International Spillovers and Guidelines for Policy Cooperation

A Welfare Theorem for National Economic Policymaking*

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Abstract

In an interconnected world, national economic policies lead to international spillover effects. This has recently raised concerns about global currency wars. We show that international spillover effects are Pareto efficient and that there is no role for global coordination if three conditions are met: (i) national policymakers act as price-takers in the international market, (ii) national policymakers possess a complete set of instruments to control transactions with the rest of the world and (iii) international financial markets are complete. Under these conditions, international spillover effects constitute pecuniary externalities that are mediated through market prices and do not give rise to inefficiency. We cover four applications: policies to correct growth externalities, aggregate demand externalities in a liquidity trap, spillovers from fiscal policy, and exchange rate stabilization policy. Conversely, if any of the three conditions we identify is violated, we show how global cooperation can improve welfare. Examples that we cover include beggar-thy-neighbor policies, countries with imperfect instruments, and incomplete global financial markets.

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1 Introduction

In a globally integrated economy, national economic policies generate international spillover effects. This has recently led to considerable controversy in international policy circles and has raised concerns about global currency wars.[1] The contribution of this paper is to provide a broad set of sufficient conditions under which national policymaking in pursuit of domestic objectives leads to Pareto efficient global outcomes. In other words, our paper extends the first welfare theorem to the interactions of national economic policymakers in a multi-country setting. Under the conditions that we identify, all international spillover effects are Pareto efficient and there is no case for international policy cooperation. Conversely, our paper describes how international policy cooperation can generate Pareto improvements if one of these conditions is violated.

We develop a multi-country general equilibrium framework that nests a broad class of open economy macro models and that is able to capture a wide range of domestic market imperfections and externalities. We assume that each country encompasses a set of optimizing private agents as well as a policymaker who has policy instruments to affect the domestic and international transactions of the economy. When the policymaker in a given country changes her policy instruments, she influences the actions of domestic consumers, which in turn leads to general equilibrium adjustments that may entail spillover effects to other economies.

We show that the spillovers created by national economic policymaking are Pareto efficient and that there is no role for global cooperation if three conditions are met: (i) national policymakers act competitively, i.e. they act as price-takers in the international market, (ii) if there are domestic externalities, national policymakers either possess a sufficient set of policy instruments to control them domestically or a complete set of external policy instruments, and (iii) international markets are complete. Under these three conditions, we can view national policymakers as competitive agents in a well-functioning global market, and so the first welfare theorem applies. Therefore international spillover effects constitute pecuniary externalities that are mediated through market prices and do not give rise to inefficiency. By implication, spillover effects by themselves should not be viewed as a sign of inefficiency.

We obtain our results by observing that, under quite general conditions, we can condense the welfare function of each country into a reduced-form welfare function $V(\cdot)$ in which the only argument is a vector of the country's international transactions. Condition (i) ensures that policymakers are price-takers and that there are no monopolistic distortions; condition (ii) of complete policy instruments guarantees that each domestic policymaker has the instruments to implement her desired exter-

[1] The term "curreny war" was coined by the Brazilian finance minister Guido Mantega in September 2010 to describe policies of the US a number of Asian countries that had the effect of depreciating their exchange rates. For detailed policy discussions on this subject see e.g. Gallagher et al. (2012), Jeanne et al. (2012), Ostry et al. (2012) and Stiglitz (2012).

nal allocation subject to a country's external budget constraint; condition (iii), that the international market is complete, ensures that the marginal rates of substitution of all countries are equated in equilibrium. As a result, we can apply the first welfare theorem at the level of domestic policymakers, interpreting $V(\cdot)$ as the utility functions of competitive agents in a complete market. Our model is general enough to allow for a wide range of domestic market imperfections, including price stickiness, financial constraints, incentive/selection constraints, missing or imperfect domestic markets, etc.

If any of the three conditions we identified is violated, spillover effects generally lead to inefficiencies, and global cooperation among national policymakers can generally improve welfare. Following the three conditions above, such cooperation works along three dimensions: (i) It ensures that countries refrain from monopolistic behavior and act with "benign neglect" towards international variables. (ii) If countries face domestic externalities but have incomplete/imperfect instruments to address them, then welfare is improved if those countries with better instruments or better-targeted instruments assist those without. (iii) If international markets are incomplete, then global coordination internalizes the redistributions created by pecuniary externalities that may lead to inefficient risk-sharing or wealth transfers. We provide examples that characterize the scope for coordination in each of the three cases.

The types of cooperation described in points (ii) and (iii) typically implement second-best equilibria that take the set of available policy instruments and markets as given but make more efficient use of them. There is also a scope for first-best policies in these areas: global cooperation may increase efficiency by creating more or better policy instruments, or by creating better insurance markets to alleviate market incompleteness.[2] Our analysis suggests that the case for global cooperation disappears if we move to the ideal of perfect external instruments and complete international markets, assuming that national economic policymakers continue to act competitively.

We apply our theoretical analysis to a number of practical applications, many of which have led to controversy in international policy circles in recent years. Our first application, chosen because it provides the simplest illustration of our results, is to analyze an economy in which international capital flows generate technological externalities. An important example that is frequently referred to by policymakers in developing economies are learning-by-exporting effects. We show that a national policymaker finds it optimal to correct technological externalities via capital account intervention. Furthermore, as long as the three conditions identified above are satisfied, this policy intervention is efficient from a global perspective, i.e. global cooperation is unnecessary to achieve a Pareto efficient oucome. We extend our analysis to domestic

[2] A number of recent reforms in the global financial architecture follow this principle: for example, Basel III provides policymakers with new counter-cyclical capital buffers (better instruments), recent IMF proposals allow countries to use capital controls (new instruments; see Ostry et al, 2011), and swap lines between advanced economy central banks provide new forms of liquidity insurance (more complete markets).

technological externalities and show that our results continue to hold even if capital account intervention is just a second-best policy measure to correct learning-by-doing externalities in the domestic economy.

Our second application describes an economy that suffers from a shortage of aggregate demand that cannot be corrected using domestic monetary policy because of a binding zero-lower-bound on nominal interest rates. In such a situation, a national economic policymaker finds it optimal to impose controls on capital inflows or to subsidize capital outflows in order to mitigate the shortage in demand. Under the three conditions discussed earlier, this behavior leads, again, to a Pareto efficient global equilibrium.

Our third application analyzes the incentives for determining the level of fiscal spending. If a domestic policymaker chooses the level of fiscal spending based on purely domestic considerations, the resulting global equilibrium is Pareto efficient under the three discussed conditions. In the context of fiscal policy, it is of particular importance to underline that the domestic policymaker needs to act competitively, i.e. with benign neglect towards the international price effects of her policy actions. In particular, if the policymaker strategically reduces her stimulus in order to manipulate its terms-of-trade, then the resulting equilibrium is inefficient and generally exhibits insufficient spending.

Our fourth application analyzes an economy in which there are heterogeneous domestic agents who face incomplete risk markets and cannot insure against fluctuations in the country's real exchange rate. We show that a national economic policymaker can improve welfare by intervening in the economy's capital account to stabilize the economy's real exchange rate, for example by imposing controls on capital inflows. This constitutes a second-best policy tool to insure domestic agents. Again, the outcome is Pareto efficient under the three conditions identified above. This illustrates that our results on global Pareto efficiency continue to hold even if a national planner pursues purely distributive objectives or implements domestic political preferences, as long as we abstain from paternalism, i.e. as long as the global planning problem that is used to evaluate Pareto efficiency respects those domestic objectives and preferences.

In each of the described applications, we show that the spillover effects from policy intervention are efficient under the idealized circumstances captured by the three conditions for efficiency. Any arguments about the desirability of global policy coordination therefore needs to be made on the basis of deviations from these conditions. Our paper therefore concludes by describing the case for coordination in each of these circumstances.

We continue our analysis by carefully examining the case for economic policy coordination when at least one of the three conditions is violated: When condition (i) is violated and policymakers internalize their market power, coordination should aim to restrict monopolistic behavior so as to maximize gains from trade. This has been well understood at least the rebuttal of mercantilism by Smith (1776).[3] This

[3]This observation also underlies much of modern optimal tariff theory. See e.g. Bagwell and

applies not only to external policy measures but also to all domestic policies that affect international prices, such as e.g. fiscal policy. We add to this literature by providing general conditions for the direction of monopolistic intervention that help to distinguish monopolistic intervention from intervention to correct domestic market imperfections. However, we also show that there are ample circumstances under which it is difficult to distinguish monopolistic from corrective intervention. Furthermore, we show that domestic policies will only be distorted with monopolistic objectives if a country faces restrictions on its set of external policy instruments. For a detailed discussion on how to achieve policy agreements to abstain from monopolistic behavior see **??**.

Condition (ii) implies that there may be a case for international policy cooperation when policymakers possess an incomplete set of policy instruments. If a country suffers from domestic externalities, we show that the global equilibrium is constrained Pareto efficient if domestic policymakers either have sufficient domestic instruments to directly correct the externalities in the domestic economy (e.g. to stimulate aggregate demand when it is insufficient) or have a complete set of external policy instruments so that the indirect effects of external transactions on domestic externalities can be corrected (e.g. to tax capital inflows when aggregate demand is insufficient). Conversely, if a country suffers from externalities and both domestic and external policy instruments are incomplete, then international policy cooperation can generally achieve a Pareto improvement. Efficient allocations require that countries with better instruments or better-targeted instruments assist those without. For example, if a country experiences externalities from capital inflows but has no instrument to control them, welfare is improved if other countries control their capital outflows.

The observation that incomplete policy instruments make it desirable to coordinate policy has a rich intellectual tradition, going back to the targets and instruments approach of Tinbergen (1952) and Theil (1968). They observed in a reduced-form setting that incomplete instruments may give rise to a role for economic policy cooperation. Our contribution to this literature is to embed the Tinbergen-Theil approach into a general equilibrium framework in which optimizing individual agents interact in a market setting. We show that many of the spillover effects that would suggest a role for cooperation in the Tinbergen-Theil framework actually constitute efficient pecuniary externalities. Once the optimizing behavior of private agents is taken into account, a wide range of spillovers can be considered as efficient.[4]

Condition (iii) requires that all international transactions take place in complete markets and at flexible prices to ensure efficiency. This is a well-known condition for

Staiger (2002) for a modern treatment in the trade literature, or Costinot et al. (2014) and De Paoli and Lipinska (2013) for recent contributions that focus on intertemporal rather than intratemporal trade. Farhi and Werning (2012) emphasize this motive for cooperation in a multi-country New Keynesian framework. Persson and Tabellini (1995) survey implications for macroeconomic policy.

[4]In the more recent literature, Jeanne (2014) provides an interesting example where the coordination of macroprudential policies is warranted because of missing policy instruments.

the first welfare theorem and has spurred a significant body of literature in the context of general equilibrium models with individual optimizing agents (see e.g. Geanakoplos and Polemarchakis, 1986; Greenwald and Stiglitz, 1986, for a general treatment). In our analysis, we interpret national economic policymakers who abstain from monopoly distortions as competitive agents who interact in an international market and show that the same efficiency results hold.[5]

The remainder of the paper is structured as follows: Section 2 introduces our model setup and section 4 examines its welfare properties and states our extension of the first welfare theorem. Section 5 provides a number of illustrations of our results. Section 6 examines the case for international policy coordination when policymakers act monopolistically, are constrained in their policy instruments or face incomplete international markets. The final section concludes.

2 Model Setup

Our model describes a multi-country economy in which each country consists of a continuum of private agents as well as a domestic policymaker. Both maximize domestic welfare subject to a set of constraints on their domestic allocations and a standard budget constraint on their external transactions. In order to create an interesting role for policy intervention, we distinguish between individual and aggregate allocations so as to capture the potential for externalities. An individual takes aggregate allocations as given, whereas the domestic planner employs her policy instruments to affect aggregate allocation so as to correct for domestic externalities. Our framework nests a wide range of open economy macro models in which there is a case for policy intervention, as we illustrate in a number of examples below.

Countries We describe a world economy with $N \geq 2$ countries indexed by $i = 1, ...N$. The mass of each country i in the world economy is $\omega^i \in [0, 1]$, where $\Sigma_{i=1}^{N} \omega^i = 1$. A country with $\omega^i = 0$ corresponds to a small open economy.

Private Agents In each country i, there is a continuum of private agents of mass 1. We denote the allocations of individual private agents by lower-case variables and the aggregate allocations of the country by upper-case variables.

A representative private agent obtains utility according to a function

$$U^i\left(x^i\right) \tag{1}$$

[5]Recent applications in an international context include Bengui (2013) who analyzes the need for coordination on liquidity policies when global markets for liquidity are incomplete, and Jeanne (2014) who analyzes a world economy in which agents are restricted to trading bonds denominated in the currency of a single country.

where x^i is a collection of domestic variables that contains all variables relevant for the utility of domestic agents, for example a vector of consumption of goods or leisure. We assume that $U^i(x^i)$ is increasing in each element of x^i and quasiconcave.

To keep notation compact, we also include all other domestic variables that do not directly yield utility but that we want to keep track of in the collection x^i, for example in models of capital accumulation the capital stock k^i held by private agents. For such variables, $\partial U^i/\partial k^i = 0$.

External Budget Constraint We denote the international transactions of private agents by a vector of net imports m^i that are traded at an international price vector Q. Private agents have initial external net worth w_0^i and may be subject to a vector of tax/subsidy instruments τ^i that is imposed by the domestic policymaker. Their budget constraint is

$$\left(1+\tau^i\right) Q \cdot m^i \leq w_0^i + T^i \tag{2}$$

where the tax vector $(1+\tau^i)$ pre-multiplies net imports element-by-element and $Q \cdot m^i$ represents the inner product of the two vectors. Any tax revenue is rebated as a lump sum transfer $T^i = \tau^i Q \cdot M^i$. If taxes are zero, then the budget constraint reduces to $Q \cdot m^i \leq w_0^i$.

Domestic Constraints We assume that the representative agent in country i is subject to a collection of constraints, which encompass domestic budget constraints and may include any financial, incentive/selection, or price-setting constraints as well as restrictions imposed by domestic policy measures,

$$f^i\left(m^i, x^i, M^i, X^i, \zeta^i, Z^i\right) \leq 0 \tag{3}$$

where ζ^i is a collection of domestic policy instruments such as taxes, subsidies, and constraints on domestic transactions; Z^i represents a collection of exogenous state variables, for example endowments, productivity shocks or initial parameters.

Observe that we include both the individual external and domestic allocations (m^i, x^i) and aggregate allocations (M^i, X^i) in the constraint to capture the potential for externalities from aggregate allocations to the choice sets of individuals, which we will analyze further in the coming sections. The choice variables of the representative agent are (m^i, x^i) and he takes all remaining variables in the constraint as given.

In summary, the optimization problem of a representative domestic agent is to choose the optimal external and domestic allocations (m^i, x^i) so as to maximize utility (1) subject to the collection of domestic and external constraints,

$$\max_{m^i, x^i} U\left(x^i\right) \quad \text{s.t.} \quad (2),\ (3) \tag{4}$$

Domestic Policymaker The domestic policymaker sets the domestic policy instruments ζ^i and external policy instruments τ^i in order to maximize the utility (1) of domestic private agents. The policymaker internalizes the consistency requirement that the allocations of the representative agent coincide with the aggregate allocations so $m^i = M^i$ and $x^i = X^i$. Furthermore, the policymaker internalizes that these allocations (m^i, x^i) have to solve the optimization problem of domestic agents as described by problem (4).

To make our setup a bit more concrete, the following examples describe how a number of benchmark open economy macro models map into our framework:

Example 1: Baseline Model of Capital Flows

Our first example is a simple model of capital flows between neoclassical endowment economies $i = 1, ...N$ with a single consumption good in infinite discrete time. Assume that the domestic variables in each economy i consist of a vector of consumption goods $x^i = \{(c_t^i)_{t=0}^{\infty}\}$ and the vector of external transactions $(m_t^i)_{t=0}^{\infty}$ denotes the imports of the consumption good in each period, which is equivalent to the trade balance. Since there is a single good, we can also interpret m_t^i as the net capital inflows in period t. Furthermore, there are no domestic policy instruments so $\zeta^i = \varnothing$ and the vector of exogenous variables Z^i consists of an exogenous endowment process $(y_t^i)_{t=0}^{\infty}$.

Assume the utility function in each country is given by $U^i(x^i) = \sum_t \beta^t u(c_t^i)$ and the domestic constraints contain one budget constraint for each time period so $f^i(\cdot) = \{f_t^i(\cdot)\}_{t=0}^{\infty}$ where $f_t^i(\cdot) = c_t^i - y_t^i - m_t^i \leq 0$. If we normalize $Q_0 = 1$ then each element of the vector Q_t represents the price of a discount bond that pays one unit of consumption good in period t, and the external budget constraint of the economy is given by (2). This fully describes our baseline setup.

The policymaker's vector of external policy instruments $\tau^i = (\tau_t^i)_{t=0}^{\infty}$ can be interpreted as either import controls or as capital controls according to the following categorization:

	$\tau_t^i < 0$	$\tau_t^i > 0$
$m_t^i < 0$ (net saving)	outflow subsidy	outflow tax
$m_t^i > 0$ (net borrowing)	inflow tax	inflow subsidy

Table 1: Interpretation of capital control τ_t^i

For example, if a country is a net importer $m_t^i > 0$ (i.e. experiences net capital inflows) in period t, a positive tax rate $\tau_t^i > 0$ raises the cost of the imports/inflows so the measure represents an inflow tax on capital.

It is also common in the open economy macro literature to keep track of the external wealth position w_t^i of a country over time and denote the external budget

constraint (2) by the law of motion of the external wealth position together with a transversality condition $\lim_{t\to\infty} Q_t w_t^i = 0$. Given the country's initial external wealth w_0^i, this law of motion can be represented as

$$\frac{w_{t+1}^i}{1+r_{t+1}} = \left(1+\xi_t^i\right)\left(w_t^i - m_t^i\right) + T_t^i$$

where the interest rate r_{t+1} is pinned down by the relative price of discount bonds $1+r_{t+1} = Q_t/Q_{t+1}$ and the period capital control by the increase in the cumulative controls $1+\xi_t^i = \left(1+\tau_{t+1}^i\right)/\left(1+\tau_t^i\right)$ between two consecutive periods. The revenue from any controls is rebated in lump sum fashion $T_t^i = \xi_t^i(M_t^i - W_t^i)$. The Arrow-Debreu formulation and the period-by-period description of our setup are equivalent, and we will use both in our applications below.

To extend our example to a stochastic economy, all that is required is to define a set of states of nature Ω and a series of probability spaces $(\Omega, \mathcal{F}_t, P_t)$ such that $\mathcal{F}_t$ defines a sigma-algebra of states measurable at time t and satisfying $\mathcal{F}_t \subseteq \mathcal{F}_{t+1}$ and P_t defines a probability measure defined over $\mathcal{F}_t$ and an associated expectations operator $E_t[\cdot] = E[\cdot|\mathcal{F}_t]$. Then we can label all variables by both time t and state of nature $\omega \in \Omega$, for example $c^i = \left(c_{\omega,t}^i\right)_{\omega\in\Omega,t=0}^{\infty}$, and define utility as $U^i(x^i) = E_0[\sum_t \beta^t u\left(c_{\omega,t}^i\right)]$. Everything else remains unchanged.

Example 2: Production Economy with Capital Flows

Next we describe capital flows between neoclassical production economies. We build on example 1 and include leisure, capital and investment in the collection of domestic variables so $x^i = \left\{(c_t^i, \ell_t^i, k_{t+1}^i, i_t^i)_{t=0}^{\infty}\right\}$ where labor in a given period is $1-\ell_t^i$. The vector of exogenous variables captures the initial capital stock and the path of productivity, $Z^i = \{k_0, (y_t^i)_{t=0}^{\infty}\}$. We extend the utility function to include leisure $U^i(x^i) = \sum_t \beta^t u(c_t^i, \ell_t^i)$. The collection of domestic constraints consists of a budget constraint $f_{t,c}^i(\cdot)$ and a capital accumulation constraint $f_{t,k}^i(\cdot)$ each period, which we denote by

$$\begin{aligned} f_{t,c}^i(\cdot) &= c_t^i + i_t^i - A_t^i\left(k_t^i\right)^{\alpha}\left(1-\ell_t^i\right)^{1-\alpha} - m_t^i \leq 0 \\ f_{t,k}^i(\cdot) &= k_{t+1} - (1-\delta)k_t^i - i_t^i \leq 0 \end{aligned}$$

As in example 1, m_t^i captures both net imports and capital inflows, and the policy measure τ_t^i can be interpreted as import taxes or as capital controls as in Table 1. The framework can be extended to stochastic shocks as described before.

Example 3: Multiple Traded Goods

The framework of our previous examples can also be extended in the direction of multiple traded goods. Expanding on our baseline model in example 1, all that is

required is to define the variables c_t^i, m_t^i, y_t^i and τ_t^i in each time period as vectors of size K capturing the consumption, net imports, endowment and inflow taxes on each good $k = 1...K$ in period t. Utility can then be written as $U^i(x^i) = \sum_t \beta^t u\left(c_{t,1}^i, ..., c_{t,K}^i\right)$.

Assuming that τ_t^i is a vector of size K supposes that the planner can set differential tax/subsidy rates on each good $k = 1...K$. Alternatively, assuming that the planner can only differentiate taxes by time period would amount to a restriction on the set of instruments of $\tau_{t,1}^i = ... = \tau_{t,K}^i \ \forall t$. We will analyze such restrictions below in Section 7.

3 Equilibrium

We analyze the equilibrium in the described world economy in two steps. The first step is the domestic optimization problem of each economy for a given external allocation (m^i, M^i) and is described in the next subsection. The second step solves for the optimal external allocation of the economy and is described in the ensuing subsection. The section ends with a formal proposition that the described two-step procedure characterizes an allocation that also solves the combined optimization problem.

3.1 Domestic Optimization Problem

Domestic Agents We describe the domestic optimization problem of a representative agent in country i for given external allocations (m^i, M^i) and for given aggregate control variables, policy variables and exogenous state variables $\left(X^i, \zeta^i, Z^i\right)$. For ease of notation, we will suppress the vector Z^i of exogenous state variables in the functions and our discussion below, though we note that all equilibrium variables depend on the exogenous vector Z^i.

We define the reduced-form utility of the representative agent as

$$v^i\left(m^i; M^i, X^i, \zeta^i\right) = \max_{x^i} U\left(x^i\right) \quad \text{s.t.} \quad f^i\left(m^i, x^i, M^i, X^i, \zeta^i, Z^i\right) \leq 0 \tag{5}$$

Denoting the shadow prices on the domestic constraints by the vector λ_d^i which is of the same size as the collection of constraints f^i, the collection of domestic optimality conditions of a representative agent is

$$U_x = \lambda_d^i f_x^i \tag{6}$$

where U_x denotes the partial derivatives of the utility function with respect to x^i and is of the same size as x^i, and f_x^i is the Jacobian of derivatives of f^i with respect to x^i and is a matrix of the size of f^i times the size of x^i.

Domestic Planner For a given aggregate external allocation M^i, a domestic planner in economy i chooses the optimal domestic policy measures ζ^i and aggregate

choice variables X^i where she internalizes the consistency conditions $x^i = X^i$ and $m^i = M^i$ as well as the implementability constraint (6). The planner's problem is

$$\max_{X^i, \zeta^i, \lambda_d^i} U\left(X^i\right) \qquad \text{s.t.} \qquad f^i\left(M^i, X^i, M^i, X^i, \zeta^i, Z^i\right) \leq 0, \qquad (6) \qquad (7)$$

We assign the shadow prices Λ_d^i to the collection of domestic constraints f^i and μ_d^i to the collection of domestic implementability constraints. The solution to this problem defines the optimal domestic policy measures $\zeta^i\left(M^i\right)$ and aggregate domestic choice variables $X^i\left(M^i\right)$.

Optimal Domestic Allocation For a given aggregate external allocation M^i, the optimal domestic allocation in country i consists of a consistent domestic allocation $x^i = X^i$ and domestic policy measures ζ^i that solve the domestic optimization problem (7) of a domestic planner and, by implication, the domestic optimization problem (5) of private agents in country i since the planner observes the implementability constraint (6).

Definition 1 (Reduced-Form Utility) *We define the reduced-form utility function of a representative agent in economy i for any pair (m^i, M^i) by*

$$V^i\left(m^i, M^i\right) = v^i\left(m^i, M^i, X^i\left(M^i\right), \zeta^i\left(M^i\right)\right) \qquad (8)$$

The reduced-form utility function $V^i\left(m^i, M^i\right)$ is also defined for off-equilibrium allocations in which m^i and M^i may differ since individual agents are in principle free to choose any allocation of m^i. In equilibrium, $m^i = M^i$ will hold.

For the remainder of our analysis, we will focus on the case where the partial derivatives of this reduced-form utility function satisfy $V_m^i > 0$ and $V_m^i + V_M^i > 0\ \forall i$: ceteris paribus, a marginal increase in individual imports m_t^i increases the welfare of an individual consumer and a simultaneous marginal increase in both individual and aggregate imports $m^i = M^i$ increases the welfare of a representative consumer. These are fairly mild assumptions that hold for the vast majority of open economy macro models, including our baseline model. (For concreteness, the reduced-form utility function in our baseline example 1 is $V^i\left(m^i, M^i\right) = \sum_t \beta^t u\left(y_t^i + m_t^i\right)$, satisfying the above marginal utility conditions since $V_{m,t}^i = V_{m,t}^i + V_{M,t}^i = \beta^t u'\left(c_t^i\right) > 0$.)

The reduced-form utility function $V^i\left(m^i, M^i\right)$ contains all the information that is required to describe external allocations and the global equilibrium.

3.2 External Allocations

Representative Agent Given the reduced-form utility $V^i\left(m^i, M^i\right)$, initial external wealth w_0^i and a vector of tax instruments τ^i on international transactions, we describe

the optimization problem of a representative consumer in country i as

$$\max_{m^i} V^i\left(m^i, M^i\right) \quad \text{s.t.} \quad (2) \tag{9}$$

Assigning scalar shadow price λ_e^i to the external budget constraint, the associated optimality conditions

$$V_m^i = \lambda_e^i\left(1+\tau^i\right)Q \tag{10}$$

describe the excess demand for each component of m^i of the representative agent as a function of the vector of world market price Q, where V_m^i is a vector of the same size as m^i and Q, and where the tax vector $(1+\tau^i)$ pre-multiplies the price vector Q in an element-by-element fashion.

Free Capital Flows We define the allocation that prevails when $\tau^i = 0\ \forall i$ as the free capital flows allocation.

Competitive Planner Next we consider how the policy instruments on international transactions ξ^i are determined. We assume a planner who acts competitively on world markets in the sense that she takes world market prices Q as given. We term this a *competitive planner* CP^i. There are two potential interpretations for such behavior: First, country i may represent a small economy with $\omega^i \approx 0$ so that it is not possible for the country to affect world market prices. Secondly, the planner may choose her optimal allocations while acting with benign neglect towards international markets. This may be the consequence of an explicitly domestic objective of the policymaker prescribed by domestic law, or because the policymaker observes a multilateral agreement to abstain from monopolistic behavior and disregard world-wide terms-of-trade effects.[6] We will analyze the behavior of a monopolistic planner who internalizes her market power in the following section, which will also discuss how to distinguish between monopolistic and competitive behavior of policymakers.

Since the planner has a full set of policy instruments to control M^i in our general setup, we can solve directly for the planner's optimal allocations and set these instruments so as to implement the allocation.

A competitive planner CP^i who faces the reduced-form utility function $V^i\left(m^i, M^i\right)$ and initial external wealth B_0^i solves

$$\max_{M^i} V^i\left(M^i, M^i\right) \quad \text{s.t.} \quad Q \cdot M^i \leq B_0^i \tag{11}$$

[6]For example, the US Federal Reserve claims to follow a policy of acting with benign neglect towards external considerations such as exchange rates, as articulated for example by Bernanke (2013). Furthermore, the G-7 Ministers and Governors proclaimed in a Statement after their March 2013 summit that "we reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates." See G-7 (2013).

Assigning a shadow price Λ_e^i to the planner's international budget constraint, the optimality condition of the planner is

$$V_m^i + V_M^i = \Lambda_e^i Q \tag{12}$$

Lemma 1 (Implementation) *The planner can implement her optimal external allocation by setting the vector of policy instruments*

$$\xi^i = -V_M^i / V_m^i \tag{13}$$

where the division V_M^i / V_m^i is performed element-by-element.

Proof. Substituting the optimal ξ^i from (13) into the optimality condition of private agents $\left(1 - \xi^i\right) V_m^i = \lambda_e^i Q$ yields the planner's optimality condition (12). ■

According to this implementation, the planner does not intervene in time periods/states of nature/goods for which $V_{M,t}^i = 0$, i.e. for which the marginal benefit is fully internalized by private agents. If $V_{M,t}^i > 0$ then $\xi_t^i < 0$ so the planner subsidizes m_t^i and vice versa for $V_{M,t}^i < 0$.

To conclude the solution to the general problem described in this section, we observe formally that the two-stage procedure that splits the problem into a domestic and external optimization problem solves the general problem. The proof of the corollary is provided in the appendix.

Corollary 1 (Two-Stage Solution Procedure) *The allocation that solves the two stages of the domestic and external optimization problem described by (7) and (11) also solves the combined planning problem in which both steps are performed simultaneously.*

Proof. See appendix A.2. ■

The intuition behind this corollary is that the competitive planner in our setup has a full set of instruments to determine the optimal external allocation; this implies that it is sufficient to solve for the optimal domestic allocation in the first step of the procedure without considering the interactions between domestic and external allocations. We will show in section 7 that this conclusion no longer holds when the planner's set of external policy instruments is imperfect or incomplete.

One practical implication of the corollary is that the two tasks of implementing the optimal domestic and external allocations of the economy can be performed by two different agencies, and the domestic agency does not need to coordinate with the agency that sets the country's external policy instruments.

Indeterminacy of Implementation Observe that there is an indeterminacy of implementation. The allocation implemented by the planner is unchanged if the policy instruments are scaled up or down by a positive constant $k > 0$: any vector $(1 - \tilde{\xi}) = k\,(1 - \xi)$ will implement the same real allocation and will simply rescale the shadow price in the optimality condition (12) by $1/k$. The intuition is that the incentive of a representative agent to shift consumption across time/states of nature/goods only depends on the relative price of goods in the vector m^i. Multiplying all prices by a constant amounts to changing the numeraire.

By the same token, if $\exists h \in (-1, \infty)$ s.t. $V_M^i = hV_m^i$, then it is not necessary for a planner to intervene since the vector of policy instruments $\tilde{\xi}^i = 0$ will implement the same equilibrium as the vector $\xi^i = -V_M^i/V_m^i = -h$. This can easily be verified by setting $k = \frac{1}{1+h}$ and applying the argument in the preceding paragraph. In the following, we will assume that $\nexists h$ that satisfies this requirement when we speak of a country that exhibits externalities $V_M^i \neq 0$.

Sequential Trading The formulation of problem (9) assumed – in Arrow-Debreu fashion – that all intertemporal trade takes place in period 0. If trading occurs sequentially, i.e. period after period, as it did in our baseline model, then the planner can implement her optimal allocation by imposing the relative tax wedge $\left(1 - \tau_{t,t+s}^i\right) = \left(1 - \xi_t^i\right) / \left(1 - \xi_{t+s}^i\right)$ between any two periods t and $t + s$.

4 Global Equilibrium and Welfare

We now turn to the determination of equilibrium at the global level. We describe the global equilibrium assuming that each individual country i is governed by a competitive planner CP^i who solves the optimization problem (11) and implements the resulting allocation:

Definition 2 (Global Equilibrium) *For a given set of initial bond holdings $\{B_0^i\}$ and reduced-form welfare functions $\{V^i\,(m^i, M^i)\}$, the global equilibrium among competitive planners CP is described by a collection of vectors of net imports $\{M^i\}$ and intertemporal prices Q such that the planner in each country i solves her optimization problem (11) and global markets clear $\sum_{i=1}^N \omega^i M^i = 0$.*

Let us next analyze the global efficiency of this equilibrium.

Proposition 1 (Efficiency of Global Equilibrium) *The global equilibrium as per Definition 2 is Pareto efficient.*

Proof. An allocation is Pareto efficient if it maximizes the weighted sum of welfare of all countries for some vector of welfare weights $\left\{\phi^i \geq 0\right\}_{i=1}^N$ subject to the global

resource constraint and the individual domestic constraints of each country,

$$\max_{\left\{M^i,X^i,\zeta^i\right\}_i} \sum_i \phi^i \omega^i U^i\left(X^i\right) \qquad \text{s.t.} \qquad \sum_i \omega^i M^i = 0, \;\; (6),$$
$$f^i\left(M^i, M^i, X^i, X^i, \zeta^i, Z^i\right) \leq 0 \;\; \forall i$$

By the definition of $V^i\left(m^i, M^i\right)$, we can restate this problem in terms of reduced-form utilities for any optimal collection of $\{M^i\}$,

$$\max_{\{M^i\}_i} \sum_i \phi^i \omega^i V^i\left(M^i, M^i\right) \qquad \text{s.t.} \qquad \sum_i \omega^i M^i = 0$$

Assigning the shadow price θ to the vector of resource constraints, the optimality condition of the global planner is

$$\phi^i\left(V_m^i + V_M^i\right) = \theta \quad \forall i$$

Any global equilibrium in per Definition 2 satisfies these optimality conditions if we assign the shadow price $\theta = Q$ and the welfare weights $\phi^i = 1/\Lambda_e^i$ where Λ_e^i is the shadow price on the external budget constraints of the external optimization problem (11) in each country i. Therefore any such equilibrium is Pareto efficient. ■

Remark The proposition is a version of the first welfare theorem. Since the planner CP^i has a complete set of tax instruments ξ^i, she can fully determine the efficient excess demand for M^i given the world market price Q. If the planner acts competitively in determining M^i, then all the conditions of the first welfare theorem apply and the resulting competitive equilibrium is Pareto efficient. Given that the planner has internalized all domestic externalities, the excess demand M^i of the country correctly reflects the country's social marginal valuation of capital flows. The marginal rates of substitution of all domestic planners are equated across countries, and the resulting equilibrium is Pareto efficient.

Compared to the global laissez-faire equilibrium, capital account interventions do create significant spillover effects, but this is not a sign of Pareto inefficiency. The spillover effects constitute pecuniary externalities that are intermediated in a complete market for M^i. As exemplified by the numerical illustrations in section **??** on the interest rate effects of capital controls, they can be quantitatively quite large. However, Pareto optimality is independent of redistributive concerns. Even though the spillover effects entail redistributions between borrowing and lending countries, proposition 1 establishes that they do not lead to Pareto inefficiency.

Efficiency of Laissez-Faire Equilibrium A straightforward corollary to Proposition 1 is that the laissez faire equilibrium is generally not Pareto efficient if there are countries subject to externalities from international capital flows.

Tatonnement and Arms Race of Capital Account Interventions The equilibrium adjustment (tatonnement) process may sometimes involve dynamics that look like an arms race. For example, assume that several countries experience negative externalities $V^i_{M,t} < 0$ from capital inflows M^i and that the absolute magnitude of these externalities increases in a convex fashion $V^i_{MM,t} < 0$ in period t. Then an exogenous shock that makes one country increase its optimal degree of intervention, will lead to greater capital flows to all other countries, which in turn increases their externalities and induces them to respond with greater intervention. This in turn will deflect capital flows back into the original country, triggering further intervention there, and so on.

Such dynamics may give the appearance of an arms race but are nonetheless efficient. As long as the conditions of Proposition 1 are satisfied, this "arms race" is simply the natural mechanism through which the efficient equilibrium is restored. In the described example, each successive round of spillovers will be smaller and the degree of intervention will ultimately converge towards its efficient levels, which involves greater intervention in all affected countries.

Pareto-Improving Capital Account Intervention If the objective of a global planner is not only to achieve Pareto efficiency but the more stringent standard of achieving a Pareto improvement, then capital controls generally need to be accompanied by transfers that compensate the countries that lose from changes in world prices/interest rates. Lump-sum transfers enable a global planner to always implement a Pareto improvement when correcting for the domestic externalities:

Proposition 2 (Pareto-Improving Capital Account Intervention, Transfers) *Starting from the laissez faire equilibrium, a global planner who identifies domestic externalities $V^i_M \neq 0$ in some countries can achieve a Pareto improvement by setting the capital account interventions $\xi^i = -V^i_M/V^i_m$ in all countries and providing compensatory international transfers $\hat{T}^i \lesseqgtr 0$ that satisfy $\sum_i \hat{T}^i = 0$.*

Proof. Denote the net imports and world prices in the laissez faire equilibrium by $\left\{M^{i,LF}\right\}_i$ and Q^{LF} and in the global planner's equilibrium that results from imposing optimal capital controls ξ^{i*} and transfers by $\left\{M^{i,GP}\right\}_i$ and Q^{GP}. Assuming the planner provides transfers $\hat{T}^i = Q^{GP} \cdot \left(M^{i,LF} - M^{i,GP}\right)$, then $\sum \hat{T}^i = 0$ since both sets of allocations (LF and GP) satisfy market clearing. Furthermore, given these transfers, consumers in each country i can still afford the allocation that prevailed in the laissez faire equilibrium. For non-zero interventions $\left\{\xi^i\right\}$, the allocation differs from the laissez faire equilibrium since the optimality conditions (10) for $\xi^i = 0$ and $\xi^i = \xi^{i*}$ differ. Given that the old allocation is still feasible but is not chosen, revealed preference implies that every country is better off under the new allocation. ■

In an international context, compensatory transfers may be difficult to implement. As an alternative, we show that a planner who can coordinate the capital control

policies of both inflow and outflow countries can correct the domestic externalities of individual economies while holding world prices and interest rates constant so that no wealth effects arise. As a result, the global planner's capital control policies constitutes a global Pareto improvement at a first-order approximation.

The following lemma demonstrates how a global planner can manipulate world prices by simultaneously adjusting the capital controls in all countries worldwide; then we show how this mechanism can be used to hold world prices fixed so as to avoid redistributive effects when correcting for externalities in a given country.

Lemma 2 *Consider a global competitive equilibrium with an allocation* $\{M^j\}_j$*, capital account interventions* $\{\xi^j\}_j$ *and world prices* Q*. A global planner can change world prices by* dQ *while keeping the bond allocations for all countries constant by moving the capital account interventions in each country* $j = 1...N$ *by*

$$d\xi^j = -\left(M^j_\xi\right)^{-1} M^j_Q dQ \tag{14}$$

Proof. We set the total differential of net imports of a given country j with respect to world prices and capital account interventions to zero,

$$dM^j = M^j_Q dQ + M^j_\xi d\xi^j = 0$$

and rearrange to obtain equation (14). ■

Corollary 2 (Pareto-Improving Capital Account Intervention, No Transfers) *Assume an exogenous marginal increase in the domestic externalities of country* i *that raises the optimal unilateral capital control by* $d\xi^{i*} > 0$ *in period* t*. A global planner can correct for this while keeping world prices constant* $dQ = 0$ *to avoid income and wealth effects by adjusting*

$$d\xi^j = -\omega^i \left(M^j_\xi\right)^{-1} M^j_Q \left(M_Q\right)^{-1} M^i_\xi d\xi^{i*} \quad \text{and} \quad d\xi^i_{t+1} = \left[I - \omega^i \left(M^i_\xi\right)^{-1} M^i_Q \left(M_Q\right)^{-1} M^i_\xi\right] d\xi^{i*}$$

In the resulting equilibrium, net imports $\{M^j\}_j$ *are altered but world prices are unchanged. By the envelope theorem, welfare is unchanged at a first-order approximation.*

Proof. If the planner implemented the unilaterally optimal increase $d\xi^{i*}_{t+1} > 0$ in capital account interventions of country i, then world prices would move by $dQ = -\omega^i \left(M_Q\right)^{-1} M^i_\xi d\xi^{i*}$. According to Lemma 2, the move in the interest rate can be undone if the capital controls of all countries $j = 1...N$ are simultaneously adjusted by $-\left(M^j_\xi\right)^{-1} M^j_Q dQ$, which delivers the first equation of the proposition. The second equation is obtained by adding the optimal unilateral change in intervention $d\xi^{i*}$ plus the adjustment given by the first equation with $j = i$. In the resulting equilibrium, the increase in the externality $d\xi^{i*}$ is corrected but the world interest rate is unchanged.

Furthermore, by the envelope theorem, for constant world prices, the change in welfare that results from a marginal change in M^j is[7]

$$dV^j\big|_{dQ=0} = \left(V_m^i + V_M^i\right) dM^j = 0$$

■

Numerical Illustration Let us illustrate the mechanics of Pareto-improving capital controls by returning to the numerical illustration in section **??**. Assuming a world economy in the steady state described there, an increase in the externality in country i that would call for an optimal unilateral response $d\tau_{t+1}^{i*}$ in the country's level of capital controls can also be corrected by setting

$$\begin{aligned} \frac{d\tau_{t+1}^{i}}{d\tau_{t+1}^{i*}} &= 1-\omega^i \\ \frac{d\tau_{t+1}^{j}}{d\tau_{t+1}^{i*}} &= -\omega^i \end{aligned}$$

In short, the country that experiences the externality corrects only a fraction $(1-\omega^i)$ of it and the rest of the world imposes a capital control to correct the remaining fraction ω^i corresponding to the country's weight in the world economy. For example, small open economies would meet the burden of adjustment by themselves since $\omega^i = 0$ and they do not affect the world interest rate. For large economies, we refer to the country weights implied by Table **??** on page **??**. For example, if China experienced a positive externality from current account surpluses that calls for a 1% unilateral subsidy to capital outflows, then a global planner who follows the described scheme would impose a 0.90% subsidy on outflows in China and a 0.10% subsidy to inflows in the rest of the world to keep the world interest rate unchanged. Similarly, if Brazil experienced a –1% externality from capital inflows, the global planner would impose a 0.97% tax on inflows to Brazil and a 0.03% tax on outflows in the rest of the world in order to keep the world interest rate stable.

In Figure **??** on page **??**, a national planner corrects for a negative externality to borrowing τ^{i*} in country i. A global planner could achieve a Pareto improvement by splitting the burden of regulating capital flows between the two countries. He would tax outflows in country j such that $1-\tau^j = R^{NP}/R^{LF}$ and tax inflows for the remaining part of the externality such that $1-\tau^i = \frac{1-\tau^{i*}}{1-\tau^j}$ in country i. As a result, the interest rate would be unchanged at R^{LF} and the welfare loss by country i,

[7]For non-infinitesimal changes in ξ^i, changes in net imports ΔM^j have second-order effects on welfare (i.e. effects that are negligible for infinitesimal changes but growing in the square of ΔM^j) even if world prices are held constant. Under certain conditions, e.g. if there are only two types of countries in the world economy, a global planner can undo these second-order effects via further adjustments in the world prices Q.

indicated by the shaded area in the figure, would be limited to the Harberger triangle between b^{NP}, b^{LF} and R^{LF}.[8]

Such a policy response shares certain characteristics with voluntary export restraints (VERs) in trade policy: If a borrowing country imposes controls on capital inflows, the world interest rate will decline and all lending countries experience negative wealth effects. However, if lenders restrict outflows by imposing controls of their own, they can keep the surplus. A global planner would share the burden of adjustment between borrower and lender in proportion to their elasticities of demand and supply so as to keep the world interest rate constant.

5 Examples and Applications

This section investigates several examples of externalities that have been used in the literature and in policy circles to motivate capital account intervention and that are relevant for our analysis of spillover effects and efficiency of equilibrium. We start with two simple examples of learning externalities that are triggered either by exporting or by producing and in which capital account interventions represent first-best and second-best policy instruments, respectively. Then we analyze aggregate demand externalities that may occur if a country experiences a liquidity trap.

Even if one is skeptical of the existence of some of the described externalities, these are important question to analyze since policymakers have explicitly invoked such externalities when they engaged in capital account interventions, exemplified by numerous comments of the Brazilian finance minister Guido Mantega (see Wheatley and Garnham, 2010).

5.1 Learning-by-Exporting Externalities

Our first and simplest example are learning-by-exporting externalities.[9] Assume a representative agent in an economy i that behaves as in our baseline model, except that the endowment income y^i_{t+1} is a function $\varphi_t(\cdot)$ of the economy's aggregate net imports M^i_t that satisfies $\varphi_t(0) = 0$ and that is continuous and decreasing $\varphi'_t(M_t) \leq 0$ to capture that higher exports increase growth,

$$y^i_{t+1} = y^i_t + \varphi_t\left(M^i_t\right) \tag{15}$$

[8] Since there are only two countries in this example, country i could be compensated for this second order loss by raising the interest rate on the remainder of its bond holdings, as described in Lemma 2, achieving an unambiguous Pareto improvement.

[9] There is a considerable theoretical literature that postulates that such effects are important for developing countries in the phase of industrialization. See for example Rodrik (2008) and Korinek and Servén (2010). In the empirical literature there have been some studies that document the existence of learning externalities, whereas others are more skeptical. For a survey see e.g. Giles and Williams (2000).

The reduced-form utility of a representative agent in country i is

$$V^i\left(m^i, M^i\right) = \sum_t \beta^t u\left(y_0 + \sum_{s=0}^{t-1} \varphi_s\left(M_s^i\right) + m_t^i\right)$$

with marginal utility of private and aggregate capital inflows of $V_{m,t}^i = \beta^t u'\left(C_t^i\right)$ and $V_{M,t}^i = \varphi_t'\left(M_t^i\right)\beta^t v_{t+1}$ where $v_{t+1}^i = \sum_{s=t+1}^{\infty} \beta^{s-t} u'\left(C_s^i\right)$ is the PDV of one unit of output growth at time $t+1$, capturing the growth externalities from exporting. Following lemma 1, a planner can implement the socially efficient allocation in economy i by imposing capital controls

$$\xi_t^i = -V_{M,t}^i / V_{m,t}^i = -\frac{\varphi_t'\left(M_t^i\right) v_{t+1}^i}{u'\left(C_t^i\right)} \geq 0 \tag{16}$$

The planner subsidizes exports (taxes imports) of capital in periods in which net exports generate positive externalities.

Observe that capital control are the first-best policy tool to internalize learning-by-exporting externalities in this framework, since they directly target net saving and hence the trade balance of the economy.

Since our model of learning-by-exporting externalities nests into the general model of section 2, it is a straightforward application of proposition 1 that the intervention of a competitive planner to internalize such externalities leads to a Pareto efficient outcome from a global perspective.

5.2 Learning-by-Doing Externalities

Capital account intervention may also serve as a second-best instrument in an economy where it would be desirable to use domestic policy measures to correct a distortion but such measures are not available.

The following example show how capital controls may serve to internalize learning-by-doing externalities in a production economy in which productivity growth is an increasing function of employment. The first-best policy instrument in such a setting is a subsidy to employment. However, if such an instrument is not available (for example, because of a lack of fiscal resources, a large informal sector, or the risk of corruption), it may be optimal to resort to capital controls as a second-best instrument to improve welfare.

Assume that the output of a representative worker in economy i is given by $y_t^i = A_t^i \ell_t^i$, where labor ℓ_t^i imposes a convex disutility $d\left(\ell_t^i\right)$ on workers. We capture learning-by-doing externalities by assuming that productivity growth A_t^i in the economy is a continuous and increasing function of aggregate employment $\psi_t\left(L_t^i\right)$ that satisfies $\psi_t'(\cdot) \geq 0$ so that

$$A_{t+1}^i = A_t^i + \psi_t\left(L_t^i\right) = A_0^i + \sum_{s=0}^{t} \psi_s\left(L_s^i\right) \tag{17}$$

In the described economy, the first-best policy instrument to internalize such learning effects would be a subsidy s_t^i to wage earnings in the amount of $s_t^i = \psi_t'\left(L_t^i\right) v_{A,t+1}/[u'\left(c_t^i\right) A_t^i]$ where $v_{A,t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} u'\left(c_s^i\right) L_s^i$ is the PDV of one unit of productivity growth starting period $t+1$.

In the absence of a policy instrument to target the labor wedge, a planner faces the implementability constraint

$$A_t^i u'\left(A_t^i L_t^i + M_t^i\right) = d'\left(L_t^i\right) \tag{18}$$

which reflects the optimal labor supply condition of individual workers. Observe that reducing M_t^i in this constraint is akin to a negative wealth effect and increases the marginal utility of consumption, which in turn serves as a second-best instrument to raise L_t^i and trigger learning-by-doing externalities.

Accounting for this implementability constraint and imposing the consistency condition $\ell_t^i = L_t^i$, a constrained planner recognizes that the reduced-form utility of the economy is

$$V\left(m^i, M^i\right) = \max_{L_t^i} \sum_t \beta^t \left\{u\left(A_t^i L_t^i + m_t^i\right) - d\left(L_t^i\right)\right\} \quad \text{s.t.} \quad (17), (18)$$

with marginal utility of private and aggregate capital inflows of $V_{m,t}^i = \beta^t u'\left(C_t^i\right)$ and $V_{M,t}^i = -\lambda_t^i \beta^t A_t^i u''\left(C_t^i\right) < 0$ where λ_t is the shadow price on the implementability constraint (18) and is given by

$$\lambda_t^i = \frac{\psi_t'\left(L_t^i\right) v_{A,t+1}}{d''\left(L_t^i\right) - \left(A_t^i\right)^2 u''\left(C_t^i\right)} > 0$$

In this expression, the positive learning externalities (in the numerator) are scaled by a term that reflects how strongly labor supply responds to changes in consumption (in the denominator).[10]

Following lemma 1, the planner can implement this second-best solution by imposing capital controls of

$$\xi_t^i = -\frac{\lambda_t A_t^i u''\left(C_t^i\right)}{u'\left(C_t^i\right)} = -\frac{\psi_t'\left(L_t^i\right) v_{A,t+1}}{u'\left(C_t^i\right)} \cdot \frac{A_t^i u''\left(C_t^i\right)}{d''\left(L_t^i\right) - \left(A_t^i\right)^2 u''\left(C_t^i\right)} \tag{19}$$

[10] Given that there are no first-best policy instruments available, the PDV of one unit of productivity growth also includes the effects of higher productivity on future labor supply decisions: on the one hand, higher productivity increases incentives to work, on the other hand it makes the agent richer and reduces the incentive to work via a wealth effect. The two effects are captured by the two expressions in square brackets,

$$v_{A,t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} \left\{u'\left(C_s^i\right) L_s^i + \lambda_s^i \left[u'\left(C_s^i\right) + A_s^i u''\left(C_s^i\right) L_s^i\right]\right\}$$

If the economy has outgrown its learning externalities, the term drops to zero.

This control reduces capital inflows and stimulates demand production, which in turn triggers learning-by-doing externalities. Observe that the first term in the expression is analogous to the optimal capital control (16) under learning-by-exporting. Using the optimality condition on labor (18), the second term can be approximated by $-\frac{1}{A_t^i[\sigma_C/\eta_L+1]}$ for small M_t^i, where η_L and σ_C are the Frisch elasticity of labor supply and the intertemporal elasticity of substitution: the second-best intervention is more desirable the more responsive the marginal utility of consumption (low σ_C) and the less responsive the marginal disutility of labor (high η_L).

Proposition 1 implies, as in the previous case, that the application of second-best capital controls lead to a globally Pareto efficient outcome. Intuitively, even though capital controls (19) are just second-best instruments, they are chosen so as to equate the marginal social benefit from indirectly triggering the LBD-externality to their marginal social cost. Reducing domestic consumption by running a trade surplus is the only way available to induce domestic agents to work harder, given the restrictions on policy instruments. Since a global planner does not have superior instruments, he cannot do better than this and chooses an identical allocation.

Remark There has been a lively debate on the multilateral desirability of capital account intervention (see e.g. IMF, 2012; Forbes et al., 2012; Ostry et al., 2012). This debate has sometimes suggested that interventions that are of a second-best nature, such as those to internalize LBD externalities in our example above, should be viewed with particular skepticism. However, our general model demonstrates that the global efficiency implications of second-best capital controls to internalize LBD-externalities are no different from other reasons to implement capital controls, given the restrictions on the set of policy instruments. A global planner who faces the same constraints on his policy instruments would implement an identical allocation.

5.3 Aggregate Demand Externalities at the ZLB

Next we study the multilateral implications of capital controls to counter aggregate demand externalities at the zero lower bound (ZLB) on nominal interest rates. We develop a stylized framework that captures the essential nature of such externalities in the spirit of Krugman (1998) and Eggertsson and Woodford (2003), adapted to an open economy framework as in Jeanne (2009).

Assume that a representative consumer in country i derives utility from consuming c_t^i units of a composite final good and experiences disutility from providing ℓ_t^i units of labor. Collecting the two time series in the vectors c^i and ℓ^i, we denote

$$U^i\left(c^i,\ell^i\right)=\sum\beta^t\left[u\left(c_t^i\right)-d\left(\ell_t^i\right)\right]$$

As is common in the New Keynesian literature, we assume that there is a continuum $z\in[0,1]$ of monopolistic intermediate goods producers who are collectively owned by consumers and who each hire labor to produce an intermediate good of variety z according to the linear function $y_t^{iz}=\ell_t^{iz}$, where labor market clearing requires

$\int \ell_t^{iz} dz = \ell_t^i$. All the varieties are combined in a CES production function to produce final output

$$y_t^i = \left(\int_0^1 \left(y_t^{iz} \right)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where the elasticity of substitution is $\varepsilon > 1$. We assume that the monopoly wedge arising from monopolistic competition is corrected by a proportional subsidy $\frac{1}{\varepsilon-1}$ that is financed by a lump-sum tax on producers. This implies that the wage income and profits of the representative agent equal final output, which in turn equals labor supply $w_t \ell_t^i + \pi_t^i = y_t^i = \ell_t^i$. In real terms and vector notation, the period budget constraints of a representative agent and the external budget constraint are given by

$$c^i = w\ell^i + \pi^i + m^i = y^i + m^i \qquad \text{and} \qquad \frac{Q}{1-\xi^i} \cdot m^i - T^i \leq b_0^i$$

The condition for the optimal labor supply of the representative agent is

$$d'\left(\ell_t^i\right) = w_t^i$$

We assume that the nominal price of one unit of consumption good follows an exogenous path $P^i = (1, P_2^i, P_3^i, ...)$ that is credibly enforced by a central bank (see Korinek and Simsek, 2013, for further motivation). This assumption precludes the central bank from committing to a future monetary expansion or future inflation in order to stimulate output in the present period.[11] The corresponding gross rate of inflation is given by $\Pi_{t+1}^i = P_{t+1}^i / P_t^i$ or by $\Pi^i = P/L(P)$ in vector notation with lag operater $L(\cdot)$. One example is a fixed inflation target $\Pi_{t+1}^i = \bar{\Pi}^i \ \forall \ t$.

Combining the ZLB constraint on the nominal interest rate $i_{t+1}^i = R_{t+1}\Pi_{t+1} - 1 \geq 0$ with the aggregate Euler equation to substitute for R_{t+1}, the ZLB in period t imposes a ceiling on aggregate period t consumption,

$$u'\left(C_t^i\right) \geq \frac{\beta}{\Pi_{t+1}^i} u'\left(C_{t+1}^i\right) \tag{20}$$

Intuitively, a binding ZLB implies that consumption it too expensive in period t compared to consumption in the following period, limiting aggregate demand in period t to the level indicated by the constraint.

In the laissez-faire equilibrium, this constraint is satisfied with strict inequality if world aggregate demand for bonds and by extension the world interest rate is sufficiently high, i.e. if $R_{t+1} \geq 1/\Pi_{t+1}^i$. Then the market-clearing wage $W_t^i = 1$ will prevail and output Y_t^i is at its efficient level determined by the optimality condition $u'(C_t^i) = d'(L_t^i)$. We call this output level potential output Y_t^{i*}.

[11] It is well known in the New Keynesian literature that the problems associated with the zero lower bound could be avoided if the monetary authority was able to commit to a higher inflation rate. See e.g. Eggertsson and Woodford (2003).

If worldwide aggregate demand declines and the world real interest rate hits the threshold $R_{t+1} = 1/\Pi^i_{t+1}$, then the ZLB on the nominal interest rate is reached and the domestic interest rate cannot fall any further. Instead, any increase in the world supply of bonds will flow to economy i, which pays a real return of $1/\Pi^i_{t+1}$ by the feature of offering liabilities with zero nominal interest rate. Given the high return on nominal bonds, consumers in economy i find that today's consumption goods are too expensive compared to tomorrow's consumption goods and consumers reduce their aggregate demand for today's consumption goods. Output is demand-determined, Y^i_t falls below potential output Y^{i*}_t in order to satisfy equation (20) and the wage falls below $W^i_t < 1$. This situation captures the essential characteristic of a liquidity trap: at the prevailing nominal interest rate of zero, consumers do not have sufficient demand to absorb both domestic output and the capital inflow M^i_t. Intermediate producers cannot reduce their prices but let domestic output adjust so that demand equals supply. If the domestic real interest rate could fall, consumers would have incentive to consume more in period t and the problem would be solved. We will show in the following that a planner in such a situation finds it optimal to erect barriers against capital inflows or encourage capital outflows in order to stimulate domestic aggregate demand.

We substitute the domestic period budget constraint and the consistency condition $x^i = X^i$ and denote the reduced-form utility maximization problem of a planner in country i as defined in section 3 by

$$V\left(m^i, M^i\right) = \max_{L^i} \sum \beta^t \left[u\left(C^i_t\right) - d\left(L^i_t\right)\right] \quad \text{s.t.} \qquad u'\left(L^i + M^i\right) \geq \frac{\beta}{\Pi^i_{t+1}} u'\left(L^i_{t+1} + M^i_{t+1}\right) \forall t$$
$$d'\left(L^i_t\right) \leq u'\left(L^i_t + M^i_t\right) \forall t$$

where the last constraint ensures that the central bank follows a time-consistent policy and cannot commit to future monetary expansion to induce workers to produce more, as we assumed earlier.

Assigning the shadow prices $\beta^t \mu_t$ and $\beta^t \gamma_t$ to the two constraints, the associated optimality conditions are

$$FOC\left(L^i_t\right) : u'\left(C^i_t\right) - d'\left(L^i_t\right) + \left[\mu_t - \mu_{t-1}/\Pi^i_t\right] u''\left(C^i_t\right) - \gamma_t \left[d''\left(L^i_t\right) - u''\left(C^i_t\right)\right] = 0$$

When the ZLB constraint is loose, the shadow prices μ_t and γ_t are zero. If the ZLB is binding in period t, then $\mu_t = \frac{u'\left(C^i_t\right) - d'\left(L^i_t\right)}{-u''\left(C^i_t\right)} > 0$ reflects the labor wedge in the economy created by the lack of demand and the second constraint is trivially satisfied so $\gamma_t = 0$. If the ZLB is loose in the ensuing period $t+1$, then the planner would like to commit to stimulate output in that period as captured by the term $-\mu_t/\Pi^i_{t+1} u''\left(C^i_{t+1}\right)$ so as to relax the ZLB constraint at date t, but we imposed the second constraint to reflect that the planner cannot commit to do this. Therefore $u'\left(C^i_{t+1}\right) = d'\left(L^i_{t+1}\right)$ in that period and the shadow price γ_{t+1} adjusts so that the optimality condition is satisfied $\gamma_{t+1} = \frac{-\mu_t u''\left(C^i_{t+1}\right)/\Pi^i_{t+1}}{d''\left(L^i_{t+1}\right) - u''\left(C^i_{t+1}\right)} > 0$.

The externalities of capital inflows in periods t and $t+1$ in such an economy are given by the partial derivatives

$$V_{M,t}(\cdot) = \left[\mu_t - \mu_{t-1}/\Pi_t^i + \gamma_t\right] u''\left(C_t^i\right)$$

If the economy experiences a liquidity trap in period t but has left the trap in period $t+1$, then $V_{M,t} = \mu_t u''(C_t^i) = -\left[u'(C_t^i) - d'(L_t^i)\right] < 0$ – the externality from a unit capital inflow is to reduce aggregate demand by one unit, which wastes valuable production opportunities as captured by the positive labor wedge $u'(C_t^i) - d'(L_t^i)$. It is optimal to set the policy instrument $\xi_t^i = 1 - \frac{d'\left(L_t^i\right)}{u'\left(C_t^i\right)} > 0$ precisely such as to reflect this social cost, thereby restricting capital inflows or encouraging capital outflows.

In the following period, it is beneficial to commit to setting $\xi_{t+1}^i < 0$ so as to restrict capital outflows or subsidize capital inflows since

$$V_{M,t+1} = -\mu_t/\Pi_{t+1}^i \left[\frac{d''\left(L_{t+1}^i\right)}{d''\left(L_{t+1}^i\right) - u''\left(C_{t+1}^i\right)}\right] u''\left(C_{t+1}^i\right) > 0$$

This has the effect of raising future consumption, which stimulates consumption during the liquidity trap by relaxing the ZLB constraint (20).[12]

Note that the capital account interventions of a planner in this setting are second-best policies since the first-best policy would be to restore price flexibility to abolish the ZLB constraint. The planner solves the optimal trade-off between foregoing efficient consumption smoothing opportunities by trading with foreigners and wasting profitable production opportunities because of the ZLB.

5.4 Level of Fiscal Spending

[To be written up.]

5.5 Exchange Rate Intervention

[To be written up.]

[12] In a time-consistent setting for capital account interventions, the planner would not be able to commit to future policy actions. The intervention during a liquidity trap would still be given by the same expression $V_{M,t} = d'\left(L_t^i\right) - u'\left(C_t^i\right)$, but after the liquidity trap has passed the planner would find $V_{M,t+1} = 0$ and no further intervention would occur.

6 Monopolistic Behavior

6.1 Optimization Problem

Assume next that there is a monopolistic planner in country i with positive mass $\omega^i > 0$ that maximizes the utility of the representative consumer U^i and internalizes that she has market power over world prices Q. We solve the problem of the monopolistic planner under the assumption that the remaining countries $j \neq i$ behave according to the competitive planning setup in section 3.2.[13]

Global market clearing requires that

$$\omega^i M^i + M^{-i}(Q) = 0 \qquad \text{with} \qquad M^{-i}(Q) = \sum_{j \neq i} \omega^j M^j(Q)$$

where $M^{-i}(Q)$ denotes the excess demand of the rest-of-the-world excluding country i and the Hessian $(\partial M^{-i}/\partial Q)$ is a negative semidefinite square matrix that captures how changing world prices Q affect imports M. Along the diagonal, $\partial M_t^{-i}/\partial Q_t$ is negative for ordinary goods since the price needs to fall for the rest-of-the-world to absorb more of a good. Off the diagonal the derivative is negative for substitutes and positive for complements. The function can be inverted to obtain an inverse rest-of-the-world excess demand function $Q^{-i}(M^{-i})$.

A monopolistic planner solves the optimization problem

$$\max V^i\left(M^i, M^i\right) \quad \text{s.t.} \quad Q^{-i}(-\omega^i M^i) \cdot M^i \leq B_0^i$$

leading to the optimality condition

$$V_m^i + V_M^i = \lambda^i Q\left(1 - \mathcal{E}_{Q,M}^i\right) \quad \text{with} \quad \mathcal{E}_{Q,M}^i = \omega^i Q_M^{-i} M^i / Q \tag{21}$$

where $\mathcal{E}_{Q,M}^i$ represents the inverse supply elasticity of imports from the rest of the world and consists of four elements: the country weight ω^i reflects the country's market power in the world market; the Hessian square matrix $Q_M^{-i} = \partial Q^{-i}/\partial M^{-i}$ captures how much world market prices have to respond to absorb an additional unit of exports from country i; this is multiplied by the vector M^i to obtain the marginal revenue accruing to country i as a result of monopolistic distortions; finally the expression is divided element-by-element by the vector Q in order to normalize it and obtain a vector of elasticities.

Lemma 3 (Monopolistic Capital Account Intervention) *The allocation of the monopolistic planner who internalizes her country's market power over world prices*

[13] Our findings can easily generalized to the case where other countries engage in monopolistic behavior or operate under laissez-faire. The only important assumption for our problem is that each country j has a well-defined and continuous demand function. This rules out, for example, discontinuous trigger strategies.

can be implemented by setting the vector of external policy instruments to

$$1 - \xi^{i,m} = \frac{1 + V_M^i / V_m^i}{1 - \mathcal{E}_{Q,M}^i} \tag{22}$$

where all divisions are performed element-by-element.

Proof. The tax $\xi_{t+1}^{i,m}$ ensures that the private optimality condition of consumers (10) replicates the planner's Euler equation (21). ■

To provide some intuition, assume that the matrix Q_M^{-i} in a country with $\omega^i > 0$ is a diagonal matrix and that there are no domestic externalities so $V_M^i = 0$.[14] Then all diagonal entries will satisfy $\partial Q_t^{-i} / \partial M_t^{-i} < 0$, reflecting that greater imports M_t^{-i} of good t require a lower world price Q_t.

If the country is a net importer $M_t^i > 0$ of good t then the elasticity $\mathcal{E}_{Q,M,t}^i$ will be negative and the optimal monopolistic tax on imports $\xi_t^{i,m} > 0$ is positive. Similarly, for goods that are net exports $M_t^i < 0$ the planner will reduce the quantity exported by a tax $\xi_t^{i,m} < 0$.

Proposition 3 (Inefficiency of Exerting Market Power) *An equilibrium in which domestic planners impose capital controls to exert market power is Pareto-inefficient.*

Proof. The result is a straightforward application of proposition 1 that optimality requires competitive behavior. ■

Returning to equation (21), a monopolistic planner equates the social marginal benefit of imports $V_m^i + V_M^i$ to the marginal expenditure $Q\left(1 - \mathcal{E}_{Q,M}^i\right)$ rather than to the world price Q times a factor of proportionality Λ_e^i. She intervenes up to the point where the marginal benefit of manipulating world prices – captured by the elasticity term $\Lambda_e^i Q \mathcal{E}_{Q,M}^i$ – equals the marginal cost of giving up profitable consumption opportunities $\Lambda_e^i Q - (V_m^i + V_M^i)$. However, giving up profitable consumption opportunities creates a deadweight loss – the planner introduces a distortion to extract monopoly rents from the rest of the global economy. The intervention constitutes a classic beggar-thy-neighbor policy.

6.2 Distinguishing Competitive and Monopolistic Behavior

The spillover effects of capital account intervention are the same, no matter what the motive for intervention. However, whether capital account interventions that are unilaterally imposed lead to Pareto efficient outcomes depends crucially on whether

[14] This would be the case, for example, if the reduced-form utility $V\left(m^i, M^i\right)$ is Cobb-Douglas in m^i. Otherwise, the off-diagonal elements of Q_M^{-i} reflect how changes in exports in one good affect the price of other goods and are positive for goods that are complements and negative for goods that are substitutes.

they correct for domestic distortions (as described in lemma 1) or whether they serve to exert market power (as described in lemma 3). Distinguishing between the two motives for intervention is therefore an essential step in determining whether global allocations are Pareto efficient or whether there is scope for global cooperation to achieve Pareto improvements.

Unfortunately there is no general recipe for answering this question. It is easy for policymakers to claim various market imperfections, domestic objectives or different political preferences to justify an arbitrary set of policy interventions in the name of domestic efficiency, and it is close to impossible for the international community to disprove them. Specifically, for any reduced-form utility function $V^i(m^i, M^i)$ and monopolistic interventions $\xi^{i,m}$ as described by lemma 3, there exists an alternative reduced-form utility function $\tilde{V}^i(m^i, M^i)$ such that the described interventions implement the optimal competitive allocation defined by

$$\tilde{V}^i(m^i, M^i) = V^i(m^i, M^i) - \xi^{i,m} \cdot V^i_m(\cdot) \cdot M^i$$

The reduced-form utility function $\tilde{V}^i(\cdot)$ can in turn be interpreted as deriving from a fundamental utility function $\tilde{U}^i(x^i)$ and a set of constraints $\tilde{f}^i(\cdot)$ that justify it.

Nonetheless, the direction of optimal monopolistic policy interventions is often instructive to determine whether it is plausible that a given intervention is for domestic or monopolistic reasons. In the following, we describe optimal monopolistic capital account interventions along a number of dimensions. If the observed interventions of a policymaker are inconsistent with these observations, then they are likely not for monopolistic reasons. Recall the definition of the elasticity $\mathcal{E}^i_{Q,M} = \omega^i Q^{-i}_M M^i/Q$, and let us discuss the various parameters:

Country Size ω^i The optimal monopolistic intervention is directly proportional to the country's weight ω^i in the world economy. Larger countries have a greater impact on the rest of the world since market clearing requires $M^{-i} = \omega^i M^i$. For example, if a small open economy with $\omega^i \approx 0$ and undifferentiated exports regulates capital in- or outflows, the reason cannot be monopolistic.

Responsiveness of Price Q^{-i}_M The second factor determining the optimal monopolistic intervention is the responsiveness of the world market price to changes in consumption, given by the Hessian $Q^{-i}_M = \partial Q/\partial M^{-i}$.

Direction and Magnitude of Flows M^i The intervention to manipulate a given price Q_t is directly proportional to the magnitude of a country's net imports M^i_t in that time period/good/state of nature and has the opposite sign of M^i_t as it aims to reduce the value of net imports $M^i_t > 0$ and increase the value of net exports $M^i_t < 0$. The larger M^i_t in absolute value, the greater the benefits from distorting the price Q_t and the more advantageous to engage in monopoly behavior. By contrast, if $M^i_t \approx 0$, the optimal monopolistic intervention is zero.

The direction and magnitude of flows has the following implications for monopolistic behavior:

- **Intertemporal trade:** If the different elements of M^i capture the trade balance $M_t^i = B_{t+1}^i/R - B_t^i$ and there is a single good per time period, then a zero trade balance $M_t^i \approx 0$ makes it impossible to engage in monopolistic behavior. By contrast, monopolistic reasons may be involved if a country with a large current account deficit $M_t^i > 0$ taxes inflows $\xi_t^i > 0$ to keep world interest rates lower or a country with a large current account surplus $M_t^i < 0$ restricts outflows $\xi_t^i < 0$ to push up interest rates.
- **Risk-sharing:** In a model of uncertainty in which $M_t^i(s_t)$ denotes different states of nature, each country has – by definition – monopoly power over its own idiosyncratic risk. Optimal risk-sharing implies greater imports in bad states and greater exports in good states of nature. A planner who exerts monopoly power would restrict risk-sharing so as to obtain a higher price for the country's idiosyncratic risk and to reduce the price of insurance from abroad. By contrast, if a country encourages insurance (e.g. by encouraging FDI and forbidding foreign currency debt; see Korinek, 2010), then the motive is unlikely to be monopolistic.
- **Intratemporal trade:** Exercising monopoly power in intratemporal trade consists of tariffs $\xi_{t,k}^i > 0$ on imported goods k with $M_{t,k}^i > 0$ and taxes on exports $\xi_{t,k}^i < 0$ for $M_{t,k}^i < 0$, as is well known from a long literature on trade policy (see e.g. Bagwell and Staiger, 2002).

In all these cases, observe that the optimal monopolistic intervention typically reduces the magnitude of capital or goods flows but does not change their direction.

It is straightforward that any price intervention ξ^i can also be implemented by an equivalent quantity restriction $\bar{M}^i$, for example by imposing a ceiling on capital inflows rather than a tax on inflows. Furthermore, under closed capital accounts, optimal monopolistic capital controls are isomorphic to reduced reserve accumulation/decumulation.[15]

6.3 Numerical Illustration

In the following we illustrate the potential for monopolistic capital account interventions in our baseline model of section 2, in which we assumed a single homogenous

[15] For example, when a country that accumulates reserves is concerned that it is not earning "sufficient" interest on its reserves because its accumulation is pushing down the world interest rate, this is non-competitive behavior and is equivalent to monopolistic capital controls.

good every period. This focuses our analysis on monopolistic distortions on a country's intertemporal terms-of-trade and complements the traditional focus in the trade literature on intratemporal terms-of-trade (see e.g. Bagwell and Staiger, 2002). Since monopolistic incentives in this framework depend on the sign and magnitude of the trade balance, we parameterize the model to replicate the trade balance of each country at $t = 2012$. For simplicity, we assume...

We parameterize $y_t^i \gtrless y_{t+1}^i$ such that each country replicates its 2012 trade balance tb_t^i at time t, as reported in Table 2, in order to smooth consumption. For simplicity, we assume zero initial wealth $b_t^i = 0$ and a constant endowment $y_s^i = 1$ for all future time periods $s \geq t+1$ so that $b_{t+1}^i / R_{t+1} = tb_t^i$ and so that all variables can be interpreted as fractions of long-run income. This implies that the world economy will be in a steady state with $\beta R = 1$ and with constant consumption in each country $c_s^i = y_s^i + (1-\beta)\, b_s^i$ for $s \geq t+1$. Relative country size is accounted for by the parameter ω^i.

We solve for the optimal monopolistic behavior of a country i under the assumption that all other countries $j \neq i$ operate under laissez-faire. Country i therefore faces an indirect period t demand $R_{t+1}^{-i}\left(b_{t+1}^i\right)$ that satisfies

$$\frac{dR_{t+1}}{db_{t+1}^i} = -\frac{\omega^i}{\sum_{j\neq i}\omega^j b_R^j} \approx -\frac{\omega^i}{\beta\sigma\left(1-\omega^i\right)}$$

at a first-order approximation around $b = 0$. In analogy to (21), the Euler equation in country i under monopoly power is

$$\left(1 + \frac{\omega^i b_{t+1}^i}{\left(1-\omega^i\right)\sigma}\right) u'\left(c_t^i\right) = \beta R_{t+1} u'\left(c_{t+1}^i\right)$$

A monopolistic planner can implement her optimal allocation by setting a capital control

$$\tau_{t+1}^{i,m} = -\frac{\omega^i}{1-\omega^i}\frac{tb_{t+1}^i R_{t+1}}{\sigma}$$

For our earlier value of the intertemporal elasticity of substitution $\sigma = 1/2$ and for small countries, the monopolistically optimal capital control is approximately twice the product of a country's size and its trade balance/GDP ratio. The resulting change in its trade balance is $\Delta^m tb_t^i = \left(1-\omega^i\right)\sigma\tau_{t+1}^{i,m}/R_{t+1}$.

We report the resulting monopolistic capital controls and effects on the trade balance in Table 2. Countries of a large size or with a large trade balance have a strong motive for monopolistic interventions. For example, China made up 11.4% of the world economy and had a trade surplus of \$232bn or 2.82% of GDP in 2012. If it optimally exerted market power over the world interest rate, it would have imposed a capital outflow tax of 0.75% (under open capital accounts) or it would have accumulated \$26bn less in foreign reserves (under closed capital accounts) compared to its optimal behavior under price-taking. As a result, the world interest rate would have

Country	y^i	ω^i	tb^i	tb^i/y^i	$\tau^{i,m}$	$\Delta^m tb^i$	$\Delta^m R$
World	\$72,216bn	...	...	...	...	...	...
United States	\$16,245bn	22.5%	-535bn	-3.29%	-1.99%	120.4bn	-0.47%
China	\$8,221bn	11.4%	232bn	2.82%	0.75%	-26.4bn	0.09%
Japan	\$5,960bn	8.3%	-104bn	-1.74%	-0.33%	8.6bn	-0.03%
Brazil	\$2,253bn	3.1%	-22bn	-0.96%	-0.06%	0.7bn	0.00%
India	\$1,842bn	2.6%	-136bn	-7.39%	-0.40%	3.5bn	-0.01%
South Korea	\$1,130bn	1.6%	41bn	3.65%	0.12%	-0.6bn	0.00%
Indonesia	\$879bn	1.2%	-2bn	-0.25%	-0.01%	0.0bn	0.00%
Turkey	\$788bn	1.1%	-42bn	-5.38%	-0.12%	0.5bn	0.00%

Table 2: Monopolistically optimal capital controls for selected countries
(Source: 2012 data from IMF IFS and author's calculations)

been 9 basis points higher.[16] Similarly, as a large importer of capital, a monopolistic policymaker in the US would have found it optimal to impose a 2% tax on capital inflows.

The table highlights that it is difficult to reconcile the capital controls observed in the real world with monopolistic motives for most countries.

Figure 1 illustrates our results in a framework of two countries i and j of equal mass for a given time period. The line $R^i(b^i)$ represents the (inverse) supply of bonds, the two lines $R^j(-b^j)$ and $R^j(-b^j) - b^j R_b$ represent the demand for bonds as well as the 'marginal revenue' curve for country i that takes into account the decline in the interest rate from supplying additional bonds. The laissez faire equilibrium is characterized by an interest rate R^{LF} and bond positions $b^i = b^{LF} = -b^j$. A monopolistic planner in country i would reduce the quantity of bonds supplied to j such that her marginal valuation $R^i(b^i)$ equals the marginal revenue derived from country j. This monopolistic equilibrium is indicated by the quantity of bonds sold b^{MP} and interest rate R^{MP}. The described policy shifts the surplus between R^{MP} and R^*, marked by the dotted area in the figure, from country j to country i. It also introduces a deadweight loss indicated by the triangular vertically-shaded area. Monopolistic capital controls constitute a classic beggar-thy-neighbor policy and are always inefficient: they introduce a distortion into the Euler equation of domestic agents, which reduces global welfare, in order to shift welfare from foreigners to domestic agents – the policy represents a "negative-sum" game overall.

[16] There are statements of Chinese policymakers that they are reducing reserve accumulation because they recognize the impact of their reserve accumulation on US Treasury yields. This is clear evidence of monopolistic considerations.

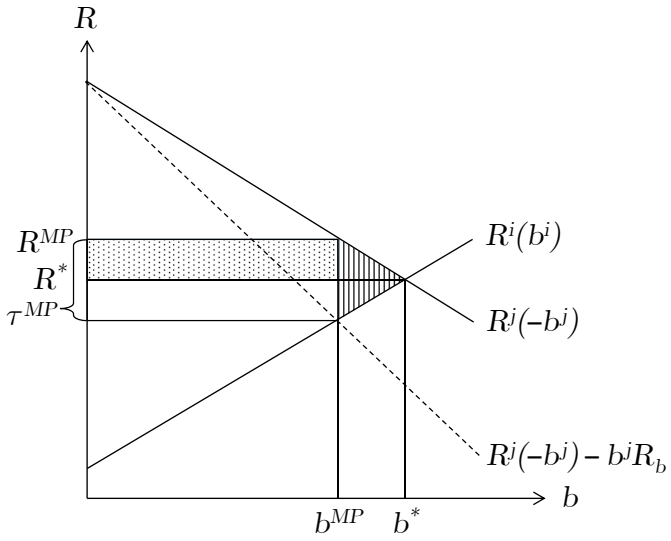


Figure 1: Optimal capital control imposed by a domestic planner to exert market power

6.4 Monopolistic Use of Domestic Policy Instruments

This section analyzes the optimal behavior of a planner who monopolistically distorts domestic policies ζ^i to improve her country's terms of trade. A planner will have incentives to do so if she faces restrictions on the set of external policy instruments, for example due to international agreements or due to technical difficulties in targeting or implementing capital controls. If the set of external policy instruments ξ^i is complete, then it would be suboptimal to distort domestic policies for monopolistic reasons, following the same arguments as in corollary 1(?).

For simplicity, we assume a monopolistic planner who does not have access to any external policy instruments so $\xi^i = 0$ and who faces no domestic targeting problems so that she can choose X^i without any restrictions. This allows us to include the vector X^i in the vector of policy instruments $\zeta^i = \{X^i, \tilde{\zeta}^i\}$ for ease of notation. A monopolistic planner under these assumptions solves[17]

$$\max_{M^i, \zeta^i, \lambda^i_e} U^i(\zeta^i) \qquad \text{s.t.} \qquad f^i(M^i, M^i, \zeta^i) \leq 0,$$
$$Q^{-i}\left(-\omega^i M^i\right) \cdot M^i \leq B^i_0, \qquad \lambda^i_d f^i_m = -\lambda^i_e Q^{-i}\left(-\omega^i M^i\right)$$

where the first constraint line contains the usual domestic constraint, and the two constraints in the second line reflect that the planner internalizes the effects of her external allocations on world market prices $Q^{-i}\left(-\omega^i M^i\right)$ and that the planner lacks

[17] We could also place a more general set of restrictions on the set of policy instruments, e.g. $\Xi\left(\xi^i\right) \leq 0$. Our main insight that a monopolistic planner would use domestic policy instruments to distort external allocations would continue to hold.

policy instruments to create a wedge in the external optimality condition of private agents.

The optimality conditions on the planner's allocations and policy instruments are

$$\begin{aligned} FOC\left(M^i\right) &: 0 = \Lambda_d^i f_{m+M}^i + \Lambda_e^i Q\left(1 - \omega^i \mathcal{E}_{Q,M}^{-i}\right) + \mu_e^i \left[\lambda_d^i f_{m(m+M)}^i - \omega^i \lambda_e^i Q_M^{-i}\right] \\ FOC\left(\zeta^i\right) &: 0 = \Lambda_d^i f_\zeta^i - \mu_d^i f_{x\zeta}^i + \mu_e^i \lambda_d^i f_{m\zeta}^i \end{aligned}$$

where we denote the sum of partial derivatives as $f_{x+X} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial X}$ and so on to condense notation. The optimality condition on λ_e^i implies that the price-weighted sum of shadow prices on the external implementability constraint (IC) is zero so $\sum_k \mu_{e,k}^i Q_k = 0$. With the exception of knife-edge cases (e.g. no monopoly power because $\omega^i = 0$), some of the shadow prices on the external IC will therefore be positive and some will be negative. In particular, if we collect the first two terms in the optimality condition on M^i in the vector $-V_{m+M}^i$, we can write the condition as

$$\mu_e^i = A^{-1}\left[V_{m+M}^i - \Lambda_e^i Q\left(1 - \omega^i \mathcal{E}_{Q,M}^{-i}\right)\right]$$

where $A = \lambda_d^i f_{m(m+M)}^i - \omega^i \lambda_e^i Q_M^{-i}$ is a negative semidefinite matrix. To interpret this expression, we assume that A is close to diagonal and that there are no domestic externalities so $V_m^i = V_{m+M}^i$.[18] In the absence of monopolistic behavior, the square brackets in this expression would be zero by the private optimality condition of consumers $V_m^i = \lambda_e^i Q$. By contrast, for a monopolistic planner, the marginal cost Q in this expression is replaced by the marginal expenditure $Q\left(1 - \omega^i \mathcal{E}_{Q,M}^{-i}\right)$. For each element k in the vector of imports, the sign of the elasticity $\mathcal{E}_{Q,M,k}^{-i}$ equals the sign of $M_k^i > 0$, e.g. for net imports the marginal expenditure is greater than the marginal cost and vice versa for net exports. The shadow prices $\mu_{e,k}^i$ capture the monopolistic cost (benefit) of increasing imports of good k and are therefore negative for net imports (positive for net exports).

[To be completed.]

7 Imperfect Instruments

This section analyzes imperfect policy tools and investigates under what circumstances such imperfections lead to a case for global coordination of capital control policies. In the previous section, we emphasized that the international spillover effects of a complete set of perfectly targeted capital controls constitute pecuniary externalities that are mediated through a well-functioning market and therefore lead to Pareto-efficient outcomes, as long as domestic policymakers act competitively.

[18] This assumption holds exactly if all utility functions are Cobb-Douglas. In the general case, the diagonal elements of the matrix are likely to be an order of magnitude larger than the off-diagonal elements unless as long as goods aren't strong complements or substitutes.

In practice policy instrument frequently differ from the perfect policy instruments that we have depicted in our analysis (see e.g. Carvalho and Marcio, 2006). In the following two subsections, we analyze two types of such distortions: implementation costs of capital controls and imperfect targeting of capital controls. We formalize both examples and analyze whether a global planner could achieve a Pareto improvement by coordinating the capital control policies of different countries in the presence of such ancillary distortions.

7.1 Costly Policy Instruments

The simplest specification of such a setup is to assume that policy instruments such as capital controls impose a resource cost $C^i(\tau)$ on the economy that represents enforcement costs or distortions arising from attempts at circumvention. Assume that the function $C^i(\cdot)$ is twice continuously differentiable and satisfies $C(0) = C'(0) = 0$ and $C''(\tau) > 0 \forall \tau$, i.e. it is convex.[19]

The optimization problem of a national policymaker, where we use the summary notation $W^i(b^i) = V^i(b^i; b^i)$, is then

$$\max_{b^i, c^i, \tau^i} u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i\left[c^i - y^i + \frac{b^i}{R} + C^i\left(\tau^i\right)\right] - \mu^i\left[\left(1-\tau^i\right)u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right]$$

The first-order conditions are

$$\begin{aligned} FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i/R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) \\ FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i\left(1-\tau^i\right)u''\left(c^i\right) \\ FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right) \end{aligned}$$

and can be combined to the optimality condition

$$u'\left(c^i\right) = \beta R W'(b)\,\frac{1 + \frac{\beta R V' u''}{(u')^2} C^{i\prime}}{1 - \frac{\beta R V''}{u'} C^{i\prime}} \tag{23}$$

We find:

Proposition 4 (Costly Policy Instruments) *If policy instruments impose a resource cost $C^i(\tau^i)$ as defined above and if $\xi^i \neq 0$, then a national planner imposes an optimal level of the same sign as ξ^i but of smaller absolute magnitude, i.e. τ^i satisfies $0 < |\tau^i| < \left|\xi^i\right|$.*

[19] Analogous results can be derived if the cost of capital controls is proportional to the amount of bond holdings, e.g. $c(\tau, b) = C(\tau b)$, which may specifically capture the costs associated with attempts at circumvention.

Proof. The planner implements the optimality condition (23) by setting the policy instrument in the decentralized optimality condition (**??**) to

$$\tau^i = \frac{\beta R \xi^i}{u'\left(c^i\right)} + \beta R C^{i\prime} \cdot \frac{V'u'' + u'V''}{\left(u'\right)^2 + \beta R V' u'' C'}$$

The first additive term corresponds to the optimal costless $\tilde{\tau}^i$. If this term is positive because the country experiences a negative externality $\xi^i > 0$ from capital inflows, then $C^{i\prime} > 0$ and the second additive term is negative, which mitigates the optimal magnitude of the instrument to $\tau^i < \tilde{\tau}^i$. (This holds as long as the denominator is positive, i.e. $\left(u'\right)^2 + \beta R V' u'' C' > 0$, which is satisfied as long as the marginal cost C' is not too large.) For $\xi^i > 0$, the second term never flips the sign of τ^i to make it negative. If it did, then $C^{i\prime}$ would switch sign as well and the second term would become positive, leading to a contradiction. The argument for $\xi^i < \tau^i < 0$ follows along the same lines. ■

7.2 Global Coordination of Costly Instruments

We next determine under what conditions the equilibrium in which each national planner imposes instruments according to equation (23) is globally Pareto efficient. In other words, if national planners follow the described rule, can a global planner achieve a Pareto improvement on the resulting equilibrium?

It turns out that the answer depends critically on the set of instruments available to the planner.

7.2.1 Global Coordination with Transfers

We analyze a global planner who maximizes global welfare in the described environment who has access to lump-sum transfers between countries. This implies that he is not bound by the period 1 budget constraints of individual countries and can undo the redistributions that stem from changes in the world interest rate.

Formally, a global planner maximizes the sum of the surplus of all nations for some set of welfare weights $\left\{\phi^i\right\}$. He internalizes that the world interest rate R is a choice variable and that the optimality conditions of individual agents (with shadow price μ^i) as well as global market clearing must hold, i.e. $\Sigma_i m^i b^i = 0$ (with shadow price ν). In addition, we include a transfer T^i in our optimization problem, which needs to satisfy global market clearing $\Sigma_i m^i T^i = 0$ (with shadow price γ). The associated Lagrangian is

$$\mathcal{L} = \sum_i \phi^i \left\{ u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i \left[c^i - y^i + b^i/R + C^i\left(\tau^i\right) - T^i\right] - \right.$$
$$\left. -\mu^i \left[\left(1-\tau^i\right) u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right] \right\} - \nu \sum_i m^i b^i - \gamma \sum_i m^i T^i$$

The first-order conditions of the global planner are

$$\begin{aligned}
FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i/R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) + m^i\nu/\phi^i \\
FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i\left(1-\tau^i\right)u''\left(c^i\right) \\
FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right) \\
FOC\left(R\right) &: \sum_i \phi^i\left\{\frac{\lambda^i b^i}{R} + \mu^i\left(1-\tau^i\right)u'\left(c^i\right)\right\} = 0 \\
FOC\left(T^i\right) &: \phi^i\lambda^i = \gamma m^i
\end{aligned}$$

The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights $\left\{\phi^i\right\}$ such that the allocations of national planners satisfy the maximization problem of the global planner. If we substitute the allocations from the Nash equilibrium, we find that the second and third optimality conditions are unchanged compared to the national planner's equilibrium and can be solved for λ^i and μ^i that are identical to the shadow prices in the Nash equilibrium between national planners. Substituting these in the optimality condition $FOC\left(b^i\right)$, we find that this condition is satisfied for all countries if we set $\nu = 0$. The fifth optimality condition is satisfied if we set $\phi^i = \gamma m^i/\lambda^i \forall i$. The Nash equilibrium among planners is therefore efficient if the described variables also satisfy the fourth optimality condition $FOC\left(R\right)$.

Proposition 5 (Coordination of Costly Instruments with Transfers) *If policy instruments to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a global planner who can engage in transfers if and only if the resulting allocation satisfies*

$$\sum_i m^i\left(1-\tau^i\right)C'\left(\tau^i\right) = 0 \tag{24}$$

Proof. The optimality condition (24) can be obtained by substituting $FOC\left(T^i\right)$ into the condition $FOC\left(R\right)$ and accounting for market clearing $\Sigma_i m^i b^i = 0$ as well as for $FOC\left(\tau^i\right)$. ■

In a Pareto-optimal allocation, the weighted average marginal distortion imposed by the policy instruments must be zero. If there are no externalities, this can be achieved by having zero instruments in all countries. Otherwise, the planner combines the policy instruments in a way that their weighted average marginal distortion is zero.

The planner's country weights ϕ^i do not show up in condition (24) since the condition is purely about efficiency, i.e. about minimizing the overall resource cost of imposing policy instruments. Since the planner has access to lump-sum transfers, she can undo any redistributions created by movements in the interest rate according to her welfare weights.

[To be completed.]

8 Conclusions

This paper has studied the effects of national economic policies in a general equilibrium model of the world economy and has delineated under what conditions economic policy coordination is desirable from a global welfare perspective. In our positive analysis, we found that the policy interventions of national economic policymakers can have significant spillover effects on other countries.

In our normative analysis, however, we emphasized that these spillover should generally be viewed as pecuniary externalities that are efficient as long as (i) national policymakers are price-takers in the international market, (ii) they possess a complete set of instruments to control transactions with the rest of the world so they can actually implement their preferred choices and (iii) international markets are complete. Under these conditions, the first welfare theorem applies at the level of national economic policymakers.

We then illustrated our result in the context of a number of different motives for intervening in external transactions. Generally, we found that if a national economic policymaker intervenes to combat national externalities, then the interventions are Pareto efficient from a global welfare perspective and there is no need for global coordination of such policies.

Our analysis provides conceptual foundations for determining under what circumstances international policy cooperation is warranted, but to determine when these circumstances are satisfied in practice is still a difficult problem. We discussed, for example, how to detect monopolistic behavior and how to identify symptoms of targeting problems. However, we also emphasized that there are frequently confounding factors that make it more difficult to identify such circumstances.

Another important research area is how to implement Pareto-improving cooperation in practice. Willett (1999) discusses a range of political economy problems that make international policy cooperation difficult. Bagwell and Staiger (2002), among others, provide an analysis of how to achieve agreements to abstain from monopolistic beggar-thy-neighbor policies if countries are sufficiently symmetric. A careful treatment of this question is beyond the scope of this article.

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A Mathematical Appendix

A.1 Bond Supply in Example 1

We denote the optimization problem of an individual consumer with bond holdings b_t^i in recursive form as

$$V_t^i\left(b_t^i\right) = \max_{b_{t+1}^i} u\left(y_t^i + b_t^i - \left(1-\tau_{t+1}^i\right) b_{t+1}^i / R_{t+1} - T_{t+1}^i\right) + \beta V_{t+1}^i\left(b_{t+1}^i\right)$$

and observe that the standard properties $V_{t+1}^{i\prime}\left(b_{t+1}^i\right) > 0 > V_{t+1}^{i\prime\prime}\left(b_{t+1}^i\right)$ hold. For given b_t^i and after substituting the government budget constraint, the Euler equation that results from this problem defines an implicit function

$$F\left(b_{t+1}^i, \tau_{t+1}^i, R_{t+1}\right) = \left(1-\tau_{t+1}^i\right) u'\left(y_t^i + b_t^i - b_{t+1}^i/R_{t+1}\right) - \beta R_{t+1} V_{t+1}^{i\prime}\left(b_{t+1}^i\right) = 0$$

$$\text{which satisfies} \quad \begin{aligned} \frac{\partial F}{\partial b_{t+1}^i} &= -\left(1-\tau_{t+1}^i\right) u''\left(c_t^i\right)/R_{t+1} - \beta R_{t+1} V_{t+1}^{i\prime\prime}\left(b_{t+1}^i\right) > 0 \\ \frac{\partial F}{\partial \tau_{t+1}^i} &= -u'\left(c_t^i\right) < 0 \\ \frac{\partial F}{\partial R_{t+1}} &= \left(1-\tau_{t+1}^i\right) u''\left(c_t^i\right) \cdot b_{t+1}^i / \left(R_{t+1}\right)^2 - \beta V_{t+1}^{i\prime}\left(b_{t+1}^i\right) \gtrless 0 \end{aligned}$$

The first partial derivative is always positive, allowing us to implicitly define a demand function $b_{t+1}^i\left(R_{t+1}, \tau_{t+1}\right)$. The second partial derivative is always negative – this is because we assumed that the revenue from capital controls is rebated so that there are only substitution effects and no income effects from capital controls.

The third partial derivative is negative as long as saving b_{t+1}^i is sufficiently high. Specifically, we re-write the condition $\partial F/\partial R_{t+1} < 0$ by employing the Euler equation to substitute for the second term and rearrange to

$$\text{or} \quad \frac{b_{t+1}^i/R_{t+1}}{c_t^i} > \frac{u'\left(c_t^i\right)}{c_t^i u''\left(c_t^i\right)} = -\sigma\left(c_t^i\right)$$

i.e. the savings/consumption ratio is greater than the negative of the elasticity of intertemporal substitution $\sigma\left(c^i\right)$, as we stated in Assumption **??**. If this inequality is satisfied then for given b_t^i, the supply function $b_{t+1}^i\left(R_{t+1}, \tau_{t+1}\right)$ is strictly increasing in R_{t+1}, which allows us to invert it into a strictly increasing inverse supply function $R_{t+1}^i\left(b_{t+1}^i, \tau_{t+1}\right)$.

Expressions in Numerical Illustration If we parameterize the model as discussed in the numerical illustration of section 2 and consider these expressions in the no-intervention steady-state in which $\beta R_{t+1} = 1$, $\tau^i_{t+1} = 0$, $b^i_{t+1} = 0$ and $y^i_t = y^i = c^i_t \forall i, t$, then we find $V'\left(b^i_{t+1}\right) = u'\left(c^i\right)$ and $V''\left(b^i_{t+1}\right) = (1-\beta)\, u''\left(c^i\right)$. As a result the partial derivative $\partial\left(b^i/y\right)/\partial\tau = \sigma$ and $\partial\left(b^i/y\right)/\partial\tau = \beta\sigma$.

A.2 Combined Optimization Problem [online]

Proof of Corollary 1 We describe the Lagrangian of the combined optimization problem of an individual agent (while suppressing the vector of exogenous parameters Z^i) as

$$w^i\left(m^i, x^i; M^i, X^i, \zeta^i, T^i\right) = \max_{m^i, x^i} U^i\left(x^i\right) \qquad -\lambda^i_d f^i\left(m^i, x^i, M^i, X^i, \zeta^i\right) - \\ -\lambda^i_e\left[\left(1-\xi^i\right) Q^i \cdot m^i - b^i_0 - T^i\right]$$

The optimality conditions are given by the vector equations

$$\begin{aligned} FOC\left(x^i\right) &: U^i_x = \lambda^i_d f^i_x \\ FOC\left(m^i\right) &: \lambda^i_d f^i_m + \lambda^i_e\left(1-\xi^i\right) Q = 0 \end{aligned}$$

These two conditions generate domestic and external implementability constraints on the problem of the combined planning problem, which is given by the Lagrangian

$$W^i\left(M^i, X^i, \zeta^i, \xi^i\right) = \max_{M^i, X^i, \zeta^i, \xi^i, \lambda^i_d, \lambda^i_e} U^i\left(X^i\right) \qquad -\Lambda^i_d f^i\left(M^i, X^i, M^i, X^i, \zeta^i\right) - \quad (A.1)$$

$$-\Lambda^i_e\left[Q \cdot m^i - b^i_0\right] - \quad (A.2)$$

$$-\mu^i_d\left[U^i_x - \lambda^i_d f^i_x\right] - \quad (A.3)$$

$$-\mu^i_e\left[\lambda^i_d f^i_m + \lambda^i_e\left(1-\xi^i\right) Q\right] \quad (A.4)$$

and delivers the associated optimality conditions

$$\begin{aligned} FOC\left(X^i\right) &: U^i_x = \Lambda^i_d\left(f^i_x + f^i_X\right) + \mu^i_d\left[U^i_{xx} - \lambda^i_d\left(f^i_{xx} + f^i_{xX}\right)\right] + \mu^i_e\lambda^i_d\left(f^i_{mx} + f^i_{mX}\right) && (A.5) \\ FOC\left(M^i\right) &: 0 = \Lambda^i_d\left(f^i_m + f^i_M\right) + \Lambda^i_e Q - \mu^i_d\lambda^i_d\left(f^i_{xm} + f^i_{xM}\right) + \mu^i_e\lambda^i_d\left(f^i_{mm} + f^i_{mM}\right) && (A.6) \\ FOC\left(\zeta^i\right) &: 0 = \Lambda^i_d f^i_\zeta - \lambda_d\mu^i_d f^i_{x\zeta} + \mu^i_e\lambda^i_d f^i_{m\zeta} && (A.7) \\ FOC\left(\xi^i\right) &: 0 = \mu^i_e\lambda^i_e Q && (A.8) \\ FOC\left(\lambda^i_d\right) &: 0 = -\mu^i_d f^i_x + \mu^i_e f^i_m && (A.9) \\ FOC\left(\lambda^i_e\right) &: 0 = \mu^i_e\left(1-\xi^i\right) Q && (A.10) \end{aligned}$$

Given the complete set of external instruments ξ^i, the system of optimality conditions $FOC\left(M^i, \xi^i, \lambda^i_e\right)$ implies that the vector of shadow prices on the external implementability constraint satisfies $\mu^i_e = 0$ – the planner sets the vector ξ^i to whichever levels she wants without facing trade-offs. By implication, the last term in the other five optimality conditions drops out, allowing us to separate the problem into two blocks.

The optimality conditions (A.5), (A.7) and (A.9) with $\mu_e^i = 0$ replicate the optimality conditions of the optimal domestic planning problem (7) in section 3.1. Together with the domestic constraint (A.1) and the domestic implementability condition (A.3), these five conditions are identical to the five conditions that pin down the optimal domestic allocation for given M^i in section 3.1 and yield identical solutions for the five domestic variables $\left(X^i, \zeta^i, \lambda_d^i, \Lambda_d^i, \mu_d^i\right)$.

Given the envelope theorem, the optimality condition (A.6) can equivalently be written as $\partial W^i / \partial M^i = dW^i / dM^i = 0$, where the latter condition coincides with the optimality condition (12) defining the optimal external allocation in section 3.2. The optimality conditions (A.8) and (A.10) for $\mu_e^i = 0$ capture that the planner can set the product $\lambda_e^i \left(1 - \xi^i\right)$ such as to precisely meet the constraints (A.4) where the planner has one scalar degree of freedom, as we also emphasized in section 3.2. The three optimality conditions together with the two constraints yield an identical set of solutions for the five external variables $\left(M^i, \xi^i, \lambda_e^i, \Lambda_e^i, \mu_e^i\right)$ as we described in section 3.2. This shows that the two-step procedure described in section 2 yields identical solutions as the combined optimization problem described here, proving the corollary.