On the Optimality of Financial Repression

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ABSTRACT

Governments tend to practice financial repression in times when fiscal conditions are poor: either when they have large amounts of accumulated debts or when the current spending needs are severe. We ask when is such repression optimal in a standard model with a financial sector constrained by a collateral constraint. We show that under commitment financial repression is not optimal but without commitment it may be.

1. Introduction

Financial repression is defined as regulation imposed by government to banks and other financial intermediary to force them to hold more government bonds than they would absent such regulation. In this paper, we investigate when, if ever, financial repression is optimal. We find that under commitment financial repression is never optimal. If, however, a government cannot commit to its policies, in particular, it cannot commit to repaying its debt then financial repression may be optimal. Moreover, we find that the more severe are the fluctuations in spending needs, the stronger is the financial repression when spending needs are high.

We are motivated by several sets of findings. The first is that financial repression has been extensively used by government as a tool to reduce the burden of government debt in the post WWII era as documented by Reinhart and Sbrancia (2011). The second is that in times of severe fiscal distress, such as in the recent financial crisis in Europe, banks have increased their holdings of their own governments debt. (See Broner, Erce, Martin, and Ventura (2014)).

We study the optimality of financial repression in a standard neoclassical model augmented to include a financial sector. We model the financial sector following Gertler and Kiyotaki (2010). Financial intermediaries (banks) channel resources from the households to firms that use such funds for investment. Bankers lack commitment in that in any period the bankers can abscond with a fraction of their assets and default on the depositors. This friction gives rise to an enforcement constraint that limits the amount of deposits that banks can raise and in turn the investment of firms.

We then consider a benevolent government that raises revenues to finance government expenditure using proportional taxes to labor income and investment and by issuing government debt that can be held by households and banks. The government can also regulate the assets holding of the banks by forcing them to hold a certain fraction of their assets as government bonds. Finally, the government can default on the debt it issues. We study the optimal policy with and without commitment on the government side.

We have several results. The first is that when the government can commit to repaying its debt, financial repression is not optimal. Specifically, the government can achieve the

Ramsey outcome without forcing banks to hold government debt. The key idea is that forcing banks to hold government debt at a below market interest rate is an inefficient way to raise revenues. Indeed, forcing banks to do so is equivalent to a tax on investment plus a tightening of the collateral constraint. Hence, whenever the collateral constraint is binding it is strictly preferable to simply directly raise the same amount of revenues through a tax on capital and leave the collateral constraint unaffected. Thus financial repression is a dominated instrument when government can commit to repay back its debt.

The second is when the government does not have commitment, the government may find it optimal to force banks to hold government debt when its fiscal needs are sufficiently high. We begin by modeling lack of commitment by the government as having the government choose policies in a Markov fashion. In the model defaulting on debt held by consumers has a strict benefit, in that it raises revenues in a lump sum fashion and it has no cost. Instead, if the government defaults on debt held by banks then it reduces the net worth available to banks for investing, tightens their collateral constraints and depresses investment in the economy. Hence, forcing the banks to hold some government debt ex-ante acts like a commitment device that ensures the government will not find it optimal to default ex-post. In the Markov equilibrium the government issues debt both to banks and to consumers to smooth tax distortions over time subject to the constraint that it has no incentive to default ex-post. We then show that in the Markov equilibrium, countries that have relatively more cyclical government spending patterns should practice more severe financial repression in times of strong fiscal needs.

Finally, we consider the best sustainable equilibrium in which governments face a sustainability constraint which specifies that after any deviation the economy reverts to a Markov equilibrium. We then show that in times of unanticipatedly high spending needs it is optimal to practice relatively more severe financial repression.

Our findings have important policy implications. Several policy makers including the current governor of the Bundesbank, Jens Weidmann, has argued argue that financial institutions should be regulated so that they are allowed to hold only small amounts of their own country's goverment bonds. Our analysis emphasizes that such a policy change may not be desirable.

2. Environment

Consider an infinite horizon economy that blends elements of Kiyotaki and Moore (1997) with that of Gertler and Kiyotaki (2010) that is composed of a household that works and runs financial intermediaries, referred to as *banks*, together with firms and a government. Households elastically supply labor and save by holding deposits in banks and government bonds and receive dividends. Banks raise deposits from households and use these deposits plus retained earnings to invest in government bonds and capital as well as pay dividends back to consumers. Firms rent capital and labor and produce output. The government finances an exogenous stream of government spending with taxes on labor income and the capital stock, sells government bonds, and can impose that banks must hold at least a certain fraction of their assets in government bonds.

The resource constraint is given by

$$(1) \quad C_t + K_{t+1} + G_t = F(K_t, L_t)$$

where C_t is aggregate consumption, K_{t+1} is the capital stock, G_t is government spending, L_t is aggregate labor, and F is a constant returns to scale production function which includes the undepreciated capital stock. Throughout we use the convention that uppercase letters denote aggregates and lower case level to denote the decisions of individual households or banks.

We follow Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012) in the formulation of households. The decision making in each household can be thought of being made by different entities: a measure 1 of workers and a measure 1 of bankers. The workers supply labor and return their wages to the household while each banker manages a bank that transfers nonnegative dividends back to the household. The household as a whole has preferences

$$(2) \quad \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

where c_t and l_t are an individual household's consumption and labor supply. Given initial

asset holdings b_{H0} and d_0 the stand-in household in the economy maximizes this utility by choosing $\{c_t, l_t, b_{Ht+1}, d_{t+1}\}$ subject to the budget constraint

$$(3) \quad c_t + q_{Bt+1} b_{Ht+1} + q_{Dt+1} d_{t+1} \leq (1 - \tau_{Lt}) w_t l_t + d_t + \delta_t b_{Ht} + X_t - (1 - \sigma)\bar{n}$$

and the restrictions that

$$(4) \quad b_{Ht+1} \geq 0, d_{t+1} \geq \bar{d}$$

where $\bar{d}$ is a large negative number. Here b_{Ht+1} and d_{t+1} are the amount of government bonds held by households and the deposits made by households in banks and q_{Bt+1} and q_{Dt+1} are the corresponding prices. Buying one unit of government bonds at t entitles the consumer to δ_t units of goods at $t+1$ where $\delta_t = 1$ signifies that the government repays its debts at $t+1$ and $\delta_t = 0$ signifies that it does not. Buying one unit of deposits at t entitles the consumer to one unit of goods paid by the bank at $t+1$. Also, w_t is the real wage, τ_{Lt} is the labor income tax, X_t are dividends paid by banks, and $\bar{n}$ is the amount of initial equity given to each newly formed banks of which there are a measure $1 - \sigma$ formed each period. The nonnegativity constraint on government debt implies that the household cannot hold negative amounts of government debt and hence borrow from the government.

The first order conditions for the household's problem can be summarized by

$$(5) \quad -\frac{U_{Lt}}{U_{Ct}} = (1 - \tau_{Lt}) w_t$$

$$(6) \quad q_{Dt+1} = \frac{\beta U_{Ct+1}}{U_{Ct}}$$

$$(7) \quad q_{Bt+1} \geq \frac{\beta U_{Ct+1}}{U_{Ct}} \delta_{t+1} \text{ with equality if } b_{Ht+1} > 0$$

Consider next the banks. At the beginning of each period an idiosyncratic random variable is realized at each existing bank. With probability σ the bank will continue in operation until the next period and with probability $1 - \sigma$ the bank ceases to exist and, by assumption, pays out all of its accumulated net worth as dividends to the household. Also at

the beginning of each period a measure $(1-\sigma)/\sigma$ of new banks are born, each of which is given an exogenously specified amount of initial equity $\bar{n}$ from households. Since only a fraction σ of these new-born banks survive until the end of the period, the measure of surviving banks is always constant at 1. This device of having banks die is a simple way to ensure that they do not build up enough equity to make the financial constraints that we will next introduce irrelevant.

In this paper we want to focus on the time inconsistency problem that comes from the desire of the government to default on the debt it issues ex-post. In order to focus on the issue we allow the government to tax the capital stock, so as to avoid a time inconsistency problem over the choice of labor versus capital taxes. The reason these capital taxes avoid this problem is that since taxes on the capital stock invested at t, namely k_{t+1} are levied at the same time these decisions are being made there is no issue of an ex post inelastic supply of previously accumulated capital that generates the standard time inconsistency problem.

Turning to the budget constraint of an individual bank note first that for any non-newborn bank the budget constraint at t is

$$(8) \quad x_t + (1+\tau_{kt})k_{t+1} + q_{Bt+1}b_{Bt+1} - q_{Dt+1}d_{t+1} \leq R_t k_t + \delta_t b_{Bt} - d_t$$

where τ_{kt} is the tax rate on investment, R_t is the rental rate for capital We will let $n_t = R_t k_t + \delta_t b_{Bt} - d_t$ denote the right-side of (8) and will refer to it as the *net worth* for the bank. For a bank that is new born at t the budget constraint is the same with the right side of (8) replace by initial net worth $\bar{n}$. This bank faces a *collateral* constraint

$$(9) \quad d_{t+1} \leq \gamma \left[R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1}\right]$$

where $0 < \gamma < 1$ non-negativity constraints on dividends and bond holdings

$$(10) \quad x_t, b_{Bt+1} \geq 0$$

and a *regulatory* constraint

$$(11)\quad b_{Bt+1} \geq \phi_t \left(R_{t+1}k_{t+1} + b_{Bt+1}\right)$$

that requires the bank to hold at least a fraction ϕ_t of its assets in government bonds. Here ϕ_t measures *financial repression*: whenever $\phi_t > 0$ we say that the government is practicing financial repression and that the higher the level that it chooses for ϕ_t the greater the degree of financial repression that it practices.

Since a bank that ceases to operate pays out its accumulated net worth as dividends, we can write the problem of a bank born at t is to maximize

$$(12)\quad \max_{\{k_{s+1}, b_{Bts+1}, d_{s+1}, x_s\}} \sum_{s=t}^{\infty} Q_{t,s}\sigma^{s-t}\left[\sigma x_s + (1-\sigma)n_s\right]$$

subject to (8)–(11)where $n_s = R_s k_s + b_{Bs} - d_s$ for $s > t$ and $n_t = \bar{n}$ where $Q_{s,t}$ is the price of a good at date t in units of a good at date s. In equilibrium, we will clearly have the

$$(13)\quad Q_{t,s} = \beta^{s-t}U_{Cs}/U_{Ct}$$

and hence the discount factor used by the bank is consistent with the rate of return on deposits in that $Q_{t,s} = q_{Dt+1} \times ... \times q_{Ds}$.

A representative firm rents capital at rate R_t from banks and hires L_t units of labor to maximize profits

$$(14)\quad \max_{K_t, L_t} F(K_t, L_t) - R_t K_t - w_t L_t.$$

The budget constraint of the government is

$$(15)\quad G_t + B_t \leq \tau_{lt} w_t L_t + \tau_{kt} K_{t+1} + q_{Bt+1} B_{t+1}$$

where B_{t+1} is bounded by some large positive constant $\bar{B}$. These bounds ensure that it is always feasible to finance any government debt by the present discounted value of tax

revenues from labor and capital. A competitive equilibrium is defined in the standard fashion. Note that the bank's problem is linear in net worth. This linearity implies that we can aggregate the assets and liabilities of banks. Hence, from now on we need only record and focus on the aggregate variables for banks, $B_{Bt+1}, D_{t+1}, K_{t+1}$, and X_t. We then define a competitive allocation by $\{C_t, L_t, K_{t+1}, B_{Bt+1}, B_{t+1}, D_{t+1}\}$. The associated price system is given by $\{w_t, R_t, q_{Bt+1}, q_{Dt+1}\}$ and the associated policies are given by $\{\tau_{lt}, \tau_{kt}, \phi_t, \delta_t\}$.

We turn next to characterizing the set of allocations and prices that can be implemented as a competitive equilibrium.[1] We start by showing that in a competitive equilibrium if a bank's collateral constraint (9) is binding then the bank will hold the minimum government debt required by regulation. Specifically, we have:

Lemma 1. In any equilibrium in which the government is repaying its debts at $t+1$, if the bank's collateral constraint is binding at t then the regulatory constraint binds at t. In particular, absent regulation $b_{Bt+1} = 0$ for all $t \geq 0$.

Proof. Let $\delta_{t+1} = 1$ and suppose, by way of contradiction, the collateral constraint binds (9) binds at t but the bank holds more debt than it is required to by regulation in that $b_{Bt+1} > \phi_t (R_{t+1} k_{t+1} + b_{Bt+1})$. Then consider the following deviation an individual bank: reduce the savings in terms of government bonds b_{Bt+1} by one unit and reduce deposits d_{t+1} by one unit. This deviation is feasible, relaxes the collateral constraint, and hence increases profits. *Q.E.D.*

The intuition for this result is that it is not optimal for the bank to borrow from consumers simply to invest in government bonds. The reason is that since the consumer can directly invest in government bonds, the bank must promise consumers the same rate of return that consumers could receive if they invested directly in these bonds. Hence, the banks makes no profits from such transactions. Moreover, because of the binding collateral constraint, holding one more unit of deposits as bonds necessarily forces the bank to lend less capital to firms and hence reduces profits.

Note that if a bank is paying strictly positive dividends in period t it is indifferent between paying the dividend in that period and reducing dividend and deposits in period t

[1] Throughout we restrict attention to economies in which in all competitive equilibria allocations are bounded and for which $\sum_t Q_{0,t}$ is finite so that the bank's problem is well defined.

and raising dividends in period $t+1$. To see this result consider a reduction in dividends x_t and deposits d_{t+1} in period t by one unit and increase dividends by $1/q_{Dt+1}$ in period $t+1$. This policy is feasible and given that $Q_{t,t+1} = q_{Dt+1}$ leaves the present value of dividends unchanged. This result immmediately implies the following lemma:

Lemma 2. Any competitive equilibrium can be implemented as an equilibrium in which dividends paid by banks conditional on survival is zero.

Next we show that without loss of generality we can restrict attention to competitive equilibria in which households and banks receive the same interest rate on government debt.

Lemma 3. Any competitive equilibrium with prices and policies given by $\{w_t, R_t, q_{Bt+1}, q_{Dt+1}\}$ and $\{\tau_{lt}, \tau_{kt}, \phi_t, \delta_t\}$ can also be supported as a competitive equilibrium with prices and policies given by $\left\{w_t, R_t, q'_{Bt+1}, q_{Dt+1}\right\}$ and $\{\tau_{lt}, \tau'_{kt}, \phi_t, \delta_t\}$ where $q'_{Bt+1} = q_{Dt+1}\delta_{t+1}$ and τ'_{kt} choosen suitably.

Proof. We first show how to choose τ'_{kt} so that it raises the same amount of revenues as did the sum of the revenues from the (possibly) repressed bond prices q_{Bt+1} and the original tax on capital income. To that end, define τ'_{kt} as follows

$$\tau_{kt}K_{t+1} + q_{Bt+1}B_{Bt+1} = \tau'_{kt}K_{t+1} + q_{Dt+1}\delta_{t+1}B_{Bt+1}$$

so that if at the original allocation the bank budget constraint (25) and the government budget constraint (24) hold at the original debt prices and tax rate on capital then they also hold at the new (unrepressed) debt prices and the new tax on capital income. Thus,

$$\tau'_{kt} = \tau_{kt} + (q_{Bt+1} - q_{Dt+1}\delta_{t+1})\,\frac{B_{Bt+1}}{K_{t+1}} \tag{16}$$

We will show that with the altered taxes on capital income and return on government debt the banks optimally choose the same allocation as in the original equilibrium. To do so we first notice that if the interest rate on bonds is lower than that on deposits and the banks hold bonds then the regulatory constraint must bind. That is, $q_{Bt+1} > q_{Dt+1}\delta_{t+1}$ implies

$B_{Bt+1} = \phi_t/(1-\phi_t)R_{t+1}K_{t+1}$ hence from (16)

$$(17)\quad \tau'_{kt} = \tau_{kt} + \frac{\phi_t}{1-\phi_t}\left(q_{Bt+1} - q_{Dt+1}\delta_{t+1}\right)R_{t+1}$$

Consider the bank's first order conditions with respect to d_{t+1} and k_{t+1}

$$(18)\quad [\sigma(1-\sigma) + \sigma\lambda_{t+1}] = \lambda_t - \frac{\mu_t}{q_{Dt+1}}$$

$$(19)\quad \lambda_t(1+\tau_{kt}) - (\mu_t\gamma - \eta_t\phi_t)R_{t+1} = Q_{t,t+1}\left[\sigma(1-\sigma) + \sigma\lambda_{t+1}\right]R_{t+1}$$

where λ_t, μ_t, and η_t are the normalized multipliers on the bank's budget constraint, the collateral constraint, and the regulatory constraint. Substituting these into each other gives

$$(20)\quad \frac{1+\tau_{kt}}{q_{Dt+1}} + \frac{(\mu_t(1-\gamma) + \eta_t\phi_t)R_{t+1}}{\lambda_t q_{Dt+1}} = R_{t+1}$$

Moreover, the first order condition with respect to government debt is

$$(21)\quad \lambda_t = \left[\lambda_t - \frac{\mu_t}{q_{Dt+1}}\right]\frac{\delta_{t+1}q_{Dt+1}}{q_{Bt+1}} + \left[\eta_t(1-\phi_t) + \mu_t\gamma\delta_{t+1}\right]\frac{1}{q_{Bt+1}}.$$

Substituting for η_t from (21) into (20) and simplifying we obtain that

$$(22)\quad R_{t+1} = \frac{1+\tau_{kt}}{q_{Dt+1}} + \frac{1-\phi_t+\phi_t\delta_{t+1}}{1-\phi_t}\frac{\mu_t(1-\gamma)R_{t+1}}{\lambda_t q_{Dt+1}} + \frac{\phi_t}{1-\phi_t}\frac{(q_{Bt+1} - q_{Dt+1}\delta_{t+1})}{q_{Dt+1}}R_{t+1}.$$

In the economy with the altered policy the relevant first order condition is

$$(23)\quad R_{t+1} = \frac{1+\tau'_{kt}}{q_{Dt+1}} + \frac{1-\phi_t+\phi_t\delta_{t+1}}{1-\phi_t}\frac{\mu'_t(1-\gamma)R_{t+1}}{\lambda'_t q_{Dt+1}}$$

Using (17) it is clear that the relevant first order condition in the economy with the altered policy is satisfied with $\mu'_t = \mu_t$ and $\lambda'_t = \lambda_t$. *Q.E.D.*

Lemmas 2 and 3 imply that without loss of generality we can restrict attention to competitive equilibria in which no dividends are paid by surviving banks and the interest rate on debt held by banks and households is the same and equal to the consumers' marginal

rate of substitution. In what follows we restrict attention to such equilibria. It will prove convenient to characterize neccesary and sufficient conditions a competitive equilibrium must satisfy.

Lemma 4. Any allocation and policies are part of a competitive equilibrium if and only if they satisfy the resource constraint (1), and

$$(24)\quad G_t + \delta_t B_t \leq \left(F_{Lt} + \frac{U_{Lt}}{U_{Ct}}\right) L_t + \tau_{kt} K_{t+1} + q_{Bt+1} B_{t+1}$$

$$(25)\quad (1+\tau_{kt})K_{t+1} + q_{Dt+1}B_{Bt+1} - q_{Dt+1}D_{t+1} = \sigma\left(F_{Kt}K_t + \delta_t B_{Bt} - D_t\right) + (1-\sigma)\bar{n}$$

$$(26)\quad D_{t+1} \leq \gamma\left[F_{Kt+1}K_{t+1} + B_{Bt+1}\right]$$

$$(27)\quad F_{Kt+1} \geq \frac{(1+\tau_{kt})U_{Ct}}{\beta U_{Ct+1}}$$

where

$$q_{Dt+1} = \beta \frac{U_{Ct+1}}{U_{Ct}}$$

and if the *capital distortion* constraint (27) holds with strict inequality at time t then the aggregate collateral constraint (26) hold with equality at t.

Proof. We will show that (24)–(27) must hold in any equilibrium. Condition (24) follows by substituting in the labor supply first order conditions into the government budget constraint. Lemma 2 and the bank's budget constraint imply (25) and (26) follows from the bank's collateral constraint. From (23) it follows that that (27) holds and if (27) holds with strict inequality the multiplier on the collateral constraint must be strictly positive so that (26) must hold with equality.

Next consider an allocation and policies that satisfy the properties described in the statement of the lemma. Let the wage and the rental rate of capital be defined by $w_t = F_{Lt}$ and $R_t = F_{Kt}$ so that the firm optimality conditions are satisfied. Use (5) to define τ_{lt} so that the households labor first order condition is satisfied. Let ϕ_t be chosen to satisfy (11) with equality. We are left to show that the bank's first order conditions are satisfied. If (27) holds with strict inequality then the relevant first order condition (23) implies that $\mu_t > 0$ and

so the collateral constraint is binding. Hence the optimal bank policy is determined by its constraints with equality. Such constraints are met with equality for the proposed allocation since the budget constraint (25) is an equality, the collateral constraint (26) holds with equality when (27) is a strict inequality and we defined ϕ_t so that the regulatory constraint holds with equality as well. If the capital distortion constraint (27) is an equality then the bank is indifferent between all the feasible policies at time t and so the proposed allocation is trivially optimal. *Q.E.D.*

At an intuitive level the capital distortion constraint captures the fact that a binding collateral constraint distorts capital over and above the distortion that arises from capital taxation. Hence, the least amount of distortion on capital is from the tax on capital alone and the marginal product of capital can be no lower than the tax adjusted marginal rate of substitution of the consumer.

3. Equilibrium with Commitment: The Ramsey Equilibrium

We turn now to characterizing the best equilibrium under commitment, namely the *Ramsey equilibrium*. This equilibrium is defined as the competitive equilibrium that yields the highest utility for consumers. Our main result is that under commitment, it is not optimal to use financial repression. The key idea is that forcing banks to hold government debt is an inefficient way to raise revenues. As we show, forcing banks to do so tightens the collateral constraint.

The Ramsey problem for this economy is to maximize

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

subject to the resource constraint, the government budget constraint, the aggregate firm budget and collateral, and capital distortion constraints. We then have the following proposition.

Proposition 1. (Financial Repression not Optimal with Commitment) The Ramsey outcome can be implemented with no financial repression, that is, $\phi_t = 0$ for all t. Moreover, if the collateral constraint (26) is binding in some period t then it is strictly optimal not to practice financial repression in that period, in that $\phi_t = 0$ and $B_{Bt+1} = 0$.

Proof. Suppose by way of contradiction that the Ramsey outcome has financial repression in that the government forces the bank to hold some government debt. Specifically, suppose that $\phi_t > 0$ so that $B_{Bt+1} > 0$. Then consider the following variation on the original allocations and policies. Reduce the amount of deposits the bank obtains from the consumer at t by exactly the amount of government debt it holds. That is, set $\tilde{D}_{t+1} = D_{t+1} - B_{Bt+1}$. Let the consumers increase their holdings of government debt by B_{Bt+1} and let the government not require that the bank holds any debt by setting $\tilde{\phi}_t = 0$. Let the rest of the allocations and policies be unchanged, in particular $\{C_t, L_t, K_{t+1}\}$ and $\{\tau_{lt}, \tau_{kt}\}$. We claim that this variation is feasible, relaxes the enforcement constraint, and supports the same allocations. To see that it is feasible note that the consumers total savings is unchanged since $\tilde{B}_{Ht+1} + \tilde{D}_{t+1} = B_{Ht+1} + D_{t+1}$ and, since from Lemma 3 we can let the rate of return on government bonds and deposits be equal, so are all future allocations. Likewise the government budget constraint is unaffected since $\tilde{B}_{Ht+1} + \tilde{B}_{Bt+1} = B_{Ht+1} + B_{Bt+1}$ is unchanged. That this variation is feasible and supports the original allocations proves that the Ramsey outcome can be implemented with no financial repression.

To prove the second part of the proposition, suppose that the bank's collateral constraint is binding at t in that the multiplier on the collateral constraint (26) is positive at t. Then clearly this variation strictly relaxes this constraint. Welfare can then be strictly improved by modifying the deviation to let the bank borrow a bit more from consumers by increasing deposits and using those deposits to increase capital. Such a variation strictly improves welfare. This proves the second part of the proposition. $Q.E.D.$

The proposition says that it is always possible to implement the Ramsey outcome with no financial repression. The intuition for the first part of the proposition is that when the collateral (26) is not binding for all t (say γ is close to 1) then banks are essentially a veil and the economy is equivalent to one in which consumers frictionlessly directly invest in firms. In this case financial repression is a redundant instrument and such repression is equivalent to directly taxing capital. In this case we could use the results from Ramsey literature on the undesirability of distorting capital that builds on the work of Chamley (xxxx) and Judd (xxxx) immediately imply that if $U(C, L) = C^{1-\sigma}/(1-\sigma) - v(L)$ then it not optimal to have financial repression from $t = 1$ onward since it is optimal to have $\tau_{kt} = 0$ from $t \geq 1$

in a standard model in which consumers directly invest in the firm. (See Chari and Kehoe (2xxx)).

The second part of the proposition is the more interesting part. It says that when the collateral constraint of the banks is binding at t then financial repression, in the sense of $\phi_t > 0$, is not just a redundant instrument but rather it is a strictly dominated instrument. Combining Lemma 3 and Proposition 1, it is clear that even though it is feasible for the government to raise revenues through financial repression by setting the interest rate on government bonds below the *market* rate of return ($U_{Ct}/\beta U_{Ct+1}$) and forcing the banks to hold such bonds by setting $\phi_t > 0$, it is inefficient to do so. Such a policy is equivalent to taxing capital income and then imposing an extra distortion on capital accumulation by tightening the banks' collateral constraints. Clearly, it is better to simply directly tax capital income and leave the banks' collateral constraint unaffected.

Notice that here we assume that both τ_{kt} and τ_{lt} are nonnegative. If we allowed the government to subsidize capital then if the distortions from the collateral constraint are sufficiently severe relative to the distortions from taxing labor the government might have an incentive to tax labor and subsidize capital so as to lessen or even remove the distortion from the collateral constraint.

4. Equilibrium Without Commitment

We are primarily interested in environments in which the government cannot commit to its policies. Our key idea is that defaulting on debt held inside the banking system disrupts financial intermediation and hence has ex post costs that are absent when the government defaults on debt held by households. These ex post costs make it optimal for the government to practice financial repression by forcing the banks to hold some of its debt.

To isolate this new idea we begin by modeling the lack of commitment as having the government choose policies in a Markovian fashion. Here the contrast between debt held by banks and debt held by households is particularly stark. If we restrict the government to only issue debt to households then in a Markov equilibrium the government will always default. Hence, the value of any such debt issued by the government is zero and in equilibrium the government has a balanced budget. In contrast, if the government is allowed to issue debt to

banks and force them to hold it, then the government does so in equilibrium. If, in addition, pari passu clauses imply that the government cannot default in a discriminatory fashion on debt held by banks and that held by households, then the government also issues debt to households. We also show that countries with more cyclical patterns of government spending find it optimal to practice higher levels of financial repression.

While we find this Markov equilibrium is helpful in isolating our new force, its predictions are extremely stark: it predicts that governments can issue debt if and only if they practice financial repression. These predictions are so stark because this way of modeling lack of commitment abstracts from the standard reputation reasons for not defaulting on debt. We then turn to our more complete model of lack of commitment in which we add our new force to a standard reputation model of debt. We capture these reputational forces by having focusing on the best sustainable equilibrium. Here even if we restrict the government to issue debt only to households there can still be positive debt in the best sustainable equilibrium: after a default by the government, private agents are convinced that the government will default from now on. Hence, the government finds defaulting costly because it leads them to be stuck in a balanced budget equilibrium. In short, these trigger strategies, interpreted as a loss of reputation, give the government incentives not to default.

When we allow governments to also issue debt to banks we then have the forces evident in the Markov equilibrium along with the standard repuational forces. Here we show that when the initial debt is sufficiently high the government has an incentive to practice financial repression and run down the level of the debt. Moreover, along this path the extent of financial repression starts high and falls over time.

A. Markov Equilibrium

We begin by modeling lack of commitment by having governments choose policies in a Markov fashion. Here if there were no costs from defaulting on government debt issued by a predecessor government, then a Markovian goverment would always do so and, in equilibrium, the government will be forced to follow a balanced budget policy. Our key idea is that in such a situation governments may find it optimal to force banks to hold government debt. The reason is that the government at t may force the bank to hold government debt is that

this government realizes that such government debt will be part of the net worth of the bank at $t+1$. Hence if the successor government at $t+1$ defaults on this debt it wipes out part of this net worth and hence impinges on the ability of the bank to intermediate funds obtained from consumers. By destroying part of the bank's ability to intermediate funds, defaulting on debt negatively affects capital accumulation and hence spills over to production side of the economy–leading to lower investment and output. Defaulting on debt, of course, also have some positive effects: by defaulting the government diminishes its need to raise distortionary labor and capital taxes. Overall, the successor government then balances these negative effects from disrupting financial intermediation against the positive effects of lowering tax distortions. As long as the negative effects are strong enough the successor government will not default.

In sum, even though financial repression is a dominated instrument under commitment, without commitment forcing banks to hold government debt may be optimal because by making it very costly to default on debt ex post, it gives assurance to private agents that they can lend to the government without fear of default. In this sense, financial repression is an endogenous way for governments to increase their credibility in servicing their debts.

To make our points in the simplest possible manner we assume that government spending deterministically fluctuates between high and low levels over time. In particular, we assume that in odd periods $G_t = G_H$ and in even periods $G_t = G_L$. These fluctuations give the government an incentive to smooth taxes by selling debt from periods with high government spending into periods of low government spending. Relative to a balanced budget allocation, selling debt in this manner allows the government to smooth tax distortions. Indeed, doing so lets the government lower the taxes in the high spending periods in which distortions are high and to raise taxes in the low spending periods in which distortions are low. We will make assumptions that guarantee that such a tax smoothing outcome occurs and that there is a steady state in which the economy follows a two period cycle.

Consider now the states. In general, because banks experience death shocks in an idiosyncratic fashion we would need to record the distribution of net worth across banks as part of the aggregate state. As we have discussed, however, bank's decisions are linear in net worth. Given this result we define the endogenous aggregate state $S = (K, D, B_B, B_H, G)$

confronting the government to record the aggregate capital stock, deposits, bonds held by banks, and bonds held by the representative household along with the exogenous aggregate state $G\,G$. A policy rule for the government is a set of functions for policies that depend on the aggregate state which we denote as $\pi(S) = (\delta, \phi, \tau_l, \tau_k, B'_B, B'_H)(S)$. The state confronting households and banks and determining prices also includes the current policies π and can be written $S_H = (S, \pi)$. In particular, let $q_D(S_H)$, $R(S_H)$, $w(S_H)$ denote the price for government bonds and deposits, the rental rate on capital, and the wage. Finally, let $S' = S'(S)$ denote the law of motion for the aggregate state.

Primal Markov Problem

Under these assumptions given some policy rule $\pi(S)$ that it will be followed in the future and future allocations and pricing rules, in light of Lemma 4 we can write the problem for the government in a Markov equilibrium in a primal form as choosing the policies δ, τ_k, τ_l and the allocations C, L, K', D', B'_B, B'_H to solve

$$\text{(M)} \quad V_M(S) = \max U(C, L) + \beta V_M(S')$$

subject to the resource constraint and

$$\text{(28)} \quad G + \delta B \leq \left(F_L + \frac{U_L}{U_C}\right) L + \tau_k K' + q_D(S_H) B'$$

$$\text{(29)} \quad (1 + \tau_k) K' + q_D \delta(S') B'_B - q_D D' = \sigma\left(F_K K + \delta B_B - D\right) + (1 - \sigma)\bar{n}$$

$$\text{(30)} \quad D' \leq \gamma\left[R(S'_H) K' + \delta(S') B'_B\right]$$

$$\text{(31)} \quad R(S'_H) \geq \frac{(1 + \tau_k)}{q_D},$$

appropriate nonnegativity constraints and upper bounds on government debt where $q_D = \beta U_C(S'_H)/U_C$, $R(S'_H) = F_K(S'_H)$, and $S'_H = (S', \pi(S'))$.

Next, note that we can rewrite this problem by setting $\delta(S') = 1$ and imposing a *no default* constraint of the form

$$(32)\quad V_M^{nd}(S') \geq V_M(K', D', 0, 0, G')$$

where $V_M^{nd}(S)$ is a constrained version of the problem defined in (M) in which we impose $\delta = 1$. This constraint ensures that the government in the current period does not choose a plan that will induce the government in the next period to default. Clearly, optimality on the part of the government in the current period implies that it will do so. The reason is that if it did choose a plan that induced default tomorrow then it would not be able to sell debt at a positive price today. That option is equivalent to choosing the quantity of debt sold today to be zero.

To obtain a sharp characterization of the Markov equilibrium we assume the utility function is quasi-linear so that $U(C, L) = C - v(L)$ and the production function is additively separable in that $F(K, L) = \omega_K K + \omega_L L$. These assumptions eliminate all the cross-partial terms and ensure simple expressions for prices. We also assume that

$$1 < \beta\omega_K < 1/\gamma$$

where the first inequality guarantees that investment is worthwhile in that the gross return on investment exceeds the discount rate and the second inequality guarantees that the collateral constraint is nontrivial. Finally, we assume that the survival rate σ and the initial net worth $\bar{n}$ are both low enough that the collateral constraint of banks always bind. Under these four assumptions the interest rate on deposits will be $q_D = \beta$. We assume these assumptions for the rest of the paper and simply refer to them as *our assumptions*.

Characterization of the Markov Equilibrium

We turn now to a detailed characterization of the Markov equilibrium using a guess and verify technique. This characterization will prove useful in two regards. First, it helps give intuition for how our new force works. Second, we will draw heavily on this characterization when we turn to characterizing the best sustainable equilibrium.

We find it convenient to transform variables and express the capital distortion con-

straint in a slightly different form. To do so first note that under our separability assumptions the capital distortion constraint can be written as as $\omega_K \geq (1+\tau_{kt})/\beta$ which when multiplied by K' gives $\omega_K K' \geq (1+\tau_{kt})K'/\beta\omega_K$. Letting $T_K = \tau_k K'$ we can write this transformed capital distortion constraint as $T_K \leq (\beta\omega_K - 1)K'$. The nonnegativity constraint on τ_k also implies that $T_K \geq 0$.

To minimize the notation, it is convenient to have the government directly choose the revenues from the labor tax T_L rather than labor L itself. To makes this transformation note that labor is determined by the consumer's first order condition $v'(L) = (1-\tau_l)\omega_L$ so that labor supply depends only on the tax rate on labor. The tax revenue from labor T_L is given by $\tau_l \omega_L L = (\omega_L - v'(L))L$. Thus, we can let the associated labor supply $\ell(T_L)$ associated with labor tax revenues T_L be implicitly defined by the solution to the equation

$$[\omega_L - v'(\ell(T_L))]\,\ell(T_L) = T_L.$$

where if multiple solutions exist we select the solution in which tax revenues are increasing in tax rates (namely, the solution on the good side of the Laffer curve). We can then let

$$W(T_L) = \omega_L \ell(T_L) - v(\ell(T_L)) \tag{33}$$

denote the *net utility from labor*, namely the part of current output that is produced by labor minus the disutility of labor. From now on we will assume that W is strictly concave, a sufficient condition for which is that the disutility of labor takes the isoelastic form $v(L) = L^{1+\eta}/(1+\eta)$ for $\eta > 0$.

Next, note that current utility can be written

$$C - v(L) = \omega_K K - G - K' + W(T_L)$$

where we have used the resource constraint implies $C = \omega_K K + \omega_L L - G - K'$. Hence, the Markov primal problem can be rewritten as

$$V_M(S) = \max_{T_L, T_K, K', B'_B, B'_H} \omega_K K - G + W(T_L) - K' + \beta V_M(S') \tag{34}$$

subject to the no default constraint (32), the nonnegativity constraints, upper bounds on government debt and

$$(35)\quad T_L + T_K + \beta(B'_B + B'_H) = G + B$$

$$(36)\quad (1 - \beta\gamma\omega_K)\, K' + T_K + \beta(1-\gamma)B'_B = \sigma N + (1-\sigma)\bar{n}$$

$$(37)\quad T_K \leq (\beta\omega_K - 1)K'$$

where we have used the binding collateral constraint to substitute out for new deposits $D' = \gamma(\omega_K K' + B'_B)$. both in (36) and in S'.

Next, under our assumptions if the capital distortion constraint is not binding, a natural conjecture is that the value function is linear in the capital stock and net worth and is separable from a function which captures the distortions due to labor and capital taxation. We assume that the capital distortion constraint is not binding and later verify that it is not. We thus conjecture and verify that that there exists a Markov equilibrium whose value can be written as

$$(38)\quad V_M(S) = \omega_K K + A_R + A_N N + \max\left\{H_M(B,G), H_M(0,G) - A_N B_B\right\}$$

where the *tax distortion* function H_M satisfies the Bellman equation

$$(39)\quad H_M(B,G) = \max_{B'_B, B', T_K, T_L} W(T_L) - \frac{A_N}{\sigma}T_K - A_B B'_B + \beta H_M(B', G')$$

subject to (35), and

$$(40)\quad A_N B'_B \geq H_M\left(0, G'\right) - H_M\left(B', G'\right)$$

where A_B, A_N,and A_R are some constants given in the Appendix that, importantly, do not depend on the level of government spending G. Note that (40) is simply a rewritten version of (32) under our conjectured value function. To see that note that under our conjecture (32)

is

$$(41)\quad \omega_K K' + A + A_N N' + H_M\left(B'_B + B'_H, G'\right) \geq \omega_K K' + A + A_N(N' - B'_B) + H_M\left(0, G'\right)$$

where $N' = (1-\gamma)\left(\omega_K K' + B'_B\right)$ which when simplified gives (40). Finally, given any solution to this problem the optimal K' is then determined from the bank's budget constraint (36).

Lemma 5. There exists a Markov equilibrium with a value given by (38).

Proof. First, we show that (39) has a solution. Let $\mathbb{T}$ be the operator defined by the right hand side of (39). Let X be the space of continuous, bounded and concave functions h defined over a compact subset of (B, G). Note that this operator maps this space into itself, it is continuous, and that the family $\mathbb{T}(X)$ is equicontinuous. To see the last property, note that for all h in X, and for all $B_2 > B_1$ we have that

$$\begin{aligned} ||\left(\mathbb{T}h\right)(B_2) - \left(\mathbb{T}h\right)(B_1)|| &\leq ||\left(\mathbb{T}h\right)(B_1) - \frac{A_N}{\sigma}(B_2 - B_1) - \left(\mathbb{T}h\right)(B_1)|| \\ &= \frac{A_N}{\sigma}||B_2 - B_1|| \end{aligned}$$

since a feasible solution at B_2 is to repay the additional debt by choosing the policies that are optimal for B_1 and just increasing capital taxes by $B_2 - B_1$. Then every $\mathbb{T}h$ in $\mathbb{T}(X)$ is Lipschitz with Lipschitz constant A_N/σ common to all elements of $\mathbb{T}(X)$. We can then apply a version of Schauder fixed point theorem (Theorem 17.4 in SLP) to conclude that $\mathbb{T}$ has a fixed point.

Next, subsituting this fixed point H_M into the the original problem (34) we can verify the conjecture and calculate the constants. *Q.E.D.*

Before we formally develop the characterization of the equilibrium, it is useful to think intuitively about the problem in (39). Since we have assumed that the net utility from labor W is concave, the cost of labor taxes defined as $W(0) - W(T_L)$ is convex in the level of those revenues T_L while the cost of capital taxes $A_N T_K/\sigma$ is linear in those revenues T_K. Thus, letting $T = T_L + T_K$ denote the total revenues raised and let $\bar{T}_L$ be the level of labor tax revenues such that the marginal cost of labor taxes equals the marginal cost of capital taxes

in that

$$(42)\quad -W'(T_{Lt}) = \frac{A_N}{\sigma}$$

we have that if $T \leq \bar{T}_L$ then all taxes are raised by labor taxes and if $T > \bar{T}_L$ then labor tax revenues are at $\bar{T}_L$ and the rest of the tax revenues are raised by capital taxes. In short, only labor taxes are used to finance revenues up to some maximal amount of revenues then, on the margin, capital taxes are used to finance any additional desired revenues. It is easy to show that if the initial debt B_0 and the level of government spending are not too high then it is never optimal to use capital income taxes. In the body of the paper we will assume this is true, that is

$$(43)\quad T_{Kt} = 0 \text{ for all } t$$

and in the appendix we show how the proofs can be extended to the general case.

Next, we turn to the dynamics of labor tax revenue. Suppose for a moment that we drop the no default constraint. The resulting problem is then simply a recursive formulation of the Ramsey problem. Then taking the first order condition B_{t+1} is (using sequential notation for simplicity)

$$(44)\quad W'(T_{Lt}) = W'(T_{Lt+1})$$

so that complete tax smoothing is optimal, in that T_{Lt} is a constant.

In the Markov equilibrium the no default constraint forces the government to bear an extra cost from debt and gives the government an incentive to front-load taxes relative to the tax smoothing outcome whenever fiscal needs, measured by $B_t + G_t$, are high. To understand the dynamics for labor tax revenues use the no-default constraint (40) to substitute for B'_B in the objective function, use envelope condition $H'_M(B, G) = W'(T_L)$, so that the first order condition for an interior B_{t+1} is

$$(45)\quad -\beta W'(T_{Lt}) + (\beta + A_B/A_N)\, W'(T_{Lt+1}) = 0$$

Since W' is negative and A_B and A_N are positive, it follows that whenever $B_{t+1} > 0$ we have that

$$(46) \quad -W'(T_{Lt}) > -W'(T_{Lt+1})$$

Clearly the size of the term A_B/A_N measures the deviation from the optimal tax smoothing in the Ramsey plan. To interpret this term consider (45) and note from inspection of (39) that A_B measures the cost of forcing the bank to hold one more unit of debt. This cost arises because to hold one more unit of debt the bank has to raise one more unit of deposits which tightens the collateral constraint and crowds out investment. To interpret A_N inspect the no default constraint (41) from which (40) was derived. Here the value of defaulting on the debt B_{t+1}, $H_M(0, G_{t+1}) - H_M(B_{t+1}, G_{t+1})$, is just balanced with the cost of defaulting on this debt $A_N B_{Bt+1}$ This cost of defaulting arises because the default lowers the net worth of the bank by B_{Bt+1} and distorts investment at $t+1$ at the utility cost of $A_N B_{Bt+1}$.

Thus, when the government issues one extra unit of total debt B_{t+1}, to keep it indifferent between defaulting and not, it must force the banks to hold just enough extra debt ΔB_{Bt+1} so that these costs and benefits balance in that $A_N \Delta B_{Bt+1} = -H'_M(B_{t+1}, G_{t+1})$. Since each additional unit of debt held by the bank has an extra utility cost of A_B we have that the extra utility cost of issuing a unit of total debt is

$$A_B \Delta B_{Bt+1} = -\frac{A_B}{A_N} H'_M(B_{t+1}, G_{t+1})$$

which by the envelope theorem equals $-A_B W'(T_{Lt+1})/A_N$.

This extra cost of issuing debt causes the Markov policy to deviate from the complete tax smoothing outcome of the Ramsey policy. Since W is concave, as long at the government is issuing debt in that $B_{t+1} > 0$, the optimality condition (46) implies that the tax revenues from labor are strictly decreasing over time. Briefly, in the Markov plan the government has an incentive to roll over less of the debt in each period than in does under the Ramsey plan because this debt generates extra costs.

Now, to figure out what such a path for taxes implies for the level of government debt iterate the government budget constraint forward to write that debt is the discounted value of

future government surpluses, which using the cyclical pattern of government spending gives

$$(47)\quad B_t = \sum_{s=0}^{\infty} \beta^s T_{Lt+s} - \frac{G_t + \beta G_{t+1}}{1-\beta^2}$$

In the next proposition we will show that debt decreases along an optimal path and the economy settles into a two period cycle in which no debt is issued when government spending is low. For debt to be issued in this cycle when government spending is high we need that the spending levels G_L and G_H are sufficiently different in that

$$(48)\quad -W'(G_H) > -W'(G_L)(1 + \frac{A_B}{\beta A_N})$$

which guarantees that the benefits of tax smoothing by issuing debt in the high state and paying it off in the low state outweigh the distortions discussed above from issuing such debt. Clearly, this condition is always satisfied if $G_L = 0$ because $W'(0) = 0$ and $-W'(G_H) > 0$. If the reverse inequality holds in (48) the economy converges to a steady state cycle in which the budget is always balanced.

Proposition 2. If W is strictly concave and (48) holds then the outcome path associated with the Markov equilibrium in (38) is such that $\{B_{2t}\}$ and $\{B_{2t+1}\}$ are decreasing. Moreover, the share of debt held by banks $r_t = B_{Bt}/B_t$ is such that $\{r_{2t}\}$ and $\{r_{2t+1}\}$ are decreasing. In the long run, independently of the level of initial debt, the economy converges to a unique steady state cyclical pattern in which no debt is issued into the high spending state and there is financial repression in the high spending state.

Proof. We start by showing that there is some period t in which $B_t = 0$ and then show that this implies that the economy converges to a two period cycle. To do so suppose by way of contradiction that $B_t > 0$ for all t. Then, (46) holds for all t and concavity of W implies that $\{T_{Lt}\}$ is decreasing. Then $\{T_{Lt}\}$ must be converging to a limit. Clearly the limit cannot be strictly positive otherwise by continuity of W' we have that at the limiting value $T_{L\infty}$, $-W'(T_{L\infty}) > -W'(T_{L\infty})$, which is a contradiction. Then it must be that T_{Lt} for some t sufficiently large, T_{Lt} is arbitrarily close to zero, so that $\sum_{s=0}^{\infty} \beta^t T_{Lt+s}$ is also arbitrarily close to 0. But then (47) implies that for sufficiently large t, $B_t = \varepsilon - \frac{G_t + \beta G_{t+1}}{1-\beta^2} < 0$ where ε is

some small value which contradicts the hypothesis that $B_t > 0$ for all t. Then it must be that there exists some finite t such that $B_t = 0$.

To see that the debt and hence the rest of the economy converges to a two-period cycle, once $B_t = 0$, note that if $G_t = G_H$ and the inherited debt at t, namely $B_t = 0$ then at $t+1$ with $G_t = 0$ it is optimal to issue $B_{t+2} = 0$. Suppose by way of contradiction that $B_{t+2} > 0$ then using the envelope condition at $t+1$ and the first order condition (45) at $t+1$ gives that

$$H'_M(B_{t+1}, G_L) = W'(B_{t+1} - \beta B_{t+2}) = \left(1 + \frac{A_B}{A_N \beta}\right) H'_M(B_{t+2}, G_H) < \left(1 + \frac{A_B}{A_N \beta}\right) H'_M(0, G_H)$$

where the inequality follows from the concavity of H_M. Likewise, the envelope condition at t and the first order condition at t imply that

$$H'_M(0, G_H) = W'(G_H - \beta B_{t+1}) = \left(1 + \frac{A_B}{A_N \beta}\right) H'_M(B_{t+1}, 0)$$

Combining these two equations gives that

$$H'_M(0, G_H) < \left(1 + \frac{A_B}{A_N \beta}\right)^2 H'_M(0, G_H)$$

which is a contradiction since $1 + A_B/(A_N\beta) > 1$. Thus, eventually the economy follows a two period cycles in the long run. It is easy also show that under (48) in the two period cycle the government issues a positive amount of debt in the low state.

We now turn to showing that debt in even periods, denoted $\{B_{2t}\}$ and debt in odd periods $\{B_{2t+1}\}$ are decreasing sequences. Notice that before the economy reaches the steady state, (46) and concavity of W implies that $\{T_{Lt}\}$ is decreasing. To see that $\{T_{Lt}\}$ is decreasing implies that these debt sequences are decreasing follows immediately since in periods t and $t+2$ the terms in government spending in (47) are the same while the present values of tax revenues are decreasing.

We now show that the fact that since the debt sequences are decreasing implies that the ratios are decreasing implies that $\{B_{B2t}/B_{2t}\}$ and $\{B_{B2t+1}/B_{2t+1}\}$ are also decreasing, where we are only referring to periods before the limit in which in the low spending state is

realized both B_{Bt+1} and B_{t+1} are zero. Since total debt $\{B_{2t}\}$ is decreasing, the no-default constraint

$$B_{Bt} = \frac{H_M(0, G_t) - H_M(B_t, G_t)}{A_N}$$

implies that the debt held in the banks $\{B_{B2t}\}$ is also decreasing since H_M is decreasing in B_t. Moreover, the share of debt held by banks $r_t = B_{Bt}/B_t$ is such that $\{r_{2t}\}$ and $\{r_{2t+1}\}$ are decreasing since

$$(49) \quad r_t = \frac{B_{Bt}}{B_t} = \frac{H_M(0, G_t) - H_M(B_t, G_t)}{A_N B_t}$$

is increasing in B_t and $\{B_{2t}\}$ is decreasing over time. That r_t is increasing in B_t immediately follows from the concavity of H_M. To see this recall that for any concave function $f(x)$, the function $(f(x) - f(0))/x$ is decreasing in x. *Q.E.D.*

Here we show that under our assumptions economies with relatively more cyclical patterns of spending have relatively more severe financial repression. Specifically, we consider two economies, a less cyclical economy that alternates between G_H and G_L and a more cyclical economy that alternates between G'_H and G'_L with $G'_H \geq G_H$ and $G'_L \leq G_L$ with at least one of the inequalities is strict. We show that in the steady state of the more cyclical economy the share of total debt that is held in banks is higher and that the resulting degree of financial repression as measured by ϕ is also higher.

Proposition 3. If W is strictly concave then in the limiting steady state financial repression is more severe in more cyclical economies in that in states with $G_t = G_H$, the degree of repression ϕ and the ratio B_B/B is increasing in G_H.

Proof. Now in a steady state in the high state the inherited debt is 0 and in the low state the tax revenues from capital are zero. Using these features we can write the first order conditions for B' in (39) starting from the high state as

$$(50) \quad -\beta W'(G_H - \beta B') + \left(\beta + \frac{A_B}{A_N}\right) W'(B' + G_L) = 0$$

where we have used the envelope condition in the low state implies that $H_M(B, G_L) =$

$W'(B+G_L)$. Since W' is a strictly decreasing function it follows that the debt issued in the high state is increasing in G_H and decreasing in G_L.

Next, we show that the ratio B_B/B is increasing in G_H and decreasing in G_L. Since r_t defined in (49) is increasing in B and $B = B(G_H)$ is increasing in G_H it follows that r is increasing in G_H. A similar argument shows that r is decreasing in G_L. *Q.E.D.*

B. Best Sustainable Equilibrium

In a Markov equilibrium there are no costs of defaulting on debt besides those that arise from the government practicing financial repression and forcing the banks to hold debt. Hence, in such an equilibrium positive level of debt can be sustained only if there is financial repression. We now turn to consider an environment where some positive amount of government debt can be sustained using reputational argument. As in classic Eaton-Gersovitz model, a default by the government triggers a reversion to the Markov equilibrium we characterized above.

Setup

We consider outcomes that can be supported by reversion to the Markov equilibrium characterized in the previous section. An outcome is sustainable with reversion to Markov if and only if it is a competitive equilibrium outcome and in all periods it safisfies the following constraint

$$(51)\quad \sum_{s=0}^{\infty} \beta^s \left[C_{t+s} - v(L_{t+s})\right] \geq V_M\left(K_t, D_t, 0, 0, G_t\right) \text{ for all } t \geq 1$$

that requires that the government weakly prefers to follow the plan rather than defaulting on its debt and follow the Markov equilibrium thereafter. Recall that $V_M\left(K_t, D_t, 0, 0, G_t\right)$, given in (38), is the value of Markov equilibrium with inherited capital K_t, deposits D_t and with no outstanding government debt, $B_B = B_H = 0$ with value given by

The best sustainable problem for a given sequence of government expenditure $\{G_t\}$ is the same as the Ramsey problem with the addition of the constraint (51). We now show that the best sustainable equilibrium with reversion to the Markov equilibrium can be expressed

as

$$(52)\quad V(S,G) = \omega_K K + A_R + A_N N + H(B,G)$$

where H the largest fixed point of a Bellman equation defined by

$$(53)\quad H(B,G) = \max_{B'_B, B', T_K, T_L} W(T_L) - \frac{A_N}{\sigma} T_K - A_B B'_B + \beta H(B', G')$$

subject to

$$G + B \leq T_L + T_K + \beta B'$$

and

$$(54)\quad H(B', G') + A_N B'_B \geq H_M(0, G')$$

where $H_M(0, G')$ is the value of the Markov programming problem (39) when the inherited debt is zero. Note that the constraint (54) referred to as the *sustainability constraint* is a simplified recursive version of (51) To see this note that this constraint follows from substituting for (52) and (38) into (51) and cancelling terms.

Lemma 6. The best sustainable equilibrium has a value given by (52).

Proof. The best sustainable value can be written recursively as

$$(55)\quad V(S,G) = \max_{T_L, T_K, K', D', B'_B, B'_H} [\omega_K K - K' + W(T_L) - G] + \beta V(S', G')$$

subject to

$$(56)\quad T_L + T_K + \beta(B'_B + B'_H) = G + B - \beta(B'_B + B'_H)$$

$$(57)\quad (1 - \beta\gamma\omega_K) K' + T_K + \beta(1-\gamma) B'_B = \sigma N + (1-\sigma)\bar{n}$$

$$(58)\quad V(S', G') \geq V_M(K', D', 0, 0, G')$$

where $D' = \gamma \left[\omega_K K' + B'_B\right]$ together with the nonnegativity constraints on B'_B, B'_H, and T_K and we have ignored the upper bound on T_K. The right hand side of (55) defines an operator, $\mathbb{T}$. Given the presence of the constraint (58), this operator $\mathbb{T}$ is not a contraction mapping but using a logic similar to Abreu, Pearce, and Stacchetti (1990) it is clear that if one start with an initial value function $V_0(S, G)$ which is pointwise larger than $V(S, G)$ and then use the operator $\mathbb{T}$ to construct a sequence of functions $V_n = \mathbb{T}^n V_0$ that converge to V in the relevant norm. In the Appendix we show that the Ramsey problem has the form

$$V_R = \omega_K K + A_R + A_N N + H_R(B, G)$$

where

$$H_R(B, G) = \frac{1}{1-\beta} W\left((1-\beta)B + \frac{G + \beta G'}{1+\beta}\right)$$

so that $H_R(B, G)$ is a concave function of B. Clearly, since the Ramsey problem solves a less-constrained version of the best sustainable problem V_R is pointwise larger than V and we can use V_R as the initial value function V_0. We need to show that the sequence of constructed functions V_n have the form $V_n = \omega_K K + A_R + A_N N + H_n(B, G)$ for some sequence of concave functions $H_n(B, G)$. To do so substitute our guess for V_n in (55) and inspect the resulting problem to conclude that the optimal B'_B, B'_H, and T_K are independent of K, D and the optimal K' is linear in $\sigma N + (1-\sigma)\bar{n}$. Using these two properties, we have that

$$\mathbb{T}V_n(S, G) = \omega_K K + A_R + A_N N + H_{n+1}(B, G) \tag{59}$$

where nonnegative B'_B, B'_H, and T_K are chosen to solve

$$H_{n+1}(B, G) = \max_{B'_B, B'_H, T_K} W(T_L) - \frac{A_N}{\sigma} T_K - A_B B'_B + H_n(B'_B + B'_H, G'),$$

subject to

$$T_L = G_t + B - T_K - \beta\left(B'_B + B'_H\right)$$

$$A_N B'_B + H_n(B, G') \geq H_M(0, G')$$

Concavity of H_{n+1}follows from concavity of H_n and W. The result in (59) implies that in the limit

$$V(K, N, B, G) = \omega_K K + A_R + A_N N + H(B, G)$$

where H, defined as the limit of H_n, is also concave. $Q.E.D.$

Note differences from Phelan and Stacchetti (200x) do not need to record marginal utilities or price like term because our assumptions about linear production and quasi-linear utility function.

Characterization of Outcome Path

In the Markov equilibrium if we start the economy in the high state with a positive amount of inherited debt, over time the inherited debt in subsequent high states will decrease and eventually converge to zero. So in the limiting behavior of the Markov equilibrium is a cycle with permanent financial repression, in that in every high state the government forces banks to hold debt.

If the economy starts with a high level of debt, in the best sustainable equilibrium debt is also decreasing over time. A key difference with the Markov equilibrium is what can happen in the limit. We focus on the most interesting case in which discount factor is fairly large. Here if the initial debt is sufficiently high, so that the Ramsey equilibrium is not sustainable, the outcomes are as follows. The government practices financial repression and runs down the debt until the debt is low enough that the sustainability constraint stops binding. At that point it curtails financial repression and Ramsey tax smoothing outcomes from then on are followed. In this sense, the model predicts qualititatively the pattern suggested by Reinhart and Sbrancia (2013): the government had high debt following WWII and practiced a long period of financial repression until fairly recently after which there was a period of little financial repression.

Our model also predicts that even if initial debt is low enough so that at normal levels of government spending there will be no financial repression, a sufficiently high unexpected

increase in government spending will lead the government to begin practicing financial repression. After unexpected increase dies out the economy will then slowly run down its debt to a point at which it again stops practicing financial repression. We find these predictions useful about thinking about some patterns in Europe .....

In order to characterize the dynamic path to a limiting cycle we need to ascertain what limiting cycles are sustainable. To this end define $\hat{B}_{LH}$ as the maximal debt that can be sold from a low state to a high state such that the unconstrained Ramsey plan is sustainable in that it satisfies

$$(60) \quad \frac{1}{1-\beta} W\left((1-\beta)\hat{B}_{LH} + \frac{\beta G_L + G_H}{1+\beta}\right) = W(G_L) + \beta H_M(0, G_H).$$

We denote the associated debt sold from high state into a low state as $\hat{B}_{HL} = \hat{B}_{LH} + (G_H - G_L)/(1+\beta)$. Here by unconstrained Ramsey plan we mean a plan in which there are no nonegativity constraints on debt. In general this plan may imply that $\hat{B}_{LH}$ is negative and hence is infeasible.

Recall that $G_t = G_H$ for $t = 0, 2, ...$ and $G_t = G_L$ for $t = 1, 3, ...$ We have the following proposition:

Proposition 4. Suppose $B_0 \geq \max\left\{\hat{B}_{LH}, 0\right\}$. Then the optimal path for debt is such that $B_{t+2} \leq B_t$ for all $t \geq 1$ so that government debt is decreasing over two period cycles. Moreover, the share of debt held by banks $r_t = B_{Bt}/B_t$ is such that $r_{t+2} \leq r_t$ for all t so that financial repression is decreasing over two period cycles. Moreover, if $\hat{B}_{LH} \geq 0$ the government practices financial repression up until some finite period T after which financial repression stops.

Proof. Letting μ_t denote the multiplier on the sustainability constraint (54) the first order conditions for the problem (53) if B_{t+1} is interior are

$$(61) \quad -\beta W'(T_{Lt}) = -\left(\beta + \mu_t\right) H'(B_{t+1}, G_{t+1}) = -\left(\beta + \mu_t\right) W'(T_{Lt+1})$$

and

$$(62) \quad -A_B + \mu_t A_N \leq 0$$

with strict inequality whenever the sustainability constraint is binding at $t+1$ in that $\mu_t > 0$.

Suppose $\{B_t\}$ is bounded away from zero for all t. from (61) $\{T_{Lt}\}$ is decreasing. The government budget constraint (47) immediately implies that $B_{t+2} \leq B_t$, with strict inequality if μ_t or μ_{t+1} strictly positive.

If $\hat{B}_{LH}$ is strictly positive then $B_t > 0$ for all t. We prove this result by showing that if $G_t = G_H$ and $B_t \geq \hat{B}_{LH}$ then $B_{t+1} \geq \hat{B}_{HL}$. Suppose by way of contradiction that $B_{t+1} < \hat{B}_{HL}$. Since $\hat{B}_{LH}$ is positive the sustainability constraint is slack at t and so (61) implies that

$$(63) \quad -W'\left(G_H + B_t - \beta B_{t+1}\right) = -H'(B_{t+1}, G_L)$$

Moreover, since

$$(64) \quad -W\left(G_H + \hat{B}_{LH} - \beta \hat{B}_{HL}\right) = -H'(\hat{B}_{HL}, G_L).$$

Using concavity of W the assumption that $B_t \geq \hat{B}_{LH}$ and the contradiction hypothesis that $B_{t+1} < \hat{B}_{HL}$ we obtain that the left side of (63) is greater than the left side of (64) and the right side of (63) is less than the righ side of (64) which gives a contradiction. Thus in all even periods B_t is strictly positive. A similar argument establishes that in all odd periods B_t is strictly positive.

Suppose next that $\hat{B}_{LH}$ is negative. We already shown that if B_t is bounded away from zero for all t then B_t is decreasing over the cycle. Let T denote the first date at which $B_T = 0$ and suppose that T is even. Then using an argument identical to the proof of Proposition 2 it is easy to show that the equilibrium follows a two period cycles in which $B_{t+2} = B_t$ for all $t \geq T$. It only remains to show that $B_{T+1} \leq B_{T-1}$. To establish this result notice that the first order condition with respect to debt is

$$(65) \quad -\beta W'(G_t + B_t - \beta B_{t+1}) \leq -\left(\beta + \mu_t\right) H'(B_{t+1}, G_{t+1})$$

with equality if B_{t+1} is strictly positive. Notice that the left side of (65) is increasing in B_t and decreasing in B_{t+1} and the right side is increasing in B_{t+1}. Thus an increase in B_t necessarily

requires that B_{t+1} must increase (weakly). Since T is the first date at which $B_T = 0$ we have that $B_{T-2} > B_T$ and it follows that $B_{T-1} \geq B_{T+1}$. These results establish that debt must be decreasing.

We now show that since the debt sequences are decreasing, repression must also be decreasing. From the sustainability constraint the share of debt held by banks $r_t = B_{Bt}/B_t$ satisfies

$$r_t = \frac{B_{Bt}}{B_t} = \max\left\{\frac{H_M(0,G_t) - H(B_t,G_t)}{A_N B_t}, 0\right\}.$$

Consider the interesting case in which r_t is strictly positive. Differentiating the expression for r_t with respect to B_t we have that

$$\begin{aligned}\frac{\partial r_t}{\partial B_t} &= \frac{-\left[H_M(0,G_t) - H(B_t,G_t)\right] - H'(B_t,G_t)B_t}{B_t^2} \\ &= \frac{\left[H(0,G_t) - H_M(0,G_t)\right] + \left[H(B_t,G_t) - H(0,G_t)\right] - H'(B_t,G_t)B_t}{B_t^2}.\end{aligned}$$

Clearly $H(0,G_t) > H_M(0,G_t)$. Concavity of H implies that $[H(B_t,G_t) - H(0,G_t)] - H'(B_t,G_t)B_t > 0$. It follows that financial repression is increasing in the debt level. Since we have shown that the debt level decreases over the cycle, it follows that repression decreases over the cycle as well.

Finally, to show that if $\hat{B}_{LH} > 0$ financial repression stops in finite time note that since $\{T_{Lt}\}$ converges to a positive level from (62) $\{\mu_t\}$ must converge to zero. Thus there is a finite T such that for $t \geq T$, $\mu_t < A_B/A_N$ and there is no financial repression. *Q.E.D.*

Remark: Proposition 4 allows for the possibility that in the best sustainable equilibrium it may be optimal to default in period zero. If it is optimal not to default then this proposition applies for all $t \geq 0$.

Response to an Unanticipated Shock

We now consider the response to a one time temporary unanticipated change in the level of spending in period 0. We want to show that for a large enough fiscal increase the government necessarily practices financial repression in the initial period. We suppose that in period $t = 0$ government spending G_0 is drawn from some distribution. For all $t \geq 1$

government spending is back to simple cyclical pattern: $G_t = G_L$ if t odd and $G_t = G_H$ if t even. The next proposition shows that it is optimal to have financial repression in the first period only if G_0 is above a critical value G^*.

For simplicity we assume that initial debt in period zero is zero. To define this critical value G^*, consider the value of G_0 such that the government is just indifferent whether or not to practice financial repression. In this case with no financial repression the level of debt B_1^* is pinned down by the sustainability constraint (54), in that

$$(66) \quad H\left(B_1^*, G_L\right) = H_M(0, G_L).$$

The first order condition with respect to debt is given by

$$(67) \quad -W'(G^* - T_K^* - \beta B_1^*) + \left(\beta + \frac{A_B}{A_N}\right) H'(B_1^*, 0) = 0$$

where T_K^* is given by

$$(68) \quad T_K^* = \begin{cases} 0 & \text{if } -\left(\beta + \frac{A_B}{A_N}\right) H'(B_1^*, 0) < \frac{A_N}{\sigma} \\ T_{\max} & \text{otherwise} \end{cases}$$

where $T_{\max}$ is the maximal capital tax revenue that can be raised, equal to $T_{\max} = (\beta\omega_K - 1) K_1$ where

$$(69) \quad K_1 = \frac{\sigma\left(\omega_K K_0 - D_0\right) + (1-\sigma)\bar{n}}{(1 - \beta\gamma\omega_K)}$$

Let B_1^*, G^*, T_K^* and K_1 be defined by (66)-(69).

Proposition 5. There is a critical value G^* such that if $G_0 \leq G^*$ there is no financial repression and $B_{B1} = 0$, if $G_0 > G^*$ then there is financial repression and $B_{B1} > 0$.

Proof. Suppose G_0 is less than G^*. Consider the first order condition for debt and no financial repression given by (61)

$$(70) \quad -\beta W'(G_0 - T_K - \beta B_1) = -\left(\beta + \mu\right) H'(B_1, G_L)$$

$$(71)\quad \mu\left[H(B_1, G_L) - H_M(0, G_L)\right] = 0$$

$$(72)\quad \mu \leq \frac{A_B}{A_N}$$

$$(73)\quad T_K = \begin{cases} 0 & \text{if } -(\beta+\mu)\,H'(B_1, G_L) < \frac{A_N}{\sigma} \\ [0, T_{\max}] & \text{if } -(\beta+\mu)\,H'(B_1, G_L) = \frac{A_N}{\sigma} \\ T_{\max} & \text{if } -(\beta+\mu)\,H'(B_1, G_L) > \frac{A_N}{\sigma} \end{cases}$$

This system can be used to determine whether a solution for B_1, T_K, μ exists. If $G_0 < G^*$ it is easy to see that the value of μ that solve this system is strictly less than A_B/A_N so that there is no financial repression. If $G_0 > G^*$ it is also easy to see that no solution exists with no financial repression. $Q.E.D.$

The intuition is staightforward: when $G_0 < G^*$ there tax smoothing motive is relatively low. So it is possible to sustain the desired level of debt held by households using standard reputational arguments without the need to force bank to hold debt and so distorting capital accumulation. When G_0 is sufficiently high – higher than the critical level G^* – trigger strategies alone cannot support enough debt (held by household) since with higher G_0 the desire to tax smooth is higher. The government finds then optimal to increase its debt issuance by forcing banks to hold some of the debt and so relaxing the sustainability constraint.

5. Discriminatory Default

In the analysis above we assumed that the government default decision is non-discriminatory. Banks and households are treated that same in the event of a default: the government defaults on both if it defaults on either. This feature is important because allows the government to credibly commit to repay debt held by households if a sufficiently large share of debt is held by banks. Here we briefly consider what happens when the government can choose different default rates on households and banks. We show that if the government can choose on default rate for households, δ_H, and a different default rate for banks, δ_B, then it will always default on households in a Markov equilibrium. Nonetheless, the government will still practice financial repression by forcing banks to hold government debt if the tax smoothing gains are sufficiently large.

Consider first the case of a Markov equilibrium. The basic idea here is the same as

when the government cannot discriminate. The key difference is that before the government could "lever up": by forcing the banks to hold a relatively small amount of debt it could make it credible that it would not default on a relatively larger amount of household debt. Now there is no such levering up: the government forces that bank to hold government debt, the private agents hold no government debt and this arrangement is optimal if the gains from tax smoothing are sufficiently large.

The following proposition is the analogue of Proposition 2:

Proposition 5. With discriminatory default no debt will be held by households. If G_H is sufficiently large and G_L is sufficiently small then it is optimal to practice financial repression. Moreover, if W is strictly concave then the outcome path associated with the Markov equilibrium with discriminatory default is such that $\{B_{2t}\}$ and $\{B_{2t+1}\}$ are decreasing. In the long run, independently of the level of initial debt, the economy converges to a unique steady state cyclical pattern in which no debt is issued into the high spending state and there is financial repression in the high spending state.

Proof. To see that households do not hold any government debt notice that there is no cost associated to defaulting on households by setting $\delta_H = 0$ but there are benefits due to a reduction in the distortions associated with labor income taxes needed to service the debt. So the government will always default on household debt. Hence households will not buy such debt and $B'_H(S, G_H) = 0$.

To see that if G_H is large enough there is financial repression and $B'_B(S, G_H) > 0$ we note that using a similar logic as in the case with non-discrimination, we can solve for the Markov policies by considering the tax distortion function $H_{M,discr}$ for the case with discriminatory default:

$$(74)\quad H_{M,discr}(B_B, G) = \max_{B'_B, T_K, T_L} W(T_L) - \frac{A_N}{\sigma} T_K - A_B B'_B + \beta H_{M,discr}(B'_B, G')$$

subject to

$$(75)\quad T_L + T_K + \beta B'_B = G + B_B$$

$$(76)\quad A_N B'_B \geq H_{M,discr}\,(0, G') - H_{M,discr}\,(B'_B, G')$$

and the non-negativity constraints where we imposed that $B'_H = 0$. Using the envelope condition $H'_{M,discr}(B_B, G) = W'(T_L)$ and letting μ be the multiplier on the no-default constraint, the first order condition for B'_B can be written as

$$(77) \quad -\beta W'(T_L) \leq -\beta W'(T'_L) + A_B - \mu \left(A_N + W'(T'_L)\right)$$

with equality if B'_B is interior. A sufficient condition for B'_B to be interior (for all $B_B \geq 0$) in the high spending state is that

$$(78) \quad -\beta W'(G_H) > -\beta W'(G_L) + A_B$$

(Note: This is a stronger condition than the sufficient condition for optimality of financial repression with non-discriminatory default, (48). In fact, a necessary condition for positive amount of debt to be feasible is that $A_N > -W'(G_L) \geq 0$ and so if (78) hold we have

$$-\beta W'(G_H) > -\beta W'(G_L) + A_B > -\beta W'(G_L) + A_B \left(\frac{-W'(G_L)}{A_N}\right)$$

and so (48) is implied by (78).)

So under (78) it is optimal to practice financial repression in the high government spending state. For the rest of the proof, note that $A_B - \mu \left(A_N + W'(T'_L)\right) > 0$ and so whenever B_{Bt} is interior we have that $-\beta W'(T_L) \leq -\beta W'(T'_L)$. To see this note that $-\mu \left(A_N + W'(T'_L)\right) \geq 0$. Suppose for contradiction that $\mu > 0$ and $A_N > -W'(T'_L)$. Then the envelope condition and the concavity of $H_{M,discr}$ imply that

$$A_N B'_B > -W'(T'_L) B'_B = -H'_{M,discr}(B'_B, G') B'_B > H_{M,discr}\left(0, G'\right) - H_{M,discr}\left(B'_B, G'\right)$$

and so the no-default constraint is slack contradicting that $\mu > 0$. Using this fact the rest of the proof is identical to the proof of Proposition 2 with the only exception that since all debt is held by banks $r_t = 1$ for all t with $B_{Bt} = B_t > 0$. *Q.E.D.*

[to be finished]

6. Numerical Illustration

[to be finished]

7. Conclusions

Financial repression has been widely practiced throughout history. In particular, financial repression is more likely when government debt is high or when governments want to issue a lot of debt. In this paper, we investigate when, if ever, financial repression is optimal. We find that under commitment financial repression is never optimal as financial repression is at best a redudundant instrument. If, however, a government cannot commit to its policies, in particular, it cannot commit to repaying its debt then financial repression may be optimal. Moreover, we find that the more severe are the fluctuations in spending needs, the stronger is the financial repression when spending needs are high. In particular, when positive amount of government debt can be sustained with standard reputational arguments, we find that financial repression is practiced only when the government spending needs are high. This paper highlights a cost associated with recent proposals to discourage banks to hold domestic government debt. In light of our theory, such proposals may not be a good idea.

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9. Appendix

Lemma 7. If $v(L) = L^{1+\eta}/(1+\eta)$ for $\eta > 0$ then $W'(0) = 0$ and $W'' < 0$.

Proof. To show that $W'(0) = 0$ and $W'' < 0$ differentiate W to get that

$$W'(T_L) = [\omega_L - v'(\ell(T_L))]\,\ell'(T_L) \tag{79}$$

and from $[\omega_L - v'(\ell(T_L))]\,\ell(T_L) = T_L$ we have

$$\ell'(T_L) = -\frac{1}{v'(\ell(T_L)) - \omega_L + v''(\ell(T_L))\ell(T_L)} < 0 \tag{80}$$

which is negative for all $G \in [0, G_{\max}]$ since we have chosen ℓ on the side on the Laffer curve in which labor tax revenues are increasing in the rates. Since labor supply is decreasing in the tax rate this in turn implies $\ell'(T_L) < 0$ and so $v'(\ell(T_L)) - \omega_L + v''(\ell(T_L))\ell(T_L) > 0$.

Using the expression (80) to substitute for ℓ' into (79) gives

$$W'(T_L) = -\frac{\omega_L - v'(\ell(T_L))}{v'(\ell(T_L)) + v''(\ell(T_L))\ell(T_L) - \omega_L}. \tag{81}$$

Moreover, it is easy to see that

$$W'(T_L) = \begin{cases} 0 & \text{if } T_L = 0 \\ < 0 & T_L \in (0, G_{\max}) \\ -\infty & \text{if } T_L = G_{\max} \end{cases} \tag{82}$$

and so $W'(0) = 0$. We can further rewrite (81) using the fact that $[\omega_L - v'(\ell(T_L))] = T_L/\ell(T_L)$ as

$$W'(T_L) = -\left[\frac{T_L/\ell(T_L)}{-T_L/\ell(T_L) + v''(\ell(T_L))\ell(T_L)}\right] \tag{83}$$

which under our functional form assumption is

$$W'(T_L) = \frac{T_L/\ell(T_L)}{T_L/\ell(T_L) - \eta v'(\ell(T_L))} \tag{84}$$

Note that W' is decreasing since the numerator, $T_L/\ell(T_L)$, is positive and increasing and the denominator, $T_L/\ell(T_L) - \eta v'(\ell(T_L)) = -v'(\ell(T_L)) + \omega_L - v''(\ell(T_L))\ell(T_L)$ is increasing and negative as argued above.[2]. $Q.E.D.$

Note that an alternative sufficient condition for this lemma is that v is convex and v'' is increasing

A. Preliminary Results for Best Sustainable Equilibrium

Here we show that under our assumptions the value for the Ramsey outcome is given by

$$(86)\quad V_R(K,D,B,G) = A_R + \omega_K K + A_N N + + \frac{1}{1-\beta}\max_T \left\{ W\left(\frac{G_H}{1+\beta} - T\right) - \frac{A_N}{\sigma}T \right\}$$

where

$$A_R = \frac{1}{1-\beta}\frac{(1-\sigma)}{\sigma}A_N\bar{n},$$

$$A_B = \frac{\beta(1-\gamma)(\beta\omega_K - 1)(1+A_N)}{1-\gamma\beta\omega_K},$$

and

$$A_N = \frac{(\beta\omega_K - 1)\sigma}{1-\beta\omega_K\left[\sigma + (1-\sigma)\gamma\right]}$$

[2]More precisely:

$$(85)\quad W''(T_L) = \left[\frac{\frac{\partial NUM(T_L)}{\partial T_L}DEN(T_L) - \frac{\partial DEN(T_L)}{\partial T_L}NUM(T_L)}{DEN(T_L)^2}\right]$$

where

$$NUM(T_L) = T_L/\ell(T_L) \geq 0 \text{ and } DEN(T_L) = T_L/\ell(T_L) - \varphi v'(\ell(T_L)) < 0$$

$$\frac{\partial NUM(T_L)}{\partial T_L} = \frac{\ell(T_L) - T_L\ell'(T_L)}{\ell(T_L)^2} > 0 \text{ because } \ell'(T_L) < 0$$

$$\frac{\partial DEN(T_L)}{\partial T_L} = \frac{\ell(T_L) - T_L\ell'(T_L)}{\ell(T_L)^2} + \varphi v''(\ell(T_L))\ell'(T_L) > 0$$

because $v'' > 0$ and $\ell' < 0$.

If it is not optimal to use capital taxes, the above expression simplifies to

$$(87)\quad V_R(K,D,B,G) = A_R + \omega_K K + A_N N + \frac{1}{1-\beta} W\left(\frac{G_H + \beta G_L}{1+\beta}\right)$$

making clear that it is optimal to smooth taxes over time. Next we prove this result.

Lemma R. Under our assumptions the value of the Ramsey outcome is given by (86).

Proof of Lemma R. The Ramsey outcome solves a relaxed version of (**??**) dropping the sustainability constaint (**??**). From the focs it follows that for all t

$$W'(T_{Lt}) = W'(T_{L0})$$

and so, since $W'' < 0$ it must be that for all t

$$T_{Lt} = T_{L0} = T_L$$

Using this in the government budget constraint (**??**) and iteating forward using the NPG condition it follows that

$$(88)\quad T_L = \left[\frac{G_H}{1-\beta^2} - \sum_{t=0}^{\infty} \beta^t T_t\right](1-\beta)$$

Consider now the "capital" component of the utility. Since we drop the sustainability constraint, it is clear from the focs that $B_{Bt+1} = 0$ and so (**??**) we have that

$$K_{t+1} = \frac{\sigma(1-\gamma)\omega_K K_t + (1-\sigma)\bar{n} - T_{Kt}}{(1-\beta\gamma\omega_K)} \text{ for } t \geq 2 \text{ and } K_1 = \frac{\sigma(1-\gamma)N_0 + (1-\sigma)\bar{n} - T_{K0}}{(1-\beta\gamma\omega_K)}$$

recursively substituting the above expression for K_{t+1} in the objective function we obtain

$$(89)\quad \sum_{t=1}^{\infty} \beta^{t-1} \left[\beta\omega_K - 1\right] K_t = A_R + A_N N_0 - \frac{A_N}{\sigma} \sum_{t=0}^{\infty} \beta^t T_t$$

So, using (88) and (89) in the objective function of the problem we have

$$(90)\quad V_R = \omega_K K + A_R + A_N N_0 - \frac{A_N}{\sigma}\sum_{t=0}^{\infty}\beta^t T_t + \max_{\{T_t\}} W\left(\left[\frac{G_H}{1-\beta^2} - \sum_{t=0}^{\infty}\beta^t T_t\right](1-\beta)\right)$$

The first order condition for T_t is

$$-W'\left(\left[\frac{G_H}{1-\beta^2} - \sum_{t=0}^{\infty}\beta^t T_t\right](1-\beta)\right) - \frac{A_N}{\sigma}\begin{cases} \leq 0 & \text{if } T_{Kt} = 0 \\ = 0 & \text{if } T_{Kt} \in (0, \beta\omega_K K_{t+1} - K_{t+1}) \\ > 0 & T_{Kt} = \beta\omega_K K_{t+1} - K_{t+1} \end{cases}$$

and so abstracting from the corner with $T_{Kt} = \beta\omega_K K_{t+1} - K_{t+1}$ it is wlog to set $T_{Kt} = T_K$ for all t so the first oder condition reduces to

$$-W'\left(\left[\frac{G_H}{1-\beta^2} - \frac{T_K}{1-\beta}\right](1-\beta)\right) - \frac{A_N}{\sigma} \leq 0$$

Note that if G_H is larger than $\bar{G}_H(\beta)$ implicitly defined as

$$-W'\left(\frac{\bar{G}_H(\beta)}{1-\beta^2}(1-\beta)\right) = \frac{A_N}{\sigma}$$

we have that $T_K > 0$. Using $T_{Kt} = T_K$ in (90) we have (86). $Q.E.D.$

Now that we have characterized the Ramsey outcome, we can find conditions such that the Ramsey outcome satisfies/does not satisfy the sustainability constraint (**??**). Note that the relevant state when the sustainability constraint is possibly binding is the low government spending state G_L. Consider the case with $G_H < \bar{G}_H(\beta)$ so there are no capital taxes along the Ramsey outcome. We can rewrite the sustainability constraint (**??**) as $V_R(S,0) \geq V_M(K, D, 0, 0, 0)$ and using the characterization of the Markov value provided in Lemma 5 and (87) we can further write

$$\omega_K K + A_R + A_N N + \frac{1}{1-\beta} W\left(\frac{G_H}{1+\beta}\right) \geq \omega_K K + A_R + A_N N + H_M(0, G_L)$$

which can be rearanged as

$$(91)\quad \frac{1}{1-\beta}W\left(\frac{G_H+\beta G_L}{1+\beta}\right) \geq H_M(0,G_L)$$

(recall that in the Ramsey outcome there are no further cost because banks do not hold government debt, $B_B = 0$). If β is sufficiently close to zero it is clear that the condition (91) is violated and so the Ramsey outcome is not sustainable. Conversely, if β is sufficiently close to one then the Ramsey outcome is indeed sustainable.