by the Heritage Foundation.

leaseholds and freeholds, and a concern with selective enforcement of one type of contract above the other. Classic rational bubbles rely on the existence of property rights strong enough that agents can expect today to sell the bubble asset at increasing prices to future generations of agents. If agents were to expect that governments would expropriate the bubble asset for sure at some future finite time or at some finite level of the bubble, then no classic rational bubble could exist. While we cannot rule out this unobservable expectation of expropriation, by testing the theory in countries with very strong property rights and on an asset, real estate, that is comparatively harder to expropriate than fiat or intangible assets, we provide a testing ground favorable to the classical rational bubble.[35]

A more nuanced possibility is that agents might expect leasehold and freehold contracts to be selectively enforced. If agents were to expect leasehold contracts to be reneged upon but freehold contracts to be upheld, then this could generate a price discrepancy between the two that resembled, mistakenly, a classic rational bubble. This effect makes our result of no classic rational bubble in the data a conservative one. On the contrary, if agents were to expect that leasehold contracts be converted into freehold ones for free, thus expropriating the rights of the underlying freeholder, our test would be biased against finding a classic rational bubble.

A long-standing tradition of strong institutional enforcement of both leasehold and freehold contracts minimizes the concern that our test could be biased by selective enforcement. Both types of contracts have been originated, traded in the secondary market, and enforced repeatedly in the past. We also stress that since both types of contracts are commonly used in private residential real estate transaction, selective enforcement would result in large scale transfers that would be politically daunting for most governments. The stability of the contracting environment is highlighted by the fact that in our sample of U.K. transactions we observe 271,661 secondary market transactions of leases that were originated before 1914, more than 100 years ago. We observe 25,497 transactions of leaseholds created more than 150 years ago. The oldest lease to transact in our U.K. sample was originally created in 1555. Similarly, in our sample of Singapore transactions we observed 25,297 transactions of leases that were originated even before independence in 1965, and enforced despite the separation from the United Kingdom. The oldest lease to transact in our sample had originally been created in 1827, only three years after the Anglo-Dutch Treaty of 1824 confirmed the status of Singapore as a British possession, and only one year after the British East India Company included Singapore in

[35]Importantly, if agents expected expropriation of the kind described above, our test would be perhaps less representative since such complete expropriation is ruled out in the theory of the classical rational bubble, but still be unbiased since it would find no bubble in a set-up where the bubble indeed cannot exist.

the newly-formed Straits Settlements. Since past experience is likely to strongly influence expectations, concerns over current expectations of future ability to freely transact each type of contracts are minimal.

4.4 Finite Maturity Leaseholds, Extensions, and Expiries

Our test for the presence of classic rational bubbles relies on the difference in maturity between 999-year leaseholds and freeholds. In this section we document that in both the U.K. and Singapore, 999-year leaseholds indeed have finite maturity and distinctly differ in this respect from freeholds.

Both in the U.K. and in Singapore, leaseholds are finite maturity contracts and at expiration the land and structure revert to the underlying freeholder. In fact, in both countries a leasehold public record is expunged on expiration from the central title registry and expired leaseholds have no validity (Burn, Cartwright and Cheshire, 2011). The reversion of ownership at expiration is enforced by the government. For example, the Minister for National Development in Singapore stated, when asked about the governments policy towards expired leases: "The value of the flats will be zero at the end of their 99-year lease."[36] Indeed, the data for shorter maturity leaseholds, which are the focus of Giglio, Maggiori and Stroebel (2014), show that market participants are aware of leasehold expiration because the pricing of shorter leases varies significantly with their maturity. For example, even relatively long leases with 300 years of unexpired maturity are priced below the extremely long leases and freeholds considered in this paper.

In the U.K., leaseholders and freeholders can in principle agree to extend the contract and increase the maturity of the lease ("lease extension"), or alternatively transfer the whole freehold interest to the leaseholder ("enfranchisement"). Since these are private market transactions among willing participants we would expect them to happen at market values: the freeholder receives a mutually-agreed compensation for the sale of part or all of the residual value of the freehold.[37] Such transactions are rare in Singapore, where the largest freeholder, the Singapore Land Authority, has made it clear that their "policy is to allow leases to expire without extension".[38]

The possibility that a contract of finite maturity, like the leasehold, could be extended at market prices by purchasing a new contract, in this case an extended lease or the

[36] Source: Gerald Giam (2014).

[37] Our data does not include such transactions because these lease extensions are not always registered with the England and Wales Land Registry.

[38] Extensions in Singapore are possible in exceptional circumstances and considered on a case by case basis where the main criteria is whether "they are in line with planning intention and help to further specific economic and social objectives". Direct quotes in this footnote and the text above are from a SLA press release available at http://www.sla.gov.sg/htm/new/new2008/new0109.htm.

freehold, does not affect our test for classic rational bubbles. In an equilibrium with classic rational bubbles, the freehold would be more expensive than the 999-year leasehold precisely by the amount of the bubble. Lease extensions that add a finite number of years to a leasehold contract that already has a maturity in excess of 700 years would be priced at zero because the new lease would still not include the bubble and would have the same fundamental value as the old lease. Similarly, enfranchisements would occur at a price that exactly equals the value of the bubble, because by acquiring the infinite maturity freehold interest, the leaseholder would obtain the bubble that is attached to it.[39]

In the U.K., but not in Singapore, leaseholders of houses in some cases have the statutory right to enfranchise or extend the lease by 50 years (Leasehold Reform Act 1967). The law stipulates that while such rights are granted, the freeholder needs to be compensated for any loss of value at market prices.[40] These statutory rights do not materially affect our empirical results. In fact, our results are consistent across the U.K. and Singapore, but no statutory rights are present in Singapore. For the U.K., the statutory rights can only adversely bias our test if enfranchisements are underpriced; but even in this case, the statutory rights could not explain our finding of a zero price difference between contracts estimated with very tight standard errors. For this to happen in the presence of a bubble, leaseholders must expect to be able to get *for sure* the freehold *for free* in court, in effect expropriating the freeholder. This runs contrary to the evidence by Giglio, Maggiori and Stroebel (2014) that the private market, and the courts, impose compensation negatively related to remaining maturity for shorter maturity leaseholds (say between 0 and 300 years). Finally, lease extensions and enfranchisement involve significant transaction costs for the leaseholder thus reducing the possible price impact of the statutory rights.[41]

[39] In this case, the existence of a liquid and competitive secondary market for leasehold and freehold contracts minimizes the concern that prices might not be competitive and reflect bilateral bargaining between the two parties.

[40] In fact, the price "shall be the amount which at the relevant time the house and premises, if sold in the open market by a willing seller, might be expected to realise." In practice, this means that if the two parties, the leaseholder and the freeholder, cannot reach an agreement in the private market, they can resort to a court tasked with determining the "market price" of the extension or enfranchisement. Court decisions, even if clearly related to a selected sample of market participants, have always been consistent with our empirical result that there is no difference in value between leases with more than 700 years and freeholds. If compensation was awarded to the freeholder, it was unrelated to the future value of the contract, i.e. the possible bubble, and instead connected to the loss of immediately payable ground rents. In none of the many court decisions we investigated did any participant raise the possibility of a classic rational bubble and the related resale option value of the freehold contract.

[41] The leaseholder has to pay the cost of valuation services, legal counseling, and legal expenses, and has to bear the uncertainty associated with a possible court trial.

4.5 Timing the Market: Maturity Choice

While the predictions of models of the classic rational bubble for trading volume are very sensitive to the specific modeling assumptions, and often indeterminate, it is still interesting to investigate whether the origination of new leases or the trading of existing ones responds meaningfully to the possible presence of a bubble.[42] For example, one could conjecture that if there were a bubble, some developers might opt to sell the new properties as freeholds to capture the bubble rather than as long dated leaseholds.

Figure IX shows that there is little yearly variation in the fraction of properties sold under each type of contract, 999-leasehold or freehold. In particular, the share of freeholds over the total of freeholds and 999-leaseholds does not respond to variations in the housing market such as the changes in the overall house price level or the price to rent ratio. Similarly, the trading activity does not respond to variation in the point estimates of our test for the bubble; recall that in any case the point estimates are both very close to zero and statistically insignificant. These facts hold both for newly minted contracts as well as for trading on the secondary market. We conclude that, consistent with the idea that there is no classic rational bubble in the data, there is no systematic variation in the type of contracts being traded or originated.

5 Conclusion

We provide a direct test for the existence of classic rational bubbles in housing markets. These bubbles rely on a violation of the transversality condition in asset valuation. We exploit the heterogeneity in ownership contracts for residential real estate in the U.K. and Singapore, which feature both infinite maturity ownership contracts (freeholds) and extremely long but finite maturity ownership contracts (leaseholds). If a bubble based on a failure of the standard no-bubble condition were present in these housing markets, it would increase the price of the freehold but not that of the extremely long leasehold. Empirically, we find that after controlling for observable property characteristics, freehold properties trade at the same price as otherwise identical leaseholds with maturities in excess of 700 years. This result holds in every year in our sample (1995-2013), and directly implies that bubbles that violate the terminal no-bubble condition were not present in these housing markets.

[42] Furthermore classic rational bubble cannot arise endogenously within a model: they are either present at the start of the model, or often assumed to arise exogenously within the model.

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Table I: Summary Statistics

Panel A: U.K.

	700+ Leaseholds			Freeholds			Δ	SE	Δ FE	SE
	Mean	Median	Std	Mean	Median	Std				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log Price (£)	11.26	11.29	0.71	11.70	11.74	0.73	-0.44	0.03	-0.01	0.01
Bedrooms	2.74	3	0.79	3.01	3	0.87	-0.26	0.02	-0.10	0.01
Bathrooms	1.16	1	0.44	1.30	1	0.58	-0.14	0.01	-0.02	0.00
Age (years)	66.6	71	43.26	58.12	50	48.75	8.47	1.32	-2.75	0.92
Size (m^2)	101.5	93	55.52	112.8	100	58.17	-11.26	0.77	-5.04	0.51
N		353,309			7,167,253					

Panel B: Singapore

	700+ Leaseholds			Freeholds			Δ	SE	Δ FE	SE
	Mean	Median	Std	Mean	Median	Std				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log Price (SG$)	13.97	13.89	0.53	14.02	13.92	0.63	-0.05	0.04	0.02	0.07
Age (years)	4.47	0	7.41	5.20	0	8.79	-0.75	0.33	0.23	1.12
Size (m^2)	175.5	134	147.3	173.1	129.0	197.1	2.72	6.19	3.84	6.36
N		26,197			174,126					

Note: Table shows summary statistics for the U.K. transaction sample (Panel A) and the Singapore transaction sample (Panel B). For each of our key hedonic characteristics, we show the mean, median and standard deviation separately for extremely long leaseholds and freeholds. We also show the average difference between extremely long leaseholds and freeholds (Δ), and the average difference (Δ FE) after controlling for the same level of fixed effects as in the hedonic pricing regressions 3. For each type of contract we report the sample size (N). All standard errors are clustered at the postcode level.

Table II: U.K.: Impact of Lease Type on Prices

	log(Transaction Price)					log(ToM)
	(1)	(2)	(3)	(4)	(5)	(6)
700+ Year Leasehold	0.001	0.001	0.001	-0.017	0.009	0.016**
	(0.005)	(0.005)	(0.005)	(0.026)	(0.062)	(0.003)
Fixed Effects	PC × Y × Prop	PC × Q × Prop	PC × M × Prop	PC × Y × Prop	PC × Y × Prop	PC × Y × Prop
Controls	✓	✓	✓	✓	✓	✓
Sample	England & Wales	England & Wales	England & Wales	London	PCL	England & Wales
R-squared	0.883	0.886	0.887	0.892	0.871	0.078
N	7,602,276	7,602,276	7,602,276	285,281	52,336	1,812,830

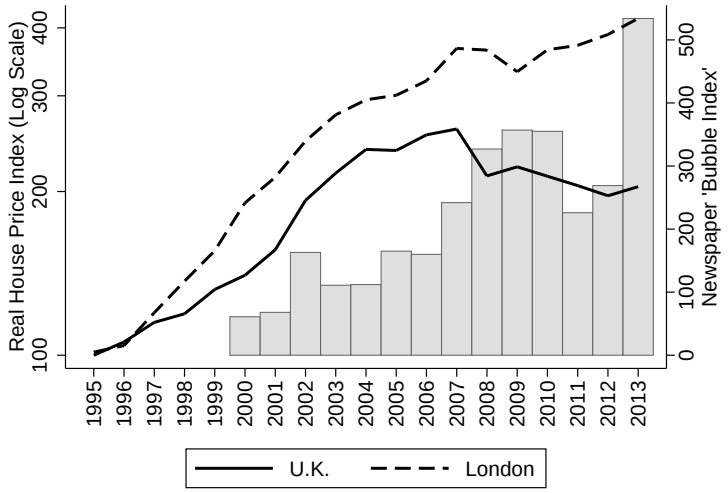
Note: Table shows results from regression 3. The dependent variable in columns (1) - (5) is the log price paid for houses in England and Wales between 1995 and 2013. The dependent variable in column (6) is the log of time on market (ToM) between listing and sale in days for those properties listed on Rightmove.co.uk. We include fixed effects at the 3-digit postcode by transaction date by property type (detached, semi-detached, terraced) level. In columns (2) and (3) the transaction date is the transaction quarter and month, respectively, in the other columns the transaction year. In columns (1) - (3) and column (6) we focus on on all transactions in England and Wales, in column (4) in the postal district of London, and in column (5) in Prime Central London (PCL). We control for property size, the number of bedrooms and bathrooms, property age, property condition, whether there is parking, and the type of heating. Standard errors are double clustered at the 3-digit postcode level and the transaction quarter level. Significance levels: * ($p<0.10$), ** ($p<0.05$), *** ($p<0.01$).

Table III: Singapore: Impact of Lease Type on Prices

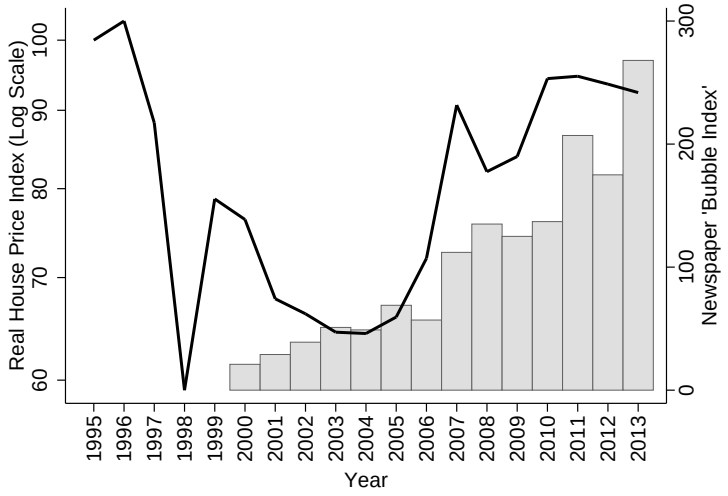
	log(Transaction Price)						log(Rent)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
700+ Year Leasehold	-0.012 (0.035)	-0.010 (0.034)	-0.007 (0.039)	-0.009 (0.039)	-0.012 (0.035)	-0.008 (0.054)	-0.008 (0.035)	-0.006 (0.034)
Fixed Effects	PC × Y × Prop × Title	PC × Q × Prop ×Title	PC × M × Prop × Title	PC × Y × Prop × Title	PC × Y × Prop	PC × Y × Prop	PC × Y × Prop × Title	PC × M × Prop × Title
Controls	✓	✓	✓	✓	✓	✓	✓	✓
Sample Restriction	·	·	·	Private Buyer	Strata Only	No Strata	· ·	· ·
R-squared	0.967	0.977	0.979	0.966	0.964	0.959	0.869	0.905
N	378,768	378,768	378,768	220,044	333,684	45,084	105,714	105,714

Note: Table shows results from regression 3. The dependent variable in columns (1) - (6) is the log price paid for properties sold by private parties in Singapore between 1995 and 2013. The dependent variable in columns (7) - (8) is the log rental listing price for all properties listed "for rent" on iProperty.com.sg between 2009 and 2013. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction date level. In columns (2) and (3) the transaction date is the transaction quarter and month, respectively, in the other columns the transaction year. In column (7) we interact with the listing year, in column (8) with the listing month. In column (4) we only focus on properties that were bought by a private individual (and not the HDB). In columns (5) and (6) we conduct the analysis for Strata and non-Strata titles separately. We control for property age, property size and the total number of units in the property. In columns (7) and (8) we cannot control for the total number of units. Standard errors are double clustered at the 5-digit postcode level and the transaction quarter level. Significance levels: * ($p<0.10$), ** ($p<0.05$), *** ($p<0.01$).

Figure I: U.K. and Singapore - House Prices and Bubble Index



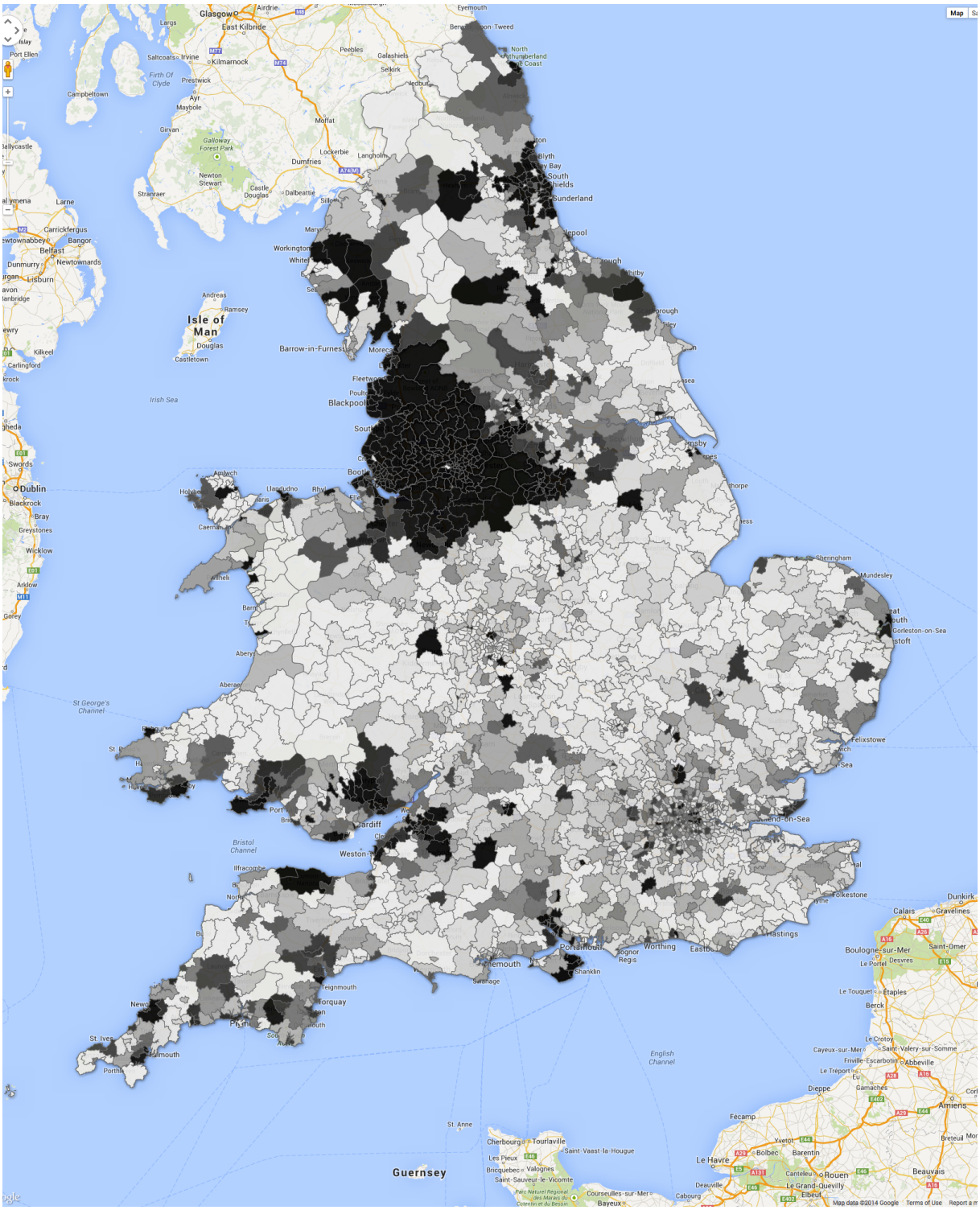
(A) U.K. and London



(B) Singapore

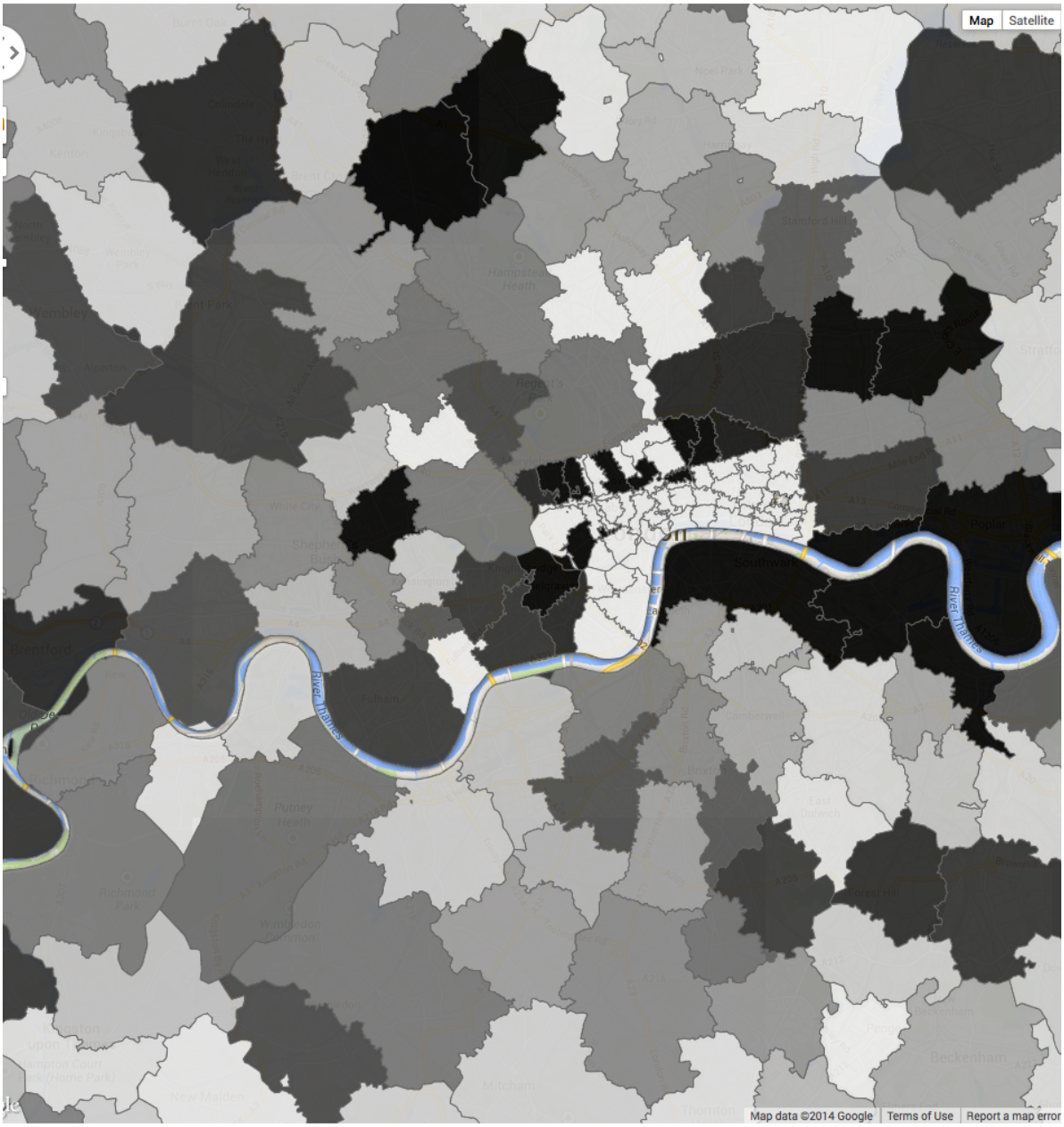
Note: Panel A shows a time series of house prices in the U.K. (solid line) and for London (dashed line) on the left scale, and a "bubble index" constructed from newspaper articles on the right scale (bars). National nominal house price data are from the Nationwide House Price Index, London house prices are from the Land Registry. We divide the nominal index by the U.K. Office of National Statistics "long term indicator of prices of consumer goods and services" to obtain a real house price index. The "bubble index" counts the number of times the phrases "real estate bubble" and "U.K." jointly appear in the following newspapers: The Sun, The Daily Mail, The Daily Telegraph, The Times, The Sunday Times, The Guardian, The Financial Times, The Independent, The Observer, The Daily Mirror, The Sunday Mirror, as reported by ProQuest. Panel B shows a time series of house prices in Singapore on the left scale (line), and a "bubble index" constructed from newspaper articles on the right scale (bars). The nominal annual house price index is from the Urban Redevelopment Authority. CPI is from Statistics Singapore. We obtain the real house price index by deflating the nominal index by CPI. The "bubble index" counts the number of times the phrases "real estate bubble" and "Singapore" jointly appear in the world's most widely-circulated English-language newspapers and periodicals, as reported by ProQuest. The "bubble index" can only be constructed from 2000 onwards.

Figure II: U.K. Sample - Distribution of 700+ Year Leaseholds



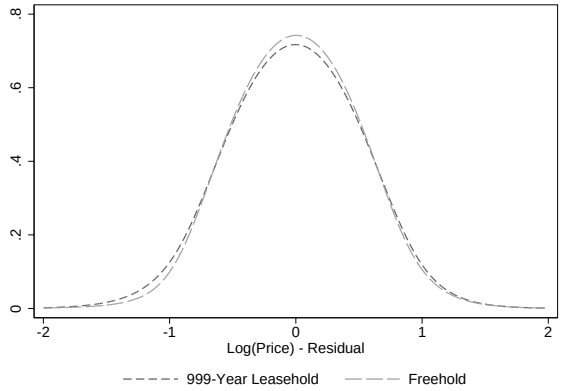
Note: The map shows for each 3-digit U.K. postcode the fraction of transactions of leaseholds with more than 700 years remaining. White indicates that the fraction was 0%. Black indicates that 2% or more of the transactions were of extremely long leaseholds, with scales of gray indicating intermediate percentages.

Figure III: U.K. Sample - Distribution of 700+ Year Leaseholds - London

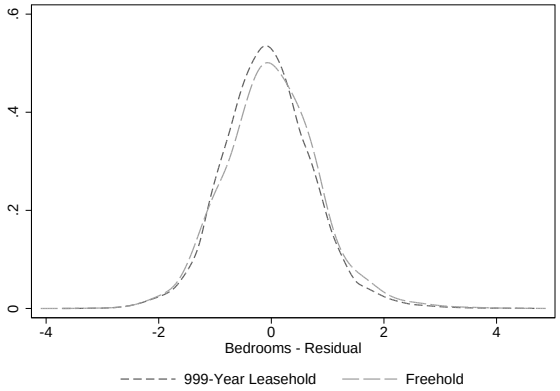


Note: The map shows for each 3-digit U.K. postcode the fraction of transactions of leaseholds with more than 700 years remaining. White indicates that the fraction was 0%. Black indicates that 2% or more of the transactions were of extremely long leaseholds, with scales of gray indicating intermediate percentages. The figure zooms in on London.

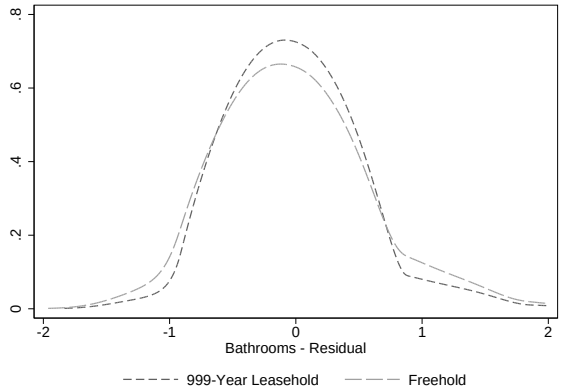
Figure IV: U.K. - Distribution of Hedonic Characteristics



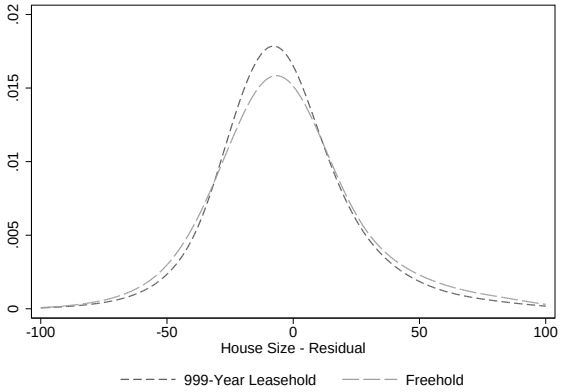
(A) Log(Price)



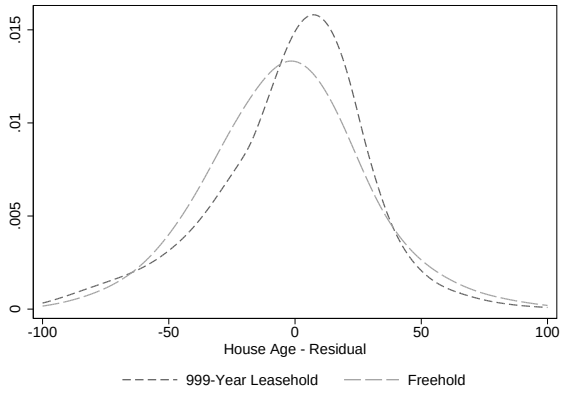
(B) Bedrooms



(C) Bathrooms



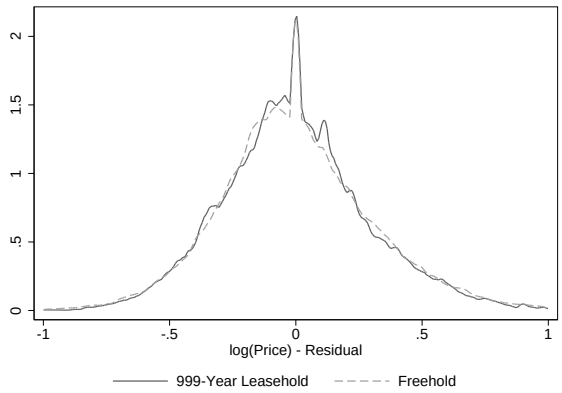
(D) Property Size



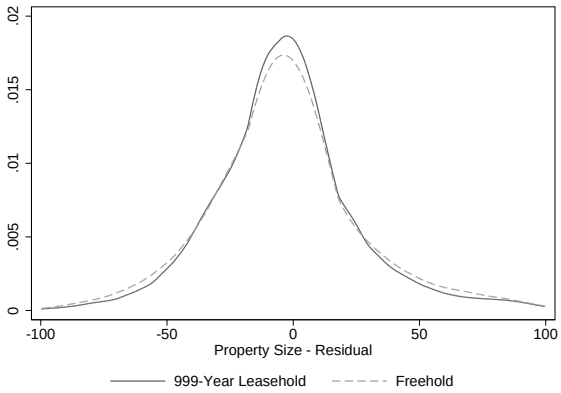
(E) Property Age

Note: Figures show the distribution of residuals from a regression of property characteristics on 3-digit postcode $\times$ transaction year $\times$ property type fixed effects, separately for freeholds and 999-year leaseholds. The sample is houses from the U.K., the characteristics plotted are: the log of the transaction price (Panel A), the number of bedrooms (Panel B), the number of bathrooms (Panel C), property size in square meters (Panel D), and property age in years (Panel E).

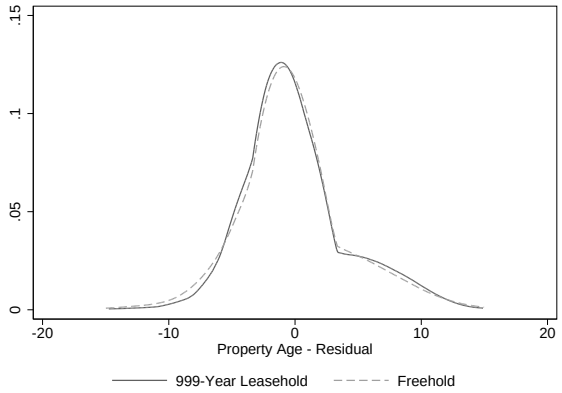
Figure V: Singapore - Distribution of Hedonic Characteristics



(A) Log(Price)



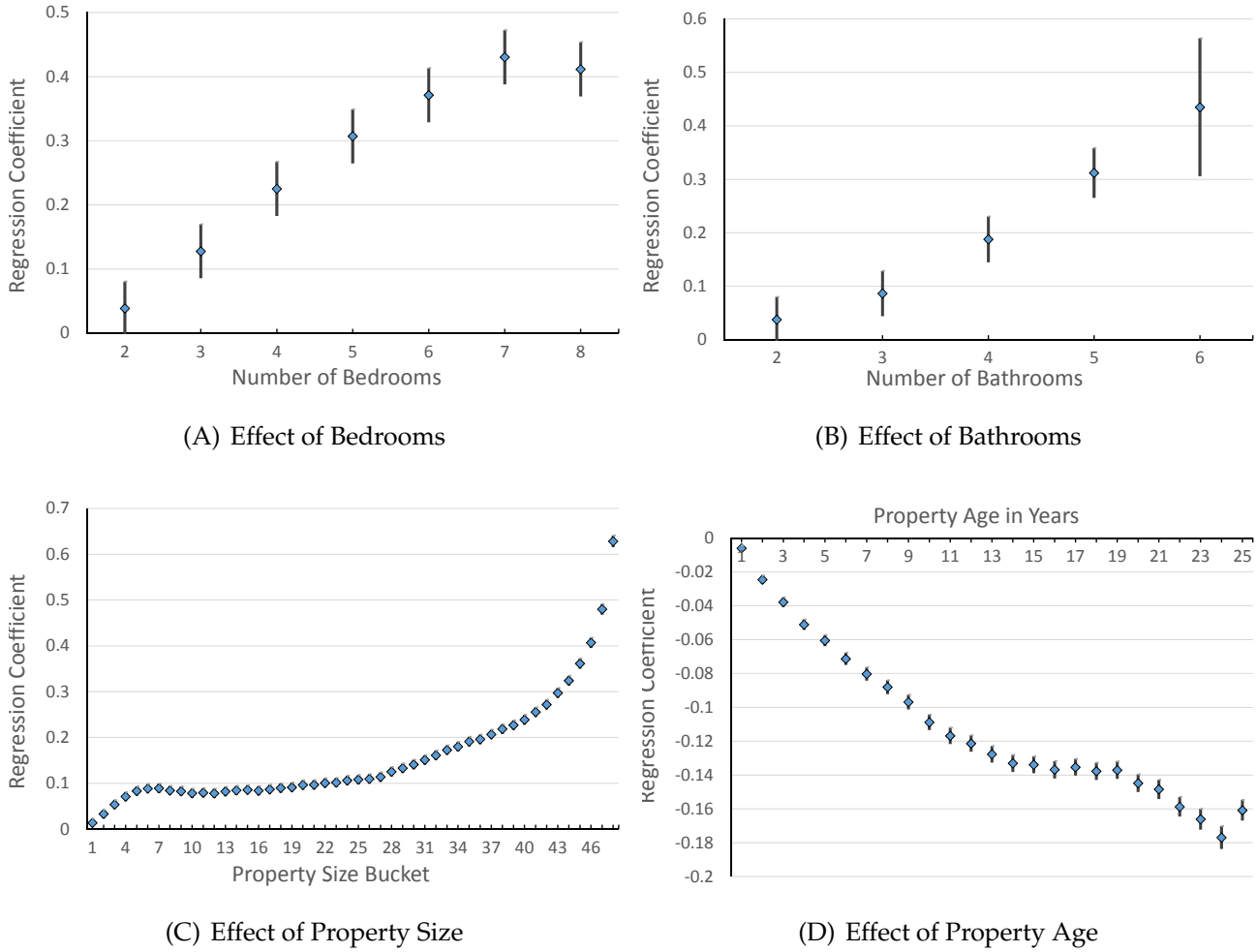
(B) Property Size



(C) Property Age

Note: Figures show the distribution of residuals from a regression of property characteristics on property type $\times$ title type (strata or land) $\times$ 5-digit postcode fixed effects, separately for freeholds and 999-year leaseholds. The sample is from Singapore, the characteristics plotted are: the log of the transaction price (Panel A), property size in square meters (Panel B), and property age in years (Panel C).

Figure VI: Price Effect of Hedonic Characteristics - U.K.



(A) Effect of Bedrooms

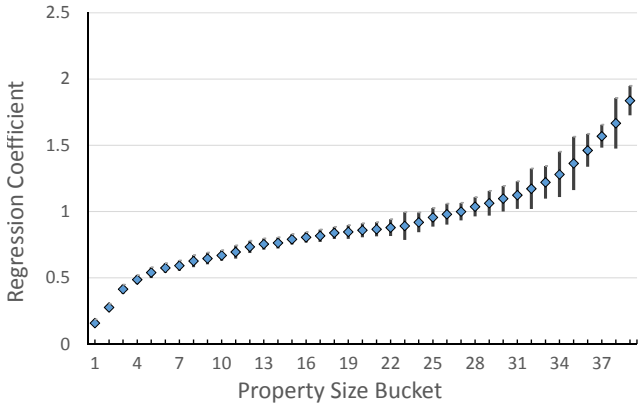
(B) Effect of Bathrooms

(C) Effect of Property Size

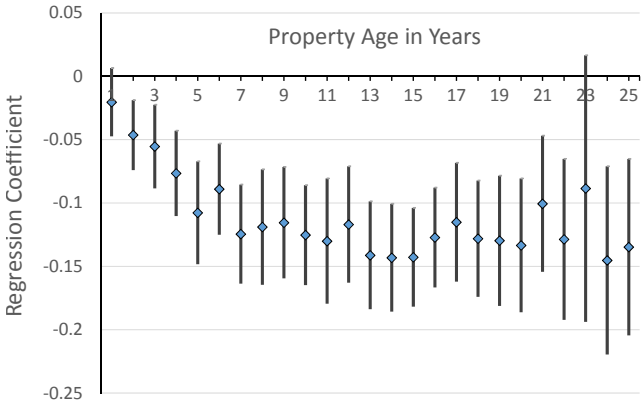
(D) Effect of Property Age

Note: Figures show the coefficients on the hedonic control variables from regression 3: Panel A shows the coefficient on the indicators for the number of bedrooms, Panel B on the indicators for the number of bathrooms, Panel C on the 50 equally-sized property size bucket indicators, and Panel D on the indicator variables for property age. The regression includes other control variables as in column (1) of Table II. For every characteristic, the first bucket is omitted from the regression to avoid collinearity. The bars show standard errors double clustered at the 3-digit postcode level and the transaction quarter level.

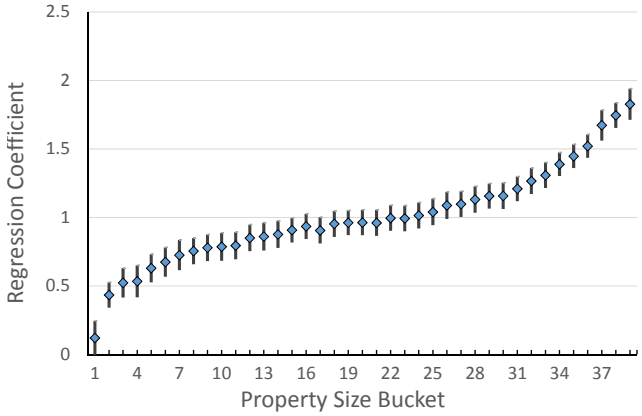
Figure VII: Singapore - Price Effect of Hedonic Characteristics



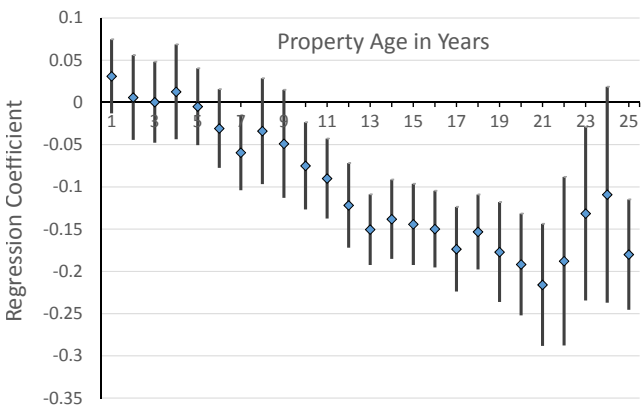
(A) For Sale: Effect of Property Size



(B) For Sale: Effect of Property Age



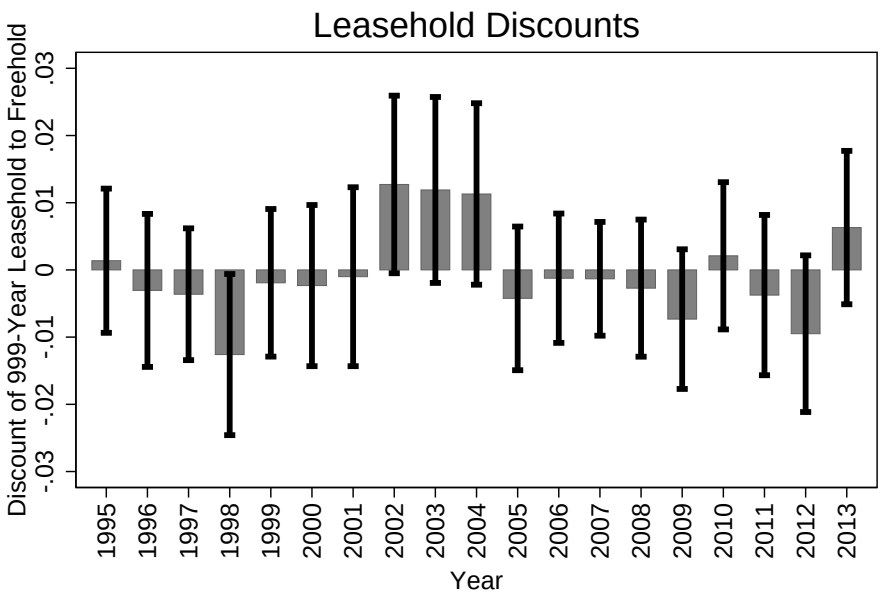
(C) For Rent: Effect of Property Size



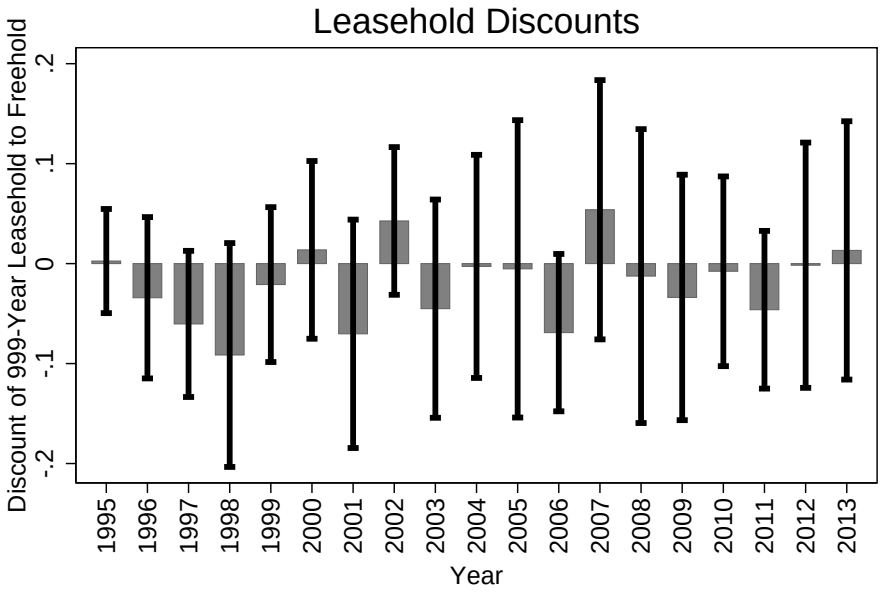
(D) For Rent: Effect of Property Age

Note: Panel A shows the coefficients from regression 3 on the 40 equally-sized property size bucket indicators where the dependent variable is the log of the transaction price; Panel B shows the coefficients on the indicator variables for property age. The regression includes other control variables and fixed effects as in column (1) of Table III. The bars show standard errors that are double clustered at the 5-digit postcode level and the transaction quarter level. Panel C shows the coefficients from regression 3 on the 40 equally-sized property size bucket indicators where the dependent variable is the log of the rental asking price; Panel D shows the coefficients on the indicator variables for property age. The regression includes other control variables and fixed effects as in column (7) of Table III. For every characteristic, the first bucket is omitted from the regression to avoid collinearity. The bars show standard errors that are double clustered at the 5-digit postcode level and the listing quarter level.

Figure VIII: Time Series of Bubble Claim



(A) U.K.



(B) Singapore

Note: The top panel shows year-by-year estimates of the discount between 700+ year leaseholds and freeholds from regression 3 for the U.K., constructed by interacting the indicator $ExtremelyLongLease_{i,t}$ with calendar year. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the 3-digit postcode level. The bottom panel shows year-by-year estimates of the discount between 700+ year leaseholds and freeholds from regression 3 for Singapore, constructed by interacting the indicator $ExtremelyLongLease_{i,t}$ with calendar year. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the 5-digit postcode level.

Figure IX: Time Series of Freehold Transaction Shares



(A) U.K.



(B) Singapore

Note: Figures show the share of transactions of freeholds in the U.K. (Panel A) and Singapore (Panel B) among the set of transactions of freeholds or leaseholds with more than 700 years remaining. The frequency is annual.

Appendix

We provide here further details on the results of Section 2.2.

Discounts in a Gordon Growth Model Following the classic valuation model of Gordon (1982) we assume that cash flows arising in each future period are discounted at a constant rate r, so that the T-period discount factor is e^{rT}. Rents grow at a constant rate g, so that they evolve according to $D_{t+s} = D_t e^{gs}$. We consider a deterministic environment only in the interest of simplicity. The price of the T-maturity leasehold, a claim to the rents for T periods, is:

$$P_t^T = \int_t^{t+T} e^{-r(s-t)} D_t e^{g(s-t)} ds = \frac{D_t}{r-g}(1 - e^{-(r-g)T}).$$

The price of the freehold, the infinite maturity claim, is: $P_t = \lim_{T\to\infty} P_t^T = \frac{D_t}{r-g}$, with $r > g$. Notice that we are deriving the fundamental value and hence imposed the no-bubble transversality condition ($B_t = 0$) in taking the limit. The price discount for a T-maturity leasehold with respect to the freehold is:

$$Disc_t^T \equiv \frac{P_t^T}{P_t} - 1 = -e^{-(r-g)T}. \tag{A.1}$$

Section 2.2 uses Equation (A.1) above to verify that, under the assumption of no bubbles, maturities (T) in excess of 700 years approximate the infinite limit of the theory well for conventional, and even conservative, choices of net discount rates ($r - g$).

Discounts in a Modern Asset Pricing Model In the more general set-up of Section 1, recall that:

$$P_t = \sum_{s=1}^{\infty} E_t[\xi_{t,t+s} D_{t+s}] + B_t, \qquad B_t \equiv \lim_{T\to\infty} E_t[\xi_{t,t+T} P_{t+T}], \tag{A.2}$$

where $\xi_{t,t+s} \equiv \prod_{j=0}^{s-1} \xi_{t+j,t+j+1}$ and $B_t = E_t[\xi_{t,t+1} B_{t+1}]$. Also recall that for a finite maturity asset, like the T-maturity leasehold we have:

$$P_t^T = \sum_{s=1}^{T} E_t[\xi_{t,t+s} D_{t+s}] \tag{A.3}$$

Subtracting A.3 from A.2 we obtain:

$$\begin{aligned}
P_t - P_t^T &= \sum_{s=1}^{\infty} E_t[\xi_{t,t+T+s} D_{t+T+s}] + B_t && \text{(A.4)}\\
&= \sum_{s=1}^{\infty} E_t[\xi_{t,t+T+s} D_{t+T+s}] + E_t[\xi_{t,t+T} B_{t+T}] && \text{(A.5)}\\
&= E_t\left[\xi_{t,t+T}\left[\sum_{s=1}^{\infty} \xi_{t+T,t+T+s} D_{t+T+s} + B_{t+T}\right]\right] && \text{(A.6)}\\
&= E_t\left[\xi_{t,t+T}\, E_{t+T}\left[\sum_{s=1}^{\infty} \xi_{t+T,t+T+s} D_{t+T+s} + B_{t+T}\right]\right] && \text{(A.7)}\\
&= E_t[\xi_{t,t+T} P_{t+T}] && \text{(A.8)}
\end{aligned}$$

Equality (A.5) makes use of the recursive nature of the bubble: $B_t = E_t[\xi_{t,t+1}B_{t+1}]$. Equality (A.6) makes use of the recursive definition of the SDF: $\xi_{t,t+s} \equiv \prod_{j=0}^{s-1} \xi_{t+j,t+j+1}$. Equality (A.7) makes use of the law of iterated expectations. Equality (A.8) makes use of the price of the freehold as in Equation (A.2) moved forward T periods. Then by using Equation (A.8), we have:

$$\lim_{T\to\infty}(P_t - P_t^T) = \lim_{T\to\infty} E_t[\xi_{t,t+T}P_{t+T}] = B_t, \tag{A.9}$$

where the last equality follows from the definition of the bubble in Equation (A.2). Equation (A.9) is the basis of the empirical test strategy described in Section 2.2.