Firm-to-Firm Trade:

Imports, Exports, and the Labor Market[1]

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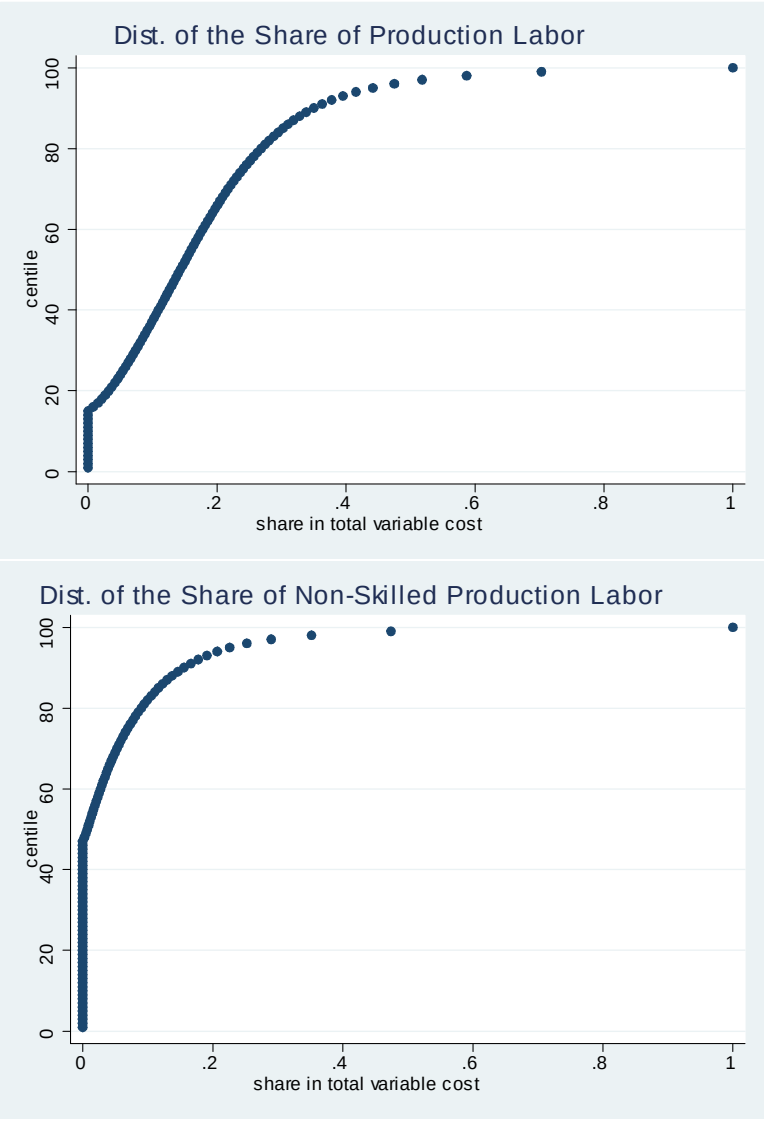
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1 Introduction

For a long time economists have exploited readily-available data on bilateral trade flows in different commodities. The aggregate nature of these data guided the first generation of quantitative trade models, which treated the sector as the relevant unit of production and intermediate demand and a representative household as the unit of final demand.

During the last two decades access to firm and plant-level data on export activity revealed the enormous heterogeneity of individual producers even within finely-defined sectors. In particular, many producers in an exporting sector don't export at all, and those that do export typically sell to only one or a small number of destinations. In response to these observations a new generation of trade models emerged in which the firm rather than the sector was the relevant production unit. This literature continued, however, to treat final demand as emanating from a representative agent. Moreover, while the literature recognized producer heterogeneity in the form of efficiency differences, it continued to treat input use as homogeneous among broad classes of firms.

Economists are now beginning to make us of data generated by customs records, the finest unit of trade transactions. These records reveal not only characteristics of the individual sellers but of their individual buyers as well. Perhaps not surprisingly, buyers are as heterogeneous as sellers, and a typical exporter sells to only one or a small number of buyers. Moreover, data on input use shows that firms are heterogeneous not only in terms of their productivity, size, and export status, but in terms of their use of different types of intermediates and workers. Figure 1 illustrates this heterogeneity in depicting the distribution of the labor share and


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Dist. of the Share of Non-Skilled Production Labor
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unskilled labor share among French manufacturing firms.[1]

To capture these additional dimensions of heterogeneity we build a general equilibrium model of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, meet sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

The model of firm-to-firm trade is complementary to the recent work of Oberfield (2013). Firm production combines the output of a number of tasks with a firm-specific efficiency level. Each task can be performed by the firm's employees or by an intermediate input purchased by the firm. The intermediates available to a firm are determined by a matching process, with the firm replacing its own workers to perform a particular task if a cheap enough intermediate is available. The distribution of prices for intermediates is itself determined by the distribution of costs of the other firms that produce them.

Our work relates to several strands in the literature. Recent papers looking at exports and labor markets (although not at imports) include Hummels, Jorgenson, Munch, and Xiang (2011), Felbermayr, Prat, and Schmerer (2008), Egger and Kreickemeier (2009), Helpman, Itskhoki, and Redding (2010), and Caliendo and Rossi-Hansberg (2012). In addition to Oberfield (2013), other theories of networks or input-output interactions include Lucas (2010),

[1] Note that many manufacturing firms have no workers at all. Even if a firm outsources all of its production, if it is the first seller of a manufactured product the firm is classified in manufacturing. For example, a fashion designer who sells clothes that she has designed is classified as a manufacturer even if she hires other firms, either at home or abroad, to make the clothes.

Acemoglu and Autor (2011), Chaney (2013), Luttmer (2013), Eaton, Eslava, Krizan, Jinkins, and Tybout (2014), and Acemoglu and Carvalho (2012). Quantitative work on exports, imports, and labor markets includes Irarrazabal, Moxnes, and Ulltveit-Moe (2010), Klein, Moser, and Urban (2010), Frias, Kaplan, and Verhoogen (2010), Kramarz (2009), Caliendo, Monte, and Rossi-Hansberg (2013), and Helpman, Itskhoki, Muendler, and Redding (2013).

2 A Model of Trade through Random Encounters

Consider $i = 1, 2, ..., \mathcal{N}$ countries. Producers in these countries make goods by combining labor and intermediates. These goods themselves may be consumed directly or used as intermediates in the production of other goods. Both final and intermediate sales of these goods occur through bilateral random meetings between agents rather than through Walrasian markets.

As in standard neoclassical trade models, each country i has an endowment L_i^l of labor of type $l = 1, 2, ...L$. As in more recent models of international trade, following Melitz (2003) and Chaney (2008), each country also has a measure potential producers of a good. The measure of potential producers in country i with efficiency $Z \geq z$ is:

$$\mu_i^Z(z) = T_i z^{-\theta}.$$

Production of a good requires performing a series of *tasks* $k = 1, 2, ..., K$, with task k having a Cobb-Douglas share β_k in production.

The measure of producers who can supply country i at a unit cost below c is given by:

$$\mu_i(c) = \Upsilon_i c^{\theta}, \tag{1}$$

where $\theta > 0$ and $\Upsilon_i \geq 0$. These suppliers could be located in country i or anywhere else.

Below we show how θ and Υ_i relate to underlying technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

A buyer can be either a consumer or a firm using a type of good for some *purpose* $k = 1, 2, ..., K$. A buyer of either type encounters various suppliers who make perfect substitutes for purpose k of this buyer. The buyer can also have purpose k fulfilled by workers of type l, whose wage is w_i^l. Worker productivity performing a task for a given buyer is given by Q, which is drawn from the distribution:

$$F(q) = \Pr[Q \leq q] = e^{-q^{-\phi}}, \tag{2}$$

where $0 < \phi \leq \theta$. We assume that $L \leq K$ and that each task can be performed by only one type of worker. But a type of worker may be able to perform multiple tasks. We denote by Ω_l the set of tasks that a worker of type l can perform. Hence, for $k \in \Omega_l$ we can denote the wage to perform task k by $w_{k,i} = w_i^l$, the wage of labor of type l. Since the various ways of performing the task are perfect substitutes, the buyer is interested in only the cheapest one.[2]

When a meeting occurs between suppliers of a good and a buyer, the buyer has all the bargaining power. Thus, the price of the good to fulfill purpose k is the cost of the lowest-cost supplier. The task is then carried out at the lowest-cost, the buyer either purchasing the good or using workers directly.

[2]While our terminology suggests that purchases are limited by a seller's ability to contact the relevant buyers, our framework can be interpreted much more broadly. The seller may be aware of a broad range of buyers, but the seller's product might not be appropriate to many of these buyers' specific purposes. We can interpret a low λ as reflecting a major idiosyncratic component to buyers' purposes or greater differentiation across producers' products. Standardization, for example, could be interpreted as raising λ.

We assume that the intensity with which a buyer in country i, seeking to fulfill need k, encounters a seller with cost c is:

$$e_{k,i}(c) = \lambda_{k,i} c^{-\varphi},$$

where $0 \leq \varphi < \theta$ captures the extent to which buyers are more aware of lower-cost sellers. Aggregating across the measure of suppliers with different costs, the number of suppliers that a buyer encounters with cost below c for purpose k is distributed Poisson with parameter

$$\begin{aligned} \rho_{k,i}(c) &= \int_0^c e_{k,i}(x) d\mu_i(x) \\ &= \int_0^c \lambda_{k,i} x^{-\varphi} \theta \Upsilon_i x^{\theta-1} dx \\ &= \frac{\theta}{\theta - \varphi} \lambda_{k,i} \Upsilon_i c^{\theta-\varphi}. \end{aligned} \tag{3}$$

Note that this Poisson parameter grows arbitrarily large with c, so that many potential suppliers are available to serve any given buyer.

Consider the cost of the lowest-cost supplier encountered by a given buyer. From the Poisson density, we know that with probability $\exp\left[-\rho_{k,i}(c_k)\right]$ the buyer will encounter no suppliers for need k with cost below c_k. Thus, with probability $1 - e^{-\rho_{k,i}(c_k)}$ the buyer will encounter at least one such supplier. The buyer also has the option of hiring type k workers directly to perform the task. This option will cost less than c_k if $w_{k,i}/Q < c_k$, which occurs with probability $1 - F(w_{k,i}/c_k)$. Need k will be carried out at a cost below c_k unless the cost of hiring workers directly and the cost of the best supplier both exceed c_k. Hence, the distribution of the cost faced by the buyer to fulfill need k is:

$$G_{k,i}(c_k) = 1 - F(w_{k,i}/c_k) e^{-\rho_{k,i}(c_k)}.$$

To work out the implications of this distribution for the resulting distribution of production

costs, we restrict

$$\theta - \varphi = \phi.$$

With this restriction, the parameter governing heterogeneity in the distribution of costs of intermediates is the same as the parameter governing heterogeneity in the distribution of worker efficiency (2) at a given task for a given buyer. In particular, the distribution of the cost to the buyer of fulfilling need k becomes:

$$G_{k,i}(c_k) = 1 - e^{-\Xi_{k,i} c_k^{\phi}}, \tag{4}$$

where

$$\Xi_{k,i} = \nu_{k,i} + w_{k,i}^{-\phi} \tag{5}$$

and

$$\nu_{k,i} = \frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i. \tag{6}$$

With probability $w_{k,i}^{-\phi}/\Xi_{k,i}$ the buyer fulfills need k by hiring workers while with probability $\nu_{k,i}/\Xi_{k,i}$ it purchases a good from the low-cost supplier.

We proceed by showing first how the cost measure (1) arises from our model of firm-to-firm trade. We then turn to consumer demand and then to intermediate demand before closing the model in general equilibrium.

2.1 Deriving the Cost Distribution

A potential producer j from country i, with efficiency $z_i(j)$, combines up to K tasks in a Cobb-Douglas constant-returns-to-scale production function. As described above, each task k

can be performed either by workers who are appropriate for that task or with an intermediate purchased from another firm.

We assume that the measure of potential producers in country i with efficiency above z is:

$$m_i(z) = T_i z^{-\theta}$$

where T_i reflects the density of producers and θ, as above, their dispersion.

The unit cost of a firm depends not only on its overall efficiency z, but on its cost of performing each task $\boldsymbol{c} = (c_1, c_2, ..., c_K)'$, where the distribution of each element of this vector is given by (4). Given z and $\boldsymbol{c}$, a potential producer's unit cost of delivering to country n is:

$$c_{ni}(j) = \frac{d_{ni} b(\boldsymbol{c})}{z},$$

where

$$b(\boldsymbol{c}) = B \prod_{k=1}^{K} (c_k)^{\beta_k},$$

where B is a constant which we employ below to cancel out a nuisance term. We assume the Cobb-Douglas parameters satisfy $\beta_k > 0$ and

$$\sum_{k=1}^{K} \beta_k = 1.$$

Here $d_{ni} \geq 1$ is the amount that has to be sent from country i to deliver one unit to country n. We set $d_{ii} = 1$ for all i.[3]

If all potential producers in i faced the same vector of costs per task $\boldsymbol{c}$, the measure that could serve market n at a cost below c would be:

$$m_i(d_{ni} b(\boldsymbol{c})/c) = T_i \left[d_{ni} b(\boldsymbol{c})\right]^{-\theta} c^{\theta}.$$

[3]We bound $\beta_k > 0$ (and $\alpha_k > 0$ below) to simplify keeping track of the number of sales a firm makes in the accounting below. The restriction could easily be dropped at the expense of complicating this accounting.

In fact, they will face different costs of carrying out tasks, drawn from (4). To solve for the measure of producers with cost below c, we need to integrate over the distribution of cost per task. In particular, the measure of potential producers from i that can deliver to market n at a unit cost below c is therefore:

$$\begin{aligned}\mu_{ni}(c) &= \int_0^\infty ... \int_0^\infty m_i(d_{ni}b(\boldsymbol{c})/c)dG_{1,i}(c_1)...dG_{K,i}(c_K) \\ &= T_i B^{-\theta} d_{ni}^{-\theta} c^\theta \prod_{k=1}^{K} \left(\int_0^\infty (c_k)^{-\theta\beta_k} \, dG_{k,i}(c_k) \right), \end{aligned} \tag{7}$$

where we have used the fact that the costs are independent across tasks.

The expected contribution of task k to $\mu_{ni}(c)$, the integral inside (7), is:

$$\begin{aligned} & \int_0^\infty (c_k)^{-\theta\beta_k} \, dG_{k,i}(c_k) \\ = & \int_0^\infty (c_k)^{-\theta\beta_k} \, \Xi_{k,i} \, (c_k)^{\phi-1} \, e^{-\Xi_{k,i} c_k^\phi} dc_k \qquad (8) \\ = & \int_0^\infty \left(\frac{x}{\Xi_{k,i}} \right)^{-\frac{\theta}{\phi}\beta_k} e^{-x} dx \\ = & \, (\Xi_{k,i})^{\tilde{\beta}_k} \, \Gamma(1-\tilde{\beta}_k), \qquad (9) \end{aligned}$$

where:

$$\tilde{\beta}_k = \frac{\theta}{\phi}\beta_k$$

and

$$\Gamma(a) = \int_0^\infty y^{a-1} e^{-y} dy$$

is the complete gamma function. Since $\Gamma(0)$ is infinite, in order to ensure that $\mu_{ni}(c)$ remains finite we need to impose the restrictions that $K > 1$ and that $\tilde{\beta}_k < 1$ for all k.

Using (9), we can solve (7) as:

$$\mu_{ni}(c) = T_i \Xi_i d_{ni}^{-\theta} c^\theta,$$

where

$$\Xi_i = \prod_{k=1}^{K} \left(\Xi_{k,i}\right)^{\tilde{\beta}_k}.$$

By setting

$$B = \left(\prod_{k=1}^{K} \Gamma(1-\tilde{\beta}_k)\right)^{1/\theta},$$

these terms drop out of all the expressions that follow.

Aggregating across all sources of supply, the measure of potential producers that can deliver a good to market n at a cost below c is:

$$\mu_n(c) = \sum_{i=1}^{\mathcal{N}} \mu_{ni}(c) = \Upsilon_n c^\theta$$

where:

$$\Upsilon_n = \sum_i T_i \Xi_i d_{ni}^{-\theta}, \tag{10}$$

showing how the parameter Υ_n posited in (1) relates to deeper parameters of technology, search, and trade costs, as well as to wages, to which we turn below.

Substituting in (6), we can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(w_{k,i}^{\phi} \nu_{k,i} + w_{k,i}^{-\phi}\right)^{\tilde{\beta}_k}. \tag{11}$$

Given wages and exogenous parameters of the model, the Υ_n are thus the solution to the set of equations (11). As discussed below, for a solution to exist requires restrictions on parameters.

The measure of potential producers from i with unit cost below c in n is $T_i \Xi_i d_{ni}^{-\theta} c^\theta$. The total measure of potential producers with unit cost below c in n is $\Upsilon_n c^\theta$. Hence the probability that a potential producer selling in n with unit cost below c is from i is:

$$\pi_{ni} = \frac{T_i \Xi_i d_{ni}^{-\theta}}{\sum_{i'} T_{i'} \Xi_{i'} d_{ni'}^{-\theta}}$$

which is independent of c.

2.1.1 Homogeneity

Holding fixed all wages by skill type, imagine scaling the technology endowment T_i of each country by the factor s while scaling the search parameters $\lambda_{k,i}$ by the factor $1/s$. Inspection of the system (11) together with the definitions (6) and (5) reveals that all the Υ_i will scale by the factor s while the $\nu_{k,i}$ and $\Xi_{k,i}$ will be unchanged. The share of labor in gross production will therefore be unchanged as will all bilateral trade shares.

When we consider general equilibrium below, it turns out that if all labor endowments L_i^l are also scaled by the factor s, equilibrium wages will remain unchanged. The number of firms selling as well as the number of firms producing in each market will also scale by the factor s, as shown below.

The notable feature of this homogeneity result is that it requires scaling the $\lambda_{k,i}$ by the factor $1/s$. Thus, if the labor forces and levels of technology are doubled, the search parameters must be cut in two if the degree of outsourcing is to remain constant. Otherwise the bigger market would generate lower priced intermediates (relative to wages) and hence a greater share of intermediates.

2.2 Consumer Demand

Denote final consumption spending, on goods or labor services, in country n by X_n^C. Consumers devote a share $\alpha_k > 0$ of this spending to fulfilling need k, where

$$\sum_{k=1}^{K} \alpha_k = 1.$$

The α_k's for consumers are the analog of the β_k's for firms.

In line with producers, a consumer can fulfill a need either with a good or by hiring labor. The lowest cost at which a household can fulfill a need is drawn from the distribution (4).

2.2.1 Final Customers per Firm

The number of *potential* final customers for need k for a potential producer supplying country n at cost c is distributed Poisson with parameter

$$e_{k,n}(c)L_n = \lambda_{k,n}c^{-\varphi}L_n.$$

Having met a final buyer, a seller with unit cost c will make the sale if and only if the buyer hasn't found a lower-cost means of fulfilling the need, by either a cheaper intermediate or by labor. The probability that a seller with cost c will make the sale is $e^{-\Xi_{k,n}c^{\phi}}$. Thus, the number of final consumers in n buying from a potential producer with unit cost c for need k is distributed Poisson with parameter $\eta^{C}_{k,n}(c)$, given by:

$$\eta^{C}_{k,n}(c) = \lambda_{k,n}L_n c^{-\varphi}e^{-\Xi_{k,n}c^{\phi}},$$

where, recall,

$$\Xi_{k,n} = \frac{\theta}{\phi}\lambda_{k,n}\Upsilon_n + w^{-\phi}_{k,n}.$$

This expression is decreasing in the producer's unit cost c since a higher cost producer typically has fewer potential customers and because each potential customer is less likely to buy from it.

Since purchases are independent across k, the number of total purchases by consumers in n from a producer with unit cost c is distributed Poisson with parameter:

$$\eta^{C}_{n}(c) = \sum_{k=1}^{K}\eta^{C}_{k,n}(c).$$

By the properties of the Poisson distribution, $\eta_n^C(c)$ is also the expected number of customers for a potential producer selling a product at unit cost c in market n.

2.2.2 Consumer Spending

A potential producer with unit cost c in market n expects revenue there of:

$$\begin{aligned} x_n^C(c) &= \sum_{k=1}^{K} \eta_{k,n}^C(c)\alpha_k \frac{X_n^C}{L_n} \\ &= \left(\sum_{k=1}^{K} \alpha_k \lambda_{k,n} c^{-\varphi} e^{-\Xi_{k,n} c^{\phi}}\right) X_n^C. \end{aligned} \tag{12}$$

Integrating (12) over the cost distribution of potential producers supplying country n, we get:

$$\begin{aligned} \int_0^{\infty} x_n^C(c) d\mu_n(c) &= X_n^C \int_0^{\infty} \left(\sum_{k=1}^{K} \alpha_k \lambda_{k,n} c^{-\varphi} e^{-\Xi_{k,n} c^{\phi}}\right) \Upsilon_n \theta c^{\theta-1} dc \\ &= X_n^C \sum_{k=1}^{K} \alpha_k \left(\int_0^{\infty} \lambda_{k,n} c^{-\varphi} e^{-\Xi_{k,n} c^{\phi}} \Upsilon_n \theta c^{\theta-1} dc\right) \\ &= X_n^C \sum_{k=1}^{K} \alpha_k \left(\int_0^{\infty} \nu_{k,n} e^{-\Xi_{k,n} c^{\phi}} \phi c^{\phi-1} dc\right) \\ &= X_n^C \sum_{k=1}^{K} \alpha_k \left(\frac{\nu_{k,n}}{\Xi_{k,n}}\right) \\ &= \left(1 - \alpha_n^L\right) X_n^C, \end{aligned}$$

where

$$\alpha_n^l = \sum_{k \in \Omega_l} \alpha_k \left(\frac{w_{k,n}^{-\phi}}{\Xi_{k,n}}\right)$$

is the share of household spending paid directly to labor of type l to perform tasks and:

$$\alpha_n^L = \sum_{l} \alpha_n^l$$

is the share of all types of labor in final spending. The share of goods in final spending is thus:

$$\Phi_n^C = (1 - \alpha_n^L)$$

2.2.3 Consumer Welfare

We can write the indirect utility of a consumer j in n spending $y_n(j) = y$ and facing costs of performing each need k given by $\boldsymbol{c} = (c_1, c_2, ..., c_K)'$ as:

$$V(j) = V(y, \boldsymbol{c}) = \frac{y}{\prod_{k=1}^{K} [c_k]^{\alpha_k}}.$$

The expected expenditure $E(V)$ needed to obtain expected utility V in market n is thus:

$$E(V) = V \prod_{k=1}^{K} \left(\int_0^{\infty} (c_k)^{\alpha_k} \, dG_{k,n}(c_k) \right).$$

In parallel to the derivation of the cost distribution, the term in parentheses above can be expressed as:

$$\begin{aligned} & \int_0^{\infty} (c_k)^{\alpha_k} \, dG_{k,n}(c_k) \\ = & \int_0^{\infty} c_k^{\alpha_k} \Xi_{k,i} \phi c_k^{\phi-1} e^{-\Xi_{k,i} c_k^{\phi}} dc_k \\ = & \int_0^{\infty} \left(\frac{x}{\Xi_{k,i}} \right)^{\tilde{\alpha}_k} e^{-x} dx \\ = & (\Xi_{k,i})^{-\tilde{\alpha}_k} \, \Gamma(1 + \tilde{\alpha}_k), \end{aligned}$$

where:

$$\tilde{\alpha}_k = \frac{1}{\phi} \alpha_k.$$

The expected expenditure function is thus:

$$E(V) = V \prod_{k=1}^{K} (\Xi_{k,i})^{-\tilde{\alpha}_k} \, \Gamma(1 + \tilde{\alpha}_k).$$

We can write the result more compactly as:

$$E(V) = \frac{AV}{\Xi_n^{\alpha}},$$

where

$$A = \prod_{k=1}^{K} \Gamma\left(1 + \tilde{\alpha}_k\right)$$

and

$$\Xi_n^{\alpha} = \prod_{k=1}^{K} \left(\Xi_{k,i}\right)^{\tilde{\alpha}_k}$$

Note how the expenditure required to achieve a given level of utility decreases with each of the $\nu_{k,i}$.

In calculating welfare, we apply the price index

$$P_n = \frac{A}{\Xi_n^{\alpha}}$$

to income in country n.

2.3 Intermediate Demand

Let M_i denote the measure of active producers, which corresponds to firms that would appear in data in country i. These firms are the potential customers for intermediates in country i, since a potential producer that makes no sales would not buy intermediates. The number of *potential* customers for an intermediate to perform task k for a firm supplying country i at cost c is thus distributed Poisson with parameter

$$e_{k,i}(c)M_i = \lambda_{k,i} c^{-\varphi} M_i.$$

If the intermediate for task k is supplied at cost c, it will be purchased with probability $e^{-\Xi_{k,i}c^{\phi}}$. Hence, the number of intermediate buyers in i from a firm with unit cost c for task k is distributed Poisson with parameter:

$$\eta_{k,i}^{I}(c) = \lambda_{k,i} M_i c^{-\varphi} e^{-\Xi_{k,i}c^{\phi}}.$$

The total number intermediate purchases in i from a producer with unit cost c is distributed Poisson with parameter:

$$\eta_i^I(c) = \sum_{k=1}^{K} \eta_{k,i}^I(c).$$

By the properties of the Poisson distribution, $\eta_i^I(c)$ is also the expected number of customers for a potential producer selling an intermediate at unit cost c in market i.

Defining y_i^M as average value of production per producer in i, a firm with cost c therefore expects revenue from intermediate sales in i of:

$$\begin{aligned} x_i^I(c) &= \sum_k \beta_k \eta_{k,i}^I(c) y_i^M \\ &= \left(\sum_k \beta_k \lambda_{k,i} c^{-\varphi} e^{-\Xi_{k,i} c^{\phi}} \right) Y_i^M, \end{aligned} \tag{13}$$

since $Y_i^M = M_i y_i^M$.

Aggregating $x_i^I(c)$ over the cost distribution for firms selling in country i, total intermediate sales there are:

$$\begin{aligned} X_i^I &= \int_0^{\infty} x_i^I(c) d\mu_i(c) = Y_i^M \int_0^{\infty} \left(\sum_k \beta_k \lambda_{k,i} c^{-\varphi} e^{-\Xi_{k,i} c^{\phi}} \right) d\mu_i(c) \\ &= Y_i^M \sum_k \beta_k \left(\int_0^{\infty} \lambda_{k,i} c^{-\varphi} e^{-\Xi_{k,i} c^{\phi}} \theta \Upsilon_i c^{\theta-1} dc \right) \\ &= Y_i^M \sum_k \beta_k \left(\int_0^{\infty} \nu_{k,i} e^{-\Xi_{k,i} c^{\phi}} \phi c^{\phi-1} dc \right) \\ &= Y_i^M \sum_k \beta_k \frac{\nu_{k,i}}{\Xi_{k,i}} \\ &= (1 - \beta_i^L) Y_i^M, \end{aligned}$$

where

$$\beta_i^l = \sum_{k \in \Omega_l} \beta_k \left(1 - \frac{\nu_{k,i}}{\Xi_{k,i}} \right) = \sum_{k \in \Omega_l} \beta_k \frac{w_{k,i}^{-\phi}}{\nu_{k,i} + w_{k,i}^{-\phi}},$$

is the share labor of type l in manufacturing production costs and:

$$\beta_i^L = \sum_l \beta_i^l$$

is the total share of labor in manufacturing costs. The share of intermediates in total costs is thus:

$$\Phi_i^I = 1 - \beta_i^L.$$

Note how the labor share depends not only on wages but on the terms $\nu_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}\Upsilon_i$. Greater access to intermediates (higher $\lambda_{k,i}$'s) or a reduction in trade barriers (lowering Υ_i) act to reduce the labor share. Figure 2 shows evidence that labor shares have fallen in many countries during a period of trade expansion.

3 Aggregate Relationships

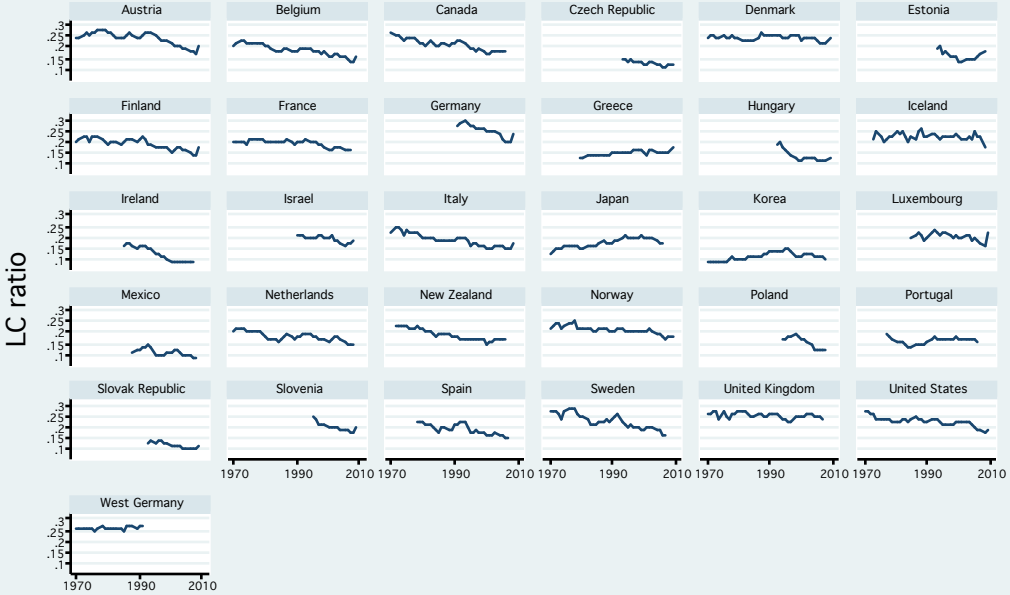
We now turn to the solution of the model, first solving for equilibrium in manufacturing production, given wages, and then to labor-market equilibrium, which determines those wages.

3.1 Manufacturing Equilibrium

Total manufacturing production in country i equals total revenue in supplying consumption goods and intermediates around the world:

$$\begin{aligned} Y_i^M &= \sum_{n=1}^{\mathcal{N}} \int_0^\infty \left[x_n^C(c') + x_n^I(c')\right] d\mu_{ni}(c') \\ &= \sum_{n=1}^{\mathcal{N}} \int_0^\infty \left[x_n^C(c') + x_n^I(c')\right] \pi_{ni} d\mu_n(c') \\ &= \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[\Phi_n^C X_n^C + \Phi_n^I Y_n^M\right]. \end{aligned}$$

Labor Compensation/Gross Production



Graphs by Country

We can write this result in matrix form as:

$$\boldsymbol{Y^M} = \boldsymbol{\Pi}\left(\boldsymbol{\Phi}^C\boldsymbol{X}^C + \boldsymbol{\Phi}^I\boldsymbol{Y^M}\right)$$

where:

$$\boldsymbol{Y^M} = \begin{bmatrix} Y_1^M \\ Y_2^M \\ . \\ . \\ . \\ Y_{\mathcal{N}}^M \end{bmatrix}, \quad \boldsymbol{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ . \\ . \\ . \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$\boldsymbol{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & ... & 0 & 0 \\ 0 & \Phi_2^j & ... & 0 & 0 \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ 0 & 0 & ... & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & ... & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I$$

and:

$$\boldsymbol{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & ... & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & ... & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & ... & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & ... & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}$$

We can then solve for $\boldsymbol{Y^M}$:

$$\boldsymbol{Y^M} = (\boldsymbol{I}_{\mathcal{N}} - \boldsymbol{\Pi\Phi}^I)^{-1}\boldsymbol{\Pi\Phi}^C\boldsymbol{X}^C$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

3.2 Labor-Market Equilibrium

With balanced trade, final spending on manufactures in country i, X_i^C, is equal to wage income Y_i^L, which corresponds in this model to GDP:

$$X_i^C = Y_i^L = \sum_l w_i^l L_i^l.$$

Equilibrium in the market for labor of type l in country i solves the expression:

$$w_i^l L_i^l = \alpha_i^l Y_i^L + \beta_i^l Y_i^M.$$

These sets of equations, for each l and i, determine w_i^l.

4 Firm-Level Implications of the Model

While delivering implications for aggregate outcomes such as trade shares, wages, and prices, our framework also delivers implications for firm entry and the firm-size distribution, which we now explore.

The number of buyers for a firm selling in n at cost c is distributed Poisson with parameter:

$$\eta_n(c) = \eta_n^C(c) + \eta_n^I(c) = (L_n + M_n)\, c^{-\varphi} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n} c^{\phi}}.$$

These customers may be buyers of final goods or intermediates to perform any need.

Now consider worldwide sales of a producer in country i. Given c, it gets a Poisson draw of customers in each market n. Its production cost c is fixed, but due to trade costs, its cost of delivery to country n is cd_{ni}. The total number of customers around the world for a producer from market i with unit cost c there is therefore distributed Poisson with parameter:

$$\begin{aligned} \eta_i^W(c) &= \sum_{n=1}^{\mathcal{N}} \eta_n(cd_{ni}) \\ &= \sum_{n=1}^{\mathcal{N}} (L_n + M_n)\, (d_{ni})^{-\varphi}\, c^{-\varphi} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n} (d_{ni})^{-\phi} c^{\phi}}. \end{aligned}$$

4.1 Measure of Producers and Sellers

In an open economy the set of firms selling in a country is not the same as the set producing there, so M_n does not correspond to N_n. Here we solve for the measure of producers.

To appear as a firm a potential producer has to sell somewhere. The probability that a potential producer from source i with unit cost c fails to make a sale anywhere is $\exp(-\eta_i^W(c))$. Integrating over the cost distribution of potential producers in source i (those from i that can deliver to i at cost c):

$$\begin{aligned} M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\ &= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc. \end{aligned}$$

Since $\eta_i^W(c)$ itself depends on the measure of customers for intermediates M_n in each market n, we need to iterate toward a solution for all the M_i's.

Having solved for the M_i's, the measure of firms selling in n can be calculated as

$$\begin{aligned} N_n &= \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_n(c) \\ &= \Upsilon_n \int_0^\infty (1 - e^{-\eta_n(c)}) \theta c^{\theta-1} dc. \end{aligned}$$

We can evaluate this integral numerically to determine the relationship between entry N_n and market size, $L_n + M_n$.

The measure of firms from i exporting to n is

$$N_{ni} = \pi_{ni} N_n = \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_{ni}(c).$$

Thus the fraction of firms from i that export to n is N_{ni}/M_i. The fraction of firms from i that sell domestically is N_{ii}/M_i, which will typically be below 1 (although perhaps close to 1).

4.2 The Distribution of Buyers per Firm

We now turn to the size distribution of firms selling in country n, with size measured by number of customers.[4] This distribution is the same for firms from any source i.

Let S_n be the integer-valued random variable for the number of customers in n that a firm sells to. From the Poisson distribution, the probability that a firm with cost c has s customers is

$$\Pr[S_n = s|c] = \frac{e^{-\eta_n(c)}\left[\eta_n(c)\right]^s}{s!},$$

for $s = 0, 1, ...$ We can integrate over the cost distribution and condition on $S_n > 0$ (since if $S_n = 0$ the firm would not be among those observed to sell in n) to get

$$\begin{aligned}\Pr[S_n &= s|S_n > 0] = \frac{1}{N_n}\int_0^\infty \frac{e^{-\eta_n(c)}\left[\eta_n(c)\right]^s}{s!} d\mu_n(c) \\ &= \frac{\Upsilon_n}{N_n s!}\int_0^\infty e^{-\eta_n(c)}\left[\eta_n(c)\right]^s \theta c^{\theta-1} dc, \end{aligned} \tag{14}$$

for $s = 1, 2, ...$

The expected number of buyers per active firm is

$$\begin{aligned} E\left[S_n|S_n > 0\right] &= \frac{1}{N_n}\int_0^\infty \eta_n(c) d\mu_n(c) \\ &= \frac{L_n + M_n}{N_n}\int_0^\infty c^{-\varphi}\left(\sum_{k=1}^K \lambda_{k,n} e^{-\Xi_{k,n}c^\phi}\right)\theta\Upsilon_n c^{\theta-1} dc \\ &= \frac{L_n + M_n}{N_n}\sum_{k=1}^K \int_0^\infty \Upsilon_n \lambda_{k,n} e^{-\Xi_{k,n}c^\phi}\theta\Upsilon_n c^{\phi-1} dc \\ &= \frac{L_n + M_n}{N_n}\sum_{k=1}^K \frac{\nu_{k,n}}{\Xi_{k,n}}\int_0^\infty e^{-x} dx \\ &= \frac{L_n + M_n}{N_n}\sum_{k=1}^K \frac{\nu_{k,n}}{\Xi_{k,n}}. \end{aligned}$$

[4] If a customer is another firm, then the size of the customer firm will also factor in to the size of the selling firm. Since such variation is totally random, not depending on cost of the selling firm, we ignore it for now.

Since $\nu_{k,n}/\Xi_{k,n}$ is the probability that a potential customer purchases a good for a purpose (rather than hiring labor), the summation on the right hand side is then expected purchases per potential customer. Thus, expected sales per firm is the product of the measure of potential customers, $L_n + M_n$, in market n and the expected number of goods purchased per potential customer, all divided by the measure of sellers in that market.

5 A Numerical Example

To illustrate some implications of the model we have solved for the equilibrium in a simple symmetric two-country example with three types of labor. Table 1 reports the parameter values we have used. Specifically, tasks performed by "nonproduction" workers cannot be outsourced ($\lambda = 0$), while those performed by "skilled" workers are difficult to outsource ($\lambda = 0.01$) and those performed by "unskilled workers are easy to outsource ($\lambda = 1$).

We solve for the equilibrium of the model under different assumptions about trade barriers d, ranging from $d = 16$, which essentially delivers autarky, to $d = 1$ (frictionless trade). Table 2 reports the results for various magnitudes of interest. Note how the skill premium rises with lower trade barriers, as producers and consumers find cheaper products to replace unskilled workers. Even though prices fall with trade barriers, the real wage of unskilled workers declines (in both countries) as trade barriers decline. Skilled workers, who are less likely to be replaced by manufactured intermediates than are unskilled workers, experience a modest increase in their real wage. Overall welfare, of course, rises.

Table 2 also shows that manufacturing value added falls relative to manufacturing gross

production as trade barriers decline.[5] The reason is that with lower trade costs, imported manufactured goods are more likely to replace skilled and unskilled production workers in some tasks, making manufacturing production more round-about. This mechanism is one interpretation of the secular decline in labor shares (in the vast majority of countries) shown in Figure 2.

[5] The first two rows of Table 2 consider nonproduction tasks (which have a 40 percent share in manufacturing output, see Table 1) to be outsourced to the service sector, with service-sector output being the value add of nonproduction workers. In this interpretation, the value added share of manufacturing gross production can be no more than 60 percent. Had we instead interpreted these task as being carried out by nonproduction labor hired directly by manufacturers, the labor share of manufacturing gross production could be as high as 100 percent (for a firm with no production tasks outsourced). These different interpretations do not matter for other implications of the model as they relate only to the accounting question of whether manufacturers are integrated or not with the suppliers of nonproduction tasks.

Table 1: Parameter Settings for Counterfactual

Parameter	symbol	value
Pareto parameter	theta	2
Technology level	T	1
Labor force (per country)	L	0.5
Labor by type (fractions of labor force):		
Nonproduction		0.6
Skilled		0.2
Unskilled		0.2
Task shares in production:	beta	
Nonproduction		0.4
Skilled		0.3
Unskilled		0.3
Task shares in consumption:	alpha	
Nonproduction		0.4
Skilled		0.3
Unskilled		0.3
Outsourcing parameters:	lambda	
Nonproduction		0
Skilled		0.01
Unskilled		1

Table 2: Counterfactual Results

	Level of iceberg trade cost					
	d=16	d=4	d=2	d=1.5	d=1.1	d=1
Mfg. value added share :						
Share of GDP	0.09	0.09	0.10	0.10	0.11	0.11
Share of mfg. gross production	0.42	0.42	0.40	0.39	0.38	0.37
Fraction of tasks outsourced:						
Skilled	0.05	0.05	0.07	0.08	0.10	0.11
Unskilled	0.55	0.56	0.58	0.61	0.64	0.66
Labor share of mfg. variable cost:						
Skilled	0.28	0.28	0.28	0.28	0.27	0.27
Unskilled	0.14	0.13	0.12	0.12	0.11	0.10
Import share	0.00	0.06	0.20	0.31	0.45	0.50
Wage:						
Nonproduction	0.81	0.82	0.83	0.84	0.86	0.87
Skilled	1.74	1.74	1.74	1.74	1.73	1.73
Unskilled	0.82	0.81	0.77	0.74	0.69	0.67
Skill premium (skilled/unskilled)	2.11	2.14	2.25	2.35	2.52	2.58
Real wage:						
Nonproduction	1.16	1.17	1.19	1.22	1.27	1.30
Skilled	2.47	2.48	2.51	2.54	2.57	2.59
Unskilled	1.17	1.16	1.12	1.08	1.02	1.00
Welfare (real per capita consumption)	1.42	1.43	1.44	1.46	1.48	1.49

1. Total payments to labor around the world are normalized to 1.

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