WHEN DOES A CENTRAL BANK'S BALANCE SHEET REQUIRE FISCAL SUPPORT?

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ABSTRACT. Using a simple, general equilibrium model, we argue that a central bank with a large balance sheet composed of long-duration nominal assets should have access to, and be willing to ask for, support for its balance sheet by the fiscal authority. Otherwise its ability to control inflation may be at risk. This need for balance sheet support — a within-government transaction — is distinct from the need for fiscal backing of inflation policy that arises even in models where the central bank's balance sheet is merged with that of the rest of the government.

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KEY WORDS: central bank's balance sheet, solvency, monetary policy.

I. INTRODUCTION

Hall and Reis (2013) and Carpenter, Ihrig, Klee, Quinn, and Boote (2013) have explored the likely path of the Federal Reserve System's balance sheet during a possible return to historically normal levels of interest rates. Both conclude that, though a period when the system's net worth at market value is negative might occur, this is unlikely, would be temporary and would not

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create serious problems.[1] Those conclusions rely on extrapolating into the future not only a notion of historically normal interest rates, but also of historically normal relationships between interest rates, inflation rates, and components of the System's balance sheet. In this paper we look at complete, though simplified, economic models in order to study why a central bank's balance sheet matters at all and the consequences of a lack of fiscal backing for the central bank. These issues are important because they lead us to think about unlikely, but nonetheless possible, sequences of events that could undermine economic stability. As recent events should have taught us, historically *ab*normal events do occur in financial markets, and understanding in advance how they can arise and how to avert or mitigate them is worthwhile.[2]

Constructing a model that allows us to address these issues requires us to specify monetary and fiscal policy behavior and to consider how demand for non-interest-bearing liabilities of the central bank (like currency, or required reserves paying zero interest) responds to interest rates. As we show below, seigniorage plays a central role in determining the possible need for fiscal support for the central bank. But with a given policy in place, seigniorage can vary widely, depending on how sharply demand for cash shrinks

[1] Christensen, Lopez, and Rudebusch (2013) study the interest rate risk faced by the Federal Reserve using probabilities for alternative interest rate scenarios obtained from a dynamic term structure model. They reach the similar conclusions as Hall and Reis (2013) and Carpenter, Ihrig, Klee, Quinn, and Boote (2013). Greenlaw, Hamilton, Hooper, and Mishkin (2013) conduct a similar exercise as Carpenter, Ihrig, Klee, Quinn, and Boote (2013), but also consider scenarios where concern about the solvency of the U.S. government lead to capital losses for the central bank.

[2] A number of recent papers, including Corsetti and Dedola (2012) and Bassetto and Messer (2013a), also study the central bank's and the fiscal authorities' balance sheets separately. Quinn and Roberds (2014) provide an interesting account of the demise of the Florin as an international reserve currency in the late 1700s and attribute such demise to the central bank's credit policies.

as inflation and interest rates rise. An equilibrium model with endogenous demand for cash is therefore required if we are to understand the sources and magnitudes of possible central bank balance sheet problems.

In the first section below we consider a stripped down model to show how the need for fiscal backing arises. In subsequent sections we make the model more realistic and calibrate it to allow simulation of the US Federal Reserve System's response to shifts in the real rate or "inflation scares".

In both the simple model and the more realistic one, we make some of the same generic points.

- Even when fiscal policy is in place that guarantees the price level is uniquely determined, it is nonetheless possible that the central bank, if its balance sheet is sufficiently impaired, may need recapitalization in order to maintain its commitment to a policy rule or an inflation target.
- A central bank's ability to earn seigniorage can make it possible for it to recover from a situation of negative net worth at market value without recapitalization from the treasury, while still maintaining its policy rule. Whether it can do so depends on the policy rule, the demand for its non-interest-bearing liabilities, and the size of the initial net worth gap.[3]
- No policy undertaken by a central bank alone, without fiscal powers, can guarantee a uniquely determined price level. Cochrane (2011) has made this point carefully.

[3]Berriel and Bhattarai (2009) study optimal policy in a setting where the central bank and the fiscal authority have separate budget constraints. Berriel and Bhattarai (2009) only consider the case where the central bank can acquire short-term assets however, which implies that solvency issues are unlikely to arise (see Bassetto and Messer (2013b)).

- In the presence of a large long-duration balance sheet, a central bank that is committed to avoiding any request for fiscal support (or a fiscal authority committed to providing none) can open the door to self-fulfilling equilibria where expectations of high inflation in the future lead to capital losses that need to be filled by generating seigniorage, thereby validating the expectations.

In the simple model we aim to explain qualitatively how the need for backing or support can arise, while in the more realistic model we try to determine how likely it is that the US Federal Reserve System will need fiscal support if interest rates return to more historically normal levels in the near future.

II. THE SIMPLE MODEL

We first consider a stripped-down model to illustrate the principles at work. A representative agent solves

$$\max_{C,B,M,F} \int_0^\infty e^{-\beta t} \log(C_t)\, dt \qquad \text{subject to} \tag{1}$$

$$C_t \cdot (1 + \psi(v_t)) + \frac{\dot{B} + \dot{M}}{P_t} + \tau_t + F_t = \rho F_t + \frac{r_t B_t}{P_t} + Y_t\,, \tag{2}$$

where C is consumption, B is instantaneous nominal bonds paying interest at the rate r, M is non-interest-bearing money, ρ is a real rate of return on a real asset F, Y is endowment income, and τ is the primary surplus (or simply lump-sum taxes, since we have no explicit government spending in this model). Velocity v_t is given by

$$v_t = \frac{P_t C_t}{M_t},\ v_t \geq 0, \tag{3}$$

and the function $\psi(.),\ \psi'(.) > 0$ captures transaction costs.

The government budget constraint is

$$\frac{\dot{B} + \dot{M}}{P_t} + \tau_t = \frac{r_t B_t}{P_t}\,. \tag{4}$$

Monetary policy is an interest-smoothing Taylor rule:

$$\dot{r} = \theta_r \cdot \left(\bar{r} + \theta_\pi \left(\frac{\dot{P}}{P} - \bar{\pi} \right) - r \right) . \tag{5}$$

The "Taylor Principle", that θ_π should exceed one, is the usual prescription for "active" monetary policy. The effective short term interest rate is given by the maximum of r_t and the lower bound on interest rates $\underline{r}$:

$$r_t^e = \max\left(r_t, \underline{r}\right). \tag{6}$$

First order conditions for the private agent are

$$\partial C: \qquad \frac{1}{C} = \lambda(1 + \psi + \psi' v) \tag{7}$$

$$\partial F: \qquad -\hat{\lambda} = \lambda(\rho - \beta) \tag{8}$$

$$\partial B: \qquad -\frac{\hat{\lambda}}{P} + \beta\frac{\lambda}{P} + \frac{\lambda}{P}\frac{\hat{P}}{P} = r\frac{\lambda}{P} \tag{9}$$

$$\partial M: \qquad -\frac{\hat{\lambda}}{P} + \beta\frac{\lambda}{P} + \frac{\lambda}{P}\frac{\hat{P}}{P} = \frac{\lambda}{P}\psi' v^2 \tag{10}$$

The $\hat{\dot{z}}_t$ notation means the time derivative of the future expected path of z at t. It exists even at dates when z has taken a jump, so long as its future path is right-differentiable. Below we also use the $\frac{\hat{d}^+}{dt}$ operator for the same concept.

We are taking the real rate ρ as exogenous, and in this simple version of the model constant. The economy is therefore being modeled as either having a constant-returns-to scale investment technology or as having access to international borrowing and lending at a fixed rate. Though we could extend the model to consider stochastically evolving Y, ρ, and other external disturbances, here we consider only surprise shifts at the $t = 0$ starting date, with perfect-foresight deterministic paths thereafter. This makes it easier to follow the logic, though it makes the time-0 adjustments unrealistically abrupt.

Besides the exogenous influences that already appear explicitly in the system above (ρ and Y), we consider an "inflation scare" variable x. This enters the agents' first order condition as a perturbation to inflation expectations. It can be reconciled with rational expectations by supposing that agents think there is a possibility of discontinuous jumps in the price level, with these jumps arriving as a Poisson process with a fixed rate. This would happen if at these jump dates monetary policy created discontinuous jumps in M. Such jumps would create temporary declines in the real value of government debt B/P which might explain why such jumps are perceived as possible. If the jump process doesn't change after a jump occurs, there is no change in velocity, the inflation rate, consumption, or the interest rate at the jump dates. Rather than solve a model that includes such jumps, we model one in which the public is wrong about this — there are no jumps, despite the expectation that there could be jumps. After a long enough period with no jumps, the public would probably change its expectations, but there is no logical contradiction in supposing that for a moderate amount of time the fact that there are no jumps does not change expectations. In fact, if we consider time-varying paths for x, in which x returns to zero after some period, there is no way to distinguish whether the "true" model is one with the assumed x path (and thus a non-zero probability of jumps in P) or one with $x \equiv 0$ if jumps do not actually occur.

The inflation scare variable changes the first order conditions above to give us

$$\partial B: \qquad -\hat{\lambda}\frac{1}{P} + \beta\frac{\lambda}{P} + \frac{\lambda}{P}\left(\frac{\hat{P}}{P} + x\right) = r\frac{\lambda}{P} \tag{9'}$$

$$\partial M: \qquad -\hat{\lambda}\frac{1}{P} + \beta\frac{\lambda}{P} + \frac{\lambda}{P}\left(\frac{\hat{P}}{P} + x\right) = \frac{\lambda}{P}\psi' v^2 \tag{10'}$$

Because the price and money jumps have no effect on interest rates or consumption, no other equations in the model need change. These first order conditions reflect the private agents' use of a probability model that includes jumps in evaluating their objective function.

We can solve the model analytically to see the impact of an unanticipated, permanent, time-0 shift in ρ (the real rate of return), x (the inflation scare variable), or $\bar{r}$ (the central bank's interest-rate target). We could also solve it numerically for arbitrary time paths of ρ, x, $\bar{r}$, *et cetera*, but we reserve such exercises for the more detailed and realistic model in subsequent sections.

Solving to eliminate the Lagrange multipliers from the first order conditions we obtain

$$\rho = r - \frac{\hat{\hat{P}}}{P} - x \tag{11}$$

$$r = \psi'(v)v^2 \tag{12}$$

$$-\frac{\hat{d^+}}{dt}\left(\frac{1}{C\cdot(1+\psi+\psi' v)}\right) = \frac{\rho-\beta}{C(1+\psi+\psi' v)}. \tag{13}$$

Using (11) and the policy rule (5), we obtain that along the path after the initial date,

$$\begin{aligned}\dot{r} &= \theta_r\cdot((\theta_\pi(r-\rho-x)-r+\bar{r}-\theta_\pi\bar{\pi})\\ &= \theta_r\cdot(\theta_\pi-1)r-\theta_r\theta_\pi(\rho+x)+\theta_r(\bar{r}-\theta_\pi\bar{\pi})\,.\end{aligned} \tag{14}$$

With the usual assumption of active monetary policy, $\theta_\pi > 1$, so this is an unstable differential equation in the single endogenous variable r. Solutions are of the form

$$r_t = E_t\left[\int_0^\infty e^{-(\theta_\pi-1)\theta_r s}\theta_r\theta_\pi(\rho_{t+s}+x_{t+s})\,ds\right] - \frac{\bar{r}-\theta_\pi\bar{\pi}}{\theta_\pi-1} + \kappa e^{(\theta_\pi-1)\theta_r t}. \tag{15}$$

In a steady state with x and ρ constant (and $\kappa = 0$), this give us

$$r = \frac{\theta_\pi(\rho+x)}{\theta_\pi-1} - \frac{\bar{r}-\theta_\pi\bar{\pi}}{\theta_\pi-1}\,. \tag{16}$$

From (12) we can find v as a function of r. Substituting the government budget constraint into the private budget constraint gives us the social resource constraint

$$C \cdot (1 + \psi(v)) + \dot{F} = \rho F + Y. \tag{17}$$

Solving this unstable differential equation forward gives us

$$F_t = E_t \left[\int_0^\infty \exp\left(- \int_0^s \rho_{t+v}\, dv \right) (Y_{t+s} - C_{t+s}(1 + \psi(v_{t+s})))\, ds \right]. \tag{18}$$

Here we do not include an exponentially explosive term because that would be ruled out by transversality in the agent's problem and by a lower bound on F. With constant ρ, x and Y, r and v are constant, and (13) then lets us conclude that C grows (or shrinks) steadily at the rate $\rho - \beta$. We can therefore use (18) to conclude that along the solution path, since ρ, Y and v are constant

$$C_t = \frac{\beta \cdot (\rho^{-1} Y + F_t)}{1 + \psi(v)}. \tag{19}$$

This lets us determine initial C_0 from the F_0 at that date. From then on C_t grows or shrinks at the rate $\rho - \beta$ and the resulting saving or dissaving determines the path of F_t from (19).

III. Unstable paths, uniqueness, fiscal backing

To this point we have not introduced a central bank balance sheet or budget constraint. We can nonetheless distinguish between monetary policy, controlling the interest rate or the money stock, and fiscal policy, controlling the level of primary surpluses. Passive fiscal policies make the primary surplus co-move positively with the level of real debt and guarantee a stable level of real debt, regardless of the time path of prices, under the assumption of stable real interest rates. Passive fiscal policies generally leave the price level indeterminate, no matter what interest rate or money stock policy is in place. Guaranteeing uniqueness of the price level requires a commitment to active fiscal policy. Nonetheless fiscal policy in the presence

of low inflation may be passive, so long as it is believed that policy would turn active if necessary to rule out explosive inflation. In the remainder of this section we make these points analytically in the simple model.

Our solution for r, given by (15), tells us that, with ρ and x constant, r could be constant, but nothing in the model to this point tells us that $\kappa \neq 0$ is impossible. To assess whether these paths are potential equilibria in the model, we need to specify fiscal policy. The standard sort of fiscal policy to accompany the type of monetary policy we have postulated (Taylor rule with $\theta_\pi > 1$) is a "passive" policy that makes primary surpluses plus seigniorage respond positively to the level of real debt. For example, we can assume

$$\frac{\dot{M}}{P} + \tau = -\phi_0 + \phi_1 \frac{B}{P}\,. \tag{20}$$

Substituting this into the government budget constraint (4) and using (11) gives us

$$\dot{b} = \left(\rho + x + \frac{\hat{\dot{P}} - \dot{P}}{P} - \phi_1\right) b + \phi_0\,. \tag{21}$$

On an equilibrium path,

$$\frac{\hat{\dot{P}}}{P} = \frac{\dot{P}}{P}\,,$$

that is, actual inflation and model-based expected inflation are equal. Thus if $\phi_1 > \rho + x$, this is a stable differential equation, with b converging to $\phi_0/(\phi_1 - \rho - x)$. In fact, any $\phi_1 > 0$ is consistent with equilibrium, even though for small values b grows exponentially. The transversality condition with respect to debt for the private agent who holds the debt is

$$E_0\left[e^{-\beta t}\frac{\lambda B}{P_t}\right] = 0\,. \tag{22}$$

From (8) λ grows at the rate $\beta - \rho$, while from (21) b grows asymptotically at $\rho + x - \phi_1$. However the E_0 in the transversality condition is the private agent's expectation operator. Since the agent believes in the possibility of price jumps, the agent thinks that the expected real return on real debt is just ρ, not $\rho + x$. Thus the agent believes that b grows asymptotically at the

rate $\rho - \phi_1$. The agent's transversality condition is therefore satisfied for any $\phi_1 > 0$. The agent in such equilibria has ever-growing wealth, but at the same time ever-growing taxes that offset that wealth, so that the agent is content with the consumption path defined by the economy's real equilibrium.[4]

A passive fiscal policy with $\phi_1 > 0$, therefore, guarantees that all conditions for a private agent optimum are met on any of the paths for prices and interest rates we have derived, including those with $\kappa > 0$. The inflation rate (not just the price level) diverges to infinity on such a path, along with the interest rate and velocity. So long as r is an increasing function of v ($\psi''(v)v^2 + 2v\psi'(v) > 0$), real balances shrink on these paths and, depending on the specification of the $\psi(v)$ function, may go to zero in finite time.

With $\kappa < 0$, the initial interest rate and inflation rate are below the level consistent with stable inflation and both the price level and the interest rate decline on an exponential path. Since negative nominal interest rates are not possible, it is impossible to maintain the Taylor rule when it prescribes, as it eventually must on such a path, negative interest rates. The simplest modification of the policy rule that accounts for this zero bound on the interest rate, has $\dot{r}$ follow the right-hand side of (5) whenever this is positive or r itself is positive, and otherwise sets $\dot{r}$ to zero. With this specification and the passive fiscal rule (20) the economy has a second steady state (assuming ϕ_1 large enough to stabilize b), at $r = 0$, $b = \phi_0/(\phi_1 - \rho - x)$. In this steady state inflation is constant at $-\rho - x$. This steady state is stable.

[4]Note that, because the realized real rate of return on debt exceeds that on real assets F, the properly discounted present value of future taxes exceeds the real value of debt on a path with $x > 0$, and may even be infinite. "Ricardian" fiscal policy does not guarantee a match between the present value of future taxes and the current real value of debt on this non-rational-expectations path for the economy.

At this point we have approximately matched the model and conclusions of Benhabib, Schmitt-Grohé, and Uribe (2001): This policy configuration produces a pair of equilibria, with only one globally stable. Because the equilibria with $\kappa \neq 0$ cannot be ruled out, and because there are many paths for the economy that converge in expectation to the stable $r = 0$ point, the price level is indeterminate.

There are reasonable beliefs on the part of agents in the model about how fiscal policy would behave at low or very high levels of inflation that would remove the $\kappa \neq 0$ policies from the set of equilibria, while leaving the $\kappa = 0$ equilibrium or something very close to it as a unique solution. Since the focus of this paper is on the possible need for fiscal support, in the form of capital injections, even on the $\kappa = 0$ paths, we postpone to an appendix detailed discussion of fiscal polices to guarantee uniqueness.

IV. Four levels of central bank balance sheet problems

So far, we have said nothing about the central bank balance sheet, but with the solution path for the economy in hand, assessing the time path of the balance sheet is straightforward. The most severe problem, which we can call level 4, is simply the possible indeterminacy of the price level. To put this in the language of the central bank balance sheet, this is the point that the central bank's assets consist of the market value of its assets and its potential seigniorage, both of which are valueless if currency is valueless. But if it holds nominal debt as assets and issues reserves and currency as liabilities, the central bank has no lever to guarantee the real value of either side of its balance sheet. If the public were to cease to accept currency in payment, it would become valueless, as would both sides of the central bank balance sheet. That this cannot happen, either suddenly or as the end point of a dynamic process, depends on fiscal commitments beyond the central bank's control. The fiscal backing required for price level determinacy

seems quite plausible in the US. In Europe, because fiscal responsibility for the Euro is divided among many countries that seem bent on frequently increasing doubts about their ability to cooperate on fiscal matters, this possibility cannot be entirely ruled out.

The next level of possible problem, level 3, arises because the notion of determinacy via a backstop fiscal commitment assumes that the central bank could maintain its commitment to an active policy rule during an inflationary excursion from the unique stable price path, up to the point that fiscal backing is triggered. If we think of a unified government budget constraint and jointly determined monetary and fiscal policy, this is not an issue. But if the central bank is concerned to maintain its policies without requiring a direct capital injection from the treasury, or possibly even without ever having to set its seigniorage payments to the treasury to zero, then this could be a problem. And of course if markets perceive that the central bank will abandon its policy rule to avoid having to seek treasury support, this undercuts the argument for price determinacy. Showing formally how these issues arise requires solving the model for time varying paths of interest rates and velocity, so it is postponed to later sections of the paper.

If the market value of the assets of the central bank fall to a value below that of their interest-bearing liabilities, it is possible that adherence to the bank's policy rule is impossible without a direct injection of capital. This is only a possibility, however, because the bank has an implicit asset in its future seigniorage. Even with assets below interest-bearing liabilities at market value, the bank may be able to meet all its interest-paying obligations and to restore the asset side of its balance sheet through accumulation of seigniorage. Whether it can do so depends on its policy rule and on the interest-elasticity of demand for currency (or more generally, for its non-interest-bearing liabilities). This issue, of whether the central bank

might require a capital injection to maintain adherence to its policy rule in a determinate-price-level equilibrium, is a level 2 balance sheet problem.

Finally, at level 1, the central bank may be solvent in the sense that with the existing policy rule its assets at market value plus future seigniorage exceed its total liabilities, yet following standard accounting rules and rules for determining how much seigniorage revenue is sent to the treasury each period may lead to episodes of zero seigniorage payments to the treasury. Extended episodes of this type might be thought to raise issues of political economy, if they led to public criticism of the central bank or to calls for revising its governance.

V. How could a central bank be "insolvent"?

A central bank in an economy with fiat money by definition can always pay its bills by printing money. In that sense it cannot be insolvent. On the other hand, paying its bills by printing money clearly could interfere with the policy objectives of the central bank, assuming it wants to control inflation. Historically there were central banks (like the US Federal Reserve in previous decades) whose liabilities were all reasonably characterized as "money". There was currency, which paid no interest, and also reserve deposits, which also paid no interest. For such a central bank it is easy to understand that "paying bills by printing money" could conflict with "restricting money growth to control inflation". Such a bank could be "insolvent" in the sense that, with money growth, and hence inflation, at the level that it targets, it is not earning enough seignorage to cover its payment obligations and will not make up the gap in the future.[5]

[5] Reis (2013) discusses a number of misconceptions about central banking, among which the notion that the central bank can have access to unlimited resources, that is, does not have to face an intertemporal budget constraint, because it can "print money."

Modern central banks, though, have interest-bearing reserve deposit liabilities as well as non-interest-bearing ones. Furthermore, in the last several years central banks have demonstrated that they can rapidly expand their reserve deposit liabilities without generating strong inflationary pressure. Such central banks can "print" interest-bearing reserves. What prevents them from meeting any payment obligations they face by creating interest-bearing reserve deposits?

The recent non-inflationary balance sheet expansions have arisen through central bank purchases of interest-earning assets by issuing interest-bearing reserves. If instead it met payment obligations — staff salaries, plant and equipment, or interest on reserves — by issuing new interest-bearing reserve deposits, there would be no flow of earnings from assets offsetting the new flow of interest on reserves. If this went on long enough, interest-bearing liabilities would come to exceed interest-bearing assets. To the extent the gap between interest earnings and interest-bearing obligations was not covered by seignorage, the bank's net liabilities would begin growing at approximately the interest rate.

In this scenario, the private sector would be holding an asset — reserves — that was growing at the real interest rate. This might be sustainable if there were some offsetting private sector liability growing at the same rate — expected future taxes, or bonds issued by the private sector and bought by the government, for example. But if we rule out these possibilities, by saying real taxes are bounded and the fiscal authority will not accumulate an exponentially growing cache of private sector debt, the exploding liabilities of the central bank violate the private sector's transversality condition. In other words, private individuals, finding their assets growing so rapidly, would try to turn those assets into consumption goods. Or in still other words, the exploding interest-bearing liabilities of the central bank would eventually cause inflation, even if money supply growth were kept low.

This conclusion is a special case of the usual analysis in the fiscal theory of the price level: increased issue of nominal bonds, unbacked by taxation, is eventually inflationary, regardless of monetary policy.

VI. Inflation scare in the simple model

Our first numerical example uses this simple model to compare a steady state with $\rho = \beta = \bar{\rho} = .01$ and $x = 0$ to one in which x jumps up to .01 at time 0. This is an "inflation scare" scenario. The 1% per year inflation scare shock produces a much larger increase in the nominal interest rate, because the increased inflation expectations shrink demand for money and thereby produce inflation, which prompts the central bank to raise rates further. If the duration of the nominal assets on the central bank's balance sheet is positive, the permanent rise in rates reduces the time 0 market value of the central bank's assets. The simple model treats the debt as of maturity 0, but this has no consequence except for the initial date capital losses, because for $t > 0$ the perfect-foresight path requires that long and short debt has the same time path of returns. We can use standard formulas to compute the nominal capital losses produced by the permanent rise in the interest rate. We show two cases: initial assets of the central bank A_0 are three times the amount of currency outstanding or six times the amount of currency outstanding with the initial deposit liabilities V_0 plus currency matching A_0 in each case.

To make the calculations, we need to specify the form of $\psi(v)$, the transactions cost function. We choose the form

$$\psi(v) = \psi_0 e^{-\psi_1/v}. \tag{23}$$

Our reasons for choosing this functional form, and the extent of its claim to realism, are discussed below in the context of the detailed model. We use $\psi_0 = .63$, $\psi_1 = 103$, as in the base case of the detailed model. We set the

policy parameters $\theta_\pi = 1.5$, $\theta_r = 1$, $\bar{r} = \rho$, $\bar{\pi} = .005$. Note that this means that the monetary authority chooses a $\bar{\pi}$ that would result in 0.5% inflation with rational expectations, but does not succeed in hitting this target in the presence of the inflation scare. It is also assumed that $Y = 1$, $F_0 = 0$, and initial $M = 1$.

TABLE 1. Change in steady state after 1% inflation scare

	r	v	m	P	dV	dpvs	C
base	0.015	7.937	0.126	7.938	0.000	0.500	1.000
new	0.045	8.671	0.115	8.098	0.066	1.167	1.000

	duration	2.5	5	10	20
proportional capital loss		0.93	0.88	0.79	0.68
	gap, A0=3	0.27	0.43	0.69	1.01
	gap, A0=6	0.47	0.80	1.31	1.96

The nominal capital losses, as a proportion of the new value of the assets, are shown in the middle panel of Table 1. There cannot be any "level 2" problem for the central bank unless the interest increase pushes its initial assets A below V. That is, it not only has to have assets less than liabilities $V + M$, where M is currency, it has to have $A < V$ in order for a level 2 problem to arise. The rise in interest rate reduces the demand for M, which has to be met either by an decrease in A through open market sales or an increase in V. This will amplify the effect on $V - A$ of the rate rise. The value of $V - A$ is shown as the "gap" lines in tjhe Table. Whether a level 2 problem actually arises then depends on the discounted present value of the seigniorage after the initial date, shown as "dpvs" in the table. For this example, even though the gap between V and A gets quite large if we assume long durations for the assets, the gap exceeds the discounted present value

of seigniorage only for durations of 10 years or more and for the (unrealistically large) balance sheet with A_0 six times outstanding currency. If we use the transactions cost parameters taken from pre-2008 data (described below with the detailed model), the gaps increase relative to the seignorage, so that fiscal support is required even for $A0 = 3$ if duration is 20, but the results are otherwise similar.

This example should make it clear that the central bank can suffer very substantial capital losses without needing direct recapitalization. On the other hand, it shows that there are drawbacks to extreme expansion of central bank holdings of long-maturity debt — an expanded balance sheet increases the probability that interest rate changes could require a direct capital injection.

This simple model has omitted two sources of seigniorage, population growth and technical progress. It therefore makes it unrealistically easy to find conditions in which fiscal support is required. The analytic solution for steady states that we have used for Table 1 can be extended to allow considering more plausible exogenous, non-constant time paths of ρ, x, etc., but only with use of numerical integration. We now expand the model to include these extra elements and calibrate the parameters and the nature of the shocks more carefully to the situation of the US Federal Reserve. Of course our ability to calibrate is limited by the sensitivity of results to the transactions cost function. We have little relevant historical experience with currency demand at low or very high interest rates. Rates were very low in the 1930's and the early 1950's, but the technology for making non-currency transactions is very different now. It is difficult to predict how much and how fast people would shift toward, say, interest-bearing pre-loaded cash cards as currency replacements if interest rates increased to historically normal levels. We can at best show ranges of plausible results.

VII. THE MODEL WITH LONG-TERM DEBT

Like the simple model, this one borrows from Sims (2005). The household planner (whose utility includes that of offspring, see Barro and Sala-i Martin (2004)) maximizes:

$$\int_0^\infty e^{-(\beta-n)t}\log(C_t)dt \tag{24}$$

where C_t is per capita consumption, β is the discount rate, and n is population growth, subject to the budget constraint:

$$C_t(1+\psi(v_t))+\dot{F}_t+\frac{\dot{V}_t+\dot{M}_t+q_t\dot{B}^P}{P_t}= \\ Ye^{\gamma t}+(\rho_t-n)F_t+(r_t-n)\frac{V_t}{P_t}+(\chi+\delta-q_t\delta-n)\frac{B^P}{P}-n\frac{M_t}{P_t}-\tau_t. \tag{25}$$

We express all variables in per-capita terms and initial population is normalized to one. F_t and B_t^P are foreign assets and long-term government bonds in the hand of the public, respectively, V_t denotes central bank reserves, M_t is currency, τ_t is lump-sum taxes, Y is an exogenous income stream growing at rate γ. Foreign assets and central bank reserves pay an exogenous real return ρ and a nominal return r_t, respectively. Long term bonds are modeled as in Woodford (2001). They are assumed to depreciate at rate δ (δ^{-1} captures the bonds average maturity) and pay a nominal coupon $\chi+\delta$.[6]

The government is divided into two distinct agencies called "central bank" and "fiscal authority". The central bank's budget constraint is

$$\left(q_t\frac{\dot{B}^C}{P_t}-\frac{\dot{V}_t+\dot{M}_t}{P_t}\right)e^{nt} \\ =\left((\chi+\delta-\delta q_t-nq_t)\frac{B^C}{P}-(r_t-n)\frac{V_t}{P_t}+n\frac{M_t}{P_t}-\tau_t^C\right)e^{nt}. \tag{26}$$

where B_t^C are long-term government bonds owned by the central bank, and τ_t^C are remittances from the central bank to the fiscal authority. The central

[6]We write the coupon as $\chi+\delta$ so that at steady state if χ equals the short term rate the bonds sell at par ($q=1$).

bank is assumed to follow the rule (5) for setting r_t, the interest on reserves. We also assume that the central bank's policy in terms of the asset side of its balance sheet B_t^C consists in an exogenous process $B_t^C = \bar{B}_t^C$. Finally, the central bank is also assumed to follow a rule for remittances, which we will describe in section VII.2. We explain there why neither the rule for B_t^C nor that for remittances will play a central role in our analysis.

Solving the central bank's budget constraint forward we can obtain its intertemporal budget constraint:

$$q\frac{B_0^C}{P_0} - \frac{V_0}{P_0} + \int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t} e^{-\int_0^t (\rho_s + x_s - n)ds} dt = \int_0^\infty \tau_t^C e^{-\int_0^t (\rho_s + x_s - n)ds} dt. \tag{27}$$

where x_s refers to the inflation scare variable discussed in section II (the inflation scare results in a premium increasing the real returns on all nominal assets, and hence enters the central bank's present discounted value calculations). Equation (27) shows that, regardless of the rule for remittances, their discounted present value $\int_0^\infty \tau_t^C e^{-\int_0^t (\rho_s + x_s - n)ds} dt$ has to equal its left hand side, namely the market value of assets minus reserves plus the discounted present value of seigniorage $\int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t} e^{-\int_0^t (\rho_s + x_s - n)ds} dt$. We can also compute the constant level of remittances $\bar{\tau}^C e^{\gamma t}$ (taking productivity growth into account) that satisfies expression (27).

$$\bar{\tau}^C = \left(\int_0^\infty e^{(\gamma + n)t - \int_0^t (\rho_s + x_s)ds} dt \right)^{-1} \left(q\frac{B^c}{P} - \frac{V}{P} + \int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t} e^{-\int_0^t (\rho_s + x_s - n)ds} dt \right). \tag{28}$$

Government debt is assumed to be held either by the central bank or the public: $B_t = B_t^C + B_t^P$. The budget constraint of the fiscal authority is

$$\left(G_t - \tau_t + (\chi + \delta - \delta q_t - n q_t)\frac{B}{P} \right) e^{nt} = \left(\tau_t^C + q_t \frac{\dot{B}}{P_t} \right) e^{nt}, \tag{29}$$

where G_t is government spending. The rule for τ_t is given by:

$$\tau_t = \phi_0 e^{\gamma t} + (\phi_1 + n + \gamma)\left(q\frac{B^P}{P} + \frac{V}{P}\right). \tag{30}$$

This rule makes the debt to GDP ratio $b_t = \left(q\frac{B^P}{P} + \frac{V}{P}\right)e^{-\gamma t}$ converge as long as $\phi_1 > \beta - n$. The initial level of foreign assets in the hand of the public, central bank reserves, and currency are F_0^P, V_0, and M_0, respectively.

As in the simple model the first order condition for the household's problem with respect to C, F^P, B, V, and M yield the Euler equation (13), the Fisher equation (11),[7] the money demand equation (12), and the arbitrage condition between reserves and long-term bonds:

$$\frac{\chi + \delta}{q} - \delta + \frac{\dot{q}}{q} = r. \tag{31}$$

The solutions for r is given by equation (15), and those for inflation $\frac{\dot{P}}{P}$ and velocity v follow from equations (11) and (12), respectively. The growth rate of consumption $\frac{\dot{C}}{C}$, is given by

$$\frac{\dot{C}}{C} = (\rho - \beta) - \frac{2\psi'(v) + v\psi''(v)}{1 + \psi(v) + v\psi'(v)}\dot{v}, \tag{32}$$

which obtains from differentiating expression (13). Differentiating the definition of velocity (3) we obtain an expression for the growth rate of currency:

$$\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{C}}{C} - \frac{\dot{v}}{v}. \tag{33}$$

The economy's resource constraint is given by

$$C(1 + \psi(v)) + \dot{F} = (Y - G)e^{\gamma t} + (\rho - n)F, \tag{34}$$

where $F = F^P + F^C$ is the aggregate amount of foreign assets held in the economy (we assume that the central banks foreign reserves F^C are zero),

[7] Note that short term debt was called B in the simple model, and was issued by the fiscal authority. Here it is called V, and is issued by the central bank.

and where we assumed $G_t = Ge^{\gamma t}$. Solving this equation forward we obtain a solution for consumption in the initial period:

$$C_0 \left(\int_0^\infty (1+\psi(v)) e^{-\int_0^t (\rho_s - \frac{\dot{C}}{C} - n) ds} dt \right) = F_0 + (Y - g) \int_0^\infty e^{(\gamma+n)t - \int_0^t \rho_s ds} dt, \tag{35}$$

Given velocity v and the level of consumption, we can compute real money balances $\frac{M}{P}$, the initial price level P_0, and seigniorage $\frac{\dot{M}}{P} + n\frac{M}{P} = \left(\frac{\dot{M}}{M} + n\right)\frac{M}{P}$ (using (33)), and the present discounted value of seigniorage

$$\int_0^\infty \left(\frac{\dot{M}}{M} + n\right) \frac{M}{P} e^{-\int_0^t (\rho_s + x_s - n) ds} dt = c_0 \int_0^\infty \left(\frac{\dot{M}}{M} + n\right) v^{-1} e^{-\int_0^t (\rho_s + x_s - \frac{\dot{C}}{C} - n) ds} dt.$$

Finally, solving (31) forward we find the current nominal value of long-term bonds

$$q_0 = (\chi + \delta) \int_0^\infty e^{-\left(\int_0^t r_s ds + \delta t\right)} dt. \tag{36}$$

VII.1. **Steady state.** At a steady state where $\bar{\rho} = \beta + \gamma$, $\bar{r} = \bar{\rho} + \bar{\pi}$, $\bar{v}$ satisfies $\bar{v}^2 \psi'(\bar{v}) = r_{ss}$. Steady state consumption is given by $\bar{C}_t = \bar{C}_0 e^{\gamma t}$ where $\bar{C}_0 = \frac{(\beta - n)F_0 + Y - G}{1 + \psi(\bar{v})}$, and real money balances are given by $\frac{M}{P}_{ss} = \frac{\bar{C}_0}{\bar{v}} e^{\gamma t}$. seigniorage is given by $(\bar{\pi} + \gamma + n)\frac{\bar{C}_0}{\bar{v}} e^{(\gamma+n)t}$ and its present discounted value is given by $(\bar{\pi} + \gamma + n)\frac{\bar{C}_0}{\bar{v}(\beta - n)}$.

VII.2. **Central bank's solvency, accounting, and the rule for remittances.** For some of the papers discussed in the introduction the issue of central bank's solvency is simply not taken into consideration: the worst that can happen is that the fiscal authority may face an uneven path of remittances, with possibly no remittances at all for an extended period. We acknowledge the possibility that remittances may have to be negative, at least at some point. This is what we mean by solvency.

Like Bassetto and Messer (2013a), we approach the issue of central bank's solvency from a present discounted value perspective. If the left hand side of equation (27) is negative, the central bank cannot face its obligations, i.e.,

pay back reserves, without the support of the fiscal authority. An interesting aspect of equation (27) is that its left hand side does *not* depend on many of aspects of central bank policy that are recurrent in debates about the fiscal consequences of central bank's balance sheet policy. For instance, the future path of B_t^C does not enter this equation: whether the central bank holds its assets to maturity or not, for instance, is irrelevant from an expected present value perspective. Intuitively, the current price q_t contains all relevant information about the future income from the asset relative to the opportunity cost r_t. Whether the central bank decides to sell the assets and realize gains or losses, or keep the assets in its portfolio and finance it via reserves, does not matter. Similarly, whether the central bank incurs negative income in any given period, and accumulates a "deferred asset", is irrelevant from the perspective of the overall present discounted value of resources transferred to the fiscal authority.[8] In fact, scenarios associated with *higher* remittances in terms of present value may well be associated with a deferred asset.

Finally, the issue of "remittances smoothing" is also, from a purely economic point of view, a non issue. In perfect foresight the central bank can always choose a perfectly smooth path of remittances (in fact, this is $\tau_t^C = \bar{\tau}^C e^{\gamma t}$). But there are accounting rules governing central banks' remittances.[9] Hence these may not be smooth and may depend on the central bank's actions, such as holding the assets to maturity or not. We recognize that the timing of remittances can matter for a variety of reasons: tax

[8] As we will see later central bank accounting does not let negative income affect capital. The budget constraint (26) implies however that negative income results in either more liabilities or less assets. To maintain capital nonetheless intact, a "deferred asset" is created on the asset side of the balance sheet.

[9] Note that the rule governing remittances matters because *past* remittances determine the current level of central bank's liabilities, which enter equation (27). Goodfriend (2014) argues that central banks involved in unconventional policies should not remit part of their income in order to build a capital buffer.

smoothing, political pressures on the central bank, *et cetera*. For this reason we assume a specific rule for remittances that very loosely matches those adopted by actual central banks and compute simulated paths of remittances under different assumptions. Appendix B discusses this rule.

VII.3. **Functional forms and parameters.** Table 2 shows the model parameters. We normalize $Y - g$ to be equal to 1, and set F_0 to 0.[10] Since we do not have investment in our model, and $F_0 = 0$, $Y - G$ in the model corresponds to national income Y minus government spending G in the data (data are from Haver analytics, mnemonics are `Y@USNA` and `G@USNA`, respectively). All real quantities discussed in the remainder of the paper should therefore be understood as multiples of $Y - G$, and their data counterparts are going to be expressed as a fraction of national income minus government spending ($ 11492 bn in 2013Q3). Our $t = 0$ corresponds to the beginning of 2014. We therefore measure our starting values for the face value of central bank assets $\frac{B^C}{P}$, reserves $\frac{V}{P}$, and currency $\frac{M}{P}$ using the January 3, 2014 H.4.1 report (`http://www.federalreserve.gov/releases/h41/`), which measures the Security Open Market Account (SOMA) assets.[11] The model parameters are chosen as follows. The discount rate β, productivity growth γ, and population growth n are 1 percent, 1 percent, and .75 percent, respectively. These values are consistent with Carpenter et al.'s assumptions of a 2% steady state real rate.

[10] Note from the steady state calculations that we could choose $F_0 \neq 0$ and use instead the normalization $(\beta - n)F_0 + y - g$=1, hence setting $F_0 \neq 0$ simply implies a different normalization.

[11] The January 3, 2014 H.4.1 reports the face value of Treasury ($ 2208.791 bn), GSE debt securities ($ 57.221 bn), and Federal Agency and GSE MBS ($ 1490.160 bn), implying that B_0^C is $ 3756.172 bn, the value of reserves V (deposits of depository institutions, $ 2374.633 bn) and currency M (Federal Reserve notes outstanding, net, $ 1194.969 bn).

The policy rule has inflation and interest rate smoothing coefficients θ_π and θ_r of 2 and 1, respectively, which are roughly consistent with those of interest feedback rules in estimated DSGE models (e.g., Del Negro, Schorfheide, Smets, and Wouters (2007); note that $\theta_r = 1$ corresponds to an interest rate smoothing coefficient of .78 for a policy rule estimated with quarterly data). The inflation target θ_π is 2 percent. As in the simple model, we use for transactions costs the functional form (23), which we repeat here for conveninence:

$$\psi(v) = \psi_0 e^{-\psi_1/v}. \tag{23}$$

This transaction cost function implies that the elasticity of money demand goes to zero for very low interest rates, consistently with the evidence in Mulligan and Sala-i Martin (2000) and Alvarez and Lippi (2009). The coefficients ψ_0 and ψ_1 used in the baseline calibration are ψ_0 =.63 and ψ_1 = 103.14. These were obtained from an OLS regression of log r on inverse velocity, which is justified by the fact that under this functional form for the transaction costs the equilibrium condition (12) implies

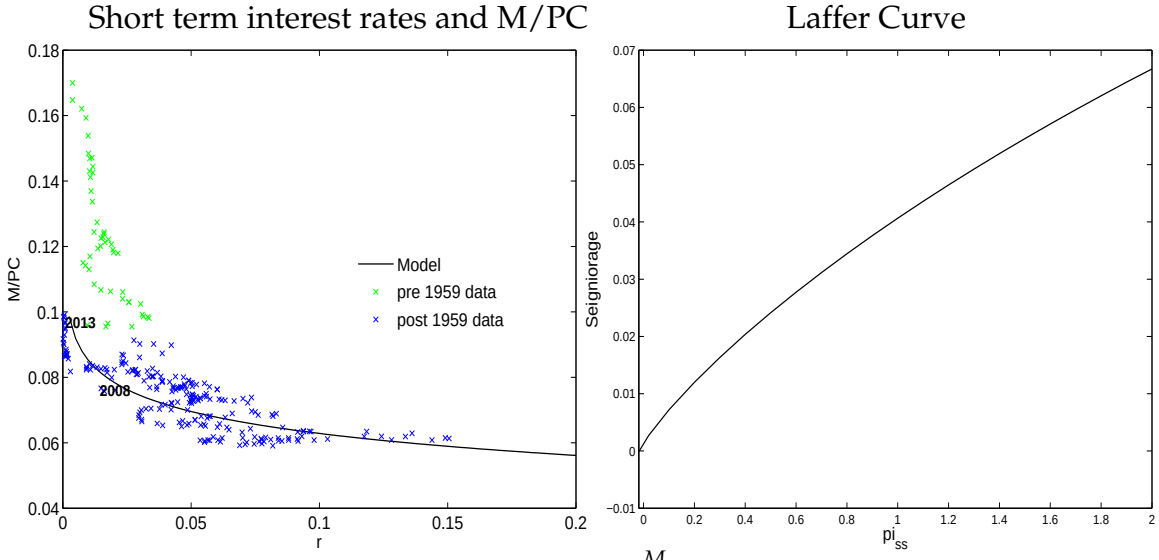
$$\log r = \log(\psi_0\psi_1) - \psi_1 v^{-1}. \tag{37}$$

The left panel of Figure 1 shows the scatter plot of quarterly $\frac{M}{PC} = v^{-1}$ and the annualized 3-month TBill rate in the data (where M is currency and PC is measured by nominal PCE)[12], where blue crosses are post-1959 data, and green crosses are 1947-1959 data, which we exclude from the estimation as they represent an earlier low-interest rate period where the transaction technology was arguably quite different. The solid black curve in the left panel of Figure 1 shows the relationship between inverse velocity and the level of interest rates implied by the model.[13] The right panel of Figure 1 shows the steady state Laffer curve as a function of inflation. The

[12]Data are from *Haver*, with mnemonics `C@USNA`, `FMCN@USECON`, and `FTBS3@USECON` for PCE, currency, and the Tbill rate, respectively.

[13]The implied transaction costs at steady state are negligible - about .04 percent of Y-G.

FIGURE 1. Money Demand and the Laffer Curve



Notes: The left panel shows a scatter plot of quarterly $\frac{M}{PC} = v^{-1}$ and the annualized 3-month TBill rate (blue crosses are post-1959 data, and green crosses are 1947-1959 data) together with relationship between inverse velocity and the level of interest rates implied by the model (solid black line). The right panel shows seigniorage as a function of steady state inflation.

figure shows that under our parameterization seigniorage is still increasing even for inflation rates of 200 percent (eventually money demand and seigniorage go to zero, but this only occurs for interest rates above 6500 percent). We also consider alternative parameterizations of currency demand. Specifically, we run the OLS regression excluding post-2008 data and obtain a substantially lower estimate of ψ_1, implying a greater sensitivity of money demand to interest rates (ψ_1= 48.17, ψ_0 = .03). Figure A-1 in the appendix shows that the Laffer curve under this parameterization appears very different from that in Figure 1, with the Laffer curve peaking at 50 percent interest rates, and money demand going to zero for r above 150 percent.

Finally, we choose χ – the average coupon on the central bank's assets – to be 3.4 percent, roughly in line with the numbers reported in figure 6 of Carpenter, Ihrig, Klee, Quinn, and Boote (2013). Chart 17 of the April 2013 FRBNY report on "Domestic Open Market Operations during 2013"[14]

[14] http://www.newyorkfed.org/markets/omo/omo2013.pdf.

shows an average duration of 6.8 years for SOMA assets (SOMA is the System Open Market Account, which represents the vast majority of the Federal Reserve balance sheet). Accordingly we set $1/\delta$ =6.8.

TABLE 2. Parameters

normalization, foreign assets			
$Y - G$	= 1	$F0$	= 0
initial assets, reserves, and currency			
$\frac{B^C}{P}$	= 0.327	$\frac{V}{P}$	= .207
$\frac{M}{P}$	= .104		
discount rate, reversion to st.st., population and productivity growth			
β	= 0.01	γ	= 0.01
φ_1	= 0.750	n	= 0.0075
monetary policy			
θ_π	= 2	θ_r	= 1
$\bar{\pi}$	= 0.02		
bonds: duration and coupon			
δ^{-1}	= 6.8	χ	= 0.034
money demand			
ψ_0	= .63	ψ_1	= 103.14

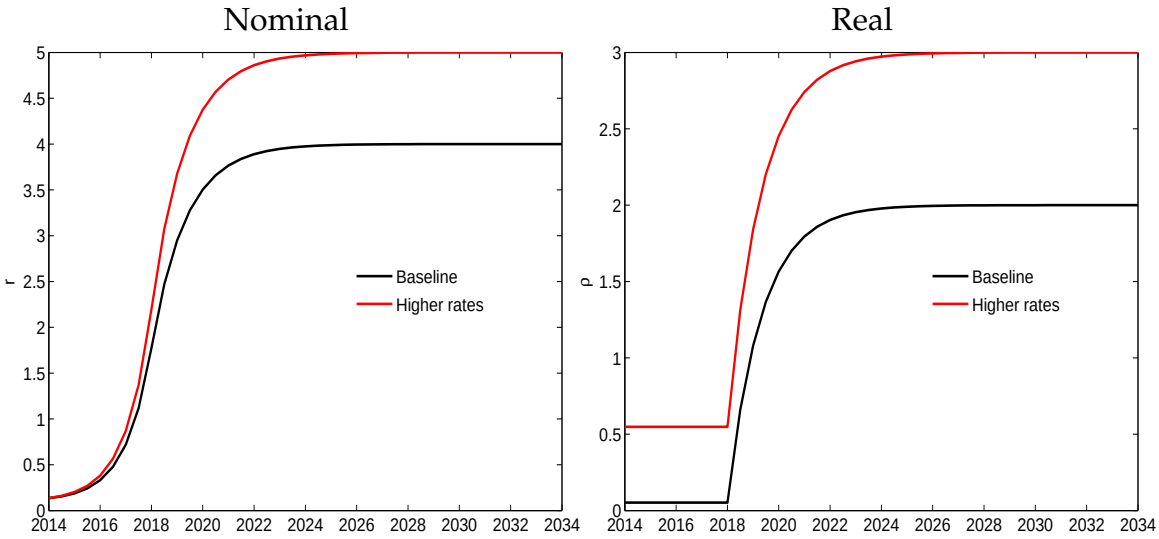
VIII. SIMULATIONS

As a baseline simulation we choose a time-varying path of short term nominal interest rates that roughly corresponds to the baseline interest path in Carpenter, Ihrig, Klee, Quinn, and Boote (2013). We generate this path by assuming that the real rate ρ_t remains at a low level ρ_0 for a period of time T_0 equal to five years, and then reverts to the steady state $\bar{\rho}$ at the rate φ_1:

$$\rho_t = \begin{cases} \rho_0, \text{ for } t \in [0, T_0] \\ \bar{\rho} + (\rho_0 - \bar{\rho}) e^{-\varphi_1 (t - T_0)}, \text{ for } t > T_0. \end{cases} \tag{38}$$

Given the path for ρ_t, equation (15) generates the path for the nominal short term rate (we set $\kappa = 0$ for the baseline simulation). The baseline paths of ρ_t, r_t and inflation π_t are shown as the solid black lines in the three panels of Figure 2.

FIGURE 2. Short term interest rates: baseline vs higher rates



Notes: The panels show the projected path of nominal (left panel) and real (right panel) short term rates under the baseline (solid black) and the "higher rates" (solid red) scenarios.

Given the path for ρ_t and r_t we can compute q and the amount of resources, both in terms of marketable assets and present value of future seigniorage, in the hands of the central bank. The first row of table 3 shows the two components of the left hand side of equation (27), namely the market value of assets minus reserves (column 1) and the discounted present value of seigniorage $\int_0^\infty (\frac{\dot{M}}{M} + n)\frac{M}{P} e^{\int_0^t (\rho_s + x_s - n)ds} dt$ (column 2). The third column shows the sum of the two, which has to equal the discounted present value of remittances $\int_0^\infty \tau^C e^{\int_0^t (\rho_s + x_s - n)ds} dt$. Last, in order to provide information about how the numbers in column 1 are constructed, column 5 shows the nominal price of long term bonds q at time 0.

Under the baseline simulation the real value of the central bank's assets minus liabilities is 14.6 percent of Y-G – which is larger than the difference

TABLE 3. Central bank's resources under different simulations

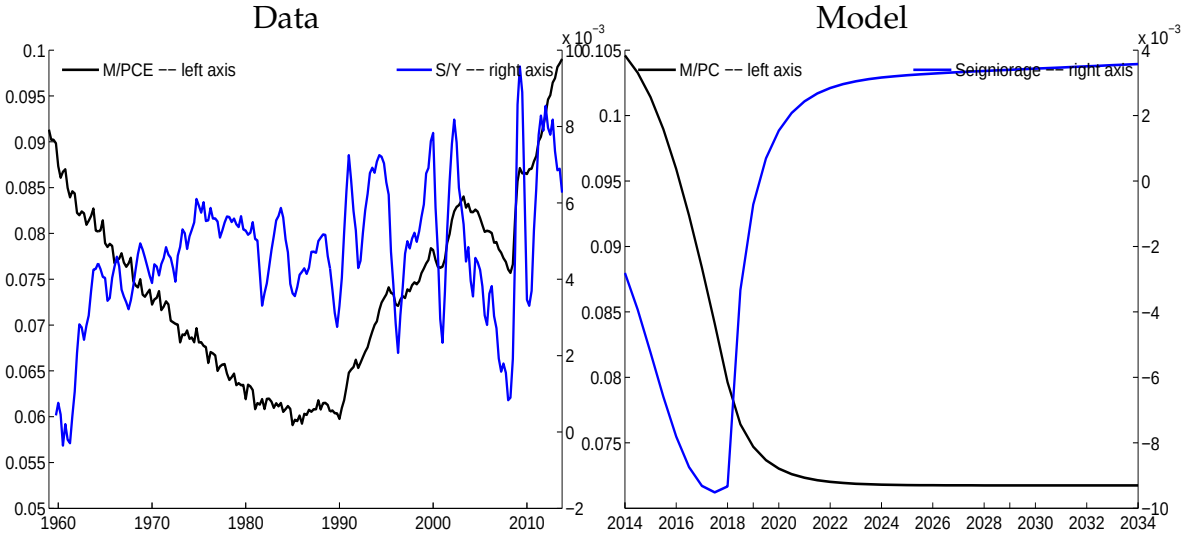
		(1) qB/P $-V/P$	(2) PDV seigniorage	(3) (1)+(2)	(4) q	(5) $\bar{B}/B$
	Baseline calibration					
(1)	Baseline scenario	0.146	1.139	1.285	1.08	
(2)	Higher rates (β)	0.130	0.181	0.311	1.06	12.62
(3)	Higher rates (γ)	0.141	1.443	1.584	1.06	60.23
(4)	Inflation scare	0.028	0.692	0.720	0.85	4.15
(5)	Explosive path	0.069	0.466	0.535	0.85	3.28
	Higher θ_π					
(6)	Inflation scare	0.048	0.599	0.647	0.90	4.54
(7)	Explosive path	-0.010	0.175	0.165	0.61	1.34
	Lower θ_π					
(8)	Inflation scare	-0.070	0.861	0.791	0.47	2.69
(9)	Explosive path	0.135	6.806	6.942	1.05	199.41

between the par value of assets minus reserves reported in table 2 given that q is above one under the baseline. Its value is 1.08, which is above the 1.04 ratio of market over par value of assets reported in Federal Reserve System (2014).[15] The discounted present value of seigniorage is almost an order of magnitude larger, however, at 114 percent of Y-G, and represents the bulk of the central bank resources (and therefore of the present discounted value of remittances), which are 128 percent of Y-G.[16]

[15]Page 23 and 29 shows the par and market (fair) value of Treasury and GSE debt securities, and Federal Agency and GSE MBS, respectively.

[16]Column 4 in Table A-1 in the appendix shows $\bar{\tau}^C$ as defined in equation (28): the constant level of remittances (accounting for the trend in productivity) that would satisfy equation (27), expressed as a fraction of Y-G like all other real variables. That is, the amount $\bar{\tau}^C$ such that $\tau_t^C = \bar{\tau}^C e^{\gamma t}$ satisfies the present value relationship. We find that the constant (in productivity units) level of remittances $\bar{\tau}^C$ that satisfies the present value relationship

FIGURE 3. seigniorage and M/PC



Notes: Paths for seigniorage (solid blue – right axis) and real money balances (solid black – left axis) in the data (left panel) and under the baseline simulation (right panel).

The left and right panels of Figures 3 show inverse velocity M/PC and seigniorage, expressed as a fraction of Y-G, in the data (1980-2013) and in the model (under the baseline simulation), respectively. A comparison of the two figures shows that the drop in M/PC as interest rates renormalize under the baseline simulation (from about .09 to .07, left axis) is roughly as large as the rise in M/PC as interest rates fell from 2008 to 2013. Partly because the model may likely over-predict the fall in currency demand, and more importantly because consumption declines (real interest rates are very low at time 0, inducing unrealistic above trend consumption), seigniorage falls to negative territory for roughly six years. After that, it converges to almost .3 percent of Y-G, a level that is in the low range of the post-1980 observations. For both reasons the numerator in the present discounted value of seigniorage reported in Table 3 for the baseline simulation is likely to be

is .29 percent of Y-G, about $ 34 bn per year, considerably lower than the amount remitted for 2013 and 2012 according to Federal Reserve System (2014) ($ 79.6 and $ 88.4 bn, respectively).

a fairly conservative estimate.[17] Table A-2 in the appendix shows the same quantities of Table 3 obtained under the alternative calibration of money demand. We see that in spite of the differences in the elasticity, the results in terms of central bank's resources are very similar.

Next, we consider alternative simulations where the economy is subject to different "shocks." In each of these simulations all uncertainty is revealed at time 0, at which point the private sector will change its consumption and portfolio decisions and prices will adjust. We will use the subscript 0^- to refer to the pre-shocks quantities and prices (that is, the time 0 quantities and prices under the baseline simulation). For each simulation, Table 3 will report the new market value of assets minus reserves in real term $(q_0 \frac{B^C_{0^-}}{P_0} - \frac{V_0}{P_0})$. By assumption the central bank will not change its assets $B^C_{0^-}$ after the new information is revealed, but the private sector will change its time 0 currency holdings given that interest rates may have changed. This necessarily leads to a change in reserves (given that central bank's assets are unchanged) equal to $\frac{V_0 - V_{0^-}}{P_0} = -\frac{M_0 - M_{0^-}}{P_0}$ in real terms (we report this quantity in column 6 of Table A-1 in the appendix).

For each scenario we also report the level of the balance sheet $\bar{B}^C$ such that, for any balance sheet size larger than $\bar{B}^C$, the present discounted value of remittances (see equation (27)) becomes negative after the shock. We refer to this situation as the central bank becoming "insolvent", in the sense that it needs resources from the fiscal authority because it suffered losses due to

[17] The left panel of Figure A-3 in the appendix shows remittances (computed as described in section B) under two scenarios for the path of assets B^C: in the first scenario (solid line) the central bank lets its assets depreciate, while in the second one it actively sells assets at a rate of 20 percent per year. These scenarios are both obviously unrealistic, since we know that the size of the balance sheet increased since the end of 2013, but highlight the fact that different paths for the balance sheet can imply different paths for remittances, even though their expected present value remains the same (this is the dotted line in Figure A-3, which shows $\bar{\tau}^C e^{\gamma t}$).

the fall in q. Specifically, assume the central bank expands its balance sheet by ΔB^C at time 0^- (right before the shock takes place) by buying assets at price q_{0^-} and pays for its purchases by expanding reserves by an amount $\Delta V = q_{0^-}\Delta B^C$. How large can ΔB^C be to still satisfy

$$\frac{q_0\left(B^C + \Delta B^C\right) - V - \Delta V}{P_0} + \frac{M_0 - M_{0^-}}{P_0} + \int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t} e^{-\int_0^t (\rho_s + x_s - n)ds} dt \geq 0 \quad (39)$$

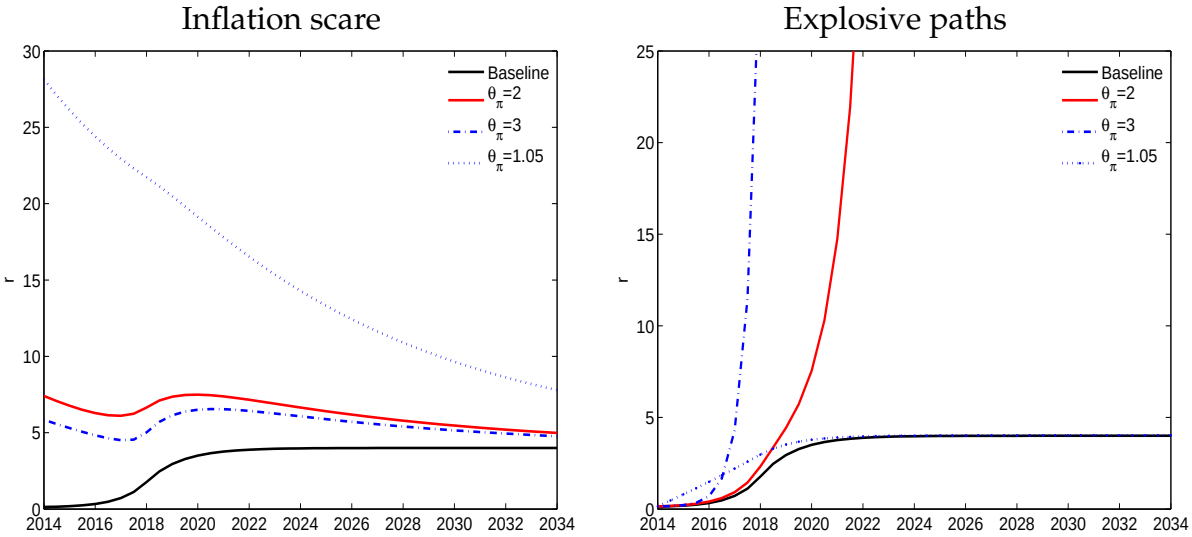
after the "shock"? We report $\bar{B}/B^C = 1 + \dfrac{\Delta B^C}{B^C}$, where B^C is the 2013Q4 level of the balance sheet reported in Table 2.

The first alternative scenario we study is a "higher rates" path similar to one considered by Carpenter, Ihrig, Klee, Quinn, and Boote (2013). Under this new path real rates converge to a 1 percent higher steady state, and so will short term nominal rates given that the central bank inflation target has not changed. We choose the new starting value for ρ, ρ_0, so that the initial rate remains at 13.5 basis points. The red solid lines in the two panels of Figure 2 show the "Higher Rates" paths for the nominal and the real short term rates, respectively. In these simulations we assume that the central bank recognizes the change in the steady state $\bar{\rho} = \beta + \gamma$, and adjusts its Taylor rule coefficient $\bar{r} = \bar{\rho} + \bar{\pi}$ accordingly.

We consider two different reasons why the new steady state ρ is higher: a higher discount rate β and a higher growth rate of technology γ. While the new value of q is the same in both cases (the interest rate path is the same), the present value of seigniorage shown in column 2, and therefore the present value of remittances shown in column 3, is quite different. In the high β case the current value of the future income from seigniorage falls by almost one order of magnitude, as future seigniorage is discounted at a higher real rate. In the high γ case the economy is growing faster, and so does money demand and future seigniorage. Table A-1 in the appendix

shows that in both cases (higher β and higher γ) the level of $\bar{\tau}^C$ is higher than in the baseline case. Carpenter, Ihrig, Klee, Quinn, and Boote (2013) take seigniorage as given and focus on the effect of the higher nominal interest rates on the value of the central bank's assets qB^C, which falls following the drop in q. The effect of the higher real rate of return on future central bank's revenues and, especially in the high γ case, on future seigniorage, trumps in our simulation the negative effect on q. [18]

FIGURE 4. "Inflation scare" and "explosive paths" scenarios: The effect on short terms rates under different inflation responses in interest rate rule



Notes: The panels show the projected path of nominal short term rates for the "inflation scare" (left panel) and the "explosive path" scenario (right panel, with $\kappa = 10^{-4}$) under different inflation responses in interest rate rule (solid red: θ_π=2, dash-and-dotted blue: θ_π=3; dotted blue: θ_π=1.05) together with the baseline projections (solid black).

Next, we consider simulations where the private sector is concerned about a sudden jump in the price level, and therefore demands a premium x_t for holding nominal bonds as described in section II ("inflation scares"

[18] This may seem surprising in the higher β case since the present value of seigniorage is lower than under the baseline simulation. However, the central bank is now earning a higher return on its assets, and can therefore afford a higher level of remittances.

scenarios). We assume that this premium follows the process

$$x_t = x_0 e^{-\chi_x t}, \tag{40}$$

with $x_0 = .04$ and $\chi_x = .1$.[19] The red solid line in the left panel of Figure 4 shows the path of the short term nominal interest rates under this scenario, which is higher than under the baseline because the higher inflation expectations force the central bank to raise rates (the corresponding paths for inflation are shown in Figure A-4 in the appendix). Row 4 of Table 3 shows the effects of this scenario on the central bank's balance sheet. The market value of assets minus reserves $q_0 \frac{B^C_{0^-}}{P_0} - \frac{V_0}{P_0}$ drops to about one fifth its baseline value, both because q falls and because a higher fraction of the central bank's liabilities becomes interest bearing relative to the baseline scenario. This happens because the private sector turns currency into reserves, driven by the higher opportunity cost of holding currency. Under our assumptions on money demand, the present discounted value of seigniorage, while lower than in the baseline case, is still sizable, and so is the present value of resources in the hands of the central bank under this scenario. As a consequence, even with a much larger balances sheet (more than four times as large) the central bank could have withstood the fall in the value of its assets without ever needing any resources from Fiscal Authority. Table A-2 in the appendix shows that these results are robust to the parameterization of money demand.

[19] Equation (14) shows that a change in x is isomorphic to changes in inflation target $\bar{\pi}$ or the constant in the Taylor rule $\bar{r}$ in terms of the path for r. Hence this "inflation scare" scenario can be alternatively thought of as resulting from the monetary authorities temporarily raising the inflation target, or mis-judging the real rate in the economy (e.g., Orphanides (2002)). However, relative to these scenarios the inflation scare implies a lower path of inflation, as therefore less seigniorage *ceteris paribus*, as equation (11) implies that there is a premium x between the nominal and the real rate r and ρ.

The quantitative results are sensitive to the inflation response in the policy reaction function. The blue dash-and-dotted and dotted blue lines in the left panel of Figure 4 show the interest rate path corresponding to an inflation coefficient θ_π of 3 and 1.05, respectively.[20] As is usually the case in stable rational expectations equilibria, a higher inflation coefficient in the interest rate rule induces a lower equilibrium response of inflation, and therefore a lower equilibrium response of interest rates – and vice versa when the inflation response is lower. When θ_π is 1.05, interest rates reach almost 30 percent. Consequently, q falls to less than half its value in the baseline scenario, and the market value of assets minus reserves $q_0 \frac{B^C_{0^-}}{P_0} - \frac{V_0}{P_0}$ falls to negative levels (see row 8 of Table 3). The implication of this finding is that under a large balance sheet the central bank may want to respond more aggressively to inflation if it is concerned about fluctuations in the values of its assets.

Even in the $\theta_\pi = 1.05$ case central bank's solvency is not an issue, however. The central bank's overall resources (column 3) are still sizable, because the higher inflation experienced under the lower θ_π policy yields greater seigniorage (column 2). In fact, the present value of remittances would remain positive even if we assumed the central bank balance sheet to be more than twice as large as the current one (column 5).

Finally, we consider explosive paths where κ in equation (15) is different from zero. The solid red line in the bottom right panel of Figure 4 shows one of these paths (with $\kappa = 10^{-4}$) under the baseline policy response. Given the rise in r_t under this scenario, q drops substantially relative to the baseline (row 5 of Table 3). The present discounted value of seigniorage also falls relative to the baseline because seigniorage goes to zero as rates become larger than 6500 percent. But it is still large enough that even with a

[20] In these simulations we change the time 0 real rate so that under the baseline scenario the nominal rate is still 13.5 basis points.

balance sheet more than three times as large as B_0^c the central bank would be solvent. This is one case where using the alternative parameterization of money demand makes a difference, however. Table A-2 in the appendix shows that under explosive paths the central bank would not be solvent under the current size of the balance sheet, which is not surprising since under this parameterization the peak of the Laffer curve is crossed at interest rates around 50 percent.

The dash-and-dotted and dotted blue lines in the bottom right panel of Figure 4 show the responses under different θ_π coefficients. In the case of unstable solutions, the inflation response coefficient in the interest rule plays the opposite role relative to the stable solution case (see Cochrane (2011)): the stronger the response, the faster inflation and interest rates explode. The market value of central bank's assets q surely falls more with a higher θ_π, and indeed $q_0 \frac{B^C_{0^-}}{P_0} - \frac{V_0}{P_0}$ falls to negative levels. The present discounted value of seigniorage also falls by more than in the $\theta_\pi = 2$ case, but not by enough to call central bank's solvency in question under these paths.

IX. Self-fulfilling solvency crises

As we have already observed, a central bank cannot guarantee determinacy of the price level in the absence of fiscal backing. Our detailed scenarios in the previous sections have all assumed (except in the $\kappa > 0$ cases) that this backing was present. But even when the backing is present, a central bank that is firmly committed to not accepting (or incapable of drawing on) fiscal support, in the sense of capital injections from the treasury, can create indeterminacy in the price level. The problem is that commitment to a policy rule that stabilizes inflation, like a Taylor rule with large coefficient on inflation, may under certain conditions require a capital injection from the treasury to be sustainable. If the central bank, to avoid the capital injection,

switches policy so as to generate more seigniorage, multiple non-explosive equilibria can arise. We give examples of this possibility in this section.

What if the public believes that, were the central bank to face the issue of solvency, it would resort to seigniorage creation? Entertaining this possibility would then lead the public to expect higher future inflation and nominal interest rates. These expectations would result in a lower value of long term nominal assets today, so that the central bank's assets qB^C could become worth less than its interest bearing liabilities V. If the current present discounted value of seigniorage is not large enough to cover this gap, the central bank may have to resort to raising more seigniorage, thereby validating the initial belief. The larger is the size of the central bank's balance sheet, and the longer its duration, the larger is the gap in $qB^C - V$ that would arise because of future expected inflation, and the likelihood of these alternative equilibria.[21]

We suppose that the central bank would like to set the inflation target equal to 2 percent and keep it there, as long as this is feasible without any recapitalization. We have already verified that if it chooses this level of inflation and the public's expectations align with this choice, the central bank solvency constraint is slack (see Table 1) — it will never require recapitalization. But the public may instead believe that the central bank will be

[21] There is a connection between self-fulfilling equilibria in this paper and the literature on currency crisis (e.g., Burnside, Eichenbaum, and Rebelo (2004)) and debt crisis (especially Calvo (1988)). In Burnside, Eichenbaum, and Rebelo (2004), government guarantees lead to the possibility of self- fulfilling speculative attack on the exchange rate regime. In this paper, the presence of long-duration assets in the central bank's balance sheet makes it vulnerable to expectations of higher future inflation, and the lack of fiscal support makes these expectations self-fulfilling because it has to resort to seigniorage. Similarly, a large, long-duration balance sheet in this model plays the same role as a large outstanding amount of debt in Calvo, in that it "may generate the seeds of indeterminacy; it may, in other words, generate a situation in which the effects of policy are at the mercy of people's expectations ... "(Calvo (1988), pg. 648).

forced by balance sheet considerations to push inflation, and thus seigniorage, higher. To keep the analysis simple in this proof-of-concept example, we suppose that there are just two possible expected inflation target paths $\bar{\pi}_t$, and two possible policy choices. Our task is to demonstrate that with the same initial balance sheet, there can in fact be two such policy/expectation pairs, in both of which inflation expectations are perfectly accurate, and in one of which the solvency constraint binds.

By definition, when the solvency constraint binds we have that

$$\frac{q_0(\bar{\pi}_t)B^C_{0^-} - V_0}{P_0(\bar{\pi}_t)} + PDVS_0(\bar{\pi}_t) = 0, \tag{41}$$

where $PDVS_0(\bar{\pi}_t) = \int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t}e^{-\int_0^t(\rho_s - n)ds}dt$: under these paths the value of assets declines and seigniorage increase just enough to compensate for the balance sheet losses. If 1) condition (41) is satisfied, 2) $\bar{\pi}_t \geq .02$ for all t, and 3) any deviation $\bar{\pi}^*_t < \bar{\pi}_t$ for some t implies $PDVS_0(\bar{\pi}^*_t) < PDVS_0(\bar{\pi}_t)$, the these paths are equilibria: the central bank would like to deviate in favor of a lower target, but such deviations would violate solvency.

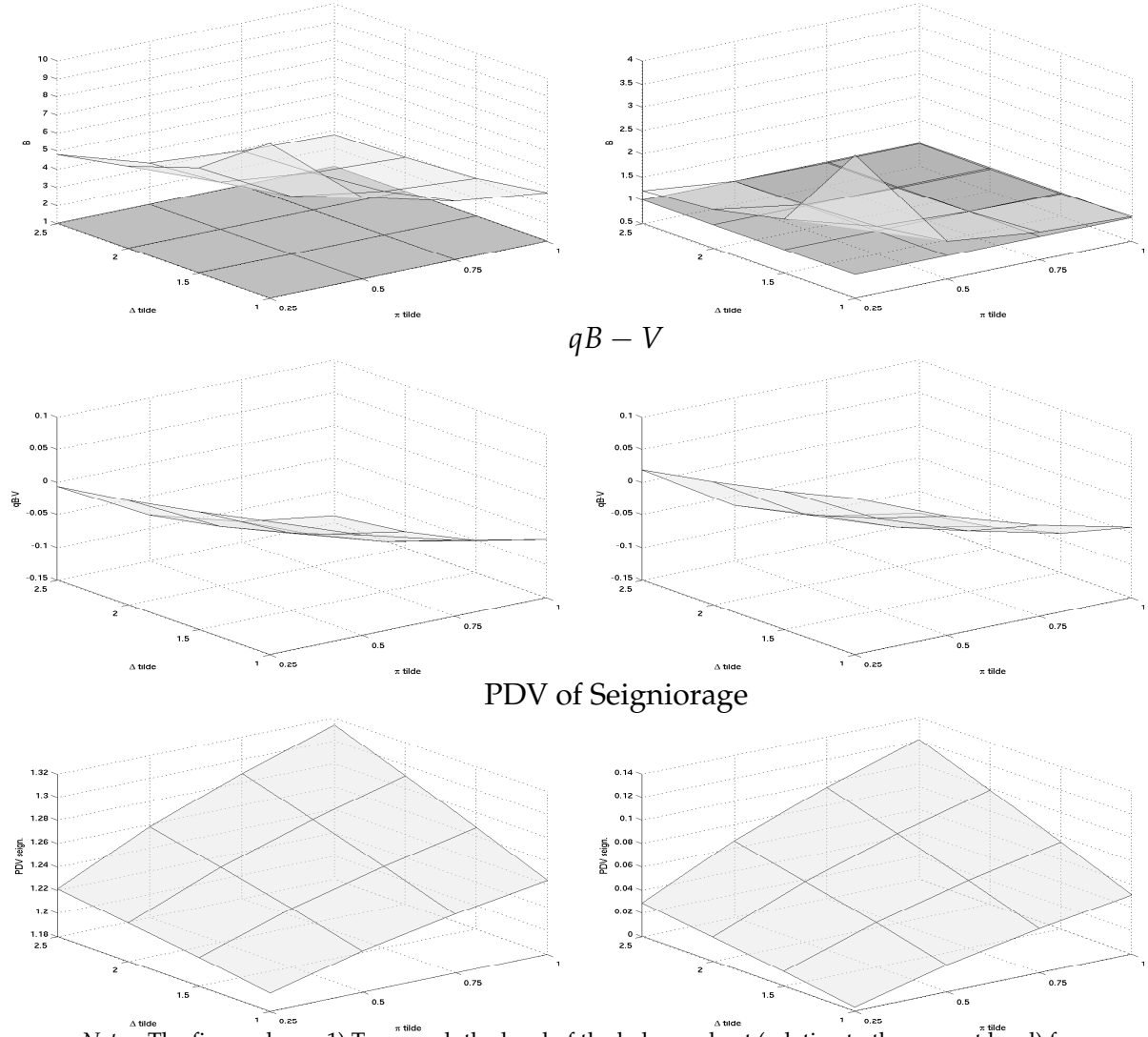
If there is the possibility of indeterminacy, these multiple equilibria can take many forms. We focus on a particular type of multiple equilibria, where agents expect that at time $t = \tilde{T}$ the central bank will change its inflation target to $\tilde{\pi}$ for a period $\tilde{\Delta}$, and revert to the old rule with inflation target $\bar{\pi}$ afterwards (for $t > \tilde{T} + \tilde{\Delta}$). The appropriately modified version of equation (15) provides the solution for the future path of interest rates. Given the path for r_t we can solve for all other endogenous variables exactly as in the model above. We can in particular obtain, under this alternative equilibrium, the value of long term assets $q_0(\tilde{\pi})$, the initial price level $P_0(\tilde{\pi})$, and the present discounted value of seigniorage in real terms at time 0, which we can call $PDVS_0(\tilde{\pi}) = \int_0^\infty (\frac{\dot{M}_t}{M_t} + n)\frac{M_t}{P_t}e^{-\int_0^t(\rho_s - n)ds}dt$.

Those triplets $(\tilde{\pi}, \tilde{T}, \tilde{\Delta})$ for which equation (41) is satified are possible self-fulfilling solvency crises, in the sense that the expectation that the central bank will switch to a new rule with target $\tilde{\pi}$ will produce a gap in the value of central bank's assets minus liabilities $\frac{q_0(\tilde{\pi})B^C_{0^-} - V_0}{P_0(\tilde{\pi})}$ that will have to be filled with future seigniorage $PDVS_0(\tilde{\pi})$. In order to generate this future seigniorage the central bank will have to validate the public expectations and switch temporarily to the rule with higher inflation target.

Alternatively, for given $(\tilde{\pi}, \tilde{T}, \tilde{\Delta})$, we find the minimum level of the balance sheet B^C for which equation (41) has a solution. The top panel left panel of Figure 5 shows this minimum balance sheet level (relative to the current level) as a function of $\tilde{\pi}$ and $\tilde{\Delta}$, with $\tilde{T}$ set to 1 year. For now, we focus on stable rational expectations solutions, that is, we set $\kappa = 0$ in equation (15). The figure shows that under the baseline money demand calibration, these threshold balance sheet limits are much larger than the current one ($\bar{B}/B > 1$). The left middle and bottom panels of Figure 5 explain why this is the case. The middle panel shows what happens to $\frac{q_0(\tilde{\pi})B^C_{0^-} - V_0}{P_0(\tilde{\pi})}$ in the alternative equilibrium under the *current* balance sheet size. The figure shows that for large enough $\tilde{\pi}$ and duration $\tilde{\Delta}$ the real value of assets minus interest bearing liabilities does become significantly negative. The bottom panel of Figure 5 show that for these values the level of seigniorage $PDVS_0(\tilde{\pi})$ overshadows this balance sheet loss, however. Hence the results in the top panel: the size of the balance sheet would have to be much larger than the current one for the balance sheet loss to be of the same size of the increase in seigniorage. In other words, under the current level of the balance sheet, the type of alternative equilibria we consider cannot arise because the increase in seigniorage triggered by the temporary higher inflation regime is larger than the balance sheet loss caused by the fall in q.[22] The Laffer curve in the

[22]We searched for alternative values of $\tilde{T}$ as well, and the results are not very different.

FIGURE 5. Self-fulfilling solvency crises

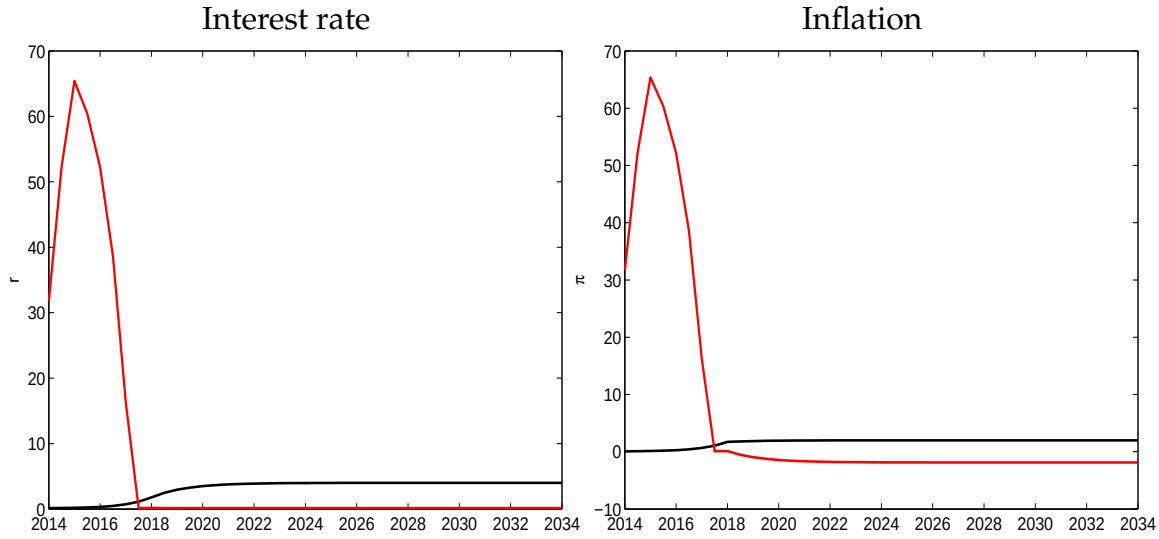


Notes: The figure shows 1) Top panel: the level of the balance sheet (relative to the current level) for which multiple equilibria are possible; 2) Middle panel: the level of $qB - V$ as a fraction of income for the current balance sheet size under alternative scenarios; 3) Bottom panel: the level of seigniorage as a fraction of income under alternative scenarios; as a function of inflation in there alternative regime ($\tilde{\pi}$), and the duration of the alternative regime ($\tilde{\Delta}$). In all simulation the alternative regime is expected to start after 1 year ($\tilde{T}$=1). The left and right figures are for $\kappa = 0$ (stable solution) and $\kappa < 0$ (downward unstable solutions), respectively. .

right panel of Figure 1 shows why the increase in seigniorage is so large under the baseline parameterization of money demand: even for large interest rates seigniorage grows almost linearly with inflation.[23]

[23]One caveat to these simulations is that we maintain the hypothesis of passive fiscal policy even under high inflation rates, which may not be realistic.

FIGURE 6. Short term interest rates and inflation under multiple equilibria with $\kappa < 0$



Notes: The panels show the projected path of nominal short term interest rates (left panel) and inflation (right panel) under the baseline scenario (solid black) and under one of the multiple equilibria scenario (solid red) obtained by setting $\tilde{T}$=1, $\tilde{\Delta}$= 2.5, $\tilde{\pi}$ = .75, and $\kappa < 0$. The value of κ so that the zero lower bound $\underline{r}$ is hit at time $t = \tilde{T} + \tilde{\Delta}$.

The panels on the right side of Figure 5 explore the situation where we allow for non stationary solutions to equation (15), and in particular we consider the case κ <0. Under this case, the interest rate is bound to eventually hit the lower limit $\underline{r}$ and to remain there forever after (in absence of the appropriate fiscal policies alluded to in section III). For each simulation, we choose κ so that the zero lower bound $\underline{r}$ is hit at time $t = \tilde{T} + \tilde{\Delta}$. Figure 6 shows the paths of interest rates and inflation associated with one such simulation (with $\tilde{T}$=1, $\tilde{\Delta}$= 2.5, and $\tilde{\pi}$ = .75). The figure shows that interest rates and inflation first rise because of the temporarily high inflation target, and then drop to $\underline{r} = .00135$ as the downward explosive root takes over. These paths are a particularly bad combination from the perspective of central bank's solvency: the initial rise in interest rates takes a toll on the value of the balance sheets assets, and the subsequent fall to $\underline{r}$ implies that

seignorage for $t \geq \tilde{T} + \tilde{\Delta}$ is very low (in fact, slightly negative since we have deflation).

The middle and bottom right panels of Figure 5 show that this is indeed the case: the value of $\frac{q_0(\tilde{\pi})B^C_{0^-} - V_0}{P_0(\tilde{\pi})}$ is only slightly higher than that obtained when κ =0 (q_0 is slightly higher in the κ <0 case as interest rates decline faster than in the κ = 0 case), and mostly negative. The present discounted value of seigniorage is much smaller in the κ <0 case, and is no longer enough to compensate for the decline in the value of the asset. As a consequence, for many of the simulations considered in Figure 5 the central bank is no longer solvent for the levels of the balance sheet reached at the end of 2013, as shown in the top right panel. The locus of points where the $\bar{B}/B$ contour intersects the dark plane marking the threshold $\bar{B}/B$=1 are equilibria under the end-of-2013 level of the central bank's balance sheet. As one can see, there are many such equilibria.[24]

The multiplicity considered in this section can be eliminated with fiscal support for the central bank's balance sheet such as the arrangement between the Bank of England and Her Majesty's Treasury, whereby all gains and losses incurred by the former as part of its asset purchase facility are transfered to the latter (see McLaren and Smith (2013)). In this model, the mere presence of fiscal support eliminates the multiplicity, without any need for actual support in equilibrium.

X. Conclusions

The large balance sheet of many central banks has raised concerns that they could suffer significant losses if interest rates rose, and might need a capital injection from the fiscal authority. This paper constructs a simple deterministic general equilibrium model, calibrated to US data, to study

[24]Figure A-5 in the appendix shows that under the alternative specification of the money demand function we reach similar conclusions.

the the impact of alternative interest rates scenarios on the central bank's balance sheet. We show that the central bank's policy rule (or, equivalently, inflation objectives), and the behavior of seigniorage under high inflation are crucial in determining whether a capital injection might be needed. We show also that a central bank that is seen as ready, in order to avoid a need for capital injection, to create seignorage by altering its inflation objectives, can thereby lose control of the price level.

We conclude that for balance sheet levels similar to the current ones of the Federal Reserve system, direct treasury support would be necessary only under what seem rather extreme scenarios. However this result depends on assumptions about demand for non-interest bearing Fed liabilities (mainly currency) that cannot be firmly grounded in empirical estimates. Higher balance sheet levels, or a lower currency demand than assumed here may force the central bank to request a capital injection in order to maintain its inflation control policy.

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APPENDIX A. PLAUSIBLE FISCAL RULES THAT DELIVER UNIQUENESS

It is easy to specify a fiscal and monetary policy pair that delivers uniqueness of the price level. For example monetary policy that pegs the interest rate at some fixed level $\bar{r}$ and the primary surplus at some fixed positive level $\bar{\tau}$. With such a combination of policies in the simple model, there is a stable equilibrium with constant inflation (zero inflation if $\bar{r} = \rho$). There are unstable solution paths for the model's differential equations, but on these unstable paths the level of real debt B/P grows exponentially upward at the

rate ρ or declines in finite time to hit zero, while inflation remains constant. In the former case, the private sector accumulates real wealth in the form of B/P while its consumption remains bounded. This cannot be optimal behavior. Formally, it is ruled out by what is called a "transversality condition". It is easy to understand that if the economy started on such a path, private sector agents would try to spend their exploding wealth, pushing the price level up. The resulting inflation would bring the level of real debt back down to the level consistent with equilibrium. In the latter case, of debt shrinking to zero in real value, agents would see their wealth steadily declining, while their tax obligations remained constant in real terms. Assuming they understand that the government will not lend to them, at least not in indefinitely large amounts, they will see their consumption as out of line with their wealth and income and will try to increase saving. This will create deflationary pressure and bring wealth back up to the level consistent with the stable equilibrium.

This type of equilibrium, with interest rate policy not keeping pace with inflation, primary surpluses not reacting to the level of real debt, are a standard result in this type of model, but the policies they assume do not look like monetary and fiscal policy in most economies. With the types of policies usually assumed (like those in the main text of this paper), the upward explosive solutions ($\kappa > 0$) do not increase or decrease private wealth. That remains stable because of the fiscal rule. These paths make the price level rise at a more rapid than exponential rate, and may even send it to infinity (i.e. a state of valueless money) in finite time. But along these paths private agents see no reason to change their behavior. In this paper's models, any initial price level above that consistent with the $\kappa = 0$ stable solution corresponds to one of these explosive equilibrium paths.

The downward explosive solutions correspond to initial price levels below that consistent with the stable solution. They, too, leave private wealth

stable. However, because of the zero lower bound, they do not imply more than exponentially downward explosion of the price level. Each initial price level below the stable-equilibrium one corresponds to a path of interest rates that declines steadily and hits the zero lower bound in finite time — earlier, the lower the initial price level. Once there, as we have assumed, monetary policy simply keeps $r = 0$. This implies, with fixed ρ, that prices decline at the constant exponential rate $-\rho$.

The upward explosive solutions are clearly undesirable — they increase transactions costs as real balances dwindle away, yet real balances in the model have no resource cost. The deflationary solutions may seem not so bad. Once $r = 0$ is reached, demand for money is satiated, transactions costs are minimized, and Friedman's "optimum quantity of money" as been achieved. Of course in a model with downward nominal rigidities, this could still be a problem. But more importantly, the indeterminacy of the price level in this deterministic model would correspond to the existence of "sunspot equilibria", continual arbitrary random fluctuations in the price level, in any stochastic version of the model. In any economy with nominal contracting, such fluctuations would be costly.

So it is desirable to find policies that guarantee a unique price level, and to be realistic we should find ones that let monetary policy look like a standard Taylor rule with $\theta_\pi > 1$ in the neighborhood of the steady state. The way the $r = \bar{r}$, $\tau = \bar{\tau}$ policy guarantees uniqueness is by making the private sector understand that when the price level is too low, real government debt becomes greater than the discounted present value of future taxes, giving them room to increase spending, and when the price level is too high, their debt holdings fall below their tax obligations, requiring them to reduce spending. Any fiscal policy with this characteristic will guarantee uniqueness of the price level.

On a path with upwardly explosive inflation, Taylor rule monetary policy, and standard "passive" fiscal policy, real debt tends steadily toward a finite steady-state value despite the explosive inflation. This means that this fiscal policy requires upwardly explosive nominal deficits to keep real debt stable despite inflation's tendency to shrink it. Any fiscal policy that commits to keeping nominal deficits smaller than this, by increasing taxes (or shrinking spending) as inflation grows, at least when it grows above some critical level, will make the explosive price path unsustainable. Such a policy will make people realize that whenever prices rise above the stable-equilibrium level, the future path of prices implied by the explosive paths will make discounted future primary surpluses (and hence tax obligations) rise above the current real value of the debt.

That budget deficits might be cut (and primary surpluses thereby increased) during an explosive inflation is eminently plausible. Policy to eliminate the deflationary indeterminacies is not quite so obvious. A fiscal policy that makes τ respond linearly, even with a tiny positive coefficient, to the inflation rate, will eliminate the upward explosions. But the downward paths do not explode; they just settle into a constant deflation rate. A positive coefficient on inflation in the rule setting τ only increases the steady state level of real debt as deflation proceeds. A stronger reaction is required to eliminate the deflation. For example, a fiscal policy that responds, even with a tiny positive coefficient, to deviation of the price *level* from its steady state value would work. It would imply eventually increasing primary deficits as the price level declined, which would achieve the result that, whenever the price level started out too low, people would see that the deflationary path puts their discounted tax obligations below their government bond wealth, leading them to spend and extinguish the deflationary path.

There is a lesson here for the advanced economies that have recently struggled with long periods of slow growth and low inflation or deflation. This is an expected result if fiscal policy is not committed to counter deflation with low or negative primary surpluses, maintained until the deflation disappears. While it seems likely that policy makers understand the need for fiscal discipline in the face of inflation, recent history suggests they do not understand the need for a strong commitment to fiscal expansion in the face of deflation.

APPENDIX B. RULE FOR REMITTANCES

The central bank is assumed to follow a rule for remittances, which embodies two principles: i) remittances cannot be negative, ii) whenever positive, remittances are such that the central bank capital measured at historical costs remains constant in nominal terms over time, that is:[25]

$$\tilde{K} = \left(\tilde{q}B^C - V - M\right) e^{nt} = \text{constant}. \tag{42}$$

The historical price $\tilde{q}$ evolves according to

$$\dot{\tilde{q}} = (q - \tilde{q}) \max\left\{0, \frac{\dot{B}^C}{B^C} + \delta + n\right\} \tag{43}$$

where the max operator is there because $\tilde{q}$ changes only if the central bank is acquiring assets (recall that bonds depreciate at a rate δ and that B^C is defined in per capita terms, so that $\dot{B}^C = -(\delta + n)B^C$ implies that the central bank is letting its assets mature). Differentiating condition (42) above and using the central bank's budget constraint (26), one obtains a condition for nominal remittances:

$$P\tau^C = (\chi - \delta(\tilde{q} - 1))\, B^C + \left(\dot{\tilde{q}} - (q - \tilde{q})\left(\frac{\dot{B}^C}{B^C} + \delta + n\right)\right) B^C - rV. \tag{44}$$

This condition resembles closely the accounting practice of central banks. The first term, $(\chi - \delta(\tilde{q} - 1))\, B^C$, measures coupon income χ net of the

[25] Hall and Reis (2013) use a similar rule, but measure capital at market prices.

amortization of historical costs $\delta(\tilde{q}-1)$, times the par value of bonds B^C. The second term equals the realized gains/losses, $-(q-\tilde{q})\left(\frac{\dot{B}^C}{B^C}+(\delta+n)\right)B^C$, from assets sales (that is, $\frac{\dot{B}^C}{B^C} \leq -(\delta+n)$), since in this case $\dot{\tilde{q}}=0$. When the central bank is acquiring assets ($\frac{\dot{B}^C}{B^C} > -(\delta+n)$) this second term is zero because $\dot{\tilde{q}}$ and $(q-\tilde{q})\left(\frac{\dot{B}^C}{B^C}+\delta+n\right)$ cancel each other out. The last term, rV, captures the interest paid on reserves. Whenever net income (the left hand side of equation 44) is negative, the central bank would have to extract negative remittances from the fiscal authority to keep its capital constant. If it cannot do this, its capital declines. Whenever internal accounting rules prevent capital from declining a deferred asset is created. Remittances remain at zero until this deferred asset is extinguished (i.e., capital is back at the original level). Hence our rule for remittances is

$$\tau_t^C = \max\left\{0, (\chi-\delta(\tilde{q}-1))\frac{B^C}{P} + \left(\dot{\tilde{q}}-(q-\tilde{q})\left(\frac{\dot{B}^C}{B^C}+(\delta+n)\right)\frac{B^C}{P}\right) - r\frac{V}{P}\right\}\mathcal{I}_{\{\tilde{K}\geq\tilde{K}_0\}}, \quad (45)$$

where $\mathcal{I}_{\{\tilde{K}\geq\tilde{K}_0\}}$ is an indicator function equal to one only if current capital $\tilde{K}$ is at least as large as initial capital $\tilde{K}_0$ (that is, the deferred asset has been extinguished). In practice central bank's capital will not be constant over time, but will likely grow along with nominal income. This implies a net influx of resources for the central bank. At the same time a fraction of net income is devoted to pay dividends on this capital.[26] Moreover the central bank

[26] In the U.S. the central bank's capital is a fixed fraction of the capital of the member banks, and dividends are 6% of capital (see Carpenter, Ihrig, Klee, Quinn, and Boote (2013)). Note also that according to our notation dividends are included in τ^C, this quantity being the total amount of resources leaving the central bank in any given period. In this sense referring to τ^C as "remittances" to the fiscal authority is not entirely appropriate.

also has operating expenses. We ignore these issues in computing the simulated path of remittances since it would further complicate the description of the remittance rule, and also quantitatively they are not very important in terms of the simulated path for remittances.

In order to compute the path for remittances implied by expression (45) we need to compute paths for $\frac{B^C}{P}$ and $\frac{V}{P}$. For the former, we make the following assumptions about the path of the central bank's assets B^C:

$$\frac{\dot{B}^C}{P} = \begin{cases} -(\delta+n)\frac{B^C}{P} & (1) \\ -(\delta+n+s)\frac{B^C}{P} & (2) \end{cases}, \text{ for } t \leq \tilde{T}, \tag{46}$$

where $\tilde{T}$ is the time when the size of reserves has reached the early 2008 level (adjusted for inflation, population and productivity growth), after which $\frac{B^C}{P}$ grows with productivity (i.e., $\frac{B^C}{P}e^{-\gamma t}$ is constant over time, yielding $\frac{\dot{B}^C}{P} = (\gamma + \frac{\dot{P}}{P})\frac{B^C}{P}$). Under assumption (1) the central bank lets its holdings of government debt mature, while under assumption (2) it sells its assets at a rate s per year (we set $s = .2$). Neither assumption is realistic in the case of the U.S. (B^C has increased in 2014!) but the point is to show that different future paths for sales can yield quite different paths for τ^C in the short run, even though the present value of resources remitted to the fiscal authority $\bar{\tau}^C$ is the same. Given remittances and the path for $\frac{B^C}{P}$ we use the budget constraint (26) to compute the evolution of reserves in real terms:

$$\begin{aligned} \left(\frac{\dot{V}}{P}\right) = (r - n - \frac{\dot{P}}{P})\frac{V}{P} \\ - (\chi + \delta - (n+\delta)q)\frac{B^C}{P} + q\frac{\dot{B}^C}{P} - \left(n + \frac{\dot{M}}{M}\right)\frac{M}{P} + \tau^C. \end{aligned} \tag{47}$$

A legitimate question (also posed by Hall and Reis (2013)) is whether a rule like (45) keeps the central bank's capital measured at market prices,

namely

$$K = \left(qB^C - V - M\right) e^{nt}. \tag{48}$$

stationary. Using the budget constraint (26) we write the evolution of detrended capital in real terms ($\frac{K}{P}e^{-(\gamma+n)t}$) as

$$d\left(\frac{K}{P}^{-(\gamma+n)t}\right) = (\rho - n - \gamma)\left(\frac{K}{P}e^{-(\gamma+n)t}\right) + \left(r\frac{M}{P} - \tau^C\right)e^{-\gamma t} \tag{49}$$

So the rule that stabilizes $\frac{K}{P}e^{-(\gamma+n)t}$ is

$$\begin{aligned}\tau^C &= r\frac{M}{P} + (\rho - n - \gamma)\left(\frac{K}{P}e^{-(\gamma+n)t}\right) \\ &= r\left(q\frac{B}{P} - \frac{V}{P}\right) - \left(\frac{\dot{P}}{P} + n + \gamma\right)\left(\frac{K}{P}e^{-(\gamma+n)t}\right).\end{aligned} \tag{50}$$

This rule has quite different implications for remittances relative to the rule (45) outside of steady state, but at steady state the two coincide. In fact, at steady state $\tilde{q} = q = 1$, $K = \tilde{K}$, and $\chi = \bar{r}$, hence the term $r\left(q\frac{B}{P} - \frac{V}{P}\right)$ coincides with the right hand side of expression (45). The remaining term $(\pi + n + \gamma)\left(\frac{K}{P}e^{-(\gamma+n)t}\right)$ accounts for the fact that capital increases with inflation, productivity, and population growth (as discussed above), a factor which we ignore in (45). If we did properly account for it, expressions (50) and (45) would be consistent with each other at least at steady state.

APPENDIX C. ADDITIONAL TABLES AND FIGURES

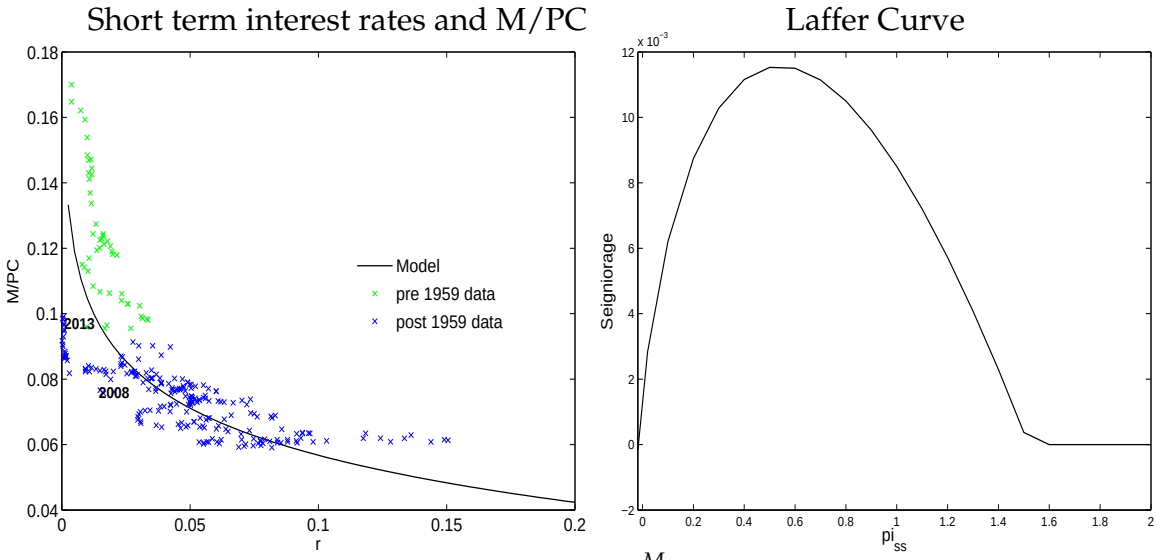
TABLE A-1. Central bank's resources under different simulations

		(1) $\frac{qB/P}{-V/P}$	(2) PDV seigniorage	(3) (1)+(2)	(4) $\bar{\tau}^C$ (average remittance)	(5) q	(6) $\Delta M/P$	(7) $\bar{B}/B$
	Baseline calibration							
(1)	Baseline scenario	0.146	1.139	1.285	0.0029	1.08		
(2)	Higher rates (β)	0.130	0.181	0.311	0.0034	1.06	-0.008	12.62
(3)	Higher rates (γ)	0.141	1.443	1.584	0.0035	1.06	0.004	60.23
(4)	Inflation scare	0.028	0.692	0.720	0.0024	0.85	-0.039	4.15
(5)	Explosive path	0.069	0.466	0.535	0.0012	0.85	-0.001	3.28
	Higher θ_π							
(5)	Inflation scare	0.048	0.599	0.647	0.0021	0.90	-0.036	4.54
(7)	Explosive path	-0.010	0.175	0.165	0.0004	0.61	-0.001	1.34
	Lower θ_π							
(8)	Inflation scare	-0.070	0.861	0.791	0.0028	0.47	-0.031	2.69
(9)	Explosive path	0.135	6.806	6.942	0.0166	1.05	0.001	199.41

TABLE A-2. Central bank's resources under different simulations – Money demand estimated on pre-2008 data

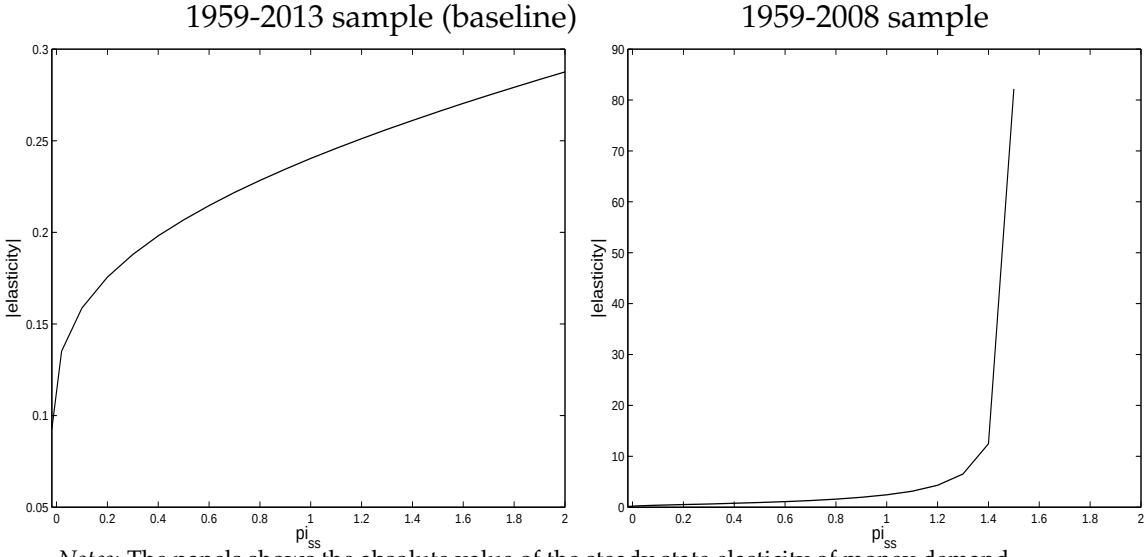
		(1) $\frac{qB/P}{-V/P}$	(2) PDV seigniorage	(3) (1)+(2)	(4) q	(5) $\bar{B}/B$
	Baseline calibration					
(1)	Baseline scenario	0.146	1.159	1.305	1.08	
(2)	Inflation scare	-0.024	0.597	0.573	0.85	3.50
(3)	Explosive path	0.068	-0.129	-0.061	0.85	0.74
	Higher θ_π					
(4)	Inflation scare	-0.003	0.384	0.381	0.90	3.08
(5)	Explosive path	-0.011	-0.160	-0.171	0.61	0.64
	Lower θ_π					
(5)	Inflation scare	-0.126	0.856	0.729	0.47	2.56
(7)	Explosive path	0.133	0.674	0.807	1.05	24.07

FIGURE A-1. Money Demand and the Laffer Curve – Money demand estimated on pre-2008 data



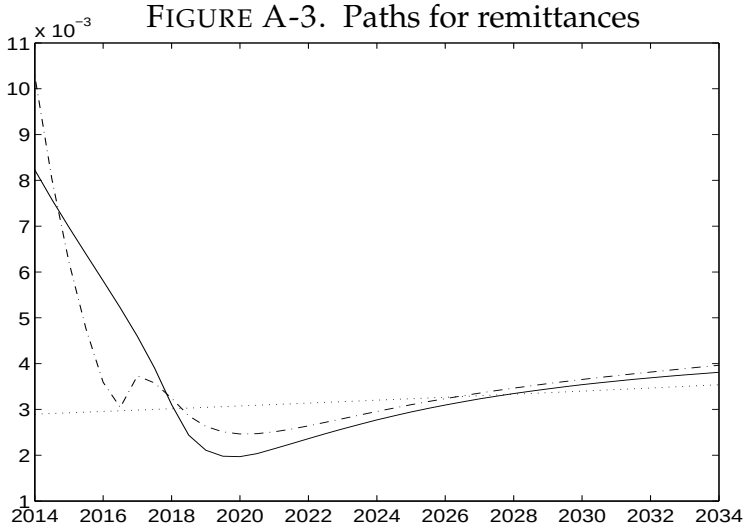
Notes: The left panel shows a scatter plot of quarterly $\frac{M}{PC} = v^{-1}$ and the annualized 3-month TBill rate (blue crosses are post-1959 data, and green crosses are 1947-1959 data) together with relationship between inverse velocity and the level of interest rates implied by the model (solid black line). The right panel shows seigniorage as a function of steady state inflation.

FIGURE A-2. Money Demand Elasticity



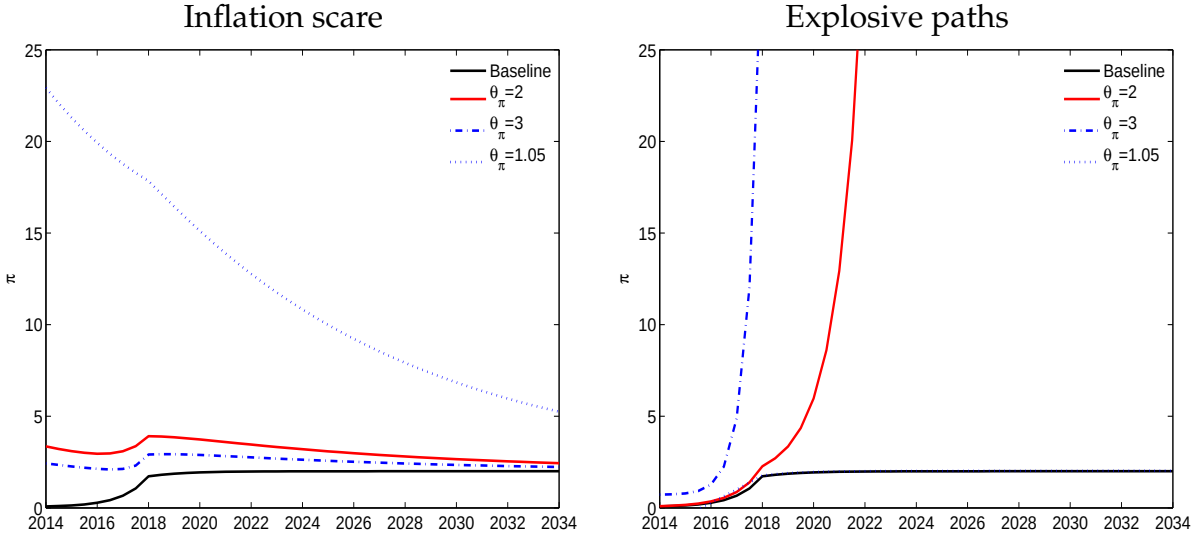
Notes: The panels shows the absolute value of the steady state elasticity of money demand ($|\frac{d\log M/P}{d\log r}|$) implied by our transaction cost functions under the baseline estimation (left panel) and under the estimation on pre-2008 data (right panel) .

FIGURE A-3. Paths for remittances



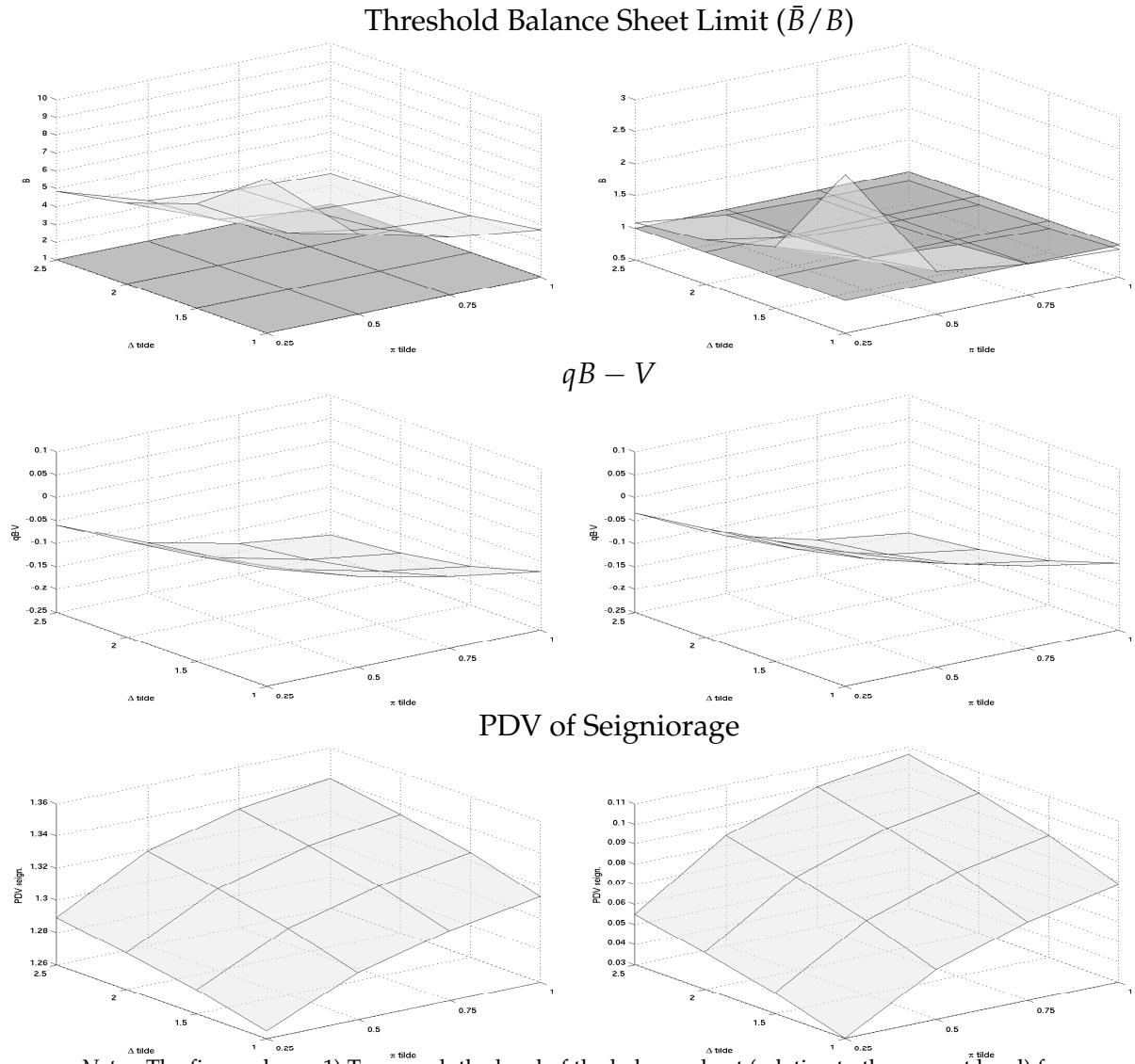
Notes: The figure shows remittances under the baseline scenario under two assumptions for the path of assets B^C: under the first assumption (solid line) the central bank lets its assets depreciate, while in the second one (dashed-and-dotted line) it actively sells assets at a rate of 20 percent per year.

FIGURE A-4. "Inflation scare" and "explosive paths" scenarios: The effect on inflation under different inflation responses in interest rate rule



Notes: The panels show the projected path of nominal short term rates for the "inflation scare" (left panel) and the "explosive path" scenario (right panel, with $\kappa = 10^{-4}$) under different inflation responses in interest rate rule (solid red: θ_π=2, dash-and-dotted blue: θ_π=3; dotted blue: θ_π=1.05) together with the baseline projections (solid black).

FIGURE A-5. Self-fulfilling solvency crises – Money demand estimated on pre-2008 data



Notes: The figure shows 1) Top panel: the level of the balance sheet (relative to the current level) for which multiple equilibria are possible; 2) Middle panel: the level of $qB - V$ as a fraction of income for the current balance sheet size under alternative scenarios; 3) Bottom panel: the level of seigniorage as a fraction of income under alternative scenarios; as a function of inflation in there alternative regime ($\tilde{\pi}$), and the duration of the alternative regime ($\tilde{\Delta}$). In all simulation the alternative regime is expected to start after 1 year ($\tilde{T}$=1). The left and right figures are for $\kappa = 0$ (stable solution) and $\kappa < 0$ (downward unstable solutions), respectively. .

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