The Pricing of the Illiquidity Factor's Systematic Risk

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1. Introduction

This paper presents a stock return factor of illiquidity, denoted *IML – Illiquid-Minus-Liquid* – intended to measure the return premium due to illiquidity. *IML* is the differential return between the quintile portfolios of the most and least illiquid stocks. Following Amihud and Mendelson's (1986) theory and evidence, the risk-adjusted mean of *IML* should be positive, and indeed this is shown to be the case. The paper focuses on the pricing of the *IML* systematic risk, β_{IML}. It tests whether stocks with higher β_{IML} have higher return after controlling for the β coefficients of the four return factors of Fama and French (1993) and Carhart (1997). The model allows for β_{IML} to be conditional on funding illiquidity, consistent with Brunnermeier and Pedersen (2009). Funding illiquidity is measured by the yield spread between BAA- and AAA-rated corporate bonds.

The results are that the conditional β_{IML} is priced. Its premium is positive and highly significant for the entire sample period – 1950-2012 – and for each of its two equal subperiods, 1950-6/1981 and 7/1981-2012. The βs of the common four risk factors do not show such a pattern. In addition, β_{IML} that is conditional on market volatility also has positive and significant premium. It follows that the illiquidity systematic risk is more strongly priced in times of adverse financial conditions.

The β_{IML} is different from the illiquidity β studied by Pastor and Stambaugh (2003), Acharya and Pedersen (2005) and Lee (2011).[1] In these studies, the systematic factor is the shocks in market-wide stock illiquidity. These shocks are significantly but weakly associated with *IML*. Importantly, the pricing of the conditional β_{IML} remains significant after controlling for the illiquidity-shocks β. To illustrate the difference between the two βs, β_{IML} is like the standard

[1] Watanabe and Watanabe (2006) and Acharya, Amihud and Bharath (2013) show that shocks in market illiquidity (as opposed to the *IML*, the illiquidity return factor) are priced in times of high market turnover or financial distress.

β of the market's excess return *RM-Rf*, whereas the illiquidity β is like the β of market volatility innovations, whose pricing is studied in Ang, Hodrick, Xing, and Zhang (2006).

IML is related to Liu's (2006) liquidity-based return factor, denoted here by *LIU*. While the two factors are correlated, they are shown to each have unique information. In the context of the Fama-French (1993) and Carhart (1997) factor model, *LIU*'s systematic risk is priced together with that of *IML*.

The paper proceeds as follows. Section 2 introduces the *IML* return factor and shows its properties, including its risk-adjusted mean, behavior over time and relation to illiquidity shocks. Section 3 presents the pricing of *IML's* systematic risk in the context of the Fama-French (1993) and Carhart (1997) factor model. Section 4 compares *IML* to *LIU*. Concluding remarks are offered in Section 5.

2. *IML* – return on Illiquid-Minus-Liquid stocks

2.1. The construction of *IML*

IML – Illiquid-Minus-Liquid – is the differential return between stocks that are most illiquid and most liquid. It is constructed as follows. The sample includes all NYSE\AMEX stocks with code 10 and 11 (common stocks) for the years 1950-2012. Stocks are sorted into portfolios using data over a portfolio formation period of three months that end in month *t*. In that period I calculate for each stock *j* the illiquidity measure $ILLIQ_{j,t}$ (Amihud (2002)), the average of the daily ratio of absolute return to dollar volume. A day is deleted by either of the following filters: (i) Price < 0, indicating a mid-point between the quoted bid and ask prices rather than a transaction price; (ii) Volume < 100; (iii) Return is -99.99, -99.00 or smaller than -1.0. A stock is included if during the entire portfolio formation period its price exceeds $5 and it

has data for more than 50 days. The stocks are then ranked by their $ILLIQ_{j,t}$ and those with the highest 1% values are deleted. In addition, for each stock I calculate $SD_{j,t}$, the standard deviation of daily returns over the portfolio-formation period.

Illiquidity and volatility ($ILLIQ_j$ and SD_j) are positively correlated across stocks,[2] each having its own effect on expected return. To avoid confounding between the two, the following procedure is employed. First, stocks that satisfy the filters and exist in month $t+1$ are sorted into three equal portfolios by their $SD_{j,t}$. Then, within each *SD* tercile, stocks are ranked by their $ILLIQ_{j,t}$ and divided into quintiles. This produces 15 (3x5) portfolios.[3] This procedure also controls for size, which is negatively correlated with volatility. (In the analyses below, the size effect is controlled for directly by the size return factor.)

Portfolio returns are calculated for month t+3 after skipping two months following the end of the portfolio formation period (month t). This mitigates the effects of reversal or momentum in stock returns following the realization of illiquidity. (For example, Ang et al. (2006) skip one month after sorting stocks by volatility.) The monthly portfolio return is the value-weighted average monthly stock return, using for weights the stock values at the end of the preceding month. For delisted stocks, the last return is adjusted following Shumway (1997).[4] Finally, the high- (low)-illiquidity return is the average return on the three highest- (lowest)-*ILLIQ* quintile portfolios across the three *SD* groups. Thus, IML_t is the *Illiquid-Minus-Liquid* stock return for month t.

[2] Stoll (1978) is the first to provide a theory tying liquidity costs to volatility. See evidence on the cross-stock correlation in Amihud (2002).

[3] Fama and French (1993) follow a similar procedure when constructing their *HML* factor. They first sort stocks by size and then by book-to-market ratio, producing 10 (2x5) portfolios.

[4] If a stock is delisted, the return used is either the last return available on CRSP, or the delisting return, if available. If the deletion code is not in 500s (which includes 500 (reason unavailable), 520 (went to OTC), 551–573 and 580 (various reasons), 574 (bankruptcy), 580 (various reasons) and 584 (does not meet exchange financial guidelines) and neither the last return is available nor delisting return is available, then the last return is set to -1.0. If delisting code is in the 500s, the delisting return is assigned to be -30%.

INSERT TABLE 1

Table 1, Panel A, presents statistics for *IML*. For the entire 1950-2012 period, the mean *IML* is 0.383% (t = 3.55), about 4.7% annually. The mean is close to the median and the fraction of positive-month *IML* values is 0.566, which is significantly different from 0.50, the chance result ($p < 0.01$). When the data are split into two equal subperiods, the mean *IML* is lower in the second subperiod, consistent with evidence in Amihud (2002) and Ben-Rephael, Kadan and Wohl (2012). However, the median *IML* and the proportion of positive *IML* returns is practically the same in both subperiods, suggesting that the lower mean *IML* in the second subperiod is due to some unusually high negative-return months. (This is discussed further below.)

The results on the positive and significance *IML* premium are consistent with those in Amihud, Hameed, Kang and Zhang (2013) for 45 countries for the years 1990-2011. The *IML* factor is constructed for each country following the procedure described here. They find that the average *IML* premium is 0.472% (t = 5.12) across the 45 sample countries (for value-weighted portfolio returns), being positive for 80% of the countries.

2.2. The risk-adjusted *IML* return

The risk-adjusted return of *IML*, *alpha*, is estimated by controlling for the returns of the factor model of Fama and French (1993) and Carhart (1997) (FFC):

$$IML_t = alpha_t + \beta_{RMrf} * RMrf_t + \beta_{SMB} * SMB_t + \beta_{HML} * HML_t + \beta_{UMD} * UMD_t \quad (1)$$

RMrf is the market excess return (over the T-bill rate), and *SMB*, *HML* and *UMD* are, respectively, the return on small-minus-big firms (size factor), on firms with high-minus-low book-to-market ratio (value factor) and on winner-minus-loser stocks (momentum factor). Table 1 Panel B presents the estimation results of model (1) for the entire sample period, 1950-2012,

and for the two equal-length subperiods 1950-6/1981 and 7/1981-2012. For the entire period, *alpha* = 0.334%, about 4% annually, with $t = 4.80$, highly significant.[5] Notably, *alpha* is close in magnitude to the mean *IML* in Panel A. The lower *alpha* in subperiod II is consistent with the observation for the mean *IML* in Panel A.

The relatively high and significant coefficient of *SMB* means that there is strong connection between *IML* and *SMB* since smaller-size stocks are also more illiquid. The negative coefficient of *RMrf* means that *IML* moves contrary to the market, and the positive coefficient of *HML* suggests a connection between the illiquidity premium and the premium on high book-to-market firms, which Fama and French (1993) suggest to be distressed. The estimated *alpha* is the excess return over the small-firm and the distressed-firm effects. At the bottom of Panel B, indicated by (i), there is *alpha* estimated from model (1) that excludes *SMB*. Then, *alpha* is naturally higher then before, 0.463 ($t = 4.19$), because it also includes the size premium. The small difference between the estimates of *alpha* with and without *SMB* in the model shows that the *IML* premium goes well beyond the size effect.

Two additional tests are performed. The first test is whether the *IML* premium is a January phenomenon, as it is known to be for the size effect (Keim (1983)). The second test is whether the *IML* premium can be explained as a micro-stocks effect which is not fully captured by *SMB*. For these tests, two variables are added to model (1): (i) *JAN*, the January dummy variable, *JAN* = 1 in January and 0 otherwise; and (ii) *MicRf*, the excess return on stocks in the smallest size quintile, calculated as the average return on the two smallest-size CRSP decile portfolios (NYSE\AMEX stocks) in excess of the risk-free rate.

The estimated *IML alpha* in model (1), which is augmented by *JAN* and *MicRf*, and these variables' coefficients are reported under (ii) at the bottom of Panel B. Throughout, *alpha*

[5] Globally across 45 countries, Amihud et al. (2013) find that the risk-adjusted return (*alpha*) is 0.439% ($t = 4.48$).

remains positive and highly significant. The coefficient of *MicRf* is positive and significant, as expected, over the entire period, though it becomes insignificant in subperiod II. The effect of *JAN* is statistically insignificant overall, being positive and insignificant in subperiod I and turning negative and significant in subperiod II.

Next, model (1) is augmented by $dMILLIQ_t$, the monthly change in market illiquidity (in logarithm). The effect of $dMILLIQ_t$ on *IML* should be negative because prices of more illiquid stocks decline more in response to rising illiquidity, as shown by Amihud (2002), Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). $MILLIQ_t$ is the monthly value-weighted average of the illiquidity of NYSE\AMEX stocks. The stock *ILLIQ* is first averaged daily across stocks and then it is averaged over the days of the month. The daily average includes all stocks with codes 10 or 11, price that exceeds $5 and trading volume of 100 shares or more. The daily mean excludes stocks whose *ILLIQ* is at the highest 1% of the sample.

Panel C of Table 1 presents estimation results of the effects on IML_t of current and lagged $dMILLIQ_t$. All coefficients are negative though only those of current and the one-lag $dMILLIQ_t$ are statistically significant. The contribution of *dMILLIQ* to the explanatory power of model (1) is small: R^2 increases by only 0.02, from 0.67 to 0.69, when *dMILLIQ* is added. Naturally, part of the effect of illiquidity shocks is captured by the return factors.

Out-of-sample $alpha_t$ of *IML* is presented in Figure 1 and its statistics are presented in Panel D of Table 1. Model (1) is estimated over a rolling window of 36 months up to month *t-1* and the estimated slope coefficients $\beta_{K,t-1}$ (*K = RMrf, SMB, HML* and *UMD*) are used to calculate $alpha_t$ conditional on month-*t* realized factor returns:

$$alpha_t = IML_t - [\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t] \ .$$

The 36-month window is then rolled forward one month and the procedure repeats. Panel D presents statistics of the series $alpha_t$ that begins in 1/1953 (the first three years are used to estimate the initial βs). The mean $alpha_t$ and its statistical significance are similar to those presented in Panel B. The bottom of Panel D, under (i), includes estimates of mean $alpha_t$ that account for the January effect, where $alpha_t$ is regressed on the dummy variable *JAN*, with an intercept. Overall, the January effect is insignificant. In the first subperiod, it is positive and significant and in the second subperiod it is negative and insignificant. The intercept remains positive and significant throughout.

INSERT FIGURE 1

Figure 1 presents the twelve-month moving average of $alpha_t$. It is mostly in positive territory with extreme negative values in the year 2000 when the dot-com bubble burst and the relatively-illiquid tech stocks incurred heavy losses. Although Nasdaq stocks are not included in the sample, there were like-wise stocks among NYSE\AMEX stocks. The two most negative values of $alpha_t$ are -9.68% on 2/2000 and -5.82% on 9/2000, with the fourth most negative value, -4.30% on 11/2000. Altogether, $alpha_t$ is negative for 11 out of the 12 months of the year 2000. Two other short periods with relatively high number of negative values of $alpha_t$ are from late 1973 through 1974 – the first oil crisis – and during 2008-2009, the time of the recent financial crisis. In these periods, illiquid stocks had abnormally low returns.

3. The pricing of *IML* systematic risk, β_{IML}

3.1. Test of a CAPM augmented by *IML*

The focus of this paper is on the pricing of *IML* systematic risk. The test employs the Fama-Macbeth (1973) procedure. In the first step, I add the liquidity factor *IML* to the Fama-

French (1993) and Carhart (1997) (FFC) model and estimate the factors' βs for each portfolio j using monthly returns.

$$(r_j\text{-}rf)_t = \beta 0_j + \beta_{RMrf,j}*(RMrf)_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HMLt + \beta_{UMD,j}*UMD_t + \beta_{IML,j}*IML_t \quad (2)$$

In the second step I regress cross-sectionally for each month t the monthly portfolio returns on the factors' βs, following the methodology of Fama and MacBeth (1973):

$$r_{jt} = \gamma 0_t + \gamma_{RMrf,t}*\beta_{RMrf,j} + \gamma_{SMB,t}*\beta_{SMB,j} + \gamma_{HML,t}*\beta_{HML,j} + \gamma_{UMD,t}*\beta_{UMD,j} + \gamma_{IML,t}*\beta_{IML,j} \quad (3)$$

The analysis employs two sets of portfolios, sorted on size and book-to-market ratio (BE/ME): Fama and French's (1993) 25 (5x5) and 100 (10x10) portfolios, denoted respectively as FF25 and FF100. Size can be viewed as an instrument for liquidity.

The FF100 portfolios provide greater dispersion of β values because they include portfolios with more extreme size and BE/ME values. However, some portfolios (at the corners of the 10x10 matrix) have very few stocks or none at all. Some portfolios have 1, 2 or 3 stocks and quite a few have 5-10 stocks. As a result, the estimated βs of some of the FF100 portfolios are estimated less precisely than those for the FF25 portfolios. This exacerbates the errors-in-the-variables problem in the β estimates, which causes the γ coefficients to be biased downward. On the other hand, as argued by Ang, Liu and Schwarz (2010), greater cross-sectional dispersion of the factor βs enables a more efficient estimation of the factor risk premia whereas aggregating stocks into larger portfolios destroys information. In what follows, all estimations are presented for both the FF25 and the FF100 portfolios.

INSERT TABLE 2

Table 2 Panel A1 presents the estimated factors' βs of model (2). The average of B_{RMrf} is close to 1.0, as expected, and that of β_{IML} is close to zero. For all factors, the cross-portfolio

dispersion of the β estimates is greater for the FF100 portfolios which, as noted above, provide greater dispersion of stock characteristics than do the FF25 portfolios. Panel A2 presents the cross-portfolio correlations between the factors' βs. The correlations are higher for FF25 because of the greater aggregation of stocks. For β_{IML}, it is most correlated with β_{IML} and β_{IML} with the correlation values being -0.48 and 0.55, respectively.

Panel B of Table 2 presents statistics of the estimated Fama-Macbeth cross-sectional coefficients γk_t, where $k = RMrf, SMB, HML, UMD$ and IML. The table presents the means and their t-statistics as well as the weighted means, where the weights are the reciprocal of the estimated standard errors from the cross-section regression. This gives higher weight to coefficients that are more-precisely estimated. Ferson and Harvey (1999, Appendix A) propose this adjustment to correct for potential heteroskedasticity in the Fama-MacBeth estimations. The table also presents the median and the fraction of positive coefficients with the associated p value of a binomial test under the null hypothesis that the fraction is 0.5, the chance result.

Consider first the results for the FF25 portfolios. The mean $\gamma_{IML,t}$ is 0.451 with $t = 1.89$, weakly significant (at 0.10 level, two-tail test). The mean of $\gamma_{IML,t}$ is close in magnitude to the mean IML_t, 0.383 (see Table 1), and the correlation between $\gamma_{IML,t}$ and IML_t is 0.45. The weighted mean of $\gamma_{IML,t}$ is significantly positive ($t = 2.49$) and the fraction of positive values of $\gamma_{IML,t}$ is significantly greater than 0.5, the chance result.

In this regression, the mean market risk premium $\gamma_{RMrf,t}$ is 1.014, statistically significant, being insignificantly higher than the mean and median of $RMrf$ for the sample period (0.590 and 0.945, respectively). The mean $\gamma_{SMB,t}$ is statistically insignificant, consistent with Amihud and Mendelson's (1986) suggestion that the illiquidity effect largely subsumes the size effect. The means of $\gamma_{HML,t}$ and $\gamma_{SMB,t}$ are positive and significant as is found in other studies.

For the FF100 portfolios, R^2 is much lower implying a worse fit of the model. This may be because of the less precise estimation of the factor βs, given the small number of stocks in many portfolios. The estimated $\gamma_{IML,t}$ is about half the size of that for FF25 and it is statistically insignificant. Here, the mean $\gamma_{RMrf,t}$ too becomes insignificantly different from zero.

3.2. Conditional *IML* systematic risk and its pricing

This section employs the *conditional* β coefficients following Ferson and Schadt (1996). By this approach, the coefficient β of *IML* is allowed to change over time and is modified to be $\beta_{IML,j} + \beta_{ZIML,j}*Z_{t-1}$. The conditioning variable is z_t and Z_t is the deviations of z_t from its unconditional mean, thus $E(Z_t) = 0$. The realized value of Z_{t-1} is known before the beginning of period t and can be used by investors when pricing stocks for that period. Models (2) and (3) are modified as follows:

$$(r_j\text{-}rf)_t = \beta 0_j + \beta_{RMrf,j}*(RMrf)_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HMLt + \beta_{UMD,j}*UMD_t + \beta_{IML,j}*IML_t + \boldsymbol{\beta_{ZIML,j}*Z_{t-1}*IML_t + \beta_{Z,j}*Z_{t-1}} \quad (2a)$$

$$r_{jt} = \gamma 0_t + \gamma_{RMrf,t}*\beta_{RMrf,j} + \gamma_{SMB,t}*\beta_{SMB,j} + \gamma_{HML,t}*\beta_{HML,j} + \gamma_{UMD,t}*\beta_{UMD,j} + \gamma_{IML,t}*\beta_{IML,j} + \boldsymbol{\gamma_{ZIML,t}*\beta_{ZIML,j} + \gamma_{Z,t}*\beta_{Z,j}} \quad (3a)$$

The effect of liquidity on asset prices is expected to change over time. Brunnermeier and Pedersen (2009) propose that funding liquidity affects market liquidity. When the funding constraint is binding, the value of liquidity rises. Watanabe and Watanabe (2008) find that the liquidity β (the sensitivity of stock returns to illiquidity shocks) is priced in times of elevated trading volume. Acharya, Amihud and Bharath (2013) show that for both stocks and bonds, illiquidity is more highly priced in times of economic distress.

Following Brunnermeier and Pedersen's (2009) idea on the role of funding liquidity, I set $Z_t = SP_t$, the yield spread between corporate bonds rated BAA and AAA (Source: the Federal Reserve Bank of St. Louis; data are monthly; 1% = 1). Higher SP_t indicates greater funding illiquidity as well as greater financial distress. The model uses the mean-adjusted yield spread, denoted $maSP_t$ (SP_t minus its mean), thus $Z_t = maSP_t$

Estimating model (2a), the mean $\beta_{ZIML,j}$ is -0.026 for the FF25 portfolios (-0.021 for FF100 portfolios), ranging from -0.269 to 0.133 for FF25 (from -0.365 to 0.470 for FF100). The correlation between $\beta_{IML,j}$ and $\beta_{ZIML,j}$ across the FF25 (FF100) portfolios is -0.122 (-0.039), statistically insignificant. The strongest correlation of $\beta_{ZIML,j}$ is with $\beta_{HML,j}$, 0.372 for FF25 (0.334 for FF100), suggesting that stocks with greater BE/ME ratio have higher *IML* systematic risk in times of funding illiquidity. Indeed, the average $\beta_{ZIML,j}$ is negative (positive) for the five lowest (highest) BE/ME portfolios of FF25, respectively. This is consistent with Acharya, Amihud and Bharath's (2013) findings that prices of high-BE/ME stocks, which are likely to be under-performing and distressed, are more negatively impacted by illiquidity in times of adverse economic conditions.

INSERT TABLE 3

The estimation results of model (3a) are presented in Table 3 Panel A. To save space, only γ_{IML}, γ_{ZIML} and $\gamma_{Z,}$ are presented. The means of both γ_{IML} and γ_{ZIML} are positive, with γ_{ZIML} being highly significant for both FF25 and FF100. The positive and significant γ_{ZIML} means that stocks whose exposure to *IML* is greater in times of high funding illiquidity have higher expected returns. Stocks with negative β_{ZIML} are more valuable (have lower expected return) because they can serve as a hedge against the *IML* risk.

Next, I set $Z_t = dSP_t = SP_t - SP_{t-1}$, the *change* in the monthly yield spread of bonds rated BAA-AAA. Higher dSP_t implies worsening funding liquidity and greater financial distress, even if SP_t is low. The mean dSP_t is 0.004 (t = 0.10), its median is practically zero and its range is -0.63% to 0.94%. Using $Z_t = dSP_t$ in model (2a), the mean of $\beta_{ZIML,j}$ is -0.139 for FF25 (-0.138 for FF100) and its dispersion is greater than before. Across the FF25 portfolios, $\beta_{ZIML,j}$ ranges between -0.688 and 0.35, and for FF100 the range is -1.042 to 0.823.

The results for the cross-section model (3a) with $Z_t = dSP_t = SP_t - SP_{t-1}$ are presented in Table 3, Panel B. For FF25, the mean of γ_{ZIML} is positive and significant, and the mean of γ_{IML} remains positive and significant. That is, the premium of the *IML* systematic risk is greater for stocks whose exposure to *IML* rises when funding liquidity worsens ($dSP_t > 0$). The economic significance of these results is illustrated as follows. The standard deviations of $\beta_{IML,j}$ and of $\beta_{ZIML,j}$ are 0.082 and 0.261, respectively. A stock whose β_{IML} is one standard deviation above the average has an annual expected return which is 0.96% higher than that of a stock whose β_{IML} is one standard deviation below the average. And if a stock's β_{ZIML} is one standard deviation above average, its annual expected return is 0.93% higher than that of a stock whose β_{ZIML} is one standard deviation below average for $Z_{t-1} = 1$.

While *IML's* conditional β is priced, this is not the case for any of the other factors's conditional β. This is tested by adding to model (2a) the interaction term $\beta_{ZRMrf} * Z_{t-1} * RMrf_t$, $\beta_{ZSMB} * Z_{t-1} * SMB_t$, $\beta_{ZHML} * Z_{t-1} * HML_t$ or $\beta_{ZUMD} * Z_{t-1} * UMD_t$, estimating the respective β and then adding this β to the cross section regression model (3a) to test whether the mean of its coefficient is significantly different from zero. The result is that the means of $\gamma_{ZRMrf,t}$, $\gamma_{ZSMB,t}$, $\gamma_{ZHML,t}$ and $\gamma_{ZUMD,t}$ are all insignificantly different from zero, all having t-statistics below 1.0. For example,

mean γ_{ZHML} = -0.065 with t = 0.73. In all these regressions, the mean $\gamma_{ZIML,t}$ remains positive and significant.

To test robustness, the sample is split into two equal subperiods of 378 months, 1950-6/1981 and 7/1981-2012. The βs of model (2a) are estimated separately for each subperiod, and the γs of model (3a) are estimated in each subperiod using that subperiod's βs. The results are consistent across the subperiods: the mean $\gamma_{ZIML,t}$ is positive and significant in both.

I test whether the premiums of *IML*'s βs, γ_{IMLt} and γ_{ZIMLt}, are a January-related phenomena. Tinic and West (1986) find that the premium of β_{RMrf} in the classic CAPM is positive only in January. Panel C of Table 3 presents results of a regression of the γ_t coefficients on *JAN* and *Non-JAN* dummy variables. For $\gamma_{IML,t}$, the mean is positive and significant in January, while in the rest of the year it is positive for FF25 and practically zero for FF100. The mean $\gamma_{ZIML,t}$ is positive and significant during the eleven non-January months, while in January it is negative and significant for FF25 and practically zero for FF100. The premium on *beta* of dSP_{t-1}, $\gamma_{Z,t}$, which is insignificant for the entire period, exhibits a January seasonality. It is negative and significant in January and it is positive and significant in the rest of the year. While dSP_t itself is slightly lower in January, this pattern is insignificant.

Other factors too display different January and non-January premiums of their β. For *HML*, mean $\gamma_{HML,t}$ is 2.097 (t = 5.08) for *JAN* and 0.179 (t = 1.71) for *Non-JAN*. (The t statistics are calculated using robust standard errors.) For the momentum factor *UMD*, the *JAN* and *Non-JAN* means of $\gamma_{UMD,t}$ switch signs, being -5.977 (t = 2.00) and 4.359 (t = 5.63), respectively. Contrary to the pattern found by Tinic and West (1986), the means $\gamma_{RMrf,t}$ for *JAN* and *Non-JAN* are -1.76 (t = 1.16) and 0.973 (t = 2.41), respectively. That is, except for January, the classic CAPM β is positively and significantly priced.

3.3. Controlling for the market illiquidity's systematic risk

The systematic risk of market illiquidity is shown by Pastor and Stambaugh (2003) to be priced across stocks. They estimate the systematic liquidity risk, denoted here β_{ILLIQ}, as the slope coefficient from a regression of monthly stock returns on monthly shocks to their measure of market illiquidity, which reflects price reversals following trading volume shocks (that are signed by the return associated with them). Pastor and Stambaugh (2003) find that stocks with more negative β_{ILLIQ} have higher risk-adjusted expected returns. β_{ILLIQ} is more negative for stocks with greater illiquidity risk, or greater exposure to market illiquidity shocks, because a rise in illiquidity leads to a rise in expected return and thus to a fall in stock price. This causes a negative relation between illiquidity shocks and contemporaneous return (see Amihud (2002)). Positive pricing of this liquidity risk means that in a cross-section regression of stock return on β_{ILLIQ}, its coefficient – denoted here as γ_{ILLIQ} – should be negative. That is, stocks with greater exposure to illiquidity shocks – more negative β_{ILLIQ} – have higher expected return. Acharya and Pedersen (2005) present this pricing of β_{ILLIQ} in the context of an asset pricing model that includes two other components of systematic liquidity risk, which are significantly priced.

The pricing of β_{ILLIQ} is different from the pricing of β_{IML}: The former is the β of illiquidity shocks whereas the latter is the β of the illiquidity *premium*. I test the effects of the two types of the illiquidity-related systematic risks by adding $\beta_{ILLIQ,j}$*$dMILLIQ_t$ to model (2a) and $\gamma_{ILLIQ,t}$*$\beta_{ILLIQ,j}$ to model (3a). It is expected that $\gamma_{ILLIQ,t} < 0$.

Across the FF25 portfolios, corr($\beta_{ILLIQ,j}$, $\beta_{IML,j}$) = -0.378, highly significant. This is reasonable: stocks with greater sensitivity to the market illiquidity premium *IML* (higher $\beta_{IML,j}$)

also have greater sensitivity to shocks in market illiquidity (more negative $\beta_{ILLIQ,j}$). Yet, $\beta_{ILLIQ,j}$ is unrelated to $\beta_{ZIML,j}$: their correlation is 0.030, practically zero.

INSERT TABLE 4

The estimation results of the augmented model (3a), presented in Table 4 Panel A, show that for both FF25 and FF100, *both* β_{ZIML} and β_{ILLIQ} are significantly priced with the proper signs (positive and negative, respectively). The negative sign of the mean γ_{ILLIQ} is consistent with the results of Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). When the models are estimated separately for each of the two subperiods, γ_{ZIML} remains consistently positive and significant in the two subperiods, whereas mean $\gamma_{ILLIQ,t}$ switches signs between the two periods and becomes insignificant. Throughout, the coefficients γ_{IML} are positive but insignificant, though by some measures, such as the fraction of positive coefficients, some estimates are significant.

The model is further augmented to allow for conditional β of illiquidity shocks, similar to the conditional β of *IML*. Model (2a) is augmented by both $\beta_{ILLIQ,j}*dMILLIQ_t$ and $\beta_{ZILLIQ,j}*Z_{t-1}*dMILLIQ_t$, and model (3a) is augmented by both $\gamma_{ILLIQ,t}*\beta_{ILLIQ,j}$ and $\gamma_{ZILLIQ,t}*\beta_{ZILLIQ,j}$. Theory suggests that both γ_{ILLIQ} and γ_{ZILLIQ} are negative. For this augmented model across the FF25 portfolios, corr($\beta_{ILLIQ,j}$, $\beta_{IML,j}$) = -0.383 and it is significant as before, while the conditional *betas*, $\beta_{ZILLIQ,j}$ and $\beta_{ZIML,j}$, are uncorrelated.

Panel B of Table 4 presents the estimation results for the augmented cross-sectional model (3a). Importantly, the mean γ_{ZIML} remains positive and significant and the mean γ_{ILLIQ} is negative and significant, as expected, for both FF25 and FF100. However, γ_{ZILLIQ} is positive for FF25, contrary to expectations, while being negative and significant for FF100, consistent with

expectations. This may be because the FF100 portfolios reflect more the effects of smaller stocks whose exposure to illiquidity shocks is greater in times of funding illiquidity.

In subperiod estimations, γ_{ZIML} remains consistently positive and significant for both subperiods and for both datasets, FF25 and FF100. The mean γ_{IML} is also positive throughout, although not statistically significant. (Its fraction of positive coefficients is significant for FF25 for both subperiods.) As for γ_{ILLIQ} and γ_{ZILLIQ}, their signs are not consistent across the two subperiods, switching between positive and negative, and they are mostly statistically insignificant.

In conclusion, the pricing of the *IML* systematic risk, interacted with funding illiquidity, remains positive and significant after the inclusion of liquidity risk variables in the model.

3.4. Augmenting the model: conditioning on market volatility

This section tests the effect on the *IML conditional* systematic risk using both funding liquidity and market volatility as condition variables. Market volatility is measured by StD_t, the standard deviation during month *t* of daily returns on the CRSP value-weighted index of NYSE\AMEX stocks. Then, $dStD_t$ is the (log) change in market volatility relative to its preceding three-month moving average:[6]

$$dStD_t = log(StD_t) - log((StD_{t-1} + StD_{t-2} + StD_{t-3})/3) \ .$$

Model (2a) is augmented by two terms: $\beta_{SIML,j} * dStD_{t-1} * IML_t$ and $\beta_{S,j} * dStD_{t-1}$. Accordingly, model (3a) is augmented by $\gamma_{SIML,t} * \beta_{SIML,j}$ and $\gamma_{S,t} * \beta_{S,j}$. Naturally, corr(dSP_t , $dStD_t$) is positive, but it is low, 0.10.

INSERT TABLE 5

[6] The first difference of $log(StD_t)$ exhibits strong reversal while $dStD_t$ as defined above does not.

The estimation results for the augmented model (3a) are presented in Table 5, Panel A. Importantly, the means of *both* $\gamma_{ZIML,t}$ and $\gamma_{SIML,t}$ are positive and statistically significant for both FF25 and FF100. That is, the systematic risk of *IML* is more highly priced in times of increased funding illiquidity *and* increased market volatility. The mean $\gamma_{IML,t}$ is positive and significant for FF25 but insignificant for FF100. In subperiod estimations, $\gamma_{ZIML,t}$ is consistently positive and significant while $\gamma_{SIML,t}$, which is always positive, is not consistently significant. The mean $\gamma_{IML,t}$ is always positive but its mean is not statistically significant; for FF25, the fraction of positive values of $\gamma_{IML,t}$ is significantly greater than 0.50 for both subperiods.

Finally, the models are estimated with *all* the variables that have been employed so far. Model (2a) is augmented by the pair $\beta_{SIML,j}*dStD_{t-1}*IML_t$ and $\beta_{S,j}*dStD_{t-1}$ and by the pair $\beta_{ZILLIQ,j}*dSP_{t-1}*dMILLIQ_t$ and $\beta_{ILLIQ,j}*dMILLIQ_t$, and model (3a) is augmented by $\gamma_{SIML,t}*\beta_{SIML,j}$ and $\gamma_{S,t}*\beta_{S,j}$ and by $\gamma_{ZILLIQ,t}*\beta_{ZILLIQ,j}$ and $\gamma_{ILLIQ,t}*\beta_{ILLIQ,j}$. The model thus allows for two conditioning variables of the β of *IML* and it also tests whether $\beta_{ILLIQ,j}$ is priced. The cross-section model has 12 coefficients, hence it is estimated only for the FF100 portfolios, and for sake of parsimony, results are presented only for the entire period.

The results, presented in Panel B, show that all the cross-sectional determinants of expected return are priced. First, consider the two conditional βs of *IML* which rise with dSP_{t-1} and with $dStD_{t-1}$. The means of both their coefficients, $\gamma_{ZIML,t}$ and $\gamma_{SIML,t}$, are positive and significant, as expected. And, the βs that are related to market illiquidity shocks, $dMILLIQ_t$, are also significantly priced with the expected negative sign: the means of both $\gamma_{ZILLIQ,t}$ and $\gamma_{ILLIQ,t}$ are negative and significant, consistent with the results of Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). In the subperiod estimations, only the mean of $\gamma_{ZIML,t}$ remains consistently

positive and significant in both subperiods, while the means of $\gamma_{SIML,t}$, $\gamma_{ZILLIQ,t}$ and $\gamma_{ILLIQ,t}$ do not have consistent signs or statistical significance through the two subperiods.

The results thus establish a robust pricing of the conditional β of *IML*.

4. Comparison with Liu's (2006) illiquidity factor

Liu (2006) proposes an illiquidity return factor, the differential return between portfolios of stocks with high and low illiquidity measured by the proportion of zero-volume days over the previous year and by stock turnover. Liu's factor, denoted *LIU*, is kindly provided by Weimin Liu for the years 1963-2005 (516 months). In addition to being based on a different measure of illiquidity, *LIU* differs from *IML* in some aspects of construction:

(i) *LIU*'s portfolio returns are *equally*-weighted, while *IML* returns are *value* weighted.

(ii) *LIU* is based on *decile* portfolios, while *IML* is based on *quintile* portfolios.

(iii) In *IML*, stocks in each liquidity quintile portfolio are pre-ranked into terciles by volatility (return standard deviation), which is also correlated with size. In *LIU* there is no pre-ranking.

Regarding (i), indeed Liu (2006, p. 642) reports that when using *value*-weighted returns, the mean return of *LIU* drops by nearly a half. And because of items (ii) and (iii), even value-weighted *LIU* reflects more the effects of extremely liquid and small stocks versus extremely liquid and large stocks compared to *IML*.

The extent of the relation between *LIU* and *IML* is presented in Table 6 Panel A, by regressing one factor on the current and lagged values of the other factor. The *LIU* regression includes three lags of *IML* because the third one is statistically significant (but none beyond that). Both regressions have a similar *alpha* (which is significant), meaning that each factor includes a return premium that is not captured by the other factor. The R^2 estimates are below 20%, which

is low. There is however a difference in the pattern of the lagged variables. In the *LIU* regression, all the coefficients of *IML* are positive with a cumulative effect of 0.775 of *IML* on *LIU* whereas in the *IML* regression, the coefficients of *LIU* switch signs from positive to negative and the cumulative effect of *LIU* is lower, 0.156.

INSERT TABLE 6

There is an important difference in the reaction of the two factors to *dMILLIQ*, as shown in Panel B. In the *IML* regression, all the *dMILLIQ* coefficients – both current and lagged – are negative with a cumulative effect of -0.101. But in the *LIU* regression, the effect of current *dMILLIQ* is positive (and highly significant), contrary to expectations. This effect switches to being negative in subsequent months, with a cumulative effect of -0.048. The longer lagged effects for *LIU* in both Panels A and B is because *LIU* reflects more the behavior of smaller and highly-illiquid stocks (see (i)-(iii) above) whose prices adjust more slowly to information.

The results in both Panels A and B suggest that the two factors, which intend to measure the illiquidity premium, have somewhat different information content. Indeed, Liu (2006, Table 1) reports that the rank correlation across stocks between his illiquidity measure and *ILLIQ* is 0.665, suggesting that the two illiquidity measures have some differences between them.

4.1. The pricing of the systematic risk of Liu's (2006) factor and of *IML*

First, I test the pricing of LIU's systematic risk by replacing in model (2) $\beta_{IML,j}*IML_t$ by $\beta_{LIU,j}*LIU$ and in model (3) $\gamma_{IML,t}*\beta_{IML,j}$ by $\gamma_{LIU,t}*\beta_{LIU,j}$. For the FF25 portfolios, the mean and weighted mean of $\gamma_{LIU,t}$ are, respectively, 0.300 (t = 1.80) and 0.365 (t = 2.37), and for FF100, the respective numbers are 0.147 (t = 0.93) and 0.170 (t = 1.16), insignificant. For this period, the results for *IML* are similar. For FF25, the mean and weighted mean of $\gamma_{IML,t}$ are, respectively,

0.417 (t = 1.49) and 0.665 (t = 2.57); for FF100, the respective numbers are 0.281 (t = 1.48) and 0.313 (t = 1.84).

To test jointly the pricing of *both* βs, I add to model (2) $\beta_{LIU,j}*LIU_t$ and to model (3) I add $\gamma_{LIU,t}*\beta_{LIU,j}$. The results are presented in Table 6 Panel A. The means of both $\gamma_{IML,t}$ and $\gamma_{LIU,t}$ are positive with varying levels of statistical significance. For FF25, the weighted means of both $\gamma_{IML,t}$ and $\gamma_{LIU,t}$ are statistically significant and for FF100, only the weighted mean of $\gamma_{IML,t}$ is significant.

Finally, allowing for conditional β, I augment model (2a) by both $\beta_{LIU,j}*LIU_t$ and $\beta_{ZLIU,j}*Z_{t-1}*LIU_t$ and model (3a) by both $\gamma_{LIU,t}*\beta_{LIU,j}$ and $\gamma_{ZLIU,t}*\beta_{ZLIU,j}$. The results are in Panel B of Table 6. For FF25, the weighted means of all γs are significant except that of $\gamma_{ZIML,t}$, although the positive-values fraction of this coefficient is significantly greater than 0.5, the chance result. For FF100, the weighted means of all γs are significant except that of γ_{LIU}, but the fraction of this coefficient's positive values is significantly greater than 0.5. The results show again that each of these two factors contributes some different information about the illiquidity premium, and the systematic risk of both contributes to the explanation of cross-sectional expected returns.

5. Conclusion

This paper presents evidence on the pricing of the systematic risk associated with the illiquidity premium. It introduces an illiquidity return factor, *IML*, the return premium on illiquid stocks compared to liquid ones, and shows that this premium is positive and significant over the period 1950-2012. The variations in this return premium over time constitute a systematic risk in the same way as other return risk factors, such as the market risk premium or the premium on stocks with high book-to-market ratio compared to stocks with low such ratio.

The analysis then shows that the systematic risk of *IML* – its β coefficient – is priced, but it is statistically significant only in times of increased funding illiquidity.

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Table 1: estimation results for the *IML* – Illiquid-Minus-Liquid – return factor

The differential returns between high and low quintile portfolios of stocks sorted on their illiquidity, measured by their average daily *ILLIQ* = |return|/dollar volume (Amihud (2002)). Stocks are sorted into three portfolios by their return standard deviation over three months, *t*-2 to *t*, then within each portfolio stocks are sorted by their *ILLIQ* into five portfolios. Value-weighted stock returns are calculated for each portfolio for month *t*+3, i.e., skipping two months after the portfolio formation period. *IML* is the average return on the three highest-*ILLIQ* quintile portfolios (across the volatility portfolios) minus the average return on the three lowest-*ILLIQ* quintile portfolios. NYSE\AMEX stocks (some filters apply).

Panel A: Statistics of monthly *IML* return. The means have associated *t*-statistics

The *p* value is of a binomial test of whether the fraction of positive returns is 0.50 (chance).

	1950-2012	1950-6/1981	7/1981-2012
Mean	0.383 (3.55)	0.476 (3.05)	0.289 (1.95)
Median	0.361	0.355	0.361
Fraction positive (*p*)	0.566 (< 0.01)	0.569 (< 0.01)	0.563 (< 0.01)
Serial correlation	-0.044	-0.034	-0.058

Panel B: Estimated *alpha* and *β* coefficients of the model

$$IML_t = alpha_t + \beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t \quad (1)$$

RMrf is the market excess return, *SMB* and *HML* are the Fama and French (1993) factors of size and book-to-market ratio and *UMD* is the Carhart (1997) momentum factor. *alpha* is the risk-adjusted excess return. The model is estimated for the entire period, 1950-2012 and for each of the two equal subperiods. The numbers are in monthly percent points (1 = 1%) with *t*-statistics in parentheses, estimated with robust standard errors (White (1980)).

The table includes at the bottom: (i) *alpha* estimates from a model that is estimated with *SMB* omitted. (ii) *alpha* estimates from a model that includes a dummy variable *JAN* = 1 in January, and *MicRf*, the average return on CRSP's smallest two size decile portfolios (for NYSE\AMEX stocks) in excess of the risk free rate

	1950-2012	1959-6/1981	7/1981-2012
alpha	**0.334 (4.80)**	**0.480 (5.12)**	**0.249 (2.53)**
β_{RMrf}	-0.259 (10.44)	-0.283 (10.44)	-0.226 (7.57)
β_{SMB}	0.796 (20.07)	0.782 (17.65)	0.769 (12.87)
β_{HML}	0.341 (9.78)	0.394 (7.28)	0.306 (7.37)
β_{UMD}	-0.090 (3.55)	-0.209 (5.10)	-0.029 (1.04)
R^2	0.67	0.70	0.67
(i) ***Alpha***, model (1) excludes *SMB*	**0.463 (4.19)**	**0.678% (4.34)**	**0.351% (2.17)**
Includes *RMrf, HML, UMD*	yes	yes	yes
(ii) ***Alpha***, model (1) with *JAN* and *MicRf*	**0.314 (4.39)**	**0.444 (4.68)**	**0.300 (2.96)**
JAN	0.064 (0.20)	0.565 (1.36)	-0.846 (2.14)
MicRf	0.081 (2.22)	0.223 (4.09)	0.080 (1.63)
Inclusdes *RMrf, SMB, HML, UMD*	yes	yes	yes

Panel C: The coefficients of *dMILLIQ*, the monthly change in log(M*ILLIQ*)
MILLIQ is market illiquidity, the monthly average of the daily value-weighted average of the *ILLIQ* of all NYSE\AMEX stocks with codes 10 or 11, price > $5 and volume of at least 100 shares. The stocks with the highest 1% of *ILLIQ* on each day are trimmed off. In addition to $dMILLIQ_t$ and its lags, the regression model includes $RMrf_t$, SMB_t, HML_t and UMD_t. The estimation is for the entire period, 1950-2012.

	Coeff. (*t*-stat)
$dMILLIQ_t$	-0.018 (3.04)
$dMILLIQ_{t-1}$	-0.018 (4.42)
$dMILLIQ_{t-2}$	-0.002 (0.45)
$dMILLIQ_{t-3}$	-0.004 (1.02)
R^2	0.69

Panel D: Out-of-sample (one-month-ahead) *alpha*, rolling estimates

$$alpha_t = IML_t - (\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t)$$

The coefficients β_K, $K = RMrf, SMB, HML$ and UMD are estimated from a regression over 36 months up to month t-1. The p values after "Fraction positive" are of a binomial test that the fraction is 0.50, the chance result.
Under (i), results are of a regression of $alpha_t$ on a constant and JAN_t , the January dummy variable. The intercept is the mean $alpha_t$ in non-January months. The t-statistics in parentheses employ robust estimation of the standard errors.

	1953-2012	**1953-6/1981**	**7/1981-2012**
Mean $alpha_t$ (t-statistic)	0.344 (5.29)	0.471 (5.05)	0.229 (2.54)
Median $alpha_t$	0.299	0.420	0.174
Fraction positive (p-value)	0.583 (< 0.01)	0.617 (< 0.01)	0.553 (0.022)
(i) Mean $alpha_t$, controlling for *January*			
Intercept	0.326 (4.98)	0.384 (4.21)	0.274 (2.93)
JAN	0.215 (0.70)	1.030 (2.15)	-0.550 (1.63)

Table 2: The pricing of the factors' systematic risk (β)

Panel A1: β coefficients of *IML* estimated with the Fama-French (1993) and Carhart (1997) factors

The dependent variable is the portfolio returns r_j, j = 1, 2, … N, N = 25 or N = 100. The portfolios are Fama and French's (1993) 25 (5x5) or 100 (10x10) portfolios of stocks sorted on size and book-to-market ratio. The table presents statistics of the βs, the slope coefficients in the model

$$(r_j\text{-}rf)_t = \beta 0_j + \beta_{RMrf,j}*(RMrf)_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HMLt + \beta_{UMD,j}*UMD_t + \beta_{IML,j}*IML_t \quad (2)$$

This model is due to Fama and French (1993) and Carhart (1997), adding *IML*, The return factor of illiquid-minus-liquid stock portfolios. See details in Table 1. The sample period is 1950-2012.

	FF25 portfolios				**FF100 portfolios**			
β of…	Mean	Std. Dev.	Min	Max	Mean	Std. Dev.	Min	Max
RMrf	1.016	0.045	0.923	1.106	1.022	0.066	0.872	1.206
SMB	0.520	0.500	-0.273	1.473	0.537	0.487	-0.375	1.636
HML	0.284	0.395	-0.395	0.837	0.292	0.393	-0.569	1.029
UMD	-0.024	0.028	-0.081	0.024	-0.034	0.056	-0.214	0.096
IML	0.015	0.082	-0.126	0.115	0.022	0.121	-0.344	0.224

Panel A2: Cross-portfolio correlations between the factors' βs

	FF25 portfolios				**FF100 portfolios**			
β of…	*RMrf*	*SMB*	*HML*	*UMD*	*RMrf*	*SMB*	*HML*	*UMD*
SMB	0.04				-0.05			
HML	-0.06	-0.14			0.08	-0.17		
UMD	-0.76	-0.12	0.03		-0.57	0.01	-0.11	
IML	-0.48	-0.15	0.16	0.55	-0.22	-0.28	0.16	0.30

Panel B: The pricing of the *IML* systematic risk (β_{IML})

Results of Fama-MacBeth monthly cross-section regressions of monthly portfolio returns on βs:

$$r_{jt} = \gamma 0_t + \gamma_{RMrf,t}*\beta_{RMrf,j} + \gamma_{SMB,t}*\beta_{SMB,j} + \gamma_{HML,t}*\beta_{HML,j} + \gamma_{UMD,t}*\beta_{UMD,j} + \gamma_{IML,t}*\beta_{IML,j} \quad (3)$$

The βs are estimated in the first pass regression, see panel A, with r_{jt}, j = 1, 2, … N, being the monthly returns on the Fama and French's (1993) portfolios, N = 25 (5x5) or N = 100 (10x10) (see panel A). The table presents statistics of the time series of γ_K, K = *RMrf, SMB, HML, UMD* and *IML*. "Wtd" is weighted mean, using as weights the reciprocal of the cross-section estimated standard error of the respective γ_K. The p value is the probability in the binomial test that the fraction of positive coefficients is greater than 0.5, the chance result.

	FF25 portfolios				**FF100 portfolios**			
Coefficient of β of…	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
RMrf	1.014 (2.12)	0.963 (2.37)	1.262	0.536 (0.027)	0.329 (1.12)	0.003 (0.01)	0.005	0.500
SMB	0.132 (1.23)	-0.763 (0.88)	-0.123	0.483 (0.837)	0.119 (1.08)	-0.068 (0.76)	-0.097	0.479
HML	0.397 (3.85)	0.392 (4.60)	0.246	0.557 (<0.01)	0.405 (3.88)	0.384 (4.35)	0.246	0.551 (<.01)
UMD	3.297 (4.53)	3.030 (5.00)	3.381	0.562 (< 0.01)	1.416 (4.45)	1.142 (4.71)	1.368	0.582 (<.01)
IML	**0.451 (1.89)**	**0.514 (2.49)**	**0.565**	**0.552 (< 0.01)**	**0.228 (1.33)**	**0.266 (1.82)**	**0.205**	**0.521 (0.130)**
R^2	0.554		0.576		0.286		0.257	

Table 3: The pricing of the *IML*'s *conditional* systematic risk

Results of Fama-MacBeth cross-section regressions of portfolio returns on βs:

$$r_{jt} = \gamma 0_t + \gamma_{RMrf,t}*\beta_{RMrf,j} + \gamma_{SMB,t}*\beta_{SMB,j} + \gamma_{HML,t}*\beta_{HML,j} + \gamma_{UMD,t}*\beta_{UMD,j} + \boldsymbol{\gamma_{IML,t}*\beta_{IML,j} + \gamma_{ZIML,t}*\beta_{ZIML,j} + \gamma_{Z,t}*\beta_{Z,j}} \quad (3a)$$

The βs are estimated in the first pass regression from the model

$$(r_j\text{-}rf)_t = \beta 0_j + \beta_{RMrf,j}*(RMrf)_t + \beta_{SMB,j}*SMB_t + \beta_{HML,j}*HMLt + \beta_{UMD,j}*UMD_t + \boldsymbol{\beta_{IML,j}*IML_t + \beta_{ZIML,j}*Z_{t-1}*IML_t + \beta_{z,j}*Z_{t-1}} \quad (2a)$$

The table presents statistics of the time series of $\gamma_{K,t}$, K =*IML, ZIML* and *Z*. The weighted mean uses as weights the reciprocal of the cross-section estimated standard error.

The test statistics are described in the legend of Table 2, Panel B.

Panel A: $\boldsymbol{Z_t = maSP_t}$, the mean-adjusted monthly yield spread SP_t between corporate bonds rated as BAA and AAA (source: the Federal Reserve Bank of St. Louis; 1% = 1).

	FF25 portfolios				**FF100 portfolios**			
Coefficient of β of…	Mean γ (t-stat)	Weighted mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Weighted mean γ (t-stat)	Median	Positive (p)
IML_t	0.388 (1.63)	0.410 (1.98)	0.548	0.543 (< 0.01)	0.148 (0.85)	0.168 (1.13)	0.239	0.525 (0.089)
$maSP_{t-1}$ *IML_t	**0.942 (4.33)**	**0.983 (5.32)**	**1.260**	**0.591** (< 0.01)	**0.435 (2.71)**	**0.478 (3.77)**	**0.480**	**0.550** (< 0.01)
$maSP_{t-1}$	0.084 (0.78)	0.160 (1.72)	0.028	0.503 (0.457)	-0.060 (0.97)	-0.027 (0.53)	-0.080	0.484 (0.201)
R^2	0.622		0.657		0.314		0.289	

Panel B: $Z_t = dSP_t = SP_t - SP_{t-1}$, the change in the monthly yield spread between corporate bonds rated as BAA and AAA.
In this application of models (2a) and (3a), the conditional systematic risk employs $Z_t = dSP_t$. In the estimations of the two subperiods, the β coefficients are estimated separately for each subperiod by model (2a). The p value is the probability under the binomial test that the fraction of positive coefficients is greater than 0.5. When the coefficient is negative, the test is whether the fraction is below 0.5.

Coefficient of β of…	FF25 portfolios				FF100 portfolios			
	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
IML_t	0.529 (2.26)	0.549 (2.69)	0.490	0.550 (< 0.01)	0.208 (1.22)	0.228 (1.56)	0.219	0.519 (0.163)
dSP_{t-1} *IML_t	**0.188 (2.96)**	**0.206 (3.83)**	**0.281**	**0.570 (< 0.01)**	**0.111 (2.93)**	**0.129 (4.01)**	**0.169**	**0.573 (< 0.01)**
dSP_{t-1}	0.015 (0.67)	0.026 (1.38)	-0.014	0.493 (0.372)	0.020 (1.55)	0.028 (2.56)	0.025	0.533 (0.037)
R^2	0.619		0.635		0.309		0.281	
Results for subperiods I and II								
Subperiod I: 1950-6/1981								
IML_t	0.473 (1.65)	0.337 (1.38)	0.369	0.545 (0.045)	0.165 (0.78)	0.187 (1.00)	0.042	0.503 (0.479)
dSP_{t-1} *IML_t	**0.160 (2.13)**	**0.195 (2.97)**	**0.217**	**0.590 (< 0.01)**	**0.061 (1.66)**	**0.081 (2.64)**	**0.078**	**0.558 (0.013)**
dSP_{t-1}	0.018 (0.89)	0.029 (1.52)	0.036	0.534 (0.099)	0.017 (1.71)	0.019 (2.06)	0.015	0.526 (0.164)
Subperiod II: 7/1981-2012								
IML_t	0.358 (1.22)	0.310 (1.19)	0.639	0.553 (0.022)	0.266 (1.22)	0.283 (1.49)	0.386	0.556 (0.017)
dSP_{t-1} *IML_t	**0.210 (3.18)**	**0.204 (3.57)**	**0.208**	**0.563 (< 0.01)**	**0.134 (2.93)**	**0.134 (2.93)**	**0.133**	**0.573 (< 0.01)**
dSP_{t-1}	-0.029 (1.11)	-0.039 (1.76)	-0.028	0.471 (0.140)	-0.004 (0.28)	-0.003 (0.23)	-0.007	0.476 (0.191)

Panel C: The coefficients in January and non-January months.
Regressions of the monthly Fama-Macbeth coefficients of γ_{IMLt}, γ_{ZIMLt} and $\gamma_{Z,t}$ from model (3a) on two dummy variables, *JAN* (= 1 in January) and *Non-JAN* = 1 – *JAN*. In parentheses are t statistics, estimated using robust standard errors.

Coefficient of…	FF25 portfolios			FF100 portfolios		
	γ_{IMLt} (IML_t)	γ_{ZIMLt} ($dSP_{t-1}*IML_t$)	γ_{Zt} (dSP_{t-1})	γ_{IMLt} (IML_t)	γ_{ZIMLt} ($dSP_{t-1}*IML_t$)	γ_{Zt} (dSP_{t-1})
JAN	1.940 (2.26)	-0.933 (3.22)	-0.493 (6.35)	3.053 (4.10)	-0.014 (0.80)	-0.119 (2.74)
Non-JAN	0.367 (1.51)	0.266 (4.17)	0.056 (2.48)	-0.050 (0.29)	0.133 (3.49)	0.033 (2.43)

Table 4: The pricing of the systematic risk of *IML* and *dMILLIQ*

Panel A: Adding $dMILLIQ_t$. Model (2a) is augmented by $\beta_{ILLIQ,j}*dMILLIQ_t$.

In the following cross-section Fama-Macbeth regressions, model (3a) is augmented by $\gamma_{ILLIQ,t}*\beta_{ILLIQ,j}$. The *p* value pertains to a test of whether the fraction of positive (negative) coefficients is greater than 0.5, the chance result, when the average coefficient is positive (negative, respectively).

Coefficient of β of…	**FF25 portfolios**				**FF100 portfolios**			
	Results for the entire sample period, 12950-2012							
Coefficient of β of…	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
IML_t	0.265 (1.06)	0.142 (0.64)	0.373	0.525 (0.089)	0.173 (1.00)	0.187 (1.27)	0.072	0.509 (0.318)
$dSP_{t-1}*IML_t$	**0.124 (2.11)**	**0.123 (2.35)**	**0.080**	**0.529 (0.059)**	**0.117 (3.10)**	**0.137 (4.25)**	**0.171**	**0.567 (< 0.01)**
dSP_{t-1}	-0.029 (1.41)	0.030 (1.66)	-0.040	0.472 (0.068)	0.017 (1.31)	0.025 (2.29)	0.021	0.520 (0.130)
$dMILLIQ_t$	**-11.057 (2.98)**	**-14.465 (4.60)**	**-12.574**	**0.427 (< 0.01)**	**-4.603 (2.61)**	**-4.794 (3.15)**	**-4.337**	**0.468 (0.044)**
R^2	0.654		0.678		0.327		0.301	
Results for subperiods I and II								
Subperiod I: 1950-6/1981								
IML_t	0.491 (1.64)	0.319 (1.22)	0.400	0.556 (0.017)	0.163 (0.77)	0.187 (1.00)	0.187	0.519 (0.252)
$dSP_{t-1}*IML_t$	**0.173 (2.55)**	**0.181 (2.99)**	**0.237**	**0.577 (< 0.01)**	**0.058 (1.59)**	**0.078 (2.55)**	**0.093**	**0.561 (0.010)**
dSP_{t-1}	0.022 (1.06)	0.027 (1.46)	0.015	0.516 (0.286)	0.016 (1.57)	0.018 (1.92)	0.011	0.534 (0.099)
$dMILLIQ_t$	**1.429 (0.40)**	**0.028 (0.01)**	**2.050**	**0.511 (0.359)**	**-1.877 (1.08)**	**-0.236 (0.15)**	**-1.186**	**0.489 (0.359)**
Subperiod II: 7/1981-2012								
IML_t	0.318 (1.06)	0.211 (0.80)	0.583	0.553 (0.022)	0.289 (1.33)	0.290 (1.53)	0.383	0.548 (0.036)
$dSP_{t-1}*IML_t$	**0.211 (3.22)**	**0.198 (3.49)**	**0.163**	**0.563 (< 0.01)**	**0.138 (3.35)**	**0.127 (3.72)**	**0.116**	**0.579 (< 0.01)**
dSP_{t-1}	-0.035 (1.33)	-0.053 (2.28)	-0.016	0.476 (0.191)	-0.005 (0.30)	-0.003 (0.25)	-0.009	0.487 (0.322)
$dMILLIQ_t$	**-2.638 (0.77)**	**-4.711 (1.63)**	**-0.317**	**0.500 (0.521)**	**1.225 (0.62)**	**0.612 (0.37)**	**3.527**	**0.524 (0.191)**

Panel B: Adding both $dMILLIQ_t$ and $dSP_{t-1}*dMILLIQ_t$.

Model (2a) is augmented by $\beta_{ILLIQ,j}*dMILLIQ_t$ and $\beta_{ZILLIQ,j}*dSP_{t-1}*dMILLIQ_t$ and model (3a) is augmented by $\gamma_{ILLIQ,t}*\beta_{ILLIQ,j}$ and $\gamma_{ZILLIQ,t}*\beta_{ZILLIQ,j}$ for the cross-section Fama-Macbeth regressions.

Coefficient of β of…	**FF25 portfolios**				**FF100 portfolios**			
	Entire sample period, 1950-2012							
Coefficient of β of…	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
IML_t	0.350 (1.36)	0.199 (0.87)	0.578	0.533 (0.037)	0.176 (1.03)	0.196 (1.33)	0.081	0.516 (0.201)
$dSP_{t-1}*IML_t$	**0.153 (2.42)**	**0.150 (2.65)**	**0.171**	**0.537 (0.023)**	**0.118 (3.13)**	**0.139 (4.31)**	**0.187**	**0.573 (< 0.01)**
dSP_{t-1}	-0.026 (1.28)	-0.025 (1.37)	-0.031	0.475 (0.089)	0.016 (1.24)	0.024 (2.25)	0.016	0.520 (0.146)
$dMILLIQ_t$	**-11.369 (3.12)**	**-14.134 (4.56)**	**-13.205**	**0.421 (< 0.01)**	**-4.314 (2.52)**	**-4.373 (2.92)**	**-3.874**	**0.458 (0.011)**
$dSP_{t-1}*dMILLIQ_t$	**0.410 (1.14)**	**0.276 (0.88)**	**0.245**	**0.509 (0.318)**	**-0.359 (1.65)**	**-0.359 (2.37)**	**-0.382**	**0.460 (0.016)**
R^2	0.654		0.678		0.338		0.311	
Subperiod I: 1950-6/1981								
IML_t	0.519 (1.72)	0.203 (1.39)	0.405	0.545 (0.045)	0.176 (0.83)	0.209 (1.12)	0.187	0.516 (0.400)
$dSP_{t-1}*IML_t$	**0.203 (2.82)**	**0.229 (3.55)**	**0.210**	**0.571 (< 0.01)**	**0.059 (1.60)**	**0.083 (2.65)**	**0.092**	**0.561 (0.010)**
dSP_{t-1}	0.008 (0.37)	0.017 (0.90)	0.013	0.508 (0.399)	0.015 (1.52)	0.018 (1.91)	0.012	0.532 (0.118)
$dMILLIQ_t$	**4.876 (1.32)**	**3.558 (1.03)**	**6.685**	**0.545 (0.045)**	**-1.613 (0.91)**	**0.181 (0.12)**	**0.300**	**0.503 (0.479)**
$dSP_{t-1}*dMILLIQ_t$	**0.862 (2.22)**	**0.837 (2.41)**	**0.927**	**0.540 (0.068)**	**0.192 (0.96)**	**0.272 (1.51)**	**0.328**	**0.534 (0.099)**
Subperiod II: 7/1981-2012								
IML_t	0.302 (1.00)	0.217 (0.81)	0.543	0.558 (0.013)	0.293 (1.35)	0.295 (1.55)	0.359	0.540 (0.068)
$dSP_{t-1}*IML_t$	**0.208 (3.25)**	**0.202 (3.62)**	**0.140**	**0.563 (< 0.01)**	**0.137 (3.31)**	**0.126 (3.70)**	**0.116**	**0.579 (< 0.01)**
dSP_{t-1}	-0.033 (1.24)	-0.056 (2.32)	-0.040	0.471 (0.140)	-0.006 (0.37)	0.005 (0.37)	-0.013	0.476 (0.191)
$dMILLIQ_t$	**-3.036 (0.96)**	**-4.294 (1.56)**	**-1.801**	**0.511 (0.359)**	**1.358 (0.72)**	**0.906 (0.56)**	**2.256**	**0.537 (0.082)**
$dSP_{t-1}*dMILLIQ_t$	**-0.469 (1.33)**	**-0.733 (2.62)**	**-0.641**	**0.450 (0.028)**	**-0.430 (1.70)**	**-0.464 (2.32)**	**-0.329**	**0.447 (0.022)**

Table 5: Interaction of *IML* with changes in market volatility

Panel A: Model (2a) is augmented by $\beta_{SIML,j}*dStD_{t-1}*IML_t$ and by $\beta_{Sj}*dStD_{t-1}$, where $dStD_t = log(StD_t) - log((StD_{t-1} + StD_{t-2} + StD_{t-3})/3)$. StD_t is the monthly market return standard deviation, calculated from daily returns on the CRSP value-weighted index of NYSE\AMEX stocks. Model (3a) is augmented by $\gamma_{SIML,t}*\beta_{SIML,j}$ and $\gamma_{S,t}*\beta_{S,j}$ for the cross-section Fama-Macbeth regressions. The parameters and test statistics are described in Table 3.

Coefficient of β of…	**FF25 portfolios**				**FF100 portfolios**			
	Results for the entire sample period, 1950-2012							
Coefficient of β of…	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
IML_t	0.757 (2.96)	0.830 (3.73)	0.812	0.546 (< 0.01)	0.257 (1.48)	0.266 (1.78)	0.305	0.522 (0.115)
$dSP_{t-1}*IML_t$	0.187 (2.86)	0.182 (3.28)	0.259	0.567 (< 0.01)	0.110 (2.95)	0.129 (4.01)	0.162	0.571 (< 0.01)
dSP_{t-1}	0.008 (0.38)	0.017 (0.93)	-0.009	0.495 (0.400)	0.021 (1.69)	0.031 (2.84)	0.021	0.522 (0.115)
$dStD_{t-1}*IML_t$	67.077 (2.63)	83.429 (3.62)	96.549	0.552 (< 0.01)	30.794 (2.27)	38.329 (3.31)	43.405	0.549 (< 0.01)
$dStD_{t-1}$	17.165 (1.85)	18.763 (2.33)	30.522	0.546 (< 0.01)	3.426 (0.73)	4.088 (1.06)	10.116	0.538 (0.019)
R^2	0.672		0.699		0.333		0.300	
Subperiod I: 1950-6/1981								
IML_t	0.523 (1.80)	0.403 (1.62)	0.529	0.561 (0.010)	0.208 (0.99)	0.222 (1.18)	0.191	0.521 (0.220)
$dSP_{t-1}*IMLt$	0.277 (3.16)	0.305 (3.94)	0.249	0.574 (< 0.01)	0.076 (1.95)	0.094 (2.92)	0.098	0.563 (< 0.01)
dSP_{t-1}	0.043 (1.97)	0.055 (2.74)	0.043	0.550 (0.028)	0.021 (2.03)	0.023 (2.43)	0.017	0.534 (0.099)
$dStD_{t-1}*IML_t$	16.737 (0.75)	13.955 (0.70)	17.293	0.505 (0.439)	23.753 (1.65)	23.821 (1.88)	16.457	0.524 (0.191)
$dStD_{t-1}$	37.915 (3.35)	36.576 (3.64)	30.795	0.558 (0.013)	7.276 (1.35)	8.275 (1.81)	8.523	0.545 (0.045)
Subperiod II: 7/1981-2012								
$IMLt$	0.448 (1.46)	0.429 (1.59)	0.727	0.550 (0.028)	0.291 (1.32)	0.305 (1.58)	0.353	0.540 (0.068)
$dSP_{t-1}*IML_t$	0.204 (2.97)	0.188 (3.17)	0.115	0.553 (0.022)	0.127 (3.07)	0.111 (3.26)	0.097	0.577 (< 0.01)
dSP_{t-1}	-0.011 (0.39)	-0.037 (1.37)	0.001	0.500 (0.521)	-0.005 (0.31)	0.001 (0.11)	-0.002	0.487 (0.322)
$dStD_{t-1}*IML_t$	52.812 (1.98)	44.424 (1.94)	59.289	0.553 (0.022)	10.803 (0.79)	15.326 (1.37)	33.447	0.558 (0.013)
$dStD_{t-1}$	3.500 (0.44)	0.070 (0.01)	4.145	0.511 (0.359)	1.376 (0.24)	0.926 (0.20)	5.009	0.524 (0.191)

Panel B: In addition to the specification described in Panel A, Model (2a) also includes $\beta_{ILLIQ,j}*dMILLIQ_t$ and $\beta_{ZILLIQ,j}*dSP_{t-1}*dMILLIQ_t$ and model (3a) also includes $\gamma_{ILLIQ,t}*\beta_{ILLIQ,j}$ and $\gamma_{ZILLIQ,t}*\beta_{ZILLIQ,j}$ for the cross-section Fama-Macbeth regressions. The model is estimated only for FF100.

Coefficient of β of…	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
IML_t	0.233 (1.34)	0.243 (1.62)	0.262	0.528 (0.068)
$dSP_{t-1}*IML_t$	0.115 (3.09)	0.136 (4.22)	0.161	0.567 (< 0.01)
dSP_{t-1}	0.015 (1.25)	0.025 (2.36)	0.012	0.524 (0.102)
$dStD_{t-1}*IML_t$	29.440 (2.17)	36.977 (3.21)	43.323	0.557 (< 0.01)
$dStD_{t-1}$	5.021 (1.03)	5.899 (1.49)	8.229	0.534 (0.032)
$dMILLIQ_t$	-4.222 (2.49)	-4.217 (2.83)	-4.124	0.458 (0.011)
$dSP_{t-1}*dMILLIQ_t$	-0.388 (1.79)	-0.445 (2.51)	-0.409	0.460 (0.016)
R^2	0.361		0.338	

Table 6: **Comparing *IML* with *LIU***

Panel A: The time-series relation between *IML* and *LIU*

The dependent variables are *IML*, defined in Table 1, and *LIU*, Liu's (2006) liquidity factor, the return on the high-minus-low decile portfolio of stocks ranked by illiquidity, measured by the proportion of zero-volume days and by turnover. *dMILLIQ* is defined in Table 1, and is mean adjusted. The data period is 1963-2005 (516 months).

Panel A			Panel B		
	LIU	***IML***		***LIU***	***IML***
alpha	0.329 (2.19)	0.366 (2.57)	*alpha*	0.694 (4.77)	0.462 (3.44)
IML_t	0.428 (9.35)		$dMILLIQ_t$	0.054 (5.86)	-0.042 (5.56)
IML_{t-1}	0.154 (3.00)		$dMILLIQ_{t-1}$	-0.044 (4.57)	-0.048 (5.26)
IML_{t-2}	0.097 (1.93)		$dMILLIQ_{t-2}$	-0.009 (0.96)	-0.012 (1.32)
IML_{t-3}	0.096 (1.96)		$dMILLIQ_{t-3}$	-0.029 (3.48)	-0.026 (3.30)
LIU_t		0.355 (7.22)	$dMILLIQ_{t-4}$	-0.020 (2.01)	0.000 (0.04)
LIU_{t-1}		-0.196 (4.79)			
LIU_{t-2}		-0.003 (0.07)			
Cumulative	0.775	0.156	Cumulative	-0.048	-0.102
R^2	0.185	0.198	R^2	0.120	0.103

Panel B: Comparison of the pricing of systematic risk with Liu's (2006) illiquidity factor
Results using Liu's (2006) illiquidity factor, LIU_t. $Z_t = dSP_t$, the change in the month yield spread between BAA and AAA corporate bonds. The test statistics are explained in Table 3.

Coefficient of β of…	**FF25 portfolios**				**FF100 portfolios**			
	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)	Mean γ (t-stat)	Wtd mean γ (t-stat)	Median	Positive (p)
(i) Adding $\beta_{LIU,j}*LIU_t$ to model (2) and $\gamma_{LIU,t}*\beta_{LIU,j}$ to model (3)								
IML_t	0.476 (1.49)	0.790 (2.69)	0.553	0.525 (0.135)	0.383 (1.90)	0.512 (2.85)	0.332	0.545 (0.024)
LIU_t	0.327 (1.98)	0.386 (2.54)	0.511	0.566 (< 0.01)	0.161 (1.03)	0.196 (1.35)	0.425	0.547 (0.019)
(ii) Adding $\beta_{LIU,j}*LIU_t$ and $\beta_{ZLIU,j}*Z_{t-1}*LIU_t$ to model (2a) and $\gamma_{LIU,t}*\beta_{LIU,j}$ and $\gamma_{ZLIU,t}*\beta_{ZLIU,j}$ to model (3a)								
IML_t	0.683 (2.13)	0.971 (3.23)	0.727	0.548 (0.015)	0.330 (1.65)	0.442 (2.45)	0.357	0.543 (0.029)
$dSP_{t-1}*IML_t$	0.110 (1.54)	0.069 (1.08)	0.144	0.539 (0.043)	0.096 (2.43)	0.114 (3.21)	0.146	0.560 (< 0.01)
dSP_{t-1}	-0.011 (0.37)	-0.003 (0.12)	-0.040	0.485 (0.255)	0.025 (1.84)	0.035 (2.84)	0.025	0.541 (0.035)
LIU_t	0.308 (1.85)	0.321 (2.08)	0.439	0.558 (< 0.01)	0.164 (1.05)	0.185 (1.27)	0.324	0.543 (0.029)
$dSP_{t-1}*LIU_t$	0.238 (2.83)	0.268 (3.57)	0.313	0.574 (< 0.01)	0.077 (1.93)	0.095 (2.64)	0.089	0.543 (0.029)

Figure 1: Moving average of $alpha_t$ over 12 months.
The monthly estimate $alpha_t$ is calculated as

$$alpha_t = IML_t - [\beta_{RMrf} * RMrf_t + \beta_{SMB} * SMB_t + \beta_{HML} * HML_t + \beta_{UMD} * UMD_t]$$

The βs are estimated over 36 months preceding month t. The data period is 1950-2012 thus $alpha_t$ begins on 1/1953. The figure presents a twelve-month moving average of $alpha_t$, thus beginning on 12/1953. The numbers on the Y-axis are %.

