Asset Markets with Heterogeneous Information

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Abstract

This paper studies competitive equilibria of economies where assets are heterogeneous and traders have heterogeneous information about them. Markets are defined by a price and a procedure for clearing trades and any asset can in principle be traded in any market. Buyers can use their information to impose acceptance rules which specify which assets they are willing to trade in each market. The set of markets where trade takes place is derived endogenously. The model can be applied to find conditions under which these economies feature fire-sales, contagion and flights to quality.

Keywords: Asymmetric information, competitive equilibrium, fire sales, expertise

JEL codes: D82, D41, G14

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1 Introduction

I study competitive asset markets where traders have different information about the assets being traded. Sellers own a portfolio of assets of heterogeneous quality and there are potential gains from trade in selling them to a group of buyers. Since Akerlof (1970), one special case has been studied in great detail: where sellers are informed and buyers are uninformed. Instead, I allow for different buyers to have different information about each of the assets.

Wilson (1980) and Hellwig (1987) first showed that in simple trading environments with asymmetric information the predictions are sensitive to the exact way that competition is modeled: the order of decisions, who proposes prices, etc. Faced with this difficulty, one approach has been to study these problems as games where all of these features are spelled out completely (Rothschild and Stiglitz 1976, Wilson 1977, Miyazaki 1977, Stiglitz and Weiss 1981, Arnold and Riley 2009). An alternative approach, which I pursue in this paper, has been to attempt to abstract from the details of how trading takes place and adapt the notion of Walrasian competitive equilibrium (Gale 1992, 1996, Dubey and Geanakoplos 2002, Bisin and Gottardi 2006) or competitive search equilibrium (Guerrieri et al. 2010, Guerrieri and Shimer 2012, Chang 2011) to settings with asymmetric information.

Existing definitions of competitive equilibrium for asymmetric information environments start by defining a set of markets with prespecified prices and allow any asset to be traded in any market; traders' decision problem is then to choose supply or demand in each market. This construct is not enough to handle environments with many differently-informed buyers. The reason is that when different assets trade in the same market, some buyers may have enough information to tell them apart while other buyers do not. Analyzing this possibility requires developing a new notion of competitive equilibrium where traders can act on this differential information in a way that's not reducible to just choosing quantities. In the equilibrium deifnition below, buyers can act on their information by imposing acceptance rules. These specify which assets the buyer is willing to buy in each market. Each buyer's acceptance rules must be consistent with his own information; if a buyer's information is not sufficient to tell two assets apart, his acceptance rule cannot discriminate beween them.

Allowing different buyers to impose their own acceptance rules in a given market can give rise to situations where there is more than one possible way to clear the market. This indeterminacy can be resolved by defining the set of all possible market-clearing algorithms and allowing traders to direct their trades to markets that use the algorithm they prefer. Thus, the set of markets is defined as the set of all price-alogrithm pairs and equilibrium is defined in terms of quantities and acceptance rules for each market.

I focus on a basic case where there are two qualities of assets, good and bad, in known proportions. Each seller owns a representative portfolio of these assets, and the source of gains from trade is that a fraction of sellers are impatient. Sellers know the quality of each asset they own but buyers cannot observe it directly. Instead, they each observe an imperfect binary signal about each asset. I first chracterize the equilibrium in a case with "false positives" only: buyers may observe good signals from bad assets but not the other way around. I then study the opposite information structure, with "false negatives" but no false positives. In both cases, buyers can be ranked by their expertise, i.e. their probability of making mistakes.

For the false positives case, the equilibrium can be characterized quite simply. All assets trade at the same price; sellers of high-quality assets can sell as many units as they choose at that price but sellers of low-quality assets face rationing. Low quality assets that are more likely to be mistaken for high quality assets face less rationing than easily detectable ones, and some assets cannot be traded at all. Only buyers who observe sufficiently informative signals choose to trade, while the rest stay out of the market.

One question of applied interest has to do with what happens if the number of impatient sellers increases. Will there be fire-sale effects, with prices falling with the number of impatient sellers? Uhlig (2010) has shown that in a pure asymmetric information case with equally uninformed buyers prices should go up with the number of impatient sellers because these are the only ones that sell high-quality assets. With differentially informed buyers, there are countervailing effects. When more assets are sold the set of active buyers must expand, so the marginal buyer has worse information; as a result, the net effect depends on the joint distribution of buyers' wealth and information quality in a way that is easily characterized. The price will drop if the density of wealth conditional on the marginal buyer's information quality is low, so that a large fall in the cutoff level of information quality is needed to absorb an increase in supply.

The model also implies that as long as two asset classes share at least part of the pool of investors, there will be contagion between them. If an increase in the number of distressed sellers in one assets class leads to a drop in prices, investors will shift towards that asset class; the set of active buyers in the other asset class must expand to make up for this so the quality of information of the marginal buyer will fall, requirng lower prices too.

For the false negatives case, different high-quality assets trade at different prices, which depend on how many buyers are able to realize that the asset is of high quality. Thus more transparent assets command a premium. In this case, an increase in the number of impatient

sellers leads to flight-to-quality effects, where the premium for the most transparent assets increases.

2 The Economy

Dates and assets

There are two periods, $t = 1$ and $t = 2$. Consumption at time t is denoted c_t.

There is nothing about the model that requires the temporal interpretation. It could be "apples" and "oranges" rather than "$t = 1$" and "$t = 2$". The key will be that oranges come in boxes called "assets" and not everyone knows how many oranges are contained in each box.

Assets are indexed by $i \in [0, 1]$. Asset i will produce $q(i) = \mathbb{I}(i \geq \lambda)$ goods at $t = 2$ for some $\lambda \in (0, 1)$. I refer to $q(i)$ as the quality of asset i. This means that a fraction $1 - \lambda$ of assets (those with indices $i \geq \lambda$) are good assets and will pay a dividend of 1 at $t = 2$ and a fraction λ (those with indices $i < \lambda$) are bad and will pay nothing.

Agents, preferences and endowments

Agents are divided into buyers and sellers. Buyers are indexed by $b \in [0, 1]$. Preferences for buyers are

$$u(c_1, c_2) = c_1 + c_2$$

and their consumption is constrained to be nonnegative. Buyer b has an endowment of $w(b)$ goods at $t = 1$, where w is a continuous, strictly positive function. Let

$$W(b) \equiv \int_b^1 w\left(\tilde{b}\right) d\tilde{b}$$

be the total endowment of buyers whose indices are at least b.

Sellers are indexed by $s \in [0, 1]$. Preferences for sellers are

$$u(c_1, c_2, s) = c_1 + \beta(s) c_2$$

with

$$\beta(s) = \mathbb{I}(s \geq \mu)$$

I refer to buyers of types $s < \mu$ as "impatient" or "distressed". Their impatience is the source of gains from trade. Each seller is endowed with a portfolio containing one unit of each asset.

I will assume that

$$W(0) \geq \mu(1-\lambda) \tag{1}$$

i.e. that the total endowment of all buyers is at least as large as the total dividends of the assets owned by distressed sellers.

Linearity in the preferences of sellers is assumed for simplicity. For sellers, it means that the decision of what to do with one asset does not depend on what the seller does with any other asset. For buyers, it means that if they choose to trade they will be at a corner solution where they spend all their endowment.

Information

Each seller knows the index i (and therefore the quality $q(i)$) of each asset he owns. Buyers do not observe i. Instead, buyer b observes a signal $x(i,b)$ whenever he analyzes asset i. If $x(\cdot,b)$ were invertible, i.e. if $x(i,b) \neq x(i',b)$ whenever $q(i) \neq q(i')$, then buyer b would be perfectly informed about asset qualitites. The interesting case arises when this is not the case for at least some buyers, who can therefore not tell apart some assets of different qualities.

I will consider two possible cases, illustrated in Figure 1. In the false positives case, buyer b observes

$$x(i,b) = \mathbb{I}(i \geq b\lambda) \tag{2}$$

When an asset is good, every buyer observes $x(i,b) = 1$. When an asset i is a bad, those buyers of types $b \leq \frac{i}{\lambda}$ will observe $x(i,b) = 1$, so they cannot distinguish it from a good asset; instead, buyers with $b > \frac{i}{\lambda}$ will observe $x(i,b) = 0$ and conclude that the asset is a bad. A buyer's type b can therefore be thought of as an index of expertise: higher values of b means that there is a smaller subset of bad assets that the buyer might misidentify as good assets. Furthermore, expertise is nested: if type b can identify that asset i is bad, then so can all types $b' > b$.

Conversely, in the false negatives case, buyer b observes

$$x(i,b) = \mathbb{I}(i \geq 1 - b(1-\lambda)) \tag{3}$$

When an asset is bad all buyers observe $x(i,b) = 0$ but when it is good only those buyers

with $b \geq \frac{1-i}{1-\lambda}$ observe $x(i,b) = 1$ and realize it is good. Again, b can be thought of as an index of expertise.

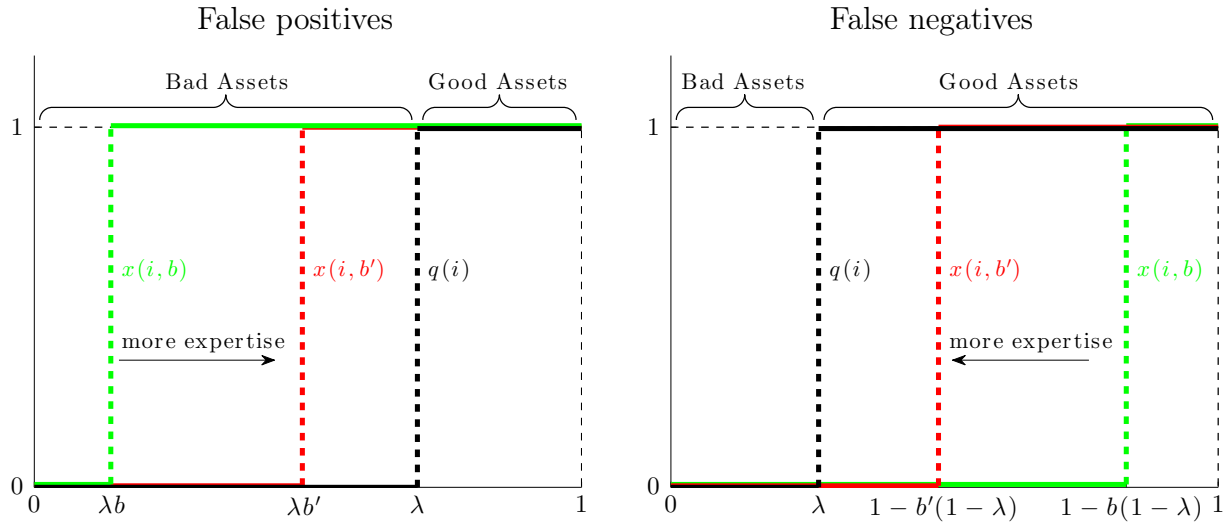


Figure 1: Information of buyers in the two examples

3 Equilibrium

Markets

There is no market for trading $t = 1$ goods against $t = 2$ goods. If there was such a market, which can be interpreted as a market for uncollateralized borrowing, then impatient sellers would borrow up to the point where $c_2 = 0$ and the gains from trade would be exhausted. Instead, the only way to achieve some sort of intertemporal trade is to trade $t = 1$ goods for assets. These assets will in turn produce $t = 2$ goods.

There are many markets, operating simultaneously, where agents can exchange goods for assets. Each market m is defined by a price $p(m)$ of assets in terms of goods and a clearing algorithm, described in more detail below. In principle, any asset can be traded in any market. Let M be the set of markets.

Gale (1996) uses a similar construct: rather than letting the price clear markets, all possible prices coexist and at each price there is pro-rata rationing of excess supply or excess demand. There are two main differences with Gale's setup. First, the current setup allows more elaborate clearing algorithms than simply rationing the long end of the market. These algorithms make it possible to describe which trades take place when different buyers place

different types of orders in the same market (more on clearing algorithms below). Second, I allow agents to trade in as many markets as they want rather than limiting them to a single market. This is meant to capture the idea of anonymous markets and is clearly more appropriate in some applications than in others.

Seller's problem

Sellers must choose how much to supply of each asset in each market. Formally, each seller chooses a function $\sigma : [0,1] \times M \to [0,1]$, where $\sigma(i,m)$ represents the number of i assets that the seller supplies in market m.

From the point of view of the sellers, markets are characterized by their prices $p(m)$ and a rationing function η.

Definition 1. A rationing function η assigns a measure $\eta(\cdot;i)$ on M to each possible asset i.

If $M_0 \subseteq M$ is a set of markets, $\eta(M_0;i)$ is the number of assets of index i that the seller will end up selling if he supplies one unit of asset i to each market $m \in M_0$. For instance, if $\eta(m;i) = \alpha$, this means that a seller who supplies one unit of asset i in market m will end up selling α units in that market. Implicit in this formulation is the idea that assets are perfectly divisible, so there is exact pro-rata rationing rather than a probability of selling an indivisible unit. η is an endogenous object, which results from clearing algorithms and from equilibrium supply and demand. Each seller simply takes it as given.

Seller s solves the following problem:

$$\max_{c_1,c_2,\sigma} u(c_1, c_2, s) \tag{4}$$

$$s.t.$$

$$c_1 = \int_{[0,1]} \left[\int_M p(m)\, \sigma(i,m)\, d\eta(m;i) \right] di \tag{5}$$

$$c_2 = \int_{[0,1]} q(i) \left[1 - \int_M \sigma(i,m)\, d\eta(m;i) \right] di \tag{6}$$

$$0 \leq \sigma(i,m) \leq 1 \quad \forall i, m \tag{7}$$

$$\int_M \sigma(i,m)\, d\eta(m;i) \leq 1 \quad \forall i \tag{8}$$

$$c_1 \geq 0 \quad c_2 \geq 0 \tag{9}$$

Constraint (5) computes how many goods the seller gets at $t = 1$ as a result of his sales. For each asset i, he supplies $\sigma(i,m)$ in market m and receives $p(m)$ for each unit he sells. Integrating across markets using measure $\eta(\cdot;i)$ and adding across all assets i results in (5). Constraint (6) computes how many goods the seller gets at $t = 2$ as a result of the assets which he does not sell. For each quality i his unsold assets are equal to his endowment of 1 minus what he sold in all markets, and each yields $q(i)$ goods. Constraint (7) says that supply is nonnegative and that he can at most attempt to sell his entire endowment of each asset in any given market. This is important when $\eta(m;i) < 1$. It rules out a strategy of offering, say, 4 units for sale when he only owns 1 because he knows that due to rationing, only 25% of the units are actually sold. Constraint (8) just says that the total sales of any given quality (added across all markets) are constrained by the seller's endowment. Note that this embodies the assumption that sellers can attempt to sell the same asset in many markets, i.e. I do *not* impose

$$\sum_{m \in M_0} \sigma(i,m) \leq 1 \quad \forall M_0 \subseteq M \text{ countable}, \forall i \tag{10}$$

If I imposed (10) instead of (8), then a unit that is offered in one market could no longer be offered in other markets, and this commitment could be used as a signal of quality. Gale (1992, 1996), Guerrieri et al. (2010), Guerrieri and Shimer (2012) and Chang (2011) all make

assumptions similar to (10) and obtain separating equilibria as a result.

The choice of $\sigma(i,m)$ for any single market m such that $\eta(m;i)=0$ has no effect on the utility obtained by the seller. The interpretation of this is that if he is not going to be able to sell, it doesn't matter whether or not he tries. Formally, this means that program (4) has multiple solutions. I am going to assume that when this is the case, the solution has to be robust to small positive $\eta(m;i)$, meaning that the seller must attempt to sell an asset in all the markets where if he could he would want to and must not attempt to sell an asset in any market where if he could he would not want to.

Definition 2. A solution to program (4) is *robust* if for every $\{i_0, m_0\}$ such that $\eta(m_0;i_0)=0$ there exists a sequence of strictly positive real numbers $\{z_n\}_{n=1}^{\infty}$ and a sequence of consumption and selling decisions c_1^n, c_2^n, σ^n such that, defining

$$\eta^n\left(\tilde{M};i\right)=\eta\left(\tilde{M},i\right)+z_n\mathbb{I}\left(m_0\in\tilde{M}\right)\mathbb{I}\left(i=i_0\right)$$

1. c_1^n, c_2^n, σ^n solve program

$$\max_{c_1,c_2,\sigma} u(c_1,c_2,s) \tag{11}$$

$$s.t.$$

$$c_1=\int_{[0,1]}\left[\int_M p(m)\,\sigma(i,m)\,d\eta^n(m;i)\right]di$$

$$c_2=\int_{[0,1]} q(i)\left[1-\int_M \sigma(i,m)\,d\eta^n(m;i)\right]di$$

$$0\le\sigma(i,m)\le 1\quad\forall i,m$$

$$\int_M \sigma(i,m)\,d\eta^n(m;i)\le 1\quad\forall i$$

$$c_1\ge 0\quad c_2\ge 0$$

2. $z_n\to 0$

3. $c_1^n\to c_1$, $c_2^n\to c_2$ and $\sigma^n(i,m)\to\sigma(i,m)$ for all i,m.

Lemma 1. *Every robust solution to program (4) satisfies*

$$\sigma(i,m) = \begin{cases} 1 & \text{if } p(m) > p^R(i) \\ 0 & \text{if } p(m) < p^R(i) \end{cases}$$

for some $p^R(i)$.

Lemma 1 implies that sellers will use a simple cutoff rule for deciding what markets to try to sell their assets in. For each asset i they will choose a reservation price $p^R(i)$. They will try to sell their entire endowment of i assets in every market where $p(m) > p^R(i)$ and will not attempt to sell i assets in any market where $p(m) < p^R(i)$. The Lemma does not exactly specify what sellers do in markets where $p(m) = p^R(i)$. They may for instance choose to attempt to sell their assets in some but not others.

Imposing rubustness in seller's decisions will rule out self-fulfilling equilibria where sellers don't supply assets at certain prices because there are no buyers and buyers do not try to buy at those prices because there are no sellers. In a robust solution, sellers will always supply their assets in markets where the price is attractive, even if they know they won't be able to sell them.

Buyer's problem

When buyers place orders in a market, they can specify both the quantity of assets that they demand and what subset of assets they are willing to accept. An example of an order will be "I offer to buy 5 assets as long as the indices i of those assets satisfy $i \geq 0.4$". I formalize the idea that buyers can be selective by defining acceptance rules:

Definition 3. An *acceptance rule* is a function $\chi : [0,1] \to \{0,1\}$.

$\chi(i) = 1$ means that a buyer is willing to accept asset i and $\chi(i) = 0$ means he is not. Buyers cannot just impose any selection rule that they want, such as accepting only the highest-quality assets. They are not necessarily able to tell different assets apart from each other since they do not observe i but just the imperfect signal $x(i,b)$. Feasible acceptance rules are those that only discriminate between assets that buyers can actually tell apart.

Definition 4. An acceptance rule χ is *feasible for buyer* b if it is measurable with respect to buyer b's information set, i.e. if

$$\chi(i) = \chi(i') \quad \text{whenever} \quad x(i,b) = x(i',b)$$

In general, since different buyers observe different signals, the set of feasible acceptance rules will be different for each of them and in equilibrium they will end up imposing different acceptance rules.

I denote the set of possible acceptance rules by X, the set of acceptance rules that are feasible for buyer b by X_b and the set of assets accepted by a rule χ by $I_\chi \equiv \{i \in [0,1] : \chi(i) = 1\}$.

Buyers must choose their entire demand pattern, which involves how much to demand in each market and what acceptance rules to impose. Formally, each buyer chooses a measure δ over markets and acceptance rules. $\delta(X_0, M_0)$ represents the number of units that the buyer demands in markets $m \in M_0$ using acceptance rules $\chi \in X_0$.

From the point of view of buyers, markets are characterized by their prices $p(m)$ and an allocation function A.

Definition 5. An *allocation function* A assigns a measure $A(\cdot; \chi, m)$ on $[0,1]$ to each acceptance rule-market pair $(\chi, m) \in X \times M$.

If $I_0 \subseteq [0,1]$, $A(I_0, \chi, m)$ represents the amount of assets $i \in I_0$ that a buyer will obtain if he demands one unit in market m and imposes acceptance rule χ. A is an endogenous object, which results from the clearing algorithms and equilibrium supply and demand. Each buyer simply takes it as given.

Buyer b solves the following problem:

$$\max_{c_1, c_2, \delta} u(c_1, c_2) \tag{12}$$

$$s.t.$$

$$c_1 = w(b) - \int_{X \times M} p(m) A([0,1]; \chi, m) \, d\delta(\chi, m) \tag{13}$$

$$c_2 = \int_{X \times M} \left(\int_{[0,1]} q(i) \, dA(i; \chi, m) \right) d\delta(\chi, m) \tag{14}$$

$$\delta(X_b, M) = \delta(X, M) \tag{15}$$

$$c_1 \geq 0 \quad c_2 \geq 0 \tag{16}$$

Constraint (13) says that $t = 1$ consumption is equal to the buyer's endowment minus what he spends on buying assets. In market m, upon demanding one asset and imposing acceptance rule χ he obtains $A([0,1]; \chi, m)$ assets and pays $p(m)$ for each of them. His total spending is given by integrating these expeditures using the demand measure he chooses.

Constraint (14) computes the total amount of $t = 2$ goods that the buyer will obtain. This is given by adding up the dividends from the assets he acquires in market m with acceptance rule χ using measure $A(\cdot; \chi, m)$ and then adding across markets and acceptance rules using measure δ. Constraint (15) restricts the buyer to place positive demand measure only on feasible acceptance rules.

Clearing algorithms

Each market is defined by a price $p(m)$ and a clearing algorithm. A clearing algorithm is a rule that determines what trades take place as a function of what trades are proposed by buyers and sellers.

To see why different clearing algorithms would lead to different results, consider the examples in Tables 1 and 2.

i	$q(i)$	$\chi(i)$ of buyer b_1	$\chi(i)$ of buyer b_2	Supply
Black	0	0	0	1.5
Red	0	1	0	1.5
Green	1	1	1	1.5
		$\delta_{b_1} = 1$	$\delta_{b_2} = 1$	

Table 1: Example of supplies and demands in a market

In the example in Table 1 there are three types of assets. Black and Red assets are bad while Green assets are good. There are two buyers in market m, with types b_1 and b_2. Type b_1 cannot tell apart Red and Green so he must either accept both of them or reject both of them; assume he is willing to accept both of them but rejects Black assets, which he can tell apart. Type b_2 can distinguish the worthless Black and Red assets from the good Green assets so he can impose that he will only accept Green assets. Each of the buyers demands a single unit. The total supply from all sellers is 1.5 units of each asset.

One possible clearing algorithm would say: "let b_1 choose first and take a represenative sample of the assets he is willing to accept; then b_2 can do the same". This would result in the following allocation and rationing functions:

$$A(i; \chi, m) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \\ 0 & 0 & \text{if } i = \text{Black} \\ 0.5 & 0 & \text{if } i = \text{Red} \\ 0.5 & 1 & \text{if } i = \text{Green} \end{cases}; \quad \eta(m; i) = \begin{cases} 0 & \text{if } i = \text{Black} \\ \frac{1}{3} & \text{if } i = \text{Red} \\ 1 & \text{if } i = \text{Green} \end{cases} \tag{17}$$

Type b_1 picks randomly from the sample excluding the rejected Black assets. Since there are equal amounts of Red and Green assets and the total exceeds his demand, he gets a measure 0.5 of each. After that, type b_2 gets to pick. He only accepts Green assets and there is one unit left, which is exactly what he wants. From the sellers' point of view, all Green assets are sold but only $\frac{1}{3}$ of Red assets and no Black assets are sold.

Another possible clearing algorithm would say "let b_2 choose first and take a representative sample of the assets he is willing to accept; then b_1 can do the same". This results in:

$$A(i;\chi,m)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black} \\ 0.75 & 0 & \text{if } i=\text{Red} \\ 0.25 & 1 & \text{if } i=\text{Green} \end{cases}; \quad \eta(m;i)=\begin{cases} 0 & \text{if } i=\text{Black} \\ \frac{1}{2} & \text{if } i=\text{Red} \\ \frac{5}{6} & \text{if } i=\text{Green} \end{cases} \tag{18}$$

After b_2 picks one unit of Green assets, there are only 0.5 units left, and there are still 1.5 units of Red assets. A respresentative sample from this remainder will give type b_1 a total of 0.75 units of Red assets and 0.25 units of Green assets.

i	$q(i)$	$\chi(i)$ of buyer b_1	$\chi(i)$ of buyer b_2	Supply
Black	0	0	0	1
Red	1	1	0	1
Green	1	1	1	1
		$\delta_{b_1}=1$	$\delta_{b_2}=1$	

Table 2: Example of supplies and demands in a market

The example in Table 2 is similar, except that Red assets are actually good so buyer b_1's rule of accepting them will not be to his detriment; also, supply of each asset is lower so assets can actually run out. Under the clearing algorithm that lets b_1 choose first, the result is:

$$A(i;\chi,m)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black} \\ 0.5 & 0 & \text{if } i=\text{Red} \\ 0.5 & 0.5 & \text{if } i=\text{Green} \end{cases}; \quad \eta(m;i)=\begin{cases} 0 & \text{if } i=\text{Black} \\ 0.5 & \text{if } i=\text{Red} \\ 1 & \text{if } i=\text{Green} \end{cases}$$

By the time b_2 gets to pick, there isn't enough supply to meet his demand, so he ends up with fewer units than he wanted. Instead, under the algorithm that lets b_2 choose first, the

result is:

$$A(i;\chi,m)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black} \\ 1 & 0 & \text{if } i=\text{Red} \\ 0 & 1 & \text{if } i=\text{Green} \end{cases};\quad \eta(m;i)=\begin{cases} 0 & \text{if } i=\text{Black} \\ 1 & \text{if } i=\text{Red} \\ 1 & \text{if } i=\text{Green} \end{cases}$$

Here all Red and Green units are allocated: the Green ones to the more selective b_2 buyer and the Red ones to the less selective b_1 buyer.

Clearly different algorithms result in different allocations and it is necessary to determine which algorithm will be used. The equilibrium definition below assumes that there exist separate markets for each possible clearing algorithm and traders can choose which of these markets they wish to trade in. To make this statement precise, I need to describe the set of possible clearing algorithms.

Definition 6. A *clearing algorithm* consists of:

1. an ordered set of rounds K
2. a measure $\omega(\cdot,\chi)$ on K with $\omega(K,\chi)=1$ for each possible acceptance rule $\chi\in X$.

Which trades will eventually take place depends on the order in which buyers' orders are executed; clearing algorithms are rules for determining this order. The algorithm defines several rounds of trading. Each buyer, depending on the acceptance rule he imposes, will execute his trades in one of those rounds, or perhaps split among several rounds. When a buyer's trade is executed, the buyer picks a representative sample of the acceptable assets, if any, that remain on sale in the market. The algorithm specifies, for each acceptance rule, in which round(s) a buyer who imposes that rule will execute his trades. Therefore a clearing algorithm can be represented by a mapping that assigns to each clearing rule a measure over the rounds of trading; the measure indicates what fraction of the requested trades will be executed in each round. The set of clearing algorithms is the set of all such mappings.

In the example from Table 1, the first clearing algorithm consists of $K=\{1,2,3\}$ and:

$$\omega(k,\chi)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \text{any other } \chi & \\ 1 & 0 & 0 & \text{if } k=1 \\ 0 & 1 & 0 & \text{if } k=2 \\ 0 & 0 & 1 & \text{if } k=3 \end{cases} \tag{19}$$

This says that the acceptance rule $\chi_1 = \{0, 1, 1\}$ will execute its trades in the first round while the acceptance rule $\chi_2 = \{0, 0, 1\}$ will execute its trades in the second round (and any other rule, which nobody in the example imposes, would come later).

The second clearing algorithm instead consists of $K = \{1, 2, 3\}$ and:

$$\omega(k, \chi) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \text{any other } \chi & \\ 0 & 1 & 0 & \text{if } k = 1 \\ 1 & 0 & 0 & \text{if } k = 2 \\ 0 & 0 & 1 & \text{if } k = 3 \end{cases} \tag{20}$$

Allocation and rationing functions result from applying each market's clearing algorithm to the demand and supply in that market. Demand consists of a measure $D(\cdot, m)$ on X. If $X_0 \subseteq X$, $D(X_0, m)$ is the total amount of assets demanded by buyers who impose acceptance rules $\chi \in X_0$ in market m. Supply is a function $S(\cdot; m)$ so that $S(i; m)$ is the total amount of assets of type i supplied by sellers.

Given demand $D(\cdot, m)$, supply $S(\cdot; m)$ and clearing algorithms $\{K, \omega\}$, the allocation function $A(\cdot; \chi, m)$ is computed as follows:

1. Denote the residual supply when the algorithm reaches round k by S^k. Let

$$a^k(i; \chi, m) = \begin{cases} \frac{\chi(i) S^k(i;m)}{\int \chi(i) S^k(i;m) di} & \text{if } \int \chi(i) S^k(i; m) di > 0 \\ \frac{\chi(i) S^k(i;m)}{\sum_i \chi(i) S^k(i;m)} & \text{if } \int \chi(i) S^k(i; m) di = 0 \text{ but } \sum_i \chi(i) S^k(i; m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

   As long as rule χ accepts some assets that remain in positive supply as of round k, then $a^k(i, \chi, m)$ defines either a density with respect to the Lebesgue measure or a discrete measure, which describes the assets received by rule χ in round k in market m. In either case, these assets constitute a representative sample of the χ-acceptable assets that remain. If no χ-acceptable assets remain as of round k, then the demand associated with rule χ is left unsatisfied.[1]

2. Residual supply is computed by subtracting from the original supply all the units that

[1] Allocating this amount might not be feasible if demand exceeds residual supply. See Appendix B for how to complete the description of an allocation algorithm for these cases. In equilibrium, this issue never arises.

were allocated to all acceptance rules up to (but not including) round k, i.e.:

$$S^k(i;m) = S(i;m) - \int_{\chi \in X} \left(\int_{j<k} a^j(i;\chi,m)\, d\omega_m(j,\chi) \right) dD(\chi,m) \tag{22}$$

3. Aggregating a^k over rounds results in:

$$a(i;\chi,m) = \int_{j \in K} a^j(i;\chi,m)\, d\omega_m(j,\chi) \tag{23}$$

$a(\cdot;\chi,m)$ is either a density with respect to the Lebesgue measure (in which case $A(\cdot;\chi,m)$ is its Lebesgue integral) or a discrete measure (in which case $A(\cdot;\chi,m) = a(\cdot;\chi,m)$).

The rationing function is then:

$$\eta(M_0;i) = \int_{\substack{m \in M_0 \\ \chi \in X}} \frac{a(i;\chi,m)}{S(i;m)} dD(\chi,m) \tag{24}$$

Equation (24) says that the number of units of asset i that can be sold in markets M_0 (per unit supplied) is computed in the following way. For each acceptance rule χ and market m, $\frac{a(i,\chi,m)}{S(i;m)}$ is the ratio between how much the buyers who impose χ get per unit of demand and how much the sellers offered. Adding up over markets and acceptance rules using the demand measure yields how many units sellers are able to sell. For instance $\eta(m;\text{Green})$ in equation (18) results from the following calculation:

$$\begin{aligned}
\eta(m;\text{Green}) &= \frac{a(\text{Green};\{0,1,1\};m)}{S(\text{Green};m)} D(\{0,1,1\};m) + \frac{a(\text{Green};\{0,0,1\};m)}{S(\text{Green};m)} D(\{0,0,1\};m) \\
&= \frac{0.25}{1.5} \times 1 + \frac{1}{1.5} \times 1 \\
&= \frac{5}{6}
\end{aligned}$$

Definition of equilibrium

The set of markets M is the set of all possible pairs of a positive price and a clearing algorithm. An equilibrium consists of:

1. Consumption and supply decisions $c_{1,s}, c_{2,s}, \sigma_s$ by sellers
2. Aggregate supply S
3. Consumption and demand decisions $c_{1,b}, c_{2,b}, \delta_b$ by buyers
4. Aggregate demand D
5. An allocation function A
6. A rationing function η

such that

1. $c_{1,s}, c_{2,s}, \sigma_s$ are a robust solution to program (4) for each seller s, taking η as given
2. $c_{1,b}, c_{2,b}, \delta_b$ solve program (12) for each buyer b, taking A as given
3. Aggregate supply and demand satisfy

$$S(i;m) = \int_{s\in[0,1]} \sigma_s(i;m)\, ds$$
$$D(X_0, M_0) = \int_{b\in[0,1]} \delta_b(X_0, M_0)\, db$$

4. A, η, S and D satisfy equations (23) and (24)

4 False Positives Case

For this information structure, there exists is an essentially unique equilibrium, which is characterized as follows. Define the less-restrictive-first (LRF) clearing algorithm as follows:

Definition 7. The *less-restrictive-first* clearing algorithm is given by the set of rounds $K = [0, 1]$ and a measure that places measure 1 on round g (and zero on all other rounds) for acceptance rules of the form $\chi(i) = \mathbb{I}(i \geq g)$ and measure 1 on round 1 (and zero on all other rounds) for any other acceptance rule.

Given the information structure, the feasible acceptance rules for buyers can only take the form of a simple cutoff rule. This could mean either only accepting assets with i above a cutoff g or only assets with i below a cutoff g. Rules of the form $\chi(i) = \mathbb{I}(i < g)$ will never be used in equilibrium but must be contemplated in order to have a complete description of the algorithm. Among the acceptance rules that only accept assets above a cutoff, the LRF algorithm orders them from least restrictive (lower cutoff for acceptance) to most restricitve (higher cutoff for acceptance). Any other acceptance rules are relegated to the last round.

Let p^* and b^* be defined by the solution to.

$$\int_{b^*}^{1} \frac{1}{\lambda(1-b)+\mu(1-\lambda)} \frac{w(b)}{p^*} db = 1 \tag{25}$$

$$p^* = \frac{\mu(1-\lambda)}{\lambda(1-b^*)+\mu(1-\lambda)} \tag{26}$$

Lemma 2. *There is a unique solution to equations (25) (26), with* $b^* \in (0,1)$ *and* $p^* \in (0,1)$.

Let market m^* be the market defined by price p^* and the LRF algorithm. An equilibrium is given by the following:

1. Supply decisions

$$\sigma_s(i,m) = \begin{cases} 1 & \text{if} \begin{cases} i < \lambda \\ p(m) \geq p^* \text{ and } s < \mu \\ \text{or} \\ p(m) \geq 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

leading to consumption

$$c_{1,s} = \begin{cases} p^* \int_0^1 \eta(m^*; i)\, di & \text{if } s < \mu \\ p^* \int_0^\lambda \eta(m^*; i)\, di & \text{if } s \geq \mu \end{cases}$$
$$c_{2,s} = \begin{cases} 0 & \text{if } s < \mu \\ 1-\lambda & \text{if } s \geq \mu \end{cases} \tag{28}$$

where $\eta(m^*; i)$ is given by (36) below.

2. Aggregate supply

$$S(i;m)=\begin{cases}1 & \text{if } \begin{cases} i<\lambda \\ \text{or} \\ p(m)\geq 1 \end{cases} \\ \mu & \text{if } i\geq\lambda,\ p(m)\in[p^*,1) \\ 0 & \text{if } i\geq\lambda,\ p(m)<p^*\end{cases} \tag{29}$$

3. Demand decisions

$$\delta_b(X_0,M_0)=\begin{cases}\frac{w(b)}{p^*} & \text{if } m^*\in M_0,\ \{\chi(i)=\mathbb{I}(i\geq\lambda b)\}\in X_0 \text{ and } b\geq b^* \\ 0 & \text{otherwise}\end{cases} \tag{30}$$

leading to consumption

$$\begin{aligned} c_1(b)&=\begin{cases} w(b) & \text{if } b<b^* \\ 0 & \text{if } b\geq b^*\end{cases} \\ c_2(b)&=\begin{cases} 0 & \text{if } b<b^* \\ \frac{w(b)}{p^*}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)} & \text{if } b\geq b^*\end{cases}\end{aligned} \tag{31}$$

4. Demand

$$D(X_0,M_0)=\begin{cases}\frac{1}{p^*}\int_{\max\{b_L,b^*\}}^{\max\{b_H,b^*\}} w(b)\,db & \text{if } m^*\in M_0 \\ 0 & \text{otherwise}\end{cases} \tag{32}$$

for X_0 of the form $X_0=\{\chi\in X:\chi(i)=\mathbb{I}(i\geq g) \text{ for } g\in[\lambda b_L,\lambda b_H]\}$.

5. Allocation function

   (a) For market m^* and $\chi(i)=\mathbb{I}(i\geq g)$ for some $g\in[0,\lambda]$:

$$a(i;\chi,m^*)=\frac{\mathbb{I}(i\in[g,\lambda))+\mu\mathbb{I}(i\geq\lambda)}{[\lambda-g]+\mu[1-\lambda]} \tag{33}$$

   (b) For market m^* and any other acceptance rule:

$$a(i;\chi,m^*)=\begin{cases}\frac{\chi(i)[1-\eta(m^*;i)]}{\int_0^\lambda\chi(i)[1-\eta(m^*;i)]di} & \text{if } \chi(i)\notin X_0 \text{ and } \int_0^\lambda\chi(i)[1-\eta(m^*;i)]\,di>0 \\ \frac{\chi(i)[1-\eta(m^*;i)]}{\sum_i\chi(i)[1-\eta(m^*;i)]} & \text{if } \chi(i)\notin X_0,\ \int_0^\lambda\chi(i)[1-\eta(m^*;i)]\,di=0 \text{ but } \sum_i\chi(i)[1-\eta(m^*;i)]>0 \\ 0 & \text{otherwise}\end{cases} \tag{34}$$

where $\eta(m^*; i)$ is given by (36) below.

(c) For any other market:

$$a(i;\chi,m) = \begin{cases} \frac{\chi(i)S(i;m)}{\int \chi(i)S(i;m)di} & \text{if } \int \chi(i) S(i;m)\, di > 0 \\ \frac{\chi(i)S(i;m)}{\sum_i \chi(i)S(i;m)} & \text{if } \int \chi(i) S(i;m)\, di = 0 \text{ but } \sum_i \chi(i) S(i;m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{35}$$

where $S(i;m)$ is given by (29) above.

6. Rationing function

$$\eta(M_0; i) = \begin{cases} 1 & \text{if } m^* \in M_0 \text{ and } i \geq \lambda \\ \int_{b^*}^{\frac{i}{\lambda}} \frac{1}{\lambda(1-b)+\mu(1-\lambda)} \frac{w(b)}{p^*} db & \text{if } m^* \in M_0 \text{ and } i \in [\lambda b^*, \lambda) \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

Proposition 1. *Equations (25)-(36) describe an equilibrium.*

The equilibrium works as follows. There is a single market m^*, with $p(m^*) = p^*$, where all trades take place. In this market, distressed sellers supply all their assets while non-distressed sellers only supply bad assets. Total supply is therefore μ of each good asset and 1 of each bad asset.

Supply decisions in markets $m \neq m^*$ have no effect on sellers' utility since $\eta(m; i) = 0$, so they are determined in equilibrium by the robustness requirement. By Lemma 1, this involves a reservation price for each asset for each seller. For good assets, it turns out that fraction that they are able to sell in market m^* is 1. Therefore distressed sellers' reservation price for them is p^*: they supply them in all markets where the price is above p^* (where if they could, they would rather sell them) but don't supply them in any markets where the price is below p^* (since they are able to sell them for sure at p^*, they don't want to sell them at a lower price). For bad assets, the fraction that can be sold in market m^* is strictly below 1. Therefore the reservation price for all sellers is 0: all sellers supply them in all markets.

The clearing algorithm in market m^* is LRF. As in example (17), being preceded by less-restrictive trades is not a problem for buyers because these trades don't change the relative proportions of acceptable assets in the residual supply faced by a more-restrictive buyer. Instead, as in example (18), any buyer faces more adverse selection if higher-b buyers have cleared before him. Therefore buyers self-select into trading in a LRF market, where

all buyers end up receiving a representative sample of the overall supply of assets they are willing to accept. Informally, one could think that a lower-b buyer would rather trade in a market where the price is $p+\varepsilon$ but he is guaranteed to be first in line than in a market where the price is p but higher-b buyers clear their trades before him.[2]

Sellers, for their part, are indifferent regarding what algorithm is used to clear trades: they just care about the price and fraction of assets they will be able to sell. Therefore they supply the same assets in all markets that have the same price.[3]

Buying from markets with prices other than p^* is not optimal for buyers. At prices lower than p^*, the supply includes only bad assets, so buyers prefer to stay away, whereas at prices above p^*, the supply of assets is exactly the same as at p^* but the price is higher.

This does not settle the question of whether a buyer chooses to buy at all. Buyers who choose to buy from market m^* can reject some of the bad assets that are on sale there, but not all of them. Consider a buyer of type b. The sample of assets he accepts includes all the good assets that are supplied, of which there are $\mu(1-\lambda)$, as well as all bad assets with indices $i \in (b\lambda, \lambda]$, which total $\lambda(1-b)$. Therefore the terms of trade (in terms of $t=2$ goods per $t=1$ good spent) for buyer b are

$$\tau(b) = \frac{1}{p^*} \frac{\mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} \tag{37}$$

$\tau(b)$ is increasing in b because the higher-b buyers can reject more of the bad assets and therefore draw from a better sample overall. Condition (26) implies that the terms of trade for type b^* are $\tau(b^*) = 1$, which leave him indifferent between buying or not. Buyers with $b > b^*$ get $\tau(b) > 1$, so they spend all their endowment buying assets and buyers with $b < b^*$ would get $\tau(b) < 1$, so they prefer not to buy at all.

The fraction of assets i that can be sold in market m^* is given by the ratio of the total allocation of that asset across of buyers to the supply of that asset. For good assets, the supply is $\mu(1-\lambda)$ and buyer b (with $b \geq b^*$) obtains $\frac{w(b)}{p} \frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$ units. Adding across buyers and imposing that all good assets get sold results in (25).

Figure 2 illustrates how the equilibrium p^* and b^* are determined. The indifference

[2]All markets besides m^* have zero demand, so no matter what the clearing algorithm, a buyer in those markets would receive a representative sample of the assets he accepts, just as in market m^*. This means that buyers are indifferent between buying in market m^* or in other markets where the price is also p^*, but sticking to m^* is one of the optimal choices.

[3]Imposing robustness in sellers' solution does not settle what sellers do about markets other with the same price as m^* and other clearing algorithms. Supplying the same assets they supply in m^* is one of the optimal choices.

condition (26) defines an upward-sloping relationship between p^* and b^*: at higher prices, terms of trade are lower so the marginal buyer needs to be more expert. The market-clearing condition (25) defines a downward-sloping relationship between p^* and b^*: at higher prices, more wealth is needed to buy the entire supply of good assets, which requires the participation of lower-b buyers.

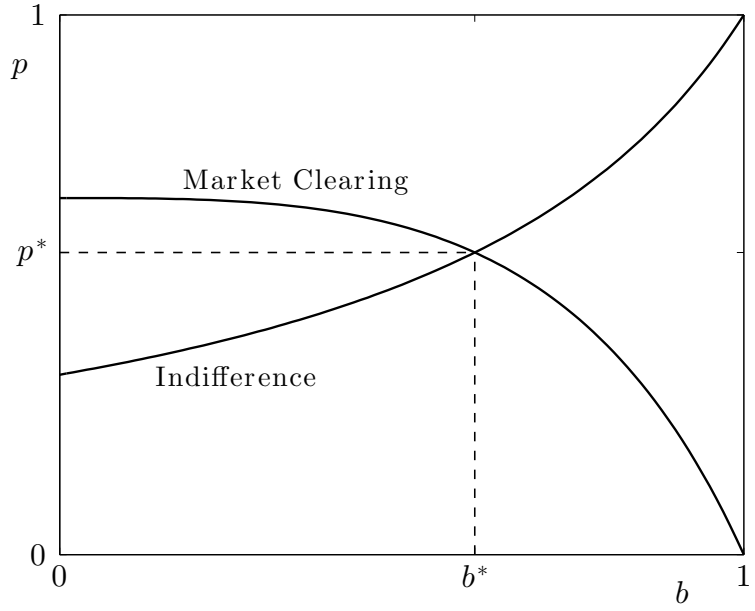


Figure 2: Equilibrium conditions. The example uses $\lambda = 0.5$, $\mu = 0.5$ and $w(b) = 0.7b^2$.

For assets $i \in (\lambda b^*, \lambda]$, the supply is 1 and buyer b obtains $\frac{w(b)}{p} \frac{1}{\lambda(1-b)+\mu(1-\lambda)}$ as long as $b \in [b^*, \frac{\lambda}{i})$; lower types demand nothing and higher types reject asset i. This implies the rationing function (36), as illustrated in Figure 3. Notice that $\eta(m^*, i)$ is continuous in $\eta(i)$. Bad assets with indices just below λ fool almost all buyers into thinking they are likely to be high quality and therefore sellers are able to sell a high fraction of them; assets with indices just above λb^* fool very few buyers and only a low fraction are sold. Assets with $i < \lambda b^*$ are rejected by all buyers who choose to trade and cannot be sold at all.

The equilibrium described above is (essentially) unique.

Proposition 2. *In any equilibrium, the price and allocations are those of the equilibrium described by equations (25)-(36).*



Figure 3: Rationing function. The example uses $\lambda = 0.5$, $\mu = 0.5$ and $w(b) = 0.7b^2$.

The proof (in Appendix A) proceeds in several steps. I first show (Lemma 3) that the logic of the example in Table 1 generalizes: as the rounds of a clearing algorithm advance, the pool of remaining assets always weakly worsens; therefore (Lemma 4) given any acceptance rule, buyers obtain the best possible terms of trade if their trades clear in the first round. This allows an easy characterization of an upper bound on the terms of trade that buyer b can obtain in any market m (Lemma 5): they can never do better than what would result from imposing $\chi(i) = \mathbb{I}(i \geq \lambda b)$ and clearing in the first round. Using this result, I show that in equilibrium it must be that all trades take place at the same price (Lemma 6): if there were more than one price, say p_H and p_L where trades take place, I can always find a market where $p \in (p_L, p_H)$ where any buyer can obtain better terms of trade than the upper bound on what he can obtain by buying at p_H.[4] I then show (Lemma 7) that in any equilibrium where there is trade at price p it must be that all distressed sellers are able to sell all the good assets at price p: otherwise buyers would be able to obtain better terms of trade at prices below p. The combination of a single price, the condition that all good assets can be sold and buyer optimization implies that equations (25) and (26) must hold.

[4]The single-price result is special to the case where the dividend paid by bad assets is exactly zero. If that dividend were $q_L > 0$, then it is easy to show that there would also be trade in at least one market with $p(m) = q_L$. Indeed, even if we maintain the assumption that the dividend of bad assets is zero, there are equilibria where as well as trading at p^*, traders trade bad assets at a price of zero. Of course, these trades don't matter for allocations.

Other equilibria besides the one described by equations (25)-(36) are possible, but they all lead to the same allocation. They only differ in terms of in which of the markets where $p(m) = p^*$ trades take place. Trades could, for instance, all take place in a market where the acceptance rule $\chi(i) = \mathbb{I}(i \in (0.3, 0.5))$ takes precedence over all others and after that the rule is less-restrictive-first. Since $\chi(i) = \mathbb{I}(i \in (0.3, 0.5))$ is not feasible for any buyers, this would make no difference for allocations. Trades could also take place in more than one market. For instance, buyers $b \in [b^*, b^* + \Delta]$ could trade in the LRF market while buyers $b \in (b^* + \Delta, 1]$ trade in a market that is less-restrictive-first but only for rules of the form $\chi(i) = \mathbb{I}(i \geq g)$ with $g > (b^* + \Delta)\lambda$. What is common to all equilibria is that all trades take place at price p^* and that no buyer trades after a more-informed buyer in the same market.

Changes in information

The model can be used to examine what happens if the quality of information changes.

Definition 8. For two otherwise identical economies where the endowment functions are w and $\tilde{w}$ respectively, with $W(0) = \tilde{W}(0)$, the economy with endowment $\tilde{w}$ has better information if $\tilde{W}(b) \geq W(b)$ for all b.

Given the way the model is parametrized, the information of each buyer is fixed; better information is represented by a shift in wealth towards higher-expertise buyers in a FOSD sense, holding total wealth constant. This is isomorphic to assuming that the wealth of each buyer is held fixed but their expertise shifts up. Conversely, a deterioration of information can equivalently be the result of the highest-expertise buyers losing their wealth or of all buyers unergoing a drop in their level of expertise.

Proposition 3. *p^* and b^* increase with better information.*

Proposition 3 provides a meaningful way to think about the effects of changes in the degree of informational asymmetry. If buyers become less expert (or, equivalently, if the more expert buyers lose wealth), the marginal buyer will become less expert and prices will drop. Increases in the informational asymmetry could be the result of traders realizing the the models that they relied on to value securities are not as accurate as they thought or, as Dang et al. (2009) argue, they could be result of negative shocks themselves.

Figure 4 shows the result graphically. A worsening of information leads to a downwad shift in the market clearing condition (25) and hence p^* and b^* must fall.

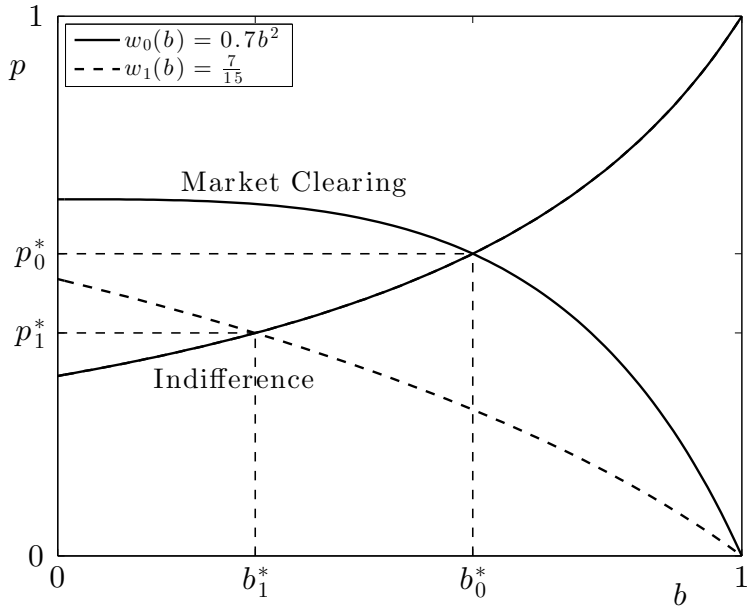


Figure 4: A worsening of information. The example uses $\lambda = 0.5$ and $\mu = 0.5$.

Fire Sales

The term "fire sales" is sometimes used to refer to situations where traders' urgency for funds leads them to sell assets at prices that are far below their usual price. Fire sales have been documented in many different markets, including used aircraft (Pulvino 1998), real estate (Campbell et al. 2011), equities (Coval and Stafford 2007), corporate bonds (Ellul et al. 2011) and convertible bonds (Mitchell et al. 2007).

Examples of traders with an urgent need of funds include hedge funds facing margin calls, banks facing runs on their deposits, etc. This distress could itself be the result about bad news about the value of the assets, in which case a drop in price is no puzzle. The question is whether the need to sell itself makes the price drop, an effect that is at the heart of a sizable literature (see Shleifer and Vishny (2011) for a recent survey).

In the context of the current model, one can ask whether an increase in μ (the fraction of sellers who are distressed) leads to a decrease in p^*. If so, then the model has the potential to explain fire sales.

Proposition 4.

1. p^* is decreasing in μ if and only if

$$w(b^*) < \left[\lambda + \frac{\mu(1-\lambda)}{1-b^*}\right] \mu(1-\lambda) \int_{b^*}^{1} \frac{w(b)}{\left[\lambda(1-b) + \mu(1-\lambda)\right]^2} db \tag{38}$$

2. If $w(b)$ is a constant, then $\frac{dp^*}{d\mu} = 0$.

In general, there are two opposing effects when more sellers become distressed, as illustrated in Figure 5. On the one hand, since $p^* < 1$, distressed sellers are the only ones who are willing to sell good assets. Other things being equal, more distressed sellers should improve the pool of assets being sold and thus lead to higher, not lower, prices. This is reflected in an upward shift in the indifference condition (26). This is the effect emphasized by Uhlig (2010), who concludes that an equally-uninformed-buyers model cannot be the entire explanation for fire-sale patterns. Indeed, with equally uninformed buyers the market clearing condition (25) is fixed and vertical, so an upward shift in the indifference condition (26) implies higher p^*.

However, more distressed sellers mean that more assets are being offered for sale. This is reflected in a downward shift in the market clearing condition (25). Given that the more expert buyers exhaust their wealth, an increased supply makes it is necessary to resort to less expert buyers. These less expert buyers are aware that they are less clever at filtering out the bad assets so, other things being equal, they will make up for this by only entering the market if prices are lower. The net effect on p^* depends on which of these two shifts is greater. In Figure 5 the second effect dominates and p^* falls.

Proposition 4 shows that which effect dominates (locally) depends on the density of wealth at the equilibrium cutoff level of expertise. If $w(b^*)$ is high, this means that a large amount of wealth would enter the market if the cutoff level of expertise was lowered slightly. In this case, the direct selection effect dominates and prices rise, meaning there are no fire sales. Instead when $w(b)$ is low, cutoff level of expertise needs to fall a lot in order to attract sufficient wealth to buy the extra units supplied. In this case, the changing-threshold effect dominates and prices fall. Interestingly, for the special case where wealth is uniformly distributed across all levels of expertise, the price is the same for any μ, so both effects cancel out.

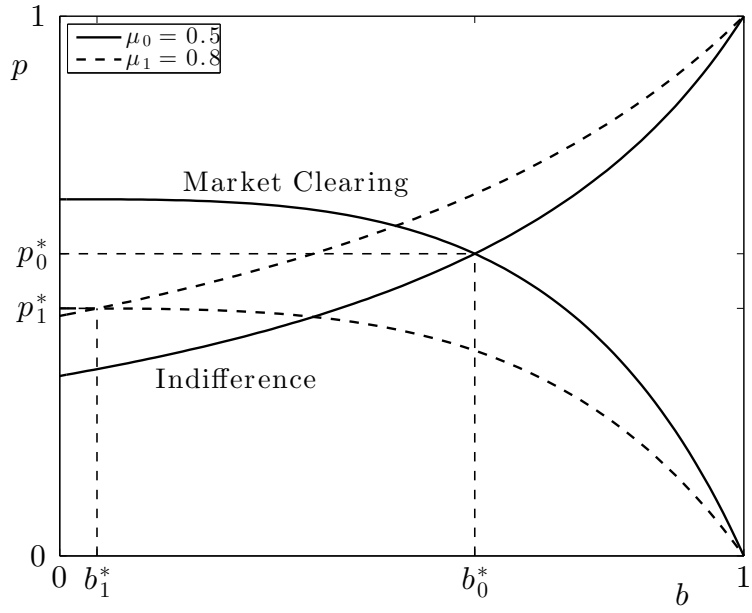


Figure 5: An increase in μ. The example uses $\lambda = 0.5$ and $w(b) = 0.7b^2$.

The model is useful for exploring the relationship among other theories of fire-sales in the existing literature. One class of theories (Shleifer and Vishny 1992, 1997, Kiyotaki and Moore 1997) emphasizes that the marginal buyer of an asset can be a second-best user with diminishing marginal product. If first-best users need to sell more units, asset prices will fall along the marginal-product curve of second-best users. This mechanism is probably better suited to explain fire sales for real assets that can be given alternative uses than for financial assets. The holder of a financial asset does not need to use his expertise and/or complementary assets in order to extract value from it, so the idea of a second-best user does not naturally fit fire-sales in financial markets. However, the current model illustrates that expertise may be relevant in the trade itself, and moving along a gradient of expertise can induce to fire-sale effects.

A second class of theories (Fostel and Geanakoplos 2008, Geanakoplos 2009) derives a diminishing-marginal-valuation schedule among potential buyers as a consequence of differences of opinion about the true value of the asset combined with borrowing constraints, even though actual payoffs from holding the asset are the same for all traders. The current setup, instead, is based on standard common-prior beliefs and the differences among buyers are in the quality of their information. Besides this basic difference, the two setups have much in common. First, changes in the identity of the marginal buyer are the key driver of changes in prices. Second, borrowing constraints are the reason why the natural buyers have limits

on the positions they can take. A maintained assumption in the current model is that the high-b buyers cannot borrow to increase the volume of assets they buy. Otherwise, $b = 1$ buyers would drive up the price all the way to 1 and reject all bad assets. One possible interpretation is that $w(b)$ represents the total resources available to buyer b after they have exhausted their borrowing capacity.[5]

A third class of theories (Allen and Gale 1994, 1998, Acharya and Yorulmazer 2008) relies on the notion of cash-in-the-market pricing. There is a given amount of purchasing-power available, so if more units are to be sold, the price must fall. But this class of models typically leave unanswered the question of why buyers with deep pockets (for instance, rich individuals) stay out of the market. The current model provides an explanation for buyers staying out of the market: even though there are good deals available for those who have expertise, those who do not have expertise are rationally worried that they are not able to select the deals among all the assets on offer. In other words, given their expertise, buying from this market does not provide excess returns, even though it does for experts.

A fourth class of theories, building on Grossman and Stiglitz (1980) and Kyle (1985) is, like the current model, based on limited information. In those models, increases in the supply of the asset as a result of "noise traders" play a similar role to increases in μ. Crucially, however, the assumption is that the aggregate net supply from noise traders is unobserved. They lead to falls in prices (over and beyond what is needed to persuade traders to hold extra units of a risky asset) because uninformed traders face a signal extraction problem: they are rationally unsure whether there has been an increase in supply or more-informed traders have received bad news. Those models have the implication that fire sales would not take place if traders were aware of the supply shocks. In the current model, instead, fire sales can take place even though the parameter μ is commonly known.

Contagion

Consider the following extension of the model to a case where there are two asset classes, A and B. For instance, A assets could be high-yield corporate bonds and B assets emerging market sovereign bonds. Each asset class contains a fraction of bad assets, λ_A and λ_B respectively. They are held by separate groups of sellers, of whom fractions μ_A and μ_B respectively are distressed. There is a common pool of buyers, with types $(b_A, b_B) \in [0,1] \times [0,1]$ and endowments $w(b_A, b_B)$. Buyers know whether an asset belongs to asset class A or

[5]Using the assets as collateral would not undo borrowing constraints because lower-b buyers (natural lenders) would not have the ability to distinguish good from bad collateral.

B and for asset i in asset class $z \in \{A, B\}$, they observe signals $x(i, b_z) = \mathbb{I}(i \geq b_z \lambda_z)$; in this formulation, b_z represents the expertise of the buyer in asset class z.

The equilibrium is characterized by a generalization of equations (25) and (26). As in the single-asset-class case, only one market for each asset class is active and all good assets held by distressed sellers are sold. Buyers have three options: buying A assets, buying B assets or not buying at all. Their decision will depend on (b_A, b_B), as illustrated in Figure 6. The terms of trade that buyers can obtain are still given by (37) so buyers will be indifferent between buying A and B assets when

$$\frac{1}{p_A^*} \frac{\mu_A (1 - \lambda_A)}{\lambda_A (1 - b_A) + \mu_A (1 - \lambda_A)} = \frac{1}{p_B^*} \frac{\mu_B (1 - \lambda_B)}{\lambda_B (1 - b_B) + \mu_B (1 - \lambda_B)} \tag{39}$$

This condition is represented by the dotted line in Figure 6. Buyers SouthEast of this line are more expert in A assets while buyers Northwest of it are more expert in B assets. Buyers with sufficiently low b_A and b_B would have $\tau < 1$ for both asset classes and stay out of the market.

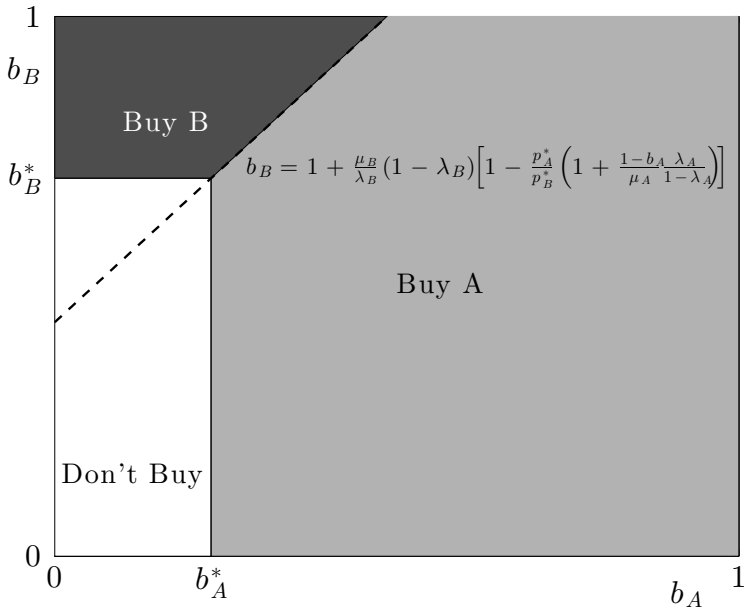


Figure 6: Buyer's decisions as a function of (b_A, b_B). The example uses $p_A = 0.7$, $p_B = 0.9$, $\mu_A = 0.2$, $\mu_B = 0.3$, $\lambda_A = 0.1$ and $\lambda_B = 0.1$.

The equilibrium p_A^*, p_B^*, b_A^* and b_B^* satisfy:

$$\frac{1}{p_A^*}\frac{\mu_A\left(1-\lambda_A\right)}{\lambda_A\left(1-b_A^*\right)+\mu_A\left(1-\lambda_A\right)}=1 \tag{40}$$

$$\int_{b_A^*}^{1}\left(\int_{0}^{\bar{b}_B\left(b_A,p_A^*,p_B^*\right)}\frac{1}{\lambda_A\left(1-b_A\right)+\mu_A\left(1-\lambda_A\right)}\frac{w\left(b_A,b_B\right)}{p_A^*}db_B\right)db_A=1 \tag{41}$$

(and symmetrically for asset class B), where

$$\bar{b}_B\left(b_A,p_A^*,p_B^*\right)\equiv\min\left\{1+\mu_B\frac{1-\lambda_B}{\lambda_B}\left[1-\frac{p_A^*}{p_B^*}\left(1+\frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right)\right],1\right\} \tag{42}$$

is derived from rearranging (39). Condition (40) is the analogue of equation (26). Buyers with A-type b_A^* are exactly indifferent between buying A assets and not buying. Condition (41) is the analogue of equation (25). Integrating all the good assets bought by buyers in the light shaded area of Figure 6 must equal the endowment of good A assets from distressed sellers.

As in the single-asset-class case, an increase in μ_A could raise or lower p_A^*, depending on the density of wealth of marginal buyers.[6] A different question is what happens to prices of A assets when μ_B increases.

Proposition 5. *Suppose p_B^* is decreasing in μ_B. Then p_A^* is decreasing in μ_B.*

If an increase in μ_B results in a fall in p_B^*, then buying B assets becomes more attractive for all buyers. Other things being equal, marginal buyers who are indifferent between buying A and B assets will shift towards buying B assets. These marginal buyers exist as long as there is positive density of wealth along the dotted line in Figure 6, i.e. as long as there are active investors whose expertise in both asset classes is sufficiently even that they are willing to shift from one asset class to the other. In order to restore equilibrium to the A market, p_A^* must fall. Thus, there will be contagion from distress of the owners of one asset class to the prices of the other asset class through the equilibrium decisions of the common pool of potential buyers. Calvo (1999) makes a related argument applied to the Russian crisis of 1998, in a model based on a signal-extraction problem in the style of Grossman and Stiglitz (1980).

[6]Here marginal buyers include both those indifferent between buying A assets and not buying and those indifferent between buying A assets and B assets, i.e. everyone on the frontier of the lightly shaded region of Figure 6.

5 False Negatives Case

I describe the equilibrium informally, relegating a formal statement, together with the proof that it is unique, to Appendix C. Thanks to Lemma 1, each seller's decision can be summarized in terms of a reservation price $p^R(i)$ for each asset. As in the false-positives case, $p^R(i) = 0$ for bad assets for all sellers and $p^R(i) = 1$ for good assets for non-distressed sellers. Unlike the false-positive case, the $p^R(i)$ for distressed sellers is different for different good assets. Distressed seller's preferences imply that $p^R(i)$ must be such that they are able to sell asset i for sure by supplying it in all markets with $p(m) \geq p^R(i)$ (otherwise they would do better by supplying it at lower prices as well); hence finding $p^R(i)$ is equivalent to finding the lowest price at which asset i trades. Unlike the false-positives case, where all trades of a given asset take place at the same price, in this case some fraction of asset i could trade at prices above $p^R(i)$ as well. Therefore to characterize the equilibrium one must find both $p^R(i)$ and any other prices at which asset i trades.

For each $i \in [\lambda, 1]$, $p^R(i)$ for for distressed sellers falls into one of three possible classes: a "cash-in-the-market" price, a "bunching" price or a "nonselective" price.

Cash-in-the-market price

The basic way to determine $p^R(i)$ is by a form of cash-in-the-market pricing. Define $\hat{b}(i)$ as

$$\hat{b}(i) \equiv \frac{1-i}{1-\lambda} \tag{43}$$

$\hat{b}(i)$ is the lowest buyer type that observes $x(i,b) = 1$, i.e. the least expert buyer who realizes that asset i is of high quality. The cash-in-the-market price $p^C(i)$ for asset i is the price such that buyer $\hat{b}(i)$ can afford to buy enough units so that all units held by distressed sellers are sold. Hence

$$p^C(i) = \frac{1}{r(i)(1-\lambda)} w\left(\hat{b}(i)\right) \tag{44}$$

where $r(i)$ is the number of units held by distressed sellers that they were not able to sell at prices above $p^C(i)$ and the term $(1-\lambda)$ is the result of a change of measure: $d\hat{b}(i) = -\frac{1}{1-\lambda} di$.[7]

As long as $p^C(i)$ defines a function that is strictly increasing and sufficiently high (in a

[7] $r(i) \equiv \mu\left[1 - \eta\left(\left\{m : p(m) > p^C(i)\right\}, i\right)\right]$, so $r(i)$ could be equal to or lower than μ (the total endowment of distressed sellers) depending on whether $\eta\left(\left\{m : p(m) > p^C(i)\right\}, i\right) > 0$, i.e. on whether it is possible to sell some units of asset i at prices above $p^C(i)$. This will be the case if nonselective pricing applies to some assets with indices above i.

sense made precise below), the logic of cash-in-the-market pricing works as follows. Each asset $i \in [\lambda, 1]$ will be supplied by distressed sellers in all markets where $p(m) \geq p^C(i)$ and in no market with a lower price, while all bad assets are supplied in all markets. Each buyer will attempt to buy assets in the cheapest market where he can find assets for which he observes $x(i,b) = 1$, i.e. where he can detect good assets, and will impose the acceptance rule $\chi(i) = \mathbb{I}(i \geq 1 - b(1-\lambda))$. Consider a market where $p = p^C(i)$. In it there will be assets in the range $[\lambda, i]$ on sale, but no assets in the range $(i, 1]$, since those can be sold at higher prices. Buyer $b = \hat{b}(i)$ will be able to see good assets in this market but buyers $b < \hat{b}(i)$ will not. Indeed, if $p^C(i)$ is strictly increasing, this is the cheapest market where buyer $\hat{b}(i)$ can detect good assets so he will spend his entire endowment in this market. Equation (44) implies that this will exhaust the remaining supply of asset i, confirming the conjecture that i will not be on sale at prices below $p^C(i)$. Notice that, because a single type of buyer demands assets at each price, it does not matter what clearing algorithm is used.

There are two reasons why assets might not actually trade at the prices described by expression (44). First, $p^C(i)$ need not be monotonic. Second, it could be so low that it makes it attractive for buyers to buy at price $p^C(i)$ and impose $\chi(i) = 1$ (i.e. accept all assets). These considerations lead to bunching and nonselective pricing respectively.

Bunching

Since the endowment function w could have any shape, the function p^C could have any shape too and need not be increasing in i. If it happens to be decreasing over some range, then the cash-in-the-market pricing logic described above breaks down. Suppose for some i, i' with $\lambda < i < i'$ it were the case that $p^C(i') < p^C(i)$. Buyer $\hat{b}(i)$ can identify both i and i' as good assets, so if asset i' is on sale at price $p^C(i')$, he would prefer to buy in that market. Therefore there would be no buyer for asset i. By this logic, if all good assets held by distressed sellers are to be sold, their reservation price must be (weakly) monotonically increasing in i, so that easier-to-recognize good assets trade at a higher price than harder-to-recognize ones.

Imposing monotonicity results in a form of bunching, where several assets trade at the same price. The range of assets that are bunched can be found as follows. Define

$$E(i,p,r) \equiv \max_{j \in [\lambda, i]} \int_{\hat{b}(i)}^{\hat{b}(j)} w(b)\, db - p \cdot r \cdot (i - j) \tag{45}$$

For a given asset i, price p and remaining supply r, $E(i,p,r)$ measures the maximum over $j<i$ of difference between the endowment of all buyers who can recognize that i is good but cannot recognize that j is good and what it would cost to buy r units of all assets in $[j,i]$ at price p. An asset can only be priced by cash-in-the-market if $E\left(i,p^C(i),r(i)\right)=0$. A strictly positive value would mean that there exists a range of buyers $\left[\hat{b}(i),\hat{b}(j)\right]$ for some $j<i$, all of whom can identify some asset in the range $[j,i]$ as a good asset (but not any asset lower than j) and whose collective endowment exceeds what is necessary to buy all assets in $[j,i]$ for a price $p^C(i)$. Since these buyers will want to spend their entire endowment buying assets, it must be that some asset in the range $[j,i]$ must be priced above $p^C(i)$; monotonicity then implies that the price of asset i exceeds $p^C(i)$.

Replacing $p^C(i)$ with a constant over any range where $E\left(i,p^C(i),r(i)\right)>0$, a procedure similar to the "ironing" algorithm of Mussa and Rosen (1978), results in a weakly monotone function that restores a version of the cash-in-the-market logic. Each buyer spends his entire endowment buying from the cheapest market where he can detect good assets and distressed sellers can sell all their good assets. In markets where there is bunching, the clearing algorithm used lets lower-b buyers, who impose more restrictive acceptance rules, trade before higher-b buyers. This ensures that there are enough good assets remaining for all buyers, as in the example in Table 2.

Note that condition (1) implies $E(1,1,\mu)>0$, i.e. the total amount of wealth is more than enough to buy all good assets from distressed sellers at a price of 1. This implies that there is necessarily a range of assets at the top that are bunched at a price of 1. In other words, the most transparent assets will be sold at no discount.

Nonselective pricing

Suppose distressed sellers offer asset $i>\lambda$ at price p. This implies that all assets in the interval $[\lambda,i)$ from distressed sellers will also be on sale in all markets where the price is p. Any buyer can decide to be "nonselective" and demand assets in the market where the price is p and the the clearing algorithm is LRF, imposing $\chi(i)=1$ (i.e. accept everything). He will obtain a representative sample from a pool of $\mu(i-\lambda)$ good assets mixed with λ bad assets and therefore obtain a fraction of

$$p^N(i)\equiv\frac{\mu(i-\lambda)}{\mu(i-\lambda)+\lambda} \tag{46}$$

good assets. If $p < p^N(i)$, this would be better than not trading. Since assumption (1) implies that some buyers do not trade, this must mean that no asset i is offered at a price below $p^N(i)$. Therefore p^N provides a lower bound on reservation prices.

When this lower bound is operative, trade will take place in markets where both selective and nonselective buyers are active. In the market where the price is $p^N(i)$, nonselective buyers will buy just enough assets (distributed pro-rata among the assets offered) so that buyer $\hat{b}(i)$ can afford to buy all the remaining i assets. Buyer $\hat{b}(i)$ can afford to buy $\frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)}$ units so nonselective buyers buy the remainining units. The fact that these buyers are nonselective implies that their purchases will include the same number of units of all assets in $[\lambda, i)$ as of asset i. Therefore the amount of asset i bought by buyer $\hat{b}(i)$ is equal to the amount of any asset $j \in [\lambda, i)$ that remains unsold. Hence, since w is assumed to be continuous:

$$r(i) = \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)} \tag{47}$$

Equation (47) implies that in any single market, nonselective buyers buy a zero measure of assets; they only buy a positive measure of assets once one aggregates over range of markets. This has a simple interpretation. Suppose asset i is such that $p^C(i) = p^N(i)$. This means that buyer $\hat{b}(i)$ can afford to buy exactly $r(i)$ assets at price $p^N(i)$. Since w is continuous, buyers $\hat{b}(j)$ for j close to i have to be very close to being able to afford the same amount. Hence a small purchase from nonselective buyers is enough for selective buyers to exhaust the supply.

By the same logic of the false-positives case, the clearing algorithm in these markets will dictate that nonselective buyers clear first.

Construction of the equilibrium

The main step in describing the equilibrium is to establish the ranges of assets over which each type of pricing will prevail. This can be done through the following iterative procedure. Supposing one knows that $\bar{i}$ is the upper limit of one type of region, the procedure finds the lower end of the region, the type of region immediately below and the prices within the region.

1. For a cash-in-the market region, the lower end is

$$\sup\left\{i < \bar{i} : p^N(i) > p^C(i) \text{ or } E\left(i, p^C(i), r(i)\right) > 0\right\} \tag{48}$$

and the region to the left is a nonselective region or a bunching region respectively, depending on which of the two conditions is met. Within the region, $p^R(i) = p^C(i)$ and $r(i) = r(\bar{i})$.

2. For a bunching region, the lower end is

$$\max\left\{i < \bar{i} : E\left(i, p^R(\bar{i}), r(\bar{i})\right) = 0\right\} \tag{49}$$

and the region to the left is always a cash-in-the-market region. Within the region $p^R(i) = p^R(\bar{i})$ and $r(i) = r(\bar{i})$.

3. For a nonselective region, the lower end is

$$\sup\left\{i < \bar{i} : \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)} > r(j) \text{ for some } j \in (i, \bar{i}) \text{ or } E\left(i, p^N(i), r(i)\right) > 0\right\} \tag{50}$$

and the region to the left is a cash-in-the-market region or a bunching region respectively, depending on which of the two conditions is met. Within the region, $p^R(i) = p^N(i)$ and $r(i) = \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)}$.

The first region is a bunching region with $\bar{i} = 1$, $p(\bar{i}) = 1$ and $r(\bar{i}) = \mu$ and, if at any point one of the sets defined by (48), (49) or (50) is empty, then the region extends up to $i = \lambda$ and p^R has been completely defined.

Figure (7) shows an example of how $p^R(i)$ is constructed. The lower left panel shows $w\left(\hat{b}(i)\right)$, which in the example is relatively high for high values of i, i.e. low-b buyers who can only recognize high-i assets have most of the wealth. The top bunching region ends at $i = 0.94$. For $i \in [0.84, 0.94]$, $p^C(i)$ is lower than 1 and increasing, so cash-in-the-market pricing prevails. For $i \in [0.74, 0.84]$, $p^C(i)$ would fall below $p^N(i)$ if $r(i)$ were constant at μ, so nonselective pricing prevails. Nonselective buyers buy a respresentative sample of all assets on offer at each price, so $r(i)$ falls from $r(0.84) = \mu = 0.5$ to $r(0.74) = 0.19$. At $i = 0.74$, $E\left(i, p^R(i), r(i)\right)$ turns positive again (lower right panel), meaning that buyers who can recognize i and lower assets as good have more than enough wealth to afford them all at p^R. Therefore there is a bunching region, with assets $i \in [0.36, 0.74]$ all trading at the same price. Finally, assets in $i \in [0.3, 0.36]$ have cash-in-the-market pricing.

Having determined p^R, the rest of the equilibrium is straightforward. Buyers for whom $p^R(1 - b(1-\lambda)) < 1$ can detect good assets in markets where the price is below 1, so they

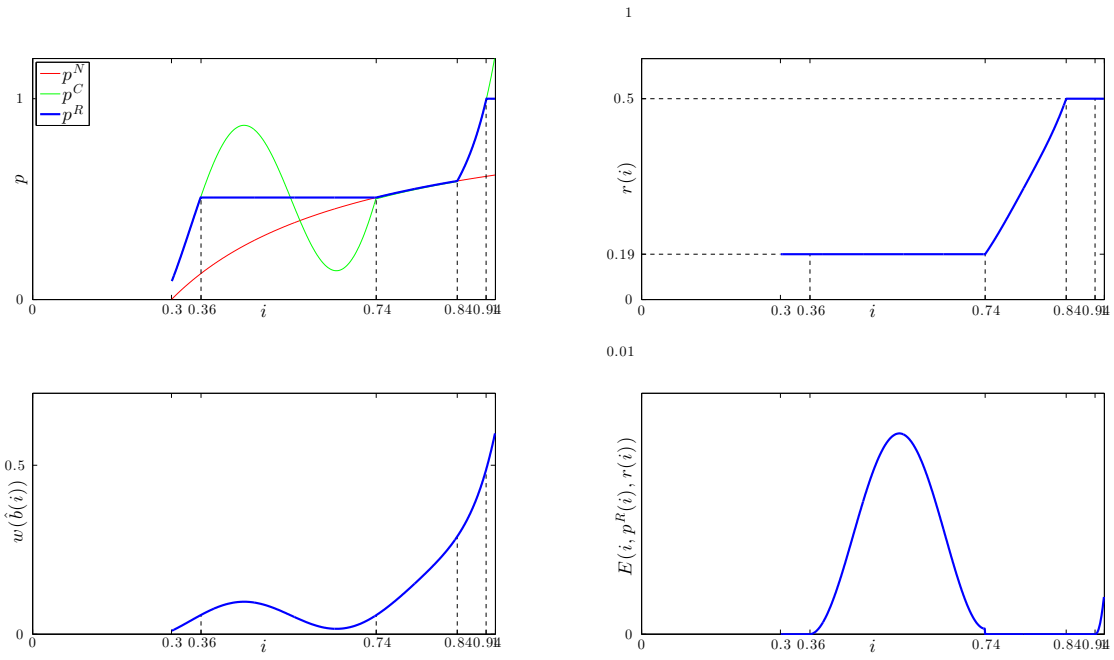


Figure 7: Construction of the reservation price p^R. The example uses $\lambda = 0.3$, $\mu = 0.5$ and $w(b) = 0.1e^{-b} + 0.5e^{-10b} + 0.05\sin(10b)$

spend their entire endowment in them; the rest are indifferent between not buying, buying (selectively) from the market where $p = 1$ and buying nonselectively from a market in a nonselective region if there is one.

Flights to Quality

What happens if μ increases? As in the false-positives case, the answer depends on parameters. Suppose first that parameters were such that there was no region of nonselective pricing, so that $r(i) = \mu$ for all i. In this case, equation (44) implies that the cash-in-the-market price falls when there is an increase in μ. This is precisely the logic of cash-in-the-market pricing, as in Allen and Gale (1994). If all assets are bought by selective buyers, their price is determined by the ratio of buyers' wealth to quantity supplied; an increase in the fraction of assets that are held by distressed sellers implies an increase in quantity supplied and necessitates a fall in prices. On the other hand, equation (46) says that nonselective prices are increasing in μ, since more distressed sellers improve the overall mix of assets that a nonselective buyer will encounter. Overall, the effect could be mixed, with prices for some assets increasing while others decrease.

Figure (8) shows the effect of an increase in μ from 0.5 to 0.7 in the example from Figure (7). For high values of i such that there are no nonselective-pricing regions to the right, then prices fall when μ increases. However, the lower bound from nonselective pricing is higher and nonselective buyers end up buying larger amounts, so for lower values of i prices rise when μ increases.

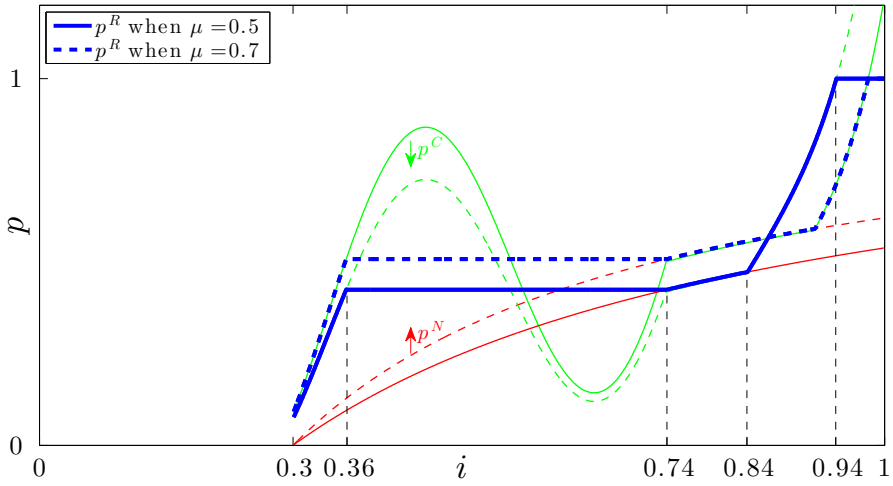


Figure 8: An increase in μ in the False-Negatives case

Some features of the example in Figure (8) turn out to hold in general.

Proposition 6.

1. *Let i_1 be the lowest i such that $p^R(i) = 1$. $p^R(i)$ is weakly decreasing in μ for any $i > i_1$ and, if $i_i > \lambda$, strictly decreasing in μ in a neighborhood of i_1.*

2. *$p^R(i)$ is increasing in μ for any i in a nonselective pricing interval.*

Part 1 of Proposition 6 can be interpreted as a model of a "flight to quality". For any given μ, there is a range of assets that are sufficiently transparent that their price is 1. But even within this range some assets are more transparent than others. If μ increases, this creates a difference between the most transparent assets, for which it is still the case that the buyers who can identify them as good have enough wealth to afford them at $p = 1$ and the slightly-less-transparent ones, for which the buyers who can identify them as good cannot afford the increased supply at the original price. This could be an explanation of why the premium for

the most transparently good assets (such as US Treasuries) over high-quality assets that may require some expertise to assess (such as high-grade corporate bonds), increases in times of market stress.

6 Final Remarks

This paper provides a theory of competitive equilibrium for economies with heterogeneous assets and heterogeneous information, as well as existence and uniqueness results and a characterization for some simple cases.

Many markets in addition to the market for financial assets fit this description. Venture capital funds may have heterogeneous ability to judge the business plans of a pool of start-ups; some sports teams may have better information than others about the talent levels in a pool of free agents; some house buyers may be better informed than others about house construction quality or neighborhood trends. Kurlat and Stroebel (2013) find evidence consistent with this sort of information heterogeneity in residential real estate.

Consistent with the simple environment, the types of trades allowed in the model are very limited: only exchanges of goods for assets. However, the same type of equilibrium notion could be applied to study the trade of more complicated, multidimensional objects such as insurance contracts. This could be useful, for instance to ask what happens when there are competitive insurance companies with different abilities to assess risks.

Moreover, the equilibrium is not equivalent to the outcome of a mechanism design problem. In this environment the information of different buyers is very highly correlated, and hence the logic of Crémer and McLean (1988) applies. A mechanism designer could obtain all buyers' information and implement allocations other than the equilibrium outcome. The equilibrium concept implicitly restricts the outcomes to those that result from buyers, in some order, choosing representative samples of acceptable assets.

Due to the assumptions on seller's preferences, with $\beta(s)$ only taking the values 0 and 1, the equilibrium allocations are Pareto Efficient: all distressed sellers sell all their good assets so gains from trade are exhausted. This would no longer be true if $\beta(s)$ took intermediate values: in that case there would be some sellers with $\beta(s) < 1$ that inefficiently kept some good assets. In Kurlat (2013) I analyze that case and show that in general expertise is socially valuable, i.e. the total surplus increases if $w(b)$ shifts towards higher values. However, the social value may be greater or smaller than the private value, so there could be both over- and under-investment in acquiring expertise.

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Appendix A: Proofs

Proof of Lemma 1

Assume the contrary. This implies that there are two markets, m and m' with $p(m') > p(m)$ such that, for some i, the seller chooses $\sigma(i, m) > 0$ and $\sigma(i, m') < 1$. There are four possible

cases:

1. $\eta(m;i) > 0$ and $\eta(m';i) > 0$. Then the seller can increase his utility by choosing supply $\tilde{\sigma}$ with $\tilde{\sigma}(i,m') = \sigma(i,m') + \varepsilon$ and $\tilde{\sigma}(i,m) = \sigma(i,m) - \varepsilon\frac{\eta(m';i)}{\eta(m;i)}$ for some positive ε.

2. $\eta(m;i) > 0$ and $\eta(m';i) = 0$. Consider a sequence such that $\eta^n(m';i) > 0$. By the argument in part 1, for any n the solution to program (11) must have either $\sigma(i,m) = 0$ or $\sigma(i,m') = 1$ (or both). Therefore either the condition that $\sigma^n(i,m) \to \sigma(i,m)$ or the condition that $\sigma^n(i,m') \to \sigma(i,m')$ in a robust solution is violated.

3. $\eta(m;i) = 0$ and $\eta(m';i) > 0$. Consider a sequence such that $\eta^n(m;i) > 0$. By the argument in part 1, for any n the solution to program (11) must have either $\sigma(i,m) = 0$ or $\sigma(i,m') = 1$ (or both). Therefore either the condition that $\sigma^n(i,m) \to \sigma(i,m)$ or the condition that $\sigma^n(i,m') \to \sigma(i,m')$ in a robust solution is violated.

4. $\eta(m;i) = \eta(m';i) = 0$. Consider a sequence such that $\eta^n(m';i) > 0$ and suppose that there is a sequence of solutions to program (11) which satisfies $\sigma^n(i,m') \to \sigma(i,m') < 1$. This implies that for any sequence such that $\eta^n(m,i) > 0$ and for any n, the solution to program (11) must have $\sigma^n(i,m) = 0$. Therefore the condition that $\sigma^n(i,m) \to \sigma(i,m) > 0$ in a robust solution is violated.

Proof of Lemma 2

Replacing (26) into (25):

$$\int_{b^*}^{1} \frac{w(b)}{\mu(1-\lambda)} \frac{\lambda(1-b^*) + \mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} db = 1 \tag{51}$$

The left-hand-side of (51) is decreasing in b^* so any solution must be unique. Setting $b^* = 1$ the left-hand-side is equal to 0 while setting $b^* = 0$

$$\int_{0}^{1} \frac{w(b)}{\mu(1-\lambda)} \frac{\lambda + \mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} db > \int_{0}^{1} \frac{w(b)}{\mu(1-\lambda)} db \geq 1$$

where the second inequality follows from the assumption (1). Hence, by continuity a solution with $b^* \in (0,1)$ exists.

Proof of Proposition 1

1. Seller optimization.

   The rationing function (36) implies that sellers will be able to sell all assets $i \in [\lambda, 1]$ and a fraction $\eta(i, m^*) < 1$ of assets $i \in [\lambda b^*, \lambda)$ in market m^*, and nothing else. A necessary and sufficient condition for a solution to program (4) is that in market m^* distressed sellers supply the maximum possible amount of assets $i \in [\lambda b^*, 1]$ and non-distressed sellers supply the maximum possible amount of assets $i \in [\lambda b^*, \lambda)$ and no assets $i \in [\lambda, 1]$. The decisions in (27) about other assets in market m^* and about all assets in markets $m \neq m^*$ are consistent with a robust solution. The level of consumption (28) then follows from the budget constraint.

2. Supply.

   (29) follows from aggregating (27) over all sellers.

3. Buyer optimization.

   Placing positive measure on any feasible acceptance rule other than $\chi(i) = x(i, b)$ in market m^* would, according to (33) and (34), result in a lower fraction of good assets, so placing positive demand only on $\chi(i) = \mathbb{I}(i \geq \lambda b)$ is optimal.

   Define the terms of trade that a buyer obtains in market m with acceptance rule χ as

$$\tau(m, \chi) \equiv \begin{cases} \frac{\int q(i) dA(i;\chi,m)}{p(m) A([0,1];\chi,m)} & \text{if } A([0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

   Let

$$\tau^{\max}(b) \equiv \max_{m \in M, \chi \in X_b} \tau(m, \chi)$$

   be the best terms of trade that buyer b can achieve, and let $M^{\max}(b)$ be the set of markets where buyer b can obtain terms of trade $\tau^{\max}$ with a feasible acceptance rule.

   Necessary and sufficient condition for buyer optimization are that buyers for whom $\tau^{\max}(b) < 1$ choose zero demand, i.e. $\delta_b(X, M) = 0$, buyers for whom $\tau^{\max}(b) > 1$ spend their entire endowment in markets in $M^{\max}(b)$ and buyers for whom $\tau^{\max}(b) = 1$ only place demand in markets in $M^{\max}(b)$. Using equation (33), a buyer b that uses

acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ in market m obtains terms of trade

$$\tau(m, \chi) = \begin{cases} \frac{1}{p(m)} \frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)} & \text{if } p(m) \geq p^* \\ 0 & \text{otherwise} \end{cases}$$

so for all buyers

$$\tau^{\max}(b) = \frac{1}{p^*} \frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$$

and the maximum is attained in any market where the price is p^*, including m^*. Together with condition (26), this implies that types $b < b^*$ have $\tau^{\max}(b) < 1$ so zero demand as specified by (30) is optimal for them while buyers with types $b \geq b^*$ have $\tau^{\max}(b) \geq 1$, so spending their entire endowment in market m^* as dictated by (30) is optimal for them too. The level of consumption (31) then follows from the budget constraint.

4. Demand.

   (32) follows from aggregating (30) over all buyers.

5. Allocation function.

   In all markets except m^* demand is zero, so for any clearing algorithm the residual supply faced by any buyer in whatever round they trade equals the original supply. For these cases, (35) follows directly from (21) and (23).

   For market m^*, the LRF algorithm together with equilibrium demand (32) implies that a buyer that imposes $\chi(i) = \mathbb{I}(i \geq g)$ will clear all his trades in round g, and for $g \leq \lambda$, residual supply in round g satisfies

$$S^g(i; m) = \alpha S(i; m)$$

   for some $\alpha > 0$ for any $i \in I_\chi$, i.e. the residual supply of assets acceptable to rule χ is positive and proportional to the original supply. Therefore the measure of assets he will obtain is the same as if he traded entirely in the first round. Therefore (33) follows from (21) and (23).

   For market m^* and rules that are not of the form $\chi(i) = \mathbb{I}(i \geq g)$ with $g \leq \lambda$ (which nobody uses in equilibrium), their trades clear after all active buyers, so the supply they face only includes assets $i < \lambda$ and is given by $S^k(i, ; m) = 1 - \eta(m^*; i)$. Therefore

(34) follows from (21) and (23).

6. Rationing function.

   (36) follows from (25) and direct application of formula (24)

Proof of Proposition 2

I first establish a series of preliminary results.

Lemma 3. *Consider an arbitrary market m and a round of clearing k. Denote the residual supply before round k by S^k and after round k by S^{k+}. Then in any equilibrium, for any $b \in [0,1]$, if $i_L \in [\lambda b, \lambda)$ and $i_H \in [\lambda, 1]$*

$$\frac{S^{k+}(i_H; m)}{S^k(i_H; m)} \leq \frac{S^{k+}(i_L; m)}{S^k(i_L; m)} \tag{53}$$

Proof. Given their information, any buyer b has three feasible acceptance rules:

$$\begin{aligned} \chi_b(i) &= \mathbb{I}(i \geq \lambda b) \\ \chi_0(i) &= 1 \\ \chi_{-b}(i) &= \mathbb{I}(i < \lambda b) \end{aligned}$$

Rule χ_{-b} will never be associated to positive demand in equilibrium, because it implies accepting only assets that are known to be bad. This means that all demand in market m will be placed on either rule χ_b, which accepts assets with a signal $x(i,b) = 1$ or rule χ_0, which accepts all assets. Let $X_b \equiv \{\chi \in X : \chi(i) = \mathbb{I}(i \geq \lambda b) \text{ for some } b \in [0,1]\}$ be the set of all rules of the form χ_b or χ_0 and define $\tilde{b}(\chi)$ by $\chi(i) \equiv \mathbb{I}\left(i \geq \lambda \tilde{b}(\chi)\right)$ for $\chi \in X_b$. Using (22) and (21), we can write the residual supply after round k as what remains after all rules in X_b get their allocation:

$$S^{k+}(i_H; m) = S^k(i_H; m) - \int_{\chi \in X_b} \left(\frac{S^k(i_H, m)}{\int_{\lambda \tilde{b}(\chi)}^1 S^k(i, m)\, di} \omega_m(k, \chi) \right) dD(\chi, m)$$

so

$$\frac{S^{k+}(i_H; m)}{S^k(i_H; m)} = 1 - \int_{\chi \in X_b} \left(\frac{1}{\int_{\lambda \tilde{b}(\chi)}^1 S^k(i, m)\, di} \omega_m(k, \chi) \right) dD(\chi, m) \tag{54}$$

Similarly,

$$S^{k+}\left(i_L;m\right)=S^{k}\left(i_L;m\right)-\int\limits_{\chi\in X_b}\left(\frac{S^{k}\left(i_L,m\right)\mathbb{I}\left(i_L\geq\lambda\tilde{b}\right)}{\int_{\lambda\tilde{b}(\chi)}^{1}S^{k}\left(i,m\right)di}\omega_m\left(k,\chi\right)\right)dD\left(\chi,m\right)$$

so

$$\frac{S^{k+}\left(i_L;m\right)}{S^{k}\left(i_L;m\right)}=1-\int\limits_{\chi\in X_b}\left(\frac{\mathbb{I}\left(i_L\geq\lambda\tilde{b}\right)}{\int_{\lambda\tilde{b}(\chi)}^{1}S^{k}\left(i,m\right)di}\omega_m\left(k,\chi\right)\right)dD\left(\chi,m\right) \tag{55}$$

Subtracting (55) from (54):

$$\frac{S^{k+}\left(i_H;m\right)}{S^{k}\left(i_H;m\right)}-\frac{S^{k+}\left(i_L;m\right)}{S^{k}\left(i_L;m\right)}=\int\limits_{\chi\in X_b}\left(\frac{\mathbb{I}\left(i_L\geq\lambda\tilde{b}\right)-1}{\int_{\lambda\tilde{b}(\chi)}^{1}S^{k}\left(i,m\right)di}\omega_m\left(k,\chi\right)\right)dD\left(\chi,m\right) \tag{56}$$

The right hand side of (56) is nonpositive, which implies (53). □

Lemma 3 states that, for any buyer b, as the rounds of clearing progress, good assets leave the pool at a (weakly) greater rate than the bad assets that buyer b might accept. This implies that, other things being equal, buyers prefer to trade in markets where their trades will clear sooner rather than later. This helps establish an upper bound on the terms of trade a buyer might obtain in a given market: the best terms of trade that a buyer can obtain in a market are those that result if his trades clear in the first round so that he obtains a representative sample of the acceptable assets supplied to that market.

Lemma 4. *Let $\tau\left(m,\chi\right)$ be defined by (52). Then in equilibrium, for any m,χ:*

$$\tau\left(m,\chi\right)\leq\frac{1}{p\left(m\right)}\frac{\int q\left(i\right)\chi\left(i\right)S\left(i;m\right)di}{\int\chi\left(i\right)S\left(i;m\right)di} \tag{57}$$

Proof. Using (52) and (23):

$$\tau\left(m,\chi\right)=\frac{1}{p\left(m\right)}\frac{\int\left[\int q\left(i\right)dA^{k}\left(i,\chi,m\right)\right]d\omega\left(k,\chi\right)}{\int A^{k}\left(\left[0,1\right],\chi,m\right)d\omega\left(k,\chi\right)}$$

For any round k:

$$A^{k}\left(\left[0,1\right],\chi,m\right)=\begin{cases}1 & \text{if } S\left(i;m\right)>0 \text{ for some } i\in I_\chi\\ 0 & \text{otherwise}\end{cases}$$

so, letting $\bar{k}$ be the highest round such that $S(i;m) > 0$ for some $i \in I_\chi$ we can rewrite

$$\begin{aligned}\tau(m,\chi) &= \frac{1}{p(m)} \frac{\int \left[\int q(i)\, dA^k(i,\chi,m)\right] d\omega(k,\chi)}{\omega\left(\left\{k \in K : k \leq \bar{k}\right\},\chi\right)} \\ &= \frac{1}{p(m)} \frac{\int \frac{\int_\lambda^1 \chi(i) S^k(i;m) di}{\int_0^1 \chi(i) S^k(i;m) di} d\omega(k,\chi)}{\omega\left(\left\{k \in K : k \leq \bar{k}\right\},\chi\right)}\end{aligned}$$

Lemma 3 implies that the term

$$\frac{\int_\lambda^1 \chi(i) S^k(i;m) di}{\int_0^1 \chi(i) S^k(i;m) di}$$

is weakly decreasing in k and therefore $\tau(m,\chi)$ is maximized if ω only places weight on the first round, which implies inequality (57). □

Lemma 4 places an upper bound on the terms of trade than can be obtained in a market given an acceptance rule. It is also possible to compute an upper bound for a given buyer who can choose among all his feasible acceptance rules.

Lemma 5. *Let*

$$\tau(m,b) \equiv \max_{\chi \in X_b} \tau(m,\chi)$$

In equilibrium, for any m, b:

$$\tau(m,b) \leq \frac{1}{p(m)} \frac{\int_\lambda^1 S(i;m) di}{\int_{\lambda b}^1 S(i;m) di}$$

Proof. For any χ, Lemma 4 implies

$$\tau(m,\chi) \leq \frac{1}{p(m)} \frac{\int q(i) \chi(i) S(i;m) di}{\int \chi(i) S(i;m) di} \tag{58}$$

The acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ satisfies $\chi \in \arg\max_{\chi \in X_b} \frac{\int q(i)\chi(i)S(i;m)di}{\int \chi(i)S(i;m)di}$ so taking the maximum in X_b on both sides of (58) implies the result. □

Knowing the upper bound on the terms of trade a buyer can obtain in a given market m is useful because if one can find a market m' where a buyer can obtain better terms of trade than the upper bound on $\tau(m,b)$, this implies that buyer b will not buy from market m. Using this fact, the following result establishes that in equilibrium all trades take place at the same price.

Lemma 6. *In equilibrium there is trade at only one positive price*

Proof. Assume the contrary, suppose there is trade in markets m_H and m_L where $p(m_H) > p(m_L) > 0$. If buyers are willing to buy in market m_L, then it means that distressed sellers are willing to sell some assets $i \geq \lambda$ at a price $p(m_L)$. Lemma 1 implies that in a robust solution to problem (4), for any m such that $p(m) > p(m_L)$, then $\sigma_s(i;m) = 1$ whenever $s < \mu$ and $i \geq \lambda$, i.e. all distressed sellers supply the maximum amount of any asset $i \geq \lambda$ (and hence $S(i;m) = \mu$) in all markets where $p > p_L$.

Fix any b and take a market m where $p(m) \in (p_L, p_H)$ and the clearing algorithm clears rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ in the first round. In such a market

$$\begin{aligned}\tau(m, \chi(i) = \mathbb{I}(i \geq \lambda b)) &= \frac{1}{p(m)} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m)\, di + \mu(1-\lambda)} \\ &> \frac{1}{p(m_H)} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m_H)\, di + \mu(1-\lambda)} \\ &\geq \tau(m_H, b) \text{ for any } m' \text{ such that } p(m') = p_H\end{aligned}$$

The first step follows from from applying (23) directly; the second from $p(m) < p(m_H)$ and the fact that, by Lemma 1, $S(i;m_H) \geq S(i;m)$ and the last step follows from Lemma 5. Therefore buyer b will not buy from any market where the price is p_H. Since this applies to all b, there can be no trade at p_H. □

Next I show that in any equilibrium where there is trade at price p^*, sellers are able to sell all their high-quality assets.

Lemma 7. *Let* $M_p = \{m \in M : p(m) = p\}$. In any equilibrium where there is trade at price p^*, $\eta(M_{p^*}; i) = 1$ *for all* $i \geq \lambda$.

Proof. Assume the contrary. Since no feasible acceptance rule for any buyer distinguishes between different high quality assets, $\eta(M_{p^*}; i) < 1$ for some $i \geq \lambda$ implies $\eta(M_{p^*}; i) < 1$ for all $i \geq \lambda$. By Lemma 6, there is no trade at any other price, which means that a fraction of high-quality assets held by distressed sellers remains unsold. Therefore in a robust solution to program (4), distressed sellers will supply $\sigma(i, m) = 1$ for all m for $i \geq \lambda$, which implies $S(i;m) = \mu$ for any $i \geq \lambda$. Now take any buyer b and consider any two markets: m, where $p(m) = p^*$ and m', where $p(m') < p^*$ and acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ clears in the

first round. The terms of trade buyer b can obtain in market m' are

$$\begin{aligned}\tau\left(m', \chi\left(i\right) = \mathbb{I}\left(i \geq \lambda b\right)\right) &= \frac{1}{p\left(m'\right)} \frac{\mu\left(1-\lambda\right)}{\int_{\lambda b}^{\lambda} S\left(i; m'\right) di + \mu\left(1-\lambda\right)} \\ &> \frac{1}{p^*} \frac{\mu\left(1-\lambda\right)}{\int_{\lambda b}^{\lambda} S\left(i; m\right) di + \mu\left(1-\lambda\right)} \\ &\geq \tau\left(m, b\right)\end{aligned}$$

where the inequalities follow in the same way as in the proof of Lemma 6. This implies that buyer b prefers to buy from market m' rather than from market m. Since this is true for all b and any m where $p\left(m\right) = p^*$, it contradicts the assumption that there is trade at p^*. □

Using Lemmas (3)-(7) Proposition 2 follows by the following argument.

Proof. Let p^* and b^* be defined by equations (25) and (26). Suppose there is an equilibrium where trade takes place at price $p_H > p^*$. For any market m where $p = p_H$, supply satisfies $S\left(i; m\right) = 1$ for $i < \lambda$ and $s\left(i, m\right) \leq \mu$ for $i \geq \lambda$. Therefore

$$\tau\left(m, b\right) \leq \frac{1}{p_H} \frac{\mu\left(1-\lambda\right)}{\lambda\left(1-b\right) + \mu\left(1-\lambda\right)}$$

so the lowest b that may be willing to buy is b_H, defined by

$$\frac{1}{p_H} \frac{\mu\left(1-\lambda\right)}{\lambda\left(1-b_H\right) + \mu\left(1-\lambda\right)} = 1$$

Equation (26) implies $b_H > b^*$. The maximum measure of high-quality assets that buyer b can get is

$$\frac{\mu\left(1-\lambda\right)}{\lambda\left(1-b\right) + \mu\left(1-\lambda\right)} \frac{w\left(b\right)}{p_H}$$

which implies that for any $i \geq \lambda$,

$$\eta\left(M_{p_H}; i\right) \leq \int_{b_H}^{1} \frac{1}{\lambda\left(1-b\right) + \mu\left(1-\lambda\right)} \frac{w\left(b\right)}{p_H} db$$

but because $p_H > p^*$ and $b_H > b^*$, equation (25) implies that $\eta\left(M_{p_H}; i\right) < 1$, which contradicts Lemma (7).

Suppose now that there is an equilibrium where trade takes place at $p_L < p^*$. For any market m where $p\left(m\right) \in \left(p_L, 1\right)$, supply satisfies $S\left(i; m\right) \leq 1$ for $i < \lambda$ and $S\left(i; m\right) = \mu$ for

$i \geq \lambda$. This is true in particular for markets where acceptance rule $\chi(i) = x(i, b)$ is cleared in the first round, so for any $p \in (p_L, 1)$, buyer b can find a market where the terms of trade are

$$\tau(m, \chi(i) = \mathbb{I}(i \geq \lambda b)) \geq \frac{1}{p} \frac{\mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)}$$

Therefore, in order for trade to only take place at p_L, it must be that all buyers with $b > b_L$ obtain terms of trade of at least

$$\frac{1}{p_L} \frac{\mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)}$$

in markets where $p = p_L$, where b_L is defined by

$$\frac{1}{p_L} \frac{\mu(1-\lambda)}{\lambda(1-b_L) + \mu(1-\lambda)} = 1$$

This would require a total of

$$\int_{b_L}^{1} \frac{\mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} \frac{w(b)}{p_L} db$$

good assets, but because $p_L < p^*$ and $b_L < b^*$, equation (25) implies this is more than $\mu(1-\lambda)$, which is the total supply of high-quality assets, so not all buyers can obtained the required terms of trade.

This means that in any equilibrium, all trades take place at $p = p^*$. The rest of the equilibrium objects follow immediately. □

Proof of Proposition 4

From equations (25) and (26), one can compute

$$\frac{dp^*}{d\mu} = \frac{\lambda(1-\lambda)}{\lambda(1-\lambda)\mu + [\lambda(1-b^*) + \mu(1-\lambda)] w(b^*)} \begin{bmatrix} \frac{1}{\lambda(1-b^*)+\mu(1-\lambda)} w(b^*)(1-b^*) \\ -\mu \int_{b^*}^{1} \frac{(1-\lambda)}{[\lambda(1-b)+\mu(1-\lambda)]^2} w(b)\, db \end{bmatrix}$$

Part 1 follows from setting $\frac{dp^*}{d\mu} < 0$ and rearranging; part 2 follows from replacing $w(b) = w, \forall b$ and simplifying.

Proof of Proposition 5

From (42),

$$\frac{\partial \bar{b}_B}{\partial \mu_B} = \begin{cases} \frac{1-\lambda_B}{\lambda_B}\left[1 - \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right)\right] & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \leq 0 \tag{59}$$

$$\frac{\partial \bar{b}_B}{\partial p_B^*} = \begin{cases} \mu_B \frac{1-\lambda_B}{\lambda_B}\frac{p_A^*}{\left(p_B^*\right)^2}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \geq 0 \tag{60}$$

$$\frac{\partial \bar{b}_B}{\partial p_A^*} = \begin{cases} -\mu_B \frac{1-\lambda_B}{\lambda_B}\frac{1}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \leq 0 \tag{61}$$

Equations (59) and (60) imply that, other things being equal, if μ_B rises and p_B^* falls, $\bar{b}_B$ falls. Using (61), his implies that for (41) to hold, either p_A^* must fall, b_A^* must fall, or both. Since equation (40) implies that b_A^* and p_A^* must move in the same direction, both fall.

Proof of Proposition 3

Define $b(x) \equiv W^{-1}(W(0) - x)$. Better information implies that $b(x)$ shifts up. Using that $\frac{db}{dx} = \frac{1}{w(b(x))}$, conditions (25) and (26) can be rewritten as:

$$\int_{b(x^*)}^{1} \frac{1}{\lambda(1 - b(x)) + \mu(1-\lambda)} \frac{1}{p^*} dx = 1 \tag{62}$$

$$p^* = \frac{\mu(1-\lambda)}{\lambda(1 - b(x^*)) + \mu(1-\lambda)} \tag{63}$$

If $b(x)$ shifts up, then either a higher p^* or a higher $b(x^*)$ is needed to restore equation (62); since equation (63) implies a positive relatioship between p^* and $b(x^*)$, this implies that they must both increase.

Proof of Proposition 6

1. By definition, $p^R(i) = 1$ for all $i > i_1$; since $p^R(i)$ is never above 1, it must be decreasing in μ. (49) and (45) imply that if $i_1 > \lambda$, then when μ increases, cash-in-the-market pricing will apply in a neighborhood of i_1. The result then follows because (44) implies $p^C(i)$ is decreasing in $r(i)$ and $r(i) = \mu$ in a neighborhood of i_1.

2. This follows directly from (46).

Appendix B: How Clearing Algorithms Process Excess Demand

Suppose that in the example from Table 1 buyer b_2 demanded 2 units instead of 1 and the clearing algorithm prescribed the measures ω given by (19). In round 2, the algorithm would attempt to assign 2 units of asset 2 to buyer b_2 when only 1 remains.

To complete the description of the algorithm, it is necessary to describe what happens if this arises. Let

$$\theta(k,i,m) \equiv \frac{S^k(i;m)}{\int\limits_{\chi\in X} a^k(i,\chi;m)\,\omega_m(k,\chi)\,dD(\chi,m)}$$

$$\theta(k,m) \equiv \left(\inf_{i\in[0,1]} \min\left\{\theta(k,i,m),1\right\}\right)$$

$\theta(k,i,m)$ is the ratio of the residual supply of assets i as of round k to the total demand that the algorithm attempts to allocate during round k. The amount the algorithm actually allocates per unit of demand is given by:

$$a^k(i,\chi,m)\,\theta(k,m)$$

This means that if in some round there is insufficient supply to meet demand for some asset, the allocation received by all acceptance rules in that round is reduced in the same proportion. This could leave some acceptable assets un-allocated. Therefore if $\theta(k,m) < 1$ and there are any assets remaining such that $\chi(i) = 1$ for an acceptance rule χ such that round k is in the support of $\omega_m(k,\chi)$, then round k is repeated until this is no longer the case.

For example, suppose the assets and acceptance rules are as in Table 1, buyer b_1 demands 2 units and buyer b_2 demands 8 units. Suppose further that the allocation algorithm is

$K = \{1, 2, 3\}$ and the following measures:

$$\omega(k, \chi) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \text{any other } \chi & \\ 0 & 0.25 & 0 & \text{if } k = 1 \\ 1 & 0.75 & 0 & \text{if } k = 2 \\ 0 & 0 & 1 & \text{if } k = 3 \end{cases}$$

In the first round, the algorithm attempts to allocate the following amounts per unit demanded

$$A^k(i; \chi, m) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \\ 0 & 0 & \text{if } i = \text{Black} \\ 0.5 & 0 & \text{if } i = \text{Red} \\ 0.5 & 1 & \text{if } i = \text{Green} \end{cases}$$

However, this results in

$$\theta(1, \text{Green}, m) \equiv \frac{1.5}{0.5 \times 1 \times 2 + 1 \times 0.25 \times 8} = 0.5$$

and therefore pro-rated demand is

$$A^k(i; \chi, m)\,\theta(k) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \\ 0 & 0 & \text{if } i = \text{Black} \\ 0.25 & 0 & \text{if } i = \text{Red} \\ 0.25 & 0.5 & \text{if } i = \text{Green} \end{cases}$$

This allocates a total of 0.5 units of each of assets Red and Green to buyer b_1 and 1 unit of Green assets to buyer b_2. This exahusts the supply of Green assets but not of Red assets, which are acceptable to buyer b_1. Therefore the first round is repeated on the remaining supply of $\{1.5, 1, 0\}$ and demands of 1 unit for buyer b_1 and 1 unit $(1 = 8 \times 0.25 - 1)$ for buyer b_2, resulting in

$$A^k(i; \chi, m) = \begin{cases} \text{if } \chi = \{0,1,1\} & \text{if } \chi = \{0,0,1\} & \\ 0 & 0 & \text{if } i = \text{Black} \\ 1 & 0 & \text{if } i = \text{Red} \\ 0 & 0 & \text{if } i = \text{Green} \end{cases}$$

which requires no further pro-rating. Since the supply of acceptable assets has been ex-

hausted, buyer b_2 receives nothing further in the second round. Overall, the allocation functions that result from this combination of supply, demand and clearing algorithm are:

$$A(i;\chi,m)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black} \\ 0.75 & 0 & \text{if } i=\text{Red} \\ 0.25 & 0.125 & \text{if } i=\text{Green} \end{cases}$$

The need to ration excess demand never arises in equilibrium. There are no constraints on the number of units a buyer can demand so any buyer that were to face rationing could overcome it by choosing higher demand.

Appendix C: Details of the False-Negatives case

Formal statement of the equilibrium

Definition 9. The *nonselective-then-more-restrictive-first* (NMR) clearing algorithm is given by the set of rounds $K=[0,1]$ and a measure that places measure 1 on round 0 (and zero on all other rounds) for rule $\chi(i)=1$; measure 1 on round $1-g$ (and zero on all other rounds) for acceptance rules of the form $\chi(i)=\mathbb{I}(i\geq g)$ for $g>0$ and measure 1 on round 1 (and zero on all other rounds) for any other acceptance rule.

The equilibrium is given by:

For any $i\in[\lambda,1]$, let $m(i)$ denote the market where the price is $p^R(i)$ (where $p^R(i)$ is as defined in the text) and the clearing algorithm is NMR. Note that because of bunching, $m(i)$ could mean the same market for different i. For any $I_0\subseteq[0,1]$, let the set of markets $M(I_0)$ be $M(I_0)=\{m(i):i\in I_0\}$. The set of active markets is $M([\lambda,1])$.

1. Supply decisions.

$$\sigma_s(i,m)=\begin{cases} 1 & \text{if } \begin{cases} i<\lambda \\ p(m)\geq p^R(i) \text{ and } s<\mu \\ \text{or} \\ p(m)\geq 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{64}$$

leading to consumption

$$c_{1,s} = \begin{cases} \int_{\lambda}^{1} \left[\int_{M([i,1])} p(m)\, d\eta(m;i) \right] di + \lambda \int_{M([\lambda,1])} p(m)\, d\eta(m;0) & \text{if } s < \mu \\ \lambda \int_{M([\lambda,1])} p(m)\, d\eta(m;0) & \text{if } s \geq \mu \end{cases}$$

$$c_{2,s} = \begin{cases} 0 & \text{if } s < \mu \\ 1-\lambda & \text{if } s \geq \mu \end{cases} \tag{65}$$

Equation (64) says that supply decisions follow the reservation price defined in the text. Equaiton (65) computes the resulting consumption using the rationing function η defined below. For distressed sellers, the first term in $c_{1,s}$ is the proceeds of trying to sell asset a good asset i in markets $M([i,1])$ and the integrating across i. The second term is the proceeds of trying to sell bad assets in markets $M([\lambda,1])$. It incorporates the fact that in equilibrium all bad assets sell at the same rate so $\eta(\cdot;i) = \eta(\cdot;0)$ for all $i \in [0,\lambda)$.

2. Aggregate supply.

$$S(i;m) = \begin{cases} 1 & \text{if } \begin{cases} i < \lambda \\ \text{or} \\ p(m) \geq 1 \end{cases} \\ \mu & \text{if } i \geq \lambda,\, p(m) \in [p^R(i), 1) \\ 0 & \text{if } i \geq \lambda,\, p(m) < p^R(i) \end{cases} \tag{66}$$

3. Demand decisions.

Let i_1 be the lowest i such that $p^R(i) = 1$. Define b_N by

$$\int_{b_N}^{\hat{b}(i_1)} w(b)\, db \equiv \mu(1 - i_1) \tag{67}$$

(a) For $b \in [b_N, 1]$:

$$\delta_b(X_0, M_0) = \begin{cases} \frac{w(b)}{p^R(1-b(1-\lambda))} & \text{if } m(1-b(1-\lambda)) \in M_0 \text{ and } \{\chi(i) = \mathbb{I}(i \geq 1-b(1-\lambda))\} \in X_0 \\ 0 & \text{otherwise} \end{cases} \tag{68}$$

(b) For $b \in [0, b_N)$

$$\delta_b\left(X_0, M\left(\left[i_L, i_H\right]\right)\right) = \begin{cases} \frac{\lambda\left[\frac{r(i_H)}{\mu} - \frac{r(i_L)}{\mu}\right] + (i_L - \lambda)[r(i_H) - r(i_L)] + \int_{i_L}^{i_H}[r(i_H) - r(i)]di}{\int_0^{b^N} w(b)db} w(b) & \text{if } \{\chi(i) = 1\} \in X_0 \\ 0 & \text{otherwise} \end{cases} \tag{69}$$

and $\delta_b\left(X_0, M_0\right) = 0$ if $M_0 \cap M\left(\left[\lambda, 1\right]\right) = \emptyset$.

leading to consumption

$$c_1(b) = \begin{cases} w(b) - \int p(m)\, d\delta_b\left(\chi(i) = 1, m\right) & \text{if } b < b^N \\ 0 & \text{if } b \geq b^N \end{cases}$$
$$c_2(b) = \begin{cases} \int p(m)\, d\delta_b\left(\chi(i) = 1, m\right) & \text{if } b < b^N \\ \frac{w(b)}{p^R(1 - b(1-\lambda))} & \text{if } b \geq b^N \end{cases} \tag{70}$$

Buyers $b \geq b_N$ spend their entire endowment buying assets in market $m\left(1 - b\left(1 - \lambda\right)\right)$, i.e. in the market for the lowest i for which they observe a good signal, and use the selective acceptance rule $\mathbb{I}\left(i \geq 1 - b\left(1 - \lambda\right)\right)$, which only accepts good assets. Buyers $b < b_N$ are nonselective and spread their demand across all markets in nonselective ranges. (69) results from noticing that in markets $M\left(\left[i_L, i_H\right]\right)$, nonselective buyers will buy enough units to bring down the number of unsold units of assets $i \in [\lambda, i_L)$ from $r\left(i_H\right)$ to $r\left(i_L\right)$; since they get assets in proportion to supply, they will bring down the number of unsold assets in $i \in [0, \lambda)$ from $\frac{r(i_H)}{\mu}$ to $\frac{r(i_L)}{\mu}$; and for assets $i \in [i_L, i_H]$ they will bring down the unsold units from $r\left(i_H\right)$ to $r\left(i\right)$. Equation (69) says that all buyers buy positive amounts and their demand is proportional to their wealth. Since they are indifferent between buying an not buying, many other patterns of demand among nonselective buyers are possible.

4. Demand.

Let $X(b_L, b_H) \equiv \{\chi \in X : \chi(i) = \mathbb{I}(i \geq 1 - b(1-\lambda)) \text{ for } b \in [b_L, b_H]\}$

$$D(X(b_L, b_H), M([i_L, i_H])) = \int_{\max\left\{b_L, b_N, \hat{b}(i_H)\right\}}^{\min\left\{b_H, \hat{b}(i_L)\right\}} \frac{w(b)}{p^R(1 - b(1-\lambda))} db$$

$$D(\chi(i) = 1, M([i_L, i_H])) = \lambda\left[\frac{r(i_H)}{\mu} - \frac{r(i_L)}{\mu}\right] + (i_L - \lambda)[r(i_H) - r(i_L)] + \int_{i_L}^{i_H} [r(i_H) - r(i)]\, di \tag{71}$$

and $D(X_0, M_0) = 0$ if $M_0 \cap M([\lambda, 1]) = \emptyset$.

5. Allocation function

   (a) For markets $m \notin M([\lambda, 1])$ or markets $m(i) \in M([\lambda, 1])$ where i falls in either a cash-in-the-market or a nonselective range:

$$a(i; \chi, m) = \begin{cases} \frac{\chi(i)S(i;m)}{\int \chi(i)S(i;m)di} & \text{if } \int \chi(i) S(i;m)\, di > 0 \\ \frac{\chi(i)S(i;m)}{\sum_i \chi(i)S(i;m)} & \text{if } \int \chi(i) S(i;m)\, di \text{ but } \sum_i \chi(i) S(i;m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{72}$$

   (b) For markets $m(i)$ where i falls in a bunching range $[i_L, i_H]$ and χ is of the form $\chi(i) = \mathbb{I}(i \geq g)$:

$$a(i; \chi, m) = \begin{cases} \frac{\chi(i)S^{1-g}(i;m)}{\int \chi(i)S^{1-g}(i;m)di} & \text{if } \int \chi(i) S^{1-g}(i;m)\, di > 0 \\ \frac{\chi(i)S^{1-g}(i;m)}{\sum_i \chi(i)S^{1-g}(i;m)} & \text{if } \int \chi(i) S^{1-g}(i;m)\, di \text{ but } \sum_i \chi(i) S^{1-g}(i;m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{73}$$

   where $S^{1-g}(i;m)$ is the solution to the differential equation

$$\frac{dS^k(i;m)}{dk} = \begin{cases} -\frac{S^k(i;m)\mathbb{I}[i \in [1-k, i_H]]}{\int_{1-k}^{i_H} S^k(j;m)dj} \frac{w\left(\frac{k}{1-\lambda}\right)}{p(m)} & \text{if } k \in [1 - i_H, 1 - i_L] \\ 0 & \text{otherwise} \end{cases} \tag{74}$$

with initial condition

$$S^0(i;m) = \begin{cases} 1 & \text{if } i < \lambda \\ \mu & \text{if } i \in [\lambda, i^H] \\ 0 & \text{otherwise} \end{cases}$$

Except for bunching markets, all buyers will draw assets from a sample that is proportional to the original supply, since their trades are never executed after another selective buyer. This results in (72). In bunching markets, buyer b trades in round $k = b(1-\lambda)$; therefore in round k supply of asset i falls in proportion to this buyer's demand $\frac{w\left(\frac{k}{1-\lambda}\right)}{p(m)}$ times the ratio between the supply of asset i and all the assets acceptable by buyer b (as long as buyer b accepts asset i). This results in the differential equation (74), which describes how the supply of each asset falls as the rounds progress.

6. Rationing function

$$\eta(M([j,1]);i) = \begin{cases} 1 - \frac{r(j)}{\mu} & \text{if } j > i \\ 1 & \text{if } j \leq i \end{cases} \tag{75}$$

and $\eta(M_0;i) = 0$ if $M_0 \cap M([\lambda,1]) = \emptyset$.

The rationing function simply says that a seller who offers an asset $i < j$ at every market with $p(m) \in \left[p^R(j), 1\right]$ will be able to sell a fraction $1 - \frac{r(j)}{\mu}$ (so that the unsold assets from the μ distressed sellers are $r(j)$), and $\frac{r(i)}{\mu}$ can be sold in market $m(i)$.

Proposition 7. *Equations (64)-(75) describe an equilibrium*

Proof.

1. Seller optimization.

   Since $r(j) > 0$, the rationing function (75) implies that in order to sell all of a seller's holdings of asset i, the reservation price must be $p^R(i)$. Therefore supply decisions (64) are optimal. Consumption (65) follows from the budget constraint and the fact that the rationing function is constant for $i < \lambda$.

2. Supply.

   (66) follows from aggregating (64) over all sellers.

3. Buyer optimization.

For $b \in [b_N, 1]$, (68) implies that each buyer only places weight on the lowest-price market m where there is an i such that $x(i,b) = 1$ and $S(i;m) > 0$; since $p(m) \leq 1$, this is optimal. For $b \in [0, b_N)$, (69) implies they only place weight on markets where nonselective pricing prevails; (46) that they are indifferent between trading or not and since the lowest-price market where there is an i such that $x(i,b) = 1$ and $S(i;m) > 0$ has $p(m) = 1$, there is no other market in which they would prefer to trade. Hence, their demand is optimal. Consumption (70) follows from the budget constraint.

4. Demand.

   (71) follows from aggregating (68) and (69) over all buyers.

5. Allocation function.

   For markets $m \notin M([\lambda, 1])$ demand is zero, so for any clearing algorithm the residual supply faced by any buyer in whatever round they trade equals the original supply. For these cases, (72) follows directly from (21) and (23).

   For markets $m(i) \in M([\lambda, 1])$ where i falls in either a cash-in-the-market or a nonselective range, the NMR algorithm together with equilibrium demand (71) implies that all buyers face a residual supply proportional to the original supply, so (72) follows from (21) and (23).

   For markets $m(i)$ where i falls in a bunching range $[i_L, i_H]$ and χ is of the form $\chi(i) = \mathbb{I}(i \geq g)$, then the differential equation (74) follows from (22), supply (66) and demand (71). Then (73) follows from applying the NMR algorithm.

6. Rationing function.

   Equation (75) follows from applying formula (36) to supply (66) and demand (71).

□

Proposition 8. *In any equilibrium, the prices and allocations are those of the equilibrium described by equations (64)-(75).*

I first establish some preliminary results. Recall that by Lemma (1), in any equilibrium there must be a price $p^R(i)$ for each asset such that distressed buyers supply asset i in all markets with $p(m) > p^R(i)$ and in no markets where $p(m) < p^R(i)$.

Lemma 8. *In any equilibrium $p^R(i)$ is nondecreasing in i over the range $[\lambda, 1]$.*

Proof. Assume the contrary. There there exist assets i, i' with $i' > i > \lambda$ such that $p^R(i') < p^R(i)$. For this to be consistent with seller optimization, it must be that

$$\eta(M_0; i') < \eta(M_0; i) < 1 = 1$$

where $M_0 = \{m : p(m) \geq p^R(i)\}$. But buyer optimization and the signal structure (3) requires that buyers only place positive demand on rules of the form $\chi(i) = \mathbb{I}(i \geq g)$. This implies that for any $M_0 \subseteq M$,

$$\eta(M_0; i') \geq \eta(M_0; i)$$

□

Lemma 9. *In any equilibrium, $\tau(m, \chi(i) = 1) \leq 1$ for all m, where τ is defined by (52).*

Proof. Assume the contrary. Since the acceptance $\chi(i) = 1$ is feasible for all buyers, this implies that all buyers will want to spend their entire wealth in some market. Condition (1) implies that this is inconsistent with equilibrium. □

The proof of Proposition (8) is as follows:

Proof. Assume that there exists an equilibrium where the reservation prices for distressed seller are $\tilde{p}^R(i) \neq p^R(i)$. Consider first the case where $\tilde{p}^R(i) > p^R(i)$ for at least one $i \in [\lambda, 1]$ and let i_0 be the highest i where this is the case.

1. If i_0 is in a cash-in-the market region, (44) implies that at price $\tilde{p}^R(i_0)$ buyer $\hat{b}(i_0)$ cannot afford to buy all the units that distressed sellers own; buyers $b > \hat{b}(i)$ can find good assets at lower prices because by Lemma (8), $\tilde{p}^R(i)$ is monotonic so they do not buy at price $\tilde{p}^R(i)$; buyers $b < \hat{b}(i_0)$ do not observe any good signals at price $\tilde{p}^R(i)$ because i_0 is the highest i where $\tilde{p}^R(i) > p^R(i)$ so no assets for which they observe good signals are on sale at $\tilde{p}^R(i)$; they could buy nonselectively but the fact that i_0 is in a cash-in-the-market region means that they would prefer not to buy. Therefore some units owned by distressed sellers will remain unsold at price $\tilde{p}^R(i)$. Given distressed seller's preferences, this implies that reservation price $\tilde{p}^R(i)$ cannot be optimal.

2. If i_0 is in a bunching region, then $\tilde{p}^R(i) > p^R(i)$ violates monotonicity.

3. If i_0 is in a nonselective region, then $\tilde{p}^R(i) > p^R(i)$ implies that nonselective buyers would prefer not to buy, so some units owned by distressed sellers will remain unsold, contradicting optimality.

Consider instead the case where $\tilde{p}^R(i) < p^R(i)$ for at least one $i \in [\lambda, 1]$ and let i_0 be the largest i where this is the case.

□

1. Suppose i_0 is in a cash-in-the market region and consider the decision of buyers of type $\hat{b}(i_0)$. Since $\tau\left(m(i_0), \chi(i) = \mathbb{I}\left(i \geq 1 - \hat{b}(i)(1-\lambda)\right)\right) > 1$, optimality requires that they spend their entire wealth. Monotonicity (Lemma 8) implies that they cannot find assets for which they observe good signals at prices below $\tilde{p}^R(i_0)$ and Lemma 9 implies that they do not want to buy nonselectively, so they must spend all their wealth buying at price $\tilde{p}^R(i_0)$. But (44) implies that there distressed sellers do not own enough units to exhaust buyers' wealth at that price, so it cannot be part of an equilibrium.

2. If i_0 is in a bunching region $[i_L, i_H]$, condition (49) together with the monotonicty condition (Lemma 8) imply that the distressed sellers' endowment of assets $i \in [i_L, i_H]$ is not enough to exhaust the wealth of buyers $b \in \left[\hat{b}(i_H), \hat{b}(i_L)\right]$, so it cannot be part of an equilibrium.

3. If i_0 is in a nonselective region, (46) implies that $\tau(m(i_0), \chi(i) = 1) > 1$, which contradicts Lemma 9.

Therefore it must be the case that in an equilibrium reservation prices are those defined in the text. The rest of the equilibrium objects follow immediately.