# Commodities as Collateral*

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**Keywords**: commodity, collateral, theory of storage, capital control, financialization

**JEL Codes**: G12, Q02

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# 1 Introduction

In this paper we propose a theory of using commodities as collateral for financing, and report evidence regarding its effect on the underlying commodities markets. This research is motivated by a recent trend that production commodities such as copper, aluminum, and iron ore are increasingly used as collateral to obtain financing, most notably in China. For example, Yuan, Layton, Currie, and Courvalin (2014) estimate that about $109 billion FX loans in China are backed by commodities as collateral.[1] This amount is equivalent to about 31% of China's total short-term FX loans and 14% of China's total FX loans. With its large volume, commodity-based financing has become a critical form of "shadow banking" in China's financial markets.

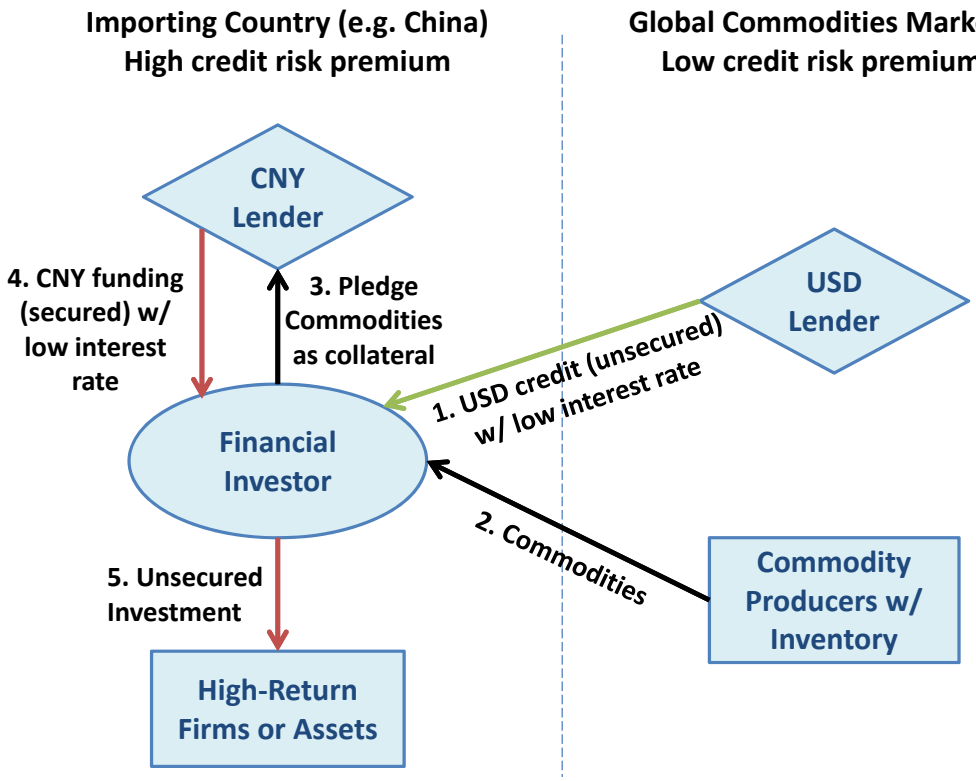
China is the second largest economy and the world leading consumer and importer of commodities. For example, China accounts for about 40% of global copper consumption and steel consumption.[2] Given the large impact of China on global commodities markets, the new trend for using commodity as collateral raise important questions. What are the impacts of using commodities as collateral on global commodities markets, such as prices, inventories, and risk premia? What does the collateral usage inform us about the underlying financial frictions and their effects on the real economy? How does the commodity-as-collateral channel interacts with the traditional theory of storage for commodities? The objective of this paper is to investigate these questions, both theoretically and empirically.

A typical financial transaction that imports commodity as collateral works as follows (see Figure 1 for an illustration and Section 2 for details). A domestic Chinese firm in the import-export business applies for US dollar credit from a bank to buy commodities in the international market. These commodities are shipped to China and pledged as collateral to raise CNY (the Chinese currency) funding on a secured basis, with a haircut. Commodities that have a higher value-to-bulk ratio (e.g., metals) are more easily used as collateral and are especially valuable for financing purposes. The CNY cash is subsequently invested, on an unsecured basis, in assets, loans or firms that generate a higher expected return. At the maturity of the US dollar loan, all legs of the trades are unwound, and the commodity is sold in the spot market. As long as the

[1] Take copper for example. The Economic Observer (2012) estimates that 90% of copper stored in the tariff-free zone in Shanghai are for financing purposes, with the total amount more than 500 thousand tons. Shanghai Metals Market, a research firm, estimates that between 400 and 600 thousand tons of copper has been used for financing in China in 2013. To put these estimates into perspective, a half million tons of copper are approximately 5.7% of China's annual copper consumption and 2.4% of world consumption in 2012.

[2] For copper statistics, see International Copper Study Group (2013). For steel statistics, see World Steel Association (2013).

Figure 1: A typical process of commodity-based financing




expected return on unsecured loans in CNY is sufficiently high, this trade is profitable in expectation. In other words, commodity is used as the means to capture the risk premium in China's credit market.

The viability and attractiveness of importing commodity as collateral relies on a couple of key frictions in China. First, Chinese banks typically require borrowers to post collateral to obtain credit. Firms that have high expected returns but do not have sufficient collateral, such as small firms, often find it difficult to borrow on an unsecured basis. Thus, some investments with high expected returns are not taken. Second, capital flows are restricted,[3] so the spread in the expected returns between China and developed countries cannot be entirely eliminated by pure financial transactions. Moreover, due to capital control, collateral commodities need to be imported, rather

[3]The capital inflows to China's financial markets from abroad are controlled by the "Qualified Foreign Institutional Investor" (QFII) program, managed by the State Administration of Foreign Exchange (SAFE). SAFE grants the QFII status to selected foreign institutions, which can then invest in China's financial markets. Each QFII has a quota on the maximum amount it can invest. According to Xinhua News Agency, as of June 2013, the overall quota for all QFIIs is $80 billion, among which the combined quota of $41.76 billion has been allocated to 229 foreign institutions. Conversely, capital outflows from China to international financial markets are controlled by the "Qualified Domestic Institutional Investor" (QDII) program, also managed by SAFE. Each QDII can invest in international financial markets, up to a specific quota.

than bought locally, to take advantage of the low dollar interest rates. Combined, financial frictions and capital control lead to a higher expected return in China that can be exploited by importing commodities as financing collateral.

We present a simple two-period model that formalizes the causes and effects of financing using commodity as collateral. In our model, a representative commodity importing country (e.g. China) buys commodities from a representative exporting country. Unsecured interest rates differ between two countries. But this gap cannot be eliminated by financial investors because of heavy restrictions of capital flows in and out of the importing country. That is, the two financial markets are effectively segmented, an extreme form of "capital immobility" (see Duffie (2010) and Duffie and Strulovici (2012)).

Although financial capital flows are heavily restricted, trades of production assets such as commodities are not. To take advantage of the gap in expected returns between countries, financial investors in the importing country engage the following sequence of trades at period 0. They borrow at the unsecured interest rate abroad and import commodities; these commodities are then pledged in the domestic market to get secured, low-interest loans, which are subsequently invested in assets with high expected returns. At the maturity of the loans, period 1, all borrowing and lending are unwound, and the collateral commodity is sold to fundamental consumers, such as firms that use commodities as input for producing other goods. The financial investor can further use the futures market in the importing country to limit the risk exposure in the spot market.

The demand for commodities as collateral has a number of important implications, as shown by the model. For example, the collateral demand for commodities increases concurrent commodity prices in both China and developed markets. It leads to lower commodity inventories in developed markets but higher inventories in China. Since owners of inventories need to hedge these positions in futures market, our model also predicts that the expected return (risk premium) for holding long positions in commodity futures increases in China but decrease in developed markets. Lastly, because the collateral demand of commodities tends to raise Chinese inventory and convenience yield together, the inventory-convenience yield relation should become significantly less negative (or more positive) in China.

We test these predictions in the data. Ideally, we would like to directly measure how much commodity is pledged as collateral, but such data could not be obtained due to the opacity of this market. Instead, we construct an indirect, theory-motivated empirical measure. Recall that the attractiveness of importing commodities as collat-

eral requires two conditions. First, the unsecured interest rate in China is sufficiently higher than that in developed markets. Second, this interest-rate spread cannot be eliminated by moving financial capital because of China's capital control. As a proxy for the unsecured interest rate spread, we use the difference between the Shanghai Interbank Offered Rate (Shibor) in CNY and London Interbank Offered Rate (Libor) in USD. As a proxy for the severity of capital control, we use the the violation of the covered interest rate parity (CIP) in the USDCNY exchange rate. Our overall proxy for the collateral demand for commodities is the product of the Shibor-Libor spread and the violation of CIP. Since the financial crisis this measure is predominantly positive and highly time-varying (see Figure 7(c) of Section 6).

We test the model predictions in the markets for eight commodities: copper, zinc, aluminum, gold, soybean, corn, fuel oil, and natural rubber. The first four constitute the metal group, and the last four constitute the nonmetal group. Our sample consists of weekly observations of prices and inventories from October 13, 2006 to November 14, 2014, in both China and developed markets. We test how the collateral demands for commoditie affect (i) commodity prices, (ii) futures risk premium, and (iii) the relation between inventory and convenience yield. In each test, we conduct eight commodity-by-commodity regressions and two panel regressions for the metal group and nonmetal group. Our theory also suggests that the predicted effects should be more evident in the metal group.

Empirical tests support our theory. In the first test, we find that a higher collateral demand for commodities significantly increases the spot commodity prices in China and in developed markets for the metal group. The economic magnitude is large. In our sample the collateral-demand-for-commodities measure has an approximate peak-to-trough range of 0.135. A fluctuation of this magnitude corresponds to a 15.0% increase in copper prices, a 13.6% increase in zinc price, and a 11.9% increase in aluminum price, all measured by dollar prices on the London Metal Exchange.

In the second test, we find that a higher collateral demand for commodities significantly increases the futures risk premium in China but reduces that in developed markets for the metal group. The panel regression for metals suggests that a one-standard-deviation increase in the collateral-demand-for-commodity measure increases the futures risk premium in China by 43 basis points per week (or 22.3% per year), but reduces futures risk premium in developed markets by 23 bps per week (or 12.0% per year).

In both the price and futures risk premium tests, we detect no statistically significant effect on the nonmetal group. This is intuitive because agricultural commodities

and oil are bulky and relatively expensive to store and ship, hence not ideal collateral.

In the third test, we find that a higher collateral demand for commodities makes the inventory-convenience yield relation significantly less negative in China. This test distinguishes our theory from the theory of storage, which predicts that inventory and convenience yield move in opposite directions. In our theory of commodity collateral, inventory and convenience yield move in the same direction in China. The evidence supporting both theories.

The results of this paper serve as a fresh reminder of how financial frictions such as capital controls can distort markets of production assets and hence the real economy. It is also a vivid example for the creation of "shadow banking" where bank lending is limited due to institutional constraints.

We caution that the current analysis does not lead to unambiguous conclusions regarding the welfare consequences of using commodities as collateral. On the one hand, we show that the collateral demand for commodity can partly crowd out the real demand for commodity. On the other hand, the high uncollateralized interest rate is presumably associated with profitable investment opportunities elsewhere in the economy. In particular, if firms can pledge commodities as collateral to relax their funding constraints, which we do not model, this practice may well reduce inefficiency caused by funding frictions. Analyzing the net welfare implication, therefore, requires a richer and more general equilibrium model, which we leave for future research.

Our theory and empirical findings are complementary to the classical theory of storage (see Working (1939), Telser (1949), Brennan (1958), Pindyck (2001), and Gorton, Hayashi and Rouwenhorst (2012), among others). For example, while the theory of storage predicts a negative relation between convenience yield and inventory, our model predicts that the collateral demands for commodity raise inventory and convenience yield simultaneously, a positive relation. Moreover, collateral demands result in a high total inventory and a high commodity price simultaneously. This is again opposite to the prediction from the theory of storage that a higher inventory indicates the abundance of commodity and hence a lower price.

This paper also relates to an emerging strand of literature on the financialization of commodities. Related papers include Masters (2008), Tang and Xiong (2012), Cheng, Kirilenko and Xiong (2014), Basak and Pavlova (2013), Henderson, Pearson and Wang (2013), and Singleton (2014), among others. In these papers, financially motivated trading on commodity futures markets, other derivatives and structured products linked to commodities tends to move underlying commodity prices. These studies focus predominantly on commodity derivatives. By contrast, an essential element of

our theory and evidence is the movement of the physical commodities. The dynamic relation between financial markets and physical commodity markets is a unique feature of our theory.

# 2 Commodities as Collateral in Practice

In this section we discuss the institutional procedures of importing commodities as collateral for financing, as well as the underlying risks and financial frictions.

## 2.1 The Institutional Procedure

A typical commodity financing transaction consists of a few steps.[4]

First, a Chinese importing firm signs a contract to buy commodity from an overseas firm. As is standard in international trade, the importing firm uses the purchase contract to apply for a letter of credit from a domestic or foreign bank. The letter of credit is typically granted in dollars at the US dollar interest rate and guarantees that the seller will be paid by the bank.[5] In order to obtain the credit, the importing firm needs to pay a margin, which is about 20% to 30% of the loan amount. The maturity of the letter of credit varies and is often between three to six months. For example, if the letter of credit is granted for six months, the importing firm needs to pay back the USD loan plus interest after six months. The importer can sell futures contracts in China to hedge the price risk of holding the commodity.

Second, the importer ships commodity to bonded warehouses in China's ports and obtain the warehouse receipts. Note that at this stage the commodity stored in bonded warehouses has not yet entered China customs, and the importer does not have to pay the associated duties yet. The warehouse receipt is subsequently provided to a domestic bank as collateral to obtain a CNY loan. A typical loan haircut is 30%, that is, the amount of the CNY loan is 70% of the market value of the commodity. Typically, the interest on the collateralized CNY loan is significantly lower than the expected return in other asset markets in China, such as real estates and short-term lending to small businesses. Effectively, the importer uses the commodity collateral to capture the spread between the collateralized and uncollateralized CNY funding rates in China.

[4] For additional overviews of the institutional arrangements of commodity financing, see Yuan, Layton and Currie (2013), Garvey and Shaw (2014), and Fu (2014).

[5] Banks that involve in commodity trade financing include BNP Paribas, Crédit Agricole, ING, Société Générale, JPMorgan, Citigroup, Standard Chartered, and HSBC, among others. See "Banks return to commodities finance", by Javier Blas and Ajay Makan, Financial Times, February 5, 2013.

Third, after three or six months, the commodity importer receives the uncollateralized return from its CNY investments and then sells the commodity stored in bonded warehouse in China's ports. The importer also closes its futures position. The proceeds of commodity sale and investment returns in its CNY investment are used to pay for the domestic bank loan in CNY (with relatively low CNY interest rates) and the foreign bank for the letter of credit (with relatively low USD interest rate). This completes a typical commodity financing transaction. As we explain in the next subsection, the financial frictions in China are sufficiently large for this series of trades to make a positive expected return. Just like in carry trades in currencies market, this expected return should not be viewed as an arbitrage but a risk premium for taking credit risk in China.

There are some variations of the above procedure. For instance, at the maturity of the CNY loan, the importing firm may re-sell the commodity in bonded warehouse to an overseas firm, again outside Chinese customs, and subsequently repeat the commodity financing procedure. This way, subsequent "importing" of commodities does not involve physical shipments because the inventories are local. Thus, each ton of imported commodity can be used to obtain financing multiple times.

An alternative arrangement involves the immediate sale of the imported commodity to the Chinese spot markets. The proceeds of the sale in CNY is then invested to obtain higher expected returns than the USD interest rates. A main difference of this procedure is that the commodity has to enter customs and incur the associated duties, and repeating this financing arrangement involves importing additional commodity, instead of recycling existing commodity in bonded warehouses.

## 2.2 Underlying Financial Frictions and Risks

This subsection discusses financial frictions that make commodity-based financing viable and profitable in the first place, as well as the associated risks.

As we discussed in the introduction, the financial frictions that give rise to commodity-based financing are twofold. First, capital flows in and out of China are strictly controlled. Investors who wish to directly participate in Chinese financial markets find it difficult to move capital across the border. Second, credit provision in China still relies heavily on banks, and the financial market is immature. Banks typically require collateral from borrowers. Due to the lack of collateral, many firms, especially small ones, have high expected returns but very limited access to financing. The combination of capital control and financial frictions leads to a relatively large credit risk

premium in China, compared to developed economies. These frictions also lead to the development of "shadow banking," i.e., lending by non-bank institutions to borrowers who need credit. Commodity-based financing is a major example.

As in other forms of shadow banking, a primary risk involved in commodity-based financing is credit risk. For example, in the third step of commodity-based financing described above, if its CNY investments default or have low realized returns, the commodity importer may not have enough financial resources to cover its USD unsecured loan and its CNY secured loan. The banks that provided collateralized credit in this process can also suffer losses if commodity prices drop by more than the haircut level.

To concretely illustrate the risks associated with commodity-based financing, Figure 2 shows the reaction of copper prices on the London Metal Exchange (LME) to two China-specific events in the first half of 2014. These two episodes also demonstrate the large scale and importance of commodity-backed financing in China.

On Wednesday, March 5, 2014, Shanghai Chaori Solar, a Chinese solar equipment producer, said it would not be able to pay the interest of $14.7 million on its corporate bonds that is due that Friday.[6] Following this announcement, the global benchmark copper price traded on LME dropped sharply by more than 8.5% over a week, from $7102.5/ton on March 5 to $6498/ton on March 12. Although the Chaori default is relatively small, it was the first ever Chinese corporate bond default, which likely led to a reassessment of corporate default risk in China. A higher default risk reduces the risk-adjusted return for importing commodities and using them as collateral.[7]

The second event is the probe by Chinese authorities of alleged frauds in the port of Qingdao (in northern China) that the same commodities like copper have been pledged to multiple banks to get multiple loans. To the best of our knowledge, the potential fraud was first reported on June 4 and unfolded in the next couple of days.[8] LME copper prices dropped by about 4% from $6930/ton on June 3 to $6660.5/ton on June 6. Since multiple pledging of collateral is likely to reduce the recovery value of commodity-backed loans in default, lenders may impose tighter lending requirements such as a higher haircut. This, in turn, reduces the attractiveness of importing commodity as collateral and associated commodity prices.[9]

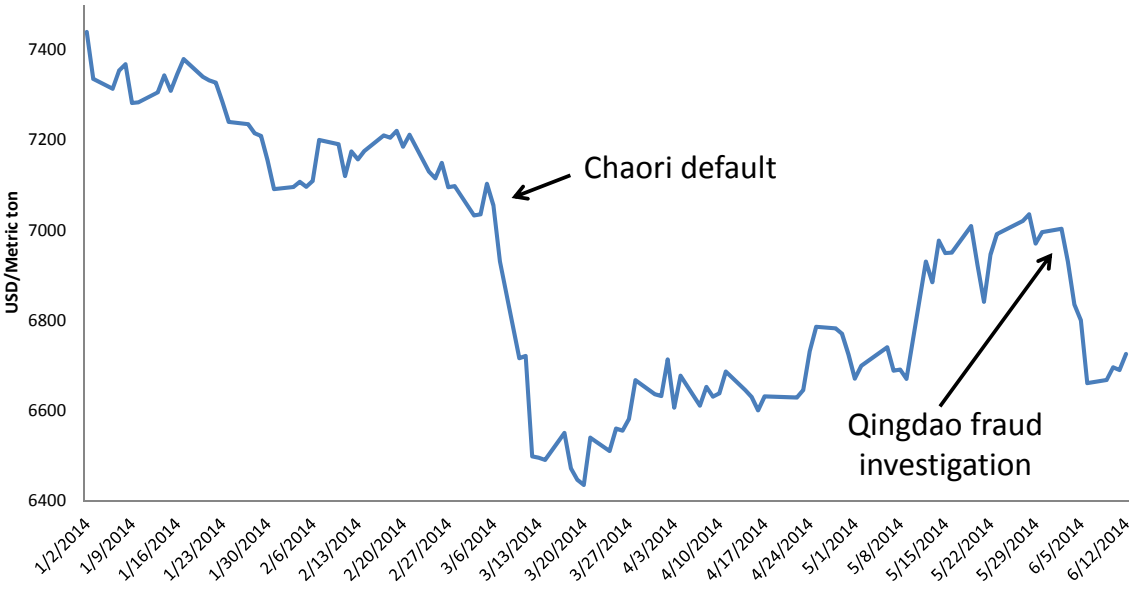
[6]See "China's Chaori Solar poised for landmark bond default," by Gabriel Wildau and Umesh Desai, Reuters, March 5, 2014.

[7]See "Copper futures fall by daily limit," by Xan Rice, Jamie Smyth, and Lucy Hornby, Financial Times, March 12, 2014 and "China Angst Slams Prices for Copper," by Ira Iosebashvili and Tatyana Shumsky, Wall Street Journals, March 10, 2014.

[8]See "Standard Bank starts probe of potential irregularities at China port," by Susan Thomas, Reuters, June 4, 2014.

[9]See "Worry plagues commodity finance trade after Chinese metals probe," by Fayen Wong and Manalo Serapio Jr, Reuters, June 8, 2014.

Figure 2: LME copper prices and two China-specific events




# 3 A Model of Commodities as Collateral

In this section we present a simple model of commodities as collateral.

There are two periods, $t \in \{0, 1\}$, and a single commodity. There is a representative commodity exporting country and a representative commodity importing country. The exporting country has a commodity supplier and a speculator. The importing country has a commodity supplier, a fundamental user of commodity for production, and a financial investor who imports commodity as collateral.

For simplicity, the commodity is priced in dollars in both countries, so currency returns are not part of our model. (Effectively, commodity importers hedge their FX exposures.) Moreover, the commodity importing country, which is meant to be modeled after China, imposes capital controls, so that its financial market and the financial market of the exporting country are segregated.

For ease of reference, Table 1 lists the exogenous and endogenous variables we use in this model. We use the superscript "e" to denote quantities and prices in the exporting country, and use the superscript "i" to denote quantities and prices in the importing country.

The rest of this section describes the model components in detail. The last subsection, Section 3.8, discusses our modeling choices and potential alternative approaches. Equilibrium solutions and implications are presented in Section 4 and Section 5.

Table 1: Key model variables

Variables in the top block are exogenous; variables in the bottom block are endogenous.

| Variable | Explanation |
|---|---|
| $r^j$, $R^j$ | The secured and unsecured interest rate in country $j \in \{e, i\}$ |
| $\delta$ | Storage cost of commodity |
| $h$ | Shipping cost of commodity |
| $G_t^e$ | Commodity production of the exporting country at time $t$ |
| $k_t$, $l$ | The fundamental demander's marginal profit of using $D_t^i$ unit of commodity is $k_t - S_t^i - lD_t^i$, where $k_t \sim N(\mu_k, \sigma_k^i)$ |
| $a$, $b$ | Commodity supply in the importing country is $a + bS_t^i$ |
| $\gamma_p^e$, $\gamma_s^e$ | Risk aversion coefficients of commodity producer and financial speculator in exporting country |
| $\gamma_d^i$, $\gamma_c^i$ | Risk aversion coefficients of fundamental commodity demander and financial player in importing country |
| $S_t^j$ | Spot commodity price in period $t$ in country $j \in \{e, i\}$ |
| $F^j$ | Futures price in country $j \in \{e, i\}$, traded at $t = 0$ and delivered at $t = 1$ |
| $I_t^e$ | Commodity inventory in the exporting country at time $t$ |
| $D_{t,f}^i$, $D_{t,d}^i$ | Fundamental demand at time $t$ of foreign and domestic commodity |
| $C_0^i$ | Collateral commodity demand at time 0, all imported |
| $\lambda$ | Lagrange multiplier associated with constraint $I_0^e \geq 0$ |
| $\eta$ | Lagrange multiplier associated with constraint $D_{0,f}^i \geq 0$ |
| $h_p^e$, $h_s^e$ | Positions of futures contracts of commodity producer and financial speculator in exporting country at time 0 |
| $h_d^i$, $h_c^i$ | Positions of futures contracts of fundamental commodity demander and financial player in importing country at time 0 |
| $\sigma_S^j$ | Volatility of $S_1^j$ for $j \in \{e, i\}$ |

## 3.1 Supplier in the Exporting Country

We directly model the net supply in the exporting country. Our model in the exporting country is largely adopted from Acharya, Lochstoer and Ramadorai (2012). Let $I_t^e$ and $G_t^e$ be the aggregate commodity inventory and production, respectively. Let $\delta \in (0, 1)$ be the cost of storage; that is, the producer can store $I$ units of the commodity at $t-1$ and receive $(1-\delta) I$ units at $t$. We also assume that the production schedule $(G_0^e, G_1^e)$ are fixed ex ante and common knowledge. (Effectively, changing production in the short term is very costly.) The inventory $I_0^e$, however, is a choice variable of the producer. Given the choice of inventory $I_0^e$, the commodity sales in period 0 and

period 1 are, respectively,

$$Q_0^e = G_0^e - I_0^e, \tag{1}$$

$$Q_1^e = G_1^e + (1-\delta) I_0^e. \tag{2}$$

In addition to selling the commodity in the spot market, the commodity supplier shorts $h_p^e$ futures contracts in the exporting country at the price of $F^e$ to hedge its inventory and production.

Therefore, the terminal wealth of the producer is

$$W_p^e = S_0^e(G_0^e - I_0^e)(1+r^e) + S_1^e(G_1^e + (1-\delta)I_0^e) - h_p^e(S_1^e - F^e), \tag{3}$$

where $r^e$ is the secured interest rate in the exporting country and $S_t^e$ is the commodity spot price in period $t$. We emphasize that $S_1^e$ is a random variable. As we elaborate shortly, $S_1^e$ is determined by the stochastic demand of the importing country in period 1. We denote by $\sigma_S^e$ the volatility (standard deviation) of $S_1^e$.

The commodity producer has a mean-variance utility of the form

$$E[W_p^e] - \frac{\gamma_p^e}{2}\mathrm{Var}[W_p^e]. \tag{4}$$

Substituting in the expression of $W_p^e$, we see that the producer solves the problem

$$\begin{aligned} \max_{\left\{I_0^e, h_p^e\right\}} \; & S_0^e \left(G_0^e - I_0^e\right)\left(1+r^e\right) + E\left[S_1^e((1-\delta)\, I_0^e + G_1^e) - h_p^e\left(S_1^e - F^e\right)\right] \\ & - \frac{\gamma_p^e}{2}\mathrm{Var}\left[S_1^e((1-\delta)\, I_0^e + G_1^e) - h_p^e\left(S_1^e - F^e\right)\right] \\ & \text{subject to: } I_0^e \geq 0. \end{aligned} \tag{5}$$

We denote by $\lambda \geq 0$ the Lagrange multiplier associated with the inventory constraint $I_0^e \geq 0$. Taking the first-order condition with respect to the inventory $I_0^e$ and futures position $h_p^e$, we get

$$I_0^e = \frac{E\left[S_1^e\right](1-\delta) - S_0^e\left(1+r^e\right) + \lambda}{\gamma_p^e\left(\sigma_S^e\right)^2(1-\delta)^2} + \frac{h_p^e - G_1^e}{(1-\delta)}, \tag{6}$$

$$h_p^e = I_0^e\,(1-\delta) + G_1^e - \frac{E\left[S_1^e - F^e\right]}{\gamma_p^e\left(\sigma_S^e\right)^2}. \tag{7}$$

If $I_0^e > 0$, $\lambda = 0$. If $I_0^e = 0$, $\lambda > 0$. The endogenous $\lambda$ affects the convenience yield of

holding the commodity.

## 3.2 Speculator in the Exporting Country

The speculators only trade futures in the exporting country, and their futures position is denoted by $h_s^e$. They have mean-variance utility and solve the following optimization problem

$$\max_{h_s^e} E\left[h_s^e\left(S_1^e - F^e\right)\right] - \frac{\gamma_s^e}{2}\mathrm{Var}\left[h_s^e\left(S_1^e - F^e\right)\right]. \tag{8}$$

The solution is

$$h_s^e = \frac{E\left[S_1^e - F^e\right]}{\gamma_s^e\left(\sigma_S^e\right)^2}. \tag{9}$$

## 3.3 Market Clearing in the Exporting Country

From (6) and (7) we obtain

$$\frac{S_0^e - F^e}{S_0^e} = \frac{\lambda}{S_0^e\left(1-\delta\right)} - \frac{r^e + \delta}{1-\delta}. \tag{10}$$

Thus, the futures price in the exporting country is

$$F^e = \frac{S_0^e\left(1+r^e\right) - \lambda}{1-\delta}, \tag{11}$$

By the futures market clearing, $h_p^e = h_s^e$, we have

$$E\left[S_1^e - F^e\right] = \frac{\gamma_s^e\gamma_p^e}{\gamma_s^e + \gamma_p^e}\left(\sigma_S^e\right)^2\left[I_0^e\left(1-\delta\right) + G_1^e\right]. \tag{12}$$

Since $F^e$ is solved, the above equation has two unknowns, $E[S_1^e]$ and $I_0^e$. These two variables cannot be determined by variables in the exporting country alone; rather, we need the demand from the importing country, which we turn to now.

## 3.4 Supplier in the Importing Country

Since the commodity supply in the importing country is not our main focus, we simply assume that the commodity supply in the importing country is given by $Q_t^i = a + bS_t^i$, where $a < 0$ and $b > 0$ are commonly known constants.

### 3.5 Fundamental Demander in the Importing Country

We model the "fundamental demander" in the importing country as a consumer who uses commodity as input to produce final goods. For simplicity, the fundamental demander does not carry inventory. At time $t$, the fundamental demander has a linearly decreasing marginal profit per unit of commodity input,

$$k_t - S_t^i - lD_t^i, \tag{13}$$

where $k_t$ is a random variable, $l$ is a constant, and $D_t^i$ is the amount of commodity input used at time $t$. At time 0, $k_0$ is commonly known, but $k_1$ is unobservable and is normally distributed $N(\mu_k, \sigma_k^i)$. This stochastic $k_1$ can be interpreted as the "fundamental shock" to the economy of the importing country, only realized at time 1. All players in our model have symmetric information and the same probability distribution about $k_1$.

The fundamental demander has three endogenous choices at time 0: the amount of commodities to import, $D_{0,f}^i$, the amount of commodities to buy in domestic market, $D_{0,d}^i$, and the amount of futures contracts to trade in the local market, $h_d^i$. The shipment of one unit of commodity across the two countries incurs the cost $h > 0$. For simplicity, shipment is instantaneous, that is, commodity purchased in the exporting country at time $t$ can be used in the importing country at time $t$ as well.

The fundamental demander's terminal wealth in period 1 that is derived from his production and trading activity in period 0 is

$$\begin{aligned} W_{d,0}^i = {} & D_{0,f}^i \left[k_0 - (S_0^e + h) - l\left(D_{0,f}^i + D_{0,d}^i\right)\right]\left(1 + r^i\right) \\ & + D_{0,d}^i \left[k_0 - S_0^i - l\left(D_{0,f}^i + D_{0,d}^i\right)\right]\left(1 + r^i\right) + h_d^i\left(S_1^i - F^i\right), \end{aligned} \tag{14}$$

where $r^i$ is the secured interest rate in the importing country, and the first and second terms are the fundamental demander's total profits of using foreign and domestic commodity supplies, respectively.

The fundamental demander has the mean-variance preference with parameter $\gamma_d^i$

and solves[10]

$$\max_{\{D^i_{0,d}, D^i_{0,f}, h^i_d\}} E[W^i_{d,0}] - \frac{\gamma^i_d}{2}\text{Var}[W^i_{d,0}], \tag{15}$$

$$\text{Subject to: } D^i_{0,f} \geq 0. \tag{16}$$

The solution to the above problem is

$$D^i_{0,f} = \frac{k_0 - (S^e_0 + h)}{2l} - D^i_{0,d} + \eta, \tag{17}$$

$$D^i_{0,d} = \frac{k_0 - S^i_0}{2l} - D^i_{0,f}, \tag{18}$$

$$h^i_d = \frac{E\left[S^i_1 - F^i\right]}{\gamma^i_s \left(\sigma^i_S\right)^2}, \tag{19}$$

where $\sigma^i_S$ is the volatility of $S^i_1$ and $\eta$ is the Lagrange multiplier associated with the constraint (16). If $D^i_{0,f} = 0$, i.e. the fundamental demander only buys commodity locally, then $\eta > 0$. If $D^i_{0,f} > 0$, then $\eta = 0$.

Similarly, we can solve the fundamental demander's problem at time 1. We denote by $D^i_{1,f}$ and $D^i_{1,d}$ the demands for foreign and domestic commodity, respectively. The terminal wealth of the fundamental demander is

$$W^i_{d,1} = D^i_{1,f}\left[k_1 - (S^e_1 + h) - l(D^i_{1,f} + D^i_{1,d})\right] + D^i_{1,d}\left[k_1 - S^i_1 - l(D^i_{1,f} + D^i_{1,d})\right]. \tag{20}$$

Since the fundamental shock $k_1$ is realized and becomes common knowledge at time 1, the fundamental demander solves

$$\max_{\{D^i_{1,d}, D^i_{1,f}\}} W^i_{d,1}. \tag{21}$$

The solution is

$$D^i_{1,d} = \frac{k_1 - S^i_1}{2l} - D^i_{1,f}, \tag{22}$$

$$D^i_{1,f} = \frac{k_1 - (S^e_1 + h)}{2l} - D^i_{1,d}. \tag{23}$$

[10]Because $k_1$ is an exogenous variable, the mean and variance of the fundamental demander's period-1 wealth that comes from his period-1 production activity are not affected by his period-0 strategy, $(D^i_{0,f}, D^i_{0,d}, h^i_d)$. Thus, we can solve his optimal strategies period by period.

### 3.6 Financial Demander in the Importing Country

The financial demander in the importing country imports commodity not for production, but to use it as collateral to get secured financing at rate $r^i$ and lend unsecured at rate $R^i > r^i$. (Without loss of generality, the interest rates $R^i$ and $r^i$ are after adjusting for the haircut imposed on the loan.) In other words, the commodity is imported as a means to capture the unsecured-secured spread, or risk premium, of $R^i - r^i$. The financial demander must first borrow unsecured in the exporting country at the rate $R^e$ to pay for the costs of commodity and shipping. Since borrowing and lending take one period, this trade must be completed at time 0. The expected time-1 profit of importing one unit of collateral commodity at time 0 is

$$S_0^i(R^i - r^i) + (1-\delta)\, E[S_1^i] - (S_0^e + h)\,(1 + R^e). \tag{24}$$

The three terms capture, respectively, the expected profit of borrowing $S_0^i$ at rate $r^i$ and lending at rate $R^i$, the proceeds from selling the remaining $(1-\delta)$ commodity at time 1, and the payment of the unsecured loan at rate $R^e$. We later specify explicit conditions under which the expected profit of importing commodity as collateral is positive. We denote by $C_0^i$ the amount of commodity imported for collateral purposes at time 0.

We emphasize that these "collateral commodities" must be imported for this trade to be viable. If the financial demander were to use domestic commodity, he must first pay the unsecured rate $R^i$, defeating the purpose of lending at $R^i$.

The financial demander also uses futures contract to hedge his position. We denote by $h_c^i$ his futures position at time 0.

The financial demander's terminal wealth at time 1 is

$$W_f^i = C_0^i \left[S_0^i(R^i - r^i) + (1-\delta)\, S_1^i - (S_0^e + h)\,(1 + R^e)\right] - h_c^i(S_1^i - F^i). \tag{25}$$

The financial demander has a mean-variance utility function with parameter $\gamma_c^i$. At time 0, he solves the problem

$$\max_{\{C_0^i, h_c^i\}} E[W_f^i] - \frac{\gamma_c^i}{2}\mathrm{Var}[W_f^i]. \tag{26}$$

where the variance comes from the uncertainty about $S_1^i$.

Solving for the optimal $C_0^i$ and $h_c^i$, we get

$$C_0^i = \frac{S_0^i(R^i - r^i) + (1-\delta)\,E\left[S_1^i\right] - \left(S_0^e + h\right)\left(1 + R^e\right)}{\gamma_c^i\left(\sigma_S^i\right)^2(1-\delta)^2} + \frac{h_c^i}{1-\delta}, \tag{27}$$

$$h_c^i = -\frac{E\left[S_1^i - F^i\right]}{\gamma_c^i\left(\sigma_S^i\right)^2} + C_0^i(1-\delta). \tag{28}$$

## 3.7 Market Clearing in the Importing Country

From (17) and (18), we get

$$S_0^i = S_0^e + h - 2l\eta. \tag{29}$$

Recall that $\eta$ is the Lagrange multiplier associated with $D_{0,f}^i \geq 0$; $\eta > 0$ whenever $D_{0,f}^i = 0$. Thus, if all commodity imports are made for financing purposes, the commodity price in the importing country is lower than that in the exporting country after adjusting for shipping costs.

From (22) and (23) we get

$$S_1^i = S_1^e + h.$$

By the market-clearing condition of the futures market, $h_d^i = h_c^i$, we have

$$C_0^i = \left(\frac{\gamma_d^i + \gamma_c^i}{\gamma_d^i\gamma_c^i}\right)\frac{E\left[S_1^i - F^i\right]}{(1-\delta)\left(\sigma_S^i\right)^2}. \tag{30}$$

For parameters considered in this paper, $C_0^i \geq 0$. Cases in which $C_0^i = 0$ are identical to the benchmark case without the collateral use of commodities. From (27) and (28), we can solve the futures price in the importing country,

$$\begin{aligned} F^i &= \frac{\left(S_0^e + h\right)\left(1 + R^e\right)}{1-\delta} - \frac{S_0^i\left(R^i - r^i\right)}{1-\delta} \\ &= \frac{1 + r^i - \left(R^i - R^e\right)}{1-\delta}S_0^i + \frac{2l\left(1 + R^e\right)}{1-\delta}\eta. \end{aligned} \tag{31}$$

## 3.8 Discussion of Model Setup

In this subsection we make a few remarks on our modeling choices.

First, in our model the futures markets of the two countries are segregated; investors cannot trade futures in both countries. This assumption is a direct consequence of capital control of the importing country, modeled after China. If investors were able to circumvent capital controls and participate directly in financial markets in both countries, importing commodities as collateral would be unnecessary. Indeed, in the

model we can show that if the collateral demanders can also trade futures contracts in the exporting country, they would not import commodities. Thus, capital control and the effective segregation of financial markets is an essential friction in the model and in reality.

Second, we have used a two-period mode, which may seemingly suggest that the unwinding of the commodity collateral trade in period 1 is mechanical. One could argue that in a multiple-period or infinite-horizon model, financial players would import commodities as collateral in every period. While this concern is reasonable, importing commodities as collateral cannot continue forever. Because commodities are imported to take advantage of the credit risk premium in China, a large amount of collateral commodities would relax small firms' funding constraint in China and start to reduce the risk premium. Once the risk premium becomes sufficiently small, commodity importing would become unattractive and finally stop (given its cost). We would expect that a more general long-horizon model, in which unsecured interest rates are endogenous of the amount of commodity collateral, would deliver qualitatively similar results as our two-period model.

Lastly, we have made a few simplifying assumptions for analytical tractability. For example, in the importing country, the fundamental commodity demanders does not keep inventory, and the commodity producer has an exogenous supply function. Relaxing these assumptions would not effect our main results because the incremental effect of the collateral demand is separate from the traditional commodity consumers and producers.

# 4 Equilibrium

In this section we characterize the equilibrium prices and quantities. We first consider the equilibrium in which demanders of collateral commodities participate in the market. Then, we consider the equilibrium without collateral demands for commodities.

## 4.1 Equilibrium with Demand for Collateral Commodity

Putting together the market-clearing conditions from the previous section, we have the following proposition.

**Proposition 1.** *Suppose that collateral demanders of commodities participate in the market. In equilibrium, the spot prices* $(S_0^e, S_1^e, S_0^i, S_1^i)$*, the inventory* $I_0^e$ *in the exporting country, and the fundamental demands* $(D_{0,d}^i, D_{1,d}^i)$ *are given by the solution to the*

*following system of equations:*

$$\begin{aligned}
a + bS_0^i &= D_{0,d}^i, && (32)\\
G_0^e - I_0^e &= D_{0,f}^i + C_0^i \\
&= \left[\frac{k_0 - (S_0^e + h)}{2l} - D_{0,d}^i + \eta\right] + \left(\frac{\gamma_d^i + \gamma_c^i}{\gamma_d^i \gamma_c^i}\right) \frac{E\left[S_1^i - F^i\right]}{(1-\delta)\left(\sigma_S^i\right)^2}, && (33)\\
E\left[S_1^e - F^e\right] &= \frac{\gamma_s^e \gamma_p^e}{\gamma_s^e + \gamma_p^e} \left(\sigma_S^e\right)^2 \left[I_0^e (1-\delta) + G_1^e\right], && (34)\\
D_{1,d}^i &= a + bS_1^i + \left(\frac{\gamma_d^i + \gamma_c^i}{\gamma_d^i \gamma_c^i}\right) \frac{E\left[S_1^i - F^i\right]}{\left(\sigma_S^i\right)^2}, && (35)\\
I_0^e (1-\delta) + G_1^e &= D_{1,f}^i \\
&= \frac{k_1 - (S_1^e + h)}{2l} - D_{1,d}^i, && (36)\\
S_1^i &= S_1^e + h, && (37)\\
S_0^i &= S_0^e + h - 2l\eta, && (38)
\end{aligned}$$

*where*

$$F^e = \frac{S_0^e(1 + r^e) - \lambda}{1 - \delta}, \quad (39)$$

$$F^i = \frac{(S_0^e + h)(1 + R^e) - S_0^i(R^i - r^i)}{1 - \delta}. \quad (40)$$

*The two Lagrange multipliers* $(\lambda, \eta)$ *satisfy:*

$$\begin{aligned}
&\textit{if } I_0^e = 0,\ \lambda > 0,\\
&\textit{if } I_0^e > 0,\ \lambda = 0;
\end{aligned}$$

*and*

$$\begin{aligned}
&\textit{if } D_{0,f}^i = 0,\ \eta = D_{0,d}^i - \frac{k_0 - (S_0^e + h)}{2l} > 0,\\
&\textit{if } D_{0,f}^i > 0,\ \eta = 0.
\end{aligned}$$

*The solution of spot prices and inventories are:*

$$S_0^i = \frac{\left[\begin{array}{c} \frac{(1-\delta)(k_0-2al)}{2l} + mq + n\left(q-h+zh\right) - \left[G_0^e\left(1-\delta\right)+G_1^e\right] \\ +\frac{n}{1-\delta}\lambda - 2l\left(om+zn\right)\eta \end{array}\right]}{v+\left(1-\delta+w\right)m+\left(1-\delta+z\right)n}, \tag{41}$$

$$S_0^e = S_0^i - h + 2l\eta. \tag{42}$$

$$S_1^i = \frac{1}{2bl+1}\left[k_1 - 2al + \left(1-\delta\right)\left(k_0-2al\right) - 2l\left(\left(1-\delta\right)G_0^e + G_1^e\right)\right] - \left(1-\delta\right)S_0^i, \tag{43}$$

$$S_1^e = S_1^i - h, \tag{44}$$

$$I_0^e = \frac{1}{1-\delta}\left[n\left(q-h+zh\right) - \left(1-\delta+z\right)nS_0^i - G_1^e - 2nlz\eta + \frac{n\lambda}{1-\delta}\right], \tag{45}$$

*where the constants* $(n, m, q, o, z, v, w)$ *are defined in the Appendix A.1. The equilibrium demands* $(C_0^i, D_{0,d}^i, D_{1,d}^i, D_{0,f}^i, D_{1,f}^i)$ *are calculated from* (32)–(36).

While the exact solutions to the four cases are provided in Appendix A.1, we now outline the main solution steps. Since each of the two Lagrange multipliers $\lambda$ and $\eta$ can be binding or non-binding, there are four possible cases.

**Case 1 ($\lambda = 0$ and $\eta = 0$, i.e., $I_0^e > 0$ and $D_{0,f}^i > 0$).**

In this case, the demand for collateral commodity does not lead to stockout or zero import by fundamental demanders. Since neither constraint is binding, the equilibrium prices and inventory are simply given by (41)–(45) after substituting in $\lambda = \eta = 0$. There are seven unknowns and seven linear equations, from which we obtain a unique solution.

**Case 2 ($\lambda = 0$ and $\eta > 0$, i.e., $I_0^e > 0$ and $D_{0,f}^i = 0$).**

In this case, the collateral demand leads to zero import by fundamental demanders. Intuitively, the collateral demand drives up the commodity price in the exporting country; if this price is above the spot price in the importing country after adjusting for shipping cost, the fundamental commodity demand in the importing country is met entirely by local commodity supply. In this case, the fundamental demanders import nothing, and $D_{0,f}^i$ is given by

$$D_{0,f}^i = \frac{k_0 - 2al - \left(2bl+1\right)S_0^i}{2l}. \tag{46}$$

Thus, $D_{0,f}^i = 0$ implies that

$$S_0^i = \frac{k_0 - 2al}{2bl+1}. \tag{47}$$

Therefore, given $\lambda = 0$, from (41) we can explicitly obtain $\eta$. After getting $S_0^i$ and

$\eta$, we can easily solve all other variables.

**Case 3 ($\lambda > 0$ and $\eta = 0$, i.e., $I_0^e = 0$ and $D_{0,f}^i > 0$).**

In this case, the collateral demand leads to zero inventory in the exporting country. This can be the case if collateral demands drive up the price in the exporting country so much that the commodity supplier does not keep any inventory. Since $I_0^e = 0$ and $\eta = 0$, combining (41) and (45), one can get

$$S_0^i = \frac{\frac{(1-\delta)(k_0-2la)}{2l} - G_0^e(1-\delta) + mq}{v + (1-\delta+w)m}. \tag{48}$$

Thus, combining (41) and (48), one can solve for $\lambda$. After getting $S_0^i$ and $\lambda$, all other variables can be easily solved.

**Case 4 ($\lambda > 0$ and $\eta > 0$, i.e., $I_0^e = 0$ and $D_{0,f}^i = 0$).**

In this case, too much collateral demand drives up the price in the exporting country and produces two effects. First, the commodity producer has a stockout. Second, the fundamental commodity demand in the importing country is met entirely by the cheaper local commodity supply (after adjusting for shipping cost). This corresponds to $I_0^e = 0$ and $D_{0,f}^i = 0$. As shown in Case 2, $D_{0,f}^i = 0$ implies that $S_0^i = \frac{k_0-2al}{2bl+1}$. Therefore, we have

$$\begin{aligned} S_0^i &= \frac{\left[\begin{array}{c} \frac{(1-\delta)(k_0-2al)}{2l} + mq + n\left(q-h+zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right] \\ + \frac{n}{1-\delta}\lambda - 2l\left(om+zn\right)\eta \end{array}\right]}{v + \left(1-\delta+w\right)m + \left(1-\delta+z\right)n} = \frac{k_0-2al}{2bl+1}, \\ I_0^e &= \frac{1}{1-\delta}\left[n\left(q-h+zh\right) - \left(1-\delta+z\right)nS_0^i - G_1^e - 2nlz\eta + \frac{n\lambda}{1-\delta}\right] = 0. \end{aligned}$$

We can solve $\lambda$ and $\eta$ from the above two equations. Then, it is easy to further solve all other variables in the equilibrium.

## 4.2 The Benchmark Equilibrium without Collateral Commodity

In the benchmark case, we exogenously shut down the collateral demand for commodities, i.e., forcing $C_0^i = 0$. This case applies, for example, if capital control is relaxed substantially so that moving financial capital into China is more efficient than importing physical commodities for financing purposes. In this benchmark case, since the exporting country is a net supplier in time 0 and time 1, its commodity must eventually be absorbed by fundamental demander in the importing country. Following

similar (but simpler) calculation as before, we have the following proposition.

**Proposition 2.** *Suppose that collateral demanders of commodities do not participate in the market. In equilibrium, the spot prices ($S_0^e$, $S_1^e$, $S_0^i$, $S_1^i$) and the inventory $I_0^e$ in the exporting country are given by the solution to the following system of equations:*

$$\begin{aligned} G_0^e - I_0^e &= \frac{k_0 - S_0^i}{2l} - a - bS_0^i, && (49)\\ I_0^e(1-\delta) + G_1^e &= \frac{\gamma_s^e + \gamma_p^e}{\gamma_s^e \gamma_p^e} \frac{E\left[S_1^e - F^e\right]}{\left(\sigma_S^e\right)^2}, && (50)\\ I_0^e(1-\delta) + G_1^e &= \frac{k_1 - S_1^i}{2l} - \left(a + bS_1^i\right), && (51)\\ S_1^i &= S_1^e + h, && (52)\\ S_0^i &= S_0^e + h - 2l\eta, && (53) \end{aligned}$$

*where*

$$F^e = \frac{S_0^e(1+r^e) - \lambda}{1-\delta}. \quad (54)$$

*The explicit equilibrium solution is provided in Appendix A.2.*

For simplicity, we restrict attention to cases such that $\lambda = 0$ and $\eta = 0$. In this case, the equilibrium $S_0^i$ is given by

$$\overline{S}_0^i \equiv \frac{\frac{(1-\delta)(k_0 - 2al)}{2l} + n(q - h + zh) - \left[G_0^e(1-\delta) + G_1^e\right]}{v + (1 - \delta + z)n}, \quad (55)$$

where the constants $(n, q, z, v)$ are defined in Appendix A, and where we use the "bar" notation to highlight the equilibrium variable in the benchmark case without collateral demand. The other spot prices, inventory, and quantities can be calculated from the equations given in Proposition 2.

In equilibrium, the restriction of $\lambda = \eta = 0$ boils down to

$$a + b\overline{S}_0^i < \frac{k_0 - \overline{S}_0^i}{2l} < G_0^e + a + b\overline{S}_0^i. \quad (56)$$

That is, at the equilibrium price $\overline{S}_0^i$, the fundamental demand in the importing country at time 0 is larger than the domestic supply but is less than the total global supply (domestic plus foreign). Hence, the exporting country still keeps positive inventory and fundamental demander imports positive amount of commodity. Evaluating the above inequality at the equilibrium $\overline{S}_0^i$, we arrive at the following technical condition:

**Technical Condition 1.**

$$\frac{k_0 - 2la - 2lG_0^e}{1+2lb} < \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right]}{v + \left(1 - \delta + z\right)n} < \frac{k_0 - 2la}{1+2lb}, \tag{57}$$

*where the constants* $(n, q, z, v, w)$ *are defined in Appendix A.*

For the collateral channel to be nontrivial, we also restrict attention to situations in which, if demanders of collateral commodity participate in the market, they import a positive amount. This amounts to the condition that

$$\overline{S}_0^i(R^i - r^i) + (1-\delta)E[\overline{S}_1^i] - (\overline{S}_0^e + h)(1 + R^e) > 0. \tag{58}$$

Evaluating the above inequality at the equilibrium prices, we get the following technical condition:

**Technical Condition 2.**

$$\frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right]}{v + \left(1 - \delta + z\right)n} < \frac{q}{1 - \delta + w}, \tag{59}$$

*where the constants* $(n, q, z, v, w)$ *are defined in Appendix A.*

Technical Condition 2 implies that a positive quantity of commodity is imported for collateral purposes in equilibrium. Because the financial investors engaging in this trade are risk-averse, the expected marginal profit of importing commodity as collateral must be positive in equilibrium. That is, we have

$$S_0(R^i - r^i) + (1-\delta)E[S_1^i] - (S_0^e + h)(1 + R^e) > 0. \tag{60}$$

Evaluating the above equation at the equilibrium prices given in Proposition 1, we have the following corollary.

**Corollary 1.** *In the equilibrium of Proposition 1,*

$$S_0^i < \frac{q - \frac{2l(1+R^e)}{1-\delta}\eta}{1 - \delta + w}. \tag{61}$$

# 5 The Effects of Demand for Collateral Commodity and Comparative Statics

In this section we compare the equilibrium of Proposition 1 to the equilibrium of Proposition 2. The difference is the effect of collateral commodity on the spot and futures prices, convenience yield, inventory, futures risk premium, and real demand for commodities. We also study how the unsecured interest rate $R^i$ in the importing country affects all these equilibrium variables in the equilibrium of Proposition 1.

## 5.1 Prices

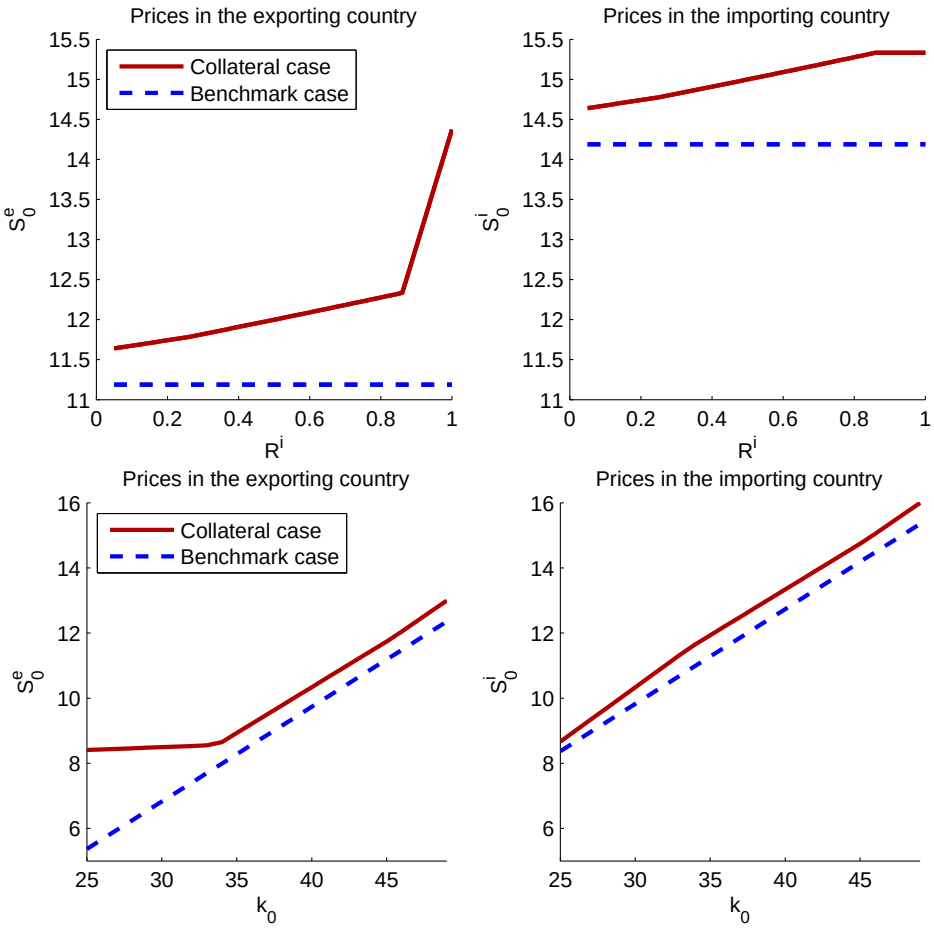
**Proposition 3** (Effect on Prices)**.** *The collateral demand for commodities:*

1. *Increases the spot prices at time 0, $S_0^e$ and $S_0^i$.*
2. *Reduces the spot prices at time 1, $S_1^e$ and $S_1^i$, for a fixed fundamental shock $k_1$.*
3. *Makes $S_0^i - (S_0^e + h) < 0$ if and only if $\eta > 0$. That is, the spot price differential can become smaller than the shipping cost if fundamental commodity demand is entirely met by local supply (i.e. $D_{0,f}^i = 0$).*
4. *Reduces the futures price $F^i$ in the importing country.*

The intuition for Proposition 3 is simple. Since there is an extra collateral demand for commodities at time 0, it will increase the spot price at time 0. However, because these extra collateral commodities are sold at time 1, they reduce the time-1 spot prices $S_1^i$ as well as the time-0 futures price $F^i$ in the importing country. In particular, if $\eta > 0$, the time-0 spot price in the exporting country becomes higher than the local spot price in the importing country plus shipping cost. Thus, it is not optimal for the fundamental demanders to import commodity; instead, they buy commodities exclusively in the local markets. The commodity supply in the exporting country is absorbed entirely by the demand for commodity collateral.

The top half of Figure 3 plots the spot prices in the exporting country and importing country as functions of the uncollateralized rate $R^i$. As shown in Proposition 3, the collateral demand increases the spot prices in both countries. Without collateral demand, the spot prices are invariant to $R^i$; with collateral demand, spot prices are weakly increasing in $R^i$. The kinks are where the constraints start to bind. In this example, if $R^i$ goes above 0.87, the spot price in the exporting country $S_0^e$ becomes so large that fundamental demand is entirely met by domestic supply (Case 4 of Proposi-

Figure 3: Spot prices as functions of $R^i$ and $k_0$, with and without collateral demand. Model parameters: $r^e = 0.01, r^i = 0.01; R^e = 0.05, \delta = 0.05, h = 3, G_0^e = 20, G_1^e = 21, k_0 = 45, u_k = 80, \sigma_k = 2, l = 0.1, a = -5, b = 10, \gamma_p^e = 2, \gamma_s^e = 2, \gamma_d^i = 5$, and $\gamma_c^i = 5$.




tion 1). Consequently, $S_0^e$ increases sharply in $R^i$, but $S_0^i$ is invariant to $R^i$. The price differential, $S_0^i - S_0^e$, becomes smaller than the shipping cost $h$.

The bottom half of Figure 3 plots the spot prices in the two countries as functions of the fundamental activity $k_0$. Naturally, a higher fundamental activity $k_0$ implies a higher spot price. Interestingly, for small $k_0$ (lower than about 34 in this example), the price in the exporting country is relatively insensitive to $k_0$. This is because collateral demand for commodity makes up for the shortfall of fundamental demand. If real activity is sufficiently low, commodity imports are entirely used as collateral, resulting in the spread $S_0^i - S_0^e$ being smaller than the shipping cost $h$, which corresponds to Case 2 of Proposition 1. Once $k_0$ becomes large enough, fundamental demand is partly met by import, and spot prices in two countries increase in $k_0$ in tandem, which corresponds

to Case 1 of Proposition 1.

## 5.2 Convenience Yield

The convenience yield of a commodity is the benefit of holding this commodity versus holding a futures for forward contract. Economically, the convenience yield can come from the real option of starting production anytime, especially if the commodity is scarce and cannot be bought quickly in the spot market. The convenience yield can be mathematically defined as the carry cost of the commodity less the spot-futures spread. In our model, the convenience yield in country $j \in \{e, i\}$ is obtained as

$$y^j = -\frac{F^j}{S_0^j} + \frac{1 + r^j}{1 - \delta}. \tag{62}$$

From (11), one can see that the convenience yield $y^e$ in the exporting country is

$$y^e = \frac{\lambda}{S_0^e (1 - \delta)}. \tag{63}$$

Note that the convenience yield arises because stockout can happen ($I_0^e = 0$) in the exporting country, which corresponds to a positive $\lambda$. This is consistent with the theory of storage, in which the convenience yield arises because of the possibility of a stockout (see, for example, Deaton and Leraque (1992, 1996) and Routledge, Seppi and Spatt (2000)).

From (31), with collateral demand, the convenience yield in the importing country is

$$y^i = \frac{R^i - R^e}{1 - \delta} - \frac{2l (1 + R^e)}{(1 - \delta) S_0^i} \eta. \tag{64}$$

It is linearly related to the spread between the uncollateralized interest rates in the two countries. This convenience yield arises because of collateral demand for financing, not real demand for production. To distinguish it from the theory of storage, we call $y^i$ "the convenience yield of collateral."

**Proposition 4** (Effect on Convenience Yield)**.** *The collateral demand for commodities:*

1. *Increases the convenience yield in the importing country and makes the futures curve exhibit more backwardation (i.e. reducing $F^i/S_0^i$).*
2. *Increases the convenience yield in the exporting country and makes the futures curve exhibit more backwardation (i.e. reducing $F^e/S_0^e$) if and only if $\lambda > 0$ (i.e. a stockout).*

The results in Proposition 4 come from the convenience yield of collateral. Intuitively, commodity as collateral offers the holders the benefit of earning the spread between collateralized and uncollateralized interest rates. If the uncollateralized interest rates are the same in both countries, $R^i = R^e$, the collateral demander will not import and hence the convenience yield of collateral is zero. If, however, $R^i > R^e$, collateral demander will benefit from importing commodity as collateral. In particular, $\eta > 0$ arises if collateral demand drives up the spot price in the exporting countries so much that the local spot price in the importing countries is lower than the foreign spot prices plus shipping cost, as shown in (29). This high spot price in the importing country corresponds to a higher convenience yield. By contrast, the convenience yield in the importing country without collateral demand is given by

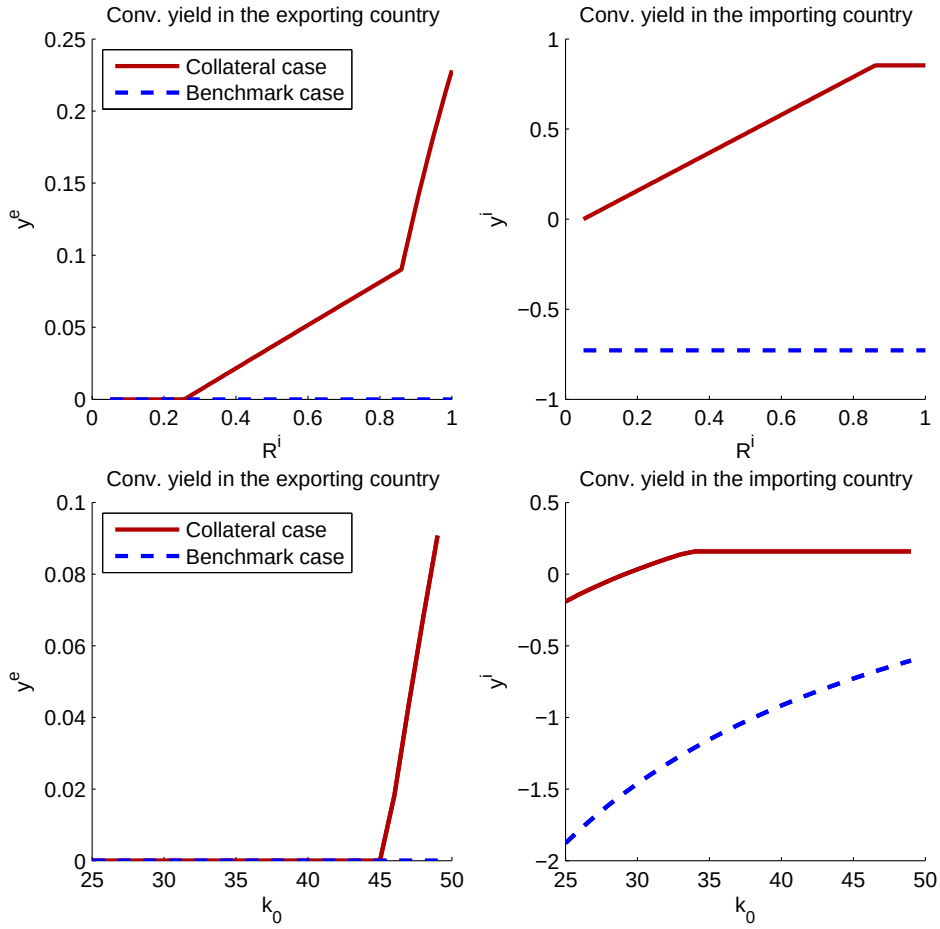
$$\overline{y}^i = -\frac{q}{\overline{S}_0^i} + (1-\delta) + \frac{1+r^i}{1-\delta}, \tag{65}$$

which is not a function of $R^e$ or $R^i$. (The equilibrium $\overline{S}_0^i$ does not depend on $R^e$ or $R^i$.) Proposition 4 reveals that in equilibrium, the collateral demand for commodity increases the convenience yield relative to the benchmark case, i.e., $y^i > \overline{y}^i$.

The effect on convenience yield is closely related to the effect on the shape of the spot-futures spread $(F^i/S_0^i)$. Since the collateral holders in the importing country need to short futures contracts to hedge their risk, the hedging activity decreases the futures price in the importing country. As shown in Proposition 3, the collateral demand increases the spot price. Thus, the futures curve exhibits more backwardation. In the mean time, the collateral demand for commodities makes stockout more likely in the exporting country. If stockout happens, the futures curve in the exporting country also exhibits more backwardation, and the convenience yield becomes positive.

The top half of Figure 4 plots the convenience yields in the two countries as functions of $R^i$. The collateral demand for commodity weakly increases the convenience yields in both countries; so does an increase in $R^i$. A positive convenience yield in the exporting country obtains only if a stockout occurs, and the speed at which $y^e$ increases in $R^i$ is elevated once the importing constraint $D_{0,f}^i \geq 0$ starts to bind. In the importing country, the convenience yield increases almost one-for-one with $R^i$ (see (64)). This increase continues until $R^i$ becomes so high that fundamental demand is entirely met by local supply, at which point the convenience yield becomes flat in $R^i$. The bottom half of Figure 4 plots the convenience yields as functions of $k_0$. As expected, a higher $k_0$ implies a higher demand at $t = 0$ and a higher convenience yield.

Figure 4: Convenience yields as functions of $R^i$ and $k_0$, with and without collateral demand. Model parameters are those of Figure 3.




## 5.3 Inventories

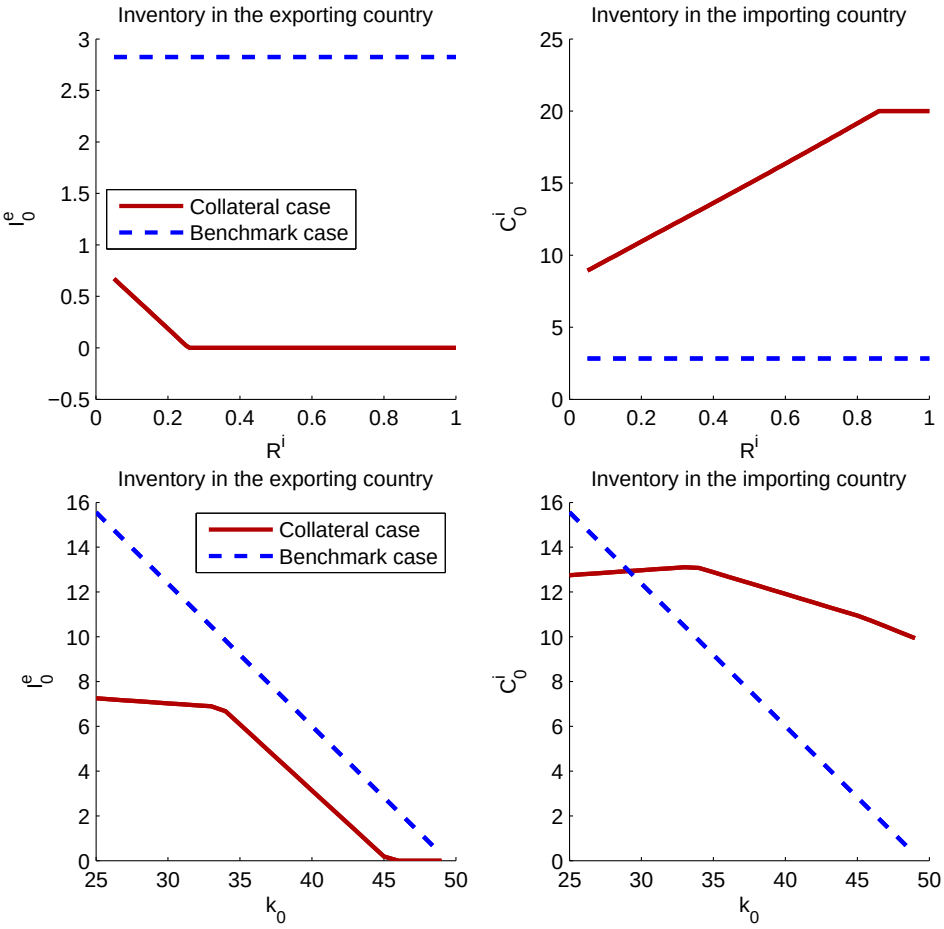
**Proposition 5** (Effect on Inventories)**.** *The collateral demand for commodities:*

1. *Reduces the inventory $I_0^e$ in the exporting country.*
2. *Increases the inventory $C_0^i$ in the importing country.*
3. *Increases the total inventory $I_0^e + C_0^i$ in both countries.*

Since the additional collateral demand increases the spot prices, the producer in the exporting country holds less inventory. Obviously, inventory in the importing country, $C_0^i$, goes up because collateral commodity must be stored. Total global inventory also goes up. Combining Proposition 4 and Proposition 5, we obtain the following corollary:

**Corollary 2.** *The collateral demand for commodities makes the correlation between inventory and convenience yield positive in the importing country.*

Figure 5: Inventories as functions of $R^i$ and $k_0$, with and without collateral demand. Model parameters are those of Figure 3.




The top half of Figure 5 plots the inventory in the exporting country and the importing country as functions of $R^i$. As we show in Proposition 5, the collateral demand for commodities reduces inventory in the exporting country but increases the inventory in the importing country.

The bottom half of Figure 5 plots the inventories as functions of $k_0$. As expected, inventory is decreasing in $k_0$ in the exporting country. Interestingly, in the importing country and the collateral case, inventory is nonmonotone in $k_0$ for small $k_0$. This is because collateral demand more than makes up for low real demand if $k_0$ is low. If $k_0$ is sufficiently high, real demands dominate and inventory decreases in real demand.

## 5.4 Commodity Futures Risk Premium

**Proposition 6** (Effect on Commodity Futures Risk Premium)**.** *The collateral demand for commodities:*

1. *Reduces the futures risk premium in the exporting country,* $E[S_1^e - F^e]$.
2. *Increases the futures risk premium in the importing country,* $E[S_1^i - F^i]$.

Equation (12) shows that the futures risk premium is proportional to $[I_0^e(1-\delta) + G_1^e]$, which can be considered as the total amount the producers need to hedge. The theory of normal backwardation as in Keynes (1923), Hirshleifer (1990) and Bessembinder (1992) argues that hedgers need to offer risk premiums in order to solicit speculators to offset their trades; thus, the futures risk premium relates positively to the amount producers hedge. Our model is consistent with this idea. The "extra" collateral demands reduces the amount of inventory $I_0^e$, which in turn reduces the futures risk premium in the exporting country. Similarly, in the importing country, the amount to hedge is just the amount of collateral in time 1, $(1-\delta)C_0^i$. Therefore, consistent with the theory of normal backwardation, our model shows that the futures risk premium in the importing country is proportional to $(1-\delta)C_0^i$. In the benchmark case, $E[S_1^i - F^i] = 0$; hence the futures risk premium is zero. In the collateral case, however, the futures risk premium is positive because $C_0^i > 0$. Thus, the futures risk premium in the importing country increases in the collateral demand for commodity.

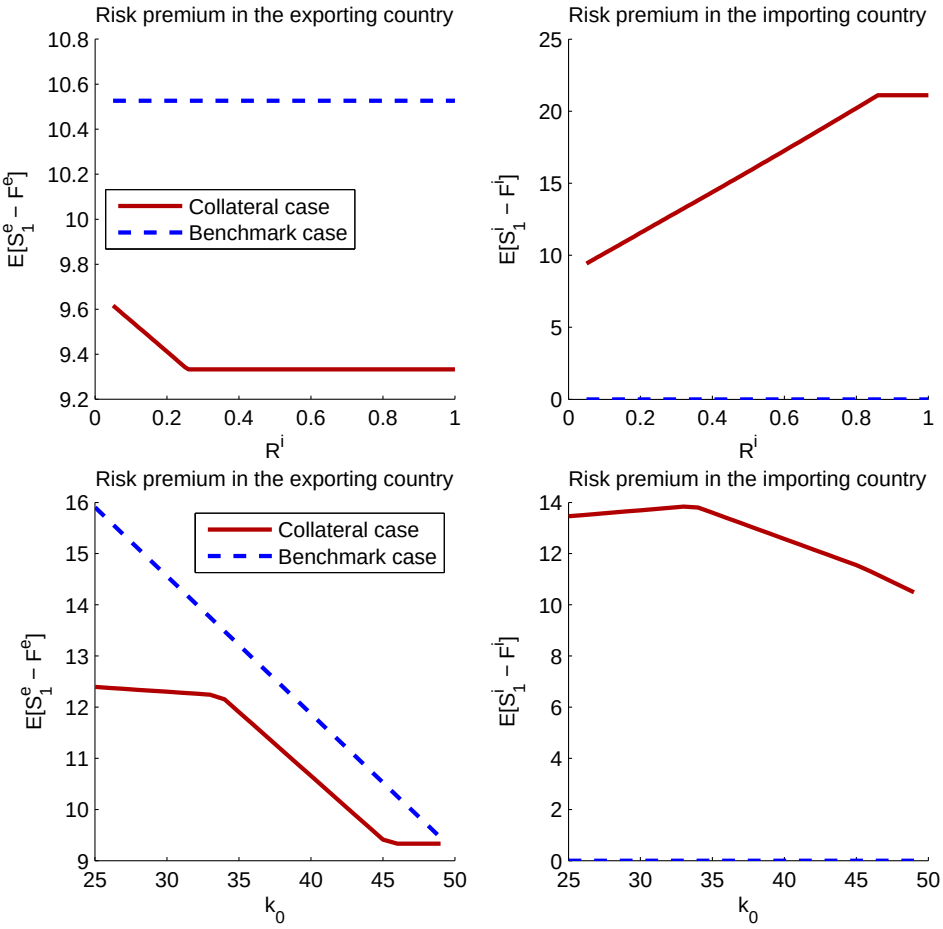
Figure 6 plots the futures risk premium in two countries as functions of $R^i$ and $k_0$. Collateral demand for commodity reduces the futures risk premium in the exporting country but increases that in the importing country. Comparing Figure 6 with Figure 5, we observe that the shapes of the futures premium resemble those of the inventories. In the exporting country, this pattern is natural because the higher is the futures risk premium, the more expensive it is for the producers to hedge, and the larger is the inventory held by producers. In the importing country, the futures risk premium is positive because collateral commodity must be sold at $t = 1$. The larger is the import for collateral purposes, the higher is the selling pressure at $t = 1$, and the higher is the futures risk premium.

## 5.5 Real Demand for Commodity

**Proposition 7** (Effect on Real Demand)**.** *For a fixed fundamental shock* $k_1$*, the collateral demand for commodities:*

1. *Reduces the real demand for commodities in the importing country at time 0.*

Figure 6: Commodity futures risk premium as functions of $R^i$ and $k_0$, with and without collateral demand. Model parameters are those of Figure 3.




2. *Increases the real demand for commodities in the importing country at time 1.*
3. *Reduces the sum of time-0 and time-1 real demands for commodities in the importing country.*

As the collateral demand increases the spot price at time 0 in the importing country, it also decreases the real demand for the commodity. As the collateral commodities are sold at time 0, the spot price at time 1 decreases in the importing country and the real demand increases. But because a larger amount of commodities is stored between the two periods, a larger deadweight loss is incurred. The total real demand is reduced by the demand for collateral commodity.

This real effect of using commodity as collateral complements to that of Kiyotaki and Moore (1997). In their model, production assets such as land and machineries can also be pledged as collateral. They show that a small, temporary negative shock to

firms' net worth can be amplified to large, persistent shock to the prices of assets and firms' investments and production. Our model is complementary in that the production asset, commodity, is a traded asset, and firms not involved in the real production can also import commodity to generate financial returns. In our model, if the production functions of the real sector is invariant to the interest rate, as we implicitly assume in Proposition 7, more financial demand for commodity crowds out the real demand by increasing commodity spot prices and by increasing the deadweight loss of commodity storage. If, however, production constraint can be relaxed by importing commodities as collateral, we may reasonably expect the collateral demand for commodity to increase total output at the cost of amplification and fragility, as in Kiyotaki and Moore (1997). The latter effect is not in our current analysis because we expect it to be similar to that modeled by Kiyotaki and Moore (1997). The welfare implications of using commodities as collateral is therefore ambiguous.

## 5.6 Comparative Statics with respect to Interest Rate $R^i$

By the same intuition as the effects of collateral commodity, we can derive the effect of raising interest rates $R^i$ in the importing country in the equilibrium of Proposition 1.

**Proposition 8.** *Holding other parameters fixed, in Case 1 of the equilibrium of Proposition 1, as the uncollateralized interest rate $R^i$ increases in the importing country:*

1. *The spot prices in importing and exporting countries at time 0, $S_0^i$ and $S_0^e$, increase.*
2. *The collateral demand $C_0^i$ increases, the inventory $I_0^e$ in the exporting country decreases, and the total inventory increases.*
3. *The convenience yield in the importing country $y^i$ increase.*
4. *The futures risk premium in the exporting country $E\left[S_1^e - F^e\right]$ decreases, and the futures risk premium in the importing country $E\left[S_1^i - F^i\right]$ increases.*

*Moreover, holding other parameters fixed:*

5 *If an increase in $R^i$ causes the constraint $I_0^e \geq 0$ to bind, then the convenience yield in the exporting country, $y^e$, increases.*

6 *If an increase in $R^i$ causes the constraint $D_{0,f}^i \geq 0$ to bind, then the spot price spread, $S_0^i - S_0^e$, decreases.*

Parts 1–4 of Proposition 8 are written for Case 1 of Proposition 1 for simplicity, but the same qualitative results hold for other three cases. The only caveat is that if $R^i$ is

sufficiently high, certain endogenous variables may become flat in $R^i$. For instance, if $\eta > 0$, $S_0^i$ and $I_0^e + C_0^i$ are invariant to $R^i$.

Proposition 8 complements existing theory and evidence on the relation between interest rate and (real) commodity prices. For example, Frankel (1986, 2008) show that high interest rates reduce the price of storable commodities by increasing the incentive for commodity extraction now rather than in the future, by decreasing firms' desire to carry inventories, and by encouraging speculators to shift out of commodity contracts and into Treasury bills. He finds a significant and negative coefficient of real commodity price on the real US interest rate, representing global monetary policy, as well as on the real interest rate differential between the non-US countries and the US, representing local variations in monetary policy. The foreign countries used in Frankel's analysis include Australia, Brazil, Canada, Chile, Mexico, New Zealand, Switzerland and UK. The first six countries are major exporting countries of commodities, whereas the last two are important commodity trading centers that hold large inventories. Frankel's results, as well as the explanation based on costs of commodity extraction and inventory, apply well in these countries.

Complementary to Frankel's work, our result focuses on the collateral channel, which applies to countries that *import* commodities to circumvents capital control. For these countries, most notably China, a higher uncollateralized interest rate is likely to increase the demand for collateral and hence increase the global price of the commodity, holding all else equal. The net effect depends on which demand, production or collateral, reacts faster and in larger magnitude to a higher interest rate in importing countries.

# 6 Empirical Evidence

In this section, we test the predictions of our model. Ideally, one would want to measure how much commodities have been pledged to lenders as financing collateral. Unfortunately, such data are unavailable, except the approximate industry estimate (see the Introduction). Instead, we take an indirect, theory-driven approach. We first construct a proxy for the attractiveness of importing commodities as collateral. Then, motivated by the predictions of our theory (Section 5), we test how this proxy affects: (i) commodity prices, and (ii) commodity futures risk premium, and (iii) the relation between inventory and convenience yield.

## 6.1 Data: Proxy for Collateral Demand for Commodities

As we have discussed so far in this paper, importing commodities as collateral requires two important conditions. First, unsecured interest rates in China must be sufficiently high relative to that in developed markets. Second, because of capital control, it is difficult to take advantage of this gap by directly moving financial capital into China.

As proxies for unsecured interest rates in China and developed markets, we use CNY Shibor, the Shanghai Interbank Offered Rate, and USD Libor, the London Interbank Offered Rate.[11] The gap in unsecured rates is:

$$\pi_t = \text{Shibor}_t - \text{Libor}_t. \tag{66}$$

In the normal case of the model (where neither Lagrange multiplier is binding), the expected profit for importing one unit of commodity as collateral can be written as (see (24)):

$$S_0^i(R^i - R^e) + (1-\delta)E[S_1^i] - S_0^i(1+r^i). \tag{67}$$

The first term of the above expression is proportional to the spread of the unsecured interest rates, $R^i - R^e$. The last two terms are the expected net cost of carrying one unit of inventory over one period, which is not related to the collateral channel. Our sample period is from October 13, 2006 to November 14, 2014, with 423 weekly observations.

Figure 7(a) plots the time-series behaviors of Libor and Shibor. While Libor and Shibor are comparable before 2009, Shibor raises substantially above Libor after 2009.

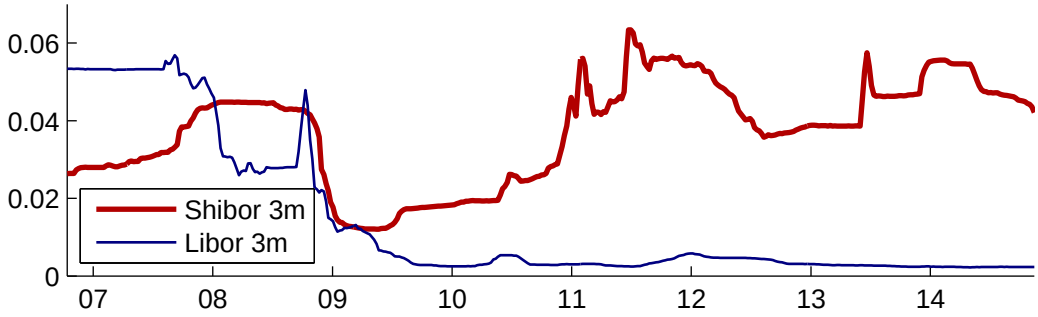
To measure the tightness of capital control in China, we use the percentage deviation from the covered interest rate parity (CIP) in the USDCNY exchange rate:

$$f_t = \frac{\text{USDCNYSpot}_t \cdot e^{(\text{Shibor}_t - \text{Libor}_t)/4}}{\text{USDCNYForward}_t} - 1, \tag{68}$$

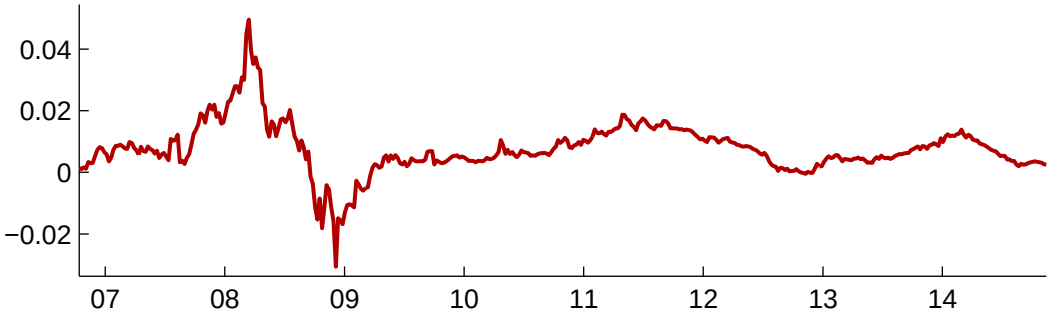
where $\text{USDCNYSpot}_t$ is the official spot USDCNY exchange rate and $\text{USDCNYForward}_t$ is the 3-month non-deliverable forward (NDF) exchange rate of USDCNY, both downloaded from Datastream. An NDF is the same as a usual forward contract except that on the delivery date, the NDF is cash settled in USD, rather than by physically delivering CNY against USD. This is because the CNY is not freely convertible and physical delivery is difficult, if possible at all. Before the development of the offshore CNY market in mid-2010, the NDF market is the predominant means for foreign in-

[11] Another measure would be the unsecured interest rates paid by nonbank firms, but such data could not be found.

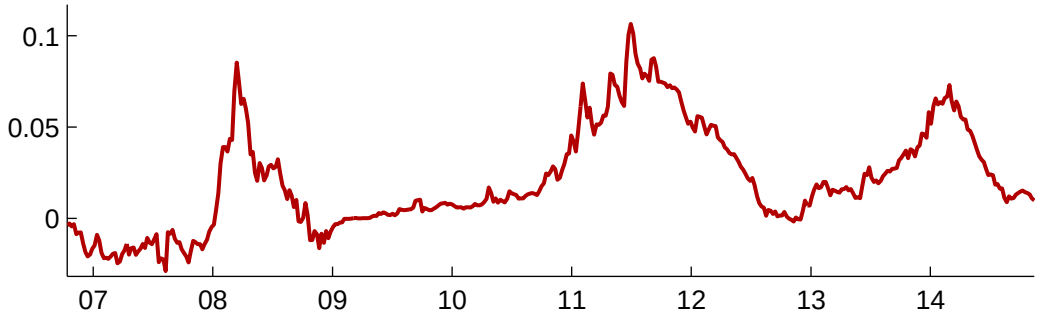
Figure 7: Collateral-demand-for-commodities proxy and its components




(a) Libor and Shibor




(b) Deviation from covered interest rate parity $f_t$




(c) Collateral-demand-for-commodities proxy $X_t$

vestors to take positions on the CNY. For more details of the USDCNY NDF, see Yu (2007) and ASIFMA (2014).

Figure 7(b) plots the time series of the the deviation from CIP. The deviation $f_t$ is positive most of the time, implying that the market expects the appreciation of CNY against USD. The sole exception is in late 2008, the depth of the crisis. Because of

capital control, this deviation from the CIP cannot be eliminated by the usual arbitrage trades, which involve buying CNY spot and selling CNY forward, both physically delivered. The higher is the deviation, the stronger is the incentive to gain access to CNY investments by circumventing capital control, such as by importing commodities. The deviation $f_t$ is thus a proxy for the tightness of capital control.

Our final proxy for the collateral demand for commodities is the product of the Shibor-Libor spread, $\pi_t$, and the deviation from covered interest rate parity, $f_t$, multiplied by 100:

$$X_t = 100\pi_t f_t. \tag{69}$$

This measure is shown in Figure 7(c). Since 2008 this measure is predominantly positive and strongly time-varying. The only periods when $X_t$ is negative are in early part of the sample, before 2008 and in late 2008, and the magnitude is small. The sample mean and standard deviation of $X_t$ are 0.021 and 0.028, respectively.

## 6.2 Data: Commodity Prices and Inventories

Commodities that we use to test the theory are selected by two criteria. First, the commodities should have active futures or forward markets in China and in developed countries (e.g. US, UK, Japan). Second, data for commodity prices and inventories should go back to at least the start of 2009, when Shibor started to raise substantially above Libor.

Applying these two criteria, we end up with eight commodities: copper, zinc, aluminum, gold, soybean, corn, fuel oil, and natural rubber. We call the first four commodities the metal group, and the last four commodities the non-metal group. We would expect the metals to be more suitable for collateral purposes as they are easier to store and have a higher value-to-bulk ratio than non-metal commodities. Thus, our model implications should be more evident in the metal group than in the nonmetal group.

For each commodity, we use the leading exchange in China and the leading exchange in developed markets as price data source. With few exceptions, we take the prices of the first and third futures contracts in both the Chinese market and developed markets.[12,13] Also with few exceptions, all price and inventory data are weekly

[12]Exceptions include the following. The price data for copper, zinc, and aluminum are obtained from LME as cash price and 3-month forward price, not futures prices. For some commodities we use the second contract. Since fuel oil futures are not available in the US, we use CME heating oil futures to proxy the fuel oil futures. (Fuel oil is one type of heating oil.)

[13]All prices are in the original currency units. We do not convert CNY to USD as CNY is not fully convertible.

Table 2: Data sources of commodities prices and inventories

*Acronyms.* SHFE: Shanghai Futures Exchanges. LME: London Metal Exchange. DCE: Dalian Commodity Exchanges. CME: CME Group. TOCOM: Tokyo Commodity Exchange. USDA: United States Department of Agriculture.

| | Price data source | | Inventory data source | |
|---|---|---|---|---|
| Commodity | China | Developed Market | China | Developed Market |
| Copper | SHFE, first and third futures | LME, cash and 3-month forward | SHFE | LME |
| Zinc | SHFE, first and third futures | LME, cash and 3-month forward | SHFE | LME |
| Aluminum | SHFE, first and third futures | LME, cash and 3-month forward | SHFE | LME |
| Gold | SHFE, first and third futures | CME, first and third futures | SHFE | CME |
| Soybean | DCE, first and third futures | CME, first and second futures | USDA | USDA |
| Corn | DCE, first and third futures | CME, first and second futures | USDA | USDA |
| Fuel oil | SHFE, first and third futures | CME, first and third futures | SHFE | CME |
| Natural rubber | SHFE, first and third futures | TOCOM, first and second futures | SHFE | TOCOM |

observations from October 13, 2006 to November 14, 2014.

Following the standard approach in the literature (see, for example, Gorton, Hayashi, and Rouwenhorst (2012)), we proxy commodities inventories by those in exchange warehouses whenever available. For our purposes of studying time variations, the inventory in exchange warehouses is a reasonable proxy for the market-wide inventory as long as they are sufficiently correlated with each other. Inventory data for copper, zinc, aluminum, gold, fuel oil, and natural rubber are obtained from various exchanges this way. Inventories of two agricultural commodities, soybean and corn, are obtained from U.S. Department of Agriculture.

Table 2 summarizes the data sources for commodity prices and inventories.

## 6.3 Commodity Prices

Proposition 3 and Proposition 8 predict that the collateral demand for commodities increases their spot prices. To test this prediction, the first step is to collect spot prices. For copper, zinc, and aluminum on the LME, cash prices are readily available. For other commodities, we follow Pindyck (2001) and infer the spot prices from traded futures prices by extrapolation.

We denote by $y_t$ the convenience yield in the Chinese market or developed markets, calculated as

$$y_t = \frac{\ln(F(t,T_1)) - \ln(F(t,T_2))}{T_2 - T_1} + \gamma_t, \tag{70}$$

where $F(t,T_1)$ and $F(t,T_2)$ are futures prices at week $t$ with maturity $T_1$ and $T_2$,

respectively, and $\gamma_t$ is the 3-month local interest rate (Shibor or Libor).

Then, for each commodity, we regress the log price change on contemporaneous changes in local convenience yield, local interest rate, and the collateral-demand-for-commodities proxy:

$$\Delta \ln(S_t) = a + b\Delta y_t + c\Delta\gamma_t + d\Delta X_t + \epsilon_t. \tag{71}$$

The local convenience yield and local interest rates are control variables for the benefit and opportunity cost of holding commodities.

Existing literature documents that commodities prices are effected by convenience yield and interest rates. For example, Pindyck (1993) argues that because the convenience yield is considered a benefit of holding commodities, spot prices should have a cointegration relation with convenience yield. Frankel (2008) shows that a higher interest rate is associated with lower commodity prices.

We also run separate panel regressions on the the metal group and nonmetal group:

$$\Delta \ln(S_{i,t}) = a_i + b\Delta y_{i,t} + c\Delta\gamma_{i,t} + d\Delta X_t + \epsilon_{i,t}. \tag{72}$$

Our theory predicts that the coefficient $d$ on $\Delta X_t$ should be positive in both China and developed markets. Because our theory makes a specific prediction about the sign of $d$, we use the one-sided critical value of 1.65 in interpreting the statistical significance of $d$ at the 5% level.

Table 3 reports the results. As predicted by our theory, the panel regression for the metal group shows a significantly positive $d$, suggesting that a higher demand to import commodities as collateral to China is associated with a higher commodity prices globally. Commodity-by-commodity regressions reveal a significantly positive $d$ for copper, zinc, and aluminum on the LME and gold in China. All other individual regressions are statistically insignificant but show the expected sign.

By contrast, the nonmetal group of commodities generally have an insignificant coefficient $d$ on $\Delta X_t$, although they all have the expected sign. This insignificance is intuitive. Because the nonmetal group of commodities are bulky and relatively expensive to store and ship, they are not as desirable collateral as metals. (In our model, agricultural commodities and oil can be viewed as having a large shipping cost $h$ and a large storage cost $\delta$.)

The price effect of commodity collateral is economically large. The minimum and maximum of $X_t$ in our sample are $-0.029$ and 0.106, with a range of about 0.135. Counting only the statistically significant entries in the metal group, we see that an in-

Table 3: Commodity spot prices

Panel (a) reports the panel regressions (72) for the metal group and nonmetal group, where we have suppressed the commodities fixed effects $\{a_i\}$. Panel (b) reports the regressions (71) for individual commodities. Standard errors are calculated using the Newey-West method with 52 lags.

| | **Metal Group** | | | | **Nonmetal Group** | | | |
|---|---|---|---|---|---|---|---|---|
| **(a) Panel** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| $\Delta y_{it}$ | 0.111 | 7.222 | 0.920 | 7.796 | 0.154 | 8.215 | 0.133 | 6.806 |
| $\Delta \gamma_{it}$ | -0.765 | -1.451 | -0.444 | -0.608 | -0.594 | -0.953 | -4.587 | -3.276 |
| $\Delta X_t$ | 0.653 | 2.630 | 0.893 | 3.899 | 0.247 | 1.266 | 0.405 | 1.469 |
| $R^2$ | 0.072 | | 0.102 | | 0.318 | | 0.195 | |
| $N$ | 1536 | | 1536 | | 1688 | | 1688 | |
| **(b) Individual** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| | **Copper** | | | | **Soybean** | | | |
| const | -0.001 | -0.516 | 0.000 | -0.156 | 0.001 | 0.441 | 0.001 | 0.436 |
| $\Delta y_{it}$ | -0.069 | -1.112 | 0.894 | 6.032 | 0.192 | 6.845 | 0.155 | 5.559 |
| $\Delta \gamma_{it}$ | 0.652 | 0.630 | -0.859 | -0.533 | -0.555 | -0.691 | -3.308 | -1.850 |
| $\Delta X_t$ | 0.772 | 1.417 | 1.114 | 2.306 | 0.436 | 1.236 | 0.512 | 1.008 |
| $R^2$ | 0.029 | | 0.057 | | 0.340 | | 0.230 | |
| $N$ | 422 | | 422 | | 422 | | 422 | |
| | **Zinc** | | | | **Corn** | | | |
| const | -0.002 | -0.761 | -0.001 | -0.556 | 0.001 | 1.826 | 0.000 | -0.018 |
| $\Delta y_{it}$ | 0.039 | 0.952 | 1.173 | 8.947 | -1.372 | -1.874 | -4.402 | -3.743 |
| $\Delta \gamma_{it}$ | -2.123 | -2.477 | -0.359 | -0.294 | 0.315 | 9.047 | 0.180 | 8.873 |
| $\Delta X_t$ | 0.987 | 1.613 | 1.009 | 1.944 | 0.001 | 0.003 | 0.548 | 0.807 |
| $R^2$ | 0.019 | | 0.140 | | 0.469 | | 0.138 | |
| $N$ | 395 | | 395 | | 422 | | 422 | |
| | **Aluminum** | | | | **Fuel Oil** | | | |
| const | -0.001 | -0.902 | -0.001 | -0.388 | 0.000 | 0.130 | 0.000 | 0.181 |
| $\Delta y_{it}$ | 0.123 | 2.886 | 1.002 | 4.979 | 0.152 | 6.306 | 0.487 | 11.848 |
| $\Delta \gamma_{it}$ | 0.412 | 0.591 | 0.046 | 0.055 | -1.925 | -2.178 | -1.277 | -1.182 |
| $\Delta X_t$ | 0.183 | 0.845 | 0.880 | 2.506 | 0.591 | 1.430 | 0.444 | 1.200 |
| $R^2$ | 0.092 | | 0.167 | | 0.470 | | 0.177 | |
| $N$ | 422 | | 422 | | 422 | | 422 | |
| | **Gold** | | | | **Rubber** | | | |
| const | 0.001 | 0.353 | 0.000 | 0.278 | -0.001 | -0.496 | -0.001 | -0.410 |
| $\Delta y_{it}$ | 0.143 | 20.638 | 0.397 | 14.376 | 0.094 | 2.046 | 0.116 | 5.532 |
| $\Delta \gamma_{it}$ | -2.140 | -3.761 | -9.096 | -2.082 | 1.246 | 0.882 | -9.118 | -4.800 |
| $\Delta X_t$ | 0.715 | 2.644 | 0.332 | 1.062 | 0.079 | 0.164 | 0.081 | 0.151 |
| $R^2$ | 0.437 | | 0.055 | | 0.049 | | 0.296 | |
| $N$ | 297 | | 297 | | 422 | | 422 | |

crease of $X_t$ of size 0.135 corresponds to 15.0% increase in copper price, 13.6% increase in zinc price, and 11.9% increase in aluminum price on the LME. The corresponding effect on gold price in China is 9.6%. These estimates reveal that China-specified frictions, reflected by the collateral demand for commodities, have important quantitative effects on global commodity prices.

## 6.4 Commodity Futures Risk Premium

Proposition 6 and Proposition 8 predict that an increase in the collateral demand for commodities tends to increase the futures risk premium in the importing country and reduce that in the exporting country. To test this prediction, we denote by

$$\theta_t = \ln(F(t,T)) - \ln(F(t-1,T)) \tag{73}$$

the excess return of holding the far-maturity futures contract for one week. For LME metals (copper, zinc, aluminum), this return is calculated from the 3-month forward with a small adjustment.[14]

Then, we run the following regression, commodity by commodity:

$$\theta_t = a + by_{t-1} + c\gamma_{t-1} + dX_{t-1} + \epsilon_t. \tag{74}$$

We control for the convenience yield $y_{t-1}$ and lagged local interest rate $\gamma_{t-1}$. Gorton, Hayashi and Rouwenhorst (2012) show that convenience yield has a positive relation with commodity risk premium. Frankel (2008) argues that high interest rates tend to reduce inventory, which, in turn, can affect convenience yield through the hedging channel.

As before, we also run the panel-data version for the metal group and nonmetal group separately:

$$\theta_{i,t} = a_i + by_{i,t-1} + c\gamma_{i,t-1} + dX_{t-1} + \epsilon_{i,t}. \tag{75}$$

Our model predicts that the coefficient $d$ on the collateral-demand-for-commodities proxy $X_t$ should be positive in the Chinese market but negative in developed markets. As before, we use the one-sided critical value of 1.65 in interpreting the statistical significance of $d$ at the 5% level.

[14]Specifically, let $F_{t,t+13}$ be the 3-month forward price of the commodity observed in week $t$. The hypothetical forward price with 14 weeks to maturity in week $t$ is approximated by $F_{t,t+14} = F_{t,t+13}e^{(\gamma_t - y_t)/52}$. In week $t+1$, the 3-month forward contract matures in week $t+14$. So the return is $\ln(F_{t+1,t+14}) - \ln(F_{t,t+14}) = \ln(F_{t+1,t+14}) - \ln(F_{t,t+13}) - \frac{1}{52}(\gamma_t - y_t)$.

Table 4: Commodity futures risk premium

Panel (a) reports the panel regressions (74) for the metal group and nonmetal group, where we have suppressed the commodities fixed effects $\{a_i\}$. Panel (b) reports the regressions (74) for individual commodities. Standard errors are calculated using the Newey-West method with 52 lags.

| | **Metal Group** | | | | **Nonmetal Group** | | | |
|---|---|---|---|---|---|---|---|---|
| **(a) Panel** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| $y_{i,t-1}$ | -0.452 | -4.030 | -0.253 | -2.382 | 0.018 | 2.004 | -0.015 | -4.183 |
| $\gamma_{i,t-1}$ | 0.009 | 1.018 | 0.001 | 0.018 | -0.354 | -2.793 | -0.020 | -0.250 |
| $X_{t-1}$ | 0.153 | 3.032 | -0.082 | -2.414 | 0.044 | 0.741 | -0.048 | -1.146 |
| $R^2$ | 0.019 | | 0.010 | | 0.023 | | 0.007 | |
| $N$ | 1536 | | 1536 | | 1688 | | 1688 | |
| **(b) Individual** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| | **Copper** | | | | **Soybean** | | | |
| const | 0.020 | 2.362 | 0.005 | 1.340 | 0.005 | 1.213 | 0.005 | 1.657 |
| $y_{i,t-1}$ | -0.004 | -0.168 | -0.082 | -1.191 | 0.021 | 1.881 | -0.022 | -4.607 |
| $\gamma_{i,t-1}$ | -0.644 | -2.381 | -0.106 | -0.715 | -0.148 | -0.895 | -0.007 | -0.050 |
| $X_{t-1}$ | 0.175 | 1.636 | -0.090 | -1.148 | -0.048 | -0.572 | -0.079 | -1.306 |
| $R^2$ | 0.025 | | 0.011 | | 0.016 | | 0.023 | |
| $N$ | 422 | | 422 | | 422 | | 422 | |
| | **Zinc** | | | | **Corn** | | | |
| const | 0.017 | 3.840 | 0.005 | 1.209 | 0.005 | 3.121 | -0.001 | -0.207 |
| $y_{i,t-1}$ | 0.039 | 2.700 | -0.037 | -0.644 | -0.007 | -1.773 | -0.010 | -0.980 |
| $\gamma_{i,t-1}$ | -0.609 | -3.224 | -0.419 | -1.726 | -0.124 | -1.989 | 0.050 | 0.389 |
| $X_{t-1}$ | 0.220 | 2.215 | -0.130 | -2.397 | 0.030 | 0.901 | 0.009 | 0.161 |
| $R^2$ | 0.024 | | 0.023 | | 0.012 | | 0.003 | |
| $N$ | 395 | | 395 | | 422 | | 422 | |
| | **Aluminum** | | | | **Fuel Oil** | | | |
| const | 0.008 | 2.712 | 0.007 | 2.189 | 0.021 | 2.333 | -0.001 | -0.320 |
| $y_{i,t-1}$ | 0.010 | 1.566 | 0.078 | 1.989 | 0.035 | 3.455 | -0.002 | -0.094 |
| $\gamma_{i,t-1}$ | -0.284 | -2.344 | -0.255 | -1.860 | -0.528 | -1.782 | 0.025 | 0.164 |
| $X_{t-1}$ | 0.097 | 1.542 | -0.070 | -1.205 | 0.134 | 1.117 | 0.049 | 0.854 |
| $R^2$ | 0.020 | | 0.012 | | 0.078 | | 0.001 | |
| $N$ | 422 | | 422 | | 422 | | 422 | |
| | **Gold** | | | | **Rubber** | | | |
| const | 0.009 | 4.358 | 0.000 | -0.094 | 0.021 | 3.671 | 0.007 | 1.201 |
| $y_{i,t-1}$ | 0.003 | 0.431 | -0.166 | -2.708 | -0.011 | -0.979 | -0.016 | -3.685 |
| $\gamma_{i,t-1}$ | -0.383 | -4.205 | -0.070 | -0.142 | -0.647 | -2.655 | -0.201 | -1.114 |
| $X_{t-1}$ | 0.196 | 3.140 | 0.027 | 0.536 | 0.121 | 0.897 | -0.211 | -2.301 |
| $R^2$ | 0.024 | | 0.005 | | 0.027 | | 0.019 | |
| $N$ | 297 | | 297 | | 422 | | 422 | |

Table 4 reports the results. For the metal group the coefficient $d$ on $X_t$ is significantly positive for China and significantly negative for developed markets. Zinc shows the highest significance in both China and the LME, followed by copper and gold (significant only in China). The coefficient $d$ for aluminum has the predicted sign but is statistically insignificant. For the nonmetal group neither the panel regression nor the individual commodity regressions show any statistical significance on $d$. These results are in line with the previous test and support our theory.

The impact of using commodities as collateral on futures risk premium is also economically large. The panel regression for the metal group suggests that a one-standard-deviation increase in $X_t$ (of size 0.028) is associated with a higher futures risk premium of 43 basis points (bps) per week in China, and a lower futures risk premium of 23 bps per week in developed markets (LME and CME). The annualized effect on futures risk premium are $+22.3\%$ in China and $-12.0\%$ in developed markets.

## 6.5 Relation between Inventory and Convenience Yield

A negative relation between inventory and convenience yield is the key element in the theory of storage. In this theory, a low inventory corresponds to a high convenience of holding commodities because it increases the real option value of starting production anytime. In our model of commodity as collateral, however, the relation is the reverse. As shown in Proposition 4, Proposition 5, and Corollary 2, an increasing collateral demand tends to simultaneously increase inventories and convenience yield in the importing country. Thus, complementary to the theory of storage, the collateral demands for commodity should make the inventory-convenience yield relation less negative in China.

We denote by $I_t$ the inventory in China or developed markets. Because inventories tend to have a time trend, we detrend the inventory level by the average inventory over the previous year:

$$\hat{I}_t = I_t - \frac{1}{52}\sum_{j=1}^{52} I_{t-j}. \tag{76}$$

The detrended inventory $\hat{I}_t$ will be our main measure of inventory. Detrending inventory is a common approach in the literature (see, for example, Gorton, Hayashi, and Rouwenhorst (2012)).

To test the inventory-convenience yield relation in the presence of collateral use of commodities, we run the following regression for both China and developed markets,

commodity by commodity:

$$y_t = a + b\hat{I}_t + c\hat{I}_t X_t + \varepsilon_t. \tag{77}$$

In addition, we run separate panel regressions for the metal group and nonmetal group. Because commodity inventories have different scales, to make sure that the coefficients are interpretable we normalize each detrended inventory by its time-series standard deviation:

$$y_{i,t} = a_i + b\frac{\hat{I}_{i,t}}{\sqrt{\text{Var}(\hat{I}_{i,t})}} + c\frac{\hat{I}_{i,t}}{\sqrt{\text{Var}(\hat{I}_{i,t})}}X_t + \varepsilon_{i,t}. \tag{78}$$

In both regressions (77) and (78), the coefficient $b$ captures the effect predicted by the theory of storage, and the coefficient $c$ captures the incremental effect predicted by our model of commodity as collateral. Our theory predicts that $c$ is positive in China, that is, the higher is benefit of importing commodities as collateral, the more positive (or the less negative) is the inventory-convenience yield relation. As before, we use the one-sided critical value of 1.65 in interpreting the statistical significance of $c$ at the 5% level.

Table 5 reports the results of regressions (77) and (78). As predicted by the theory, the panel regression on the metal group in China shows a significantly positive coefficient $c$ on $\hat{I}_{i,t}X_t$. It reveals that the collateral use of commodities has a significant impact on the convenience yield-inventory relation. The same result is observed for all four commodities in the metal group: copper, zinc, aluminum, and gold. By contrast, the coefficient $c$ for the nonmetal group is generally insignificant, in both the panel regression and individual commodity regressions. Overall, these results are consistent with the previous two tests and support our theory.

Table 5: Relation between convenience yield and detrended inventory

Panel (a) reports the panel regressions (78) for the metal group and the nonmetal group, where we have suppressed the commodities fixed effects $\{a_i\}$. Panel (b) reports the regressions (77) for each individual commodity. Standard errors are calculated using the Newey-West method with 52 lags.

| | **Metal Group** | | | | **Nonmetal Group** | | | |
|---|---|---|---|---|---|---|---|---|
| **(a) Panel** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| $\hat{I}_{i,t}$ | -0.078 | -5.970 | -0.012 | -3.986 | -0.029 | -1.864 | -0.027 | -1.618 |
| $\hat{I}_{i,t} \cdot X_t$ | 0.804 | 2.505 | 0.116 | 1.296 | -0.211 | -0.688 | -0.115 | -0.334 |
| $R^2$ | 0.330 | | 0.411 | | 0.169 | | 0.058 | |
| $N$ | 1488 | | 1488 | | 1640 | | 1640 | |
| **(b) Individual** | China | | Developed Market | | China | | Developed Market | |
| **Regressions** | coef | t-stat | coef | t-stat | coef | t-stat | coef | t-stat |
| | **Copper** | | | | **Soybean** | | | |
| const | 0.094 | 4.531 | 0.020 | 1.687 | 0.117 | 3.874 | 0.085 | 2.121 |
| $\hat{I}_{i,t}$ | -4.951 | -4.306 | -0.122 | -2.345 | -0.004 | -0.285 | -0.005 | -0.401 |
| $\hat{I}_{i,t} \cdot X_t$ | 65.614 | 2.383 | 1.731 | 0.978 | 0.264 | 0.922 | 0.189 | 0.919 |
| $R^2$ | 0.293 | | 0.091 | | 0.005 | | 0.004 | |
| $N$ | 423 | | 423 | | 423 | | 423 | |
| | **Zinc** | | | | **Corn** | | | |
| const | -0.025 | -1.620 | -0.031 | -4.837 | -0.038 | -1.254 | -0.031 | -0.762 |
| $\hat{I}_{i,t}$ | -1.175 | -8.318 | -0.114 | -2.006 | 0.004 | 0.562 | 0.000 | 0.092 |
| $\hat{I}_{i,t} \cdot X_t$ | 9.321 | 3.283 | 0.806 | 0.800 | -0.191 | -1.289 | -0.060 | -1.986 |
| $R^2$ | 0.334 | | 0.155 | | 0.040 | | 0.012 | |
| $N$ | 344 | | 344 | | 423 | | 423 | |
| | **Aluminum** | | | | **Fuel Oil** | | | |
| const | 0.024 | 1.035 | -0.037 | -2.836 | -0.121 | -3.388 | -0.039 | -2.497 |
| $\hat{I}_{i,t}$ | -1.151 | -3.890 | -0.024 | -2.942 | -0.434 | -3.361 | -5.249 | -3.089 |
| $\hat{I}_{i,t} \cdot X_t$ | 11.401 | 1.778 | -0.372 | -0.493 | 0.069 | 0.021 | 33.702 | 0.792 |
| $R^2$ | 0.379 | | 0.092 | | 0.027 | | 0.279 | |
| $N$ | 423 | | 423 | | 423 | | 423 | |
| | **Gold** | | | | **Rubber** | | | |
| const | 0.006 | 0.858 | -0.004 | -6.079 | 0.056 | 1.650 | 0.018 | 0.295 |
| $\hat{I}_{i,t}$ | -0.153 | -4.024 | 0.001 | 1.215 | -3.856 | -2.991 | -0.018 | -1.226 |
| $\hat{I}_{i,t} \cdot X_t$ | 6.094 | 3.973 | -0.077 | -2.330 | 7.239 | 0.201 | 0.093 | 0.324 |
| $R^2$ | 0.095 | | 0.012 | | 0.201 | | 0.025 | |
| $N$ | 298 | | 298 | | 371 | | 371 | |

# 7 Conclusion

In this paper we propose and test a theory of using commodities as collateral for financing. This practice has become increasingly important in recent years, particularly China. In the presence of capital controls and financing frictions, financial investors import commodities and pledge them as collateral to capture a credit risk premium. A simple model shows that the collateral demand for commodities increases the concurrent commodity spot prices globally, as well as inventory, convenience yield, and commodities futures risk premium in the importing country (e.g. China).

To test the model predictions, we construct an empirical proxy for the collateral demand for commodities. It is the product of two components. The first component is the Shibor-Libor spread, which is a proxy for the gap in unsecured interest rates between China and developed markets. The second component is the deviation from the covered interest rate parity in the USD-CNY exchange rate, which is a proxy for the tightness of capital control across Chinese borders.

Empirical tests strongly support our theory. A higher collateral demand for commodities is associated with (i) higher metal prices globally, (ii) a higher futures risk premium in China and a lower futures risk premium in developed markets, and (iii) a less negative inventory-convenience yield relation in China. The economic magnitude is also large. For example, the estimates suggest that the collateral demand for commodities can explain up to 11.9%–15.0% price increase of major industrial metals since 2007.

The mechanism and predictions of this paper are complementary to and distinct from those of the theory of storage. For example, a key prediction by the theory of storage is a negative relation between convenience yield and inventory. By contrast, when commodities are imported as collateral, spot price, inventory, and convenience yield all move in the same direction in the importing country. In addition, our results also contribute to the recent literature on the financialization of commodities. We show that collateral demands for commodities have quantitatively important implications for commodity prices and futures risk premium. Lastly, this paper concretely illustrates unintended consequences of capital control on commodities markets through the collateral channel.

# Appendix

# A Equilibrium Solutions

For the simplicity of notations, we define the constants $(m, n, q, v, w, z, o)$ as follows:

$$m = \frac{1}{\left(\sigma_S^i\right)^2}\left(\frac{\gamma_d^i+\gamma_c^i}{\gamma_d^i\gamma_c^i}\right), \tag{79}$$

$$n = \frac{1}{\left(\sigma_S^e\right)^2}\left(\frac{\gamma_s^e+\gamma_p^e}{\gamma_s^e\gamma_p^e}\right), \tag{80}$$

$$q = \frac{1}{2bl+1}\left[\mu_k - 2al + (1-\delta)(k_0 - 2al) - 2l\left((1-\delta)G_0^e + G_1^e\right)\right], \tag{81}$$

$$v = \frac{(1-\delta)(2bl+1)}{2l}, \tag{82}$$

$$w = \frac{1+R^e+r^i-R^i}{1-\delta}, \tag{83}$$

$$z = \frac{1+r^e}{1-\delta}, \tag{84}$$

$$o = \frac{1+R^e}{1-\delta}. \tag{85}$$

## A.1 Solution to Model with Demand for Collateral Commodity (Proposition 1)

By canceling out $D_{0,f}^i$ and $D_{0,d}^i$ in the system of seven equations, we get a system of five equations:

$$G_0^e - I_0^e = \left[\frac{k_0 - S_0^i}{2l} - a - bS_0^i\right] + \left(\frac{\gamma_d^i+\gamma_c^i}{\gamma_d^i\gamma_c^i}\right)\frac{E\left[S_1^i - F^i\right]}{(1-\delta)\left(\sigma_S^i\right)^2}, \tag{86}$$

$$I_0^e(1-\delta) + G_1^e = \frac{\gamma_s^e+\gamma_p^e}{\gamma_s^e\gamma_p^e}\frac{E\left[S_1^e - F^e\right]}{\left(\sigma_S^e\right)^2}, \tag{87}$$

$$I_0^e(1-\delta) + G_1^e = \frac{k_1 - S_1^i}{2l} - \left(a + bS_1^i + \left(\frac{\gamma_d^i+\gamma_c^i}{\gamma_d^i\gamma_c^i}\right)\frac{E\left[S_1^i - F^i\right]}{\left(\sigma_S^i\right)^2}\right), \tag{88}$$

$$S_1^i = S_1^e + h, \tag{89}$$

$$S_0^i = S_0^e + h - 2l\eta. \tag{90}$$

Our solution strategy is to first write $S_0^e$, $S_1^e$, $S_1^i$, $F^e$ and $F^i$ as functions of $S_0^i$ and then solve for $S_0^i$.

From (88) and (89) we get

$$\left(\sigma_S^i\right)^2 = \left(\sigma_S^e\right)^2 = \frac{(\sigma_k)^2}{(2lb+1)^2}. \tag{91}$$

From (86) and (88) we get

$$\begin{aligned} E\left[S_1^i\right] &= \frac{1}{2bl+1}\left[\mu_k - 2al + (1-\delta)\left(k_0 - 2al\right) - 2l\left(\left(1-\delta\right)G_0^e + G_1^e\right)\right] - (1-\delta)\,S_0^i \\ &= q - (1-\delta)S_0^i, \qquad (92) \\ E\left[S_1^e\right] &= E\left[S_1^i\right] - h. \qquad (93) \end{aligned}$$

The futures prices are given by

$$\begin{aligned} F^e &= \frac{S_0^e\left(1+r^e\right)-\lambda}{1-\delta} = \frac{\left(S_0^i - h + 2l\eta\right)\left(1+r^e\right)-\lambda}{1-\delta} \qquad (94) \\ &= zS_0^i - hz + 2lz\eta - \frac{\lambda}{1-\delta}, \qquad (95) \\ F^i &= \frac{\left(1+R^e+r^i-R^i\right)}{1-\delta}S_0^i + \frac{2l\left(1+R^e\right)}{1-\delta}\eta. \qquad (96) \end{aligned}$$

Equations (86) and (87) can be rewritten as

$$\begin{aligned} G_0^e - I_0^e &= \left[\frac{k_0 - S_0^i}{2l} - a - bS_0^i\right] + \frac{m}{(1-\delta)}E\left[S_1^i - F^i\right], \qquad (97) \\ I_0^e\left(1-\delta\right) + G_1^e &= nE\left[S_1^e - F^e\right]. \qquad (98) \end{aligned}$$

Substituting in the expressions of $E[S_1^e]$, $E[S_1^i]$, $F^e$ and $F^i$, we have

$$\begin{aligned} (1-\delta)\,G_0^e + G_1^e &= (1-\delta)\left[\frac{k_0 - S_0^i}{2l} - a - bS_0^i\right] + mE\left[S_1^i - F^i\right] + nE\left[S_1^e - F^e\right] \\ &= \frac{(1-\delta)\left(k_0 - 2al\right)}{2l} - vS_0^i \qquad (99) \\ &\quad + mq - (1-\delta+w)\,mS_0^i - 2lmo\eta \\ &\quad + n\left(q - h + zh\right) - (1-\delta+z)\,nS_0^i - 2lnz\eta + \frac{n\lambda}{1-\delta}. \end{aligned}$$

Thus,

$$\begin{aligned} S_0^i &= \frac{\left[\begin{array}{c} \frac{(1-\delta)(k_0-2al)}{2l} + mq + n\left(q-h+zh\right) - \left[G_0^e\left(1-\delta\right)+G_1^e\right] \\ + \frac{n}{1-\delta}\lambda - 2l\left(om+zn\right)\eta \end{array}\right]}{v + (1-\delta+w)\,m + (1-\delta+z)\,n}, \qquad (100) \\ S_0^e &= S_0^i - h + 2l\eta. \qquad (101) \end{aligned}$$

By (86) and (88), the time-1 prices are

$$\begin{aligned} S_1^i &= \frac{1}{2bl+1}\left[k_1 - 2al + (1-\delta)\left(k_0 - 2al\right) - 2l\left((1-\delta)\,G_0^e + G_1^e\right)\right] - (1-\delta)\,S_0^i, \\ &= E[S_1^i] + \frac{k_1-\mu_k}{1+2bl} = q - (1-\delta)S_0^i + \frac{k_1-\mu_k}{1+2bl}, \qquad (102) \\ S_1^e &= S_1^i - h. \qquad (103) \end{aligned}$$

By (87), the inventory in the exporting country is

$$I_0^e = \frac{1}{1-\delta}\left[n\left(q-h+zh\right) - (1-\delta+z)\,nS_0^i - G_1^e - 2nlz\eta + \frac{n\lambda}{1-\delta}\right]. \qquad (104)$$

Furthermore,

$$C_0^i = \frac{m}{1-\delta}\left[q - (1-\delta+w)\,S_0^i - 2lo\eta\right]. \qquad (105)$$

## A.2 Solution to the Benchmark Case (Proposition 2)

This case corresponds to $C_0^i = 0$, so equations (86)–(90) change to

$$\begin{aligned} G_0^e - I_0^e &= \left[\frac{k_0 - S_0^i}{2l} - a - bS_0^i\right], \qquad (106) \\ I_0^e\,(1-\delta) + G_1^e &= \frac{\gamma_s^e + \gamma_p^e}{\gamma_s^e\gamma_p^e}\frac{E\left[S_1^e - F^e\right]}{\left(\sigma_S^e\right)^2}, \qquad (107) \\ I_0^e\,(1-\delta) + G_1^e &= \frac{k_1 - S_1^i}{2l} - \left(a + bS_1^i\right), \qquad (108) \\ S_1^i &= S_1^e + h, \qquad (109) \\ S_0^i &= S_0^e + h - 2l\eta. \qquad (110) \end{aligned}$$

The futures prices are given by

$$\begin{aligned} F^e &= \frac{S_0^e\,(1+r^e) - \lambda}{1-\delta} = \frac{\left(S_0^i - h + 2l\eta\right)(1+r^e) - \lambda}{1-\delta} \\ &= \frac{1+r^e}{1-\delta}S_0^i - \frac{\left(h - 2l\eta\right)(1+r^e)}{1-\delta} - \frac{\lambda}{1-\delta}, \qquad (111) \\ F^i &= E\left[S_1^i\right], \qquad (112) \end{aligned}$$

where the expression of $F^i$ follows from (27), (28) and (19) after imposing $C_0^i = 0$.

Using the same constants $(n, q, v, w, z)$ defined in the previous section, following similar procedure of Proposition 1, we thus have:

$$\begin{aligned} S_0^i &= \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q-h+zh\right) - \left[G_0^e\,(1-\delta) + G_1^e\right] + \frac{n\lambda}{1-\delta} - 2nzl\eta}{v + (1-\delta+z)\,n}, \qquad (113) \\ S_0^e &= S_0^i - h + 2l\eta. \qquad (114) \end{aligned}$$

In this benchmark case, we restrict attention to situations in which neither the constraints binds, that is, $\eta = \lambda = 0$. Thus, the solution of the model is

$$S_0^i = \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right]}{v + \left(1 - \delta + z\right)n}, \tag{115}$$

$$S_0^e = S_0^i - h, \tag{116}$$

$$I_0^e = \frac{1}{1-\delta}\left[n\left(q - h + zh\right) - G_1^e - \left(1 - \delta + z\right)nS_0^i\right], \tag{117}$$

$$S_1^i = E[S_1^i] + \frac{k_1 - \mu_k}{2bl+1} = q - (1-\delta)S_0^i + \frac{k_1 - \mu_k}{2bl+1}, \tag{118}$$

$$S_1^e = S_1^i - h = q - (1-\delta)S_0^i + \frac{k_1 - \mu_k}{2bl+1} - h. \tag{119}$$

# B Proofs

## B.1 Proof of Proposition 3 (Prices)

1. We prove this item for the four cases one by one.

   **Case 1** $(\lambda = 0, \eta = 0)$**:** Technical Condition 2 implies that

   $$\begin{aligned}\overline{S}_0^i &= \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right]}{v + \left(1 - \delta + z\right)n} \\ &< \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right] + mq}{v + \left(1 - \delta + z\right)n + m(1 - \delta + w)},\end{aligned}$$

   where the right-hand side is the equilibrium $S_0^i$ with collateral demand. Thus, the demand for collateral commodities increases $S_0^i$ and hence $S_0^e$ (since $S_0^e = S_0^i - h$).

   **Case 2** $(\lambda = 0, \eta > 0)$**:** In this case, the spot price $S_0^i = \frac{k_0-2al}{2bl+1}$, and $\eta > 0$. Technical Condition 1 implies that $\overline{S}_0^i < \frac{k_0-2al}{2bl+1} = S_0^i$. Furthermore, $\eta > 0$ implies $S_0^e = S_0^i - h + 2l\eta > S_0^i - h > \overline{S}_0^i - h = \overline{S}_0^e$.

   **Case 3** $(\lambda > 0, \eta = 0)$**:** Similar with case 1, combined with $\lambda > 0$, Technical Condition 2 implies that

   $$\begin{aligned}\overline{S}_0^i &= \frac{\frac{(1-\delta)(k_0-2al)}{2l} + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right]}{v + \left(1 - \delta + z\right)n} \\ &< \frac{\frac{(1-\delta)(k_0-2al)}{2l} + mq + n\left(q - h + zh\right) - \left[G_0^e\left(1-\delta\right) + G_1^e\right] + \frac{n}{1-\delta}\lambda}{v + \left(1 - \delta + w\right)m + \left(1 - \delta + z\right)n},\end{aligned}$$

   where the right-hand side is the equilibrium $S_0^i$ with collateral demand. Thus, $\overline{S}_0^i < S_0^i$. Since $S_0^e = S_0^i - h$, $\overline{S}_0^e < S_0^e$.

   **Case 4** $(\lambda > 0, \eta > 0)$**:** The proof is the same as Case 2.

2. With or without the demand for collateral commodity, we have $S_1^i = q - (1 -$

$\delta)S_0^i + (k_1 - \mu_k)/(2bl+1)$ and $S_1^e = S_1^i - h$, both of which decrease in $S_0^i$. Thus, the demand for collateral commodity decreases the spot prices at time 1.

3. In Cases 1 and 3, $\eta = 0$ and $S_0^e = S_0^i - h$. In Cases 2 and 4, $\eta > 0$, and hence $S_0^i - (S_0^e + h) < 0$.

4. With collateral commodity, $F^i$ is given by

$$F^i = \frac{1 + R^e - R^i + r^i}{1-\delta} S_0^i + \frac{2l(1+R^e)}{1-\delta}\eta = wS_0^i + \frac{2l(1+R^e)}{1-\delta}\eta.$$

Without collateral commodity,

$$\overline{F}^i = q - (1-\delta)\overline{S}_0^i > q - (1-\delta)S_0^i.$$

Then, by Corollary 1,

$$F^i - \overline{F}^i < wS_0^i + \frac{2l(1+R^e)}{1-\delta}\eta - q + (1-\delta)S_0^i < 0.$$

## B.2 Proof of Proposition 4 (Convenience Yield)

1. We prove this item for the four cases one by one.

**Case 1** ($\lambda = 0, \eta = 0$)**:** In the benchmark case, $\overline{F}^i = E[\overline{S}_1^i] = q - (1-\delta)\overline{S}_0^i$. Also, from previous proposition, we know $\overline{S}_0^i < S_0^i$. So,

$$\frac{\overline{F}^i}{\overline{S}_0^i} > \frac{F^i}{S_0^i} = \frac{q}{S_0^i} - (1-\delta) > w = \frac{F^i}{S_0^i},$$

where the last inequality follows from Technical Condition 2.

**Case 2** ($\lambda = 0, \eta > 0$)**:** Note that $\frac{F^i}{S_0^i} = w + \frac{2l(1+R^e)}{(1-\delta)S_0^i}\eta$. In the benchmark case, $\frac{\overline{F}^i}{\overline{S}_0^i} = \frac{q}{\overline{S}_0^i} - (1-\delta)$. But Corollary 1 implies that

$$\frac{q}{S_0^i} - (1-\delta) > w + \frac{2l\,(1+R^e)}{(1-\delta)S_0^i}\eta.$$

Hence, as $\overline{S}_0^i < S_0^i$, we have

$$\frac{\overline{F}^i}{\overline{S}_0^i} = \frac{q}{\overline{S}_0^i} - (1-\delta) > \frac{q}{S_0^i} - (1-\delta) > w + \frac{2l\,(1+R^e)}{(1-\delta)S_0^i}\eta = \frac{F^i}{S_0^i}.$$

**Case 3** ($\lambda > 0, \eta = 0$)**:** The proof is the same as Case 1.

**Case 4** ($\lambda > 0, \eta > 0$)**:** The proof is the same as Case 2.

In sum, the demand for collateral commodity makes the futures curve exhibit more backwardation, or less contango, in the importing country.

2. In Cases 1 and 2, the demand for collateral commodity does not lead to a stockout in the exporting country, which is the case here. Thus, the convenience yield remains zero in the exporting country, and $\frac{F^e}{S_0^e} = \frac{1+r^e}{1-\delta}$ does not change. In Cases 3 and 4, by (63),

$$\frac{F^e}{S_0^e} = \frac{(1+r^e)}{1-\delta} - \frac{\lambda}{S_0^e} < \frac{(1+r^e)}{1-\delta} = \frac{\overline{F}^e}{\overline{S}_0^e}.$$

Thus, the demand for collateral commodity can increases the convenience yield from zero to positive if and only if $\lambda > 0$.

## B.3 Proof of Proposition 5 (Inventory)

1. We prove this item for the four cases respectively.

**Case 1 ($\lambda = 0, \eta = 0$):** From (45), $I_0^e$ is linearly decreasing in $S_0^i$, so $I_0^e$ is reduced by collateral demand for commodity.

**Case 2 ($\lambda = 0, \eta > 0$):** From (45), $I_0^e$ is linearly decreasing in $S_0^i$ and $\eta$, so $I_0^e$ is reduced by collateral demand for commodity.

**Case 3 ($\lambda > 0, \eta = 0$) and Case 4 ($\lambda > 0, \eta > 0$):** $I_0^e = 0$, so it is smaller than that in the benchmark case.

2. This is obvious because without collateral demand the inventory in the importing country is zero.

3. Equation (33) indicates that the total inventory $I_0^e + C_0^i$ can be expressed as

$$I_0^e + C_0^i = G_1 - \frac{k_0 - 2al}{2l} + \frac{2bl+1}{2l} S_0^i$$

Since $S_0^i$ is higher with collateral demand, so is $I_0^e + C_0^i$.

## B.4 Proof of Proposition 6 (Commodity Futures Risk Premium)

From (12), one can see that the futures risk premium in the importing country $E\left[S_1^e - F^e\right]$ relates positively to the inventory level $I_0^e$. Since $I_0^e$ is smaller in the case with collateral, $E\left[S_1^e - F^e\right]$ is smaller. Furthermore, (35) and (36) show that the futures risk premium $E\left[S_1^i - F^i\right]$ is negatively correlated with the inventory level $I_0^e$ and $S_1^i$, both of which become smaller in the collateral case. Thus, $E\left[S_1^i - F^i\right]$ is larger with collateral.

## B.5 Proof of Proposition 7 (Real Demand)

1. The fundamental demand for commodities in the importing country at time 0 is $\frac{k_0 - S_0^i}{2l}$, since $S_0^i$ is smaller in the benchmark case. So the fundamental demand in the importing country is smaller in the collateral case than the benchmark case.

2. The fundamental demand for commodities in the importing country at time 1 is $\frac{k_1 - S_1^i}{2l}$, since $S_1^i$ is larger in the benchmark case. So the fundamental demand in the importing country is larger in the collateral case than the benchmark case.

3. The total fundamental demand is

$$\frac{k_0 - S_0^i}{2l} + \frac{k_1 - S_1^i}{2l} = \frac{1}{2l}\left[k_0 + k_1 - \left(S_0^i + q - (1-\delta)S_0^i + \frac{k_1 - \mu_k}{1 + 2bl}\right)\right],$$

which is decreasing in $S_0^i$. Hence, collateral demand reduces the total demand at time 0 and time 1.

## B.6 Proof of Proposition 8 (comparative statics of $R^i$)

As $R^i$ increases, one can see that $w$ in (79) to (85) decreases, and no other parameters are affected by $w$.

1. From (41), it is easy to derive that in Case 1 of Proposition 1, a smaller $w$ causes a higher $S_0^i$. As $S_0^e = S_0^i - h$ in Case 1, a smaller $w$ also causes $S_0^e$ to increase.
2. From (33), the total inventory positively depend on $S_0^i$. Thus, a higher $R^i$ causes a higher $S_0^i$ and hence a higher total inventory. As shown in (45), the inventory in the exporting country negatively depends on $S_0^i$ and hence decreases in $R^i$. From (30), one can see that the collateral demand depends on the futures risk premium $E\left[S_1^i - F^i\right] = q - (1 - \delta + w)\, S_0^i$. It is easy to show that a smaller $w$ results in a larger $E\left[S_1^i - F^i\right]$, which in turn causes a larger collateral demand.
3. (64) shows that the convenience yield in the importing country directly depends on $R^i$; a higher $R^i$ results in a higher convenience yield.
4. As shown previously, the futures risk premium in the importing country $E\left[S_1^i - F^i\right]$ increases in $R^i$. The futures risk premium in the exporting country $E\left[S_1^e - F^e\right]$ depends on the inventory level $I_0^e$, as shown in (12). A higher $R^i$ causes a lower $I_0^e$ and hence a lower futures risk premium in the exporting country.
5. If an increase in $R^i$ implies a stockout in the exporting country, then $y^e$ increases from zero to positive.
6. If an increase in $R^i$ implies that all fundamental demand is met by domestic supply, then the spot price spread $S_0^i - S_0^e$ decreases from $h$ to a smaller amount.

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